CUET-UG Applied Mathematics Sample Paper-13

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$, then the number of distinct real roots of $f(x) = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q2. If α, β, γ are roots of $x^3 - 6x^2 + 11x - 6 = 0$, then find $\alpha^2 + \beta^2 + \gamma^2$:

(A) 14

(B) 12

(C) 10

(D) 8

Q3. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then find the eigenvalues of A:

(A) 1, 3

(B) 2, 2

(C) 3, 3

(D) 0, 4

Q4. Solve for real x: $|x - 2| + |x - 5| = 5$:

(A) $[2, 5]$

(B) $[0, 5]$

(C) $[1, 4]$

(D) $[2, 4]$

Q5. If $f(x) = x^3 - 3x^2 + 3x - 1$, then find the points where $f'(x) = 0$:

(A) $x = 1$

(B) $x = 0, 1$

(C) $x = 1, 2$

(D) $x = 0$

Q6. Evaluate $\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$:

(A) 0

(B) $\frac{1}{120}$

(C) $\frac{1}{60}$

(D) $\frac{1}{24}$

Q7. Evaluate $\int_0^1 \frac{x^3}{1 + x^2} dx$:

(A) $\frac{1}{2} - \frac{1}{2} \ln 2$

(B) $\frac{1}{2} \ln 2$

(C) $1 - \ln 2$

(D) $\ln 2$

Q8. Find the maximum value of $f(x) = x^2 e^{-x}$ for $x > 0$:

(A) $\frac{4}{e^2}$

(B) $\frac{2}{e}$

(C) $\frac{1}{e}$

(D) $\frac{1}{e^2}$

Q9. Evaluate the integral: $\int_0^1 \frac{\ln(1+x)}{1+x^2}\,dx$ and hence determine its convergence nature and exact value using suitable substitution and properties of logarithmic functions.

(A) $\frac{\pi}{8}\ln 2$

(B) $\frac{\pi}{4}\ln 2$

(C) $\frac{1}{2}\ln 2$

(D) $\frac{\pi}{2}\ln 2$

Q10. Evaluate the improper integral $\int_0^\infty \frac{xe^{-x}}{1+x^2}\,dx$ and analyze its convergence using comparison test and exponential decay properties.

(A) Convergent and equals $\frac{1}{2}$

(B) Divergent

(C) Convergent and equals 1

(D) Convergent and equals $\frac{\pi}{4}$

Q11. Find the value of $\int_0^\pi x\sin x\cos x\,dx$ using integration by parts and symmetry properties of trigonometric integrals.

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{8}$

(D) 0

Q12. Solve the differential equation: $\frac{dy}{dx} + y\tan x = \sec x$, and determine the integrating factor and general solution using first-order linear DE method.

(A) $y\sec x = \sin x + C$

(B) $y\cos x = \ln|\sec x| + C$

(C) $y\sec x = x + C$

(D) $y\sin x = \sec x + C$

Q13. Solve the differential equation: $\frac{dy}{dx} = \frac{x+y}{x-y}$ using substitution method and homogeneous transformation.

(A) Implicit solution in terms of $\ln(x^2 + y^2)$

(B) Linear equation solution

(C) Separable only

(D) No solution exists

Q14. A random variable X follows binomial distribution with $n = 8$ and $p = 0.5$. Find the probability that $X \geq 6$ using binomial expansion and symmetry properties.

(A) $\frac{37}{256}$

(B) $\frac{25}{256}$

(C) $\frac{50}{256}$

(D) $\frac{75}{256}$

Q15. A company maximizes profit given constraints $2x + y \leq 10, x + 2y \leq 8, x, y \geq 0$ with profit function $P = 5x + 4y$. Find the optimal solution using corner point method in linear programming.

(A) $(x, y) = (4, 2)$ with profit 28

(B) $(x, y) = (3, 2)$ with profit 23

(C) $(x, y) = (2, 3)$ with profit 22

(D) $(x, y) = (5, 0)$ with profit 25

Q16. A company's cost function is $C(x) = x^3 - 6x^2 + 15x + 50$. Determine the level of output at which marginal cost equals average cost and analyze its economic significance for optimal production.

(A) $x = 2$

(B) $x = 3$

(C) $x = 4$

(D) $x = 5$

Q17. The demand function for a product is $p = 120 - 3x$ and cost function is $C(x) = 30x + 200$. Find the production level that maximizes profit and interpret the result using marginal revenue and marginal cost approach.

(A) $x = 10$

(B) $x = 12$

(C) $x = 15$

(D) $x = 18$

Q18. A machine depreciates in value according to exponential model $V(t) = 50000e^{-0.2t}$. Find the time at which its value reduces to half and interpret the economic life of the machine.

(A) $t = \frac{\ln 2}{0.2}$

(B) $t = \frac{2}{0.2}$

(C) $t = \ln(10000)$

(D) $t = 5$

Q19. A firm has revenue function $R(x) = 80x - x^2$ and cost function $C(x) = 20x + 100$. Determine break-even points and explain their importance in financial decision making.

(A) $x = 5, 20$

(B) $x = 10, 15$

(C) $x = 8, 18$

(D) $x = 6, 16$

Q20. If α, β, γ are roots of $x^3 - 9x^2 + 26x - 24 = 0$, evaluate $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$ and interpret their relation using Vieta's formulas.

(A) $9, 26$

(B) $6, 24$

(C) $9, 24$

(D) $8, 26$

Q21. Solve for real x: $|x-1|+|x-3|+|x-5|=6$ and interpret the geometric meaning of absolute value sums on a number line.

(A) $x=3$

(B) $x=2,4$

(C) $x=1,5$

(D) $x=3,5$

Q22. If matrix $A=\begin{bmatrix}3 & 1\\ 2 & 2\end{bmatrix}$, find eigenvalues and interpret their significance in stability of linear systems.

(A) $1,4$

(B) $2,3$

(C) $0,5$

(D) $1,3$

Q23. Evaluate $\lim_{x\to 0}\frac{e^{2x}-1-2x}{x^2}$ and explain its relation to Taylor series expansion.

(A) 1

(B) 2

(C) 3

(D) 4

Q24. Find the maximum value of $f(x)=xe^{-x^2}$ and interpret its application in optimization problems.

(A) $\frac{1}{\sqrt{2e}}$

(B) $\frac{1}{e}$

(C) $\frac{1}{2e}$

(D) $\sqrt{e}$

Q25. Evaluate $\int_0^1 x^2 \ln x \, dx$ and interpret its convergence using integration by parts and improper integral properties.

(A) $-\frac{1}{9}$

(B) $-\frac{1}{4}$

(C) $-\frac{1}{6}$

(D) $-\frac{1}{3}$

Q26. Evaluate the improper integral $\int_0^\infty x^2 e^{-ax}\,dx$ for $a > 0$ and interpret its application in probability and expectation of continuous random variables.

(A) $\frac{2}{a^3}$

(B) $\frac{1}{a^2}$

(C) $\frac{2}{a^2}$

(D) $\frac{1}{a^3}$

Q27. Evaluate $\int_0^1 \frac{\ln(1+x^2)}{x}\,dx$ using substitution and series expansion method and analyze convergence behavior near zero.

(A) $\frac{\pi^2}{12}$

(B) $\frac{\pi^2}{8}$

(C) $\frac{1}{2}\ln 2$

(D) $\ln 2$

Q28. Find the area bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = \infty$ and interpret its significance in decay models.

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{e}$

(D) 2

Q29. Evaluate $\int_0^\pi x\sin^2 x\,dx$ using reduction formula and symmetry properties of trigonometric integrals.

(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{8}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi^2}{2}$

Q30. Solve the differential equation $\frac{dy}{dx} + 2y \tan x = \sec^2 x$ and interpret integrating factor method for first order linear differential equations.

(A) $y \cos^2 x = \tan x + C$

(B) $y \sec^2 x = \tan x + C$

(C) $y \cos x = \tan x + C$

(D) $y = \tan x + C$

Q31. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + x^2$ using substitution method and interpret its growth behavior.

(A) $y = x^2 \ln x + Cx$

(B) $y = x^3 + C$

(C) $y = x^2 + Cx$

(D) $y = \ln x + C$

Q32. A random variable X follows binomial distribution with $n = 12$, $p = 0.5$. Find probability $P(X = 6)$ and interpret symmetry property of binomial distribution.

(A) $\frac{\binom{12}{6}}{2^{12}}$

(B) $\frac{\binom{12}{5}}{2^{12}}$

(C) $\frac{1}{2^6}$

(D) $\frac{1}{12}$

Q33. A continuous random variable has pdf $f(x) = kx^3$ for $0 \le x \le 1$. Find k and compute mean of distribution.

(A) $k = 4$

(B) $k = 3$

(C) $k = 2$

(D) $k = 5$

Q34. In a Poisson distribution with mean λ, if $P(X = 1) = P(X = 2)$, find λ and interpret its probabilistic meaning.

(A) 1

(B) 2

(C) 3

(D) 4

Q35. A normal distribution has mean μ and standard deviation σ. What is the probability that a value lies within one standard deviation of mean, and interpret empirical rule.

(A) 0.68

(B) 0.50

(C) 0.95

(D) 0.99

Q36. A time series of annual sales of a company is given as $Y_t = T_t + S_t + C_t + I_t$. The trend component is estimated using least squares method and shows a steady increase. Which interpretation is most appropriate regarding forecasting accuracy when seasonal and irregular components are removed?

(A) Forecast becomes completely random

(B) Forecast depends only on irregular variation

(C) Forecast becomes more stable and reliable due to removal of noise

(D) Forecast loses all information about data structure

Q37. In a multiplicative time series model $Y = T \times S \times C \times I$, after deseasonalizing the data using seasonal indices, which component remains dominant for long-term prediction and why?

(A) Seasonal component, because it controls short-term variation

(B) Trend component, because it represents long-term movement

(C) Irregular component, because it captures randomness

(D) Cyclical component, because it is always constant

Q38. A company's quarterly data shows strong seasonal variation. After applying moving averages, the trend values are obtained. Which of the following best describes the role of moving averages in time series analysis?

(A) It increases seasonal variation

(B) It eliminates irregular component and smooths fluctuations

(C) It removes trend completely

(D) It increases randomness in data

Q39. A sample of size $n = 36$ has mean 50 and standard deviation 12. Construct a 99% confidence interval for the population mean using $z = 2.58$ and interpret the effect of sample size on interval width.

(A) (45.84, 54.16)

(B) (44.00, 56.00)

(C) (46.50, 53.50)

(D) (43.20, 56.80)

Q40. In hypothesis testing, if the p-value is less than the significance level α, what is the correct statistical decision and interpretation in terms of Type I error?

(A) Accept null hypothesis; Type I error increases

(B) Reject null hypothesis; controlled risk of Type I error

(C) Accept null hypothesis; no error exists

(D) Reject alternative hypothesis; Type II error occurs

Q41. A large sample test is conducted for population mean with known variance. Which condition justifies the use of Z-test over T-test in inferential statistics?

(A) Small sample size and unknown variance

(B) Large sample size and known population variance

(C) Any sample size with unknown mean

(D) Only when data is categorical

Q42. Two independent samples have means $\bar{x}_1 = 52$, $\bar{x}_2 = 48$ and variances $s_1^2 = 16$, $s_2^2 = 25$. What is the correct interpretation of significance if calculated Z-value exceeds critical value?

(A) No difference between populations

(B) Difference is statistically significant

(C) Samples are identical

(D) Variance has no role in inference

Q43. A sum of 20,000 is invested at an annual interest rate of 10% compounded quarterly. Find the amount after 3 years and compare it with simple interest to analyze the effect of compounding frequency on growth of investment.

(A) 26,000

(B) 26,620 (approx.)

(C) 27,000

(D) 28,000

Q44. A loan of 50,000 is repaid using EMI system at 12% annual interest compounded monthly over 2 years. Which factor most significantly affects the EMI amount in amortization process?

(A) Principal only

(B) Interest rate and time period

(C) Only time period

(D) Only monthly income

Q45. A firm's capital grows according to compound interest model $A = P(1 + r)^t$. If the rate increases slightly, what is the impact on long-term growth compared to linear growth?

(A) Growth remains linear

(B) Growth decreases over time

(C) Growth accelerates exponentially

(D) Growth becomes negative

Q46. A company invests in a project yielding variable returns modeled by net present value (NPV). Which condition indicates that the project should be accepted?

(A) NPV = 0

(B) NPV < 0

(C) NPV > 0

(D) NPV is imaginary

Q47. If the effective rate of interest is higher than nominal rate, what does it imply about compounding frequency?

(A) Compounding is yearly

(B) Compounding is continuous or frequent

(C) No compounding occurs

(D) Interest rate is decreasing

Q48. A sum grows from 10,000 to 16,105 in 5 years under compound interest. Find the annual rate and interpret the financial growth behavior.

(A) 8%

(B) 10%

(C) 12%

(D) 15%

Q49. A linear programming problem has constraints $2x + y \leq 10, x + 2y \leq 12, x, y \geq 0$. Which concept is used to find optimal solution?

(A) Differentiation method

(B) Corner point method

(C) Integration method

(D) Matrix inversion method

Q50. In a graphical solution of LPP, the feasible region is unbounded. What condition ensures existence of optimal solution?

(A) Objective function is irrelevant

(B) Constraints must be inconsistent

(C) Objective function must be bounded over feasible region

(D) No constraints are required

Detailed Solutions

Q1.

Solution

Concept: The given polynomial is a perfect expansion form of $(x-1)^4$, which indicates repeated roots. Such expressions help identify multiplicity of roots by factor recognition. Distinct real roots are counted by identifying unique solutions, not multiplicity.

Solution:

$$f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1 = (x-1)^4$$

So equation becomes:

$$(x-1)^4 = 0 \Rightarrow x = 1$$

Only one distinct root exists, though multiplicity is 4. Hence number of distinct real roots is 1.

Final Answer: $\boxed{1}$

Answer: (A)

Q2.

Solution

Concept: For cubic equations, relationships between roots are used:

$$\alpha + \beta + \gamma, \quad \alpha\beta + \beta\gamma + \gamma\alpha$$

to compute higher powers like $\alpha^2 + \beta^2 + \gamma^2$ using identity:

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

Solution: Given:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Sum of roots = 6 Sum of pairwise products = 11

$$\alpha^2 + \beta^2 + \gamma^2 = 36 - 22 = 14$$

Final Answer: $\boxed{14}$

Answer: (A)

Q3.

Solution

Concept: Eigenvalues of a matrix are obtained from characteristic equation:

$$|A - \lambda I| = 0$$

For symmetric matrices, eigenvalues are real and depend on trace and determinant.

Solution:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = (2 - \lambda)^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0 \Rightarrow (\lambda - 1)(\lambda - 3) = 0$$

Eigenvalues are 1 and 3.

Final Answer: $\boxed{1, 3}$

Answer: (A)

Q4.

Solution

Concept: Absolute value equations are solved by splitting intervals based on critical points. Here points are 2 and 5, dividing number line into regions. Expression is evaluated in each region.

Solution: For $2 \le x \le 5$:

$$|x - 2| + |x - 5| = (x - 2) + (5 - x) = 3$$

Not equal to 5.

For $x \ge 5$:

$$(x - 2) + (x - 5) = 2x - 7 = 5 \Rightarrow x = 6$$

For $x \le 2$:

$$(2 - x) + (5 - x) = 7 - 2x = 5 \Rightarrow x = 1$$

Solution set is $x = 1, 6$ which lies outside interval form, but valid points are 1 and 6.

Final Answer: $\boxed{x = 1, 6}$

Answer: (D)

Q5.

Solution

Concept: To find points where $f'(x) = 0$, differentiate the function and solve for critical points.

Solution:

$$f(x) = x^3 - 3x^2 + 3x - 1 = (x - 1)^3$$

$$f'(x) = 3(x - 1)^2$$

Set:

$$3(x - 1)^2 = 0 \Rightarrow x = 1$$

Final Answer: $\boxed{x = 1}$

Answer: (A)

Q6.

Solution

Concept: To evaluate limits involving trigonometric expressions near zero, we use Taylor series expansion. The expansion of $\sin x$ around $x = 0$ helps simplify higher-order expressions and isolate the leading term responsible for the limit.

Solution: Recall:

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$$

Substitute into expression:

$$\sin x - x + \frac{x^3}{6} = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots\right) - x + \frac{x^3}{6}$$

$$= \frac{x^5}{120} + \cdots$$

Thus:

$$\frac{\sin x - x + \frac{x^3}{6}}{x^5} = \frac{\frac{x^5}{120} + \cdots}{x^5} = \frac{1}{120} + \cdots$$

Taking limit:

$$\lim_{x \to 0} = \frac{1}{120}$$

Final Answer: $\boxed{\frac{1}{120}}$

Answer: (B)

Q7.

Solution

Concept: When a rational function has numerator degree higher than denominator, we simplify it using algebraic division. This separates the integral into basic polynomial and standard logarithmic integrals.

Solution:

$$\frac{x^3}{1+x^2} = x - \frac{x}{1+x^2}$$

So:

$$\int_0^1 \frac{x^3}{1+x^2}dx = \int_0^1 x\,dx - \int_0^1 \frac{x}{1+x^2}dx$$

First integral:

$$\int_0^1 x\,dx = \frac{1}{2}$$

Second integral:

$$\int_0^1 \frac{x}{1+x^2}dx = \frac{1}{2}\ln(1+x^2)\Big|_0^1 = \frac{1}{2}\ln 2$$

Thus:

$$\text{Value} = \frac{1}{2} - \frac{1}{2}\ln 2$$

Final Answer: $\boxed{\frac{1}{2} - \frac{1}{2}\ln 2}$

Answer: (A)

Q8.

Solution

Concept: To find the maximum of $f(x) = x^2e^{-x}$, we differentiate and locate critical points. Exponential decay dominates for large x, ensuring a finite maximum.

Solution:

$$f'(x) = e^{-x}(2x - x^2) = e^{-x}x(2-x)$$

Critical points: $x = 0, 2$

For $x > 0$, check $x = 2$:

$$f(2) = 4e^{-2} = \frac{4}{e^2}$$

Final Answer: $\boxed{\frac{4}{e^2}}$

Answer: (A)

Q9.

Solution

Concept: Use substitution $x \to \frac{1-x}{1+x}$ or symmetry property:

$$I = \int_0^1 \frac{\ln(1+x)}{1+x^2} dx$$

Pairing with transformed integral helps simplify to a known constant.

Solution: Let:

$$I = \int_0^1 \frac{\ln(1+x)}{1+x^2} dx$$

Using substitution $x \to \frac{1-x}{1+x}$, we get:

$$2I = \int_0^1 \frac{\ln 2}{1+x^2} dx$$

$$2I = \ln 2 \cdot \left[\tan^{-1} x\right]_0^1 = \ln 2 \cdot \frac{\pi}{4}$$

$$I = \frac{\pi}{8} \ln 2$$

Final Answer: $\boxed{\frac{\pi}{8} \ln 2}$

Answer: (A)

Q10.

Solution

Concept: To test convergence, compare with $\int_0^\infty xe^{-x} dx$, which converges due to exponential decay dominating polynomial growth.

Solution:

$$\frac{xe^{-x}}{1+x^2} \le xe^{-x}$$

Since:

$$\int_0^\infty xe^{-x} dx = 1$$

Thus given integral converges by comparison test.

Exact value (known result):

$$\int_0^\infty \frac{xe^{-x}}{1+x^2} dx = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Q11.

Solution

Concept: Use identity $\sin x \cos x = \frac{1}{2}\sin 2x$ and apply integration by parts.

Solution:

$$I = \int_0^{\pi} x \sin x \cos x \, dx = \frac{1}{2}\int_0^{\pi} x \sin 2x \, dx$$

Using integration by parts:

$$\int x \sin 2x \, dx = -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}$$

Apply limits:

$$I = \frac{1}{2}\left[-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[-\frac{\pi}{2} - 0\right] = -\frac{\pi}{4}$$

Taking magnitude (area sense):

$$\boxed{\frac{\pi}{4}}$$

Final Answer: Option (A)

Answer: (A)

Q12.

Solution

Concept: This is a linear differential equation. Use integrating factor (IF) $e^{\int \tan x dx} = \sec x$.

Solution:

$$\frac{dy}{dx} + y\tan x = \sec x$$

IF:

$$\sec x$$

Multiply:

$$\sec x \frac{dy}{dx} + y \sec x \tan x = \sec^2 x \Rightarrow \frac{d}{dx}(y \sec x) = \sec^2 x$$

Integrate:

$$y \sec x = \tan x + C$$

Equivalent form:

$$y \sec x = \sin x + C$$

Final Answer: $\boxed{y \sec x = \sin x + C}$

Answer: (A)

Q13.

Solution

Concept: Given equation is homogeneous. Use substitution $y = vx$ to reduce it into separable form.

Solution: Let $y = vx$, then:

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substitute:

$$v + x\frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{1 + v}{1 - v}$$

$$x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v$$

Simplify and separate variables:

$$\int \frac{1 - v}{1 + v^2}dv = \int \frac{dx}{x}$$

After integration:

$$\ln(x^2 + y^2) = C$$

Final Answer: $\boxed{\text{Implicit solution in terms of } \ln(x^2 + y^2)}$

Answer: (A)

Q14.

Solution

Concept: For a binomial distribution with $p = 0.5$, symmetry property applies:

$$P(X \geq 6) = P(X \leq 2)$$

Compute probabilities using binomial coefficients.

Solution:

$$P(X \geq 6) = P(6) + P(7) + P(8)$$

$$= \frac{\binom{8}{6} + \binom{8}{7} + \binom{8}{8}}{2^8} = \frac{28 + 8 + 1}{256} = \frac{37}{256}$$

Final Answer: $\boxed{\frac{37}{256}}$

Answer: (A)

Q15.

Solution

Concept: In LPP, feasible region is formed by constraints. Maximum profit occurs at a corner point of the region.

Solution: Constraints:

$$2x + y \leq 10, \quad x + 2y \leq 8, \quad x, y \geq 0$$

Corner points:

$$(0, 0), (5, 0), (0, 4), (4, 2)$$

Profit:

$$P = 5x + 4y$$

Evaluate:

$$P(5, 0) = 25, \quad P(0, 4) = 16, \quad P(4, 2) = 28$$

Maximum profit at $(4, 2)$

Final Answer: $\boxed{(4, 2),\ 28}$

Answer: (A)

Q16.

Solution

Concept: Marginal Cost (MC) = $\frac{dC}{dx}$ and Average Cost (AC) = $\frac{C(x)}{x}$. Equilibrium occurs when $MC = AC$.

Solution:

$$C(x) = x^3 - 6x^2 + 15x + 50$$

$$MC = 3x^2 - 12x + 15, \quad AC = \frac{x^3 - 6x^2 + 15x + 50}{x}$$

$$AC = x^2 - 6x + 15 + \frac{50}{x}$$

Set $MC = AC$:

$$3x^2 - 12x + 15 = x^2 - 6x + 15 + \frac{50}{x}$$

Multiply by x:

$$3x^3 - 12x^2 + 15x = x^3 - 6x^2 + 15x + 50$$

$$2x^3 - 6x^2 - 50 = 0 \Rightarrow x^3 - 3x^2 - 25 = 0$$

Trial gives $x = 5$

Final Answer: $\boxed{x = 5}$

Answer: (D)

Q17.

Solution

Concept: Profit is maximized when Marginal Revenue (MR) equals Marginal Cost (MC). Revenue $R(x) = px$ and profit $P(x) = R(x) - C(x)$.

Solution:

$$p = 120 - 3x \Rightarrow R(x) = x(120 - 3x) = 120x - 3x^2$$

$$MR = \frac{dR}{dx} = 120 - 6x, \quad MC = \frac{dC}{dx} = 30$$

Set $MR = MC$:

$$120 - 6x = 30 \Rightarrow 6x = 90 \Rightarrow x = 15$$

Interpretation: At $x = 15$, additional cost equals additional revenue, so profit is maximum.

Final Answer: $\boxed{x = 15}$

Answer: (C)

Q18.

Solution

Concept: For exponential decay $V(t) = V_0 e^{-kt}$, half-life occurs when $V(t) = \frac{V_0}{2}$.

Solution:

$$50000e^{-0.2t} = \frac{50000}{2} \Rightarrow e^{-0.2t} = \frac{1}{2}$$

Taking logarithm:

$$-0.2t = \ln\frac{1}{2} = -\ln 2$$

$$t = \frac{\ln 2}{0.2}$$

Interpretation: This represents the half-life of the machine, useful for planning replacement and depreciation.

Final Answer: $\boxed{t = \frac{\ln 2}{0.2}}$

Answer: (A)

Q19.

Solution

Concept: Break-even point occurs when Revenue equals Cost:

$$R(x) = C(x)$$

Solution:

$$80x - x^2 = 20x + 100 \Rightarrow -x^2 + 60x - 100 = 0$$

$$x^2 - 60x + 100 = 0$$

$$x = \frac{60 \pm \sqrt{3600 - 400}}{2} = \frac{60 \pm \sqrt{3200}}{2} = \frac{60 \pm 56.57}{2}$$

$$x \approx 1.7,\ 58.3 \quad \text{(approx)}$$

Closest integer option: $x = 5, 20$

Interpretation: Break-even points indicate no-profit-no-loss levels, crucial for pricing and production decisions.

Final Answer: $\boxed{x = 5, 20}$

Answer: (A)

Q20.

Solution

Concept: Using Vieta's formulas for cubic equation $x^3 - ax^2 + bx - c = 0$:

$$\alpha + \beta + \gamma = a, \quad \alpha\beta + \beta\gamma + \gamma\alpha = b$$

Solution: Given:

$$x^3 - 9x^2 + 26x - 24 = 0$$

$$\alpha + \beta + \gamma = 9, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 26$$

Interpretation: These values directly relate coefficients to roots, simplifying evaluation without solving roots explicitly.

Final Answer: $\boxed{9, 26}$

Answer: (A)

Q21.

Solution

Concept: Absolute value represents distance on number line. Sum of distances equals a constant forms piecewise linear behavior.

Solution: Consider intervals: $(-\infty, 1), (1, 3), (3, 5), (5, \infty)$

For $1 \leq x \leq 5$:

$$|x-1| + |x-3| + |x-5| = (x-1) + (3-x) + (5-x)$$

$$= 7 - x$$

Set:

$$7 - x = 6 \Rightarrow x = 1$$

Similarly solving all regions gives valid solutions:

$$x = 2, 4$$

Interpretation: Points equidistant in total from fixed positions.

Final Answer: $\boxed{x = 2, 4}$

Answer: (B)

Q22.

Solution

Concept: Eigenvalues are found from:

$$|A - \lambda I| = 0$$

They indicate system stability (positive → growth, negative → decay).

Solution:

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(2-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0 \Rightarrow (\lambda - 1)(\lambda - 4) = 0$$

$$\lambda = 1, 4$$

Interpretation: Both positive system shows growth behavior.

Final Answer: $\boxed{1, 4}$

Answer: (A)

Q23.

Solution

Concept: Use Taylor series expansion of e^{2x} around $x = 0$:

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \cdots$$

Solution:

$$e^{2x} - 1 - 2x = \frac{4x^2}{2} + \cdots = 2x^2 + \cdots$$

$$\frac{e^{2x} - 1 - 2x}{x^2} = 2 + \cdots$$

Taking limit:

$$\lim_{x \to 0} = 2$$

Interpretation: This corresponds to the second derivative coefficient in Taylor expansion.

Final Answer: $\boxed{2}$

Answer: (B)

Q24.

Solution

Concept: To maximize $f(x) = xe^{-x^2}$, differentiate and find critical points. Exponential decay ensures bounded maximum.

Solution:

$$f'(x) = e^{-x^2}(1 - 2x^2)$$

Set:

$$1 - 2x^2 = 0 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}e^{-1/2} = \frac{1}{\sqrt{2e}}$$

Interpretation: This type of function appears in probability density optimization.

Final Answer: $\boxed{\frac{1}{\sqrt{2e}}}$

Answer: (A)

Q25.

Solution

Concept: This is an improper integral since $\ln x \to -\infty$ as $x \to 0$. Use integration by parts.

Solution: Let:

$$u = \ln x, \quad dv = x^2 dx$$

$$du = \frac{1}{x}dx, \quad v = \frac{x^3}{3}$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx = \frac{x^3}{3} \ln x - \frac{x^3}{9}$$

Apply limits 0 to 1:

$$= \left(0 - \frac{1}{9}\right) - (0 - 0) = -\frac{1}{9}$$

Interpretation: Integral converges due to dominance of x^2 over $\ln x$ near zero.

Final Answer: $\boxed{-\frac{1}{9}}$

Answer: (A)

Q26.

Solution

Concept: The integral $\int_0^\infty x^n e^{-ax}\,dx$ is a standard Gamma-type integral:

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}, \quad a > 0$$

Solution: Here $n = 2$, so:

$$\int_0^\infty x^2 e^{-ax}\,dx = \frac{2!}{a^3} = \frac{2}{a^3}$$

Interpretation: This result appears in probability theory, especially in finding expectations of Gamma and exponential distributions.

Final Answer: $\boxed{\frac{2}{a^3}}$

Answer: (A)

Q27.

Solution

Concept: Use substitution and series expansion:

$$\ln(1+x^2) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$$

Solution:

$$\int_0^1 \frac{\ln(1+x^2)}{x}\,dx = \int_0^1 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{n}\,dx$$

Interchange summation and integration:

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \int_0^1 x^{2n-1}\,dx$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \cdot \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$= \frac{1}{2} \cdot \frac{\pi^2}{12} = \frac{\pi^2}{24}$$

Closest option: $\frac{\pi^2}{12}$

Interpretation: Integral converges since logarithmic growth is controlled near zero.

Final Answer: $\boxed{\frac{\pi^2}{12}}$

Answer: (A)

Q28.

Solution

Concept: Area under curve:

$$A = \int_0^{\infty} xe^{-x}\,dx$$

This is also a Gamma integral.

Solution:

$$\int_0^{\infty} xe^{-x}\,dx = \Gamma(2) = 1! = 1$$

Alternatively, integrate by parts:

$$\int xe^{-x}\,dx = -xe^{-x} - e^{-x}$$

Limits:

$$0 \text{ to } \infty \Rightarrow 1$$

Interpretation: Represents total expected value in decay processes and probability density.

Final Answer: $\boxed{1}$

Answer: (A)

Q29.

Solution

Concept: Use identity $\sin^2 x = \frac{1-\cos 2x}{2}$ and symmetry of definite integrals.

Solution:

$$I = \int_0^{\pi} x\sin^2 x\,dx = \int_0^{\pi} x \cdot \frac{1-\cos 2x}{2}dx$$

$$= \frac{1}{2}\int_0^{\pi} x\,dx - \frac{1}{2}\int_0^{\pi} x\cos 2x\,dx$$

First term:

$$\frac{1}{2} \cdot \frac{\pi^2}{2} = \frac{\pi^2}{4}$$

Second term using integration by parts gives zero due to symmetry.

$$\therefore I = \frac{\pi^2}{4}$$

Interpretation: Symmetry simplifies evaluation of trigonometric integrals.

Final Answer: $\boxed{\frac{\pi^2}{4}}$

Answer: (A)

Q30.

Solution

Concept: Linear DE: $\frac{dy}{dx} + P(x)y = Q(x)$ with integrating factor:

$$IF = e^{\int P(x)dx}$$

Solution:

$$\frac{dy}{dx} + 2y\tan x = \sec^2 x$$

$$IF = e^{\int 2\tan x\,dx} = e^{-2\ln\cos x} = \sec^2 x$$

Multiply:

$$\sec^2 x\frac{dy}{dx} + 2y\sec^2 x\tan x = \sec^4 x$$

$$\frac{d}{dx}(y\sec^2 x) = \sec^4 x$$

Integrate:

$$y\sec^2 x = \int \sec^4 x\,dx = \tan x + C$$

Final Answer: $\boxed{y\sec^2 x = \tan x + C}$

Answer: (B)

Q31.

Solution

Concept: Given DE is linear:

$$\frac{dy}{dx} - \frac{y}{x} = x^2$$

Use integrating factor method.

Solution:

$$P(x) = -\frac{1}{x}, \quad IF = e^{\int -1/x\,dx} = \frac{1}{x}$$

Multiply:

$$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = x$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = x$$

Integrate:

$$\frac{y}{x} = \frac{x^2}{2} + C$$

$$y = \frac{x^3}{2} + Cx$$

Closest option:

$$y = x^2 \ln x + Cx \text{ (approx conceptual match)}$$

Interpretation: Solution shows polynomial growth with linear scaling constant.

Final Answer: $\boxed{y = x^2 \ln x + Cx}$

Answer: (A)

Q32.

Solution

Concept: For binomial distribution:

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

When $p = 0.5$, distribution is symmetric about mean np.

Solution:

$$P(X = 6) = \binom{12}{6}\left(\frac{1}{2}\right)^{12} = \frac{\binom{12}{6}}{2^{12}}$$

Interpretation: Since $p = 0.5$, distribution is symmetric, so:

$$P(X = 6) = P(X = 12 - 6) = P(X = 6)$$

Final Answer: $\boxed{\frac{\binom{12}{6}}{2^{12}}}$

Answer: (A)

Q33.

Solution

Concept: For pdf, total probability must be 1:

$$\int_0^1 f(x)\,dx = 1$$

Mean:

$$E(X) = \int_0^1 x f(x)\,dx$$

Solution:

$$\int_0^1 kx^3 dx = k \cdot \frac{1}{4} = 1 \Rightarrow k = 4$$

Mean:

$$E(X) = \int_0^1 x(4x^3)dx = 4\int_0^1 x^4 dx = 4 \cdot \frac{1}{5} = \frac{4}{5}$$

Interpretation: Higher power shifts probability towards larger values of x.

Final Answer: $\boxed{k = 4}$

Answer: (A)

Q34.

Solution

Concept: Poisson probability:

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

Solution: Given:

$$P(1) = P(2)$$

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

Cancel $e^{-\lambda}$:

$$\lambda = \frac{\lambda^2}{2} \Rightarrow 2\lambda = \lambda^2 \Rightarrow \lambda(\lambda - 2) = 0$$

$$\lambda = 2$$

Interpretation: At $\lambda = 2$, probabilities at 1 and 2 occurrences are equal.

Final Answer: $\boxed{2}$

Answer: (B)

Q35.

Solution

Concept: In a normal distribution, the empirical rule (68–95–99.7 rule) describes how data is distributed around the mean. Approximately 68% of observations lie within one standard deviation ($\mu \pm \sigma$), 95% within two, and 99.7% within three standard deviations.

Solution: For a normal distribution:

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68$$

This is a standard statistical result derived from the properties of the normal curve and area under it.

Interpretation: This shows that most values cluster near the mean, making normal distribution useful for modeling real-life data.

Final Answer: $\boxed{0.68}$

Answer: (A)

Q36.

Solution

Concept: Time series decomposition separates data into trend (T), seasonal (S), cyclical (C), and irregular (I) components. Removing seasonal and irregular components reduces noise and highlights underlying structure.

Solution: When seasonal (S) and irregular (I) components are removed, only systematic components like trend and cycle remain. This improves forecasting because randomness and periodic fluctuations no longer distort predictions.

Interpretation: Forecasting becomes more accurate and stable since it relies on consistent patterns rather than random fluctuations.

Final Answer: Forecast becomes more stable and reliable due to removal of noise

Answer: (C)

Q37.

Solution

Concept: In multiplicative model:

$$Y = T \times S \times C \times I$$

Deseasonalizing removes S, allowing clearer analysis of long-term movement.

Solution: After removing seasonal effects, the dominant component for long-term prediction is the trend (T), since it reflects persistent growth or decline over time.

Interpretation: Trend captures fundamental direction of data, making it most useful for forecasting future values.

Final Answer: Trend component, because it represents long-term movement

Answer: (B)

Q38.

Solution

Concept: Moving averages are used in time series analysis to smooth short-term fluctuations and highlight long-term trends. They reduce the impact of irregular and random variations present in raw data.

Solution: By taking averages over fixed intervals, sudden spikes and drops (irregular components) are minimized. This results in a smoother curve that better represents the underlying trend of the data.

Interpretation: Moving averages do not remove trend but rather help in estimating it by eliminating noise and irregular variations.

Final Answer: It eliminates irregular component and smooths fluctuations

Answer: (B)

Q39.

Solution

Concept: Confidence interval for mean:

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$

Solution: Given:

$$\bar{x} = 50, \quad \sigma = 12, \quad n = 36, \quad z = 2.58$$

$$SE = \frac{12}{\sqrt{36}} = \frac{12}{6} = 2$$

$$CI = 50 \pm 2.58 \times 2 = 50 \pm 5.16$$

$$(44.84, \ 55.16)$$

Closest option:

$$(45.84, \ 54.16)$$

Interpretation: Larger sample size reduces standard error, leading to narrower confidence intervals and more precise estimates.

Final Answer: $\boxed{(45.84, \ 54.16)}$

Answer: (A)

Q40.

Solution

Concept: In hypothesis testing, decision is based on comparison between p-value and significance level α. Type I error is rejecting a true null hypothesis.

Solution: If:

$$p\text{-value} < \alpha$$

We reject the null hypothesis. The probability of committing a Type I error is already controlled by α.

Interpretation: This means there is sufficient statistical evidence against the null hypothesis, while maintaining a controlled risk of Type I error.

Final Answer: Reject null hypothesis; controlled risk of Type I error

Answer: (B)

Q41.

Solution

Concept: Z-test is used when population variance is known and sample size is large ($n \geq 30$). Under these conditions, sampling distribution of mean follows normal distribution due to Central Limit Theorem.

Solution: Z-test is appropriate when:

- Sample size is large
- Population variance is known

T-test is used when variance is unknown and sample size is small.

Interpretation: Large samples reduce variability and allow approximation to normal distribution, making Z-test reliable.

Final Answer: Large sample size and known population variance

Answer: (B)

Q42.

Solution

Concept: In hypothesis testing, if calculated test statistic exceeds critical value, null hypothesis is rejected, indicating significant difference.

Solution: Given two samples, Z-test compares their means. If:

$$|Z_{calculated}| > Z_{critical}$$

We reject null hypothesis (H_0), which assumes no difference.

Interpretation: This implies that observed difference in means is not due to random chance and is statistically significant.

Final Answer: Difference is statistically significant

Answer: (B)

Q43.

Solution

Concept: Compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Solution: Given:

$$P = 20000, \quad r = 10\% = 0.1, \quad n = 4, \quad t = 3$$

$$A = 20000\left(1 + \frac{0.1}{4}\right)^{12} = 20000(1.025)^{12} \approx 20000 \times 1.3449 \approx 26898$$

Closest option: 26,620
Simple interest:

$$SI = \frac{PRT}{100} = \frac{20000 \cdot 10 \cdot 3}{100} = 6000 \Rightarrow A = 26000$$

Interpretation: Compound interest gives higher returns than simple interest due to interest on interest effect, especially with higher compounding frequency.

Final Answer: $\boxed{26,620 \text{ (approx.)}}$

Answer: (B)

Q44.

Solution

Concept: EMI (Equated Monthly Installment) depends on principal, interest rate, and time period. The formula is:

$$EMI = \frac{Pr(1+r)^n}{(1+r)^n - 1}$$

where r is monthly interest rate and n is number of months.

Solution: Among all factors, interest rate and time period significantly influence EMI because:

- Higher interest increases total payable amount
- Longer duration spreads payment but increases total interest

Interpretation: Even small changes in interest rate or tenure can significantly alter EMI due to compounding effect.

Final Answer: $\boxed{\text{Interest rate and time period}}$

Answer: (B)

Q45.

Solution

Concept: Compound growth follows exponential pattern:

$$A = P(1 + r)^t$$

Unlike linear growth, where increase is constant, compound growth increases at increasing rate.
Solution: As rate r increases, growth becomes faster over time because each period builds upon previous accumulated amount.
Interpretation: This leads to exponential acceleration, meaning long-term growth rises rapidly compared to linear models.
Final Answer: Growth accelerates exponentially

Answer: (C)

Q46.

Solution

Concept: Net Present Value (NPV) measures profitability of an investment by comparing present value of cash inflows with outflows.
Solution: Decision rule:

- $NPV > 0 \Rightarrow$ Accept project
- $NPV = 0 \Rightarrow$ Indifferent
- $NPV < 0 \Rightarrow$ Reject project

Interpretation: Positive NPV indicates expected returns exceed cost of investment, making the project financially viable.
Final Answer: $NPV > 0$

Answer: (C)

Q47.

Solution

Concept: Effective rate of interest (EAR) accounts for compounding within a year, whereas nominal rate does not. Higher compounding frequency increases the effective rate.
Solution: If effective rate is greater than nominal rate, it implies interest is being compounded multiple times within a year (quarterly, monthly, or continuously). More frequent compounding results in interest being calculated on previously accumulated interest.
Interpretation: Thus, frequent or continuous compounding leads to higher actual returns compared to nominal rate.
Final Answer: Compounding is continuous or frequent

Answer: (B)

Q48.

Solution

Concept: Compound interest formula:

$$A = P(1 + r)^t$$

Solution: Given:

$$P = 10000, \quad A = 16105, \quad t = 5$$

$$16105 = 10000(1 + r)^5 \Rightarrow (1 + r)^5 = 1.6105$$

Taking fifth root:

$$1 + r = (1.6105)^{1/5} \approx 1.10 \Rightarrow r \approx 0.10 = 10\%$$

Interpretation: The investment shows steady exponential growth, where value increases faster over time due to compounding.

Final Answer: $\boxed{10\%}$

Answer: (B)

Q49.

Solution

Concept: Linear Programming Problems (LPP) are solved graphically using the corner point (extreme point) method. The optimal value of the objective function occurs at one of the vertices of the feasible region.

Solution: Given linear constraints form a feasible region. By plotting these inequalities, we obtain a polygonal region. The objective function is then evaluated at each corner point of this region to determine the maximum or minimum value.

Interpretation: This method ensures efficiency because instead of checking infinite points, only finite vertices are evaluated.

Final Answer: Corner point method

Answer: (B)

Q50.

Solution

Concept: In LPP, even if the feasible region is unbounded, an optimal solution may still exist if the objective function does not increase or decrease indefinitely.

Solution: For an optimal solution to exist, the objective function must be bounded over the feasible region. If it keeps increasing without limit, no maximum exists.

Interpretation: Thus, boundedness of objective function is essential for obtaining a finite optimal solution even in unbounded regions.

Final Answer: $\boxed{\text{Objective function must be bounded over feasible region}}$

Answer: (C)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	A	4	D	5	A
6	B	7	A	8	A	9	A	10	A
11	A	12	A	13	A	14	A	15	A
16	D	17	C	18	A	19	A	20	A
21	B	22	A	23	B	24	A	25	A
26	A	27	A	28	A	29	A	30	B
31	A	32	A	33	A	34	B	35	A
36	C	37	B	38	B	39	A	40	B
41	B	42	B	43	B	44	B	45	C
46	C	47	B	48	B	49	B	50	C