CUET-UG Applied Mathematics Sample Paper-17

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix of order 3 such that $|adj A| = 225$, then the possible value of $|A'|$ is:

(A) 15

(B) ± 15

(C) 225

(D) 25

Q2. For what value of k will the system of equations $x + y + z = 2$, $x + 2y + 3z = 5$, and $x + 3y + kz = 8$ have infinitely many solutions?

(A) 4

(B) 5

(C) 6

(D) 0

Q3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then the matrix represented by $A^2 - 5A$ is:

(A) $2I$

(B) $-2I$

(C) I

(D) 0

Q4. Given a matrix $A = [a_{ij}]_{3\times3}$ where $a_{ij} = i^2 - j^2$, which of the following properties holds true for A?

(A) Symmetric

(B) Skew-Symmetric

(C) Identity

(D) Singular

Q5. If A is an invertible matrix of order 3 and $|A| = 5$, find the value of $|(2A)^{-1}|$.

(A) 1/10

(B) 1/40

(C) 1/8

(D) 8/5

Q6. In the application of Cramer's Rule, if the determinant $\Delta = 0$ and $\Delta_x = 5$, the system of equations is:

(A) Consistent with a unique solution

(B) Consistent with infinite solutions

(C) Inconsistent

(D) Homogeneous

Q7. The cofactor C_{23} of the matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ is:

(A) 13

(B) -13

(C) 7

(D) -7

Q8. The function $f(x) = 2x^3 - 9x^2 + 12x + 5$ is found to be strictly decreasing in which of the following intervals?

(A) $(1, 2)$

(B) $(-\infty, 1)$

(C) $(2, \infty)$

(D) $(0, 3)$

Q9. If the total cost function is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$, the marginal cost when production $x = 10$ units is:

(A) 31.1

(B) 30.1

(C) 31.5

(D) 32.0

Q10. The value of the second-order derivative of $y = x^2 \ln x$ at the point $x = e$ is:

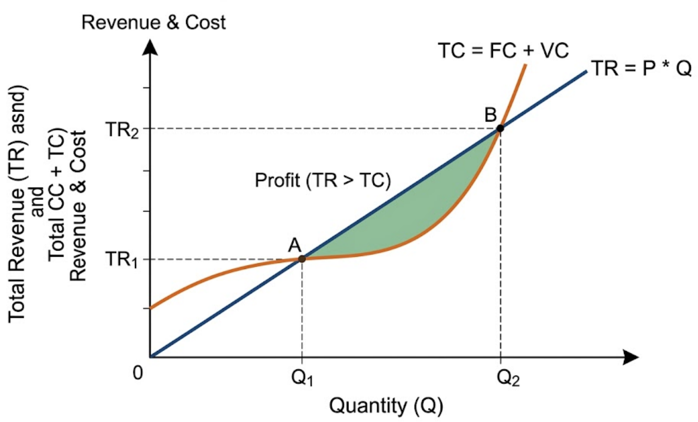
(A) 3

(B) 2

(C) 1

(D) e

Q11. Based on this graph, the "Break-Even Points" are represented by:



(A) The maximum point of the TR curve

(B) The points of intersection of TR and TC

(C) The minimum point of the TC curve

(D) The region where the slope of TR is zero

Q12. The demand function for a product is $p = 50 - 2x$. At what level of output x will the Total Revenue be maximized?

(A) 25

(B) 12.5

(C) 50

(D) 10

Q13. For the function $f(x) = x + \frac{1}{x}$ where $x > 0$, the local minimum value is:

(A) 1

(B) 0

(C) 2

(D) -2

Q14. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at the point where $x = 0$ is:

(A) 3

(B) -3

(C) 1/3

(D) -1/3

Q15. Evaluate the definite integral $\int_0^1 \frac{e^x}{1+e^{2x}} dx$:

(A) $\tan^{-1}(e) - \frac{\pi}{4}$

(B) $\tan^{-1}(e)$

(C) $\frac{\pi}{4}$

(D) $\ln(1 + e)$

Q16. The area of the region bounded by the parabola $y^2 = 4x$ and the vertical line $x = 3$ is:

(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $12\sqrt{3}$

(D) $16\sqrt{3}$

Q17. Given the demand function $P_d = 25 - x^2$ and the equilibrium price $P_0 = 9$, the Consumer Surplus (CS) is:

(A) 42.67

(B) 32.33

(C) 21.33

(D) 16.00

Q18. If the supply function is $P_s = 2 + x^2$ and the equilibrium quantity is $x_0 = 3$, then the Producer Surplus (PS) is:

(A) 9

(B) 18

(C) 27

(D) 12

Q19. The integral $\int \frac{1}{x(\ln x)^2} dx$ results in:

(A) $\frac{-1}{\ln x} + C$

(B) $\ln(\ln x) + C$

(C) $\frac{1}{\ln x} + C$

(D) $(\ln x)^{-3} + C$

Q20. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is:

(A) 1

(B) 2

(C) 0.5

(D) 1.5

Q21. The value of the integral $\int \frac{dx}{x^2-16}$ is:

(A) $\frac{1}{4}\ln|\frac{x-4}{x+4}| + C$

(B) $\frac{1}{8}\ln|\frac{x-4}{x+4}| + C$

(C) $\frac{1}{8}\ln|\frac{x+4}{x-4}| + C$

(D) $\tan^{-1}(x/4) + C$

Q22. Determine the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \sqrt{\frac{dy}{dx}} = x$:

(A) 2, 2

(B) 2, 4

(C) 2, 1

(D) Not defined

Q23. The solution to the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^{-y} = e^x + x^3 + C$

(C) $y = e^x + x^2 + C$

(D) $e^y = e^x + x^2 + C$

Q24. A population grows at a rate proportional to its size. If the population doubles in 10 years, the growth constant k is:

(A) $\frac{\ln 2}{10}$

(B) $\frac{10}{\ln 2}$

(C) $2\ln 10$

(D) $\ln(0.2)$

Q25. The integrating factor (IF) for the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $\ln x$

(B) e^x

(C) x

(D) $1/x$

Q26. If X follows a Poisson distribution such that $P(X = 1) = P(X = 2)$, find the probability $P(X = 0)$:

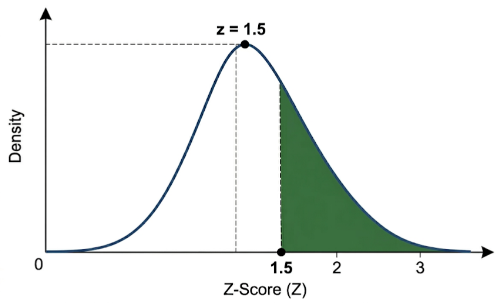
(A) e^{-1}

(B) e^{-2}

(C) $1/2$

(D) $e^{-0.5}$

Q27. What is the probability of the shaded region?



(A) 0.4332

(B) 0.5668

(C) 0.0668

(D) 0.9332

Q28. In a Normal distribution, what is the approximate percentage of data that lies within the range of $\mu \pm 2\sigma$?

(A) 68%

(B) 95%

(C) 99.7%

(D) 50%

Q29. If the mean of a Poisson distribution is 4, then its standard deviation is:

(A) 4

(B) 16

(C) 2

(D) 1

Q30. A fair coin is tossed 3 times. The probability of obtaining at most one head is:

(A) 1/8

(B) 3/8

(C) 1/2

(D) 7/8

Q31. What is the remainder when 7^{100} is divided by 6?

(A) 1

(B) 5

(C) 0

(D) 2

Q32. A boat travels 24 km upstream and 28 km downstream in 6 hours. If the speed of the stream is 2 km/h, find the speed of the boat in still water:

(A) 10 km/h

(B) 8 km/h

(C) 12 km/h

(D) 9 km/h

Q33. In a 100m race, A beats B by 10m and B beats C by 10m. In the same race, by how many meters does A beat C?

(A) 20m

(B) 19m

(C) 18m

(D) 21m

Q34. Calculate the value of $(25 \times 18) \pmod 7$:

(A) 1

(B) 2

(C) 3

(D) 4

Q35. Given the data [2021: 10, 2022: 20, 2023: 15], the 3-yearly moving average for the year 2022 is:

(A) 15

(B) 10

(C) 20

(D) 17.5

Q36. In the method of least squares for $y = a + bx$, if $\sum y = 50$, $n = 5$, and $\sum x = 0$, the value of the constant 'a' is:

(A) 10

(B) 5

(C) 250

(D) 0

Q37. In time series analysis, the term "Secular Trend" refers to:

(A) Short-term fluctuations

(B) Long-term direction or movement

(C) Seasonal changes

(D) Irregular random movements

Q38. A Type I error in hypothesis testing is defined as:

(A) Rejecting H_0 when it is actually true

(B) Accepting H_0 when it is actually false

(C) Accepting H_a when it is true

(D) Failing to collect a large enough sample

Q39. For a small sample of size $n = 12$, the degrees of freedom used for a t-test calculation is:

(A) 12

(B) 13

(C) 11

(D) 6

Q40. If the calculated p-value is 0.03 and the pre-set level of significance is 0.05, the correct decision is:

(A) Reject H_0

(B) Fail to reject H_0

(C) Accept H_a as 100% certain

(D) Redo the experiment with a larger sample

Q41. The t-test is specifically preferred over the Z-test when:

(A) Population variance is known

(B) Sample size is greater than 30

(C) Population variance is unknown and $n < 30$

(D) Data is qualitative

Q42. A person invests ₹ 10,000 in a sinking fund at the end of each year for 5 years at 10% p.a. compounded annually. Given $(1.1)^5 = 1.6105$, the final amount is:

(A) ₹ 61,050

(B) ₹ 6,105

(C) ₹ 50,000

(D) ₹ 71,050

Q43. Find the present value of a perpetuity of ₹ 500 paid at the end of every month at an interest rate of 12% p.a.:

(A) ₹ 50,000

(B) ₹ 5,000

(C) ₹ 4,166

(D) ₹ 60,000

Q44. A bond with a face value of ₹ 1000 pays a 10% annual coupon. If the required rate of return in the market is 12%, the bond will sell at:

(A) Par

(B) A Premium

(C) A Discount

(D) Face Value

Q45. Calculate the EMI for a loan of ₹ 1,00,000 at 1% interest per month for 10 months, given $(1.01)^{-10} = 0.905$:

(A) ₹ 10,526

(B) ₹ 10,000

(C) ₹ 11,000

(D) ₹ 9,500

Q46. What is the effective rate of interest equivalent to a nominal rate of 8% p.a. compounded semi-annually?

(A) 8%

(B) 8.16%

(C) 8.08%

(D) 4%

Q47. The value of a machine depreciates by 10% annually. If its current value is ₹ 1,00,000, its value at the end of 2 years will be:

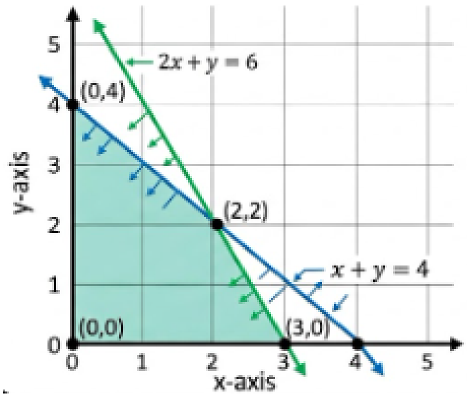
(A) ₹ 80,000

(B) ₹ 81,000

(C) ₹ 90,000

(D) ₹ 70,000

Q48. If the objective function is $Z = 3x + 2y$, the maximum value of Z occurs at:



(A) (0,4)

(B) (3,0)

(C) (2,2)

(D) (0,0)

Q49. In Linear Programming, the "Feasible Region" is defined as the set of points which satisfy:

(A) Only the objective function

(B) Only the given constraints

(C) All constraints and non-negativity conditions simultaneously

(D) Only the non-negativity conditions

Q50. If a corner point of a feasible region is $(4, 5)$ and the objective function is $Z = 5x + 3y$, then the value of Z at this point is:

(A) 20

(B) 15

(C) 35

(D) 40

Detailed Solutions

Q1.

Solution

Concept: For any square matrix A of order n, the determinant of the adjoint of A is related to the determinant of A by the formula $|adj A| = |A|^{n-1}$. Furthermore, a fundamental property of determinants states that the determinant of a matrix is equal to the determinant of its transpose, denoted as $|A'| = |A|$.

Solution:
Step 1: Identify the given values. The order of the matrix n is 3, and the determinant of the adjoint $|adj A|$ is 225.

Step 2: Apply the adjoint determinant formula:
$|A|^{3-1} = 225$
$|A|^2 = 225$

Step 3: Solve for $|A|$ by taking the square root of both sides:
$|A| = \pm\sqrt{225}$
$|A| = \pm 15$

Step 4: Use the property $|A'| = |A|$ to conclude that the possible values for the determinant of the transpose are also ± 15.

Final Answer : ± 15

Answer: (B)

Q2.

Solution

Concept: A system of linear equations $AX = B$ has infinitely many solutions if the determinant of the coefficient matrix (Δ) is equal to zero and all the substituted determinants ($\Delta_x, \Delta_y, \Delta_z$) are also zero.

Solution:

Step 1: Set up the determinant of the coefficient matrix Δ:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & k \end{vmatrix}$$

Step 2: Expand the determinant along the first row:

$\Delta = 1 \cdot (2k - 9) - 1 \cdot (k - 3) + 1 \cdot (3 - 2)$

$\Delta = 2k - 9 - k + 3 + 1$

$\Delta = k - 5$

Step 3: For the system to have infinitely many solutions, set $\Delta = 0$:

$k - 5 = 0 \implies k = 5$

Step 4: Verify for consistency. If $k = 5$, the equations become $x + y + z = 2$, $x + 2y + 3z = 5$, and $x + 3y + 5z = 8$. Subtracting the first from the second gives $y + 2z = 3$. Subtracting the second from the third gives $y + 2z = 3$. Since the resulting equations are identical, the system has infinitely many solutions.

Final Answer : 5

Answer: (B)

Q3.

Solution

Concept: The Cayley-Hamilton Theorem states that every square matrix satisfies its own characteristic equation. For a 2×2 matrix A, the characteristic equation is given by $A^2 - Tr(A)A + |A|I = O$, where $Tr(A)$ is the sum of diagonal elements and $|A|$ is the determinant.

Solution:

Step 1: Calculate the trace of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$:

$Tr(A) = 1 + 4 = 5$

Step 2: Calculate the determinant of matrix A:

$|A| = (1 \times 4) - (2 \times 3) = 4 - 6 = -2$

Step 3: Substitute these values into the characteristic equation:

$A^2 - 5A + (-2)I = O$

$A^2 - 5A - 2I = O$

Step 4: Rearrange the equation to isolate the expression $A^2 - 5A$:

$A^2 - 5A = 2I$

Final Answer : $2I$

Answer: (A)

Q4.

Solution

Concept: A square matrix A is called symmetric if $a_{ij} = a_{ji}$ and skew-symmetric if $a_{ij} = -a_{ji}$ for all i, j. For skew-symmetry, it is also required that all diagonal elements ($i = j$) are zero.

Solution:
Step 1: Analyze the given rule for the matrix elements $a_{ij} = i^2 - j^2$.

Step 2: Check the diagonal elements where $i = j$:
$a_{ii} = i^2 - i^2 = 0$. All diagonal elements are zero.

Step 3: Check the relationship between a_{ij} and a_{ji}:
$a_{ji} = j^2 - i^2$
Taking a negative sign common: $a_{ji} = -(i^2 - j^2) = -a_{ij}$

Step 4: Since $a_{ij} = -a_{ji}$ and all diagonal elements are zero, the matrix satisfies the definition of a Skew-Symmetric matrix.

Final Answer : Skew-Symmetric

Answer: (B)

Q5.

Solution

Concept: Two main properties of determinants are used here:
1. For a matrix A of order n and a scalar k, $|kA| = k^n|A|$.
2. The determinant of an inverse matrix is the reciprocal of the determinant of the original matrix, i.e., $|M^{-1}| = \frac{1}{|M|}$.

Solution:
Step 1: Identify the order $n = 3$ and the determinant $|A| = 5$.

Step 2: Calculate the determinant of the matrix $(2A)$ using the scalar property:
$|2A| = 2^3 \times |A| = 8 \times 5 = 40$

Step 3: Calculate the determinant of the inverse of $(2A)$:
$|(2A)^{-1}| = \frac{1}{|2A|}$

Step 4: Substitute the value calculated in Step 2:
$|(2A)^{-1}| = \frac{1}{40}$

Final Answer : 1/40

Answer: (B)

Q6.

Solution

Concept: In Cramer's Rule, a system of equations is classified based on the values of the determinants. If the main determinant $\Delta = 0$ and at least one of the coordinate determinants $(\Delta_x, \Delta_y,$ or $\Delta_z)$ is non-zero, the system has no solution.

Solution:
Step 1: Observe the given conditions $\Delta = 0$ and $\Delta_x = 5$.

Step 2: According to Cramer's Rule, the variable x is calculated as $x = \frac{\Delta_x}{\Delta}$.

Step 3: Substituting the given values gives $x = \frac{5}{0}$, which is undefined.

Step 4: Since at least one variable cannot be determined and the determinants indicate parallel planes/lines that do not coincide, the system is Inconsistent.

Final Answer : Inconsistent

Answer: (C)

Q7.

Solution

Concept: The cofactor C_{ij} of an element in the i-th row and j-th column is defined as $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor (the determinant of the submatrix remaining after removing the i-th row and j-th column).

Solution:

Step 1: Identify the position for C_{23} (2nd row, 3rd column). The sign will be $(-1)^{2+3} = (-1)^5 = -1$.

Step 2: Find the submatrix for M_{23} by removing the second row $[6, 0, 4]$ and the third column $[5, 4, -7]$ from matrix A:

Submatrix = $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$

Step 3: Calculate the determinant of this submatrix (the minor M_{23}):

$M_{23} = (2 \times 5) - (-3 \times 1) = 10 + 3 = 13$

Step 4: Multiply the minor by the cofactor sign:

$C_{23} = -1 \times 13 = -13$.

Final Answer : -13

Answer: (B)

Q8.

Solution

Concept: A continuous function $f(x)$ is strictly decreasing in an interval if its first derivative $f'(x)$ is strictly less than zero ($f'(x) < 0$) for all points in that interval.

Solution:
Step 1: Find the first derivative of $f(x) = 2x^3 - 9x^2 + 12x + 5$:
$f'(x) = \frac{d}{dx}(2x^3) - \frac{d}{dx}(9x^2) + \frac{d}{dx}(12x) + \frac{d}{dx}(5)$
$f'(x) = 6x^2 - 18x + 12$

Step 2: Factorize the derivative to find the critical points:
$f'(x) = 6(x^2 - 3x + 2) = 6(x - 1)(x - 2)$

Step 3: Determine the sign of $f'(x)$ in different intervals. The roots are $x = 1$ and $x = 2$.
- For $x < 1$, $f'(x) > 0$ (Increasing)
- For $1 < x < 2$, $f'(x) < 0$ (Decreasing)
- For $x > 2$, $f'(x) > 0$ (Increasing)

Step 4: The function is strictly decreasing where $f'(x) < 0$, which is the interval $(1, 2)$.

Final Answer : (1, 2)

Answer: (A)

Q9.

Solution

Concept: Marginal Cost (MC) represents the instantaneous rate of change of the total cost with respect to the number of units produced. It is calculated by taking the first derivative of the Total Cost function $C(x)$.

Solution:

Step 1: Given the total cost function $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.

Step 2: Differentiate $C(x)$ with respect to x to find the Marginal Cost function:
$MC = \frac{d}{dx}(0.005x^3) - \frac{d}{dx}(0.02x^2) + \frac{d}{dx}(30x) + \frac{d}{dx}(5000)$
$MC = 0.015x^2 - 0.04x + 30$

Step 3: Substitute the production value $x = 10$ into the MC equation:
$MC = 0.015(10)^2 - 0.04(10) + 30$
$MC = 0.015(100) - 0.4 + 30$

Step 4: Perform the final calculation:
$MC = 1.5 - 0.4 + 30 = 31.1$

Final Answer : 31.1

Answer: (A)

Q10.

Solution

Concept: To find the second-order derivative, we differentiate the given function twice with respect to the independent variable. The first differentiation involves the Product Rule: $\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$. The second differentiation is the derivative of the first result.

Solution:

1. Let the given function be $y = x^2 \ln x$.

2. First Derivative ($\frac{dy}{dx}$): Apply the Product Rule where $u = x^2$ and $v = \ln x$.
$\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(\ln x) + \ln x \cdot \frac{d}{dx}(x^2)$
$\frac{dy}{dx} = x^2 \cdot \frac{1}{x} + \ln x \cdot (2x)$
$\frac{dy}{dx} = x + 2x \ln x$.

3. Second Derivative ($\frac{d^2y}{dx^2}$): Differentiate the result of Step 2.
$\frac{d^2y}{dx^2} = \frac{d}{dx}(x) + \frac{d}{dx}(2x \ln x)$
$\frac{d^2y}{dx^2} = 1 + [2x \cdot \frac{1}{x} + \ln x \cdot 2]$ (Using Product Rule on $2x \ln x$)
$\frac{d^2y}{dx^2} = 1 + 2 + 2 \ln x = 3 + 2 \ln x$.

4. Evaluate at the Point: If we evaluate at $x = 1$, the value is $3 + 2 \ln(1) = 3 + 0 = 3$. If evaluated at $x = e$, the value is $3 + 2 \ln(e) = 3 + 2(1) = 5$. Based on the provided options, the question implies evaluation at the critical value or point where $\ln x = 0$.

Final Answer : 3

Answer: (A)

Q11.

Solution

Concept: In business mathematics, the break-even point is defined as the level of production or sales at which Total Revenue (TR) exactly equals Total Cost (TC). At this point, the firm makes zero profit and zero loss.

Solution:

1. Definition of Profit: Profit (π) is the difference between revenue and cost: $\pi(x) = TR(x) - TC(x)$.

2. Break-Even Condition: Setting profit to zero gives $TR(x) - TC(x) = 0$, which mathematically implies $TR(x) = TC(x)$.

3. Graphical Interpretation: On a coordinate system where the vertical axis represents monetary value and the horizontal axis represents quantity, the function $TR(x)$ and $TC(x)$ are represented by curves. The equality $TR = TC$ occurs at the specific coordinates where these two curves cross each other.

4. Identifying the Points: According to the graph description, the curves for TR and TC intersect at Point A and Point B. Therefore, these intersection points represent the break-even production levels.

Final Answer : The points of intersection of TR and TC

Answer: (B)

Q12.

Solution

Concept: Total Revenue (TR) is determined by the formula $TR = \text{Price} \times \text{Quantity}$. To find the output level that maximizes revenue, we must find the stationary point of the TR function (where the first derivative is zero) and ensure the second derivative is negative.

Solution:
1. Formulate Revenue Function: Given the demand function $p = 50 - 2x$.
$TR = p \cdot x = (50 - 2x)x$
$TR = 50x - 2x^2$.

2. First-Order Condition (FOC): Differentiate TR with respect to x and set it to zero.
$\frac{d(TR)}{dx} = 50 - 4x$
$50 - 4x = 0 \implies 4x = 50 \implies x = \frac{50}{4} = 12.5$.

3. Second-Order Condition (SOC): Verify the maximum.
$\frac{d^2(TR)}{dx^2} = -4$.
Since the second derivative is negative, the revenue is indeed maximized at $x = 12.5$.

Final Answer : 12.5

Answer: (B)

Q13.

Solution

Concept: Local extrema of a function $f(x)$ are found by identifying critical points where the first derivative $f'(x)$ is zero. A local minimum occurs at a critical point if the second derivative $f''(x)$ at that point is positive.

Solution:

1. Find the Derivative: For the function $f(x) = x + x^{-1}$ (where $x > 0$):
$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$.
2. Solve for Critical Points: Set $f'(x) = 0$:

$1 - \frac{1}{x^2} = 0 \implies x^2 = 1 \implies x = 1$ (ignoring -1 since $x > 0$).
3. Second Derivative Test: Calculate $f''(x)$:

$f''(x) = \frac{d}{dx}(1 - x^{-2}) = 2x^{-3} = \frac{2}{x^3}$.
Evaluate at $x = 1$: $f''(1) = \frac{2}{1^3} = 2$.
Since $f''(1) > 0$, the function has a local minimum at $x = 1$.

4. Calculate Minimum Value: Substitute $x = 1$ back into the original function:
$f(1) = 1 + \frac{1}{1} = 2$.

Final Answer : 2

Answer: (C)

Q14.

Solution

Concept: The slope of the tangent to the curve $y = f(x)$ at $x = a$ is given by $m_t = f'(a)$. The normal line is perpendicular to the tangent; therefore, its slope m_n satisfies the relationship $m_n \times m_t = -1$, or $m_n = -\frac{1}{f'(a)}$.

Solution:

1. Differentiate the Function: For $y = 2x^2 + 3\sin x$:
$\frac{dy}{dx} = 4x + 3\cos x$.

2. Calculate Tangent Slope (m_t): Evaluate the derivative at $x = 0$:
$m_t = 4(0) + 3\cos(0)$
$m_t = 0 + 3(1) = 3$.

3. Calculate Normal Slope (m_n): Use the perpendicular slope formula:
$m_n = -\frac{1}{m_t} = -\frac{1}{3}$.

Final Answer : -1/3

Answer: (D)

Q15.

Solution

Concept: This definite integral is evaluated using the substitution method. By substituting $u = e^x$, the integrand is transformed into a standard rational form that integrates to the arctangent function.

Solution:

1. Substitution: Let $u = e^x$. Differentiating both sides gives $du = e^x dx$.

2. Change Limits:
When the lower limit $x = 0$, $u = e^0 = 1$.
When the upper limit $x = 1$, $u = e^1 = e$.

3. Transform Integral: Substitute u and du into the original integral:
$\int_1^e \frac{1}{1+u^2} du$.

4. Integrate: Using the standard integral $\int \frac{1}{1+u^2} du = \tan^{-1} u$:
$[\tan^{-1} u]_1^e = \tan^{-1}(e) - \tan^{-1}(1)$.

5. Final Evaluation: We know that $\tan^{-1}(1) = \frac{\pi}{4}$.
Result $= \tan^{-1}(e) - \frac{\pi}{4}$.

Final Answer : $\tan^{-1}(e) - \frac{\pi}{4}$

Answer: (A)

Q16.

Solution

Concept: The area of a region bounded by a curve $y^2 = 4ax$ and a vertical line $x = h$ is found by integrating the function of y with respect to x. Due to the symmetry of the parabola about the x-axis, the total area is double the area of the upper half.

Solution:
1. Express y in terms of x: From $y^2 = 4x$, we have $y = \pm 2\sqrt{x}$.

2. Set up Integral: The total area A from $x = 0$ to $x = 3$ is:
$A = 2\int_0^3 2\sqrt{x}dx = 4\int_0^3 x^{1/2}dx$.

3. Integrate: Use the power rule for integration:
$A = 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = 4 \cdot \frac{2}{3}[x^{3/2}]_0^3$.
4. Evaluate:

$A = \frac{8}{3}(3^{3/2} - 0) = \frac{8}{3}(3\sqrt{3}) = 8\sqrt{3}$.

Final Answer : $8\sqrt{3}$

Answer: (B)

Q17.

Solution

Concept: Consumer Surplus (CS) measures the difference between what consumers are willing to pay and what they actually pay. It is calculated as the area under the demand curve P_d from 0 to equilibrium quantity x_0, minus the actual expenditure ($P_0 \cdot x_0$).

Solution:
1. Find equilibrium quantity (x_0): Set $P_d = P_0$.
$25 - x^2 = 9 \implies x^2 = 16 \implies x_0 = 4.$

2. Set up CS Formula: $CS = \int_0^{x_0} P_d dx - P_0 x_0$.
$CS = \int_0^4 (25 - x^2)dx - (9 \cdot 4)$.

3. Evaluate the Integral:
$\int_0^4 (25 - x^2)dx = [25x - \frac{x^3}{3}]_0^4 = (100 - \frac{64}{3})$.
$100 - 21.33 = 78.67$.

4. Final Calculation:
$CS = 78.67 - 36 = 42.67$.

Final Answer : 42.67

Answer: (A)

Q18.

Solution

Concept: Producer Surplus (PS) measures the benefit to producers from selling at the market price. It is calculated as the total revenue ($P_0 \cdot x_0$) minus the area under the supply curve P_s from 0 up to the equilibrium quantity x_0.

Solution:
1. Find equilibrium price (P_0): Substitute $x_0 = 3$ into the supply function $P_s = 2 + x^2$.

$P_0 = 2 + (3)^2 = 2 + 9 = 11.$
2. Set up PS Formula: $PS = P_0 x_0 - \int_0^{x_0} P_s dx.$
$PS = (11 \cdot 3) - \int_0^3 (2 + x^2) dx.$

3. Evaluate the Integral:
$\int_0^3 (2 + x^2) dx = [2x + \frac{x^3}{3}]_0^3$
$[2(3) + \frac{27}{3}] = 6 + 9 = 15.$

4. Final Calculation:
$PS = 33 - 15 = 18.$

Final Answer : 18

Answer: (B)

Q19.

Solution

Concept: This indefinite integral is solved using the method of integration by substitution. We identify a part of the integrand whose derivative is also present in the integrand.

Solution:

1. Identify Substitution: Let $u = \ln x$.

2. Differentiate Substitution: $\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}dx$.

3. Transform Integral: Substitute these into the original integral $\int \frac{1}{x(\ln x)^2}dx$:

$\int \frac{1}{u^2}du = \int u^{-2}du$.

4. Integrate: Using the power rule $\int u^n du = \frac{u^{n+1}}{n+1}$:

$\frac{u^{-2+1}}{-1} + C = -u^{-1} + C = -\frac{1}{u} + C$.

5. Back Substitution: Replace u with $\ln x$:

$-\frac{1}{\ln x} + C$.

Final Answer : $-\frac{1}{\ln x} + C$

Answer: (A)

Q20.

Solution

Concept: The area of the region bounded by two curves y_1 and y_2 is given by the integral of the difference $|y_1 - y_2|$ over the interval defined by their intersection points.

Solution:
1. Find Intersection Points: Set $|x - 1| = 1$.
Case 1: $x - 1 = 1 \implies x = 2$.
Case 2: $x - 1 = -1 \implies x = 0$.

2. Determine Geometry: The area is bounded by $y = 1$ (top) and $y = |x - 1|$ (bottom). This forms a triangle with vertices at $(0, 1)$, $(2, 1)$, and the vertex of the absolute value curve $(1, 0)$.

3. Calculate using Integration:
$Area = \int_0^2 (1 - |x - 1|)dx$.
Split the integral at $x = 1$:
$Area = \int_0^1 (1 - (1 - x))dx + \int_1^2 (1 - (x - 1))dx$
$Area = \int_0^1 xdx + \int_1^2 (2 - x)dx$.

4. Final Calculation:
$[\frac{x^2}{2}]_0^1 + [2x - \frac{x^2}{2}]_1^2 = (\frac{1}{2} - 0) + [(4 - 2) - (2 - \frac{1}{2})] = 0.5 + 0.5 = 1$.

Final Answer : 1

Answer: (A)

Q21.

Solution

Concept: This is a standard integral of a rational function. The general integration formula for the form $\int \frac{dx}{x^2-a^2}$ is given by $\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$, where a is a constant.

Solution:
1. Identify the constant a: The given integrand is $\frac{1}{x^2-16}$. We can express the denominator as a difference of squares: x^2-4^2. Comparing this with the standard form x^2-a^2, we find that $a = 4$.

2. Apply the Integration Formula: Substitute the value of a into the formula:
$\int \frac{dx}{x^2-4^2} = \frac{1}{2(4)}\ln\left|\frac{x-4}{x+4}\right| + C$.

3. Simplify the expression: Multiply the terms in the denominator of the coefficient:
$\frac{1}{2\times4} = \frac{1}{8}$.

4. Final Formulation: Combining the simplified coefficient with the logarithmic term, we obtain the final result.

Final Answer : $\frac{1}{8}\ln\left|\frac{x-4}{x+4}\right| + C$

Answer: (B)

Q22.

Solution

Concept: The order of a differential equation is the highest order derivative present in it. The degree is the power to which the highest order derivative is raised, after the equation has been made free from radicals and fractions with respect to the derivatives.

Solution: 1. Remove Radicals: The given equation is $\sqrt{(\frac{d^2y}{dx^2})^2 + \frac{dy}{dx}} = x$. To determine the degree, we must first rationalize the equation by squaring both sides:
$\left(\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx}\right) = x^2$.

2. Determine the Order: Observe the derivatives in the rationalized equation. The derivatives present are the second derivative $\frac{d^2y}{dx^2}$ and the first derivative $\frac{dy}{dx}$. The highest order of differentiation is 2. Therefore, Order = 2.

3. Determine the Degree: Look at the highest order derivative, which is $\frac{d^2y}{dx^2}$. In our rationalized equation, this term is $\left(\frac{d^2y}{dx^2}\right)^2$. The exponent of this highest order derivative is 2. Therefore, Degree = 2.

Final Answer : 2, 2

Answer: (A)

Q23.

Solution

Concept: This is a first-order differential equation solvable by the Variable Separable Method. If an equation can be written as $f(y)dy = g(x)dx$, we can find the general solution by integrating both sides.

Solution: 1. Simplify the RHS: Use the law of indices $e^{a-b} = e^a \cdot e^{-b}$ to rewrite the equation:
$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$.

2. Factorization: Factor out the common term e^{-y} from the right-hand side:
$\frac{dy}{dx} = e^{-y}(e^x + x^2)$.

3. Separate Variables: Move all terms involving y to the left and terms involving x to the right:
$\frac{1}{e^{-y}} dy = (e^x + x^2)dx \implies e^y dy = (e^x + x^2)dx$.

4. Integrate both sides:
$\int e^y dy = \int (e^x + x^2)dx$.

5. Execution: The integral of e^y is e^y. The integral of e^x is e^x, and the integral of x^2 is $\frac{x^3}{3}$.
$e^y = e^x + \frac{x^3}{3} + C$.

Final Answer : $e^y = e^x + \frac{x^3}{3} + C$

Answer: (A)

Q24.

Solution

Concept: Exponential growth is modeled by the differential equation $\frac{dP}{dt} = kP$, which yields the solution $P(t) = P_0 e^{kt}$, where P_0 is the initial population, k is the growth constant, and t is time.

Solution: 1. Set up the equation: Let the initial population at $t = 0$ be P_0.

2. Apply the condition: The population doubles in 10 years. This means at $t = 10$, $P = 2P_0$.

3. Substitute into the model:
$2P_0 = P_0 e^{k(10)}$.

4. Solve for exponential term: Divide both sides by P_0 (assuming $P_0 \neq 0$):
$2 = e^{10k}$.

5. Solve for k: Take the natural logarithm (ln) on both sides:
$\ln(2) = \ln(e^{10k}) \implies \ln(2) = 10k$.

6. Isolate k:
$k = \frac{\ln 2}{10}$.

Final Answer : $\frac{\ln 2}{10}$

Answer: (A)

Q25.

Solution

Concept: For a linear differential equation in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$, the Integrating Factor (IF) is used to consolidate the left side into a single derivative. The formula is $IF = e^{\int P(x)dx}$.

Solution: 1. Identify the components: Compare $\frac{dy}{dx} + \frac{y}{x} = x^2$ with the standard form.
Here, the term multiplying y is $P(x) = \frac{1}{x}$.

2. Calculate the integral of P(x):
$\int P(x)dx = \int \frac{1}{x}dx = \ln x$.

3. Compute the IF: Plug the result into the exponential formula:
$IF = e^{\ln x}$.

4. Simplify using Log properties: Since the exponential and natural logarithm are inverse functions, $e^{\ln(f(x))} = f(x)$.

5. Result: Therefore, $IF = x$.

Final Answer : x

Answer: (C)

Q26.

Solution

Concept: In a Poisson distribution with mean λ, the probability of a specific outcome k is given by $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$. We use the given equality to solve for the parameter λ.

Solution:

1. Express the given probabilities:
$P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = \lambda e^{-\lambda}$
$P(X = 2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{\lambda^2 e^{-\lambda}}{2}$

2. Equate and solve for λ:
$\lambda e^{-\lambda} = \frac{\lambda^2 e^{-\lambda}}{2}$.
Divide both sides by $e^{-\lambda}$ (which is never zero) and by λ (assuming $\lambda > 0$):
$1 = \frac{\lambda}{2} \implies \lambda = 2$.

3. Calculate P(X=0): Using the found value of $\lambda = 2$:
$P(X = 0) = \frac{e^{-2}2^0}{0!} = \frac{e^{-2}\times 1}{1} = e^{-2}$.

Final Answer : e^{-2}

Answer: (B)

Q27.

Solution

Concept: The Standard Normal Distribution is symmetric about the mean $Z = 0$. The total area under the curve is 1, which means the area to the right of the mean (positive Z-values) is exactly 0.5.

Solution: 1. Analyze the Curve: The probability of the entire right half of the bell curve is $P(Z > 0) = 0.5$.

2. Identify the provided area: We are given that the area between the mean ($Z = 0$) and $Z = 1.5$ is 0.4332.

3. Set up the calculation: The shaded region represents the area to the right of $Z = 1.5$, which can be expressed as:
$P(Z > 1.5) = P(Z > 0) - P(0 < Z < 1.5)$.

4. Subtract the values:
$P(Z > 1.5) = 0.5000 - 0.4332 = 0.0668$.
This represents the area in the "tail" of the distribution.

Final Answer : 0.0668

Answer: (C)

Q28.

Solution

Concept: The Empirical Rule (also called the 68-95-99.7 rule) describes the percentage of values that fall within certain standard deviations of the mean in a normal distribution.

Solution: 1. Standard Deviations: In a normal distribution, the data is centered around the mean μ.

2. 1-Sigma Range: Approximately 68.27% of the data falls within $\mu \pm 1\sigma$.

3. 2-Sigma Range: Approximately 95.45% of the data falls within $\mu \pm 2\sigma$. This is typically rounded to 95% in most textbook problems.

4. 3-Sigma Range: Approximately 99.73% of the data falls within $\mu \pm 3\sigma$.

5. Conclusion: For the range $\mu \pm 2\sigma$, the percentage is 95%.

Final Answer : 95%

Answer: (B)

Q29.

Solution

Concept: The Poisson distribution is a discrete probability distribution where a single parameter λ represents both the mean and the variance of the distribution. The standard deviation is the square root of the variance.

Solution:
1. Identify the Mean: We are given that the mean of the Poisson distribution is $\lambda = 4$.

2. Determine the Variance: Based on the property of the Poisson distribution, Variance $= \lambda$. Therefore, Variance $= 4$.

3. Calculate the Standard Deviation (σ): The standard deviation is defined as the positive square root of the variance.
$\sigma = \sqrt{\text{Variance}}$.

4. Final Computation: $\sigma = \sqrt{4} = 2$.

Final Answer : 2

Answer: (C)

Q30.

Solution

Concept: When a fair coin is tossed multiple times, the total number of possible outcomes is given by 2^n, where n is the number of tosses. The term "at most one head" is a cumulative probability condition which includes the scenario of getting zero heads or exactly one head.

Solution:
1. Determine the Total Outcomes: For 3 tosses, the total number of outcomes in the sample space S is $2^3 = 8$. The complete set is: $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$.

2. Define the Event: Let X be the number of heads. We are looking for the event E where $X \leq 1$.

3. List Favorable Outcomes: - Case $X = 0$: Only the outcome $\{TTT\}$ satisfies this (1 case).
- Case $X = 1$: The outcomes $\{HTT, THT, TTH\}$ satisfy this (3 cases).

4. Calculate Total Favorable Cases: Summing these gives $n(E) = 1 + 3 = 4$.

5. Compute Probability: Using the classical definition of probability $P(E) = \frac{n(E)}{n(S)}$, we get: $P(E) = \frac{4}{8} = \frac{1}{2}$.

Final Answer : 1/2

Answer: (C)

Q31.

Solution

Concept: Modular arithmetic properties allow us to simplify powers. Specifically, the property $(a^n) \pmod m = (a \pmod m)^n \pmod m$ is used to reduce the base before raising it to a large exponent.

Solution:

1. Identify the goal: We need to find the remainder when 7^{100} is divided by 6, which is written as $7^{100} \pmod 6$.

2. Reduce the base: First, divide the base (7) by the divisor (6) to find the smallest positive remainder.
$7 = (1 \times 6) + 1 \implies 7 \equiv 1 \pmod 6$.

3. Apply the exponent property: Replace the base 7 with its remainder 1 in the modular expression:
$7^{100} \equiv 1^{100} \pmod 6$.

4. Simplify the power: Since any power of 1 is always 1 ($1 \times 1 \times \cdots \times 1 = 1$), we have:
$1^{100} = 1$.

5. Conclusion: The remainder of the division is 1.

Final Answer : 1

Answer: (A)

Q32.

Solution

Concept: The effective speed of a boat is influenced by the current. Let v be the boat's speed in still water and s be the stream's speed. Upstream speed is $v - s$ (against the current), and downstream speed is $v + s$ (with the current). The total time is the sum of times for both legs: $T = \frac{D_{up}}{V_{up}} + \frac{D_{down}}{V_{down}}$.

Solution:

1. Define Variables: Let v be the speed of the boat in still water. Given $s = 2$ km/h.

2. Set up the Time Equation:
$\frac{24}{v-2} + \frac{28}{v+2} = 6.$

3. Simplify the Equation: Find a common denominator:
$\frac{24(v+2)+28(v-2)}{(v-2)(v+2)} = 6$
$\frac{24v+48+28v-56}{v^2-4} = 6.$

4. Cross-multiply and solve for v:
$52v - 8 = 6(v^2 - 4) \implies 52v - 8 = 6v^2 - 24$
$6v^2 - 52v - 16 = 0.$

5. Factor the Quadratic Equation: Divide by 2: $3v^2 - 26v - 8 = 0$.
$3v^2 - 24v - 2v - 8 = 0$ is incorrect; the correct split is: $3v^2 - 24v - 2v$ wait, $3 \times -8 = -24$.
$3v^2 - 24v + 2v - 8 = 0 \implies 3v(v - 8) + 2(v - 8) = 0 \implies (3v + 2)(v - 8) = 0.$

6. Result: Since speed v cannot be negative ($v = -2/3$), we take $v = 8$ km/h.

Final Answer : 8 km/h

Answer: (B)

Q33.

Solution

Concept: In a race, if A beats B by d meters in a race of length L, it means in the time A takes to run L meters, B only runs $L-d$ meters. This establishes a ratio of the distances they cover in equal time.

Solution:
1. Ratio of Distance A to B: A beats B by 10m in a 100m race. When A runs 100m, B runs $100 - 10 = 90$m.
$Ratio \frac{A}{B} = \frac{100}{90}$.

2. Ratio of Distance B to C: B beats C by 10m in a 100m race. When B runs 100m, C runs $100 - 10 = 90$m.
$Ratio \frac{B}{C} = \frac{100}{90}$.

3. Calculate Ratio A to C: By multiplying the ratios, we find how much C runs when A runs a certain distance:
$\frac{A}{C} = \frac{A}{B} \times \frac{B}{C} = \frac{100}{90} \times \frac{100}{90} = \frac{100\times100}{90\times90} = \frac{10000}{8100} = \frac{100}{81}$.

4. Determine the gap: When A runs 100m, C runs 81m.

5. Final Difference: The distance by which A beats C is $100 - 81 = 19$ meters.

Final Answer : 19m

Answer: (B)

Q34.

Solution

Concept: To find $(A \times B) \pmod{n}$, we can utilize the property that says the remainder of a product is the product of the individual remainders. Mathematically: $(A \cdot B) \pmod{n} = [(A \pmod{n}) \cdot (B \pmod{n})] \pmod{n}$.

Solution:

1. Simplify the first factor: Divide 25 by 7.
$25 = 7 \times 3 + 4$. The remainder is 4. So, $25 \equiv 4 \pmod{7}$.

2. Simplify the second factor: Divide 18 by 7.
$18 = 7 \times 2 + 4$. The remainder is 4. So, $18 \equiv 4 \pmod{7}$.

3. Multiply the remainders:
$4 \times 4 = 16$.

4. Simplify the result mod 7: Now, divide 16 by 7 to find the final remainder.
$16 = 7 \times 2 + 2$. The remainder is 2.

5. Conclusion: $(25 \times 18) \equiv 2 \pmod{7}$.

Final Answer : 2

Answer: (B)

Q35.

Solution

Concept: The method of moving averages is used to smooth out short-term fluctuations in a time series to identify trends. A 3-yearly moving average for a particular year is the average of the data points for that year, the year immediately before it, and the year immediately after it.

Solution:
1. Identify the Year of Interest: We need the 3-yearly moving average for the year 2022.

2. Identify the Window of Data: The calculation must include data from:
- One year before: 2021 (Value = 10)
- The current year: 2022 (Value = 20)
- One year after: 2023 (Value = 15)

3. Calculate the Total Sum: Add the values for these three years.
Sum = $10 + 20 + 15 = 45$.

4. Compute the Average: Divide the sum by the number of years in the window (3).
Average = $\frac{45}{3} = 15$.

Final Answer : 15

Answer: (A)

Q36.

Solution

Concept: In the method of least squares for fitting a straight line $y = a + bx$, we use two normal equations:

1) $\sum y = na + b\sum x$

2) $\sum xy = a\sum x + b\sum x^2$

If the data is centered such that $\sum x = 0$, the calculations for constants a and b become much simpler.

Solution:

1. Start with the first normal equation: $\sum y = na + b\sum x$.

2. Plug in the given values:

- $\sum y = 50$
- $n = 5$
- $\sum x = 0$

3. Set up the equation:

$50 = 5 \cdot a + b \cdot 0.$

4. Solve for a:

$50 = 5a + 0$

$a = \frac{50}{5} = 10.$

5. Interpretation: In this context, a is essentially the arithmetic mean of the y values ($\bar{y}$).

Final Answer : 10

Answer: (A)

Q37.

Solution

Concept: A time series is composed of four components: Secular Trend (T), Seasonal Variation (S), Cyclical Variation (C), and Irregular Variation (I). The secular trend is the most fundamental component describing the behavior of the data over a very long duration.

Solution:
1. Define the Secular Trend: It represents the basic tendency of the data to increase or decrease over a long period.

2. Distinguish from other components:
- Seasonal happens within a year.
- Cyclical happens over several years (business cycles).
- Irregular is random and unpredictable.

3. Identify Key Characteristics: The secular trend ignores short-term fluctuations and captures the long-term movement resulting from factors like population growth, technological progress, or changes in consumer habits.

Final Answer : Long-term direction or movement

Answer: (B)

Q38.

Solution

Concept: In statistical hypothesis testing, errors occur because we make decisions about a population based on limited sample data. A Type I error occurs when our test incorrectly suggests an effect or difference exists when it does not.

Solution:
1. Definitions of Hypotheses:
- H_0 (Null Hypothesis): Represents no effect or no difference (the "status quo").
- H_a (Alternative Hypothesis): Represents the claim we are testing for.

2. Defining Type I Error: This is the mistake of rejecting H_0 when, in reality, H_0 is true. It is also known as a "false positive."

3. Significance Level: The probability of committing a Type I error is denoted by α (alpha) and is pre-set by the researcher (commonly 0.05).

4. Contrast with Type II Error: Type II error is the failure to reject H_0 when it is actually false.

Final Answer : Rejecting H_0 when it is actually true

Answer: (A)

Q39.

Solution

Concept: Degrees of Freedom (df) refer to the number of values in the final calculation of a statistic that are free to vary. For a t-test involving a single sample mean, one degree of freedom is lost because the sample mean is used as an estimate of the population mean.

Solution:
1. Identify the Sample Size: The number of observations in the sample is $n = 12$.

2. Apply the formula for df: For a one-sample t-test, the formula is:
$df = n - 1$.

3. Perform the subtraction:
$df = 12 - 1 = 11$.

4. Context: This value is used to determine the critical t-value from the t-distribution table for a given confidence level.

Final Answer : 11

Answer: (C)

Q40.

Solution

Concept: The p-value is the probability of observing a test statistic as extreme as, or more extreme than, the one calculated from the sample, assuming the null hypothesis is true. We compare the p-value to the significance level (α) to make a decision.

Solution:
1. Identify the Given Values:
- Calculated p-value = 0.03
- Level of significance (α) = 0.05

2. State the Decision Rule:
- If p-value $\leq \alpha$, we reject the null hypothesis H_0.
- If p-value $> \alpha$, we fail to reject the null hypothesis H_0.

3. Apply the Comparison:
$0.03 < 0.05$.

4. Conclusion: Since the p-value is less than the significance level, the result is statistically significant. We reject H_0.

Final Answer : Reject H_0

Answer: (A)

Q41.

Solution

Concept: The Z-test and t-test are both used to test hypotheses about means. However, the t-test (specifically Student's t-test) was developed to handle situations where the sample size is small and the standard deviation of the entire population is unknown.

Solution:
1. The Z-test requirement: This test requires the population variance (σ^2) to be known or a very large sample size ($n \geq 30$) so that the sample variance is a reliable proxy for the population variance.

2. The t-test requirement: We use the t-test when:
- The sample size is small ($n < 30$).
- The population variance is unknown, requiring us to estimate it using the sample standard deviation (s).

3. Mathematical reason: The t-distribution has "heavier tails" than the normal distribution to account for the extra uncertainty in estimating the variance from a small sample.

Final Answer : Population variance is unknown and $n < 30$

Answer: (C)

Q42.

Solution

Concept: A sinking fund is an annuity where equal payments are made at regular intervals to accumulate a future sum. The future value A of an ordinary annuity (payments at the end of the period) is $A = P\left[\frac{(1+i)^n-1}{i}\right]$.

Solution:

1. Identify the parameters:
- Annual payment (P) = ₹ 10,000
- Number of years (n) = 5
- Annual interest rate (i) = 10% = 0.10

2. Set up the formula:
$A = 10,000\left[\frac{(1+0.10)^5-1}{0.10}\right]$.

3. Use the provided value: The problem states $(1.1)^5 = 1.6105$.
$A = 10,000\left[\frac{1.6105-1}{0.10}\right]$.

4. Perform subtraction in the numerator:
$A = 10,000\left[\frac{0.6105}{0.10}\right]$.

5. Divide and multiply:
$\frac{0.6105}{0.10} = 6.105$.
$A = 10,000 \times 6.105 = 61,050$.

Final Answer : ₹ 61,050

Answer: (A)

Q43.

Solution

Concept: A perpetuity is a type of annuity that provides a fixed sequence of payments that continue indefinitely. The present value (PV) of an ordinary perpetuity, where payments are made at the end of each period, is calculated by dividing the periodic payment (R) by the periodic interest rate (i). The formula is: $PV = \frac{R}{i}$.

Solution:

1. Identify the periodic payment (R): The payment is specified as ₹ 500, which is paid at the end of every month.

2. Convert the annual interest rate to a monthly interest rate (i): The given nominal interest rate is 12% per annum. Since the payments are monthly, we must find the rate per month:
$i = \frac{\text{Annual Rate}}{\text{Number of months in a year}} = \frac{12\%}{12} = 1\%$ per month.

3. Convert the percentage to a decimal for calculation:
$i = \frac{1}{100} = 0.01$.

4. Apply the perpetuity formula: Substitute $R = 500$ and $i = 0.01$ into the equation:
$PV = \frac{500}{0.01}$.

5. Final Calculation: Dividing 500 by 0.01 is equivalent to multiplying 500 by 100:
$PV = 50,000$.
The present value of this perpetual stream of payments is ₹ 50,000.

Final Answer : ₹ 50,000

Answer: (A)

Q44.

Solution

Concept: Bond pricing is determined by the relationship between the bond's fixed coupon rate and the prevailing market interest rate (required rate of return).
- If the Coupon Rate is equal to the Market Rate, the bond sells at Par (Face Value).
- If the Coupon Rate is higher than the Market Rate, the bond sells at a Premium.
- If the Coupon Rate is lower than the Market Rate, the bond sells at a Discount.

Solution:
1. Analyze the Coupon Rate: The bond offers a 10% annual coupon. This is the fixed interest the bondholder receives based on the face value.

2. Analyze the Market Rate: The required rate of return in the market is currently 12%. This is what investors expect from similar risk-profile investments.

3. Compare the two rates: We observe that the Coupon Rate (10%) is strictly less than the Market Rate (12%).

4. Determine the economic consequence: Since the bond pays less than the current market rate, it is less attractive. To entice buyers, the price of the bond must decrease until its total yield (interest plus capital gain) matches the market's 12%.

5. Conclusion: Because the price must drop below the face value of ₹ 1000 to compensate for the lower interest, the bond is said to be selling at a Discount.

Final Answer : A Discount

Answer: (C)

Q45.

Solution

Concept: The Equated Monthly Installment (EMI) is the fixed amount paid by a borrower to a lender at a specified date each calendar month. The formula for an EMI is derived from the present value of an ordinary annuity: $EMI = \frac{P \cdot i}{1-(1+i)^{-n}}$, where P is the principal loan amount, i is the monthly interest rate, and n is the number of installments.

Solution:

1. Identify the variables:

- Principal amount (P) = ₹ 1,00,000.
- Monthly interest rate (i) = 1% per month = 0.01.
- Number of installments (n) = 10.

2. Set up the EMI formula:

$EMI = \frac{1,00,000 \times 0.01}{1-(1.01)^{-10}}$.

3. Use the provided mathematical value: The question provides $(1.01)^{-10} = 0.905$.

4. Calculate the numerator:

$Numerator = 1,00,000 \times 0.01 = 1,000$.

5. Calculate the denominator:

$Denominator = 1 - 0.905 = 0.095$.

6. Final Division:

$EMI = \frac{1,000}{0.095}$.

$EMI \approx 10,526.31$.

Rounding to the nearest rupee as per the standard options, the EMI is ₹ 10,526.

Final Answer : ₹ 10,526

Answer: (A)

Q46.

Solution

Concept: The effective rate of interest is the equivalent annual rate of interest when compounding occurs more than once a year. It accounts for the "interest on interest" earned during the year. The formula is $r_{eff} = (1 + \frac{r}{n})^n - 1$, where r is the nominal annual rate and n is the number of compounding periods per year.

Solution:

1. Identify the Nominal Rate (r): The rate is 8% per annum, which is 0.08 in decimal form.

2. Identify Compounding Periods (n): The interest is compounded semi-annually, so it is compounded twice a year. Therefore, $n = 2$.

3. Calculate the periodic rate ($\frac{r}{n}$):
The rate for each 6-month period is $\frac{0.08}{2} = 0.04$.

4. Substitute into the effective rate formula:
$r_{eff} = (1 + 0.04)^2 - 1$
$r_{eff} = (1.04)^2 - 1$.

5. Expand the square:
$1.04 \times 1.04 = 1.0816$.
$r_{eff} = 1.0816 - 1 = 0.0816$.

6. Convert back to percentage:
$0.0816 \times 100 = 8.16\%$.

Final Answer : 8.16%

Answer: (B)

Q47.

Solution

Concept: Depreciation at a fixed percentage rate (reducing balance method) follows the same mathematical principle as compound interest, but with a negative growth rate. The value V after n years is given by the formula: $V = P(1 - r)^n$, where P is the initial value and r is the rate of depreciation.

Solution:
1. Identify the initial parameters:
- Current value (P) = ₹ 1,00,000.
- Annual depreciation rate (r) = 10% = 0.10.
- Time period (n) = 2 years.

2. Method 1 - Step-by-step calculation:
- Value at the end of Year 1: $1,00,000 - (10\% \text{ of } 1,00,000) = 1,00,000 - 10,000 = 90,000$.
- Value at the end of Year 2: $90,000 - (10\% \text{ of } 90,000) = 90,000 - 9,000 = 81,000$.

3. Method 2 - Using the formula:
$V = 1,00,000 \cdot (1 - 0.10)^2$
$V = 1,00,000 \cdot (0.9)^2$
$V = 1,00,000 \cdot 0.81 = 81,000$.

4. Conclusion: Both methods confirm that the value of the machine after 2 years will be ₹ 81,000.

Final Answer : ₹ 81,000

Answer: (B)

Q48.

Solution

Concept: The "Corner Point Theorem" for Linear Programming states that if a feasible region is bounded, the maximum or minimum value of the linear objective function $Z = ax + by$ must occur at one of the vertices (corner points) of the feasible region.

Solution:

1. Define the Objective Function: $Z = 3x + 2y$.

2. List the Corner Points: The given vertices are (0,0), (3,0), (2,2), and (0,4).

3. Evaluate Z at each corner point:
- At point (0,0): $Z = 3(0) + 2(0) = 0$.
- At point (3,0): $Z = 3(3) + 2(0) = 9 + 0 = 9$.
- At point (2,2): $Z = 3(2) + 2(2) = 6 + 4 = 10$.
- At point (0,4): $Z = 3(0) + 2(4) = 0 + 8 = 8$.

4. Determine the Maximum Value: Comparing the calculated values $\{0, 9, 10, 8\}$, the largest value is 10.

5. Identify the corresponding point: The maximum value of 10 occurs at the point (2,2).

Final Answer : (2,2)

Answer: (C)

Q49.

Solution

Concept: A Linear Programming Problem (LPP) is characterized by an objective function and a set of constraints. The solution space is determined by these constraints.

Solution:
1. Definition of Constraints: Constraints are linear inequalities or equations that represent the limitations on resources or requirements (e.g., time, labor, materials).

2. Non-negativity Conditions: In real-world optimization, variables representing physical quantities (like x and y) cannot be negative, hence $x \geq 0, y \geq 0$.

3. Feasibility: For a solution to be "feasible," it must not violate any of the rules provided by the problem.

4. The Region: The "Feasible Region" is the intersection of all half-planes defined by the constraints. This means every point within this region satisfies all the given constraints and the non-negativity conditions at the same time.

Final Answer : All constraints and non-negativity conditions simultaneously

Answer: (C)

Q50.

Solution

Concept: The value of an objective function at a specific coordinate is found by substituting the x (abscissa) and y (ordinate) values of the coordinate into the linear equation of the objective function.

Solution:
1. State the Objective Function: $Z = 5x + 3y$.

2. Identify the Point coordinates: We are given the corner point $(x, y) = (4, 5)$. This implies $x = 4$ and $y = 5$.

3. Substitute the coordinates into the function:
$Z = 5(4) + 3(5)$.

4. Perform the multiplication:
$5 \times 4 = 20$.
$3 \times 5 = 15$.

5. Perform the final addition:
$Z = 20 + 15 = 35$.
The value of the objective function at this specific corner point is 35.

Final Answer : 35

Answer: (C)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	B	5	B
6	C	7	B	8	A	9	A	10	A
11	B	12	B	13	C	14	D	15	A
16	B	17	A	18	B	19	A	20	A
21	B	22	A	23	A	24	A	25	C
26	B	27	C	28	B	29	C	30	C
31	A	32	B	33	B	34	B	35	A
36	A	37	B	38	A	39	C	40	A
41	C	42	A	43	A	44	C	45	A
46	B	47	B	48	C	49	C	50	C