CUET-UG Applied Mathematics Sample Paper-14

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix of order 3 such that $|A| = 4$, then the value of $|2A|$ is:

(A) 8

(B) 16

(C) 32

(D) 64

Q2. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are:

(A) $k = -6, a = -4, b = -9$

(B) $k = 6, a = 4, b = 9$

(C) $k = -6, a = 4, b = 9$

(D) $k = -6, a = -4, b = 9$

Q3. For a 3×3 matrix A, if $|adjA| = 64$, then $|A|$ can be:

(A) ± 8

(B) ± 4

(C) ± 64

(D) ± 16

Q4. If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is:

(A) 1

(B) -1

(C) 0

(D) 3

Q5. The function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing in the interval:

(A) $(2, 3)$

(B) $(-\infty, 2)$

(C) $(3, \infty)$

(D) $(-\infty, 2) \cup (3, \infty)$

Q6. If the radius of a circle is increasing at the rate of 0.5 cm/s, the rate of increase of its area when the radius is 10 cm is:

(A) 5π cm^2/s

(B) 10π cm^2/s

(C) 20π cm^2/s

(D) 15π cm^2/s

Q7. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $1/\sqrt{2}$

Q8. For the curve $y = x^2 - 4x + 5$, the coordinates of the point where the tangent is parallel to the x-axis are:

(A) $(2, 1)$

(B) $(1, 2)$

(C) $(4, 5)$

(D) $(0, 5)$

Q9. The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 1

(D) $\log 2$

Q10. The area bounded by the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is:

(A) 1 sq. unit

(B) 2 sq. units

(C) π sq. units

(D) 4 sq. units

Q11. $\int e^x(\tan x + \sec^2 x)dx$ is equal to:

(A) $e^x \tan x + C$

(B) $e^x \sec x + C$

(C) $e^x \sec^2 x + C$

(D) $e^x(\tan x + \sec x) + C$

Q12. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt{\frac{dy}{dx}} = x$ are respectively:

(A) 2, 1

(B) 2, 2

(C) 1, 2

(D) 2, not defined

Q13. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y = \tan^{-1} x + C$

(B) $y - x = C(1 + xy)$

(C) Both A and B

(D) $\tan^{-1} y \cdot \tan^{-1} x = C$

Q14. Two cards are drawn at random from a pack of 52 cards. The probability that both are hearts is:

(A) $1/17$

(B) $1/26$

(C) $13/52$

(D) $1/16$

Q15. In a LPP, if the objective function is $Z = ax + by$ and the corner points of the feasible region are $(0, 10), (5, 5), (15, 15), (0, 20)$, the minimum value occurs at $(5, 5)$. Which condition must a and b satisfy?

(A) $a > b$

(B) $a = b$

(C) $a < b$

(D) $a + b = 0$

Q16. What is the value of $(24 \times 32) \pmod 7$?

(A) 1

(B) 2

(C) 3

(D) 5

Q17. A boatman can row 48 km downstream in 4 hours and 36 km upstream in 6 hours. The speed of the stream is:

(A) 3 km/h

(B) 6 km/h

(C) 9 km/h

(D) 12 km/h

Q18. In a 100m race, A runs at 8 km/h. If A gives B a start of 4 m and still beats him by 15 seconds, the speed of B is:

(A) 5.76 km/h

(B) 6 km/h

(C) 4.5 km/h

(D) 6.24 km/h

Q19. If $x \equiv 3 \pmod 5$, then $x^2 + 2x + 1 \pmod 5$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q20. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, then A^{-1} using cofactors is:

(A) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$

Q21. For the system $3x + y = 5$ and $2x - y = 0$, the value of x using Cramer's rule is:

(A) 1

(B) 2

(C) 5

(D) 0

Q22. A system of equations $AX = B$ is inconsistent if:

(A) $|A| \neq 0$

(B) $|A| = 0$ and $(adjA)B = 0$

(C) $|A| = 0$ and $(adjA)B \neq 0$

(D) $|A| = 0$

Q23. The total cost function is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. The marginal cost at $x = 10$ is:

(A) 31.1

(B) 30.7

(C) 31.5

(D) 30.3

Q24. A demand function is $p = 40 - 5x$. The revenue is maximum when x is:

(A) 4

(B) 8

(C) 2

(D) 5

Q25. The average cost function AC is minimum when:

(A) $MC = AC$

(B) $MC = 0$

(C) $AC = 0$

(D) $MC > AC$

Q26. The demand function is $p = 50 - 2x$. If the market price is 30, the Consumer Surplus is:

(A) 100

(B) 200

(C) 50

(D) 150

Q27. The supply function is $p = 10 + 3x$. If the equilibrium quantity is $x = 4$, the Producer Surplus is:

(A) 24

(B) 48

(C) 12

(D) 36

Q28. The total revenue function is $R(x) = 100x - 0.5x^2$. The change in revenue when production increases from 10 to 20 units is:

(A) 850

(B) 925

(C) 750

(D) 1000

Q29. Consumer Surplus represents:

(A) Total Utility - Total Expenditure

(B) Total Revenue - Total Cost

(C) Price × Quantity

(D) Profit - Fixed Cost

Q30. The rate of decay of a radioactive substance is $\frac{dy}{dt} = -ky$. If the amount reduces to half in 100 years, the value of k is:

(A) $\frac{\log 2}{100}$

(B) $\frac{100}{\log 2}$

(C) $100 \log 2$

(D) $\log 2$

Q31. If the growth of a population follows $\frac{dP}{dt} = 0.02P$ and $P(0) = 500$, the population after 50 years is:

(A) $500e$

(B) $500e^2$

(C) 1000

(D) $500/e$

Q32. In a Poisson distribution, if the mean is 2, the variance is:

(A) 4

(B) $\sqrt{2}$

(C) 2

(D) 1

Q33. If X is a Poisson variable such that $P(X = 0) = 0.2$, then $P(X = 1)$ is:

(A) $0.2 \log_e 5$

(B) $0.2 \log_e 2$

(C) 0.1

(D) 0.4

Q34. For a Normal distribution $N(\mu, \sigma^2)$, the area to the left of $Z = 0$ is:

(A) 0.5

(B) 1.0

(C) 0.68

(D) 0.95

Q35. A standard normal variable Z has:

(A) Mean = 1, Variance = 0

(B) Mean = 0, Variance = 1

(C) Mean = μ, Variance = σ

(D) Mean = 0, Variance = 0

Q36. In a time series, "Seasonal variations" repeat every:

(A) 1 year or less

(B) 5 years

(C) 10 years

(D) Long term

Q37. The trend line $y = 20 + 1.5x$ is fitted to a data with 2015 as the origin ($x = 0$). The predicted value for 2020 is:

(A) 27.5

(B) 26.0

(C) 29.0

(D) 25.5

Q38. The 3-year moving average for the sequence 10, 12, 14, 16, 18 starting from the second term is:

(A) 12, 14, 16

(B) 11, 13, 15

(C) 13, 15, 17

(D) 10, 12, 14

Q39. A Null Hypothesis H_0 is:

(A) A hypothesis of no difference

(B) A hypothesis of significant difference

(C) Always false

(D) The same as the alternative hypothesis

Q40. Type II error occurs when we:

(A) Reject H_0 when it is true

(B) Accept H_0 when it is true

(C) Accept H_0 when it is false

(D) Reject H_0 when it is false

Q41. For a sample size of $n = 16$, the degrees of freedom for a t-test is:

(A) 16

(B) 17

(C) 15

(D) 32

Q42. In a t-test, the population must be:

(A) Binomial

(B) Normal

(C) Poisson

(D) Discrete

Q43. The EMI of a loan is calculated using the formula (where P is principal, i is rate per period, n is no. of periods):

(A) $P \times i \times \frac{(1+i)^n}{(1+i)^n-1}$

(B) $P \times i \times \frac{(1+i)^n-1}{(1+i)^n}$

(C) $P \times \frac{(1+i)^n}{i}$

(D) $P \times (1 + i)^n$

Q44. The present value of a perpetuity of Rs. 1200 payable at the end of every year at 8% interest per annum is:

(A) Rs. 15,000

(B) Rs. 12,000

(C) Rs. 18,000

(D) Rs. 20,000

Q45. A sinking fund is:

(A) Created to pay off a future liability

(B) Used for daily expenses

(C) A type of tax

(D) A bank overdraft

Q46. If the nominal rate of interest is 12% p.a. compounded quarterly, the effective rate of interest is:

(A) 12.55%

(B) 12.36%

(C) 12%

(D) 12.48%

Q47. The value of a bond that pays Rs. 50 coupons annually for 3 years and has a face value of Rs. 1000, given the discount rate is 5%, is:

(A) Rs. 1000

(B) Rs. 950

(C) Rs. 1050

(D) Rs. 1100

Q48. An amount of Rs. 2000 is invested in a sinking fund at the end of each year for 5 years at 10% interest. The amount accumulated is:

(A) Rs. 12,210.20

(B) Rs. 10,000

(C) Rs. 11,051

(D) Rs. 13,420

Q49. In a LPP, the constraints $x + y \leq 5, x \geq 0, y \geq 0$ define a feasible region in the shape of a:

(A) Square

(B) Triangle

(C) Circle

(D) Quadrilateral

Q50. The mathematical formulation of an LPP includes:

(A) Objective function

(B) Constraints

(C) Non-negativity restrictions

(D) All of the above

Detailed Solutions

Q1.

Solution

Concept: For any square matrix A of order n and a scalar k, the property of determinants states that $|kA| = k^n|A|$. This is because the scalar k is multiplied to every element in each of the n rows, and factoring it out from the determinant requires pulling it out once for each row.

Solution: Given:

- Order of matrix A, $n = 3$
- Determinant $|A| = 4$
- Scalar $k = 2$

Using the property $|kA| = k^n|A|$:

$$|2A| = 2^3 \times |A|$$

$$|2A| = 8 \times 4 = 32$$

Final Answer: The value of $|2A|$ is 32.

Answer: (C)

Q2.

Solution

Concept: Scalar multiplication of a matrix involves multiplying every individual element of the matrix by the constant k. For two matrices to be equal, their corresponding elements must be equal.

Solution: Given $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$, then $kA = \begin{bmatrix} k(0) & k(2) \\ k(3) & k(-4) \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$. We are given $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$. Comparing the corresponding elements:

(a) From the a_{22} position: $-4k = 24 \implies k = -6$.

(b) From the a_{12} position: $2k = 3a \implies 2(-6) = 3a \implies -12 = 3a \implies a = -4$.

(c) From the a_{21} position: $3k = 2b \implies 3(-6) = 2b \implies -18 = 2b \implies b = -9$.

Thus, $k = -6, a = -4, b = -9$.

Final Answer: $k = -6, a = -4, b = -9$

Answer: (A)

Q3.

Solution

Concept:
For a square matrix of order n:

$$|adj(A)| = |A|^{n-1}$$

Solution:
Given A is a 3×3 matrix, so $n = 3$.

$$|adj(A)| = |A|^{3-1} = |A|^2$$

Given:

$$|adj(A)| = 64$$

So,

$$|A|^2 = 64$$

$$|A| = \pm 8$$

Final Answer:

$$|A| = \pm 8$$

Answer: (A)

Q4.

Solution

Concept:
For any skew-symmetric matrix A of odd order:

$$|A| = 0$$

Solution:
Given A is a skew-symmetric matrix of order 3 (which is odd).
Using the property:

$$|A| = 0$$

Final Answer:

$$|A| = 0$$

Answer: (C)

Q5.

Solution

Concept: A function $f(x)$ is strictly decreasing in an interval where its first derivative is negative, i.e., $f'(x) < 0$. The critical points, where $f'(x) = 0$, help define the intervals of increase and decrease.

Solution: First, find the derivative of $f(x) = 2x^3 - 15x^2 + 36x + 1$:

$$f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$$

$$f'(x) = 6x^2 - 30x + 36$$

To find the critical points, set $f'(x) = 0$:

$$6(x^2 - 5x + 6) = 0$$

$$6(x - 2)(x - 3) = 0$$

The critical points are $x = 2$ and $x = 3$. These points divide the real line into three intervals: $(-\infty, 2)$, $(2, 3)$, and $(3, \infty)$.

We test the sign of $f'(x)$ in these intervals:

- For $x \in (-\infty, 2)$, choose $x = 0$: $f'(0) = 36 > 0$ (Increasing)
- For $x \in (2, 3)$, choose $x = 2.5$: $f'(2.5) = 6(2.5 - 2)(2.5 - 3) = 6(0.5)(-0.5) = -1.5 < 0$ (Decreasing)
- For $x \in (3, \infty)$, choose $x = 4$: $f'(4) = 6(4 - 2)(4 - 3) = 12 > 0$ (Increasing)

Since $f'(x) < 0$ in the interval $(2, 3)$, the function is strictly decreasing there.

Final Answer: The interval is (2, 3).

Answer: (A)

Q6.

Solution

Concept: This problem involves related rates of change. The area of a circle A is related to its radius r by the formula $A = \pi r^2$. When the radius changes with respect to time t, the rate of change of the area $\frac{dA}{dt}$ can be found using the chain rule: $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$.

Solution: Let r be the radius and A be the area of the circle. We are given the rate of increase of the radius:

$$\frac{dr}{dt} = 0.5 \text{ cm/s}$$

The area of the circle is:

$$A = \pi r^2$$

Differentiating both sides with respect to time t:

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2)$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Now, substitute the given values $r = 10$ cm and $\frac{dr}{dt} = 0.5$ cm/s:

$$\frac{dA}{dt} = 2\pi(10)(0.5)$$

$$\frac{dA}{dt} = 2\pi(5) = 10\pi$$

The rate of increase of the area is 10π cm^2/s.

Final Answer: 10π cm^2/s

Answer: (B)

Q7.

Solution

Concept:

The maximum value of $\sin x + \cos x$ is:

$$\sqrt{2}$$

(using identity or amplitude method)

Solution:

$$f(x) = \sin x + \cos x$$

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Thus,

$$\text{Maximum value} = \sqrt{2}$$

Final Answer:

$$\sqrt{2}$$

Answer: (C)

Q8.

Solution

Concept:

Tangent is parallel to x-axis when:

$$\frac{dy}{dx} = 0$$

Solution:

Given:

$$y = x^2 - 4x + 5$$

Differentiate:

$$\frac{dy}{dx} = 2x - 4$$

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Set derivative equal to zero:

$$2x - 4 = 0 \Rightarrow x = 2$$

Substitute in equation:

$$y = 2^2 - 4(2) + 5 = 4 - 8 + 5 = 1$$

Point:

$$(2, 1)$$

Final Answer:

$$(2, 1)$$

Answer: (A)

Q9.

Solution

Concept: The definite integral of a function $f(x)$ represents the signed area between the graph of the function and the x-axis over a given interval $[a, b]$. The standard integral formula for $\frac{1}{1+x^2}$ is the inverse trigonometric function $\arctan(x)$ (or $\tan^{-1} x$).

Solution: The given integral is:

$$I = \int_0^1 \frac{dx}{1+x^2}$$

Using the standard integration formula $\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$:

$$I = \left[\tan^{-1} x\right]_0^1$$

Substituting the upper and lower limits:

$$I = \tan^{-1}(1) - \tan^{-1}(0)$$

We know that:

- $\tan^{-1}(1) = \frac{\pi}{4}$ (since $\tan \frac{\pi}{4} = 1$)
- $\tan^{-1}(0) = 0$ (since $\tan 0 = 0$)

Thus:

$$I = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: The value of the integral is $\frac{\pi}{4}$.

Answer: (B)

Q10.

Solution

Concept: The area bounded by a curve $y = f(x)$, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by the definite integral $\int_a^b |f(x)|\,dx$. Since $\sin x \geq 0$ for all x in the interval $[0, \pi]$, the area is simply the integral of the function.

Solution: We need to find the area A under the curve $y = \sin x$ from $x = 0$ to $x = \pi$:

$$A = \int_0^{\pi} \sin x\,dx$$

The integral of $\sin x$ is $-\cos x$. Applying the limits of integration:

$$A = [-\cos x]_0^{\pi}$$

$$A = -(\cos \pi - \cos 0)$$

We know that $\cos \pi = -1$ and $\cos 0 = 1$:

$$A = -(-1 - 1)$$

$$A = -(-2) = 2$$

The area bounded by the curve is 2 sq. units.

Final Answer: 2 sq. units

Answer: (B)

Q11.

Solution

Concept:

Use product rule in reverse:

$$\frac{d}{dx}(e^x \tan x) = e^x \tan x + e^x \sec^2 x$$

Solution:

Given:

$$\int e^x(\tan x + \sec^2 x)\,dx$$

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Thus,

$$\int e^x(\tan x + \sec^2 x)\,dx = e^x \tan x + C$$

Final Answer:

$$e^x \tan x + C$$

Answer: (A)

Q12.

Solution

Concept:

Order = highest order derivative present.

Degree = power of the highest order derivative (after removing radicals/fractions of derivatives).

If the equation is not a polynomial in derivatives, degree is not defined.

Solution:

Given:

$$\frac{d^2y}{dx^2} + \sqrt{\frac{dy}{dx}} = x$$

Highest order derivative is:

$$\frac{d^2y}{dx^2}$$

So, order = 2.

Since $\sqrt{\frac{dy}{dx}}$ involves a fractional power of derivative, the equation is not polynomial in derivatives.

Hence, degree is not defined.

Final Answer:

Order = 2, Degree = not defined

Answer: (D)

Q13.

Solution

Concept:

This is a separable differential equation:

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Solution:

Integrate both sides:

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1} y = \tan^{-1} x + C$$

This can also be written in another equivalent form:

$$y - x = C(1 + xy)$$

Thus, both forms represent the general solution.

Final Answer:

Both A and B

Answer: (C)

Q14.

Solution

Concept: The probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes. When drawing multiple items without replacement, the total number of ways to choose r items from n is given by the combination formula ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$.

Solution: A standard deck of cards contains 52 cards, and there are 13 cards in each suit (Hearts, Diamonds, Clubs, and Spades).

(a) **Total outcomes:** The number of ways to draw 2 cards from 52 is:

$$^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 26 \times 51 = 1326$$

(b) **Favorable outcomes:** The number of ways to draw 2 hearts from the 13 available hearts is:

$$^{13}C_2 = \frac{13 \times 12}{2 \times 1} = 13 \times 6 = 78$$

(c) **Probability:**

$$P(\text{Both Hearts}) = \frac{^{13}C_2}{^{52}C_2} = \frac{78}{1326}$$

Simplifying the fraction:

$$\frac{78 \div 78}{1326 \div 78} = \frac{1}{17}$$

Alternatively, using the multiplication rule for dependent events:

$$P(\text{1st is Heart}) \times P(\text{2nd is Heart}|\text{1st was Heart}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$

Final Answer: The probability is 1/17.

Answer: (A)

Q15.

Solution

Concept:
In Linear Programming, the optimal value occurs at a corner point. For minimum value, compare $Z = ax + by$ at given vertices.

Solution:
Evaluate Z at all corner points:

$$(0, 10) \Rightarrow Z = 10b$$

$$(5, 5) \Rightarrow Z = 5a + 5b = 5(a + b)$$

$$(15, 15) \Rightarrow Z = 15(a + b)$$

$$(0, 20) \Rightarrow Z = 20b$$

For minimum at $(5, 5)$:

$$5(a + b) < 10b \Rightarrow a + b < 2b \Rightarrow a < b$$

Also:

$$5(a + b) < 20b \Rightarrow a < 3b \quad \text{(already satisfied if } a < b\text{)}$$

Thus required condition:

$$a < b$$

Final Answer:

$$a < b$$

Answer: (C)

Q16.

Solution

Concept:

Use modular arithmetic:

$$(a \times b) \bmod m = [(a \bmod m)(b \bmod m)] \bmod m$$

Solution:

$$24 \equiv 3 \pmod 7, \quad 32 \equiv 4 \pmod 7$$

$$(24 \times 32) \bmod 7 = (3 \times 4) \bmod 7 = 12 \bmod 7 = 5$$

Final Answer:

$$5$$

Answer: (D)

Q17.

Solution

Concept:

Downstream speed = $u + v$, upstream speed = $u - v$

Solution:

Downstream speed:

$$\frac{48}{4} = 12 \text{ km/h}$$

Upstream speed:

$$\frac{36}{6} = 6 \text{ km/h}$$

Let boat speed = u, stream speed = v:

$$u + v = 12, \quad u - v = 6$$

Add:

$$2u = 18 \Rightarrow u = 9$$

$$v = 12 - 9 = 3$$

Final Answer:

$$3 \text{ km/h}$$

Answer: (A)

Q18.

Solution

Concept:

Use time = distance/speed and relative comparison.

Solution:

Speed of A = 8 km/h = $\frac{40}{18} = \frac{20}{9}$ m/s

Time taken by A for 100 m:

$$t_A = \frac{100}{20/9} = 45 \text{ s}$$

B runs only 96 m.

Given A beats B by 15 s:

$$t_B = 45 + 15 = 60 \text{ s}$$

Speed of B:

$$= \frac{96}{60} = 1.6 \text{ m/s}$$

Convert to km/h:

$$1.6 \times \frac{18}{5} = 5.76 \text{ km/h}$$

Final Answer:

$$5.76 \text{ km/h}$$

Answer: (A)

Q19.

Solution

Concept: Modular arithmetic allows us to substitute congruent values into polynomial expressions. If $a \equiv b \pmod{m}$, then $f(a) \equiv f(b) \pmod{m}$ for any polynomial f with integer coefficients. Additionally, the expression $x^2 + 2x + 1$ can be simplified using the algebraic identity $(x + 1)^2$.

Solution: Given that $x \equiv 3 \pmod{5}$. We want to find the value of $x^2 + 2x + 1 \pmod{5}$.

Method 1: Direct Substitution Substitute $x = 3$ into the expression:

$$3^2 + 2(3) + 1 = 9 + 6 + 1 = 16$$

Now, find $16 \pmod{5}$:

$$16 = 5 \times 3 + 1 \implies 16 \equiv 1 \pmod{5}$$

Method 2: Using Algebraic Identity Notice that $x^2 + 2x + 1 = (x + 1)^2$. Substitute the congruence $x \equiv 3 \pmod{5}$:

$$(x + 1)^2 \equiv (3 + 1)^2 \pmod{5}$$

$$(x + 1)^2 \equiv 4^2 \pmod{5}$$

$$(x + 1)^2 \equiv 16 \pmod{5}$$

Since $16 \equiv 1 \pmod{5}$, the result is 1.

Final Answer: The value is 1.

Answer: (B)

Q20.

Solution

Concept:

For a 2×2 matrix:

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solution:

Given:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Determinant:

$$|A| = (1)(2) - (1)(1) = 1$$

Adjoint:

$$\text{adj}(A) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Final Answer:

$$\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Answer: (A)

Q21.

Solution

Concept: Cramer's Rule is a method for solving a system of linear equations using determinants. For a system of two equations in x and y, the value of x is given by $x = \frac{D_x}{D}$, where D is the determinant of the coefficient matrix and D_x is the determinant obtained by replacing the x-column with the constant terms.

Solution: The given system of equations is:

$$3x + y = 5$$
$$2x - y = 0$$

1. **Find the determinant of the coefficients (D):**

$$D = \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = (3 \times -1) - (1 \times 2) = -3 - 2 = -5$$

2. **Find the determinant for x (D_x):** Replace the first column (coefficients of x) with the constants $(5, 0)$:

$$D_x = \begin{vmatrix} 5 & 1 \\ 0 & -1 \end{vmatrix} = (5 \times -1) - (1 \times 0) = -5 - 0 = -5$$

3. **Calculate the value of x:**

$$x = \frac{D_x}{D} = \frac{-5}{-5} = 1$$

Final Answer: The value of x is 1.

Answer: (A)

Q22.

Solution

Concept:
System $AX = B$ is inconsistent if:

$$|A| = 0 \text{ and } (adjA)B \neq 0$$

Solution:
This is the condition for no solution.
Final Answer:

$$|A| = 0 \text{ and } (adjA)B \neq 0$$

Answer: (C)

Q23.

Solution

Concept:

Marginal cost = derivative of cost function:

$$MC = \frac{dC}{dx}$$

Solution:

Given:

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$$

Differentiate:

$$C'(x) = 0.015x^2 - 0.04x + 30$$

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At $x = 10$:

$$C'(10) = 0.015(100) - 0.04(10) + 30 = 1.5 - 0.4 + 30 = 31.1$$

Final Answer:

$$31.1$$

Answer: (A)

Q24.

Solution

Concept: Total Revenue (R) is defined as the product of the price (p) and the quantity sold (x), expressed as $R = p \times x$. To find the value of x that maximizes revenue, we determine the stationary point by setting the first derivative of the revenue function to zero ($\frac{dR}{dx} = 0$) and ensuring the second derivative is negative ($\frac{d^2R}{dx^2} < 0$).

Solution: Given the demand function:

$$p = 40 - 5x$$

The Revenue function R is:

$$R = p \cdot x = (40 - 5x)x = 40x - 5x^2$$

To find the maximum revenue, we differentiate R with respect to x:

$$\frac{dR}{dx} = \frac{d}{dx}(40x - 5x^2) = 40 - 10x$$

Set the first derivative to zero to find the critical point:

$$40 - 10x = 0 \implies 10x = 40 \implies x = 4$$

To verify this is a maximum, we check the second derivative:

$$\frac{d^2R}{dx^2} = -10$$

Since the second derivative is negative ($-10 < 0$), the revenue is indeed maximized at $x = 4$.

Final Answer: The revenue is maximum when x is 4.

Answer: (A)

Q25.

Solution

Concept:

Average Cost is minimum when:

$$MC = AC$$

Solution:

This is a standard economic result: the marginal cost curve intersects the average cost curve at its minimum point.

Final Answer:

$$MC = AC$$

Answer: (A)

Q26.

Solution

Concept:

Consumer Surplus = Area under demand curve above price line.

Solution:

Given:

$$p = 50 - 2x, \quad p = 30$$

Find equilibrium quantity:

$$30 = 50 - 2x \Rightarrow x = 10$$

Consumer Surplus:

$$= \frac{1}{2} \times 10 \times (50 - 30) = \frac{1}{2} \times 10 \times 20 = 100$$

Final Answer:

$$100$$

Answer: (A)

Q27.

Solution

Concept:

Producer Surplus = Area above supply curve and below price line.

Solution:

Given:

$$p = 10 + 3x, \quad x = 4$$

Market price:

$$p = 10 + 3(4) = 22$$

Producer Surplus:

$$= \frac{1}{2} \times 4 \times (22 - 10) = \frac{1}{2} \times 4 \times 12 = 24$$

Final Answer:

$$24$$

Answer: (A)

Q28.

Solution

Concept: The change in total revenue when production level changes from x_1 to x_2 is calculated by finding the difference between the revenue values at those two points, denoted as $\Delta R = R(x_2) - R(x_1)$. This represents the total additional income generated by increasing the output.

Solution: Given the total revenue function:

$$R(x) = 100x - 0.5x^2$$

We need to calculate the change in revenue as production increases from $x = 10$ to $x = 20$.

1. Calculate the revenue at $x = 20$:

$$R(20) = 100(20) - 0.5(20)^2$$

$$R(20) = 2000 - 0.5(400)$$

$$R(20) = 2000 - 200 = 1800$$

2. Calculate the revenue at $x = 10$:

$$R(10) = 100(10) - 0.5(10)^2$$

$$R(10) = 1000 - 0.5(100)$$

$$R(10) = 1000 - 50 = 950$$

3. Calculate the change in revenue (ΔR):

$$\Delta R = R(20) - R(10)$$

$$\Delta R = 1800 - 950 = 850$$

The change in revenue is 850 units.

Final Answer: 850

Answer: (A)

Q29.

Solution

Concept:

Consumer Surplus:

$$= \text{Total Utility} - \text{Total Expenditure}$$

Solution:

This is the correct definition of consumer surplus.

Final Answer:

Total Utility - Total Expenditure

Answer: (A)

Q30.

Solution

Concept: The rate of radioactive decay is proportional to the amount of substance present. The solution to the differential equation $\frac{dy}{dt} = -ky$ is the exponential decay function $y(t) = y_0 e^{-kt}$. The half-life $T_{1/2}$ is the time required for the substance to reduce to half its initial value, related to k by the formula $k = \frac{\ln 2}{T_{1/2}}$.

Solution: Given the differential equation $\frac{dy}{dt} = -ky$, we integrate to find $y(t) = y_0 e^{-kt}$. According to the problem, at $t = 100$, $y = \frac{y_0}{2}$. Substituting these values:

$$\frac{y_0}{2} = y_0 e^{-100k} \implies \frac{1}{2} = e^{-100k}$$

Taking natural logarithms on both sides:

$$\ln\left(\frac{1}{2}\right) = -100k$$

$$-\ln 2 = -100k \implies k = \frac{\ln 2}{100}$$

Assuming log represents the natural logarithm as per the options provided, $k = \frac{\log 2}{100}$.

Final Answer: The value of k is $\frac{\log 2}{100}$.

Answer: (A)

Q31.

Solution

Concept: The differential equation $\frac{dP}{dt} = kP$ represents exponential growth, where the rate of change of the population is proportional to the population size at any time t. The general solution for this equation is $P(t) = P_0e^{kt}$, where P_0 is the initial population.

Solution: Given the growth rate:

$$\frac{dP}{dt} = 0.02P$$

Separating the variables and integrating:

$$\int \frac{1}{P}\,dP = \int 0.02\,dt$$

$$\ln P = 0.02t + C \implies P(t) = P_0e^{0.02t}$$

Using the initial condition $P(0) = 500$:

$$500 = P_0e^{0.02(0)} \implies P_0 = 500$$

Thus, the population equation is $P(t) = 500e^{0.02t}$. To find the population after $t = 50$ years:

$$P(50) = 500e^{0.02\times 50}$$

$$P(50) = 500e^1 = 500e$$

Final Answer: The population after 50 years is $500e$.

Answer: (A)

Q32.

Solution

Concept: A Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space. A unique characteristic of the Poisson distribution is that its mean (λ) and its variance (σ^2) are equal.

Solution: For a Poisson distribution with parameter λ:

$$\text{Mean}(E[X]) = \lambda$$

$$\text{Variance}(Var(X)) = \lambda$$

Given that the mean is 2, we have $\lambda = 2$. Therefore, the variance is also 2.

Final Answer: The variance is 2.

Answer: (C)

Q33.

Solution

Concept: The probability mass function (PMF) of a Poisson distribution with parameter λ is given by $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$. To find specific probabilities, we first determine the value of λ using the given information.

Solution: Given $P(X = 0) = 0.2$:

$$\frac{e^{-\lambda}\lambda^0}{0!} = 0.2 \implies e^{-\lambda} = 0.2$$

To solve for λ, we take the natural logarithm (ln or $\log_e$) of both sides:

$$-\lambda = \ln(0.2) = \ln\left(\frac{1}{5}\right) = -\ln 5$$

Thus, $\lambda = \log_e 5$. Now, we find $P(X = 1)$:

$$P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-\lambda} \cdot \lambda$$

Substituting $e^{-\lambda} = 0.2$ and $\lambda = \log_e 5$:

$$P(X = 1) = 0.2 \log_e 5$$

Final Answer: The probability $P(X = 1)$ is $0.2 \log_e 5$.

Answer: (A)

Q34.

Solution

Concept: The Normal distribution is a symmetric, bell-shaped continuous probability distribution. The Standard Normal distribution (Z) has a mean (μ) of 0 and a variance (σ^2) of 1. Because the distribution is perfectly symmetrical about the mean, exactly half of the total area under the curve lies to the left of the mean, and the other half lies to the right.

Solution: The total area under the probability density function curve for any continuous distribution is 1. In a Standard Normal distribution $N(0, 1)$, the mean is $Z = 0$. Since the curve is symmetrical about $Z = 0$:

$$P(Z < 0) = P(Z > 0) = \frac{1}{2} = 0.5$$

Therefore, the area to the left of $Z = 0$ is 0.5.

Final Answer: The area to the left of $Z = 0$ is 0.5.

Answer: (A)

Q35.

Solution

Concept: The Standard Normal Distribution is a special case of the Normal Distribution where the data is centered and scaled. It is often used to calculate probabilities and percentiles using Z-tables. By definition, any normal variable $X \sim N(\mu, \sigma^2)$ can be transformed into a standard normal variable Z using the formula $Z = \frac{X-\mu}{\sigma}$.
Solution: A Standard Normal Variable, denoted as Z, is defined by two fixed parameters:

- **Mean (μ):** The center of the distribution is 0.
- **Variance (σ^2):** The spread or scale of the distribution is 1 (which also means the standard deviation σ is 1).

This results in the probability density function being centered at zero with a total area of 1 underneath the bell curve.
Final Answer: Mean = 0, Variance = 1

Answer: (B)

Q36.

Solution

Concept: In time series analysis, components are categorized by their duration and nature. Seasonal variations refer to periodic fluctuations that are caused by factors like weather, holidays, or academic calendars. These fluctuations occur within a specific, predictable window of time.
Solution: Seasonal variations are defined as patterns that repeat at regular intervals within a period of **one year or less**. Examples include:

- Increased ice cream sales during summer (annual cycle).
- Increased department store sales every December (annual cycle).
- High electricity demand during specific hours of the day (daily cycle, sometimes termed "diurnal").

Longer fluctuations (like those lasting 2–10 years) are classified as "Cyclical variations," while long-term trends are called "Secular trends."
Final Answer: 1 year or less

Answer: (A)

Q37.

Solution

Concept: In time series analysis, a linear trend line is represented by the equation $y = a + bx$, where y is the predicted value, a is the intercept at the origin, b is the slope (rate of change), and x is the time variable measured from a specific origin year.

Solution: The given trend line equation is:

$$y = 20 + 1.5x$$

The origin is given as the year 2015, which means for 2015, $x = 0$. To find the predicted value for the year 2020, we calculate the value of x:

$$x = 2020 - 2015 = 5$$

Now, substitute $x = 5$ into the trend equation:

$$y = 20 + 1.5(5)$$

$$y = 20 + 7.5 = 27.5$$

The predicted value for 2020 is 27.5.

Final Answer: 27.5

Answer: (A)

Q38.

Solution

Concept: A 3-year moving average is a smoothing technique used in time series to reduce short-term fluctuations. It is calculated by taking the arithmetic mean of three consecutive values. The result is usually placed at the center (the second year) of the three-year period.

Solution: Given the sequence: 10, 12, 14, 16, 18. We calculate the 3-year moving averages as follows:

(a) Average centered at the 2nd term: $\frac{10+12+14}{3} = \frac{36}{3} = 12$

(b) Average centered at the 3rd term: $\frac{12+14+16}{3} = \frac{42}{3} = 14$

(c) Average centered at the 4th term: $\frac{14+16+18}{3} = \frac{48}{3} = 16$

The sequence of moving averages starting from the second term (where the first average is positioned) is 12, 14, 16.

Final Answer: 12, 14, 16

Answer: (A)

Q39.

Solution

Concept: In statistical hypothesis testing, the Null Hypothesis (H_0) is a fundamental assumption used for testing data. It serves as a baseline for the test and is typically formulated to be challenged by the Alternative Hypothesis (H_1 or H_a).

Solution: The Null Hypothesis (H_0) represents a statement of "status quo" or "no effect." Specifically:

- It is a hypothesis of **no difference** between population parameters or no relationship between variables.
- It assumes that any observed difference in a sample is due to chance or sampling error rather than a real underlying effect.
- The goal of a statistical test is often to see if there is enough evidence to reject H_0 in favor of a significant difference (the Alternative Hypothesis).

Final Answer: A hypothesis of no difference

Answer: (A)

Q40.

Solution

Concept: In hypothesis testing, errors are classified based on the decision made relative to the actual truth of the Null Hypothesis (H_0). A Type I error (α) is "falsely rejecting" a true H_0, while a Type II error (β) is "falsely accepting" (failing to reject) a false H_0.

Solution: To identify the correct error type, we look at the standard definitions:

- **Type I Error:** Rejecting H_0 when it is actually true (False Positive).
- **Type II Error:** Accepting (Failing to reject) H_0 when it is actually false (False Negative).

In this case, the question asks for Type II error, which corresponds to accepting the null hypothesis even though it is incorrect.

Final Answer: Accept H_0 when it is false

Answer: (C)

Q41.

Solution

Concept: Degrees of freedom (df) in statistics represent the number of independent pieces of information that go into the calculation of a statistic. For a standard one-sample t-test, the degrees of freedom are calculated based on the sample size n.

Solution: The formula for degrees of freedom for a single sample t-test is:

$$df = n - 1$$

Given the sample size:

$$n = 16$$

Substituting the value into the formula:

$$df = 16 - 1 = 15$$

This reduction by one accounts for the fact that the sample mean is estimated from the data, which "uses up" one degree of freedom.

Final Answer: 15

Answer: (C)

Q42.

Solution

Concept: Student's t-test is a parametric statistical test used to determine if there is a significant difference between the means of two groups. For the results of a t-test to be valid, certain underlying assumptions must be met regarding the distribution of the data.

Solution: One of the fundamental assumptions of the t-test is the **Assumption of Normality**. This states that the observations within each sample must be drawn from a population that follows a **Normal distribution**. This is especially critical for small sample sizes ($n < 30$), as the t-distribution was specifically derived to handle normally distributed populations where the population standard deviation is unknown.

Final Answer: Normal

Answer: (B)

Q43.

Solution

Concept: Equated Monthly Installment (EMI) is a fixed payment amount made by a borrower to a lender at a specified date each calendar month. EMIs are used to pay off both interest and principal each month so that over a specified number of years, the loan is paid off in full.

Solution: The standard formula for calculating EMI is derived from the present value of an annuity. The formula is:

$$E = P \times i \times \frac{(1+i)^n}{(1+i)^n - 1}$$

Where:

- P = Principal loan amount.
- i = Interest rate per period (usually monthly interest rate, i.e., Annual Rate / 12 / 100).
- n = Number of periods (total number of monthly installments).

Option (A) correctly represents this mathematical relationship.

Final Answer: $P \times i \times \frac{(1+i)^n}{(1+i)^n - 1}$

Answer: (A)

Q44.

Solution

Concept: A perpetuity is an annuity that continues indefinitely. The present value (PV) of a perpetuity represents the current value of all future payments. For a perpetuity where payments are made at the end of each period, the formula is $PV = \frac{R}{i}$, where R is the periodic payment and i is the interest rate per period.

Solution: Given:

- Annual payment (R) = Rs. 1200
- Annual interest rate (i) = 8% $= \frac{8}{100} = 0.08$

Using the formula for the present value of a perpetuity:

$$PV = \frac{R}{i}$$

$$PV = \frac{1200}{0.08}$$

$$PV = \frac{1200 \times 100}{8} = 150 \times 100 = 15,000$$

The present value is Rs. 15, 000.

Final Answer: Rs. 15, 000

Answer: (A)

Q45.

Solution

Concept: A sinking fund is a financial strategy used by organizations and individuals to accumulate a specific sum of money over a period of time. This is done by making regular periodic contributions into an interest-bearing account.
Solution: A sinking fund is specifically established for the purpose of:

- Repaying a long-term debt or bond at maturity.
- Replacing a wasting asset (like machinery) at the end of its useful life.
- Meeting any known **future liability**.

It is not used for daily operational expenses (working capital) nor is it a form of taxation or a credit facility like an overdraft.
Final Answer: Created to pay off a future liability

Answer: (A)

Q46.

Solution

Concept: The effective rate of interest (r_e) is the actual interest rate earned or paid after compounding over a specific period. It is related to the nominal rate (r) and the number of compounding periods per year (m) by the formula $r_e = \left(1 + \frac{r}{m}\right)^m - 1$.
Solution: Given:

- Nominal rate (r) = 12% = 0.12
- Compounded quarterly (m) = 4

Substituting these values into the formula:

$$r_e = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$r_e = (1 + 0.03)^4 - 1$$

$$r_e = (1.03)^4 - 1$$

Calculating $(1.03)^4$:

$$(1.03)^2 = 1.0609$$

$$(1.03)^4 = 1.0609 \times 1.0609 \approx 1.125508$$

Thus, $r_e = 1.125508 - 1 = 0.125508$. Converting to percentage: 12.55%.
Final Answer: The effective rate of interest is 12.55%.

Answer: (A)

Q47.

Solution

Concept: The value of a bond is the present value of its future cash flows, which include periodic coupon payments and the face value (par value) returned at maturity. If the coupon rate of a bond is equal to the discount rate (required rate of return), the bond will always trade at its face value.

Solution: Given:

- Annual Coupon (C) = Rs. 50
- Face Value (F) = Rs. 1000
- Discount rate $(r) = 5\% = 0.05$
- Time $(n) = 3$ years

The value V is calculated as:

$$V = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C+F}{(1+r)^3}$$

$$V = \frac{50}{1.05} + \frac{50}{1.1025} + \frac{1050}{1.157625}$$

$$V = 47.619 + 45.351 + 907.029 = 1000$$

Since the coupon rate $(50/1000 = 5\%)$ matches the discount rate (5%), the bond value is equal to the face value.

Final Answer: The value of the bond is Rs. 1000.

Answer: (A)

Q48.

Solution

Concept: A sinking fund is an annuity where regular payments are made to accumulate a future sum. The accumulated amount (A) is calculated using the future value of an ordinary annuity formula: $A = R \times \frac{(1+i)^n - 1}{i}$, where R is the periodic payment, i is the interest rate, and n is the number of periods.

Solution: Given:

- Annual investment (R) = Rs. 2000
- Interest rate (i) = 10% = 0.10
- Number of years (n) = 5

Substituting the values into the formula:

$$A = 2000 \times \frac{(1 + 0.10)^5 - 1}{0.10}$$

$$A = 2000 \times \frac{(1.1)^5 - 1}{0.10}$$

Since $(1.1)^5 = 1.61051$:

$$A = 2000 \times \frac{1.61051 - 1}{0.10}$$

$$A = 2000 \times \frac{0.61051}{0.10}$$

$$A = 2000 \times 6.1051 = 12,210.20$$

Final Answer: The amount accumulated is Rs. 12,210.20.

Answer: (A)

Q49.

Solution

Concept: In Linear Programming Problems (LPP), the feasible region is the set of all points (x, y) that satisfy all given inequalities simultaneously. The non-negativity restrictions $x \geq 0$ and $y \geq 0$ confine the region to the first quadrant of the Cartesian plane.

Solution: We analyze the boundary lines of the given inequalities:

(a) $x = 0$ is the y-axis.

(b) $y = 0$ is the x-axis.

(c) $x + y = 5$ is a line passing through $(5, 0)$ and $(0, 5)$.

The region $x \geq 0$ and $y \geq 0$ starts at the origin $(0, 0)$. The constraint $x + y \leq 5$ represents the area below the line $x + y = 5$. The intersection of these three constraints forms a closed region with vertices at $(0, 0)$, $(5, 0)$, and $(0, 5)$. A polygon with three vertices is a triangle.

Final Answer: The feasible region is in the shape of a Triangle.

Answer: (B)

Q50.

Solution

Concept: Linear Programming is a mathematical method used to determine the best possible outcome (such as maximum profit or minimum cost) in a given mathematical model whose requirements are represented by linear relationships.

Solution: The mathematical formulation of any Linear Programming Problem (LPP) consists of three essential components:

- **Objective Function:** A linear function $Z = ax + by$ that needs to be maximized or minimized.
- **Constraints:** A set of linear inequalities or equations that limit the values of the decision variables based on available resources.
- **Non-negativity restrictions:** Requirements that the decision variables must be greater than or equal to zero $(x, y \geq 0)$, as physical quantities (like products or time) cannot be negative.

Since all three are necessary for a complete LPP model, the correct choice is "All of the above."

Final Answer: All of the above

Answer: (D)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	A	4	C	5	A
6	B	7	C	8	A	9	B	10	B
11	A	12	D	13	C	14	A	15	C
16	D	17	A	18	A	19	B	20	A
21	A	22	C	23	A	24	A	25	A
26	A	27	A	28	A	29	A	30	A
31	A	32	C	33	A	34	A	35	B
36	A	37	A	38	A	39	A	40	C
41	C	42	B	43	A	44	A	45	A
46	A	47	A	48	A	49	B	50	D