CUET UG Applied Mathematics Sample Paper - 11

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. A and B together can complete a work in 12 days. A alone can do it in 20 days. If B works only for half a day daily, in how many days will A and B together complete the work?

(A) 10 days

(B) 11 days

(C) 15 days

(D) 20 days

Q2. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then the value of $A^2 - 6A + 5I$ is:

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Q3. The area of the region bounded by $y^2 = 4x$ and the line $x = 3$ is:

(A) $4\sqrt{3}$ sq. units

(B) $8\sqrt{3}$ sq. units

(C) $16\sqrt{3}$ sq. units

(D) 12 sq. units

Q4. If the probability of a defective bolt is 0.1, find the mean and standard deviation for a distribution of 400 bolts.

(A) 40, 6

(B) 40, 36

(C) 40, 6.32

(D) 10, 3

Q5. An annuity in which payments continue forever is called:

(A) Annuity Due

(B) Ordinary Annuity

(C) Perpetuity

(D) Deferred Annuity

Q6. Find the value of $3^{50} \pmod 7$.

(A) 1

(B) 2

(C) 4

(D) 5

Q7. The cost function is $C(x) = 100 + 5x + 0.1x^2$. The average cost is minimum when x is:

(A) 10

(B) $\sqrt{1000}$

(C) 100

(D) 31.62

Q8. A man rows 750 m in 675 seconds against the stream and returns in $7\frac{1}{2}$ minutes. His speed in still water is:

(A) 3 km/h

(B) 4 km/h

(C) 5 km/h

(D) 6 km/h

Q9. In a normal distribution, the relation between Mean Deviation (MD) and Standard Deviation (σ) is:

(A) $MD = \frac{4}{5}\sigma$

(B) $MD = \frac{2}{3}\sigma$

(C) $MD = \sigma$

(D) $MD = 1.25\sigma$

Q10. A person invests ₹ 5,000 every year in a savings scheme at 10% p.a. compound interest. The amount standing to his credit at the end of 3 years is:

(A) ₹ 16,500

(B) ₹ 16,550

(C) ₹ 15,000

(D) ₹ 17,000

Q11. The elasticity of demand η_d for the demand function $x = 20 - 2p$ at $p = 5$ is:

(A) 1

(B) 0.5

(C) 2

(D) 1.5

Q12. The integrating factor of $\frac{dy}{dx} + \frac{2}{x}y = x$ is:

(A) x^2

(B) $2\ln x$

(C) e^x

(D) $1/x^2$

Q13. Sinking fund is a fund created for:

(A) Daily expenses

(B) Repayment of a long-term liability

(C) Paying dividends

(D) Increasing sales

Q14. For a binomial distribution with $n = 10$ and $p = 0.5$, the distribution is:

(A) Positively skewed

(B) Negatively skewed

(C) Symmetric

(D) Leptokurtic

Q15. In an LPP, the constraints $x + y \leq 5, x \geq 0, y \geq 0$ define a region in the shape of a:

(A) Square

(B) Rectangle

(C) Triangle

(D) Circle

Q16. A time series pattern that repeats itself every year is:

(A) Trend
(B) Seasonal
(C) Cyclical
(D) Irregular

Q17. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A
(B) $I - A$
(C) I
(D) $3A$

Q18. Two pipes can fill a cistern in 12 min and 15 min respectively. A third pipe can empty it in 6 min. If the first two are kept open for 5 min and then the third is also opened, the time taken to empty the cistern is:

(A) 30 min
(B) 33 min
(C) 45 min
(D) 15 min

Q19. $\int \frac{dx}{\sqrt{a^2-x^2}}$ is equal to:

(A) $\sin^{-1}(x/a) + C$
(B) $\frac{1}{a}\sin^{-1}(x/a) + C$
(C) $\cos^{-1}(x/a) + C$
(D) $\ln|x + \sqrt{a^2 - x^2}| + C$

Q20. The nominal rate of interest is 12% p.a. If compounding is done quarterly, the effective rate of interest is:

(A) 12%

(B) 12.55%

(C) 12.68%

(D) 13%

Q21. A "Type I Error" occurs when:

(A) We accept a false null hypothesis

(B) We reject a true null hypothesis

(C) We reject a false null hypothesis

(D) We accept a true null hypothesis

Q22. A boatman goes 2 km against the current of the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water?

(A) 40 min

(B) 1 hour

(C) 1 hr 15 min

(D) 1 hr 30 min

Q23. The total area under the standard normal curve is:

(A) 0

(B) 0.5

(C) 1

(D) ∞

Q24. EMI stands for:

(A) Every Month Installment

(B) Equated Monthly Installment

(C) Equal Money Income

(D) External Money Interest

Q25. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y$ is:

(A) 3

(B) 2

(C) 4

(D) 1

Q26. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $(a + c) \equiv (b + d) \pmod{m}$ is:

(A) Always true

(B) Only true if m is prime

(C) Always false

(D) True only for addition, not multiplication

Q27. Any point in the feasible region that maximizes or minimizes the objective function is called:

(A) Basic solution

(B) Feasible solution

(C) Optimal solution

(D) Boundary solution

Q28. The Consumer Surplus is represented by the area:

(A) Above the price line and below the demand curve

(B) Below the price line and above the supply curve

(C) Below the demand curve and above the x-axis

(D) Above the supply curve and below the x-axis

Q29. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is:

(A) A

(B) $-I$

(C) I

(D) 0

Q30. In a Poisson distribution, if $P(X = 0) = k$, then the mean λ is:

(A) $\ln k$

(B) $-\ln k$

(C) e^k

(D) $1/k$

Q31. If the growth of a population follows the law $\frac{dP}{dt} = kP$, the population at any time t is:

(A) $P = P_0 e^{kt}$

(B) $P = P_0 + kt$

(C) $P = P_0 \ln(kt)$

(D) $P = P_0 e^{-kt}$

Q32. The solution of $x\frac{dy}{dx} = y$ is:

(A) $y = x + C$

(B) $y = Cx$

(C) $y = e^x + C$

(D) $x^2 + y^2 = C$

Q33. Fisher's Index Number is considered ideal because it satisfies:

(A) Time Reversal Test

(B) Factor Reversal Test

(C) Both (A) and (B)

(D) Neither (A) nor (B)

Q34. A card is drawn from a pack of 52 cards. The probability that it is a king or a heart is:

(A) 17/52

(B) 16/52

(C) 4/13

(D) 3/13

Q35. The scrap value of an asset is also known as:

(A) Book value

(B) Market value

(C) Residual value

(D) Face value

Q36. A large sample is generally defined as one where n is:

(A) > 100

(B) > 30

(C) < 30

(D) > 10

Q37. $15 \equiv 1 \pmod{x}$. How many positive integer values can x take (where $x > 1$)?

(A) 2

(B) 4

(C) 3

(D) 1

Q38. The point at which the Total Revenue equals Total Cost is called:

(A) Equilibrium point

(B) Break-even point

(C) Optimal point

(D) Peak point

Q39. For any square matrix A, $(A^T)^T$ is:

(A) A^T

(B) $-A$

(C) A

(D) I

Q40. Flat interest rate is always ______ the effective interest rate on a reducing balance.

(A) Lower than

(B) Higher than

(C) Equal to

(D) Half of

Q41. $\int_1^e \frac{\ln x}{x} dx$ is:

(A) 1

(B) 0.5

(C) e

(D) 2

Q42. The Z-score for a value x is calculated as:

(A) $(x - \mu)/\sigma$

(B) $(x + \mu)/\sigma$

(C) $\sigma/(x - \mu)$

(D) $(x - \sigma)/\mu$

Q43. Which of the following can be used to smooth a time series?

(A) Correlation

(B) Moving Averages

(C) Regression

(D) Integration

Q44. A square matrix A is invertible if and only if A is:

(A) Singular

(B) Non-singular

(C) Symmetric

(D) Skew-symmetric

Q45. $\int e^x (f(x) + f'(x)) dx$ is:

(A) $e^x f'(x) + C$

(B) $e^x f(x) + C$

(C) $f(x) + C$

(D) $e^x + f(x) + C$

Q46. The standard error of the mean is:

(A) σ/n

(B) $\sigma/\sqrt{n}$

(C) σ^2/n

(D) $\sqrt{\sigma/n}$

Q47. A man can row 9 km/h in still water and find that it takes him thrice as much time to row up than as to row down the same distance in the river. The speed of the current is:

(A) 2 km/h

(B) 3 km/h

(C) 4.5 km/h

(D) 5 km/h

Q48. The value of an asset at the end of its useful life is ₹ 0. This is common in:

(A) Real estate

(B) Intangible assets like patents

(C) Antique cars

(D) Land

Q49. If two rows of a determinant are identical, the value of the determinant is:

(A) 1

(B) -1

(C) 0

(D) Infinity

Q50. $\int_0^{\pi/4} \sec^2 x dx$ is:

(A) 1

(B) 0

(C) $\pi/4$

(D) $\sqrt{2}$

Detailed Solutions

Q1.

Solution

Concept:
This problem involves work and time. The fundamental principle is that the rate of work is the reciprocal of the time taken to complete the task. If two people work together, their combined rate is the sum of their individual rates. If one person works only for a fraction of a day, their effective rate for that day is reduced by that same fraction.

Solution:
1. Let the total work be 1 unit. 2. The combined rate of A and B is:

$$\text{Rate}_{A+B} = \frac{1}{12} \text{ units/day}$$

3. The individual rate of A is:

$$\text{Rate}_{A} = \frac{1}{20} \text{ units/day}$$

4. Find the individual rate of B:

$$\text{Rate}_{B} = \text{Rate}_{A+B} - \text{Rate}_{A} = \frac{1}{12} - \frac{1}{20}$$

Taking the LCM of 12 and 20 (which is 60):

$$\text{Rate}_{B} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30} \text{ units/day}$$

5. According to the condition, B works only for half a day. So, B's new effective rate is:

$$\text{Rate}'_{B} = \frac{1}{2} \times \frac{1}{30} = \frac{1}{60} \text{ units/day}$$

6. The new combined rate of A and B is:

$$\text{New Rate}_{A+B} = \text{Rate}_{A} + \text{Rate}'_{B} = \frac{1}{20} + \frac{1}{60}$$

Taking the LCM as 60:

$$\text{New Rate}_{A+B} = \frac{3+1}{60} = \frac{4}{60} = \frac{1}{15} \text{ units/day}$$

7. Time taken to complete the work = $\frac{1}{\text{New Rate}_{A+B}}$ = 15 days.
Final Answer: The work will be completed in 15 days.

Answer: (C)

Q2.

Solution

Concept:

The Cayley-Hamilton Theorem states that every square matrix satisfies its own characteristic equation. For a 2×2 matrix A, the characteristic equation is given by:

$$|A - \lambda I| = 0$$

which simplifies to $\lambda^2 - \text{tr}(A)\lambda + |A| = 0$. Replacing λ with A gives the matrix equation $A^2 - \text{tr}(A)A + |A|I = O$.

Solution:

1. Given matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. 2. Calculate the Trace of A (sum of diagonal elements):

$$\text{tr}(A) = 2 + 4 = 6$$

3. Calculate the Determinant of A:

$$|A| = (2 \times 4) - (3 \times 1) = 8 - 3 = 5$$

4. According to the Cayley-Hamilton Theorem, A satisfies:

$$A^2 - \text{tr}(A)A + |A|I = O$$

5. Substitute the calculated values:

$$A^2 - 6A + 5I = O$$

6. Here, O represents the null matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Final Answer: The value is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Answer: (A)

Q3.

Solution

Concept:
The area of a region bounded by a curve $y^2 = 4ax$ and a vertical line $x = h$ is found using integration. Since the parabola $y^2 = 4x$ is symmetric about the x-axis, the total area is twice the area of the upper half. The area is given by:

$$\text{Area} = 2\int_0^h y\,dx$$

Solution:
1. The given parabola is $y^2 = 4x$, which implies $y = \pm 2\sqrt{x}$. 2. The region is bounded by $x = 0$ to $x = 3$. 3. Total Area $A = 2 \times \int_0^3 2\sqrt{x}\,dx$:

$$A = 4\int_0^3 x^{1/2}\,dx$$

4. Perform the integration:

$$A = 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = 4 \times \frac{2}{3}\left[x^{3/2}\right]_0^3$$

5. Evaluate the limits:

$$A = \frac{8}{3}[3^{3/2} - 0] = \frac{8}{3}[3\sqrt{3}]$$

6. Simplify the expression:

$$A = 8\sqrt{3}\text{ sq. units}$$

Final Answer: The area is $8\sqrt{3}$ sq. units.

Answer: (B)

Q4.

Solution

Concept:
This problem follows a Binomial Distribution where n is the number of trials and p is the probability of success (defect). The Mean (μ) is given by:

$$\mu = np$$

The Standard Deviation (σ) is given by:

$$\sigma = \sqrt{npq}$$

where $q = 1 - p$.
Solution:
1. Given $n = 400$ and $p = 0.1$. 2. Calculate the probability of non-defective bolts (q):

$$q = 1 - 0.1 = 0.9$$

3. Calculate the Mean:

$$\text{Mean} = np = 400 \times 0.1 = 40$$

4. Calculate the Variance (npq):

$$\text{Variance} = 400 \times 0.1 \times 0.9 = 40 \times 0.9 = 36$$

5. Calculate the Standard Deviation:

$$\sigma = \sqrt{36} = 6$$

6. Thus, the mean is 40 and the standard deviation is 6.
Final Answer: The mean and standard deviation are 40 and 6 respectively.

Answer: (A)

Q5.

Solution

Concept:
An annuity is a series of equal payments made at regular intervals. Annuities are classified based on their duration and timing of payments. A perpetuity is a special type of annuity where the periodic payments begin on a fixed date and continue indefinitely (forever).

Solution:
1. **Ordinary Annuity:** Payments made at the end of each period for a fixed term. 2. **Annuity Due:** Payments made at the beginning of each period for a fixed term. 3. **Deferred Annuity:** An annuity where payments start after a certain lapse of time. 4. **Perpetuity:** The defining characteristic of a perpetuity is that the term is infinite ($n \to \infty$). There is no end date for the payments. Examples include certain types of government bonds and scholarship funds.

Final Answer: An annuity that continues forever is called a Perpetuity.

Answer: (C)

Q6.

Solution

Concept:
To find $a^n \pmod{m}$, we look for cycles in the remainders or use Fermat's Little Theorem. Fermat's Little Theorem states that if p is a prime and a is not divisible by p, then $a^{p-1} \equiv 1 \pmod{p}$.

Solution:
1. We need to find $3^{50} \pmod{7}$. Here $a = 3$ and $p = 7$. 2. By Fermat's Little Theorem, since 7 is prime:

$$3^{7-1} \equiv 3^6 \equiv 1 \pmod{7}$$

3. We can express the exponent 50 in terms of the cycle 6:

$$50 = (6 \times 8) + 2$$

4. Therefore:

$$3^{50} = (3^6)^8 \times 3^2$$

5. Substitute the modular value ($3^6 \equiv 1$):

$$3^{50} \equiv (1)^8 \times 3^2 \pmod{7}$$

$$3^{50} \equiv 1 \times 9 \pmod{7}$$

6. Since $9 \div 7$ leaves a remainder of 2:

$$3^{50} \equiv 2 \pmod{7}$$

Final Answer: The value is 2.

Answer: (B)

Q7.

Solution

Concept:

Average Cost (AC) is the cost per unit of output, calculated as $AC = C(x)/x$. To minimize AC, we find its derivative with respect to x, set it to zero, and solve for x. This occurs at a point where Average Cost equals Marginal Cost.

Solution:

1. Given Total Cost function: $C(x) = 100 + 5x + 0.1x^2$. 2. Find the Average Cost function AC:

$$AC = \frac{C(x)}{x} = \frac{100}{x} + 5 + 0.1x$$

3. To find the minimum, differentiate AC with respect to x and set to 0:

$$\frac{d(AC)}{dx} = -\frac{100}{x^2} + 0.1 = 0$$

4. Solve the equation:

$$\frac{100}{x^2} = 0.1$$

$$x^2 = \frac{100}{0.1} = 1000$$

5. Take the square root:

$$x = \sqrt{1000}$$

6. This can be simplified to $10\sqrt{10} \approx 31.62$. Based on the options, the exact form is required.

Final Answer: The average cost is minimum when $x = \sqrt{1000}$.

Answer: (B)

Q8.

Solution

Concept:
Let the speed of the man in still water be x km/h and the speed of the stream be y km/h. - Speed upstream $(U) = x - y$. - Speed downstream $(D) = x + y$. - Speed in still water $(x) = \frac{D+U}{2}$.
Solution:
1. **Upstream:** Distance = 750 m = 0.75 km. Time = 675 seconds = $\frac{675}{3600}$ hours = $\frac{3}{16}$ hours.

$$U = \frac{0.75}{3/16} = 0.75 \times \frac{16}{3} = \frac{3}{4} \times \frac{16}{3} = 4 \text{ km/h}$$

2. **Downstream:** Distance = 750 m = 0.75 km. Time = 7.5 minutes = $\frac{7.5}{60}$ hours = $\frac{1}{8}$ hours.

$$D = \frac{0.75}{1/8} = 0.75 \times 8 = 6 \text{ km/h}$$

3. **Speed in Still Water:**

$$x = \frac{D+U}{2} = \frac{6+4}{2} = \frac{10}{2} = 5 \text{ km/h}$$

Final Answer: The speed in still water is 5 km/h.

Answer: (C)

Q9.

Solution

Concept:
In a Normal Distribution, there is a constant proportional relationship between different measures of dispersion. Mean Deviation (MD) is approximately 0.8 times the Standard Deviation (σ).
Solution:
1. For a perfectly normal distribution, the probability density function follows a specific mathematical curve. 2. The Mean Deviation about the mean is calculated as $E[|X - \mu|]$. 3. For the Normal Distribution, this integral evaluates exactly to:

$$MD = \sigma\sqrt{\frac{2}{\pi}}$$

4. Numerically, $\sqrt{\frac{2}{\pi}} \approx \sqrt{\frac{2}{3.14159}} \approx \sqrt{0.6366} \approx 0.7979$. 5. This value is commonly approximated as 0.8 or the fraction $4/5$. 6. Therefore:

$$MD = \frac{4}{5}\sigma$$

Final Answer: The relation is $MD = \frac{4}{5}\sigma$.

Answer: (A)

Q10.

Solution

Concept:

This is a problem of the **Amount of an Ordinary Annuity**. When a person invests a fixed sum R at the end of every year for n years at interest rate i, the total accumulated amount A is:

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Solution:

1. Given periodic payment R = ₹ 5,000. 2. Rate of interest $r = 10\%$, so $i = 0.1$. 3. Time period $n = 3$ years. 4. Substitute values into the formula:

$$A = 5000\left[\frac{(1+0.1)^3 - 1}{0.1}\right]$$

5. Calculate $(1.1)^3$:

$$1.1 \times 1.1 \times 1.1 = 1.331$$

6. Continue the calculation:

$$A = 5000\left[\frac{1.331 - 1}{0.1}\right]$$

$$A = 5000\left[\frac{0.331}{0.1}\right]$$

$$A = 5000 \times 3.31$$

7. Final result:

$$A = 16550$$

Final Answer: The amount at the end of 3 years is ₹ 16,550.

Answer: (B)

Q11.

Solution

Concept:

The point elasticity of demand (η_d) measures the responsiveness of the quantity demanded to a change in price. It is calculated as:

$$\eta_d = \left| \frac{p}{x} \cdot \frac{dx}{dp} \right|$$

Where x is the quantity, p is the price, and $\frac{dx}{dp}$ is the derivative of the demand function with respect to price.

Solution:

1. Given demand function: $x = 20 - 2p$. 2. Differentiate x with respect to p:

$$\frac{dx}{dp} = \frac{d}{dp}(20 - 2p) = -2$$

3. Find the quantity x when the price $p = 5$:

$$x = 20 - 2(5) = 20 - 10 = 10$$

4. Substitute p, x, and $\frac{dx}{dp}$ into the elasticity formula:

$$\eta_d = \left| \frac{5}{10} \cdot (-2) \right|$$

5. Calculate the value:

$$\eta_d = \left| \frac{1}{2} \cdot (-2) \right| = | - 1| = 1$$

6. Since $\eta_d = 1$, the demand is said to be unitary elastic at this price.

Final Answer: The elasticity of demand is 1.

Answer: (A)

Q12.

Solution

Concept:
For a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$, the Integrating Factor (IF) is a function used to solve the equation. It is defined as:

$$IF = e^{\int P\,dx}$$

Multiplying the entire equation by this factor makes the left side a perfect derivative of $(y \cdot IF)$.
Solution:
1. Given differential equation: $\frac{dy}{dx} + \frac{2}{x}y = x$. 2. Identify P by comparing it with the standard form:

$$P = \frac{2}{x}$$

3. Set up the integral for the exponent:

$$\int P\,dx = \int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx$$

4. Perform the integration:

$$2\ln x = \ln(x^2)$$

5. Calculate the Integrating Factor:

$$IF = e^{\ln(x^2)}$$

6. Using the identity $e^{\ln(f(x))} = f(x)$:

$$IF = x^2$$

Final Answer: The integrating factor is x^2.

Answer: (A)

Q13.

Solution

Concept:
A sinking fund is a financial strategy where a company or individual sets aside a fixed amount of money at regular intervals into a separate account. This account earns interest over time, allowing the fund to grow to a target amount needed for a specific future purpose.

Solution:
1. Companies often take long-term loans or issue debentures/bonds to raise capital. 2. These liabilities must be paid back in a lump sum after a specific period (e.g., 10 or 20 years). 3. To avoid financial strain at the time of maturity, a **Sinking Fund** is created. 4. By contributing small amounts periodically, the business ensures it has enough cash to "sink" or retire the debt when it falls due. 5. It is also commonly used for the replacement of aging assets (depreciation fund).

Final Answer: It is created for the repayment of a long-term liability.

Answer: (B)

Q14.

Solution

Concept:
The symmetry of a Binomial Distribution depends on the probability of success (p). - If $p < 0.5$, the distribution is positively skewed (skewed to the right). - If $p > 0.5$, the distribution is negatively skewed (skewed to the left). - If $p = 0.5$, the distribution is perfectly symmetric.

Solution:
1. Given parameters: $n = 10$ and $p = 0.5$. 2. Calculate the probability of failure q:

$$q = 1 - p = 1 - 0.5 = 0.5$$

3. Since $p = q$, the probability of getting k successes is exactly the same as the probability of getting $n - k$ successes. For example: $P(X = 2) = P(X = 8)$. 4. This balance around the mean ($np = 5$) results in a bell-shaped, mirror-image histogram. 5. Therefore, the distribution is symmetric regardless of the value of n, as long as $p = 0.5$.

Final Answer: The distribution is Symmetric.

Answer: (C)

Q15.

Solution

Concept:
In Linear Programming, constraints represent boundaries on a coordinate plane. Non-negativity constraints ($x \geq 0, y \geq 0$) restrict the region to the first quadrant. A linear inequality like $Ax + By \leq C$ defines a half-plane.

Solution:
1. The constraints are: - $x \geq 0$ (Region to the right of the y-axis) - $y \geq 0$ (Region above the x-axis) - $x + y \leq 5$ (Region below the line $x + y = 5$) 2. Find the intercepts of the line $x + y = 5$: - When $x = 0$, $y = 5$. Point: $(0, 5)$. - When $y = 0$, $x = 5$. Point: $(5, 0)$. 3. The origin $(0, 0)$ also satisfies the inequality $0 + 0 \leq 5$. 4. The intersection of these three half-planes forms a closed region with vertices at $(0, 0)$, $(5, 0)$, and $(0, 5)$. 5. A shape with three vertices is a triangle. Specifically, this is a right-angled isosceles triangle.

Final Answer: The region is in the shape of a Triangle.

Answer: (C)

Q16.

Solution

Concept:
A time series is a set of observations taken at specified times, usually at equal intervals. The variations in time series are classified into four components. The **Seasonal Variation** refers to periodic movements in a time series that repeat regularly within a period of one year or less.

Solution:
1. **Trend:** Long-term direction of the data over many years. 2. **Seasonal Variation:** Fluctuations that repeat annually, monthly, or weekly due to seasons or customs (e.g., increased sales during festivals or summer). 3. **Cyclical Variation:** Long-term oscillations (upward and downward movements) extending over several years (Business cycles). 4. **Irregular Variation:** Unpredictable, random fluctuations (e.g., floods, strikes, wars). 5. The question describes a pattern that repeats "every year." This periodicity within the 12-month calendar defines it as seasonal.

Final Answer: The pattern is Seasonal.

Answer: (B)

Q17.

Solution

Concept:

An **Idempotent Matrix** is a square matrix A such that $A^2 = A$. This property allows us to simplify higher powers of the matrix, as $A^3 = A^2 \cdot A = A \cdot A = A^2 = A$. In general, $A^n = A$ for all positive integers n.

Solution:

1. We need to simplify $(I + A)^3 - 7A$. 2. Expand $(I + A)^3$ using the binomial expansion (since I and A commute):

$$(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$$

3. Use the properties of the identity matrix ($I^n = I$ and $IA = A$):

$$(I + A)^3 = I + 3A + 3A^2 + A^3$$

4. Since A is idempotent ($A^2 = A$ and $A^3 = A$):

$$(I + A)^3 = I + 3A + 3A + A$$

5. Combine the like terms:

$$(I + A)^3 = I + 7A$$

6. Now, substitute this back into the original expression:

$$(I + 7A) - 7A = I$$

Final Answer: The expression is equal to I.

Answer: (C)

Q18.

Solution

Concept:

The rate of filling or emptying is the fraction of the tank handled per unit of time. - Filling rate is positive (+). - Emptying rate is negative (-). - Net rate = Sum of individual rates.

Solution:

1. Rate of Pipe 1 (R_1) = $1/12$ per min. 2. Rate of Pipe 2 (R_2) = $1/15$ per min. 3. Combined rate of $(1 + 2) = 1/12 + 1/15 = (5 + 4)/60 = 9/60 = 3/20$ per min. 4. They work for 5 minutes. Volume filled = $5 \times (3/20) = 3/4$ of the cistern. 5. Now, Pipe 3 is opened. Rate of Pipe 3 (R_3) = $-1/6$ per min. 6. New net rate = $R_1 + R_2 + R_3 = 3/20 - 1/6$. 7. Find the LCM of 20 and 6 (which is 60):

$$\text{Net Rate} = \frac{9 - 10}{60} = -1/60 \text{ per min}$$

8. The negative sign indicates the cistern is now being emptied at a rate of $1/60$ of the total capacity per minute. 9. Time to empty the filled volume ($3/4$):

$$\text{Time} = \frac{\text{Volume}}{\text{Rate}} = \frac{3/4}{1/60} = \frac{3}{4} \times 60 = 45 \text{ min}$$

Final Answer: The time taken to empty the cistern is 45 min.

Answer: (C)

Q19.

Solution

Concept:
This is a standard integral formula derived from trigonometric substitution. By substituting $x = a\sin\theta$, we can transform the algebraic expression into a trigonometric one that is easily integrable.

Solution:
1. Let $x = a\sin\theta \implies dx = a\cos\theta\, d\theta$. 2. Substitute into the integral:

$$\int \frac{a\cos\theta\, d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} = \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2(1-\sin^2\theta)}}$$

3. Use the identity $1 - \sin^2\theta = \cos^2\theta$:

$$\int \frac{a\cos\theta\, d\theta}{a\cos\theta} = \int 1\, d\theta$$

4. Integrate:

$$\theta + C$$

5. From the initial substitution, $\sin\theta = x/a$, so $\theta = \sin^{-1}(x/a)$. 6. Thus:

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}(x/a) + C$$

Final Answer: The integral is $\sin^{-1}(x/a) + C$.

Answer: (A)

Q20.

Solution

Concept:

The **Effective Rate of Interest** (r_e) is the actual interest earned or paid in a year when compounding occurs more than once annually. It is given by:

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

where r is the nominal annual rate and m is the number of compounding periods per year.

Solution:

1. Given nominal rate $r = 12\% = 0.12$. 2. Given compounding is quarterly, so $m = 4$. 3. Substitute into the formula:

$$r_e = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

4. Calculate the periodic rate:

$$r_e = (1 + 0.03)^4 - 1 = (1.03)^4 - 1$$

5. Expand $(1.03)^4$:

$$1.03 \times 1.03 = 1.0609$$

$$1.0609 \times 1.0609 \approx 1.12550881$$

6. Calculate r_e:

$$r_e = 1.1255 - 1 = 0.1255$$

7. Convert to percentage:

$$0.1255 \times 100 = 12.55\%$$

Final Answer: The effective rate of interest is 12.55%.

Answer: (B)

Q21.

Solution

Concept:

In statistical hypothesis testing, we make decisions about a population based on sample data. Two types of errors can occur during this process: - **Type I Error (α):** Occurs when the null hypothesis (H_0) is true, but we incorrectly reject it (False Positive). - **Type II Error (β):** Occurs when the null hypothesis (H_0) is false, but we fail to reject it (False Negative).

Solution:

1. The null hypothesis (H_0) represents the "status quo" or the assumption of no effect. 2. A Type I error is often considered more serious in certain contexts (like a criminal trial where an innocent person is convicted). 3. The probability of committing a Type I error is denoted by α, which is also known as the **level of significance** of the test. 4. If a researcher sets $\alpha = 0.05$, they are accepting a 5% risk of rejecting a true null hypothesis. 5. Therefore, the definition specifically matches "rejecting a true null hypothesis."

Final Answer: A Type I error occurs when we reject a true null hypothesis.

Answer: (B)

Q22.

Solution

Concept:

Let the speed of the boatman in stationary (still) water be u km/h and the speed of the current be v km/h. - Speed upstream (U) = $u - v$ - Speed downstream (D) = $u + v$ - Speed = $\frac{\text{Distance}}{\text{Time}}$

Solution:

1. **Against the current (Upstream):** Distance = 2 km, Time = 1 hour.

$$u - v = \frac{2}{1} = 2 \text{ km/h} \quad \text{—(1)}$$

2. **Along the current (Downstream):** Distance = 1 km, Time = 10 minutes = $\frac{10}{60} = \frac{1}{6}$ hours.

$$u + v = \frac{1}{1/6} = 6 \text{ km/h} \quad \text{—(2)}$$

3. **Solve for u (Still water speed):** Add equations (1) and (2):

$$(u - v) + (u + v) = 2 + 6$$

$$2u = 8 \implies u = 4 \text{ km/h}$$

4. **Calculate time for 5 km in stationary water:** Time = $\frac{\text{Distance}}{u} = \frac{5 \text{ km}}{4 \text{ km/h}} = 1.25$ hours. 5. **Convert to minutes:** 1 hour + 0.25×60 min = 1 hour 15 minutes.

Final Answer: It will take 1 hr 15 min.

Answer: (C)

Q23.

Solution

Concept:
The Normal Distribution is a continuous probability distribution. For any continuous probability density function $f(x)$, the total area under the curve from $-\infty$ to $+\infty$ must represent the total probability of all possible outcomes.
Solution:
1. In probability theory, the sum of all possible probabilities in a sample space must equal 1. 2. For the standard normal distribution (mean $\mu = 0$, standard deviation $\sigma = 1$), the function is:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

3. The integral of this function over the entire real line is:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = 1$$

4. Visually, the bell curve is symmetric, with 0.5 area to the left of the mean and 0.5 area to the right, totaling exactly 1.
Final Answer: The total area under the standard normal curve is 1.

Answer: (C)

Q24.

Solution

Concept:
In financial mathematics, when a loan is repaid through a series of fixed monthly payments that include both principal and interest components, the payment amount is standardized.
Solution:
1. EMI is a very common term in banking and personal finance. 2. It stands for **Equated Monthly Installment**. 3. It is "equated" because the amount remains the same every month throughout the loan tenure (assuming fixed interest rates). 4. Early in the loan, a larger portion of the EMI goes toward interest. As the principal is paid down, a larger portion of the EMI goes toward the principal. 5. The formula used is $EMI = P \cdot r \cdot \frac{(1+r)^n}{(1+r)^n - 1}$, where P is principal, r is monthly interest, and n is number of months.
Final Answer: EMI stands for Equated Monthly Installment.

Answer: (B)

Q25.

Solution

Concept:
The **Order** of a differential equation is the highest derivative present. The **Degree** is the power to which the highest-order derivative is raised, provided the equation is a polynomial in its derivatives (i.e., no derivatives are inside radicals or denominators).

Solution:
1. Observe the given equation:

$$\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y$$

2. Identify the highest order derivative: The derivatives are $\frac{d^3y}{dx^3}$ (3rd order), $\frac{d^2y}{dx^2}$ (2nd order), and $\frac{dy}{dx}$ (1st order). 3. The highest order is 3. 4. Now, find the power (exponent) of this highest order derivative: The term is $\left(\frac{d^3y}{dx^3}\right)^2$. 5. The power of the 3rd order derivative is 2. 6. Note: Even though the 1st order derivative is raised to the 4th power, it does not determine the degree of the equation because it is not the highest order derivative.

Final Answer: The degree is 2.

Answer: (B)

Q26.

Solution

Concept:
The properties of congruence modulo m are similar to the properties of equality. If two numbers are congruent to two other numbers under the same modulus, their sums, differences, and products are also congruent. Specifically: If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $(a + c) \equiv (b + d) \pmod{m}$.

Solution:
1. $a \equiv b \pmod{m}$ means that $(a - b)$ is divisible by m. We can write $a - b = k_1 m$ for some integer k_1. 2. $c \equiv d \pmod{m}$ means that $(c - d)$ is divisible by m. We can write $c - d = k_2 m$ for some integer k_2. 3. We want to check $(a + c) \equiv (b + d) \pmod{m}$. This requires $(a + c) - (b + d)$ to be divisible by m. 4. Rearrange the expression:

$$(a + c) - (b + d) = (a - b) + (c - d)$$

5. Substitute the integer multiples:

$$(a - b) + (c - d) = k_1 m + k_2 m = (k_1 + k_2)m$$

6. Since the result is an integer multiple of m, it is divisible by m. 7. This property holds for any integer m and any integers a, b, c, d. It is a fundamental theorem of modular arithmetic.

Final Answer: The statement is Always true.

Answer: (A)

Q27.

Solution

Concept:
In Linear Programming Problems (LPP), the feasible region contains all points that satisfy the constraints. The **Fundamental Theorem of Linear Programming** states that if an optimal solution exists, it must occur at one of the corner points (vertices) of the feasible region.

Solution:
1. **Feasible solution:** Any point that satisfies all constraints. 2. **Basic solution:** A solution where the number of non-zero variables equals the number of constraints. 3. **Optimal solution:** A feasible solution that achieves the maximum (or minimum) value of the objective function (e.g., maximum profit or minimum cost). 4. The objective function $Z = ax + by$ represents a family of parallel lines. As we move these lines across the feasible region, the last point of contact (or first, for minimization) will typically be a vertex. 5. Therefore, the specific point that optimizes the goal is called the optimal solution.

Final Answer: Such a point is called an Optimal solution.

Answer: (C)

Q28.

Solution

Concept:
Consumer Surplus (CS) is the economic measure of the difference between what consumers are willing to pay for a good and what they actually pay (the market price). It represents the "savings" or extra utility gained by the consumer.

Solution:
1. On a graph, the Demand Curve represents the maximum price consumers are willing to pay for various quantities. 2. The Market Price (P_0) is a horizontal line. 3. Consumers who were willing to pay more than P_0 receive a "surplus" benefit. 4. Mathematically, $CS = \int_0^{x_0} D(x)\,dx - (P_0 \cdot x_0)$. 5. Geometrically, this is the area bounded above by the demand curve and below by the price line $P = P_0$, from $x = 0$ to the equilibrium quantity x_0. 6. It is always the area **above the price line and below the demand curve**.

Final Answer: The Consumer Surplus is the area above the price line and below the demand curve.

Answer: (A)

Q29.

Solution

Concept:
Matrix multiplication is performed by taking the dot product of the rows of the first matrix with the columns of the second. For A^2, we calculate $A \times A$.

Solution:
1. Given matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. This is a permutation matrix. 2. Calculate A^2:

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

3. Perform the multiplication for each element: - Row 1, Col 1: $(0 \times 0) + (1 \times 1) = 1$ - Row 1, Col 2: $(0 \times 1) + (1 \times 0) = 0$ - Row 2, Col 1: $(1 \times 0) + (0 \times 1) = 0$ - Row 2, Col 2: $(1 \times 1) + (0 \times 0) = 1$
4. The resulting matrix is:

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. This is the definition of the Identity matrix, denoted by I.
Final Answer: A^2 is equal to I.

Answer: (C)

Q30.

Solution

Concept:

The Poisson distribution is defined by the probability mass function:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where λ is the mean of the distribution and x is the number of occurrences $(0, 1, 2, \ldots)$.

Solution:

1. Given that $P(X = 0) = k$. 2. Substitute $x = 0$ into the Poisson formula:

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!}$$

3. Since $\lambda^0 = 1$ and $0! = 1$:

$$P(X = 0) = e^{-\lambda}$$

4. Therefore, we have the equation:

$$k = e^{-\lambda}$$

5. To solve for λ, take the natural logarithm (ln) of both sides:

$$\ln k = \ln(e^{-\lambda})$$

$$\ln k = -\lambda$$

6. Multiply by -1 to isolate λ:

$$\lambda = -\ln k$$

Final Answer: The mean λ is $-\ln k$.

Answer: (B)

Q31.

Solution

Concept:
The differential equation $\frac{dP}{dt} = kP$ describes a situation where the rate of change of a population is directly proportional to the current population size. This is the fundamental model for unrestricted exponential growth.

Solution:

1. Separate the variables P and t:

$$\frac{1}{P} dP = k\, dt$$

2. Integrate both sides:

$$\int \frac{1}{P} dP = \int k\, dt$$

$$\ln |P| = kt + C$$

3. To solve for P, exponentiate both sides:

$$P = e^{kt+C} = e^{kt} \cdot e^{C}$$

4. Let e^C be a new constant, P_0, which represents the initial population at $t = 0$:

$$P = P_0 e^{kt}$$

5. This indicates that the population grows (if $k > 0$) or decays (if $k < 0$) exponentially over time.

Final Answer: The population at any time t is $P = P_0 e^{kt}$.

Answer: (A)

Q32.

Solution

Concept:
A first-order differential equation is solved by separating the variables such that all terms involving y are on one side and all terms involving x are on the other.

Solution:
1. Given equation: $x\frac{dy}{dx} = y$. 2. Rearrange the terms to separate x and y:

$$\frac{1}{y}dy = \frac{1}{x}dx$$

3. Integrate both sides:

$$\int \frac{1}{y}dy = \int \frac{1}{x}dx$$

4. The integral of $1/u$ is $\ln|u|$:

$$\ln|y| = \ln|x| + \ln|C|$$

(We use $\ln|C|$ as the constant of integration for easier simplification). 5. Use the property of logarithms $\ln A + \ln B = \ln(AB)$:

$$\ln|y| = \ln|Cx|$$

6. Remove the logarithms from both sides:

$$y = Cx$$

7. This represents a family of straight lines passing through the origin.

Final Answer: The solution is $y = Cx$.

Answer: (B)

Q33.

Solution

Concept:

Fisher's Ideal Index Number is defined as the geometric mean of Laspeyres' Index (L) and Paasche's Index (P). It is widely regarded as "ideal" because it satisfies several important statistical tests that other index numbers fail.

Solution:

1. **Time Reversal Test:** Requires that the index for period 1 with base period 0 should be the reciprocal of the index for period 0 with base period 1 ($I_{01} \times I_{10} = 1$). 2. **Factor Reversal Test:** Requires that the product of the price index and the quantity index should equal the value ratio ($P \times Q = V$). 3. Laspeyres' and Paasche's methods typically fail both tests. 4. Fisher's index is constructed specifically to correct the biases inherent in those two methods, and it mathematically satisfies both the Time Reversal and Factor Reversal tests. 5. Because it meets these rigorous criteria, it is termed the "Ideal" index.

Final Answer: It satisfies both the Time Reversal and Factor Reversal tests.

Answer: (C)

Q34.

Solution

Concept:

The Addition Rule for Probability for two events A and B is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

where $P(A \cap B)$ represents the probability of both events occurring simultaneously.

Solution:

1. Total number of cards in a pack = 52. 2. Let event A be drawing a "King": There are 4 kings in a deck. So, $P(A) = 4/52$. 3. Let event B be drawing a "Heart": There are 13 hearts in a deck. So, $P(B) = 13/52$. 4. Let $(A \cap B)$ be drawing the "King of Hearts": There is only 1 such card. So, $P(A \cap B) = 1/52$. 5. Apply the addition rule:

$$P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$P(A \cup B) = \frac{17 - 1}{52} = \frac{16}{52}$$

6. Simplify the fraction by dividing by 4:

$$P = 4/13$$

Final Answer: The probability is 4/13.

Answer: (C)

Q35.

Solution

Concept:
When an asset is purchased, it has an estimated "useful life." At the end of this period, the asset may no longer be efficient for its intended use, but it may still have some value as waste or for resale.
Solution:
1. **Book Value:** The value of an asset as recorded in account books (Cost minus accumulated depreciation). 2. **Market Value:** The price at which an asset can currently be sold in the open market. 3. **Residual (or Scrap) Value:** The estimated amount that an entity can obtain from disposing of an asset after deducting the estimated costs of disposal, at the end of its useful life. 4. For example, if a machine is used for 10 years, its residual value is the price its metal components might fetch when sold to a scrap dealer. 5. Therefore, "Scrap Value" and "Residual Value" are synonymous terms in accounting and finance.
Final Answer: It is also known as Residual value.

Answer: (C)

Q36.

Solution

Concept:
The p-value approach is a common method for reaching a conclusion in hypothesis testing. The p-value represents the probability of observing a test statistic as extreme as, or more extreme than, the one calculated from the sample, assuming the null hypothesis (H_0) is true.
Solution:
1. Before conducting a test, we set a significance level α (common values are 0.05 or 0.01). 2. The decision rule is: - If **p-value $\leq \alpha$**: The result is statistically significant. We have enough evidence to **Reject H_0**. - If **p-value $> \alpha$**: The result is not statistically significant. We **Fail to Reject H_0**. 3. In this context, a small p-value indicates that the sample evidence is strongly inconsistent with the null hypothesis. 4. When $n > 30$, we typically use the Z-test, and the p-value is the area under the normal curve beyond the calculated Z-score.
Final Answer: If the p-value is less than α, we Reject H_0.

Answer: (A)

Q37.

Solution

Concept:

The notation $a \equiv b \pmod{x}$ means that when a is divided by x, the remainder is b. Mathematically, this implies that x is a divisor of the difference $(a - b)$.

Solution:

1. Given: $15 \equiv 1 \pmod{x}$. 2. This implies x divides $(15 - 1)$. 3. So, x must be a factor of 14. 4. List the positive factors of 14:

$$1, 2, 7, 14$$

5. The question specifies that x must be an integer and $x > 1$. 6. The values that satisfy $x > 1$ are:

$$\{2, 7, 14\}$$

7. Counting these values, there are 3 such integers.

Final Answer: There are 3 positive integer values for x.

Answer: (C)

Q38.

Solution

Concept:

The **Break-even Point** is a fundamental concept in business and economics. it represents the level of production or sales volume at which the business neither makes a profit nor incurs a loss.

Solution:

1. **Total Revenue (TR):** The total money coming in from sales $(Price \times Quantity)$. 2. **Total Cost (TC):** The sum of Fixed Costs and Variable Costs. 3. **Profit (π):** $\pi = TR - TC$. 4. At the break-even point:

$$\text{Profit} = 0 \implies TR - TC = 0 \implies TR = TC$$

5. Below this point, the business operates at a loss. Above this point, every additional unit sold contributes to profit. 6. Graphically, it is the intersection of the Total Revenue curve and the Total Cost curve.

Final Answer: This point is called the Break-even point.

Answer: (B)

Q39.

Solution

Concept:
The transpose of a matrix (denoted as A^T) is obtained by interchanging its rows and columns. If we transpose a matrix twice, we are essentially reversing the first operation.

Solution:
1. Let A be a matrix of order $m \times n$. 2. The first transpose A^T will have the order $n \times m$. The element at (i, j) in A moves to (j, i) in A^T. 3. Applying the transpose operation again to A^T: The element at (j, i) in A^T moves back to (i, j) in $(A^T)^T$. 4. The resulting matrix will have the original order $m \times n$. 5. Therefore, $(A^T)^T = A$. This is known as the **involution property** of the transpose operation.

Final Answer: $(A^T)^T$ is equal to A.

Answer: (C)

Q40.

Solution

Concept:
Interest rates can be quoted as a "Flat Rate" or an "Effective Rate" (Reducing Balance). - **Flat Rate:** Interest is calculated on the full original principal for the entire loan term. - **Reducing Balance:** Interest is calculated only on the remaining unpaid principal.

Solution:
1. Suppose you borrow ₹ 1,000 at a 10% flat rate for 1 year. You pay ₹ 100 interest. 2. If you pay it back in installments, you are effectively using less than ₹ 1,000 on average over the year, but still paying interest on the full ₹ 1,000. 3. To achieve the same ₹ 100 interest payout on a **reducing balance** (where your average principal is roughly half), the interest rate would have to be nearly double (around 18-20%). 4. Therefore, for any given numerical value, a flat interest rate is always **higher than** the equivalent effective interest rate because it ignores the principal repayments.

Final Answer: Flat interest rate is always higher than the effective interest rate.

Answer: (B)

Q41.

Solution

Concept:
The integral $\int \frac{\ln x}{x} dx$ can be solved using the method of substitution. Since the derivative of $\ln x$ is $1/x$, this substitution simplifies the integrand significantly. For a definite integral, we must also change the limits or evaluate the antiderivative at the original limits.

Solution:
1. Let $u = \ln x$. 2. Differentiate u with respect to x:

$$du = \frac{1}{x}dx$$

3. Change the limits of integration: - When $x = 1$, $u = \ln(1) = 0$. - When $x = e$, $u = \ln(e) = 1$. 4. Substitute u and du into the integral:

$$\int_1^e \frac{\ln x}{x}dx = \int_0^1 u\, du$$

5. Integrate:

$$\left[\frac{u^2}{2}\right]_0^1$$

6. Evaluate at the limits:

$$\frac{1^2}{2} - \frac{0^2}{2} = 0.5$$

Final Answer: The value of the integral is 0.5.

Answer: (B)

Q42.

Solution

Concept:
The Z-score (or standard score) is a dimensionless quantity that indicates how many standard deviations an observation or datum is above or below the mean of the population. It allows for the comparison of scores from different normal distributions by "standardizing" them.

Solution:
1. Let x be the value of a raw score. 2. Let μ be the mean of the population. 3. Let σ be the standard deviation of the population. 4. The distance of the score from the mean is calculated as $(x - \mu)$. 5. To find how many standard deviations this distance represents, we divide by σ:

$$Z = \frac{x - \mu}{\sigma}$$

6. A positive Z-score indicates the value is above the mean, while a negative Z-score indicates it is below the mean. A Z-score of 0 is exactly at the mean.

Final Answer: The Z-score is calculated as $(x - \mu)/\sigma$.

Answer: (A)

Q43.

Solution

Concept:
"Smoothing" a time series refers to the process of removing short-term fluctuations (noise or seasonal/irregular variations) to highlight the underlying long-term trend.
Solution:
1. Time series data often looks "jagged" due to day-to-day or month-to-month changes. 2. **Moving Averages** involve taking the average of a specific number of consecutive data points and replacing the central point with this average. 3. As the window moves through the data, the averaging process cancels out the high and low peaks, resulting in a "smoother" curve. 4. For example, a 3-year moving average for year t would be:

$$\text{MA}_t = \frac{Y_{t-1} + Y_t + Y_{t+1}}{3}$$

5. This is one of the most widely used techniques for trend identification.
Final Answer: Moving Averages can be used to smooth a time series.

Answer: (B)

Q44.

Solution

Concept:
A square matrix A has an inverse (A^{-1}) if and only if there exists a matrix B such that $AB = BA = I$. The existence of this inverse depends entirely on the value of the determinant of the matrix.
Solution:
1. The formula for the inverse of a matrix is:

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

2. This formula involves division by the determinant $|A|$. 3. If $|A| = 0$, the division is undefined, and the inverse does not exist. Such a matrix is called **Singular**. 4. If $|A| \neq 0$, the division is possible, and the inverse exists. Such a matrix is called **Non-singular**. 5. Therefore, being non-singular is the necessary and sufficient condition for a matrix to be invertible.
Final Answer: A square matrix is invertible if it is Non-singular.

Answer: (B)

Q45.

Solution

Concept:

This is a special integral form in calculus. When an integral is of the form $\int e^x[f(x) + f'(x)]dx$, it can be solved quickly using the product rule of differentiation in reverse.

Solution:

1. Consider the derivative of the product $e^x f(x)$:

$$\frac{d}{dx}[e^x f(x)] = e^x \frac{d}{dx}[f(x)] + f(x)\frac{d}{dx}[e^x]$$

2. Using the property $\frac{d}{dx}(e^x) = e^x$:

$$\frac{d}{dx}[e^x f(x)] = e^x f'(x) + e^x f(x)$$

3. Factor out e^x:

$$\frac{d}{dx}[e^x f(x)] = e^x[f(x) + f'(x)]$$

4. By the Fundamental Theorem of Calculus, integrating the right side gives the original product back:

$$\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$$

Final Answer: The integral is $e^x f(x) + C$.

Answer: (B)

Q46.

Solution

Concept:
The **Standard Error of the Mean** (SE) measures the dispersion of sample means around the population mean. It provides an estimate of how much the sample mean is likely to vary from the actual population mean.

Solution:
1. If we take all possible samples of size n from a population with standard deviation σ, the distribution of those sample means is called the sampling distribution. 2. The standard deviation of this sampling distribution is the Standard Error. 3. Mathematically, it is derived as:

$$SE = \frac{\sigma}{\sqrt{n}}$$

4. This formula shows that as the sample size n increases, the standard error decreases. This means larger samples provide a more precise estimate of the population mean. 5. In practice, if the population standard deviation σ is unknown, we use the sample standard deviation s as an estimate.

Final Answer: The standard error of the mean is $\sigma/\sqrt{n}$.

Answer: (B)

Q47.

Solution

Concept:

Let the speed of the man in still water be u and the speed of the current be v. - Downstream speed $(D) = u + v$ - Upstream speed $(U) = u - v$ - Time is inversely proportional to speed for a constant distance: $t \propto 1/s$.

Solution:

1. Given $u = 9$ km/h. 2. The problem states that time upstream (t_u) is thrice the time downstream (t_d):

$$t_u = 3 \times t_d$$

3. Since Time = Distance/Speed, and distance d is constant:

$$\frac{d}{u-v} = 3 \times \frac{d}{u+v}$$

4. Cancel d and cross-multiply:

$$u + v = 3(u - v)$$

$$u + v = 3u - 3v$$

5. Rearrange the terms to group u and v:

$$v + 3v = 3u - u$$

$$4v = 2u$$

6. Substitute $u = 9$:

$$4v = 2(9) = 18$$

$$v = 18/4 = 4.5 \text{ km/h}$$

Final Answer: The speed of the current is 4.5 km/h.

Answer: (C)

Q48.

Solution

Concept:
The value of an asset at the end of its useful life is its residual or scrap value. While physical assets usually have some scrap value (like metal in a car), certain assets are projected to have zero value.
Solution:
1. **Real Estate:** Buildings might depreciate, but the land usually appreciates. Value is rarely zero. 2. **Antique Cars:** These often increase in value over time. 3. **Intangible Assets (Patents/Copyrights):** A patent gives a legal right for a specific period (e.g., 20 years). Once that period expires, the technology enters the public domain. The legal "asset" ceases to exist and has a financial value of ₹ 0 to the original holder. 4. Therefore, intangible assets are the most common example where the residual value is explicitly set to zero at the end of their legal life.
Final Answer: This is common in Intangible assets like patents.

Answer: (B)

Q49.

Solution

Concept:
One of the fundamental properties of determinants is related to the dependency of rows or columns. If a determinant has two rows (or two columns) that are identical, the matrix is singular.
Solution:
1. Consider a 2×2 determinant with identical rows:

$$\begin{vmatrix} a & b \\ a & b \end{vmatrix} = (a \times b) - (b \times a) = 0$$

2. For larger matrices, if two rows are identical, they are linearly dependent. 3. In any determinant, if we subtract one identical row from another (an elementary row operation), we get a row of all zeros. 4. Any determinant with a row (or column) of zeros has a value of 0. 5. This property holds regardless of the size of the matrix.
Final Answer: The value of the determinant is 0.

Answer: (C)

Q50.

Solution

Concept:

The fundamental trigonometric integral $\int \sec^2 x\, dx = \tan x + C$ is based on the derivative of the tangent function. To solve the definite integral, we evaluate $\tan x$ at the given bounds.

Solution:

1. Identify the antiderivative:

$$\int \sec^2 x\, dx = \tan x$$

2. Apply the limits of integration from 0 to $\pi/4$:

$$[\tan x]_0^{\pi/4}$$

3. Evaluate at the upper limit:

$$\tan(\pi/4) = 1$$

4. Evaluate at the lower limit:

$$\tan(0) = 0$$

5. Calculate the difference:

$$1 - 0 = 1$$

Final Answer: The value of the integral is 1.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	A	5	C
6	B	7	B	8	C	9	A	10	B
11	A	12	A	13	B	14	C	15	C
16	B	17	C	18	C	19	A	20	B
21	B	22	C	23	C	24	B	25	B
26	A	27	C	28	A	29	C	30	B
31	A	32	B	33	C	34	C	35	C
36	A	37	C	38	B	39	C	40	B
41	B	42	A	43	B	44	B	45	B
46	B	47	C	48	B	49	C	50	A