CUET-UG Applied Mathematics Sample Paper-18

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a non-singular matrix of order 3 such that $A^2 = 3A$, then the value of the determinant $|A|$ is:

(A) 3

(B) 9

(C) 27

(D) 81

Q2. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, then the value of $|3AB|$ is:

(A) -9

(B) -81

(C) -27

(D) 243

Q3. For a matrix A, if $A^{-1} = \frac{1}{k}adj(A)$, then the value of k must be:

(A) $|A|$

(B) $|A|^2$

(C) $1/|A|$

(D) k^2

Q4. The system of linear equations $x - cy - cz = 0$, $cx - y + cz = 0$, $cx + cy - z = 0$ has a non-trivial solution if the value of c is:

(A) $1/2$

(B) -1

(C) 0

(D) Both (A) and (B)

Q5. If the matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then the value of a is:

(A) $\pm 1/\sqrt{2}$

(B) $\pm 1/\sqrt{3}$

(C) $\pm 1/2$

(D) ± 1

Q6. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then the value of α is:

(A) ± 3

(B) ± 2

(C) ± 5

(D) 0

Q7. The inverse of a symmetric matrix, if it exists, is always:

(A) Diagonal

(B) Skew-symmetric

(C) Symmetric

(D) Identity

Q8. The total revenue function is $R(x) = 100x - 0.5x^2$. The value of x for which the marginal revenue is 20 is:

(A) 40

(B) 80

(C) 60

(D) 100

Q9. The function $f(x) = x^x$ has a stationary point at x equal to:

(A) e

(B) $1/e$

(C) 1

(D) $\ln(2)$

Q10. The interval in which the function $f(x) = xe^{x(1-x)}$ is increasing is:

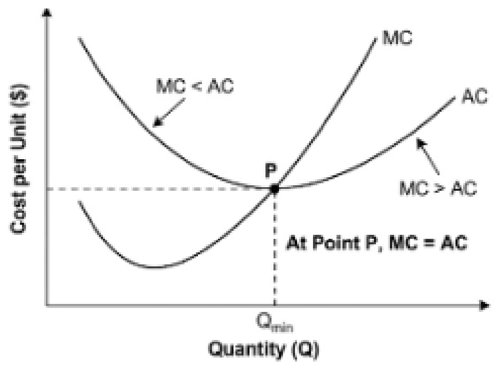
(A) $(-1/2, 1)$

(B) $(-1, 1/2)$

(C) $(-\infty, \infty)$

(D) $(0, 2)$

Q11. Which of the following is true?



(A) Total Cost is minimum

(B) Average Cost is minimum

(C) Marginal Cost is maximum

(D) Profit is maximum

Q12. If the demand function is $p = 400 - 2x - 3x^2$, the marginal revenue when $x = 10$ is:

(A) 250

(B) -540

(C) -500

(D) 400

Q13. The maximum value of $f(x) = \frac{\ln x}{x}$ for $x > 0$ occurs at $x =$:

(A) 1

(B) e

(C) $1/e$

(D) e^2

Q14. The cost function $C(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 10$. The value of x where the marginal cost is minimum is:

(A) 3

(B) 2

(C) 9

(D) 0

Q15. Evaluate $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$:

(A) $\ln|\sin x + \cos x| + C$

(B) $-\ln|\sin x + \cos x| + C$

(C) $\ln|\sin x - \cos x| + C$

(D) $\tan x + C$

Q16. The area bounded by the curve $y = x^3$, the x-axis and the lines $x = -1$ and $x = 1$ is:

(A) 0

(B) $1/2$

(C) $1/4$

(D) 2

Q17. The demand function for a commodity is $p = 10 - 2x$. If the equilibrium price is 4, find the Consumer Surplus:

(A) 9

(B) 18

(C) 4.5

(D) 6

Q18. The supply function is $p = 2 + x$. If the market equilibrium quantity is 5, the Producer Surplus is:

(A) 12.5

(B) 25

(C) 7.5

(D) 10

Q19. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q20. Find $\int e^x(\frac{1}{x} - \frac{1}{x^2})dx$:

(A) $\frac{e^x}{x^2} + C$

(B) $\frac{e^x}{x} + C$

(C) $e^x \ln x + C$

(D) $-\frac{e^x}{x} + C$

Q21. If the marginal revenue is $MR = 20 - 4x - 3x^2$, the total revenue function $R(x)$ (assuming $R(0) = 0$) is:

(A) $20x - 2x^2 - x^3$

(B) $20 - 4x - x^3$

(C) $20x - 4x^2 - 3x^3$

(D) $10x^2 - x^3$

Q22. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $y - x = C(1 + xy)$

(C) $x + y = C(1 - xy)$

(D) $\ln(1 + y^2) = \ln(1 + x^2) + C$

Q23. The differential equation representing the family of curves $y = ae^{bx}$ is of order:

(A) 1

(B) 2

(C) 3

(D) 0

Q24. In a bacterial culture, the rate of increase of bacteria is proportional to the number present. If the number triples in 5 hours, the time taken to become 9 times the original is:

(A) 10 hours

(B) 15 hours

(C) 7.5 hours

(D) 12 hours

Q25. The degree of the differential equation $\frac{d^2y}{dx^2} + \sin(\frac{dy}{dx}) = 0$ is:

(A) 1

(B) 2

(C) 0

(D) Not Defined

Q26. A random variable X follows Poisson distribution such that $P(X = k) = \frac{e^{-3}3^k}{k!}$. The variance of this distribution is:

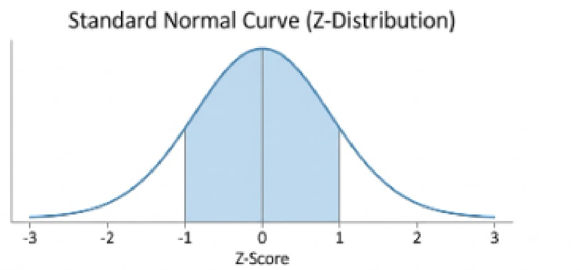
(A) $\sqrt{3}$

(B) 9

(C) 3

(D) e^{-3}

Q27. Given that the area under the curve from $Z = 0$ to $Z = 1$ is 0.3413, what is the probability represented by the shaded region?



(A) 0.3413

(B) 0.6826

(C) 0.1587

(D) 0.8413

Q28. If the Z-score for a value x in a normal distribution is -2, it means the value x is:

(A) 2 units below the mean

(B) 2 standard deviations below the mean

(C) 2 units above the mean

(D) 2 standard deviations above the mean

Q29. In a binomial distribution, if the mean is 4 and the variance is 3, then the number of trials n is:

(A) 12

(B) 16

(C) 8

(D) 10

Q30. The probability that a Poisson variable X takes a positive value is $1 - e^{-2}$. The mean of the distribution is:

(A) 1

(B) 2

(C) e

(D) 0

Q31. What is the last digit of 2^{50}?

(A) 2

(B) 4

(C) 8

(D) 6

Q32. A boat can travel at 5 km/h in still water. If the speed of the stream is 3 km/h, the time taken to go 16 km downstream is:

(A) 4 hours

(B) 8 hours

(C) 2 hours

(D) 5 hours

Q33. In a 500m race, A beats B by 50m. In a 600m race, B beats C by 60m. By how many meters will A beat C in a 500m race?

(A) 95m

(B) 110m

(C) 100m

(D) 85m

Q34. Find x if $3x \equiv 2 \pmod 5$:

(A) 1

(B) 2

(C) 3

(D) 4

Q35. Using a 4-yearly moving average, how many data points are lost (cannot be calculated) at the beginning and end of the series combined?

(A) 2

(B) 3

(C) 4

(D) 5

Q36. If the trend equation is $y = 20 + 3x$ (with origin 2018 and x unit = 1 year), the predicted value for 2023 is:

(A) 35

(B) 38

(C) 32

(D) 41

Q37. The additive model of a time series is expressed as:

(A) $Y = T \times S \times C \times I$

(B) $Y = T + S + C + I$

(C) $Y = T + S - C + I$

(D) $Y = T \times S + C$

Q38. The probability of committing a Type II error is denoted by:

(A) α

(B) β

(C) $1 - \alpha$

(D) $1 - \beta$

Q39. A researcher wants to test if the mean weight of a sample of 25 students is significantly different from 60kg. The population standard deviation is unknown. Which test should be used?

(A) Z-test

(B) t-test

(C) F-test

(D) Chi-square test

Q40. The "Power of a Test" is defined as:

(A) α

(B) β

(C) $1 - \beta$

(D) $1 - \alpha$

Q41. If the null hypothesis $H_0 : \mu = 50$ is tested against $H_a : \mu \neq 50$, it is a:

(A) Right-tailed test

(B) Left-tailed test

(C) Two-tailed test

(D) One-tailed test

Q42. The present value of a bond that pays ₹ 100 annually forever (perpetuity) at a discount rate of 5% is:

(A) ₹ 500

(B) ₹ 2,000

(C) ₹ 1,000

(D) ₹ 10,000

Q43. A sinking fund is created to accumulate ₹ 5,00,000 in 10 years. If the interest rate is 8% p.a. compounded annually, the periodic payment R is given by:

(A) $R = \frac{5,00,000 \times 0.08}{(1.08)^{10} - 1}$

(B) $R = \frac{5,00,000 \times 1.08}{(1.08)^{10} - 1}$

(C) $R = 5,00,000(1.08)^{10}$

(D) $R = \frac{5,00,000}{(1.08)^{10}}$

Q44. If a bond is trading "at par", it means:

(A) Coupon Rate > Yield

(B) Coupon Rate < Yield

(C) Coupon Rate = Yield

(D) The bond has no coupons

Q45. The nominal rate of interest is 12% p.a. compounded quarterly. The effective rate of interest is:

(A) 12%

(B) 12.55%

(C) 12.68%

(D) 13%

Q46. An annuity where payments are made at the beginning of each period is called:

(A) Ordinary Annuity

(B) Annuity Due

(C) Perpetuity

(D) Deferred Annuity

Q47. The value of a property is ₹ 20,00,000. It appreciates at 5% p.a. Its value after 3 years will be:

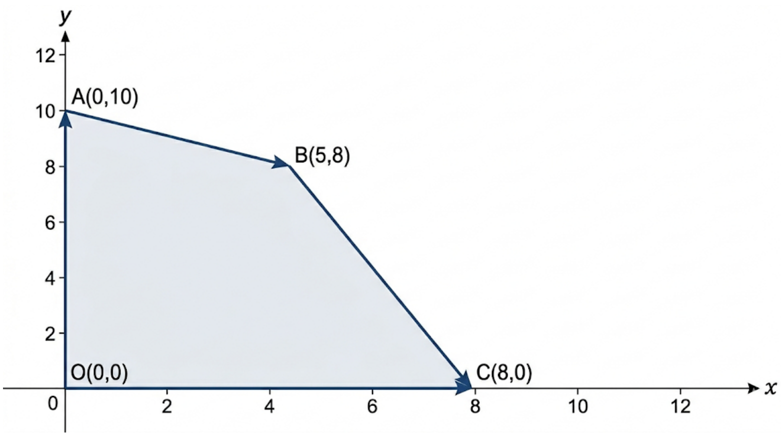
(A) ₹ 23,15,250

(B) ₹ 23,00,000

(C) ₹ 21,50,000

(D) ₹ 24,00,000

Q48. The objective function is $Z = 10x + 12y$. At which vertex is the objective function maximized?



(A) A(0, 10)

(B) B(5, 8)

(C) C(8, 0)

(D) O(0,0)

Q49. In an LPP, if the objective function is parallel to one of the constraint lines, the system may have:

(A) No solution

(B) Unique solution

(C) Infinite solutions

(D) Exactly two solutions

Q50. Which of the following constraints is redundant if the objective is to maximize $Z = x + y$ subject to $x \leq 5, x \leq 10, y \leq 8, x, y \geq 0$?

(A) $x \leq 5$

(B) $x \leq 10$

(C) $y \leq 8$

(D) $x \geq 0$

Detailed Solutions

Q1.

Solution

Concept: For a square matrix A of order n:

1. $|A^m| = (|A|)^m$
2. $|kA| = k^n|A|$

A non-singular matrix has $|A| \neq 0$, which means it is invertible.

Solution: Given that A is a non-singular matrix of order 3 and satisfies the equation $A^2 = 3A$.

Taking determinants on both sides of the equation:
$|A^2| = |3A|$

Now applying determinant properties:
$|A^2| = (|A|)^2$ (using $|A^m| = (|A|)^m$)
$|3A| = 3^3|A| = 27|A|$ (since order of A is 3)

Substituting these results:
$(|A|)^2 = 27|A|$

Rewriting the equation:
$(|A|)^2 - 27|A| = 0$

Factorizing:
$|A|(|A| - 27) = 0$

So, $|A| = 0$ or $|A| = 27$.

Since A is non-singular, its determinant cannot be zero. Hence, we discard $|A| = 0$.

Final Answer : "27"

Answer: (C)

Q2.

Solution

Concept: Important determinant properties:
1. $|AB| = |A||B|$ (determinant of a product)
2. $|kA| = k^n|A|$ for a matrix of order n, where k is a scalar

Solution: Given that A and B are square matrices of order 3 with $|A| = -1$ and $|B| = 3$.

We need to find the value of $|3AB|$.

First, consider the scalar multiple:
Since AB is also a matrix of order 3, applying the property gives:
$|3AB| = 3^3|AB| = 27|AB|$

Next, evaluate $|AB|$ using the product property:
$|AB| = |A||B|$

Substituting this into the equation:
$|3AB| = 27(|A||B|)$

Now replace the given values:
$|3AB| = 27(-1 \times 3)$

Simplifying:
$|3AB| = 27(-3) = -81$

Thus, the required determinant is -81.

Final Answer : "-81"

Answer: (B)

Q3.

Solution

Concept: The inverse of a square matrix A, denoted by A^{-1}, is a unique matrix such that $AA^{-1} = A^{-1}A = I$, where I is the identity matrix. The inverse exists if and only if A is a non-singular matrix (i.e., its determinant $|A| \neq 0$).
The standard formula for calculating the inverse of an invertible matrix A is: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$ where $\text{adj}(A)$ represents the adjugate (also known as the adjoint) of matrix A. The adjugate matrix is the transpose of the cofactor matrix of A.

Solution: We are given a particular relationship for the inverse of a matrix A:
$A^{-1} = \frac{1}{k}\text{adj}(A)$ (Given Equation)

We also know the fundamental definition and formula for the inverse of any invertible matrix A:
$A^{-1} = \frac{1}{|A|}\text{adj}(A)$ (Standard Formula)

Our goal is to find the value of k. To do this, we can directly compare the "Given Equation" with the "Standard Formula."
Comparing the two expressions for A^{-1}:
$\frac{1}{k}\text{adj}(A) = \frac{1}{|A|}\text{adj}(A)$

For this equality to hold true for any invertible matrix A (meaning $\text{adj}(A)$ is not the zero matrix and $|A| \neq 0$), the scalar coefficients multiplying $\text{adj}(A)$ on both sides must be equal.
Therefore, we can equate the scalar factors:
$\frac{1}{k} = \frac{1}{|A|}$

To solve for k, we can simply take the reciprocal of both sides of this equation:
$k = |A|$

Thus, the value of k must be equal to the determinant of matrix A. This makes sense as the determinant is the unique scalar value that appears in the denominator of the standard inverse formula.

Final Answer : "$|A|$"

Answer: (A)

Q4.

Solution

Concept: A homogeneous system $AX = 0$ has a non-trivial solution only if $|A| = 0$.

Solution: Given system:

$x - cy - cz = 0, \quad cx - y + cz = 0, \quad cx + cy - z = 0$

Coefficient matrix:

$$A = \begin{bmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{bmatrix}$$

For non-trivial solutions, set $|A| = 0$.

Expanding determinant:

$$|A| = 1\begin{vmatrix} -1 & c \\ c & -1 \end{vmatrix} - (-c)\begin{vmatrix} c & c \\ c & -1 \end{vmatrix} + (-c)\begin{vmatrix} c & -1 \\ c & c \end{vmatrix}$$

Evaluating minors:

$1 - c^2, \quad -c - c^2, \quad c^2 + c$

Substituting:

$|A| = (1 - c^2) + c(-c - c^2) - c(c^2 + c)$

Simplifying:

$|A| = 1 - 3c^2 - 2c^3 = 0$

$\Rightarrow 2c^3 + 3c^2 - 1 = 0$

Checking values:

$c = \frac{1}{2}$ and $c = -1$ satisfy the equation.

Final Answer : "Both (A) and (B)"

Answer: (D)

Q5.

Solution

Concept: A matrix A is orthogonal if $A^T A = I$, meaning its rows (and columns) form an orthonormal set. Thus: 1. Each row vector has unit length (dot product with itself = 1) 2. Different row vectors are mutually perpendicular (dot product = 0)

Solution: Given: $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$

Let: $R_1 = [0, 2b, c],\ R_2 = [a, b, -c],\ R_3 = [a, -b, c]$

Step 1: Apply normalization condition
$R_1 \cdot R_1 = 4b^2 + c^2 = 1 \quad \ldots(1)$ $R_2 \cdot R_2 = a^2 + b^2 + c^2 = 1 \quad \ldots(2)$ $R_3 \cdot R_3 = a^2 + b^2 + c^2 = 1 \quad \ldots(3)$
Step 2: Apply orthogonality condition
$R_1 \cdot R_2 = 2b^2 - c^2 = 0 \Rightarrow c^2 = 2b^2 \quad \ldots(4)$ $R_2 \cdot R_3 = a^2 - b^2 - c^2 = 0 \quad \ldots(5)$
Step 3: Solve equations
Substitute $c^2 = 2b^2$ into (1): $4b^2 + 2b^2 = 1 \Rightarrow 6b^2 = 1 \Rightarrow b^2 = \frac{1}{6}$

Then: $c^2 = 2b^2 = \frac{1}{3}$

Now substitute into (2): $a^2 + \frac{1}{6} + \frac{1}{3} = 1$ $a^2 + \frac{1}{2} = 1$ $a^2 = \frac{1}{2}$

Step 4: Final value $a = \pm\frac{1}{\sqrt{2}}$

Conclusion: All orthogonality conditions are satisfied.

Final Answer : "$\pm 1/\sqrt{2}$"

Answer: (A)

Q6.

Solution

Concept: For any square matrix A: $|A^n| = (|A|)^n$ This property simplifies determinant calculations of matrix powers.

Solution: Given: $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}, \quad |A^3| = 125$

Step 1: Find $|A|$ $|A| = \alpha^2 - 4$

Step 2: Apply determinant property $|A^3| = (|A|)^3 = 125$

Step 3: Solve equation $(\alpha^2 - 4)^3 = 125$
Taking cube root: $\alpha^2 - 4 = 5$
$\alpha^2 = 9$
Step 4: Final values $\alpha = \pm 3$

Conclusion: Both positive and negative values satisfy the condition.

Final Answer : "±3"

Answer: (A)

Q7.

Solution

Concept: 1. Symmetric Matrix: A square matrix A is defined as symmetric if it is equal to its own transpose. That is, $A = A^T$. This means the elements a_{ij} are equal to a_{ji} for all i, j.
2. Inverse of a Matrix: The inverse of a square matrix A, denoted by A^{-1}, exists if and only if A is non-singular ($|A| \neq 0$). It satisfies $AA^{-1} = A^{-1}A = I$, where I is the identity matrix.
3. Properties of Transpose and Inverse: A crucial property relating transpose and inverse is that the transpose of the inverse of a matrix is equal to the inverse of its transpose:
$(A^{-1})^T = (A^T)^{-1}$.
Solution: Let A be a symmetric matrix. By the definition of a symmetric matrix, this means:
$A = A^T \quad$ (Equation 1)

We are asked about the nature of the inverse of A, A^{-1}, assuming it exists. For A^{-1} to exist, A must be invertible (non-singular, so $|A| \neq 0$).
We want to determine if A^{-1} is symmetric. For A^{-1} to be symmetric, it must satisfy the condition:
$(A^{-1})^T = A^{-1}$

Let's use the property linking transpose and inverse:
For any invertible matrix A, we know that $(A^{-1})^T = (A^T)^{-1}$.

Now, substitute Equation 1 ($A = A^T$) into this property. Wherever we see A^T, we can replace it with A:
$(A^T)^{-1} = (A)^{-1}$

Therefore, combining these steps, we have:
$(A^{-1})^T = (A)^{-1}$

This final result, $(A^{-1})^T = A^{-1}$, directly shows that the transpose of the inverse of A is equal to the inverse of A itself. By definition, this means that A^{-1} is a symmetric matrix.

In conclusion, if a symmetric matrix has an inverse, its inverse will always also be symmetric.

Final Answer : "Symmetric"

Answer: (C)

Q8.

Solution

Concept: In economics, marginal revenue (MR) is the additional revenue generated by selling one more unit of a good or service. Mathematically, if $R(x)$ is the total revenue function (where x is the quantity of units sold), then the marginal revenue function $MR(x)$ is the first derivative of the total revenue function with respect to x:
$MR(x) = \frac{dR}{dx}$

Solution: We are given the total revenue function:
$R(x) = 100x - 0.5x^2$

Our first step is to find the marginal revenue function, $MR(x)$, by differentiating $R(x)$ with respect to x.
$MR(x) = \frac{dR}{dx} = \frac{d}{dx}(100x - 0.5x^2)$

Apply the rules of differentiation:
The derivative of $100x$ with respect to x is $100 \times \frac{d}{dx}(x) = 100 \times 1 = 100$.
The derivative of $0.5x^2$ with respect to x is $0.5 \times \frac{d}{dx}(x^2) = 0.5 \times (2x) = x$.

So, the marginal revenue function is:
$MR(x) = 100 - x$

Next, we are told that the value of x for which the marginal revenue is 20. This means we set $MR(x)$ equal to 20:
$MR(x) = 20$
$100 - x = 20$

Finally, we solve this simple linear equation for x:
$x = 100 - 20$
$x = 80$

Therefore, when 80 units are sold, the marginal revenue is 20.

Final Answer : "80"

Answer: (B)

Q9.

Solution

Concept: A stationary (critical) point occurs where $f'(x) = 0$. For functions like x^x, where the variable appears in both base and exponent, logarithmic differentiation is used.

Solution: Let $y = x^x$. Taking natural logarithm:

$$\ln y = x \ln x$$

Differentiate both sides (implicit + product rule):

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

Multiply by y:

$$\frac{dy}{dx} = x^x(\ln x + 1)$$

For stationary point:

$$x^x(\ln x + 1) = 0$$

Since $x^x > 0$ for $x > 0$,

$$\ln x + 1 = 0 \Rightarrow \ln x = -1$$

$$x = e^{-1} = \frac{1}{e}$$

Final Answer : "$1/e$"

Answer: (B)

Q10.

Solution

Concept: A function $f(x)$ is increasing on an interval if $f'(x) > 0$. Finding the derivative involves the product rule $(uv)' = u'v + uv'$ and the chain rule for exponential functions.

Solution: Given $f(x) = xe^{x(1-x)}$, we rewrite the exponent as $x - x^2$:

$$f(x) = xe^{x-x^2}$$

Let $u = x$ and $v = e^{x-x^2}$.
Then $u' = 1$ and $v' = e^{x-x^2} \cdot \frac{d}{dx}(x - x^2) = e^{x-x^2}(1 - 2x)$.
Applying the product rule:

$$f'(x) = (1)e^{x-x^2} + (x)e^{x-x^2}(1 - 2x)$$

Factor out e^{x-x^2}:

$$f'(x) = e^{x-x^2}[1 + x(1 - 2x)] = e^{x-x^2}(1 + x - 2x^2)$$

For the function to be increasing, we require $f'(x) > 0$. Since $e^{x-x^2} > 0$ for all x, we solve:

$$1 + x - 2x^2 > 0$$

Rearranging and multiplying by -1 (which reverses the inequality sign):

$$2x^2 - x - 1 < 0$$

Factoring the quadratic:

$$(2x + 1)(x - 1) < 0$$

The roots are $x = -1/2$ and $x = 1$. The expression is negative between the roots. Therefore, $f(x)$ is increasing in the interval $(-1/2, 1)$.

Final Answer : "$(-1/2, 1)$"

Answer: (A)

Q11.

Solution

Concept: In microeconomics, the relationship between Marginal Cost (MC) and Average Cost (AC) is fundamental. AC is defined as Total Cost (TC) divided by quantity (x). Mathematically, the MC curve intersects the AC curve at the minimum point of the AC curve.

Solution: Let $AC = \frac{C(x)}{x}$. To find the minimum of AC, we find its derivative and set it to zero:

$$\frac{d}{dx}(AC) = \frac{x \cdot C'(x) - C(x)}{x^2} = 0$$

Since $C'(x)$ is the definition of Marginal Cost (MC), we have:

$$x \cdot MC - C(x) = 0 \implies MC = \frac{C(x)}{x}$$

Because $\frac{C(x)}{x}$ is AC, this confirms that $MC = AC$ at the stationary point of the AC curve.
In a typical U-shaped cost structure:
1. When $MC < AC$, the cost of the next unit pulls the average down (AC falls).
2. When $MC > AC$, the cost of the next unit pulls the average up (AC rises).
3. At the intersection $MC = AC$ (Point P), AC stops falling and starts rising, meaning it is at its minimum.

Final Answer : "Average Cost is minimum"

Answer: (B)

Q12.

Solution

Concept: Total Revenue (TR) is found by multiplying price (p) by quantity (x). Marginal Revenue (MR) is the derivative of Total Revenue with respect to x: $MR = \frac{d(TR)}{dx}$.

Solution: Given the demand function: $p = 400 - 2x - 3x^2$.
Step 1: Find the Total Revenue function:

$$TR = p \cdot x = (400 - 2x - 3x^2)x = 400x - 2x^2 - 3x^3$$

Step 2: Find the Marginal Revenue function by differentiating TR:

$$MR = \frac{d}{dx}(400x - 2x^2 - 3x^3) = 400 - 4x - 9x^2$$

Step 3: Calculate the value of MR at $x = 10$:

$$MR(10) = 400 - 4(10) - 9(10)^2$$

$$MR(10) = 400 - 40 - 9(100)$$

$$MR(10) = 400 - 40 - 900 = -540$$

The marginal revenue is -540.

Final Answer : "-540"

Answer: (B)

Q13.

Solution

Concept: To find the maximum of $f(x)$, we calculate $f'(x)$ using the quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$. We then find critical points and use the first derivative test to check for a maximum.

Solution: Given $f(x) = \frac{\ln x}{x}$.
Let $u = \ln x$ and $v = x$. Then $u' = 1/x$ and $v' = 1$.
Applying the quotient rule:

$$f'(x) = \frac{(1/x)(x) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

Set $f'(x) = 0$ to find the critical point:

$$1 - \ln x = 0 \implies \ln x = 1 \implies x = e$$

To verify it is a maximum, check the sign of $f'(x)$:
- If $x < e$, $\ln x < 1$, so $f'(x) > 0$ (function is increasing).
- If $x > e$, $\ln x > 1$, so $f'(x) < 0$ (function is decreasing).

Since the derivative changes from positive to negative at $x = e$, the function reaches its maximum at $x = e$.

Final Answer : "e"

Answer: (B)

Q14.

Solution

Concept: Marginal Cost (MC) is defined as the first derivative of the Total Cost ($C(x)$). To find the value of x where MC is minimum, we differentiate the MC function and set it to zero.

Solution: Given $C(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 10$.
Step 1: Derive the Marginal Cost function:

$$MC(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x^2 + 9x + 10\right) = x^2 - 6x + 9$$

Step 2: Find the derivative of $MC(x)$ to find the minimum:

$$MC'(x) = \frac{d}{dx}(x^2 - 6x + 9) = 2x - 6$$

Step 3: Set the derivative to zero:

$$2x - 6 = 0 \implies x = 3$$

Step 4: Verify the minimum using the second derivative test:

$$MC''(x) = 2$$

Since $MC''(3) = 2 > 0$, the function is concave up at $x = 3$, confirming it is a minimum point for Marginal Cost.

Final Answer : "3"

Answer: (A)

Q15.

Solution

Concept: This integral is solved using u-substitution. The goal is to recognize that the derivative of the denominator is closely related to the numerator.
Formula: $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$.

Solution: We want to evaluate $I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$.
Let $u = \sin x + \cos x$.
Then $du = (\cos x - \sin x) dx$.
Comparing this to the numerator: $\sin x - \cos x = -(\cos x - \sin x)$.
Therefore, the numerator part $(\sin x - \cos x) dx$ becomes $-du$.
The integral simplifies to:

$$I = \int \frac{-du}{u} = -\int \frac{1}{u} du$$

$$I = -\ln|u| + C$$

Substituting back $u = \sin x + \cos x$:

$$I = -\ln|\sin x + \cos x| + C$$

Final Answer : "$-\ln|\sin x + \cos x| + C$"

Answer: (B)

Q16.

Solution

Concept: The area bounded by $y = f(x)$, the x-axis, and $x = a$ to $x = b$ is $\int_a^b |f(x)|dx$. Since $y = x^3$ is an odd function, it is negative on the interval $[-1, 0]$ and positive on $[0, 1]$.
To find the geometric area, we integrate the absolute values separately.

Solution:

$$Area = \int_{-1}^{1} |x^3|dx = \int_{-1}^{0} (-x^3)dx + \int_{0}^{1} x^3 dx$$

Evaluate the first part:

$$\int_{-1}^{0} -x^3 dx = \left[-\frac{x^4}{4}\right]_{-1}^{0} = (0) - (-\frac{(-1)^4}{4}) = \frac{1}{4}$$

Evaluate the second part:

$$\int_{0}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{0}^{1} = \frac{1}{4} - 0 = \frac{1}{4}$$

Sum the two parts:

$$Area = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Final Answer : "1/2"

Answer: (B)

Q17.

Solution

Concept: Consumer Surplus (CS) is the area between the demand curve $p = D(x)$ and the equilibrium price p_e up to the equilibrium quantity x_e:

$$CS = \int_0^{x_e} (D(x) - p_e)dx$$

Solution: Given the demand function $p = 10 - 2x$ and equilibrium price $p_e = 4$.
Step 1: Find the equilibrium quantity x_e by substituting $p = 4$:

$$4 = 10 - 2x_e \implies 2x_e = 6 \implies x_e = 3$$

Step 2: Set up the integral for CS:

$$CS = \int_0^3 (10 - 2x - 4)dx = \int_0^3 (6 - 2x)dx$$

Step 3: Evaluate the integral:

$$CS = [6x - x^2]_0^3$$

$$CS = (6(3) - 3^2) - (6(0) - 0^2)$$

$$CS = (18 - 9) - 0 = 9$$

The Consumer Surplus is 9.

Final Answer : "9"

Answer: (A)

Q18.

Solution

Concept: Producer Surplus (PS) is the excess amount that producers receive over the minimum price they are willing to accept. Graphically, it is the area between the supply curve and the equilibrium price line from 0 to x_e. It can be calculated using: $PS = \int_0^{x_e} [p_e - S(x)]dx$ For a linear supply curve, this area forms a triangle.

Solution: Given supply function: $p = S(x) = 2 + x$, and $x_e = 5$

Step 1: Find equilibrium price: $p_e = 2 + 5 = 7$

Step 2: Apply PS formula: $PS = \int_0^5 [7 - (2 + x)]dx = \int_0^5 (5 - x)dx$

Step 3: Evaluate integral: $\int (5 - x)dx = 5x - \frac{x^2}{2}$

$PS = \left[5x - \frac{x^2}{2}\right]_0^5 = 25 - \frac{25}{2} = \frac{25}{2} = 12.5$

Conclusion: Producer surplus equals the triangular area under price line and above supply curve.

Final Answer : "12.5"

Answer: (A)

Q19.

Solution

Concept: Using symmetry property: $\int_0^a f(x)dx = \int_0^a f(a - x)dx$ Also, $\sin(\frac{\pi}{2} - x) = \cos x$ helps simplify expressions.

Solution: Let: $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Using symmetry: $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Add both: $2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$

Simplify: $2I = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

Conclusion: Symmetry converts complex integral into a simple constant integral.

Final Answer : "$\pi/4$"

Answer: (B)

Q20.

Solution

Concept: If integrand is of form $e^x[f(x) + f'(x)]$, then: $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$.

Solution: $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

Step 1: Identify function: Let $f(x) = \frac{1}{x}$

Step 2: Differentiate: $f'(x) = -\frac{1}{x^2}$

Step 3: Recognize pattern: Integrand = $e^x[f(x) + f'(x)]$

Step 4: Apply formula: $I = e^x f(x) + C = \frac{e^x}{x} + C$

Verification (optional): Can also be solved using integration by parts, giving same result.

Conclusion: Pattern recognition simplifies integration significantly.

Final Answer : "$\frac{e^x}{x} + C$"

Answer: (B)

Q21.

Solution

Concept: In calculus and economics, Marginal Revenue (MR) is defined as the rate of change of Total Revenue (R) with respect to the quantity sold (x). Mathematically, this means $MR(x) = \frac{dR}{dx}$. To find the Total Revenue function $R(x)$ from a given Marginal Revenue function $MR(x)$, we need to perform the inverse operation of differentiation, which is integration: $R(x) = \int MR(x)dx$ Since integration results in an arbitrary constant of integration, C, we need an initial condition to determine its specific value. A common initial condition for revenue functions is $R(0) = 0$, meaning no revenue is generated when no units are sold.

Solution: We are given the marginal revenue function:
$MR(x) = 20 - 4x - 3x^2$

Step 1: Find the Total Revenue function $R(x)$ by integrating $MR(x)$.
$R(x) = \int (20 - 4x - 3x^2)dx$
Apply the power rule of integration ($\int x^n dx = \frac{x^{n+1}}{n+1} + C$) and the constant rule ($\int kdx = kx + C$) to each term in the integrand:
$\int 20dx = 20x$
$\int -4xdx = -4\frac{x^{1+1}}{1+1} = -4\frac{x^2}{2} = -2x^2$
$\int -3x^2dx = -3\frac{x^{2+1}}{2+1} = -3\frac{x^3}{3} = -x^3$

Combining these integrated terms, the general Total Revenue function is:
$R(x) = 20x - 2x^2 - x^3 + C$
where C is the constant of integration.

Step 2: Use the initial condition $R(0) = 0$ to find the value of C.
Substitute $x = 0$ and $R(x) = 0$ (since $R(0) = 0$) into the general $R(x)$ function:
$0 = 20(0) - 2(0)^2 - (0)^3 + C$
$0 = 0 - 0 - 0 + C$
$C = 0$

Step 3: Write the specific Total Revenue function.
Substitute the value $C = 0$ back into the general total revenue function:
$R(x) = 20x - 2x^2 - x^3$

Final Answer : "$20x - 2x^2 - x^3$"

Answer: (A)

Q22.

Solution

Concept: A separable differential equation can be solved by arranging variables on opposite sides and integrating.

Solution: Given: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

Separate variables: $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$.

Integrate both sides: $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$.

Using $\int \frac{1}{1+t^2} dt = \tan^{-1} t$: $\tan^{-1} y = \tan^{-1} x + C$.

Rewriting: $\tan^{-1} y - \tan^{-1} x = C$.

Taking tangent on both sides and simplifying: $\frac{y-x}{1+xy} = C \Rightarrow y - x = C(1 + xy)$.

Final Answer : "$y - x = C(1 + xy)$"

Answer: (B)

Q23.

Solution

Concept: The order of a differential equation is the highest order derivative present.

Solution: Given: $y = ae^{bx}$ with two constants a, b.

Differentiate once: $y' = abe^{bx} = by$.

Differentiate again: $y'' = by'$.

From $y' = by$, we get $b = \frac{y'}{y}$.

Substitute into y'': $y'' = \frac{y'}{y} \cdot y' = \frac{(y')^2}{y}$.

Thus, differential equation: $yy'' = (y')^2$.

Highest derivative is y'', so order = 2.

Final Answer : "2"

Answer: (B)

Q24.

Solution

Concept: Exponential growth is modeled by $N = N_0 e^{kt}$, where k is a constant.

Solution: Given: bacteria triples in 5 hours.

$3N_0 = N_0 e^{5k} \Rightarrow 3 = e^{5k}$ Taking logs: $\ln 3 = 5k \Rightarrow k = \frac{\ln 3}{5}$.

Now for $N = 9N_0$: $9 = e^{kt}$
Taking logs: $\ln 9 = kt$

Since $\ln 9 = 2\ln 3$, substitute: $2\ln 3 = \frac{\ln 3}{5} t$

Cancel $\ln 3$: $2 = \frac{t}{5} \Rightarrow t = 10$ hours.

Final Answer : "10 hours"

Answer: (A)

Q25.

Solution

Concept: For a differential equation, two fundamental characteristics are its order and its degree.
1. Order: The order of a differential equation is the order of the highest derivative present in the equation. For example, $\frac{dy}{dx}$ is a first-order derivative, and $\frac{d^2y}{dx^2}$ is a second-order derivative.
2. Degree: The degree of a differential equation is the power of the highest-order derivative present in the equation, provided that the differential equation can be expressed as a polynomial in its derivatives. This means that derivatives (like $y', y'', \ldots$) must only appear with integer powers and must NOT be part of a transcendental function (such as $\sin(\cdot)$, $\cos(\cdot)$, $e^{(\cdot)}$, $\ln(\cdot)$, $\sqrt{\cdot}$, etc.). If any derivative is part of a transcendental function, or raised to a fractional/negative power, then the equation is not a polynomial in its derivatives, and its degree is considered to be not defined.

Solution: Given the differential equation:
$\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$

Step 1: Identify the highest order derivative.
The derivatives present in the equation are:
$\frac{d^2y}{dx^2}$ (which is a second-order derivative)
$\frac{dy}{dx}$ (which is a first-order derivative)
The highest order derivative appearing in the equation is $\frac{d^2y}{dx^2}$.
Therefore, the order of the differential equation is 2.

Step 2: Determine if the equation is a polynomial in its derivatives.
For the degree to be defined, the differential equation must be expressible as a polynomial in terms of its derivatives. This means that each derivative term should appear only with integer powers and not be an argument of any transcendental function.
In the given equation, the term $\sin\left(\frac{dy}{dx}\right)$ involves the first derivative $\frac{dy}{dx}$ *inside* a sine function. The sine function is a transcendental function.
Because of the presence of the derivative within a transcendental function, this differential equation cannot be written in a polynomial form with respect to its derivatives.

Therefore, the degree of this differential equation is not defined.

Final Answer : "Not Defined"

Answer: (D)

Q26.

Solution

Concept: The Poisson distribution is a discrete probability distribution that models the number of times an event occurs in a fixed interval of time or space, given that these events occur with a known constant mean rate and independently of the time since the last event.
The probability mass function (PMF) of a Poisson distribution is given by:
$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$
where:
k is the number of occurrences of the event ($k = 0, 1, 2, \ldots$)
e is Euler's number (approximately 2.71828)
λ (lambda) is the average rate of event occurrences, which represents the mean number of events in the given interval.

A crucial characteristic of the Poisson distribution is that its mean and variance are both equal to the parameter λ.
Mean $E[X] = \lambda$
Variance $\text{Var}(X) = \lambda$

Solution: We are given the probability mass function for a random variable X that follows a Poisson distribution:
$P(X = k) = \frac{e^{-3}3^k}{k!}$

We need to find the variance of this distribution.

Step 1: Identify the parameter λ from the given PMF.
Let's compare the given PMF with the standard Poisson PMF formula:
Standard PMF: $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$ Given PMF: $P(X = k) = \frac{e^{-3}3^k}{k!}$

By direct comparison of the two formulas, we can clearly identify the parameter λ: $\lambda = 3$.

Step 2: Determine the variance of the distribution.
For a Poisson distribution, the variance is equal to its parameter λ.
Variance $\text{Var}(X) = \lambda$
Since we have identified $\lambda = 3$:
$\text{Var}(X) = 3$.

Therefore, the variance of this Poisson distribution is 3.

Final Answer : "3"

Answer: (C)

Q27.

Solution

Concept: The standard normal distribution is symmetric about its mean ($Z = 0$), and probabilities correspond to areas under the curve. Equal intervals on either side of 0 have equal areas.

Solution: Given: $P(0 \le Z \le 1) = 0.3413$.

By symmetry of the normal curve: $P(-1 \le Z \le 0) = 0.3413$.

Hence, total probability between $Z = -1$ and $Z = 1$: $P(-1 \le Z \le 1) = 0.3413 + 0.3413 = 0.6826$.

Thus, about 68.26% of values lie within ± 1 standard deviation.

Final Answer : "0.6826"

Answer: (B)

Q28.

Solution

Concept: A Z-score measures how far a value lies from the mean in terms of standard deviations.

Solution: Given $Z = -2$. Using: $Z = \frac{x-\mu}{\sigma}$

So, $x - \mu = -2\sigma \Rightarrow x = \mu - 2\sigma$.

This shows the value is below the mean, and the magnitude 2 indicates it is 2 standard deviations away.

Hence, the value lies 2 standard deviations below the mean.

Final Answer : "2 standard deviations below the mean"

Answer: (B)

Q29.

Solution

Concept: For a binomial distribution: Mean $= np$, Variance $= np(1-p)$.

Solution: Given: Mean $= 4$, Variance $= 3$.

So, $np = 4$ and $np(1-p) = 3$.

Substitute $np = 4$ into the variance equation: $4(1-p) = 3$

Solving: $1 - p = \frac{3}{4} \Rightarrow p = \frac{1}{4}$.

Now substitute p into $np = 4$: $n \cdot \frac{1}{4} = 4 \Rightarrow n = 16$.

Thus, the number of trials is 16.

Final Answer : "16"

Answer: (B)

Q30.

Solution

Concept: For a Poisson random variable X, the probability mass function (PMF) is given by $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$, where $k = 0, 1, 2, \ldots$ represents the number of occurrences, and λ is the mean of the distribution.
The sum of all probabilities for a discrete probability distribution must equal 1:
$\sum_{k=0}^{\infty} P(X = k) = 1$.
The probability that a Poisson variable X takes a positive value is $P(X > 0)$. This can be calculated as $1 - P(X = 0)$, since X can only take non-negative integer values.

Solution: Let X be a Poisson variable with mean λ.
The probability that X takes a positive value is given as $1 - e^{-2}$.
So, $P(X > 0) = 1 - e^{-2}$.

We know that $P(X > 0)$ can also be expressed as $1 - P(X = 0)$.
Therefore, $1 - P(X = 0) = 1 - e^{-2}$.

From this equation, we can deduce:
$P(X = 0) = e^{-2}$.

Now, let's use the Poisson PMF formula for $k = 0$:
$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!}$
Recall that $\lambda^0 = 1$ (for $\lambda \neq 0$) and $0! = 1$.
So, $P(X = 0) = \frac{e^{-\lambda}\cdot 1}{1}$
$P(X = 0) = e^{-\lambda}$.

Now, we have two expressions for $P(X = 0)$:
$e^{-\lambda} = e^{-2}$.

By comparing the exponents, we can find the value of λ:
$-\lambda = -2$
$\lambda = 2$.

The mean of a Poisson distribution is equal to its parameter λ.
Therefore, the mean of the distribution is 2.

Final Answer : "2"

Answer: (B)

Q31.

Solution

Concept: To find the last digit of a large power of an integer, we observe the pattern of the last digits of successive powers of that integer. The last digits usually repeat in a cycle.
For powers of 2, we are interested in the last digit of 2^n. This is equivalent to finding $2^n \pmod{10}$.

Solution: We need to find the last digit of 2^{50}.
Let's list the last digits of the first few powers of 2:
$2^1 = 2$ (last digit is 2)
$2^2 = 4$ (last digit is 4)
$2^3 = 8$ (last digit is 8)
$2^4 = 16$ (last digit is 6)
$2^5 = 32$ (last digit is 2)
$2^6 = 64$ (last digit is 4)

We can see that the last digits follow a repeating cycle: 2, 4, 8, 6.
The length of this cycle is 4.

To find the last digit of 2^{50}, we need to determine where 50 falls in this cycle. We do this by finding the remainder when the exponent (50) is divided by the length of the cycle (4). $50 \div 4 = 12$ with a remainder of 2.

The remainder is 2. This means the last digit of 2^{50} will be the same as the 2nd digit in our cycle.
The cycle is:
1st digit: 2
2nd digit: 4
3rd digit: 8
4th digit: 6

Since the remainder is 2, the last digit of 2^{50} is 4.
If the remainder were 0, the last digit would be the last digit in the cycle (6).

Final Answer : "4"

Answer: (B)

Q32.

Solution

Concept: This is a problem involving relative speeds in water (boats and streams).
Speed in still water (V_b): The speed of the boat in the absence of a current.
Speed of the stream (V_s): The speed of the water current.
Speed downstream: When the boat travels with the current, its effective speed is the sum of its speed in still water and the speed of the stream. $V_{downstream} = V_b + V_s$.
Speed upstream: When the boat travels against the current, its effective speed is the difference between its speed in still water and the speed of the stream. $V_{upstream} = V_b - V_s$.
The relationship between distance, speed, and time is: $Time = \frac{Distance}{Speed}$.

Solution: Given information:
Speed of the boat in still water (V_b) = 5 km/h.
Speed of the stream (V_s) = 3 km/h.
Distance to travel downstream (D) = 16 km.

Step 1: Calculate the speed of the boat downstream.
When the boat travels downstream, its speed is aided by the stream.
$V_{downstream} = V_b + V_s$
$V_{downstream} = 5$ km/h + 3 km/h
$V_{downstream} = 8$ km/h

Step 2: Calculate the time taken to go 16 km downstream.
Using the formula $Time = \frac{Distance}{Speed}$:
$Time = \frac{16 \text{ km}}{8 \text{ km/h}}$
$Time = 2$ hours

Therefore, the time taken to go 16 km downstream is 2 hours.

Final Answer : "2 hours"

Answer: (C)

Q33.

Solution

Concept: In races, for the same time interval, the ratio of distances covered equals the ratio of speeds.

Solution: A beats B by 50m in a 500m race $\Rightarrow$ when A runs 500m, B runs 450m.
So, $\frac{V_A}{V_B} = \frac{500}{450} = \frac{10}{9}$.

B beats C by 60m in a 600m race $\Rightarrow$ when B runs 600m, C runs 540m.
So, $\frac{V_B}{V_C} = \frac{600}{540} = \frac{10}{9}$.

Thus, $\frac{V_A}{V_C} = \frac{10}{9} \times \frac{10}{9} = \frac{100}{81}$.

Now, when A runs 500m: $\frac{500}{D_C} = \frac{100}{81} \Rightarrow D_C = \frac{500\times 81}{100} = 405\text{m}$.

Hence, A beats C by: $500 - 405 = 95\text{m}$.

Final Answer : "95m"

Answer: (A)

Q34.

Solution

Concept: A congruence $ax \equiv b \pmod{m}$ means ax leaves remainder b when divided by m.

Solution: Given: $3x \equiv 2 \pmod{5}$.

Testing values (mod 5):
$x = 0 \Rightarrow 3x = 0$
$x = 1 \Rightarrow 3$
$x = 2 \Rightarrow 6 \equiv 1$
$x = 3 \Rightarrow 9 \equiv 4$
$x = 4 \Rightarrow 12 \equiv 2$ (satisfies condition)

Thus, $x = 4$.

Alternatively, inverse of 3 modulo 5 is 2:
$x \equiv 2 \times 2 = 4 \pmod{5}$.

Final Answer : "4"

Answer: (D)

Q35.

Solution

Concept: A moving average smooths time series data by averaging k consecutive observations. To compute a k-year moving average, we need a complete set of k values, so some data points at the beginning and end cannot be used.

Solution: Here, $k = 4$ (4-year moving average).

Each moving average requires 4 consecutive values. For even values of k, the averages are first computed and then centered, which results in loss of data points at both ends.

Number of data points lost at each end $= \frac{k}{2} = \frac{4}{2} = 2$.

So, Loss at beginning = 2
Loss at end = 2

Total data points lost = 2 + 2 = 4.

Thus, 4 original observations do not get a corresponding moving average value.

Final Answer : "4"

Answer: (C)

Q36.

Solution

Concept: A trend equation in time series analysis is used to model the long-term movement in data. It is typically a linear equation of the form $y = a + bx$, where:
y is the predicted value of the time series variable.
x is the time period (often an integer representing a year, quarter, or month relative to an origin).
a is the y-intercept (the predicted value when $x = 0$).
b is the slope (the average change in y per unit change in x).
The "origin" specifies the time point at which $x = 0$.
To predict a value for a future year, we first need to determine the corresponding x value for that year relative to the given origin.

Solution: Given the trend equation: $y = 20 + 3x$.
Origin: 2018 (which means $x = 0$ corresponds to the year 2018).
x unit = 1 year.

We need to predict the value for the year 2023.

Step 1: Determine the value of x for the year 2023.
The x value represents the number of years from the origin (2018).
x = Year of prediction − Origin Year
$x = 2023 - 2018$
$x = 5$

Step 2: Substitute the calculated x value into the trend equation.
$y = 20 + 3x$
Substitute $x = 5$:
$y = 20 + 3(5)$
$y = 20 + 15$
$y = 35$

Therefore, the predicted value for the year 2023 is 35.

Final Answer : "35"

Answer: (A)

Q37.

Solution

Concept: A time series is a sequence of data points indexed in time order. Time series analysis involves decomposing the data into several components. The two primary models for time series decomposition are the additive model and the multiplicative model.
The components typically considered are:
Y: The observed value of the time series.
T: Trend component (long-term movement).
S: Seasonal component (regular, recurring fluctuations within a specific period, like a year). C: Cyclical component (long-term oscillations, often associated with business cycles, but less regular than seasonal).
I (or R for Residual/Irregular): Irregular or random component (unpredictable, short-term fluctuations).

The way these components are combined defines the model:
Additive Model: Assumes that the components are summed together. It is suitable when the absolute magnitude of the fluctuations (seasonal, cyclical, irregular) does not vary with the level of the trend. $Y = T + S + C + I$.
Multiplicative Model: Assumes that the components are multiplied together. It is suitable when the magnitude of the fluctuations increases or decreases proportionally with the level of the trend. $Y = T \times S \times C \times I$.

Solution: The question asks for the expression of the additive model of a time series.
Based on the definition of the additive model:
The observed value Y is expressed as the sum of its components: Trend (T), Seasonal (S), Cyclical (C), and Irregular (I).
$Y = T + S + C + I$

Comparing this with the given options:
(A) $Y = T \times S \times C \times I$: This is the multiplicative model.
(B) $Y = T + S + C + I$: This is the additive model.
(C) $Y = T + S - C + I$: This is not a standard model.
(D) $Y = T \times S + C$: This is not a standard model.

Therefore, option (B) correctly represents the additive model of a time series.

Final Answer : "$Y = T + S + C + I$"

Answer: (B)

Q38.

Solution

Concept: In hypothesis testing, we formulate a null hypothesis (H_0) and an alternative hypothesis (H_1 or H_A). After conducting a statistical test, we make a decision to either reject or fail to reject the null hypothesis. There are two types of errors that can be made in this decision process:
1. Type I Error (False Positive): This occurs when we incorrectly reject the null hypothesis (H_0) when it is actually true. The probability of committing a Type I error is denoted by α (alpha), which is also known as the significance level.
2. Type II Error (False Negative): This occurs when we incorrectly fail to reject (or accept) the null hypothesis (H_0) when it is actually false. The probability of committing a Type II error is denoted by β (beta).
The power of a test is $1 - \beta$, which is the probability of correctly rejecting a false null hypothesis.

Solution: The question asks for the probability of committing a Type II error.
Based on the definitions in hypothesis testing:
α is the probability of a Type I error.
β is the probability of a Type II error.
$1 - \alpha$ is the confidence level.
$1 - \beta$ is the power of the test.

Therefore, the probability of committing a Type II error is denoted by β.

Final Answer : "β"

Answer: (B)

Q39.

Solution

Concept: Choosing the correct statistical test for hypothesis testing depends on several factors, including:

1. Nature of the data: Is it categorical or continuous?
2. Number of samples: One sample, two samples, or more?
3. Parameter being tested: Mean, proportion, variance?
4. Population standard deviation (σ): Is it known or unknown?
5. Sample size (n): Is it large ($n \geq 30$) or small ($n < 30$)?

For testing a hypothesis about a single population mean (μ):
Z-test: Used when the population standard deviation (σ) is known, or when the sample size (n) is large ($n \geq 30$) (in which case the sample standard deviation s can be used as a good estimate for σ due to the Central Limit Theorem).
t-test: Used when the population standard deviation (σ) is unknown and the sample size (n) is small ($n < 30$). In this scenario, we use the sample standard deviation (s) as an estimate for σ, and the t-distribution (which accounts for the additional uncertainty due to estimating σ) is appropriate. The t-test is more robust for small samples. F-test: Used for comparing variances of two or more populations, or in ANOVA (Analysis of Variance) to compare means of three or more groups.
Chi-square test: Used for categorical data, e.g., for testing goodness-of-fit, independence between two categorical variables, or homogeneity.

Solution: We are given the following information:
A researcher wants to test if the mean weight of a sample is significantly different from 60kg (testing a hypothesis about a single population mean).
Sample size (n) = 25 students. This is a small sample size ($n < 30$).
The population standard deviation (σ) is unknown.

Based on the concepts:
Since the sample size is small ($n = 25$) and the population standard deviation is unknown, the appropriate test to use for testing a hypothesis about the population mean is the t-test. The t-test uses the sample standard deviation to estimate the unknown population standard deviation and relies on the t-distribution, which is more suitable for small samples.

Final Answer : "t-test"

Answer: (B)

Q40.

Solution

Concept: In hypothesis testing, we aim to make a decision about a null hypothesis (H_0). There are two types of errors we can make:
1. Type I Error (α): Rejecting the null hypothesis when it is actually true (false positive). The probability of this error is the significance level, α.
2. Type II Error (β): Failing to reject the null hypothesis when it is actually false (false negative). The probability of this error is denoted by β.

The power of a test is a measure of its ability to correctly detect an effect if that effect truly exists. More formally, it is the probability of correctly rejecting a false null hypothesis.
Power = $P(\text{Reject } H_0 \mid H_0 \text{ is false})$.
Since failing to reject H_0 when it's false is a Type II error (with probability β), the probability of correctly rejecting H_0 when it's false must be the complement of β.
Power = $1 - \beta$.

Solution: The question asks for the definition of the "Power of a Test".
Based on the definitions in hypothesis testing:
α represents the probability of a Type I error.
β represents the probability of a Type II error.
$1 - \alpha$ represents the confidence level (the probability of correctly failing to reject a true null hypothesis).
$1 - \beta$ represents the power of the test (the probability of correctly rejecting a false null hypothesis).

Therefore, the Power of a Test is defined as $1 - \beta$.

Final Answer : "$1 - \beta$"

Answer: (C)

Q41.

Solution

Concept: In hypothesis testing, the alternative hypothesis (H_a or H_1) dictates the type of test (one-tailed or two-tailed).
One-tailed test: Used when the alternative hypothesis specifies a direction (either greater than or less than).
Right-tailed test: $H_a : \mu > \mu_0$ (testing if the mean is significantly greater than a specified value μ_0). The rejection region is entirely in the right tail of the sampling distribution.
Left-tailed test: $H_a : \mu < \mu_0$ (testing if the mean is significantly less than a specified value μ_0). The rejection region is entirely in the left tail of the sampling distribution.
Two-tailed test: Used when the alternative hypothesis states that the parameter is simply "not equal to" a specified value, without specifying a direction. $H_a : \mu \neq \mu_0$ (testing if the mean is significantly different from μ_0). The rejection region is split between both tails of the sampling distribution.

Solution: We are given the null hypothesis (H_0) and the alternative hypothesis (H_a):
$H_0 : \mu = 50$
$H_a : \mu \neq 50$

The alternative hypothesis $H_a : \mu \neq 50$ states that the population mean (μ) is simply different from 50. It does not specify whether μ is greater than 50 or less than 50. This "not equal to" sign indicates that we are interested in deviations in either direction from 50.

Therefore, this type of test is a two-tailed test. The rejection region for such a test would be divided into two parts, one in the upper tail and one in the lower tail of the sampling distribution.

Final Answer : "Two-tailed test"

Answer: (C)

Q42.

Solution

Concept: A bond that pays a fixed amount annually forever is known as a perpetuity.
The present value (PV) of a perpetuity is the current worth of a series of infinite equal payments (annuities). The formula for the present value of a perpetuity is:
$PV = \frac{C}{r}$
where:
C is the amount of the periodic payment (coupon payment).
r is the discount rate (or required rate of return) per period.

Solution: Given information:
Annual payment (C) = ₹ 100.
Discount rate (r) = 5% p.a. = 0.05.
The payments are made annually forever (perpetuity).

Step 1: Apply the formula for the present value of a perpetuity.
$PV = \frac{C}{r}$
Substitute the given values into the formula:
$PV = \frac{100}{0.05}$

Step 2: Calculate the present value.
$PV = \frac{100}{0.05} = \frac{100}{\frac{5}{100}} = 100 \times \frac{100}{5} = \frac{10000}{5}$ $PV = 2000$

Therefore, the present value of the bond (perpetuity) is ₹ 2,000.

Final Answer : "₹ 2,000"

Answer: (B)

Q43.

Solution

Concept: A sinking fund is an annuity in which a series of equal payments are made at regular intervals to accumulate a specific future sum of money by a certain date. The payments are typically made at the end of each period (ordinary annuity).

The formula for the future value (FV) of an ordinary annuity (sinking fund) is:

$FV = R\left[\frac{(1+i)^n-1}{i}\right]$

where: FV is the future value of the annuity (the target amount to be accumulated).

R is the periodic payment (the amount of each installment).

i is the interest rate per period.

n is the total number of periods.

The question asks for the periodic payment R. We need to rearrange the formula to solve for R.

$R = FV\left[\frac{i}{(1+i)^n-1}\right]$

Solution: Given information:

Future Value (FV) = ₹ 5,00,000 (target amount to accumulate).

Time period (n) = 10 years.

Interest rate = 8% p.a. compounded annually.

So, interest rate per period (i) = 8Number of periods (n) = 10.

Step 1: Identify the appropriate formula for periodic payment in a sinking fund.

The formula for the periodic payment R in a sinking fund (ordinary annuity to accumulate a future sum) is:

$R = FV\left[\frac{i}{(1+i)^n-1}\right]$

Step 2: Substitute the given values into the formula.

$FV = 5,00,000$

$i = 0.08$

$n = 10$

$R = 5,00,000\left[\frac{0.08}{(1+0.08)^{10}-1}\right]$

$R = 5,00,000\left[\frac{0.08}{(1.08)^{10}-1}\right]$

This can also be written as:

$R = \frac{5,00,000\times0.08}{(1.08)^{10}-1}$

Comparing this with the given options, option (A) matches exactly.

Final Answer : "$R = \frac{5,00,000\times0.08}{(1.08)^{10}-1}$"

Answer: (A)

Q44.

Solution

Concept: A bond's price depends on its coupon rate and market yield.
- If Coupon Rate = Yield $\Rightarrow$ bond trades at par.
- If Coupon Rate > Yield $\Rightarrow$ bond trades at premium.
- If Coupon Rate < Yield $\Rightarrow$ bond trades at discount.

Solution: The question asks the condition for a bond trading "at par".

When a bond is at par, its market price equals its face value. This occurs when the return offered by the bond (coupon rate) matches the market return (yield).

Checking options:
(A) Coupon Rate > Yield $\Rightarrow$ premium (incorrect)
(B) Coupon Rate < Yield $\Rightarrow$ discount (incorrect)
(C) Coupon Rate = Yield $\Rightarrow$ at par (correct)
(D) No coupons $\Rightarrow$ zero-coupon bond, usually trades at discount

Hence, the correct condition is when Coupon Rate equals Yield.

Final Answer : "Coupon Rate = Yield"

Answer: (C)

Q45.

Solution

Concept: The effective rate of interest is the actual annual rate of interest earned or paid over a year, taking into account the effect of compounding. It differs from the nominal (stated) rate when interest is compounded more than once a year.
The formula for the effective rate of interest (i_e) is:
$i_e = \left(1 + \frac{i}{m}\right)^m - 1$
where:
i is the nominal annual interest rate (as a decimal).
m is the number of compounding periods per year.

Solution: Given information:
Nominal rate of interest (i) = 12% p.a. = 0.12.
Compounding frequency: compounded quarterly.
So, the number of compounding periods per year (m) = 4 (since there are 4 quarters in a year).

Step 1: Apply the formula for the effective rate of interest.
$i_e = \left(1 + \frac{i}{m}\right)^m - 1$
Substitute the given values into the formula:
$i_e = \left(1 + \frac{0.12}{4}\right)^4 - 1$

Step 2: Calculate the effective rate of interest.
First, calculate the term inside the parenthesis:
$\frac{0.12}{4} = 0.03$
So, $(1 + 0.03)^4 - 1 = (1.03)^4 - 1$
Calculate $(1.03)^4$:
$(1.03)^4 = 1.03 \times 1.03 \times 1.03 \times 1.03 \approx 1.12550881$
Now, subtract 1:
$i_e = 1.12550881 - 1$
$i_e = 0.12550881$

To express this as a percentage, multiply by 100:
$i_e \approx 12.55\%$

Comparing this with the options, 12.55% is the closest value.

Final Answer : "12.55%"

Answer: (B)

Q46.

Solution

Concept: An annuity is a series of equal payments made or received at regular intervals over a specified period. The classification of an annuity depends on when these payments are made within each period.
Ordinary Annuity (or Annuity in Arrears): Payments are made at the end of each period. This is the most common type of annuity.
Annuity Due: Payments are made at the beginning of each period. Because payments are made earlier, they typically earn more interest than payments in an ordinary annuity, resulting in higher future values or lower present values for the same series of payments.
Perpetuity: An annuity where payments continue indefinitely (forever).
Deferred Annuity: An annuity where payments begin after a specified period of time (deferral period) has passed. No payments are made during the deferral period.

Solution: The question asks about an annuity where payments are made at the beginning of each period.
Based on the definitions of different types of annuities:
An Ordinary Annuity involves payments at the end of each period.
An Annuity Due involves payments at the beginning of each period.
A Perpetuity involves payments forever.
A Deferred Annuity involves payments starting after a delay.

Therefore, an annuity where payments are made at the beginning of each period is called an Annuity Due.

Final Answer : "Annuity Due"

Answer: (B)

Q47.

Solution

Concept: When the value of an asset (like property) appreciates at a constant percentage rate per annum, its future value can be calculated using the compound interest formula. This is because the appreciation in each period is calculated on the value of the asset at the beginning of that period, similar to how interest compounds.

The formula for future value (FV) under compound appreciation is:

$FV = PV(1 + r)^n$

where:

PV is the present value (initial value) of the property.

r is the annual appreciation rate (as a decimal).

n is the number of years (periods).

Solution: Given information:

Present value of the property (PV) = ₹ 20,00,000.

Appreciation rate (r) = 5Number of years (n) = 3 years.

Step 1: Apply the compound appreciation formula.

$FV = PV(1 + r)^n$

Substitute the given values into the formula:

$FV = 20,00,000(1 + 0.05)^3$

$FV = 20,00,000(1.05)^3$

Step 2: Calculate the future value.

First, calculate $(1.05)^3$:

$(1.05)^3 = 1.05 \times 1.05 \times 1.05 = 1.157625$

Now, multiply this by the present value:

$FV = 20,00,000 \times 1.157625$

$FV = 2,315,250$

Therefore, the value of the property after 3 years will be ₹ 23,15,250.

Final Answer : "₹ 23,15,250"

Answer: (A)

Q48.

Solution

Concept: In LPP, the optimal value of the objective function is obtained at the vertices of the feasible region.

Solution: Vertices: A(0,10), B(5,8), C(8,0), O(0,0).
Objective function: $Z = 10x + 12y$.

Evaluate Z at each vertex:

At A(0,10): $Z = 10(0) + 12(10) = 120$
At B(5,8): $Z = 10(5) + 12(8) = 50 + 96 = 146$
At C(8,0): $Z = 10(8) + 12(0) = 80$
At O(0,0): $Z = 0$

Comparing all values, the maximum is 146 at B(5,8).

Final Answer : "B(5, 8)"

Answer: (B)

Q49.

Solution

Concept: If the objective function is parallel to a constraint line forming an edge of the feasible region, then all points on that edge give the same optimal value.

Solution: When the objective function line is parallel to a boundary of the feasible region and coincides with it at the optimum, it does not touch just one point but an entire edge.

Thus, every point on that segment satisfies the optimal condition, leading to infinitely many solutions.

Final Answer : "Infinite solutions"

Answer: (C)

Q50.

Solution

Concept: A constraint is redundant if its removal does not affect the feasible region.

Solution: Given constraints: $x \le 5$, $x \le 10$, $y \le 8$, $x, y \ge 0$.

Observe that any value of x satisfying $x \le 5$ will automatically satisfy $x \le 10$.

Thus, $x \le 10$ does not impose any additional restriction on the feasible region and can be removed without changing it.

Hence, it is a redundant constraint.

Final Answer : "$x \le 10$"

Answer: (B)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	D	5	A
6	A	7	C	8	B	9	B	10	A
11	B	12	B	13	B	14	A	15	B
16	B	17	A	18	A	19	B	20	B
21	A	22	B	23	B	24	A	25	D
26	C	27	B	28	B	29	B	30	B
31	B	32	C	33	A	34	D	35	C
36	A	37	B	38	B	39	B	40	C
41	C	42	B	43	A	44	C	45	B
46	B	47	A	48	B	49	C	50	B