CUET 2026 May 20 Mathematics Shift 2

Question Paper (Memory-Based) With Solution

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. Find the integrating factor (I.F.) for the linear differential equation:

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$$

(A) $1+x^2$

(B) $\ln(1+x^2)$

(C) $\frac{1}{1+x^2}$

(D) e^{x^2}

Correct Answer: (A) $1+x^2$

Solution:

Concept: A first-order linear differential equation written in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$ can be solved by multiplying through by an Integrating Factor (I.F.), which is defined as:

$$\text{I.F.} = e^{\int P(x)\,dx}$$

Step 1: Identify the function $P(x)$ from the given differential equation.

By comparing our equation directly to the standard linear layout, we find the coefficient function of y:

$$P(x) = \frac{2x}{1+x^2}$$

Step 2: Integrate $P(x)$ with respect to x.

To evaluate $\int \frac{2x}{1+x^2}\,dx$, we use integration by substitution. Let $u = 1 + x^2$, which means its differential is $du = 2x\,dx$:

$$\int \frac{2x}{1+x^2}\,dx = \int \frac{du}{u} = \ln|u| = \ln(1+x^2)$$

Step 3: Raise the integrated function as a power of base e.

Substitute the result back into the exponential format to finalize our integrating factor:

$$\text{I.F.} = e^{\ln(1+x^2)}$$

Using the fundamental logarithmic identity $e^{\ln(f(x))} = f(x)$, the expression simplifies perfectly:

$$\text{I.F.} = 1 + x^2$$

Quick Tip: Whenever the numerator is the exact derivative of the denominator inside an exponent, the integrating factor simplifies directly to the denominator function itself: $e^{\int \frac{f'(x)}{f(x)}\,dx} = e^{\ln|f(x)|} = f(x)$.

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2. Determine the sum of the order and the degree of the following differential equation:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{frac32} = k\frac{d^2y}{dx^2}$$

(A) 4

(B) 3

(C) 5

(D) The degree is undefined.

Correct Answer: (A) 4

Solution:

Concept: The **order** of a differential equation is the order of the highest derivative present in the equation. The **degree** is the power of the highest-order derivative when the differential equation is written as a polynomial in its derivatives. This means all fractional exponents and radicals affecting the derivative terms must be cleared first.

Step 1: Clear the fractional radical exponent from the expression.

The equation contains a fractional exponent of $\frac{3}{2}$ on the left side. To turn this into a standard polynomial form, we square both sides of the entire equation:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}\right)^2 = \left(k\frac{d^2y}{dx^2}\right)^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$$

Step 2: Identify the order and degree from the rationalized equation.

1. Look for the highest-order derivative present: The term $\frac{d^2y}{dx^2}$ is a second-order derivative, so:

$$\text{Order} = 2$$

2. Look for the exponent power attached to this highest-order derivative: The term $\left(\frac{d^2y}{dx^2}\right)$ is raised to the power of 2, so:

$$\text{Degree} = 2$$

Step 3: Calculate the sum of the two metrics.

Adding our isolated values together:

$$\text{Sum} = \text{Order} + \text{Degree} = 2 + 2 = 4$$

Quick Tip: Never find the degree while fractional roots are still visible in the equation. Always clear out denominators in powers by raising the entire equation to the necessary scaling integer value first.

3. Find the area of the region enclosed by the ellipse given parametrically by the coordinates $x = 2\sin\theta$ and $y = 3\cos\theta$ where $0 \le \theta \le 2\pi$.

(A) 6π

(B) 12π

(C) 5π

(D) 13π

Correct Answer: (A) 6π

Solution:

Concept: The total area bounded by a standard Cartesian ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the formula:

$$\text{Area} = \pi a b$$

where a and b represent the lengths of the semi-major and semi-minor axes respectively.

Step 1: Convert the parametric equations into standard Cartesian form.

We are given the parametric coordinates:

$$x = 2\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{x}{2}$$

$$y = 3\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{y}{3}$$

Using the fundamental trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, we substitute our fractional expressions into the equation:

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1 \quad \Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{9} = 1$$

Step 2: Extract the lengths of the semi-axes.

Comparing our derived Cartesian equation to the standard ellipse layout $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

$$a^2 = 4 \quad \Rightarrow \quad a = 2$$

$$b^2 = 9 \quad \Rightarrow \quad b = 3$$

Step 3: Evaluate the area formula using these semi-axes.

Substitute the values of a and b directly into the area equation:

$$\text{Area} = \pi \cdot 2 \cdot 3 = 6\pi$$

Quick Tip: For parametric coordinate boundaries that fit the form $x = a\sin\theta$ and $y = b\cos\theta$, the coefficients are always your semi-axes values. You can skip the Cartesian conversion entirely and solve directly using Area $= \pi ab$.

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4. Evaluate the indefinite integral using pattern-based substitution:

$$\int \frac{\ln x - 1}{(\ln x)^2}\, dx$$

(A) $\frac{x}{\ln x} + C$

(B) $x \ln x + C$

(C) $\frac{\ln x}{x} + C$

(D) $\frac{1}{\ln x} + C$

Correct Answer: (A) $\frac{x}{\ln x} + C$

Solution:

Concept: Integrals containing logarithmic functions can be simplified by substituting $t = \ln x$. This transforms the expression into a classical exponential template:

$$\int e^t \left[f(t) + f'(t)\right] dt = e^t f(t) + C$$

Step 1: Apply integration by substitution and change the variables.

Let $t = \ln x$. This exponential rewrite gives $x = e^t$. Differentiating both sides with respect to t:

$$dx = e^t\, dt$$

Now, substitute these terms into our original integral:

$$I = \int \frac{t-1}{t^2} \cdot (e^t\,dt) = \int e^t \left(\frac{t}{t^2} - \frac{1}{t^2} \right) dt$$

$$I = \int e^t \left(\frac{1}{t} - \frac{1}{t^2} \right) dt$$

Step 2: Match the transformed integral to the standard exponential pattern.

Let's define the internal function as $f(t) = \frac{1}{t}$. Differentiating this function using the power rule:

$$f'(t) = \frac{d}{dt}(t^{-1}) = -1 \cdot t^{-2} = -\frac{1}{t^2}$$

Our integral matches the standard pattern perfectly: $\int e^t [f(t) + f'(t)]\,dt$. Evaluating this structural form gives:

$$I = e^t f(t) + C = e^t \left(\frac{1}{t} \right) + C = \frac{e^t}{t} + C$$

Step 3: Substitute original variables back into the solution.

Replace t with $\ln x$ and e^t with x to find the final answer:

$$I = \frac{x}{\ln x} + C$$

Quick Tip: Whenever an integral features $\ln x$ mixed across fractions, using the substitution $x = e^t$ will instantly convert a tricky logarithmic problem into a straightforward exponential pattern recognition exercise.

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5. If A is a square matrix of order 3 such that its determinant value is $|A| = -2$, find the value of the scalar-scaled determinant $|4A|$.

(A) -128

(B) -8

(C) -24

(D) 128

Correct Answer: (A) -128

Solution:

Concept: For any square matrix A of order n and a scalar multiplier constant k, pulling the scalar factor outside the determinant brackets requires raising it to the power of the matrix order:

$$|k \cdot A| = k^n \cdot |A|$$

Step 1: Identify the order and the scalar factor from the problem.

From the problem statement, we isolate the key metrics:

- Matrix order, $n = 3$
- Scalar multiplier factor, $k = 4$
- Original determinant value, $|A| = -2$

Step 2: Apply the scalar scaling property formula.

Substitute our values directly into the property formula:

$$|4A| = 4^3 \cdot |A|$$

Calculate the value of the cubed scalar term:

$$4^3 = 4 \cdot 4 \cdot 4 = 64$$

Step 3: Compute the final determinant value.

Multiply the expanded scalar by the original determinant value:

$$|4A| = 64 \cdot (-2) = -128$$

Quick Tip: Never multiply a determinant directly by a scalar without checking the matrix order first. The scalar multiplier must always be raised to the power of the order (k^n) before completing the calculation.

6. Find the general solution of the separable differential equation:

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^{-y} = e^x + x^3 + C$

(C) $e^y = e^x + 2x + C$

(D) $y = \ln\left(e^x + \frac{x^3}{3}\right) + C$

Correct Answer: (A) $e^y = e^x + \frac{x^3}{3} + C$

Solution:

Concept: A separable differential equation can be solved by grouping all terms containing the dependent variable y on one side of the equation and all terms containing the independent variable x on the other side, allowing you to integrate each side directly:

$$\int h(y)\,dy = \int g(x)\,dx$$

Step 1: Separate the mixed terms using exponent rules.

Using the laws of exponents, rewrite the exponential term on the right side:

$$e^{x-y} = e^x \cdot e^{-y}$$

Substitute this back into the differential equation:

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$$

Factor out the common term e^{-y} from the right side:

$$\frac{dy}{dx} = e^{-y}\left(e^x + x^2\right)$$

Step 2: Move variables to opposite sides of the equation.

Multiply both sides by dx and divide by e^{-y} to group the variables:

$$\frac{1}{e^{-y}}\,dy = \left(e^x + x^2\right)dx \quad \Rightarrow \quad e^y\,dy = \left(e^x + x^2\right)dx$$

Step 3: Integrate both sides to find the general solution.

Set up and evaluate the integrals on both sides:

$$\int e^y \, dy = \int \left(e^x + x^2\right) dx$$

Integrating using standard rules:

$$e^y = e^x + \frac{x^3}{3} + C$$

Quick Tip: When dealing with mixed exponential terms like e^{x-y}, factoring out the negative exponent component is almost always the fastest way to separate variables cleanly.

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7. Let A be a square matrix of order 3 such that $|A| = 5$. Find the value of the determinant of its adjoint matrix, $|\text{adj}(A)|$.

(A) 25

(B) 5

(C) 125

(D) 15

Correct Answer: (A) 25

Solution:

Concept: The determinant of an adjoint matrix is directly linked to the determinant of the original matrix through the standard exponent property:

$$|\text{adj}(A)| = |A|^{n-1}$$

where n represents the spatial order of the square matrix.

Step 1: Identify the order and original determinant value.

From the problem description:

- Matrix order, $n = 3$
- Original determinant value, $|A| = 5$

Step 2: Substitute the values into the adjoint property formula.

Plugging our isolated metrics into the exponent template:

$$|\text{adj}(A)| = 5^{3-1}$$

Simplify the exponent calculation:

$$|\text{adj}(A)| = 5^2 = 25$$

Thus, the determinant value of the adjoint matrix is exactly 25.

Quick Tip: The exponent for the adjoint property is always one less than the matrix order $(n-1)$. For a 3×3 matrix, simply square the original determinant value to find the answer immediately.

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8. In a Linear Programming Problem (LPP), if the objective function to maximize is $Z = 3x + 4y$ and the corner points of the feasible bounded region are $(0,0), (4,0), (2,3)$, and $(0,4)$, find the maximum value of Z.

(A) 18

(B) 16

(C) 12

(D) 22

Correct Answer: (B) 16

Solution:

Concept: According to the Fundamental Theorem of Linear Programming, the optimal value (maximum or minimum) of a linear objective function across a bounded feasible region always occurs at one of the boundary corner vertices (vertex points).

Step 1: Evaluate the objective function Z at each corner point.

We calculate the value of $Z = 3x + 4y$ at each of the four given vertices:

1. At point $(0,0)$:

$$Z = 3(0) + 4(0) = 0$$

2. At point $(4,0)$:

$$Z = 3(4) + 4(0) = 12$$

3. At point $(2,3)$:

$$Z = 3(2) + 4(3) = 6 + 12 = 18$$

4. At point $(0,4)$:

$$Z = 3(0) + 4(4) = 16$$

Step 2: Compare the calculated values to identify the maximum result.

Reviewing our list of results: $\{0, 12, 18, 16\}$. The largest value is 18, which occurs at the coordinate vertex $(2,3)$.

Quick Tip: Always test every single corner point listed in the problem. Even if one coordinate looks large (like $(0,4)$), a balanced interior vertex (like $(2,3)$) can often generate a higher overall value depending on the objective weights.

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9. Find the integrating factor (I.F.) for the linear differential equation:

$$\frac{dy}{dx} - y\tan x = e^x$$

(A) $\cos x$

(B) $\sec x$

(C) $-\cos x$

(D) $\ln|\cos x|$

Correct Answer: (A) $\cos x$

Solution:

Concept: For a linear differential equation written in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$, the Integrating Factor is calculated using the formula:

$$\text{I.F.} = e^{\int P(x)\,dx}$$

Be sure to include any negative signs when identifying the function $P(x)$.

Step 1: Isolate the function $P(x)$ with its proper sign.

By comparing our given equation to the standard layout, the coefficient function of y includes the negative sign:

$$P(x) = -\tan x$$

Step 2: Integrate the function $P(x)$ with respect to x.

Evaluate the indefinite integral of the negative tangent function component:

$$\int P(x)\,dx = \int -\tan x\,dx = -\int \tan x\,dx$$

Using standard trigonometric integration rules, we know that $\int \tan x\,dx = \ln|\sec x|$:

$$-\int \tan x\,dx = -\ln|\sec x|$$

Using logarithmic properties to simplify the negative coefficient into an exponent:

$$-\ln|\sec x| = \ln\left|(\sec x)^{-1}\right| = \ln\left|\frac{1}{\sec x}\right| = \ln|\cos x|$$

Step 3: Evaluate the exponential expression to find the I.F.

Plugging this result back into our exponential base formula:

$$\text{I.F.} = e^{\ln|\cos x|} = \cos x$$

Quick Tip: Remember to carry the negative sign along with your $P(x)$ function. Missing the negative sign will lead you to calculate $\sec x$ instead of the correct answer, which is a common multiple-choice trap.

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10. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ given the initial boundary condition that $y = 2$ when $x = 1$.

(A) $y = 2x$

(B) $y = x + 1$

(C) $y = x^2$

(D) $y = \frac{2}{x}$

Correct Answer: (A) $y = 2x$

Solution:

Concept: A particular solution is found by first separating variables and integrating to determine the general solution with an integration constant C, and then substituting the initial boundary conditions to solve for the exact value of that constant.

Step 1: Separate the variables and integrate both sides.

Group the y variables on the left and the x variables on the right:

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

Set up and evaluate the integrals:

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx \quad \Rightarrow \quad \ln|y| = \ln|x| + \ln|C|$$

Using logarithmic properties to combine the right side:

$$\ln|y| = \ln|Cx| \quad \Rightarrow \quad y = Cx$$

Step 2: Substitute the boundary conditions to find the constant C.

We are given that $y = 2$ when $x = 1$. Substitute these coordinates into our general solution:

$$2 = C \cdot (1) \quad \Rightarrow \quad C = 2$$

Step 3: Write out the final particular solution equation.

Replace the constant C with its calculated value of 2 in our general equation:

$$y = 2x$$

Quick Tip: When every term in a separated integral results in a natural logarithm, writing your integration constant as $\ln C$ instead of just C makes simplifying the logarithms much cleaner.
