CUET 2026 May 21 Mathematics Shift 2

Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

- The examination will be conducted in Computer-Based Test (CBT) mode.
- Each question carries +5 marks for correct answer and -1 mark for wrong answer.
- The total number of questions are 50.
- Duration of the exam is 1 hour (60 minutes).

1. Find the shortest distance between the two skew lines whose vector equations are given by:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3hatj + 2\hat{k})$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

(A) $\frac{3}{\sqrt{19}}$

(B) 0

(C) 9

(D) $\sqrt{151}$

Correct Answer: (A) $\frac{3}{\sqrt{19}}$

Solution:

Concept: The shortest distance (d) between two skew lines represented by the vector equations $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is determined by calculating the projection of the position difference vector along the common perpendicular direction vector:

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Step 1: Extract the position and direction coordinates from the equations.

From the provided line equations, isolate the independent parameters:

- Line 1: $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$
- Line 2: $\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

Now, calculate the displacement vector connecting the two starting points:

$$\vec{a}_2 - \vec{a}_1 = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Step 2: Compute the cross product vector $\vec{b}_1 \times \vec{b}_2$.

Set up the component matrix determinant using standard base unit vectors:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

Expanding along the top row:

$$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3-(-6)) = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

Calculate its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + 3^2 + 9^2} = \sqrt{81+9+81} = \sqrt{171} = 3\sqrt{19}$$

Step 3: Evaluate the scalar dot product and solve for distance d.

Multiply the vector terms calculated in our earlier steps:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})$$

$$= (3)(-9) + (3)(3) + (3)(9) = -27 + 9 + 27 = 9$$

Plug these completed values back into the shortest distance projection formula:

$$d = \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$$

Quick Tip: To perform the vector cross product quickly and accurately during exams, always write down your grid layout explicitly. Missing a single negative sign during the middle $-\hat{j}$ row expansion is the most common cause of calculation errors.

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2. Evaluate the definite integral using standard definite integral properties:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\,dx$$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) π

(D) 0

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Concept: According to the King's Property (P_4) of definite integrals, substituting the sum of the boundary limits minus the independent variable leaves the net value of the definite integral completely unchanged:

$$I = \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Step 1: Apply the boundary property to create an alternative equation.

Let our initial integral equation be designated as equation (1):

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\,dx \quad \cdots (1)$$

Apply the definite boundary property by replacing every x variable with $\left(0 + \frac{\pi}{2} - x\right) = \frac{\pi}{2} - x$:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5(\frac{\pi}{2} - x)}{\sin^5(\frac{\pi}{2} - x) + \cos^5(\frac{\pi}{2} - x)}\,dx$$

Using standard co-function reduction identities where $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$, rewrite the equation:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\cos^5 x + \sin^5 x} \, dx \quad \cdots (2)$$

Step 2: Add both equations together to simplify the integrand expression.

Add equation (1) and equation (2) together. Since they share identical integration boundaries and denominators, combine their numerators directly:

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} \, dx$$

The matching numerator and denominator terms cancel out completely, leaving an integrand value of 1:

$$2I = \int_0^{\frac{\pi}{2}} 1 \, dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Step 3: Isolate the single integral variable I.

Divide both sides of the equation by 2 to find the final value:

$$2I = \frac{\pi}{2} \quad \Rightarrow \quad I = \frac{\pi}{4}$$

Quick Tip: For any definite integral problem bounded precisely from 0 to $\frac{\pi}{2}$ that follows the standard structural form $\frac{f(\sin x)}{f(\sin x)+f(\cos x)}$, the answer will always evaluate to exactly half of the upper bound: $**\frac{\pi}{4}**$.

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3. If A is a skew-symmetric matrix of order 5×5, what is the value of its determinant $|A|$?

(A) 0

(B) 1

(C) -1

(D) 5

Correct Answer: (A) 0

Solution:

Concept: A square matrix is classified as skew-symmetric if its transpose equals its negative ($A^T = -A$). We can find its scalar determinant value by combining the scaling properties of determinants with transpose laws.

Step 1: Apply determinant operations across the matrix definition.

Take the determinant of both sides of the structural definition equation:

$$|A^T| = |-A|$$

Step 2: Apply standard determinant property scaling rules.

1. The determinant of any matrix is equal to the determinant of its transpose: $|A^T| = |A|$.
2. Factoring out a scalar multiplier of -1 from a matrix determinant requires raising it to the power of the matrix order n: $|-A| = (-1)^n|A|$.

Substitute these core properties back into our expression:

$$|A| = (-1)^n|A|$$

Step 3: Evaluate the expression for the given odd matrix order $n = 5$.

The problem states that the matrix has a spatial size order of 5. Since 5 is an odd integer, the factor $(-1)^5 = -1$:

$$|A| = -|A| \quad \Rightarrow \quad |A| + |A| = 0 \quad \Rightarrow \quad 2|A| = 0 \quad \Rightarrow \quad |A| = 0$$

This proves that the determinant of any odd-order skew-symmetric matrix is always zero.

Quick Tip: This is a pure property question that requires no manual arithmetic calculations. If you see the words **skew-symmetric** combined with an **odd order** number (like $1, 3, 5$, or 7), select **0** immediately to save time.

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4. Two independent events A and B have individual probabilities of occurring given by $P(A) = 0.4$ and $P(B) = 0.5$. Find the probability that at least one of these two events occurs.

(A) 0.70

(B) 0.90

(C) 0.20

(D) 0.30

Correct Answer: (A) 0.70

Solution:

Concept: The phrase "probability of occurrence of at least one event" refers to the union of the two sets, represented as $P(A \cup B)$. According to the probability addition theorem:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For independent events, their joint intersection is the direct product of their separate individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

Step 1: Calculate the intersection probability for the independent events.

Since events A and B are explicitly stated to operate independently, calculate their overlapping intersection:

$$P(A \cap B) = P(A) \cdot P(B) = 0.4 \times 0.5 = 0.20$$

Step 2: Apply values to the probability addition theorem.

Plug the individual parameters along with our calculated intersection value into the addition formula:

$$P(A \cup B) = 0.4 + 0.5 - 0.20$$

Simplify the addition and subtraction steps:

$$P(A \cup B) = 0.90 - 0.20 = 0.70$$

Quick Tip: Never simply add independent probabilities together $(0.4 + 0.5 = 0.9)$ without checking for overlap. Unless the problem explicitly states that the events are **mutually exclusive**, you must always calculate and subtract their intersection point.

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5. Find the general solution of the following homogeneous differential equation:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$$

(A) $y^2 = 2x^2 \ln|x| + Cx^2$

(B) $y = x \ln|x| + C$

(C) $y^2 = x^2 + C$

(D) $y = e^x + Cx$

Correct Answer: (A) $y^2 = 2x^2 \ln|x| + Cx^2$

Solution:

Concept: A first-order homogeneous differential equation can be solved by substituting $y = vx$. Differentiating this product with respect to x using the product rule transforms the derivative component:

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

This substitution transforms the homogeneous structure into a basic separable format.

Step 1: Apply the substitution parameters to rewrite the equation.

Divide both terms in the numerator by the denominator to simplify the expression:

$$\frac{dy}{dx} = \frac{x^2}{xy} + \frac{y^2}{xy} = \frac{x}{y} + \frac{y}{x}$$

Substitute $y = vx$ (which means $\frac{y}{x} = v$ and $\frac{x}{y} = \frac{1}{v}$), and replace $\frac{dy}{dx}$:

$$v + x\frac{dv}{dx} = \frac{1}{v} + v$$

Step 2: Separate the variables and integrate both sides.

Subtract the common scalar variable v from both sides to isolate the differential terms:

$$x\frac{dv}{dx} = \frac{1}{v} \quad \Rightarrow \quad v\,dv = \frac{1}{x}\,dx$$

Set up and evaluate the indefinite integrals on both sides:

$$\int v\,dv = \int \frac{1}{x}\,dx \quad \Rightarrow \quad \frac{v^2}{2} = \ln|x| + C'$$

Step 3: Substitute original variables back to find the final equation.

Replace v with its original fractional definition, $v = \frac{y}{x}$:

$$\frac{\left(\frac{y}{x}\right)^2}{2} = \ln|x| + C' \quad \Rightarrow \quad \frac{y^2}{2x^2} = \ln|x| + C'$$

Multiply through by $2x^2$ to isolate our dependent variable terms:

$$y^2 = 2x^2 \ln|x| + 2C'x^2$$

Combine the constant terms by letting $C = 2C'$ to find the final general solution:

$$y^2 = 2x^2 \ln|x| + Cx^2$$

Quick Tip: When verifying solutions for homogeneous equations, check the polynomial degrees across your final terms. Every term in the solution should maintain a balanced quadratic degree distribution when factoring variables out.

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6. Find the angle θ between the line $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-3}{2}$ and the plane $2x - y + 2z = 7$.

(A) $\sin^{-1}\left(\frac{8}{9}\right)$
(B) $\cos^{-1}\left(\frac{8}{9}\right)$
(C) $\frac{\pi}{2}$
(D) $\sin^{-1}\left(\frac{2}{3}\right)$

Correct Answer: (A) $\sin^{-1}\left(\frac{8}{9}\right)$

Solution:

Concept: The angle θ between a straight line with a directional heading vector $\vec{b}$ and a flat plane with a surface normal vector $\vec{n}$ is calculated using the sine projection product formula:

$$\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$$

Step 1: Extract the directional components from the line and plane equations.

- Line direction vector coefficients (from denominators): $\vec{b} = 1\hat{i} - 2\hat{j} + 2\hat{k}$
- Plane normal vector coefficients (from variable modifiers): $\vec{n} = 2\hat{i} - 1\hat{j} + 2\hat{k}$

Step 2: Calculate the scalar dot product and vector magnitudes.

1. Compute the dot product:

$$\vec{b} \cdot \vec{n} = (1)(2) + (-2)(-1) + (2)(2) = 2 + 2 + 4 = 8$$

2. Compute the magnitude of the line's direction vector:

$$|\vec{b}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

3. Compute the magnitude of the plane's normal vector:

$$|\vec{n}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Step 3: Substitute these components into the sine formula to solve for θ.

$$\sin\theta = \frac{8}{3 \times 3} = \frac{8}{9} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{8}{9}\right)$$

Quick Tip: Remember the unique rule for mixed line-and-plane problems: finding the angle between a line and a plane uses **$\sin\theta$**, whereas finding the angle between two lines or two planes uses **$\cos\theta$**.

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7. Let a relation R be defined on the set of all natural numbers $\mathbb{N}$ by the rule: $(a, b) \in R$ if and only if $a + b$ is an even number. This relation R is classified as an:

(A) Equivalence Relation

(B) Symmetric but not Transitive Relation

(C) Reflexive but not Symmetric Relation

(D) Anti-symmetric Relation

Correct Answer: (A) Equivalence Relation

Solution:

Concept: A relation is classified as an **Equivalence Relation** if and only if it satisfies three independent properties simultaneously: it must be Reflexive, Symmetric, and Transitive.

Step 1: Verify the Reflexive property.

A relation is reflexive if $(a, a) \in R$ for every element. Substitute a into our addition rule:

$$a + a = 2a$$

Since $2a$ is always a multiple of 2, the sum is always an even number for any natural number, making the relation **Reflexive**.

Step 2: Verify the Symmetric property.

A relation is symmetric if $(a, b) \in R$ implies $(b, a) \in R$. If $a + b$ is even, then because addition is commutative $(a + b = b + a)$, the expression $b + a$ must also yield the exact same even value, proving the relation is **Symmetric**.

Step 3: Verify the Transitive property.

A relation is transitive if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$. 1. If $a + b$ is even, both numbers must share the same parity (either both are even or both are odd).

2. If $b + c$ is even, then since b's parity is fixed, c must also share that same parity.

This matching parity ensures that a and c are either both even or both odd, meaning their sum $a + c$ will always be an even number. Since the relation is reflexive, symmetric, and transitive, it is an Equivalence Relation.

Quick Tip: Parity relationships (checking if a sum or difference matches an even number or is divisible by an integer) are classic templates that almost always evaluate to **Equivalence Relations** in competitive exam modules.

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8. Find the total area of the bounded region enclosed between the parabola curve $y^2 = 8x$ and its vertical latus rectum boundary line.

(A) $\frac{32}{3}$ square units

(B) $\frac{16}{3}$ square units

(C) 8 square units

(D) $\frac{64}{3}$ square units

Correct Answer: (A) $\frac{32}{3}$ square units

Solution:

Concept: The standard parabola equation $y^2 = 4ax$ features a focal point at coordinates $(a, 0)$. Its latus rectum is the vertical boundary line running perpendicular to its axis through this focus, defined by the line equation $x = a$.

Step 1: Identify the boundary limit coordinates.

Compare our given equation $y^2 = 8x$ to the standard parabola layout $y^2 = 4ax$:

$$4a = 8 \quad \Rightarrow \quad a = 2$$

This places our vertical latus rectum boundary line at $x = 2$. The integration limits for the bounded region run from the origin vertex ($x = 0$) to this boundary line ($x = 2$).

Step 2: Set up the integration equation using symmetry properties.

Because the parabola curve is perfectly symmetric across the x-axis, calculate the total area by doubling the area of the upper quadrant slice:

$$\text{Area} = 2\int_0^2 y\,dx = 2\int_0^2 \sqrt{8x}\,dx = 2\cdot\sqrt{8}\int_0^2 x^{\frac{1}{2}}\,dx = 4\sqrt{2}\int_0^2 x^{\frac{1}{2}}\,dx$$

Step 3: Evaluate the definite integral using the power rule.

$$\text{Area} = 4\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = 4\sqrt{2}\cdot\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 = \frac{8\sqrt{2}}{3}\left(2^{\frac{3}{2}} - 0\right)$$

Since $2^{\frac{3}{2}} = 2\sqrt{2}$, substitute this back into the expression:

$$\text{Area} = \frac{8\sqrt{2}}{3}\cdot 2\sqrt{2} = \frac{16\cdot 2}{3} = \frac{32}{3}\text{ square units}$$

Quick Tip: You can save valuable time during your test by using this parabola integration shortcut: the total area enclosed between any standard parabola $y^2 = 4ax$ and its latus rectum line is always exactly $**\frac{8}{3}a^2**$.

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9. If the direction vectors of two straight lines are given by $\vec{m} = \hat{i} + \hat{j} + 0\hat{k}$ and $\vec{n} = \hat{i} + 0\hat{j} + \hat{k}$, calculate the value of $\cos\phi$ representing the acute angle between them.

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) 0

(D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept: The cosine of the angle ϕ between two intersecting lines depends on the scalar dot product of their direction vectors divided by the product of their individual vector magnitudes:

$$\cos\phi = \frac{|\vec{m}\cdot\vec{n}|}{|\vec{m}||\vec{n}|}$$

Step 1: Calculate the scalar dot product of the direction vectors.

Multiply corresponding vector components together and sum the results:

$$\vec{m}\cdot\vec{n} = (1)(1)+(1)(0)+(0)(1) = 1+0+0 = 1$$

Step 2: Calculate the individual magnitudes of both vectors.

1. Magnitude of vector $\vec{m}$:

$$|\vec{m}| = \sqrt{1^2+1^2+0^2} = \sqrt{1+1} = \sqrt{2}$$

2. Magnitude of vector $\vec{n}$:

$$|\vec{n}| = \sqrt{1^2+0^2+1^2} = \sqrt{1+1} = \sqrt{2}$$

Step 3: Substitute these values into the angle equation.

Plug the calculated values back into the directional cosine template:

$$\cos\phi = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

Quick Tip: Always use absolute value bars for the numerator when calculating the angle between lines. This ensures your cosine value remains positive, which guarantees an acute angle result ($\leq 90^\circ$) in line with standard geometric conventions.

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10. A pair of standard unbiased six-sided dice are rolled simultaneously. Given that the sum of the numbers showing is exactly 6, what is the conditional probability that one of the dice shows the number 2?

(A) $\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{6}$

(D) $\frac{2}{36}$

Correct Answer: (A) $\frac{2}{5}$

Solution:

Concept: Conditional probability measures the likelihood of an event occurring given that a prior condition has already taken place. We can solve this directly by restricting our sample space denominator to only include outcomes that satisfy the given condition:

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$

Step 1: Determine the size of the restricted condition sample space $n(B)$.

The condition states that the sum of the numbers showing on the two dice must equal exactly 6. List all possible coordinate pairs that satisfy this sum requirement:

$$B = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$$

Counting these outcomes gives our restricted sample space size:

$$n(B) = 5$$

Step 2: Count the outcomes that satisfy both conditions simultaneously, $n(A \cap B)$.

Identify which of the pairs in our restricted sample space contain the number 2 on at least one of the dice:

$$A \cap B = \{(2,4), (4,2)\}$$

Counting these favorable outcomes yields:

$$n(A \cap B) = 2$$

Step 3: Evaluate the conditional probability ratio.

Divide the number of favorable outcomes by the size of our restricted sample space:

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{2}{5}$$

Quick Tip: For conditional probability problems, avoid using the default sample space of 36 as your denominator. Reducing your sample space immediately based on the given condition phrase makes counting favorable pairs much simpler and less prone to errors.
