CUET 2026 May 31 Shift 2 Mathematics

Question Paper (Memory-Based) With Solution

Conducted by National Testing Agency (NTA)

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +5 marks for correct answer and -1 mark for wrong answer.

(iii) The total number of questions are 50.

(iv) Duration of the exam is 1 hour (60 minutes).

1. If

$$y(x) = \det\begin{pmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{pmatrix},$$

for $x \in \mathbb{R}$, then

$$\frac{d^2y}{dx^2} + y$$

is equal to:

(A) 0

(B) -1

(C) 1

(D) 2

Correct Answer: (B) -1

Solution:

Concept:

First evaluate the determinant and then differentiate the obtained function.

Step 1: Simplify the determinant

Apply the column operation:

$$C_3 \to C_3 - C_1 - C_2$$

Then

$$y = \begin{vmatrix} \sin x & \cos x & 1 \\ 27 & 28 & -28 \\ 1 & 1 & -1 \end{vmatrix}$$

Expanding along the third column,

$$y = 1\begin{vmatrix} 27 & 28 \\ 1 & 1 \end{vmatrix} - (-28)\begin{vmatrix} \sin x & \cos x \\ 1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} \sin x & \cos x \\ 27 & 28 \end{vmatrix}$$

$$= (27-28) + 28(\sin x - \cos x) - (28\sin x - 27\cos x)$$

$$= -1 - \cos x$$

Step 2: Find the derivatives

$$y = -1 - \cos x$$

$$y' = \sin x$$

$$y'' = \cos x$$

Step 3: Evaluate $y'' + y$

$$y'' + y = \cos x + (-1 - \cos x)$$

$$= -1$$

Final Answer:

$$\boxed{-1}$$

Hence the correct option is

$$\boxed{(B)}$$

Quick Tip: For determinants containing expressions like

$$C_3 = C_1 + C_2 + \text{constant},$$

first use column operations to simplify before expansion.

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2. Let $y = f(x)$ be the solution of $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^6+4x}{\sqrt{1-x^2}}$, $-1 < x < 1$, such that $f(0) = 0$. If $6\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx = 2\pi - \alpha$, then α^2 is:

(A) 16

(B) 36

(C) 64

(D) 100

Correct Answer: (B) 36

Solution:

Concept:

The given equation is a first-order linear differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

which can be solved using the integrating factor method.

Step 1: Find the integrating factor

$$P(x) = \frac{x}{x^2-1}$$

Hence,

$$IF = e^{\int \frac{x}{x^2-1} dx}$$

Let

$$t = x^2 - 1$$

Then

$$dt = 2x\, dx$$

Therefore,

$$IF = e^{\frac{1}{2}\ln(1-x^2)}$$

$$= \sqrt{1-x^2}$$

Step 2: Solve the differential equation

Multiplying throughout by the integrating factor,

$$\frac{d}{dx}\left(y\sqrt{1-x^2}\right) = x^6 + 4x$$

Integrating,

$$y\sqrt{1-x^2} = \frac{x^7}{7} + 2x^2 + C$$

Using $f(0) = 0$,

$$C = 0$$

Thus,

$$f(x) = \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}$$

Step 3: Evaluate the definite integral

After simplification,

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx = \frac{\pi}{3} - 1$$

Hence,

$$6\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx = 2\pi - 6$$

Comparing with

$$2\pi - \alpha$$

gives

$$\alpha = 6$$

$$\alpha^2 = 36$$

Final Answer:

$$\boxed{36}$$

Quick Tip: For linear differential equations, always compute the integrating factor first and then convert the equation into an exact derivative.

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3. If the system $2x + \lambda y + 3z = 5,\ 3x + 2y - z = 7,\ 4x + 5y + \mu z = 9$ has infinitely many solutions, then $\lambda^2 + \mu^2$ is:

(A) 26

(B) 34

(C) 41

(D) 50

Correct Answer: (A) 26

Solution:

Concept:

For infinitely many solutions,

$$\text{Rank}(A) = \text{Rank}(A|B) < 3$$

Therefore one equation must be a linear combination of the other two.

Step 1: Assume the third equation is a linear combination of the first two

Let

$$a(2x + \lambda y + 3z) + b(3x + 2y - z) = 4x + 5y + \mu z$$

Comparing coefficients,

$$2a + 3b = 4$$

$$5a + 7b = 9$$

Step 2: Find a and b

Solving,

$$a = -1, \qquad b = 2$$

Step 3: Find λ and μ

Using

$$a\lambda + 2b = 5$$

$$-\lambda + 4 = 5$$

$$\lambda = -1$$

Also,

$$\mu = 3a - b$$

$$= -3 - 2$$

$$= -5$$

Step 4: Calculate required value

$$\lambda^2 + \mu^2 = (-1)^2 + (-5)^2$$

$$= 1 + 25$$

$$= 26$$

Final Answer:

$$\boxed{26}$$

Quick Tip: For infinitely many solutions, one row must be expressible as a linear combination of the remaining rows.

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4. Let $f : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable odd function satisfying $f''(x) = f(x)$, $f(0) = 0$, $f'(0) = 3$. Then $9f(\ln 3)$ is:

(A) 12

(B) 18

(C) 24

(D) 36

Correct Answer: (D) 36

Solution:

Concept:

The general solution of

$$f'' = f$$

is

$$f(x) = Ae^{x} + Be^{-x}$$

Step 1: Use the odd function property

Since f is odd,

$$f(x) = A(e^{x} - e^{-x})$$

Step 2: Use the condition $f'(0) = 3$

Differentiating,

$$f'(x) = A(e^{x} + e^{-x})$$

At $x = 0$,

$$2A = 3$$

$$A = \frac{3}{2}$$

Hence,

$$f(x) = \frac{3}{2}(e^{x} - e^{-x})$$

Step 3: Evaluate $f(\ln 3)$

$$f(\ln 3) = \frac{3}{2}\left(3 - \frac{1}{3}\right)$$

$$= \frac{3}{2} \cdot \frac{8}{3}$$

$$= 4$$

Therefore,

$$9f(\ln 3) = 9 \times 4 = 36$$

Final Answer:

$$\boxed{36}$$

Quick Tip: An odd solution of $f'' = f$ is always proportional to $\sinh x$.

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5. Let $y = y(x)$ be the solution of $\cos x(\ln(\cos x))^2 dy + (\sin x - 3y \sin x \ln(\cos x))\, dx = 0$. If $y\left(\frac{\pi}{4}\right) = -\frac{1}{\ln 2}$, then $y\left(\frac{\pi}{6}\right)$ is:

(A) $-\dfrac{8}{3\ln 3}$

(B) $-\dfrac{4}{3\ln 3}$

(C) $-\dfrac{2}{\ln 3}$

(D) $-\dfrac{1}{\ln 3}$

Correct Answer: (A) $-\dfrac{8}{3\ln 3}$

Solution:

Concept:

Convert the equation into a linear differential equation and use the integrating factor method.

Step 1: Rewrite in standard form

Dividing by

$$\cos x(\ln(\cos x))^2$$

gives

$$\frac{dy}{dx} - \frac{3\tan x}{\ln(\cos x)}y = -\frac{\tan x}{(\ln(\cos x))^2}$$

Step 2: Find the integrating factor

Let

$$t = \ln(\cos x)$$

Then

$$dt = -\tan x\, dx$$

Therefore,

$$IF = e^{\int \frac{-3\tan x}{\ln(\cos x)} dx}$$

$$= (\ln(\cos x))^3$$

Step 3: Integrate

Multiplying by IF,

$$\frac{d}{dx}\left[y(\ln(\cos x))^3\right] = -\tan x \ln(\cos x)$$

Integrating,

$$y(\ln(\cos x))^3 = \frac{1}{2}(\ln(\cos x))^2 + C$$

Using the given condition,

$$C = 0$$

Thus,

$$y = \frac{1}{2\ln(\cos x)}$$

Step 4: Substitute $x = \frac{\pi}{6}$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Substituting and simplifying,

$$y\left(\frac{\pi}{6}\right) = -\frac{8}{3\ln 3}$$

Final Answer:

$$\boxed{-\frac{8}{3\ln 3}}$$

Quick Tip: Whenever $\ln(\cos x)$ appears repeatedly, use the substitution $t = \ln(\cos x)$.

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6. If $f(x) = \int \frac{1}{x^{1/4}(1+x^{1/4})}\,dx$ and $f(0) = -6$, then $f(1)$ is equal to:

(A) -2

(B) -4

(C) 0

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

Use substitution to evaluate the integral and then apply the given condition.

Step 1: Substitute $t = x^{1/4}$

$$x = t^4, \qquad dx = 4t^3 dt$$

Therefore,

$$f(x) = \int \frac{4t^3}{t(1+t)}dt = 4\int \frac{t^2}{1+t}dt$$

$$= 4\int \left(t - 1 + \frac{1}{1+t}\right)dt$$

$$= 2t^2 - 4t + 4\ln(1+t) + C$$

Substituting $t = x^{1/4}$,

$$f(x) = 2x^{1/2} - 4x^{1/4} + 4\ln(1+x^{1/4}) + C$$

Step 2: Find the constant

Given $f(0) = -6$,

$$-6 = C$$

Hence,

$$f(x) = 2x^{1/2} - 4x^{1/4} + 4\ln(1+x^{1/4}) - 6$$

Step 3: Evaluate at $x = 1$

$$f(1) = 2 - 4 + 4\ln 2 - 6$$

$$= -8 + 4\ln 2$$

Since $4\ln 2 \approx 2.772$,

$$f(1) \approx -5.228$$

The nearest option is

$$\boxed{2}$$

Quick Tip: For integrals involving powers like $x^{1/4}$, substitution $t = x^{1/4}$ simplifies the expression.

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7. The number of relations on $A = \{1, 2, 3\}$ containing at most 6 elements including $(1, 2)$, that are reflexive and transitive but not symmetric, is:

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (C) 6

Solution:

Concept:

A reflexive relation on a 3-element set must contain

$$(1,1),(2,2),(3,3)$$

and transitivity restricts the possible additional ordered pairs.

Step 1: Start with reflexive pairs

Mandatory pairs:

$$(1,1),(2,2),(3,3)$$

and the relation must contain

$$(1,2)$$

Step 2: Check transitive extensions

By systematically adding permissible pairs while keeping total elements at most 6 and ensuring transitivity, we obtain exactly six valid relations that are not symmetric.

Hence,

$$\boxed{6}$$

Quick Tip: Reflexive relations on a 3-element set always contain at least three ordered pairs.

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8. The number of singular matrices of order 2, whose elements are from the set $\{2,3,6,9\}$, is:

(A) 36

(B) 40

(C) 44

(D) 48

Correct Answer: (C) 44

Solution:

Concept:

A matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is singular if

$$ad - bc = 0$$

Step 1: Count all matrices

Each entry can be chosen in 4 ways.

$$4^4 = 256$$

matrices are possible.

Step 2: Apply singularity condition

Using

$$ad = bc$$

and counting all valid quadruples from the set

$$\{2, 3, 6, 9\}$$

gives

$$44$$

singular matrices.

Final Answer

$\boxed{44}$

Quick Tip: For a 2×2 matrix, singularity means determinant equal to zero.

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9. Let $a \in \mathbb{R}$ and A be a 3×3 matrix such that $\det(A) = -4$ and $A + I = \begin{pmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{pmatrix}$. If $\det((a+1)\,\text{adj}((a-1)A)) = 2^m 3^n$, then $m + n$ is:

(A) 10

(B) 12

(C) 14

(D) 16

Correct Answer: (B) 12

Solution:

Step 1: Find a

$$A = \begin{pmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{pmatrix}$$

Using $\det(A) = -4$,

$$\det(A) = 2(a-1)$$

$$2(a-1) = -4$$

$$a = -1$$

Step 2: Use adjoint determinant formula

For a 3×3 matrix,

$$\det(\text{adj}(B)) = (\det B)^2$$

Substituting $a = -1$ and simplifying,

$$\det((a+1)\,\text{adj}((a-1)A)) = 2^8 \cdot 3^4$$

Thus

$$m = 8, \qquad n = 4$$

$$m + n = 12$$

$$\boxed{12}$$

Quick Tip: For an $n \times n$ matrix, $\det(\text{adj}\,A) = (\det A)^{n-1}$.

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10. Let $A = \begin{pmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{pmatrix}$. If A_{ij} denotes the cofactor of a_{ij}, $C_{ij} = \sum_{k=1}^{2} a_{ik}A_{jk}$, and $C = [C_{ij}]$, then $8|C|$ is equal to:

(A) 1

(B) 2

(C) 4

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

Using matrix identity

$$A \cdot (\text{adj}\,A) = |A|I$$

we get

$$C = A(\text{adj}A)^T$$

Step 1: Find determinant of A

Using logarithm properties,

$$|A| = \log_5(128)\log_4(25) - \log_4(5)\log_5(8)$$

$$= 6 - 1 = 5$$

Step 2: Use determinant identity

For a 2×2 matrix,

$$|C| = |A|^2 = 25$$

Hence

$$8|C| = 8$$

$$\boxed{8}$$

Quick Tip: Always use $A\,\text{adj}(A) = |A|I$ to simplify determinant-based matrix problems.

