CUET 2026 May 18 Shift 2 Mathematics

Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. Match the following integrals in Column I with their corresponding results in Column II:

Column I	Column II
A. $\int \frac{dx}{x^2 - a^2}$	I. $\frac{1}{2a} \log\left\|\frac{a+x}{a-x}\right\| + C$
B. $\int \frac{dx}{a^2 - x^2}$	II. $\log\left\|x + \sqrt{x^2 - a^2}\right\| + C$
C. $\int \frac{dx}{\sqrt{x^2 - a^2}}$	III. $\frac{1}{2a} \log\left\|\frac{x-a}{x+a}\right\| + C$
D. $\int \frac{dx}{\sqrt{a^2 - x^2}}$	IV. $\sin^{-1}\frac{x}{a} + C$

(A) $A - I,\ B - III,\ C - IV,\ D - II$

(B) $A - III,\ B - I,\ C - II,\ D - IV$

(C) $A - III,\ B - II,\ C - I,\ D - IV$

(D) $A - IV,\ B - II,\ C - III,\ D - I$

Correct Answer: (B) $A - III,\ B - I,\ C - II,\ D - IV$

Solution:

Concept:

This problem is based on standard integrals involving algebraic expressions of the forms:

$$x^2 - a^2, \qquad a^2 - x^2, \qquad \sqrt{x^2 - a^2}, \qquad \sqrt{a^2 - x^2}$$

These are extremely important standard results in Integral Calculus and are frequently used in competitive examinations such as CUET, JEE Main, and JEE Advanced.

The important standard integrals used in this question are:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

We now match each integral carefully one by one.

Step 1: Matching Integral A

We are given:

$$A = \int \frac{dx}{x^2 - a^2}$$

Recall the standard result:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$$

This exactly matches expression III.

Therefore,

$$A \longrightarrow III$$

Step 2: Matching Integral B

We are given:

$$B = \int \frac{dx}{a^2 - x^2}$$

Using the standard formula:

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

This matches expression I.

Hence,

$$B \longrightarrow I$$

Step 3: Matching Integral C

We are given:

$$C = \int \frac{dx}{\sqrt{x^2 - a^2}}$$

Using the standard integral:

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

This corresponds exactly to expression II.

Therefore,

$$C \longrightarrow II$$

Step 4: Matching Integral D

We are given:

$$D = \int \frac{dx}{\sqrt{a^2 - x^2}}$$

Using the standard trigonometric integral formula:

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

This matches expression IV.

Hence,

$$D \longrightarrow IV$$

Step 5: Writing the final matching

Collecting all the obtained matches:

$$A \rightarrow III$$

$$B \rightarrow I$$

$$C \rightarrow II$$

$$D \rightarrow IV$$

Thus the correct option is:

$$\boxed{(B)\, A-III,\ B-I,\ C-II,\ D-IV}$$

Quick Tip: Always memorize the following four standard integrals together because they are closely related and frequently asked in matching-type questions:

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$$

These formulas are among the most repeated results in indefinite integration problems.

2. If $R = \{(x, y) \mid x, y \in \mathbb{Z},\ x^2 + y^2 \leq 4\}$ is a relation in $\mathbb{Z}$, then the domain of R is:

(A) $\{0, 1, 2\}$

(B) $\{0, -1, -2\}$

(C) $\{-2, -1, 0, 1, 2\}$

(D) $\{-1, 0, 2\}$

Correct Answer: (C) $\{-2, -1, 0, 1, 2\}$

Solution:

Concept:

A relation R from a set A to a set B is a collection of ordered pairs.

The **domain** of a relation consists of all first elements of the ordered pairs belonging to the relation.

If

$$R = \{(x, y) \mid \text{condition on } x \text{ and } y\}$$

then the domain is the set of all values of x for which at least one value of y exists satisfying the given condition.

In this problem:

$$R = \{(x, y) \mid x, y \in \mathbb{Z},\ x^2 + y^2 \leq 4\}$$

Since both x and y are integers, we must find all integer values of x for which there exists at least one integer y satisfying:

$$x^2 + y^2 \leq 4$$

Step 1: Understanding the given inequality

We are given:

$$x^2 + y^2 \leq 4$$

Since squares are always non-negative,

$$x^2 \leq 4$$

because $y^2 \geq 0$.
Now solve:

$$x^2 \leq 4$$

Taking square roots,

$$-2 \leq x \leq 2$$

Since $x \in \mathbb{Z}$, the possible integer values are:

$$x = -2, -1, 0, 1, 2$$

Now we must check whether for each of these values there exists at least one integer value of y satisfying the inequality.

Step 2: Checking each possible value of x

$\underline{x = -2}$

Substituting into the inequality:

$$(-2)^2 + y^2 \leq 4$$

$$4 + y^2 \leq 4$$

$$y^2 \leq 0$$

Thus,

$$y = 0$$

which is an integer.
Hence, $x = -2$ belongs to the domain.

$\underline{x = -1}$

$$(-1)^2 + y^2 \le 4$$

$$1 + y^2 \le 4$$

$$y^2 \le 3$$

Possible integer values:

$$y = -1, 0, 1$$

Hence, at least one integer y exists.
Therefore, $x = -1$ belongs to the domain.

$\underline{x = 0}$

$$0^2 + y^2 \le 4$$

$$y^2 \le 4$$

Possible integer values:

$$y = -2, -1, 0, 1, 2$$

Hence, $x = 0$ belongs to the domain.

$\underline{x = 1}$

$$1 + y^2 \le 4$$

$$y^2 \le 3$$

Possible integer values:

$$y = -1, 0, 1$$

Thus, $x = 1$ belongs to the domain.

$\underline{x = 2}$

$$2^2 + y^2 \leq 4$$

$$4 + y^2 \leq 4$$

$$y^2 \leq 0$$

Hence,

$$y = 0$$

Therefore, $x = 2$ also belongs to the domain.

Step 3: Writing the domain

All possible values of x are:

$$\{-2, -1, 0, 1, 2\}$$

Therefore, the domain of the relation R is:

$$\boxed{\{-2, -1, 0, 1, 2\}}$$

Hence, the correct option is:

$$\boxed{(C)}$$

Quick Tip: To find the domain of a relation defined by an equation or inequality involving x and y:

- Focus on the first coordinate x.
- Find all values of x for which at least one valid value of y exists.
- For inequalities involving squares, use the fact that:

$$x^2 \geq 0, \qquad y^2 \geq 0$$

to simplify the range quickly.

In integer-based relations, always check whether the required values are integers.

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3. If $R = \{(x, y) \mid x, y \in \mathbb{R},\ x^2 + y^2 = 1\}$ is a relation in $\mathbb{R}$, then R is:

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Equivalence Relation

Correct Answer: (B) Symmetric

Solution:

Concept:

A relation R on a set A may possess different properties such as reflexivity, symmetry, and transitivity.

The important definitions are:

- **Reflexive Relation:** A relation R is reflexive if:

$$(x, x) \in R \quad \text{for every } x \in A$$

- **Symmetric Relation:** A relation R is symmetric if:

$$(x, y) \in R \Rightarrow (y, x) \in R$$

- **Transitive Relation:** A relation R is transitive if:

$$(x, y) \in R \text{ and } (y, z) \in R \Rightarrow (x, z) \in R$$

- **Equivalence Relation:** A relation is called an equivalence relation if it is simultaneously:

$$\text{reflexive, symmetric, and transitive}$$

We now check each property carefully for the given relation.

Step 1: Understanding the given relation

The relation is:

$$R = \{(x, y) \mid x, y \in \mathbb{R},\ x^2 + y^2 = 1\}$$

This means an ordered pair (x, y) belongs to R only when:

$$x^2 + y^2 = 1$$

Geometrically, this represents all points lying on the unit circle centered at the origin.

Step 2: Checking whether the relation is reflexive

For reflexivity, we must have:

$$(x, x) \in R \quad \text{for every } x \in \mathbb{R}$$

Substitute $y = x$ into the relation:

$$x^2 + x^2 = 1$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\frac{1}{\sqrt{2}}$$

Thus, only two specific real numbers satisfy the condition.

But reflexivity requires the condition to hold for *every* real number.

For example, take $x = 0$:

$$0^2 + 0^2 = 0 \neq 1$$

Hence,

$$(0,0) \notin R$$

Therefore, the relation is **not reflexive**.

Step 3: Checking whether the relation is symmetric

Suppose:

$$(x,y) \in R$$

Then by definition,

$$x^2 + y^2 = 1$$

Now interchange x and y:

$$y^2 + x^2 = 1$$

Since addition is commutative,

$$y^2 + x^2 = x^2 + y^2$$

Therefore,

$$(y,x) \in R$$

Hence, whenever $(x,y) \in R$, we also have $(y,x) \in R$.

Therefore, the relation is **symmetric**.

Step 4: Checking whether the relation is transitive

For transitivity, we require:

$$(x, y) \in R \text{ and } (y, z) \in R \Rightarrow (x, z) \in R$$

We test this using a counterexample.
Take:

$$(x, y) = (1, 0)$$

Then,

$$1^2 + 0^2 = 1$$

Hence,

$$(1, 0) \in R$$

Now take:

$$(y, z) = (0, 1)$$

Then,

$$0^2 + 1^2 = 1$$

Thus,

$$(0, 1) \in R$$

Now check whether:

$$(1, 1) \in R$$

We compute:

$$1^2 + 1^2 = 2$$

Since:

$$2 \neq 1$$

we get:

$$(1,1) \notin R$$

Thus, the transitive condition fails.

Therefore, the relation is **not transitive**.

Step 5: Checking whether it is an equivalence relation

An equivalence relation must be:

$$\text{reflexive} + \text{symmetric} + \text{transitive}$$

But this relation is only symmetric and is neither reflexive nor transitive.

Hence, it is **not an equivalence relation**.

Final Conclusion:

The given relation satisfies only the symmetric property.

Therefore, the correct answer is:

$$\boxed{(B) \text{ Symmetric}}$$

Quick Tip: For relation-property questions:

- Reflexive $\Rightarrow$ check whether (x,x) belongs to the relation for all elements.
- Symmetric $\Rightarrow$ interchange x and y and verify whether the condition remains unchanged.
- Transitive $\Rightarrow$ usually test with a counterexample.
- Equivalence relation requires all three properties simultaneously.

Relations involving expressions like $x^2 + y^2$ are often symmetric because swapping variables does not change the equation.

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4. The value of $\cot^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ **is:**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{2\pi}{3}$
(D) $\frac{\pi}{3}$

Correct Answer: (D) $\frac{\pi}{3}$

Solution:

Concept:
This problem is based on inverse trigonometric functions together with standard trigonometric identities.
The important identities used are:

$$\cos 2\theta = 1 - 2\sin^2\theta$$

and

$$\cot^{-1} x = \theta \iff \cot\theta = x$$

where the principal value of $\cot^{-1} x$ lies in:

$$(0, \pi)$$

The strategy is:

- First simplify the inverse trigonometric expression.
- Then evaluate the trigonometric function inside the bracket.
- Finally determine the angle whose cotangent gives the obtained value.

Step 1: Evaluating the inverse sine expression
We are given:

$$\sin^{-1}\frac{1}{2}$$

We know that:

$$\sin\frac{\pi}{6} = \frac{1}{2}$$

Therefore,

$$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

Substituting this into the given expression:

$$\cot^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$$

$$= \cot^{-1}\left[2\cos\frac{\pi}{3}\right]$$

Step 2: Evaluating the cosine value

We know:

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

Therefore,

$$2\cos\frac{\pi}{3} = 2 \times \frac{1}{2} = 1$$

Thus the expression becomes:

$$\cot^{-1}(1)$$

Step 3: Finding the angle whose cotangent is 1

We now determine the principal value angle θ such that:

$$\cot\theta = 1$$

We know:

$$\cot\frac{\pi}{4} = 1$$

Therefore,

$$\cot^{-1}(1) = \frac{\pi}{4}$$

Step 4: Comparing with the options

Hence the value of the given expression is:

$$\boxed{\frac{\pi}{4}}$$

Therefore, the correct option is:

$$\boxed{(B)}$$

Quick Tip: Whenever expressions involve inverse trigonometric functions:

- First convert the inverse trigonometric quantity into a standard angle.
- Then simplify step-by-step using standard trigonometric values.
- Remember principal values:

$$\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cot^{-1} x \in (0, \pi)$$

Important standard values:

$$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\cot^{-1}(1) = \frac{\pi}{4}$$

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5. The value of $\cos\left(\frac{\pi}{6} - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ is equal to:

(A) $-\frac{\sqrt{1}}{2}$

(B) $-\frac{1}{\sqrt{2}}$

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Correct Answer: (C) $-\frac{1}{2}$

Solution:

Concept:

This problem involves inverse trigonometric functions together with compound angle identities.

The main ideas used are:

- Evaluating the principal value of an inverse cosine function.
- Using the cosine subtraction identity:

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

- Using standard trigonometric values.

For inverse cosine:

$$\cos^{-1} x \in [0, \pi]$$

Hence we must always choose the angle lying in this interval.

Step 1: Evaluating the inverse cosine term

We are given:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

We know that:

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

But the value inside the inverse cosine is negative:

$$-\frac{\sqrt{3}}{2}$$

Now cosine is negative in the second quadrant.

The angle in the interval $[0, \pi]$ whose cosine is:

$$-\frac{\sqrt{3}}{2}$$

is:

$$\frac{5\pi}{6}$$

Therefore,

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

Substituting into the given expression:

$$\cos\left(\frac{\pi}{6} - \frac{5\pi}{6}\right)$$

Step 2: Simplifying the angle

Subtract the fractions:

$$\frac{\pi}{6} - \frac{5\pi}{6} = -\frac{4\pi}{6} = -\frac{2\pi}{3}$$

Thus the expression becomes:

$$\cos\left(-\frac{2\pi}{3}\right)$$

Step 3: Using the even property of cosine

We know:

$$\cos(-\theta) = \cos\theta$$

Therefore,

$$\cos\left(-\frac{2\pi}{3}\right) = \cos\frac{2\pi}{3}$$

Now,

$$\cos\frac{2\pi}{3} = -\frac{1}{2}$$

Hence,

$$\cos\left(\frac{\pi}{6} - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = -\frac{1}{2}$$

Final Answer:

$$\boxed{-\frac{1}{2}}$$

Therefore, the correct option is:

$$\boxed{(C)}$$

Quick Tip: Always remember the principal value range of inverse cosine:

$$\cos^{-1} x \in [0, \pi]$$

When cosine is negative, the angle usually lies in the second quadrant.

Important standard values:

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

Also remember that cosine is an even function:

$$\cos(-\theta) = \cos\theta$$

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6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$**, then:**

(A) $A^{-1} = \frac{1}{11}A$

(B) $A^{-1} = \frac{1}{19}A$

(C) $A^{-1} = -\frac{1}{19}A$

(D) $A^{-1} = \frac{1}{7}A$

Correct Answer: (C) $A^{-1} = -\frac{1}{19}A$

Solution:

Concept:

For any square matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

its inverse exists only when:

$$|A| = ad - bc \neq 0$$

The inverse of a 2×2 matrix is given by:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Another important idea used in this question is recognizing whether the adjoint matrix becomes a scalar multiple of the original matrix itself.

We now solve the question carefully step-by-step.

Step 1: Writing the given matrix

We are given:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

We first compute the determinant of A.

Step 2: Finding the determinant of the matrix

For a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant is:

$$|A| = ad - bc$$

Here,

$$a = 2, \quad b = 3, \quad c = 5, \quad d = -2$$

Therefore,

$$|A| = (2)(-2) - (3)(5)$$

$$= -4 - 15$$

$$= -19$$

Since:

$$|A| = -19 \neq 0$$

the inverse exists.

Step 3: Finding the inverse using the standard formula

Using:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

we get:

$$A^{-1} = \frac{1}{-19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

Now take negative sign common from the matrix:

$$= -\frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

But the matrix inside the bracket is exactly A.

Hence,

$$A^{-1} = -\frac{1}{19}A$$

Step 4: Comparing with the given options

We obtained:

$$A^{-1} = -\frac{1}{19}A$$

This matches option (C).

Final Answer:

$$\boxed{A^{-1} = -\frac{1}{19}A}$$

Therefore, the correct option is:

$$\boxed{(C)}$$

Quick Tip: For a 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

always remember:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A shortcut for objective questions:

- Compute the determinant first.
- Check whether the adjoint matrix is proportional to the original matrix.
- This often allows very quick matching with the options.

Also note:

$$AA^{-1} = I$$

where I is the identity matrix.

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7. The value of λ for which the following system of equations has unique solution:

$$\lambda x + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-\lambda x + y + 2z = -1$$

are:

(A) $\lambda \neq \frac{5}{2}$

(B) $\lambda \neq \frac{3}{2}$

(C) $\lambda \neq \frac{7}{2}$

(D) $\lambda \neq -\frac{7}{2}$

Correct Answer: (A) $\lambda \neq \frac{5}{2}$

Solution:

Concept:
A system of linear equations has a unique solution if and only if the determinant of its coefficient matrix is non-zero.
For a system:

$$AX = B$$

the condition for unique solution is:

$$|A| \neq 0$$

where A is the coefficient matrix.
If:

$$|A| = 0$$

then the system may have either infinitely many solutions or no solution.
Thus, in this problem, we first form the coefficient matrix and then compute its determinant.

Step 1: Writing the coefficient matrix
The given system is:

$$\lambda x + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-\lambda x + y + 2z = -1$$

Therefore, the coefficient matrix is:

$$A = \begin{bmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{bmatrix}$$

For a unique solution:

$$|A| \neq 0$$

Step 2: Evaluating the determinant

Compute:

$$|A| = \begin{vmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{vmatrix}$$

Expand along the first row:

$$|A| = \lambda \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ -\lambda & 2 \end{vmatrix} + (-1) \begin{vmatrix} 1 & 2 \\ -\lambda & 1 \end{vmatrix}$$

Now evaluate each minor carefully.

<u>First Minor</u>

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = (2)(2) - (1)(1)$$

$$= 4 - 1$$

$$= 3$$

Thus first term becomes:

$$\lambda(3) = 3\lambda$$

<u>Second Minor</u>

$$\begin{vmatrix} 1 & 1 \\ -\lambda & 2 \end{vmatrix} = (1)(2) - (1)(-\lambda)$$

$$= 2 + \lambda$$

Thus second term becomes:

$$-3(2 + \lambda)$$

$$= -6 - 3\lambda$$

<u>Third Minor</u>

$$\begin{vmatrix} 1 & 2 \\ -\lambda & 1 \end{vmatrix} = (1)(1) - (2)(-\lambda)$$

$$= 1 + 2\lambda$$

Thus third term becomes:

$$(-1)(1 + 2\lambda)$$

$$= -1 - 2\lambda$$

Step 3: Combining all terms

Adding all three parts:

$$|A| = 3\lambda + (-6 - 3\lambda) + (-1 - 2\lambda)$$

$$= 3\lambda - 6 - 3\lambda - 1 - 2\lambda$$

$$= -7 - 2\lambda$$

$$= -(2\lambda + 7)$$

Step 4: Applying the condition for unique solution

For unique solution:

$$|A| \neq 0$$

Therefore,

$$-(2\lambda + 7) \neq 0$$

$$2\lambda + 7 \neq 0$$

$$2\lambda \neq -7$$

$$\lambda \neq -\frac{7}{2}$$

Final Answer:

$$\boxed{\lambda \neq -\frac{7}{2}}$$

Hence, the correct option is:

$$\boxed{(D)}$$

Quick Tip: For a system of linear equations:

$$AX = B$$

- Unique solution exists when:

$$|A| \neq 0$$

- No unique solution occurs when:

$$|A| = 0$$

In determinant expansion:

$$\begin{vmatrix} a & b & c \end{vmatrix}$$

always remember the sign pattern:

$$+ \quad - \quad +$$

while expanding along a row or column.

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8. If B is a non-singular 4×4 matrix and A is its adjoint such that $|A| = 125$, then $|B|$ is:

(A) 5

(B) 25

(C) 125

(D) 625

Correct Answer: (A) 5

Solution:

Concept:

For a square matrix B of order n, an important property relating the determinant of a matrix and the determinant of its adjoint is:

$$|\operatorname{adj}(B)| = |B|^{n-1}$$

where:

- $\text{adj}(B)$ denotes the adjoint (adjugate) of matrix B,
- n is the order of the matrix.

This is one of the most important determinant properties used in matrix algebra.
Since the matrix is non-singular:

$$|B| \neq 0$$

Therefore all determinant properties are valid.

Step 1: Identifying the order of the matrix
We are given that B is a:

$$4 \times 4$$

matrix.
Hence the order of the matrix is:

$$n = 4$$

Also A is the adjoint of B.
Thus:

$$A = \text{adj}(B)$$

We are given:

$$|A| = 125$$

Step 2: Using the determinant property of adjoint matrix
The standard formula is:

$$|\text{adj}(B)| = |B|^{n-1}$$

Substituting $n = 4$:

$$|\text{adj}(B)| = |B|^{4-1}$$

$$|\operatorname{adj}(B)| = |B|^3$$

But:

$$A = \operatorname{adj}(B)$$

Therefore:

$$|A| = |B|^3$$

Since $|A| = 125$, we get:

$$125 = |B|^3$$

Step 3: Finding $|B|$

Taking cube root on both sides:

$$|B| = \sqrt[3]{125}$$

We know:

$$125 = 5^3$$

Therefore:

$$|B| = 5$$

Final Answer:

$$\boxed{|B| = 5}$$

Hence, the correct option is:

$$\boxed{(A)}$$

Quick Tip: For a square matrix A of order n, always remember the important identity:

$$|\operatorname{adj}(A)| = |A|^{n-1}$$

Special cases:

$$2 \times 2 \Rightarrow |\operatorname{adj}(A)| = |A|$$

$$3 \times 3 \Rightarrow |\operatorname{adj}(A)| = |A|^2$$

$$4 \times 4 \Rightarrow |\operatorname{adj}(A)| = |A|^3$$

This formula is extremely important in objective matrix questions.

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9. If A is a square matrix of order 3 such that $|2(\operatorname{adj} A)| = 288$, then the value of $|A|$ is:

(A) 144

(B) 36

(C) ± 12

(D) ± 6

Correct Answer: (D) ± 6

Solution:

Concept:
This problem uses important determinant properties involving scalar multiplication and adjoint matrices.
The two standard results required are:

- If A is a square matrix of order n, then:

$$|kA| = k^n |A|$$

where k is a scalar.

- For a square matrix A of order n:

$$|\operatorname{adj}(A)| = |A|^{n-1}$$

Since the matrix is of order 3, we will substitute $n = 3$ carefully into these formulas.

Step 1: Using the determinant property for scalar multiplication

We are given:

$$|2(\operatorname{adj}A)| = 288$$

Since A is a matrix of order 3, the adjoint matrix adj(A) is also of order 3.

Using:

$$|kB| = k^n|B|$$

for an $n \times n$ matrix B, we get:

$$|2(\operatorname{adj}A)| = 2^3|\operatorname{adj}A|$$

because the order is 3.

Therefore,

$$8|\operatorname{adj}A| = 288$$

Step 2: Finding the determinant of the adjoint matrix

Divide both sides by 8:

$$|\operatorname{adj}A| = \frac{288}{8}$$

$$= 36$$

Thus,

$$|\operatorname{adj}A| = 36$$

Step 3: Using the adjoint determinant property

For a matrix of order n:

$$|\operatorname{adj} A| = |A|^{n-1}$$

Since the order is 3:

$$|\operatorname{adj} A| = |A|^{3-1}$$

$$|\operatorname{adj} A| = |A|^{2}$$

But we already found:

$$|\operatorname{adj} A| = 36$$

Hence,

$$|A|^2 = 36$$

Step 4: Finding the value of $|A|$

Taking square root on both sides:

$$|A| = \pm\sqrt{36}$$

$$|A| = \pm 6$$

Final Answer:

$$\boxed{|A| = \pm 6}$$

Therefore, the correct option is:

$$\boxed{(D)}$$

Quick Tip: Always remember these two very important determinant properties:

$$|kA| = k^n|A|$$

where n is the order of the matrix.

Also,

$$|\operatorname{adj}(A)| = |A|^{n-1}$$

For a 3×3 matrix specifically:

$$|\operatorname{adj}(A)| = |A|^2$$

These formulas are extremely common in determinant-based objective questions.

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10. If A is a square matrix of order 3 and $|A| = -3$, then the value of $|2AA^T|$ is:

(A) -36

(B) 72

(C) -72

(D) 36

Correct Answer: (B) 72

Solution:

Concept:

This problem is based on important determinant properties involving:

- Scalar multiplication of matrices,
- Product of matrices,
- Determinant of transpose matrices.

The important formulas used are:

$$|AB| = |A||B|$$

$$|A^T| = |A|$$

and for an $n \times n$ matrix:

$$|kA| = k^n |A|$$

where n is the order of the matrix.

Since the matrix is of order 3, the scalar multiplication factor will be raised to power 3.

Step 1: Writing the given information

We are given:

$$|A| = -3$$

We need to find:

$$|2AA^T|$$

Since A is of order 3, the matrix AA^T is also of order 3.

Step 2: Using the scalar multiplication determinant property

Using:

$$|kB| = k^n |B|$$

for an $n \times n$ matrix.

Here:

$$n = 3, \qquad k = 2$$

Therefore,

$$|2AA^T| = 2^3 |AA^T|$$

$$= 8|AA^T|$$

Step 3: Evaluating $|AA^T|$

Using the determinant property of product:

$$|AA^T| = |A||A^T|$$

Also, determinant of transpose equals determinant of the original matrix:

$$|A^T| = |A|$$

Hence,

$$|AA^T| = |A||A|$$

$$= |A|^2$$

Given:

$$|A| = -3$$

Therefore,

$$|AA^T| = (-3)^2$$

$$= 9$$

Step 4: Substituting into the required expression

We found:

$$|2AA^T| = 8|AA^T|$$

and

$$|AA^T| = 9$$

Therefore,

$$|2AA^T| = 8 \times 9$$

$$= 72$$

Final Answer:

$$\boxed{72}$$

Therefore, the correct option is:

$$\boxed{(B)}$$

Quick Tip: For determinant problems involving transpose and products, remember these key formulas:

$$|AB| = |A||B|$$

$$|A^T| = |A|$$

$$|AA^T| = |A|^2$$

Also, for an $n \times n$ matrix:

$$|kA| = k^n|A|$$

For a 3×3 matrix specifically:

$$|2A| = 2^3|A| = 8|A|$$
