CUET 2026 May 22 Mathematics Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for a correct answer and -1 mark for a wrong answer.

(iii) The total number of questions is 50.

(iv) Duration of the examination is 1 hour (60 minutes).

1. Find the total area of the region bounded by the parabola $y^2 = 4x$ and the straight line $y = x$.

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) 2

Correct Answer: (B) $\frac{8}{3}$

Solution:

Concept: The area bounded between two intersecting curves $y_1 = f(x)$ and $y_2 = g(x)$ from $x = a$ to $x = b$ is calculated by integrating the difference between the upper curve and the lower curve:

$$\text{Area} = \int_a^b \left(y_{\text{upper}} - y_{\text{lower}}\right)\, dx$$

Finding the accurate boundary limits requires solving the equations simultaneously to locate their physical intersection coordinates.

Step 1: Find the intersection points of both curves.

Substitute the line equation $y = x$ directly into the parabola equation $y^2 = 4x$:

$$x^2 = 4x \quad \Rightarrow \quad x^2 - 4x = 0 \quad \Rightarrow \quad x(x-4) = 0$$

This yields the intersection x-coordinates at $x = 0$ and $x = 4$. The corresponding points are $(0,0)$ and $(4,4)$.

Step 2: Set up and evaluate the definite integral framework.

In the interval $[0,4]$, the parabola curve $y = 2\sqrt{x}$ forms the upper boundary, while the line $y = x$ forms the lower boundary. Set up the difference integration:

$$\text{Area} = \int_0^4 \left(2\sqrt{x} - x\right) dx = \int_0^4 \left(2x^{\frac{1}{2}} - x\right) dx$$

Apply the standard integration power rules:

$$= \left[2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4 = \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$$

Step 3: Evaluate the definite integral limits.

Substitute the upper limit $x = 4$ into the integrated expression (the lower limit at 0 vanishes):

$$\text{Area} = \left(\frac{4}{3}(4)^{\frac{3}{2}} - \frac{4^2}{2}\right) = \left(\frac{4}{3}(8) - \frac{16}{2}\right) = \frac{32}{3} - 8$$

Take the common denominator to isolate the final fractional area:

$$\text{Area} = \frac{32 - 24}{3} = \frac{8}{3} \text{ square units}$$

Quick Tip: Save time on standard calculus formats by memorizing this shortcut: the area enclosed between a standard parabola $y^2 = 4ax$ and a passing line $y = mx$ always evaluates precisely to **$\frac{8}{3}\frac{a^2}{m^3}$**.

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2. Determine the sum of the order and the degree of the differential equation given by: $y = x\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

(A) 2

(B) 4

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept: The structural properties of a differential equation are defined as follows:

- **Order:** The highest derivative present in the equation.
- **Degree:** The power index of the highest-order derivative, provided the equation is expressed as a rational polynomial in terms of its derivatives. Fractional radicals wrapping derivative terms must be cleared before determining the degree.

Step 1: Isolate and clear the radical expression from the equation.

Move the polynomial terms to the left-hand side to isolate the square root:

$$y - x\frac{dy}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Square both sides of the equation to clear the radical fraction:

$$\left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

Step 2: Expand the polynomial and determine the order and degree.

Expanding the left side using the binomial identity gives:

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = 1 + \left(\frac{dy}{dx}\right)^2$$

The highest derivative appearing in this expanded expression is the first derivative $\frac{dy}{dx}$, which gives an **Order = 1**. The highest power exponent attached to this derivative component is 2, which gives a **Degree = 2**.

Step 3: Calculate the final requested sum value.

Sum the isolated parameters:

$$\text{Sum} = \text{Order} + \text{Degree} = 1 + 2 = 3$$

Quick Tip: Never read the degree of a differential equation directly if any derivative sits inside a radical sign, fraction, or non-polynomial function. Always isolate and clear the radical term by squaring or multiplying out denominators first.

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3. If the three vectors $\vec{a} = \hat{i} + \lambda\hat{j} + \hat{k}$, $\vec{b} = \hat{j} + \hat{k}$, and $\vec{c} = \hat{i} + \hat{j}$ are coplanar, find the exact value of the scalar constant λ.

(A) 0

(B) 1

(C) -2

(D) 2

Correct Answer: (D) 2

Solution:

Concept: Three non-zero vectors are coplanar (meaning they lie in the exact same geometric plane) if and only if their Scalar Triple Product evaluates to exactly zero:

$$\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix} = \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

This scalar product can be computed efficiently by setting the vector components into a 3×3 matrix row layout and evaluating its determinant.

Step 1: Set up the matrix determinant using the vector components.

Extract the directional coefficients from vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ to form the rows of the matrix:

$$\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

Step 2: Expand the determinant along the first row.

Evaluate the 2x2 sub-determinants across the top row:

$$1 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \lambda \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

Simplify the cross-multiplication pairs:

$$1(0-1)-\lambda(0-1)+1(0-1)=0$$

$$-1+\lambda-1=0$$

Step 3: Isolate the unknown variable λ.

Combine the constant numerical values:

$$\lambda-2=0 \quad \Rightarrow \quad \lambda=2$$

Quick Tip: The Scalar Triple Product measures the volume of a parallelopiped spanned by three vectors. When vectors are coplanar, the volume collapses to zero, which is why equating the determinant to **0** works perfectly.

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4. A straight vector line makes equal acute angles $\alpha=\beta=\gamma$ with all three primary coordinate axes. Find the absolute value of $\cos\alpha$.

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{3}$

(C) $\frac{1}{\sqrt{2}}$

(D) 1

Correct Answer: (A) $\frac{1}{\sqrt{3}}$

Solution:

Concept: If a line forms directional orientation angles α, β, and γ with the positive x, y, and z axes respectively, its direction cosines are defined as $l=\cos\alpha$, $m=\cos\beta$, and $n=\cos\gamma$. These parameters must satisfy the fundamental geometric identity:

$$l^2+m^2+n^2=1 \quad \Rightarrow \quad \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

Step 1: Apply the equal angle condition to the direction cosine identity.

Since the problem states that the line is oriented at equal angles to all three axes, substitute

$\alpha = \beta = \gamma$ into the equation:

$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

Combine the matching terms:

$$3\cos^2\alpha = 1$$

Step 2: Isolate the value of $\cos\alpha$.

Divide both sides by the scalar factor 3:

$$\cos^2\alpha = \frac{1}{3}$$

Take the square root of both sides. Because the problem specifies an **acute angle** ($\alpha \leq 90°$), the cosine value must be positive:

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

Quick Tip: Since $\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$, remember the related identity for sine terms by substituting $\cos^2\theta = 1-\sin^2\theta$, which gives: **$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$**.

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5. Simplify the inverse trigonometric expression to find its principal value: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$

(A) $\frac{\pi}{2}$

(B) $\tan^{-1}\left(\frac{2}{5}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept: To combine two inverse tangent terms, apply the standard tangent summation identity:

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

This identity is valid provided that the product of the arguments satisfies the domain constraint $xy < 1$.

Step 1: Verify the domain constraint rule for the arguments.

Identify the arguments from the problem: $x = \frac{1}{2}$ and $y = \frac{1}{3}$. Check their product:

$$xy = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Since $\frac{1}{6} < 1$, the standard summation identity can be applied safely.

Step 2: Substitute the arguments into the summation formula.

Plug the fractions into the identity template:

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\cdot\frac{1}{3}\right)}\right)$$

Simplify the fractional expressions in both the numerator and the denominator:

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$$

This simplifies to a single unit argument:

$$= \tan^{-1}(1)$$

Step 3: Determine the final principal value angle.

The unique angle in the principal branch interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent value equals 1 is $\frac{\pi}{4}$ (40°):

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Quick Tip: Always check the product xy before using the tangent addition formula. If the product evaluates to greater than one ($xy > 1$), you must use the alternative identity track: **$\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$**.

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6. Under what condition will a linear programming system containing objective function variables have no common feasible region?

(A) When the objective function coefficients are set strictly to zero.

(B) When the system of linear constraints is mutually inconsistent, meaning there is no overlapping coordinate space that satisfies all conditions at once.

(C) When all constraints are written using strictly non-negative bounding limits.

(D) When the optimal solution point matches one of the outer corner coordinates.

Correct Answer: (B) When the system of linear constraints is mutually inconsistent, meaning there is no overlapping coordinate space that satisfies all conditions at once.

Solution:

Concept: In Linear Programming Problems (LPP), the **feasible region** is the shared collection of all coordinate points on a graph that satisfy every constraint inequality simultaneously.

Step 1: Analyze the geometric structure of constraint half-planes.

Each individual constraint inequality forms a shaded half-plane boundary line across a coordinate grid. For a feasible region to exist, these shaded areas must overlap, creating a shared boundary space.

Step 2: Evaluate the cause of an infeasible state.

If the constraint requirements conflict with each other (for example, requiring $x \leq 2$ and $x \geq 5$ at the same time), their shaded regions will point in opposite directions and never overlap. This mutual inconsistency means there are zero coordinate points that satisfy all conditions at once, resulting in **no feasible region**.

Quick Tip: When an LPP system produces no overlapping feasible region, it is classified as an **infeasible solution space**, meaning it is impossible to find a maximum or minimum value for the objective function.

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7. Find the total area of the region bounded between the curve $y = x^3$, the x-axis, and the vertical lines $x = -1$ and $x = 1$.

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept: When calculating the geometric area under a curve that dips below the x-axis within the integration interval, you cannot integrate continuously across the entire range. Doing so allows negative values below the axis to cancel out positive values above it. Instead, you must split the integral at the origin crossing point and sum the absolute values of the separate region areas:

$$\text{Total Area} = \int_a^c |f(x)|\,dx + \int_c^b f(x)\,dx$$

Step 1: Identify the behavior of the cubic function across the boundaries.

The function $y = x^3$ crosses the x-axis at $x = 0$.

- Across the left interval section $[-1, 0]$, the function values are negative ($y \leq 0$).
- Across the right interval section $[0, 1]$, the function values are positive ($y \geq 0$).

Step 2: Set up the split definite integration equations.

Apply absolute value adjustments to the negative area section by introducing an explicit negative sign:

$$\text{Area} = \int -1^0(-x^3)\,dx + \int_0^1 x^3\,dx$$

Apply the power integration rule to evaluate each section:

$$= \left[-\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^4}{4}\right]_0^1$$

Step 3: Evaluate the limits to find the final area.

Substitute the boundary numbers into the integrated terms:

$$\text{Area} = \left(0 - \left(-\frac{(-1)^4}{4}\right)\right) + \left(\frac{(1)^4}{4} - 0\right) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \text{ square units}$$

Quick Tip: Always look at the symmetry of the function. Because $y = x^3$ is an odd function, integrating it directly from -1 to 1 gives **0**. For geometric area, utilize its structural balance by integrating the positive slice and doubling it: $2 \times \int_0^1 x^3\,dx = \frac{1}{2}$.

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8. Determine the degree of the following differential equation: $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

(A) Not defined

(B) 2

(C) 3

(D) 1

Correct Answer: (B) 2

Solution:

Concept: The degree of a differential equation is the power index of its highest-order derivative component, calculated only after the equation has been cleared of any fractional exponents or radicals affecting its derivatives.

Step 1: Clear the fractional exponent from the equation structure.

The left side of the equation features a fractional power exponent of $\frac{3}{2}$. To clear the denominator radical, square both sides of the equation:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

This simplifies the expression to a clean polynomial format:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 2: Identify the highest-order derivative and its exponent index.

Look across the transformed equation to find its derivative properties:

- The highest-order derivative present is the second derivative $\frac{d^2y}{dx^2}$, which means the **Order = 2**.
- The power exponent attached directly to this highest-order derivative term is 2, which gives a **Degree = 2**.

Quick Tip: Be careful not to look at the larger power of 3 on the left side of the equation. The degree is determined strictly by the exponent on the **highest-order derivative** ($\frac{d^2y}{dx^2}$), not by larger exponents attached to lower-order derivatives.

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9. If the straight line equation $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ runs completely parallel to the plane surface given by $Ax + 2y + 3z = 5$, find the value of the coefficient A.

(A) 9

(B) 0

(C) -9

(D) -4

Correct Answer: (C) -9

Solution:

Concept: If a straight line runs parallel to a flat plane, the line's directional movement vector $\vec{m}$ must run perpendicular to the plane's surface normal vector $\vec{n}$. As a result, their scalar dot product must evaluate to exactly zero:

$$\vec{m} \cdot \vec{n} = 0$$

Step 1: Extract the directional component vectors from the equations.

- Extract the line's direction vector components from the denominators: $\vec{m} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
- Extract the plane's normal vector components from the variable coefficients: $\vec{n} = A\hat{i} + 2\hat{j} + 3\hat{k}$

Step 2: Set up the scalar dot product equation and equate it to zero.

Multiply the corresponding vector components and sum them up:

$$\vec{m} \cdot \vec{n} = (2)(A) + (3)(2) + (4)(3) = 0$$

Simplify the arithmetic multiplication terms:

$$2A + 6 + 12 = 0$$

$$2A + 18 = 0$$

Step 3: Isolate the unknown coefficient variable A.

Move the constant numbers across the equality sign and divide by the coefficient:

$$2A = -18 \quad \Rightarrow \quad A = -9$$

Quick Tip: Remember this geometric inversion rule: when a line is **parallel** to a plane, you use the **perpendicular dot product condition** ($a_1a_2 + b_1b_2 + c_1c_2 = 0$) because the plane's normal vector runs perpendicular to its surface.

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10. Find the maximum value of the linear objective function $Z = 3x + 4y$ subject to the system constraints: $x + y \leq 4$, $x \geq 0$, and $y \geq 0$.

(A) 12

(B) 16

(C) 24

(D) 0

Correct Answer: (B) 16

Solution:

Concept: According to the Corner Point Theorem of Linear Programming, the optimal maximum or minimum value of an objective function always occurs at one of the corner vertices of the bounded feasible region.

Step 1: Identify and map the corner vertices of the feasible region.

The non-negativity constraints ($x \geq 0$, $y \geq 0$) restrict the feasible region to the first quadrant. Find where the primary constraint line $x + y = 4$ intersects both axes:

- Setting $y = 0$ gives the x-intercept at $(4, 0)$.
- Setting $x = 0$ gives the y-intercept at $(0, 4)$.

The boundary lines meet at the origin, forming a closed triangular feasible region with corner vertices at: $(0, 0)$, $(4, 0)$, and $(0, 4)$.

Step 2: Evaluate the objective function Z at each vertex point.

Plug the coordinate numbers for each corner point into the objective equation $Z = 3x + 4y$:

- At vertex $(0, 0)$: $Z = 3(0) + 4(0) = 0$
- At vertex $(4, 0)$: $Z = 3(4) + 4(0) = 12$
- At vertex $(0, 4)$: $Z = 3(0) + 4(4) = 16$

Step 3: Identify the absolute maximum value.

Comparing our results shows that the absolute maximum value is 16, which occurs at the corner coordinate position $(0, 4)$.

Quick Tip: To locate the maximizing corner point quickly without full math checks, compare the variable coefficients in the objective function. Since y carries a larger weight coefficient (4) than x (3), maximizing the y-coordinate budget at $(0, 4)$ will naturally yield the highest value.
