CUET PG 2025 Mathematics Question Paper with Solutions

Time Allowed: 1 Hour 30 Mins Maximum Marks:300 **Total Questions:**75

General Instructions

Read the following instructions very carefully and strictly follow them:

- 1. The examination duration is 90 minutes. Manage your time effectively to attempt all questions within this period.
- 2. The total marks for this examination are 300. Aim to maximize your score by strategically answering each question.
- 3. There are 75 mandatory questions to be attempted in the Agro forestry paper. Ensure that all questions are answered.
- 4. Questions may appear in a shuffled order. Do not assume a fixed sequence and focus on each question as you proceed.
- 5. The marking of answers will be displayed as you answer. Use this feature to monitor your performance and adjust your strategy as needed.
- 6. You may mark questions for review and edit your answers later. Make sure to allocate time for reviewing marked questions before final submission.
- 7. Be aware of the detailed section and sub-section guidelines provided in the exam. Understanding these will aid in effectively navigating the exam.

1. If p is a prime number and a group G is of the order p², then G is:

- (A) trivial
- (B) non-abelian
- (C) non-cyclic
- (D) either cyclic of order p² or isomorphic to the product of two cyclic groups of order p each

2. If
$$S = \lim_{n \to \infty} \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{n^2}\right)$$
, then S is equal to:

- (A) 0
- (B) $\frac{1}{4}$ (C) $\frac{1}{2}$
- (D) 1

3. Let R be the planar region bounded by the lines x=0, y=0 and the curve $x^2+y^2=4$ in the first quadrant. Let C be the boundary of R, oriented counter clockwise. Then, the value of $\oint_C x(1-y)dx+(x^2-y^2)dy$ is equal to:

- (A) 0
- (B) 2
- (C) 4
- (D) 8

4. Let [x] be the greatest integer function, where x is a real number, then $\int_0^1 \int_0^1 \int_0^1 ([x] + [y] + [z]) dx dy dz =$

- (A) 0
- (B) $\frac{1}{3}$
- (C) 1
- (D) 3

5. Let V(F) be a finite dimensional vector space and $T: V \to V$ be a linear transformation. Let R(T) denote the range of T and N(T) denote the null space of T. If $rank(T) = rank(T^2)$, then which of the following are correct?

- A. N(T) = R(T)
- $B. N(T) = N(T^2)$
- C. $N(T) \cap R(T) = \{0\}$
- $D. R(T) = R(T^2)$
- (A) A, B and D only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only

6. Match List-I with List-II and choose the correct option:

LIST-I	LIST-II
A. The solution of an ordinary	I. singular solution
differential equation of order 'n' has	
B. The solution of a differential equation which	II. complete primitive
contains no arbitrary constant is	
C. The solution of a differential equation which	III. 'n' arbitrary constants
is not obtained from the general solution is	
D. The solution of a differential equation	IV. particular solution
containing as many as arbitrary constants	
as the order of a differential equation is	

- (A) A I, B II, C III, D IV
- (B) A I, B III, C II, D IV
- (C) A I, B II, C IV, D III
- (D) A III, B IV, C I, D II
- 7. For the function $f(x,y) = x^3 + y^3 3x 12y + 12$, which of the following are correct:
- A. minima at (1,2)
- B. maxima at (-1,-2)
- C. neither a maxima nor a minima at (1,-2) and (-1,2)
- D. the saddle points are (-1,2) and (1,-2)
- (A) A, B and D only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only

8. Which of the following statement is true:

- (A) Continuous image of a connected set is connected
- (B) The union of two connected sets, having non-empty intersection, may not be a connected set
- (C) The real line \mathbb{R} is not connected
- (D) A non-empty subset X of \mathbb{R} is not connected if X is an interval or a singleton set

9. Match List-I with List-II and choose the correct option:

LIST-I (Function)	LIST-II (Value)
A. $\int_{\gamma} \frac{1}{z-a} dz$, where $\gamma : z-a = r, r > 0$	I. $-4 + 2i\pi$
B. $\int_{\gamma} \frac{z+2}{z} dz$, where $\gamma : z = 2e^{it}, 0 \le t \le \pi$	II. $2i\pi(e^4 - e^2)$
C. $\int_{\gamma} \frac{e^{2z}}{(z-1)(z-2)} dz$, where $\gamma : z = 3$	III. $2i\pi$
D. $\int_{\gamma} \frac{z^2 - z + 1}{2(z - 1)} dz$, where $\gamma : z = 2$	IV. $i\pi$

- (A) A I, B II, C III, D IV
- (B) A I, B III, C II, D IV
- (C) A III, B I, C II, D IV
- (D) A III, B IV, C I, D II
- 10. Consider the following: Let f(z) be a complex valued function defined on a subset $S \subset \mathbb{C}$ of complex numbers. Then which of the following are correct?
- A. The order of a zero of a polynomial equals to the order of its first non-vanishing derivative at that zero of the polynomial
- B. Zeros of non-zero analytic function are isolated
- C. Zeros of f(z) are obtained by equating the numerator to zero if there is no common factor in the numerator and the denominator of f(z)
- D. Limit points of zeros of an analytic function is an isolated essential singularity
- (A) A, B and D only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only
- 11. Which of the following are correct?
- A. A set $S = \{(x,y)|xy \le 1 : x,y \in \mathbb{R}\}$ is a convex set
- **B.** A set $S = \{(x,y)|x^2 + 4y^2 \le 12 : x,y \in \mathbb{R}\}$ is a convex set
- C. A set $S = \{(x,y)|y^2 4x \le 0 : x,y \in \mathbb{R}\}$ is a convex set
- **D.** A set $S = \{(x, y) | x^2 + 4y^2 \ge 12 : x, y \in \mathbb{R} \}$ is a convex set
- (A) B and C only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only
- 12. The value of $\lim_{n\to\infty}(\sqrt{4n^2+n}-2n)$ is:
- (A) $\frac{1}{2}$
- (B) 0

- (C) $\frac{1}{4}$
- (D) 1

13. Match List-I with List-II and choose the correct option:

10. Material Elst 1 with Elst 11 and choose the confect option.	
LIST-I (Set)	LIST-II (Supremum/Infimum)
A. $S = \{2, 3, 5, 10\}$	I. Inf $S = 2$
B. $S = (1, 2] \cup [3, 8)$	II. Sup $S = 5$, Inf $S = -5$
C. $S = \{2, 2^2, 2^3, \dots, 2^n, \dots\}$	III. Sup $S = 10$, Inf $S = 2$
D. $S = \{x \in \mathbb{Z} : x^2 \le 25\}$	IV. Sup $S = 8$, Inf $S = 1$

- (A) A I, B II, C III, D IV
- (B) A I, B III, C II, D IV
- (C) A I, B II, C IV, D III
- (D) A III, B IV, C I, D II

14. Match List-I with List-II and choose the correct option:

LIST-I (Differential Equation)	LIST-II (Integrating Factor)
$\mathbf{A.} \ (y - y^2)dx + xdy = 0$	I. $\tan x$
B. $(xy + y + e^x)dx + (x + e^x)dy = 0$	II. $\frac{1}{x^2y^2}$
dx + g = 0	III. e^x
D. $(2xy^2 + y)dx + (2y^3 - x)dy = 0$	IV. $\frac{1}{y^2}$

- (A) A I, B II, C IV, D III
- (B) A II, B III, C I, D IV
- (C) A I, B II, C III, D IV
- (D) A III, B IV, C I, D II

15. The function $f(z) = |z|^2$ is differentiable, at

- (A) z = 0
- (B) for all $z \in \mathbb{C}$
- (C) no $z \in \mathbb{C}$
- (D) $z \neq 0$

16. If C is the positively oriented circle represented by |z|=2, then $\int_C \frac{e^{2z}}{z-4} dz$ is:

- (A) $\frac{2\pi i}{3}$
- (B) πi

- (C) $\frac{4\pi i}{3}$ (D) $\frac{8\pi i}{3}$

17. Let f be a continuous function on \mathbb{R} and $F(x) = \int_{x-2}^{x+2} f(t)dt$, then F'(x) is

- (A) f(x-2) f(x+2)
- (B) f(x-2)
- (C) f(x+2)
- (D) f(x+2) f(x-2)

18. If C is a triangle with vertices (0,0), (1,0) and (1,1) which are oriented counter clockwise, then $\oint_C 2xydx + (x^2 + 2x)dy$ is equal to:

- (A) $\frac{1}{2}$
- (B) 1
- (C) $\frac{3}{2}$
- (D) 2

19. The integral domain of which cardinality is not possible:

- A. 5
- B. 6
- C. 7
- D. 10
- (A) A and B only
- (B) A and C only
- (C) B and D only
- (D) C and D only

20. Let $m,n\in\mathbb{N}$ such that m< n and $P_{m\times n}(\mathbb{R})$ and $Q_{n\times m}(\mathbb{R})$ are matrices over real numbers and let $\rho(V)$ denotes the rank of the matrix V. Then, which of the following are NOT possible.

- **A.** $\rho(PQ) = n$
- **B.** $\rho(QP)=m$
- C. $\rho(PQ) = m$
- D. $\rho(QP) = \lfloor (m+n)/2 \rfloor$, where $\lfloor \rfloor$ is the greatest integer function

- (A) A and D only
- (B) B and C only
- (C) A, C and D only
- (D) A, B and C only

21. Which of the following are subspaces of vector space \mathbb{R}^3 :

- **A.** $\{(x, y, z) : x + y = 0\}$
- **B.** $\{(x, y, z) : x y = 0\}$
- **C.** $\{(x, y, z) : x + y = 1\}$
- **D.** $\{(x, y, z) : x y = 1\}$
- (A) A and C only
- (B) A, B and C only
- (C) A and B only
- (D) A and D only

22. Maximize Z = 2x + 3y, subject to the constraints:

$$x + y \le 2$$

$$2x + y \le 3$$

$$x, y \ge 0$$

- (A) 5
- (B) 6
- (C) 7
- (D) 10

23. Which one of the following mathematical structure forms a group?

- (A) $(\mathbb{N}, *)$, where a * b = a for all $a, b \in \mathbb{N}$
- (B) $(\mathbb{Z},*)$, where a*b=a-b, for all $a,b\in\mathbb{Z}$
- (C) $(\mathbb{R}, *)$, where a * b = a + b + 1, for all $a, b \in \mathbb{R}$
- (D) $(\mathbb{R}, *)$, where a * b = |a|b, for all $a, b \in \mathbb{R}$

24. If
$$A=\begin{pmatrix}2&4&1\\0&2&-1\\0&0&1\end{pmatrix}$$
 satisfies $A^3+\mu A^2+\lambda A-4I_3=0$, then the respective values of λ and μ are:

- (A) -5, 8
- (B) 8, -5
- (C) 5, -8
- (D) -8, 5

25. Let A and B be two symmetric matrices of same order, then which of the following statement are correct:

- A. AB is symmetric
- B. A+B is symmetric
- C. $A^TB = AB^T$
- **D.** $BA = (AB)^T$
- (A) A, B and D only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only

26. Match List-I with List-II and choose the correct option:

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LIST-I (Infinite Series)	LIST-II (Nature of Series)
A. $12 - 7 - 3 - 2 + 12 - 7 - 3 - 2 + \dots$	I. convergent
B. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$	II. oscillatory
	III. divergent
D. $\sum_{n=1}^{n=0} \frac{1}{n(1+\frac{1}{n})}$	IV. conditionally convergent

- (A) A I, B II, C III, D IV
- (B) A II, B I, C III, D IV
- (C) A II, B IV, C III, D I
- (D) A II, B IV, C I, D III

27. Consider the function $f(x,y) = x^2 + xy^2 + y^4$, then which of the following statement is correct:

- (A) f(x,y) has neither a maxima nor a minima at the origin (0,0)
- (B) f(x,y) has a minimum value at the origin (0,0)
- (C) origin (0, 0) is a saddle point of f(x, y)
- (D) f(x,y) has a maximum value at the origin (0,0)

28. If $f(z) = (x^2 - y^2 - 2xy) + i(x^2 - y^2 + 2xy)$ and f'(z) = cz, where c is a complex

constant, then |c| is equals to:

- (A) $\sqrt{3}$
- (B) $\sqrt{2}$
- (C) $3\sqrt{3}$
- (D) $2\sqrt{2}$

29. Match List-I with List-II and choose the correct option:

20. Widden Eist I with Eist II and choose the correct option.	
LIST-I (Differential)	LIST-II (Order/degree / nature)
A. $\left(y + x \left(\frac{dy}{dx}\right)^2\right)^{5/3} = x \frac{d^2y}{dx^2}$	I. order $= 2$, degree $= 2$, non-linear
$\mathbf{B.} \left(\frac{d^2y}{dx^2}\right)^{1/3} = \left(y + \frac{dy}{dx}\right)^{1/2}$	II. order $= 1$, degree $= 1$, linear
$\mathbf{C.} \ y = x \frac{dy}{dx} + \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{1/2}$	III. order $= 2$, degree $= 3$, non-linear
D. $(2+x^3)\frac{dy}{dx} = (e^{\sin x})^{1/2} + y$	IV. order $= 1$, degree $= 2$, non-linear

- (A) A III, B I, C II, D IV
- (B) A I, B III, C II, D IV
- (C) A III, B I, C IV, D II
- (D) A III, B IV, C I, D II

30. The solution of the differential equation $\frac{xdy-ydx}{xdx+ydy} = \sqrt{x^2+y^2}$ is:

- (A) $\frac{x}{y} = \sin^{-1} \sqrt{1 x^2} + C$; where C is a constant (B) $\sqrt{x^2 + y^2} = \tan^{-1} \frac{y}{x} + C$; where C is a constant (C) $1 + x^2 = \tan^{-1}(y) + C$; where C is a constant

- (D) $y = x \tan(\sqrt{x^2 + y^2}) + C$; where C is a constant

31. If, $I_n = \int_{-\pi}^{\pi} \frac{\cos(nx)}{1+2^x} dx, n = 0, 1, 2, \ldots$, then which of the following are correct: A. $I_n = I_{n+2}$, for all $n = 0, 1, 2, \ldots$

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- B. $I_n = 0$, for all n = 0, 1, 2, ...C. $\sum_{n=1}^{10} I_n = 2^{10}$ D. $\sum_{n=1}^{10} I_n = 0$

- (A) A, B and D only
- (B) A and C only
- (C) B and D only
- (D) A, C and D only

32. The number of maximum basic feasible solution of the system of equations AX = b, where A is m × n matrix, b is n × 1 column matrix and rank of A is $\rho(A) = m$, is:

- (A) m+n
- (B) m-n
- (C) mn
- (D) nC_m

33. If $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$, for $(x, y, z) \in \mathbb{R}^3$, then $\oiint_S \vec{F} \cdot d\vec{S}$, where S is the surface of the cube formed by $x = \pm 1, y = \pm 1, z = \pm 1$, is

- (A) 24
- (B) 6
- (C) 1
- (D) 0

34. Find the residue of $(67 + 89 + 90 + 87) \pmod{11}$

- (A) 3
- (B) 0
- (C) 2
- (D) 1

35. The solution of the differential equation $(xy^3 + y)dx + (2x^2y^2 + 2x + 2y^4)dy = 0$ is:

- (A) $3xy^2 + 6y^4x 2y^6 + C$, where C is an arbitrary constant
- (B) $3xy^4 + 3xy^2 + y^6 + C$, where C is an arbitrary constant
- (C) $6xy^2 2y^4x + C$, where C is an arbitrary constant
- (D) $3x^2y^4 + 6xy^2 + 2y^6 + C$, where C is an arbitrary constant

36. If G is a cyclic group of order 12, then the order of Aut(G) is:

- (A) 1
- (B) 5
- (C) 4

(D) 77

37. Which of the following function is discontinuous at every point of \mathbb{R} ?

(A)
$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$$
(B) $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$
(C) $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 2x, & \text{if } x \text{ is irrational} \end{cases}$
(D) $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational} \end{cases}$

(B)
$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

(C)
$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 2x, & \text{if } x \text{ is irrational} \end{cases}$$

(D)
$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational} \end{cases}$$

38. If the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, $\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$ are linearly dependent, then the value of p is:

- (A) 2
- (B) 4
- (C) 1
- (D) 6

39. If \vec{F} is a vector point function and ϕ is a scalar point function, then match List-I with List-II and choose the correct option:

LIST-I	LIST-II
A. div (grad ϕ)	I. $\frac{1}{2}\nabla F^2 - (\vec{F}\cdot\nabla)\vec{F}$
B. curl (grad ϕ)	II. grad(div \vec{F}) – $ abla^2 \vec{F}$
C. $\vec{F} \times \text{curl } \vec{F}$	III. $\vec{0}$
D. curl (curl \vec{F})	IV. $\nabla \cdot \nabla \phi$

- (A) A-I, B-II, C-III, D-IV
- (B) A-IV, B-III, C-II, D-I
- (C) A-I, B-II, C-IV, D-III
- (D) A-IV, B-III, C-I, D-II

40. The value of integral $\oint_C \frac{z^3-z}{(z-z_0)^3}dz$, where z_0 is outside the closed curve C described in the positive sense, is

- (A) 1
- (B) 0
- (C) $-\frac{8\pi i}{3}e^{-2}$ (D) $\frac{2\pi i}{3}e^{2}$

41. The solution of the differential equation $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$, is

- (A) $x^3 + 6x^2y 6xy^2 y^3 + C = 0$
- (B) $x^3 6x^2y 6xy^2 + y^3 + C = 0$
- (C) $x^3 6x^2y 6xy^2 y^3 + C = 0$ (D) $x^3 + 6x^2y + 6xy^2 + y^3 + C = 0$

42. If $x \in \mathbb{R}$ and a particular integral (P.I.) of $(D^2 - 2D + 4)y = e^x \sin x$ is $\frac{1}{2}e^x f(x)$, then f(x) is:

- (A) an increasing function on $[0, \pi]$
- (B) a decreasing function on $[0, \pi]$
- (C) a continuous function on $[-2\pi, 2\pi]$
- (D) not differentiable function at x = 0

43. The value of $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ is:

- (A) $\frac{\pi^2}{2}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{8}$

44. The orthogonal trajectory of the cardioid $r = a(1 - \cos \theta)$, where 'a' is an arbitrary constant is:

- (A) $r = b(1 + \cos \theta)$, where b is an arbitrary constant
- (B) $r = b(1 \cos \theta)$, where b is an arbitrary constant
- (C) $r = b(1 + \sin \theta)$, where b is an arbitrary constant
- (D) $r = b(1 \sin \theta)$, where b is an arbitrary constant

45. If \vec{F} be the force and C is a non-closed arc, then $\int_C \vec{F} \cdot d\vec{r}$ represents:

- (A) Flux
- (B) Circulation
- (C) Work done
- (D) Conservative field

46. In Green's theorem, $\oint_C (x^2ydx + x^2dy) = \iint_R f(x,y)dxdy$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1) and R is the region bounded by a simple closed curve C in the x-y plane, then f(x,y)is equal to:

- (A) $x x^2$
- (B) $2x x^2$
- $(C) y x^2$
- (D) $2y x^2$

47. The value of $\int_0^{1+i} (x^2 - iy) dz$, along the path $y = x^2$ is:

- $\begin{array}{l} ({\rm A}) \ \frac{5}{6} \frac{1}{6}i \\ ({\rm B}) \ \frac{5}{6} + \frac{1}{6}i \\ ({\rm C}) \ \frac{1}{6} \frac{5}{6}i \\ ({\rm D}) \ \frac{1}{6} + \frac{5}{6}i \end{array}$

48. Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined as $f(x,y) = \begin{cases} \frac{x}{\sqrt{x^2 + y^2}} & ; (x,y) \neq (0,0) \\ 1 & ; (x,y) = (0,0) \end{cases}$, then which of the following statement is true?

- (A) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist
- (B) f(x,y) is continuous but not differentiable
- (C) f(x,y) is differentiable function
- (D) f(x,y) have removable discontinuity

49. The system of linear equations x + y + z = 6, x + 2y + 5z = 10, $2x + 3y + \lambda z = \mu$ has a unique solution, if

- (A) $\lambda \neq 16, \mu = 6$
- (B) $\lambda = 6, \mu = 16$
- (C) $\lambda = 6, \mu \neq 16$
- (D) $\lambda \neq 6, \mu \in \mathbb{R}$

50. For the given linear programming problem,

Minimum Z = 6x + 10y

subject to the constraints

$$x \ge 6$$
; $y \ge 2$; $2x + y \ge 10$; $x, y \ge 0$,

the redundant constraints are:

- (A) $x \ge 6, 2x + y \ge 10$
- (B) $2x + y \ge 10$
- (C) $x \ge 6, y \ge 2, x \ge 0, y \ge 0$
- (D) $y \ge 2, x \ge 0$
- 51. Which of the following statements are true for group of permutations?
- A. Every permutation of a finite set can be written as a cycle or a product of disjoint cycles
- B. The order of a permutation of a finite set written in a disjoint cycle form is the least common multiple of the lengths of the cycles
- C. If A_n is a group of even permutation of n-symbol (n > 1), then the order of A_n is n!
- D. The pair of disjoint cycles commute
- (A) A, B and D only
- (B) A, B and C only
- (C) A, B, C and D
- (D) B, C and D only
- 52. A ring (R, +, .), where all elements are idempotent is always:
- (A) a commutative ring
- (B) not an integral domain
- (C) a field
- (D) an integral domain with unity

53. If $v = \sin^{-1}\left(\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right)$, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}$ is equal to:

- (A) $\frac{12}{\tan v}$ (B) $\frac{1}{12} \tan v$ (C) $-\frac{1}{12} \tan v$ (D) $\frac{-12}{\tan v}$

54. Let D be the region bounded by a closed cylinder $x^2 + y^2 = 16$, z = 0 and z = 4, then the value of $\iiint_D (\nabla \cdot \vec{v}) dV$, where $\vec{v} = 3x^2 \hat{i} + 6y^2 \hat{j} + z \hat{k}$, is:

- (A) 64π
- (B) 128π
- (C) $\frac{64\pi}{3}$
- (D) 48π

55. The value of the double integral $\iint_R e^{x^2} dx dy$, where R is a region given by $2y \le x \le 2$ and $0 \le y \le 1$, is:

- (A) $(e^4 1)$
- (B) $\frac{1}{4}(e^4 1)$ (C) $\frac{1}{4}(e^4 + 1)$ (D) $\frac{1}{2}(e^4 1)$

56. Let A be a 2×2 matrix with det(A) = 4 and trace(A) = 5. Then the value of $trace(A^2)$ is:

- (A) 10
- (B) 13
- (C) 17
- (D) 18

57. A complete solution of $y'' + a_1y' + a_2y = 0$ is $y = b_1e^{-x} + b_2e^{-3x}$, where a_1, a_2, b_1 and b_2 are constants, then the respective values of a_1 and a_2 are:

- (A) 3, 3
- (B) 3, 4
- (C) 4, 3

(D) 4, 4

58. If 'a' is an imaginary cube root of unity, then $(1-a+a^2)^5+(1+a-a^2)^5$ is equal to:

- (A) 4
- (B) 5
- (C) 32
- (D) 16

59. The solution of the differential equation $xdy - ydx = (x^2 + y^2)dx$, is

- (A) $y = \tan(x + c)$; where c is an arbitrary constant
- (B) $x = y \tan(x + c)$; where c is an arbitrary constant
- (C) $y = x \tan^{-1}(y+c)$; where c is an arbitrary constant
- (D) $y = x \tan(x + c)$; where c is an arbitrary constant

60. The value of $\iiint_E \frac{dx\,dy\,dz}{x^2+y^2+z^2}$, where E: $x^2+y^2+z^2\leq a^2$, is

- (A) πa
- (B) $2\pi a$
- (C) $4\pi a$
- (D) $8\pi a$

61. The given series $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots (p > 0)$ is conditionally convergent, if 'p' lies in the interval:

- (A) (0,1]
- (B) [0,1]
- (C) $(1, \infty)$
- (D) $[1,\infty)$

62. Which of the following set of vectors forms the basis for \mathbb{R}^3 ?

- (A) $S = \{(1, 1, 1), (1, 0, 1)\}$
- (B) $S = \{(1, 1, 1), (1, 2, 3), (2, -1, 1)\}$

(C)
$$S = \{(1, 2, 3), (1, 3, 5), (1, 0, 1), (2, 3, 0)\}$$

(D)
$$S = \{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$$

63. If p is a prime number and O(G) denotes the order of a group G and p divides O(G), then group G has an element of order p. Then, this is a statement of

- (A) Lagrange's Theorem
- (B) Sylow's Theorem
- (C) Euler's Theorem
- (D) Cauchy's Theorem

64. If U and W are distinct 4-dimensional subspaces of a vector space V of dimension 6, then the possible dimensions of $U \cap W$ is:

- (A) 1 or 2
- (B) exactly 4
- (C) 3 or 4
- (D) 2 or 3

65. Which of the following forms a linear transformation:

- (A) $T: \mathbb{R}^2 \to \mathbb{R}, T(x,y) = xy$
- (B) $T: \mathbb{R}^2 \to \mathbb{R}^3, T(x,y) = (x+1,2y,x+y)$
- (C) $T: \mathbb{R}^3 \to \mathbb{R}^2, T(x, y, z) = (|x|, 0)$
- (D) $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x + y, x)$

66. Let f(x) = |x| + |x-1| + |x+1| be a function defined on \mathbb{R} , then f(x) is:

- (A) differentiable for all $x \in \mathbb{R}$
- (B) differentiable for all $x \in \mathbb{R}$ other than x = -1, 0, 1
- (C) differentiable only for x = -1, 0, 1
- (D) not differentiable at any real point

67. Let f(x) be a real valued function defined for all $x \in \mathbb{R}$, such that $|f(x) - f(y)| \le (x - y)^2$, for all $x, y \in \mathbb{R}$, then

- (A) f(x) is nowhere differentiable
- (B) f(x) is a constant function
- (C) f(x) is strictly increasing function in the interval [0,1]
- (D) f(x) is strictly increasing function for all $x \in \mathbb{R}$

68. The value of the integral $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$ is:

- (A) 1
- (B) $\frac{3}{2}$
- (C) 0
- (D) $\frac{1}{2}$

69. If, $u = y^3 - 3x^2y$ be a harmonic function then its corresponding analytic function f(z), where z = x + iy, is:

- (A) $f(z) = z^2 + C$; where C is an arbitrary constant
- (B) $f(z) = i(z^2 + C)$; where C is an arbitrary constant
- (C) $f(z) = z^3 + C$; where C is an arbitrary constant
- (D) $f(z) = i(z^3 + C)$; where C is an arbitrary constant

70. The value of v_3 for which the vector $\vec{v} = e^y \sin x \hat{i} + e^y \cos x \hat{j} + v_3 \hat{k}$ is solenoidal, is:

- (A) $2ze^y \cos x$
- $(B) -2ze^y \cos x$
- (C) $-2e^y \cos x$
- (D) $2e^y \sin x$

71. The value of the integral $\iint_R (x+y) dy dx$ in the region R bounded by x=0, x=2, y=x, y=x+2, is

- (A) 3
- (B) 8
- (C) 12
- (D) 16

- 72. For any Linear Programming Problem (LPP), choose the correct statement:
- A. There exists only finite number of basic feasible solutions to LPP
- B. Any convex combination of k different optimum solution to a LPP is again an optimum solution to the problem
- C. If a LPP has feasible solution, then it also has a basic feasible solution
- D. A basic solution to AX = b is degenerate if one or more of the basic variables vanish
- (A) A, B and C only
- (B) A, C and D only
- (C) A, B and D only
- (D) A, B, C and D

73. Which of the following are correct:

- A. Every infinite bounded set of real number has a limit point
- B. The set $S = \{x : 0 < x \le 1, x \in \mathbb{R}\}$ is a closed set
- C. The set of whole real numbers is open as well closed set
- **D.** The set $S = \{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \dots\}$ is neither open set nor closed set
- (A) A, B and C Only
- (B) A, C and D Only
- (C) B, C and D Only
- (D) D Only

74. Match List-I with List-II and choose the correct option:

LIST-I (Function)	LIST-II (Expansion)
A. $\log(1-x)$	I. $1 + \frac{1}{3} + \frac{1}{6} + \frac{3}{40} + \frac{15}{336} + \dots$
B. $\sin^{-1} x$	II. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$
C. log 2	III. $x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots, -1 < x \le 1$
D. $\frac{\pi}{2}$	IV. $-x - \frac{x^2}{2} - \frac{x^3}{3} - \dots, -1 \le x < 1$

- (A) A-IV, B-III, C-II, D-I
- (B) A-III, B-IV, C-I, D-II
- (C) A-III, B-IV, C-II, D-I
- (D) A-I, B-II, C-III, D-IV

75. The locus of point z which satisfies $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is:

(A)
$$x^2 + y^2 - 2y + 1 = 0$$

(B)
$$3x^2 + 3y^2 + 10x + 3 \ge 0$$

(C)
$$3x^2 + 3y^2 + 10x + 3 = 0$$

(C)
$$3x^2 + 3y^2 + 10x + 3 = 0$$

(D) $x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$