CUET PG 2026 Mathematics Question Paper with Solutions(Memory Based)

Time Allowed :1 Hours 30 min	**Maximum Marks :**300	**Total Questions :**75

General Instructions

1. The exam lasts 90 minutes (1 hour 30 minutes).
2. There are 75 Multiple Choice Questions (MCQs) to be answered.
3. +4 marks for every correct answer. -1 mark (negative marking) for every incorrect answer. 0 marks for unanswered or un-attempted questions.
4. For any discrepancy in questions, the English version is considered final (except for language-specific papers).
5. Click one of the four options to choose an answer.
6. You must click "Save & Next" to confirm your response. Only saved answers are considered for evaluation.
7. Use "Mark for Review & Next" to flag a question for later. You can unselect or change your answer using the "Clear Response" button.
8. All calculations must be done on the Rough Sheets provided at the centre. These must be returned to the invigilator after the exam.

1. What is the dimension of the vector space of all $n \times n$ real symmetric matrices?
(A) n^2
(B) $\frac{n(n+1)}{2}$
(C) $\frac{n(n-1)}{2}$
(D) $2n$

Correct Answer: (B) $\frac{n(n+1)}{2}$

Solution:
Concept: A real symmetric matrix satisfies:

$$A^T = A$$

This means the entries are symmetric about the main diagonal.
Step 1: Count diagonal elements
There are n diagonal elements:

$$a_{11}, a_{22}, \dots, a_{nn}$$

Step 2: Count off-diagonal elements
For $i \neq j$, we have:

$$a_{ij} = a_{ji}$$

So each pair contributes only one independent element.

Number of such pairs:

$$\frac{n(n-1)}{2}$$

Step 3: Total independent elements

$$n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$$

Hence, the dimension is:

$$\frac{n(n+1)}{2}$$

Quick Tip

Symmetric matrix dimension = diagonal + upper triangle
$= n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$

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2. If a function $f(x)$ is continuous on a closed interval $[a, b]$, is it necessarily uniformly continuous?
(A) Yes, always uniformly continuous
(B) No, never uniformly continuous
(C) Only if differentiable
(D) Only if bounded

Correct Answer: (A) Yes, always uniformly continuous

Solution:

Concept: Heine–Cantor Theorem
A very important result in real analysis states:

- If a function is continuous on a **closed and bounded interval** $[a, b]$,
- Then it is **uniformly continuous** on that interval.

Step 1: Understand continuity vs uniform continuity

- **Continuity:** For every point x, we can find a δ depending on x.
- **Uniform continuity:** A single δ works for all $x \in [a, b]$.

Step 2: Apply the theorem
Since the function is continuous on a **closed interval** $[a, b]$, it satisfies:

$$\text{Uniform continuity on } [a, b]$$

Step 3: Why closed interval matters
Closed intervals are:

- Bounded

- Contain all limit points

This ensures no "escape" of function behavior at endpoints.

Conclusion:

The function is always uniformly continuous.

> **Quick Tip**
>
> Continuous on closed interval $\Rightarrow$ Uniformly continuous (Always true!)

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3. What is the value of the limit $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$?
(A) 1
(B) 0
(C) e
(D) ∞

Correct Answer: (C) e

Solution:

Concept: Definition of e
The number e is defined as:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Step 1: Recognize the standard limit
The given expression is a direct standard limit used to define e.
Step 2: Intuition behind the limit
This expression arises in:

- Compound interest problems
- Continuous growth models

As n increases, the quantity:

$$\left(1+\frac{1}{n}\right)^n$$

approaches a fixed number.
Step 3: Numerical idea

$$n = 1 \Rightarrow 2, \quad n = 2 \Rightarrow 2.25, \quad n = 10 \Rightarrow 2.593, \quad n \to \infty \Rightarrow 2.718...$$

Step 4: Final value

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \approx 2.718$$

Conclusion:

Limit value is e

Quick Tip

Always remember:

$$\left(1+\frac{1}{n}\right)^n \to e$$

A standard and very important limit in calculus.

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4. How many elements of order 5 are there in a cyclic group of order 25?
(A) 1
(B) 4
(C) 5
(D) 10

Correct Answer: (B) 4

Solution:

Concept: Order of elements in a cyclic group
In a cyclic group of order n, the number of elements of order d (where $d \mid n$) is given by Euler's totient function:

$$\phi(d)$$

Step 1: Identify the group order

$$n = 25 = 5^2$$

Step 2: Find valid divisors
Possible orders divide 25:

$$1, 5, 25$$

We need elements of order 5.
Step 3: Apply Euler's totient function

$$\phi(5) = 5 - 1 = 4$$

Step 4: Interpretation
There are exactly 4 generators of the subgroup of order 5.
Conclusion:

$$\text{Number of elements of order } 5 = 4$$

Quick Tip

In cyclic groups: Number of elements of order $d = \phi(d)$

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5. If A is a 3×3 matrix with eigenvalues $1, 2, 3$, what is the determinant of A^2?

(A) 6
(B) 12
(C) 18
(D) 36

Correct Answer: (D) 36

Solution:

Concept 1: Determinant and eigenvalues
The determinant of a matrix is equal to the product of its eigenvalues:

$$\det(A) = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$$

Step 1: Compute determinant of A
Given eigenvalues:

$$1, 2, 3$$

$$\det(A) = 1 \times 2 \times 3 = 6$$

Concept 2: Determinant of powers
For any square matrix:

$$\det(A^k) = (\det A)^k$$

Step 2: Apply the formula for A^2

$$\det(A^2) = (\det A)^2 = 6^2 = 36$$

Step 3: Alternative understanding
Eigenvalues of A^2 are:

$$1^2, 2^2, 3^2 = 1, 4, 9$$

$$\det(A^2) = 1 \times 4 \times 9 = 36$$

Conclusion:

$$\det(A^2) = 36$$

Quick Tip

Two useful results:

- Product of eigenvalues = determinant
- $\det(A^k) = (\det A)^k$

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6. Which theorem states that every bounded sequence in $\mathbb{R}^n$ has a convergent subsequence?
(A) Mean Value Theorem
(B) Bolzano–Weierstrass Theorem
(C) Rolle's Theorem
(D) Taylor's Theorem

Correct Answer: (B) Bolzano–Weierstrass Theorem

Solution:

Concept: Bolzano–Weierstrass Theorem
This is a fundamental theorem in real analysis which states:

- Every bounded sequence in $\mathbb{R}^n$ has at least one convergent subsequence.

Step 1: Understand bounded sequence
A sequence $\{x_n\}$ is bounded if:

$$\exists M > 0 \text{ such that } \|x_n\| \leq M \text{ for all } n$$

Step 2: Apply the theorem
If the sequence is bounded, then it cannot "escape to infinity," so there exists a subsequence that converges.
Step 3: Conclusion

$$\text{Bolzano–Weierstrass Theorem guarantees convergence of a subsequence}$$

Quick Tip

Bounded sequence $\Rightarrow$ Convergent subsequence (always!)

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7. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?
(A) 0
(B) 1
(C) ∞
(D) e

Correct Answer: (C) ∞

Solution:

Concept: Radius of convergence using Ratio Test
For a power series:

$$\sum a_n x^n$$

the radius of convergence R is found using:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Step 1: Identify coefficients

$$a_n = \frac{1}{n!}$$

Step 2: Apply Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{(n+1)!} \cdot n! = \frac{1}{n+1}$$

Step 3: Take the limit

$$\lim_{n\to\infty} \frac{1}{n+1} = 0$$

Step 4: Interpretation
Since the limit is 0 for all x, the series converges for all real x.
Conclusion:

$$R = \infty$$

> **Quick Tip**
>
> $\sum \frac{x^n}{n!} = e^x$
> Exponential series converges for all real numbers x

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8. Is the set of all rational numbers $\mathbb{Q}$ a countable or uncountable set?
(A) Finite set
(B) Countable set
(C) Uncountable set
(D) Empty set

Correct Answer: (B) Countable set

Solution:

Concept: Countability of sets
A set is called:

- **Countable** if its elements can be put in one-to-one correspondence with $\mathbb{N}$
- **Uncountable** if this is not possible

Step 1: Form of rational numbers
Every rational number can be written as:

$$\frac{p}{q}, \quad p \in \mathbb{Z},\ q \in \mathbb{N}$$

Step 2: Arrangement idea
We can arrange all such fractions in a grid and traverse them diagonally (Cantor's method), ensuring every rational number is listed.
Step 3: Conclusion
Since all rational numbers can be listed in a sequence:

$$\mathbb{Q} \text{ is countable}$$

Quick Tip

Even though rationals are infinite, they are still countable!

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9. If $T : V \to W$ is a linear transformation, what is the relationship between rank(T), nullity(T), and dim(V)?
(A) $\text{rank}(T) + \text{nullity}(T) = \dim(W)$
(B) $\text{rank}(T) \times \text{nullity}(T) = \dim(V)$
(C) $\text{rank}(T) + \text{nullity}(T) = \dim(V)$
(D) $\text{rank}(T) = \text{nullity}(T)$

Correct Answer: (C) $\text{rank}(T) + \text{nullity}(T) = \dim(V)$

Solution:

Concept: Rank–Nullity Theorem
This is a fundamental theorem in linear algebra which states:

$$\text{rank}(T) + \text{nullity}(T) = \dim(V)$$

Step 1: Understand terms

- **Rank**: Dimension of the image (range) of T
- **Nullity**: Dimension of the kernel (null space) of T
- **dim(V)**: Dimension of domain space

Step 2: Interpretation
The theorem splits the domain space into:

- Part mapped to zero (kernel)
- Part mapped to image

Step 3: Conclusion

$$\text{rank}(T) + \text{nullity}(T) = \dim(V)$$

Quick Tip

Always remember:

$$\text{Rank} + \text{Nullity} = \text{Dimension of domain}$$

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10. What is the condition for a group G to be Abelian based on the commutator subgroup?

(A) Commutator subgroup is equal to G
(B) Commutator subgroup is trivial
(C) Commutator subgroup is infinite
(D) Commutator subgroup is cyclic

Correct Answer: (B) Commutator subgroup is trivial

Solution:

Concept: Commutator Subgroup
The commutator of two elements $a, b \in G$ is:

$$[a, b] = aba^{-1}b^{-1}$$

The commutator subgroup G' (or $[G, G]$) is generated by all such commutators.
Step 1: Abelian group definition
A group is Abelian if:

$$ab = ba \quad \forall a, b \in G$$

Step 2: Effect on commutators
If $ab = ba$, then:

$$[a, b] = e$$

(identity element)
Step 3: Conclusion
All commutators are identity $\Rightarrow$ commutator subgroup is trivial:

$$G' = \{e\}$$

Quick Tip

Group is Abelian $\Leftrightarrow$ Commutator subgroup is trivial

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11. What is the value of the integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx$?
(A) 0
(B) 1
(C) $\sqrt{\pi}$
(D) π

Correct Answer: (C) $\sqrt{\pi}$

Solution:

Concept: Gaussian Integral
The integral:

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx$$

is a standard result known as the Gaussian integral.
Step 1: Define the integral
Let:

$$I = \int_{-\infty}^{\infty} e^{-x^2}\,dx$$

Step 2: Square the integral

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$

Step 3: Convert to polar coordinates

$$x^2 + y^2 = r^2, \quad dx\,dy = r\,dr\,d\theta$$

$$I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\,dr\,d\theta$$

Step 4: Evaluate the integral

$$\int_0^{\infty} e^{-r^2} r\,dr = \frac{1}{2}$$

$$I^2 = 2\pi \cdot \frac{1}{2} = \pi$$

Step 5: Final result

$$I = \sqrt{\pi}$$

Quick Tip

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

A very important standard result in probability and analysis.

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12. In a metric space, is every Cauchy sequence necessarily a convergent sequence?
(A) Yes, always
(B) No, not always
(C) Only in finite spaces
(D) Only for bounded sequences

Correct Answer: (B) No, not always

Solution:

Concept: Cauchy sequence and completeness

- A sequence is **Cauchy** if its terms get arbitrarily close to each other.
- A sequence **converges** if it approaches a limit in the space.

Step 1: Key idea
In general metric spaces:

$$\text{Cauchy sequence} \not\Rightarrow \text{convergent}$$

Step 2: Special case
If the space is **complete**, then:

$$\text{Every Cauchy sequence converges}$$

Step 3: Example
In $\mathbb{Q}$, a Cauchy sequence may converge to an irrational number, which is not in $\mathbb{Q}$, so it does not converge in that space.
Conclusion:

$$\text{Not every Cauchy sequence is convergent (unless space is complete)}$$

> Quick Tip
>
> Cauchy $\Rightarrow$ Convergent only in complete spaces

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13. What are the possible values for the rank of a 4×3 matrix?
(A) $0, 1, 2, 3, 4$
(B) $1, 2, 3, 4$
(C) $0, 1, 2, 3$
(D) Only 3

Correct Answer: (C) $0, 1, 2, 3$

Solution:

Concept: Rank of a matrix
The rank of a matrix is the maximum number of linearly independent rows or columns.
Step 1: General rule
For an $m \times n$ matrix:

$$\text{rank} \leq \min(m, n)$$

Step 2: Apply to given matrix
Matrix size:

$$4 \times 3$$

$$\min(4, 3) = 3$$

Step 3: Possible values

Rank can be any integer from 0 up to 3:

$$0, 1, 2, 3$$

Step 4: Interpretation

- Rank 0: zero matrix
- Rank 3: full column rank

Conclusion:

$$\text{Possible ranks} = 0, 1, 2, 3$$

> Quick Tip
>
> Rank of $m \times n$ matrix $\leq \min(m, n)$

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14. What is the order of the group of permutations S_3?

(A) 3
(B) 6
(C) 9
(D) 12

Correct Answer: (B) 6

Solution:

Concept: Symmetric Group

The symmetric group S_n consists of all permutations of n elements.

Step 1: Formula for order

$$|S_n| = n!$$

Step 2: Apply for S_3

$$|S_3| = 3! = 3 \times 2 \times 1 = 6$$

Step 3: Interpretation

There are 6 possible ways to arrange 3 distinct elements.

Conclusion:

$$|S_3| = 6$$

> Quick Tip
>
> Order of symmetric group $S_n = n!$

15. Which partial differential equation represents the Laplace equation in two dimensions?

(A) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$
(B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
(C) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
(D) $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$

Correct Answer: (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Solution:

Concept: Laplace Equation
The Laplace equation is a second-order partial differential equation widely used in physics and engineering.
Step 1: Standard form in two dimensions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Step 2: Interpretation
It describes:

- Steady-state heat distribution
- Electrostatic potential
- Fluid flow

Step 3: Identify correct option
Option (B) matches the standard Laplace equation.
Conclusion:

$$\text{Laplace equation in 2D is } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Quick Tip

Laplace equation = sum of second partial derivatives equals zero