CUET-UG Applied Mathematics Sample Paper-2

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. What is the remainder when 7^{103} is divided by 25?

(A) 7

(B) 18

(C) 21

(D) 24

Q2. A boat can travel 36 km upstream in 6 hours and 60 km downstream in 5 hours. What is the speed of the boat in still water?

(A) 9 km/h

(B) 10.5 km/h

(C) 6 km/h

(D) 12 km/h

Q3. In a 1000m race, A beats B by 100m and B beats C by 100m. By how many meters does A beat C?

(A) 200m

(B) 190m

(C) 210m

(D) 180m

Q4. If $x \equiv 3 \pmod 7$ and $y \equiv 4 \pmod 7$, then what is $(x+y)^2 \pmod 7$?

(A) 0

(B) 1

(C) 4

(D) 6

Q5. If A is a square matrix of order 3 and $|A| = 5$, then find $|adj(A)|$.

(A) 5

(B) 15

(C) 25

(D) 125

Q6. For what value of k does the system of equations $x + y + z = 2$, $x + 2y + 3z = 5$, $x + 3y + kz = 8$ have infinitely many solutions?

(A) 4

(B) 5

(C) 6

(D) 0

Q7. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A^T = I$ if the value of α is:

(A) $\pi/6$

(B) $\pi/3$

(C) π

(D) $3\pi/2$

Q8. The total cost function is $C(x) = \frac{1}{3}x^3 + 5x^2 - 12x + 10$. At what level of output x is the marginal cost minimum?

(A) $x = 5$

(B) $x = -5$

(C) $x = 10$

(D) $x = 0$

Q9. The function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing in the interval:

(A) $(2, 3)$

(B) $(-\infty, 2)$

(C) $(3, \infty)$

(D) $(-\infty, 3)$

Q10. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Q11. The demand function for a commodity is $p = 25 - x^2$. If the equilibrium price is 9, the Consumer Surplus (CS) is:

(A) 32/3

(B) 64/3

(C) 128/3

(D) 16/3

Q12. The area bounded by the curve $y^2 = 4x$ and the line $x = 3$ is:

(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $12\sqrt{3}$

(D) $16\sqrt{3}$

Q13. Find $\int \frac{1}{x(x^5+1)}dx$.

(A) $\frac{1}{5}\log|\frac{x^5}{x^5+1}| + C$

(B) $\log|\frac{x^5}{x^5+1}| + C$

(C) $5\log|\frac{x^5+1}{x^5}| + C$

(D) $\frac{1}{5}\log|\frac{x^5+1}{x^5}| + C$

Q14. The order and degree of the differential equation $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = \sin y$ are:

(A) Order 2, Degree 1

(B) Order 2, Degree 3

(C) Order 1, Degree 3

(D) Order 3, Degree 2

Q15. A population grows at a rate proportional to the current population P. If the population doubles in 10 years, the growth constant k is:

(A) $\frac{\log 2}{10}$

(B) $10\log 2$

(C) $\frac{10}{\log 2}$

(D) $\log 20$

Q16. In a Poisson distribution, if the Mean is 4, what is the Standard Deviation?

(A) 4

(B) 16

(C) 2

(D) 1

Q17. For a Normal Distribution, the area under the curve between $Z = -\infty$ and $Z = 0$ is:

(A) 0

(B) 0.5

(C) 1.0

(D) 0.95

Q18. If X follows a Poisson distribution such that $P(X = 1) = P(X = 2)$, then $P(X = 0)$ is:

(A) e^{-2}

(B) e^{-1}

(C) $1/e$

(D) e^{-4}

Q19. The present value of a perpetuity of ₹ 500 per month at 12% per annum is:

(A) ₹ 50,000

(B) ₹ 5,000

(C) ₹ 6,000

(D) ₹ 4,166

Q20. A sinking fund is created for redeeming debentures of ₹ 5,00,000 at the end of 10 years. How much should be invested annually at 8% p.a. (Given $(1.08)^{10} = 2.158$)?

(A) ₹ 34,542

(B) ₹ 43,170

(C) ₹ 50,000

(D) ₹ 32,000

Q21. The EMI for a loan of ₹ 1,00,000 at 1% per month for 12 months is calculated using which formula?

(A) $P \times r \times \frac{(1+r)^n}{(1+r)^n - 1}$

(B) $P \times r \times (1 + r)^n$

(C) $\frac{P \times r \times n}{100}$

(D) $\frac{P}{(1+r)^n}$

Q22. A Null Hypothesis is rejected when the p-value is:

(A) Greater than the significance level (α)

(B) Less than the significance level (α)

(C) Equal to 1

(D) Greater than 0.5

Q23. The t-test is typically used when the sample size is:

(A) Large ($n > 30$)

(B) Small ($n < 30$) and population SD is unknown

(C) Any size, provided population is normal

(D) Only for infinite populations

Q24. In LPP, the objective function $Z = ax + by$ is maximized at:

(A) Any point in the feasible region

(B) Only the origin

(C) The corner points of the feasible region

(D) The center of the feasible region

Q25. Which of the following is a constraint for "non-negative variables" in LPP?

(A) $x, y < 0$

(B) $x, y \leq 0$

(C) $x, y \geq 0$

(D) $x + y \geq 0$

Q26. In the method of Least Squares for a straight line trend $Y = a + bX$, if $\sum X = 0$, the value of 'a' is:

(A) $\sum Y/n$

(B) $\sum XY / \sum X^2$

(C) $\sum Y$

(D) $n / \sum Y$

Q27. A 3-yearly moving average is used to:

(A) Eliminate seasonal variations

(B) Smooth out short-term fluctuations

(C) Find the exact value of future data

(D) Determine the cyclical irregular movements

Q28. A man can row 6 km/h in still water. If the speed of the current is 2 km/h, it takes him 3 hours to row to a place and come back. How far is the place?

(A) 8 km

(B) 10 km

(C) 12 km

(D) 9 km

Q29. If the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then the value of $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) $5I$

(D) 0

Q30. Find the value of x such that the matrix $\begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$ is singular.

(A) $1, -2$

(B) $0, 1$

(C) $-1, 2$

(D) 3, 0

Q31. The demand function for a product is $p = 100 - 2x$. At what value of x is the Total Revenue (TR) maximized?

(A) 25

(B) 50

(C) 100

(D) 20

Q32. If the Marginal Revenue (MR) is 25 and the elasticity of demand with respect to price is 5, the price (p) of the commodity is:

(A) 31.25

(B) 20.00

(C) 125.00

(D) 50.00

Q33. The supply function for a commodity is $p = 2 + x^2$. If the equilibrium quantity is $x_0 = 3$, find the Producer Surplus (PS).

(A) 12

(B) 18

(C) 9

(D) 27

Q34. Evaluate $\int_0^1 \frac{e^x}{1+e^x} dx$.

(A) $\log(1 + e)$

(B) $\log(\frac{1+e}{2})$

(C) $\log(e)$

(D) $e - 1$

Q35. A 1,000 bond with a 7% annual coupon rate has 3 years to maturity. If the required yield is 8%, the market value of the bond will be:

(A) Exactly 1,000

(B) Greater than 1,000

(C) Less than 1,000

(D) 1,070

Q36. An equipment costs 10,00,000 and has a scrap value of 1,00,000 after 10 years. What is the annual contribution to a sinking fund at 5% p.a. to replace it? (Use $\frac{1}{s_{\overline{n}|i}} = 0.0795$ for $n = 10, i = 0.05$)

(A) 71,550

(B) 79,500

(C) 80,000

(D) 90,000

Q37. A person pays 1,200 per month towards a loan EMI. If the interest rate is 12% p.a. (1% per month) and the loan period is 2 years, the principal amount is:

(A) 24,000

(B) 25,492

(C) 21,243

(D) 28,800

Q38. The present value of a deferred perpetuity of 10,000 per year, starting 5 years from now, at an interest rate of 10% is:

(A) 1,00,000

(B) 68,301

(C) 62,092

(D) 90,909

Q39. In a test of hypothesis, the "Level of Significance" (α) represents:

(A) The probability of Type II error

(B) The probability of Type I error

(C) The power of the test

(D) The probability of accepting a true null hypothesis

Q40. For a sample of size $n = 16$ from a normal population with mean 50 and variance 25, what is the standard error of the mean?

(A) 1.25

(B) 5.0

(C) 0.8

(D) 1.56

Q41. When the sample size is small ($n < 30$) and the population standard deviation is unknown, which distribution is used for testing the mean?

(A) Normal Distribution

(B) Poisson Distribution

(C) t-Distribution

(D) Chi-square Distribution

Q42. For a Binomial distribution, if the Mean is 4 and the Variance is 3, what is the value of n?

(A) 12

(B) 16

(C) 20

(D) 8

Q43. In a Normal Distribution, approximately what percentage of data falls within ± 2 standard deviations of the mean?

(A) 68%

(B) 99.7%

(C) 95.4%

(D) 50%

Q44. Using the method of 4-yearly centered moving averages, the first moving average value is assigned to:

(A) The end of Year 2

(B) The middle of Year 3

(C) The start of Year 1

(D) The end of Year 3

Q45. In the trend line $Y = 20 + 1.5X$ (origin 2015, X units = 1 year), the estimated value for 2020 is:

(A) 21.5

(B) 27.5

(C) 30.5

(D) 25.0

Q46. The region represented by $x + y \leq 6, 2x + y \leq 8, x \geq 0, y \geq 0$ has a corner point at:

(A) (4, 0)

(B) (0, 8)

(C) (6, 0)

(D) (4, 4)

Q47. In LPP, if the objective function Z has the same maximum value at two corner points, then:

(A) There is no optimal solution.

(B) There are only two optimal solutions.

(C) Every point on the line segment joining these two points is optimal.

(D) The solution is at the origin.

Q48. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is:

(A) $y = x + C$

(B) $y = Cx$

(C) $x^2 + y^2 = C$

(D) $y = \log x + C$

Q49. A bacteria culture triples in size every 2 hours. If the initial count is N, the count after 6 hours is:

(A) $3N$

(B) $9N$

(C) $27N$

(D) $6N$

Q50. If the total cost function is $C(x) = 5x^2 + 20x + 500$, at what level of output is the Average Cost (AC) equal to the Marginal Cost (MC)?

(A) $x = 10$

(B) $x = 100$

(C) $x = 5$

(D) $x = 20$

Detailed Solutions

Q1.

Solution

Concept: To find the remainder of a large power a^n divided by m: 1. Use Euler's Totient Theorem: $a^{\phi(m)} \equiv 1 \pmod{m}$ if $\gcd(a, m) = 1$. 2. For $m = 25$, the totient function is $\phi(25) = 25\left(1 - \frac{1}{5}\right) = 20$. 3. Reduce the exponent n modulo $\phi(m)$. **Solution:** 1. Identify Variables: $a = 7, n = 103, m = 25$. Check $\gcd(7, 25) = 1$, so Euler's Theorem applies. 2. Calculate $\phi(25)$: $\phi(25) = 20$. This implies $7^{20} \equiv 1 \pmod{25}$. 3. Reduce the Exponent: Divide the exponent 103 by 20: $103 = (20 \times 5) + 3$. Therefore, $7^{103} = (7^{20})^5 \times 7^3$. $7^{103} \equiv (1)^5 \times 7^3 \pmod{25}$. 4. Calculate Final Remainder: $7^3 = 343$. Divide 343 by 25: $343 = 25 \times 13 + 18$. $343 \equiv 18 \pmod{25}$. **Final Answer:** 18

Answer: (B)

Q2.

Solution

Concept: Let u be the speed of the boat in still water and v be the speed of the stream (current). 1. Upstream Speed (s_u): The effective speed is $(u - v)$. 2. Downstream Speed (s_d): The effective speed is $(u + v)$. 3. Speed of Boat in Still Water (u): $u = \frac{s_d + s_u}{2}$. **Solution:** 1. Calculate Upstream Speed (s_u): Distance = 36 km, Time = 6 hours. $s_u = \frac{\text{Distance}}{\text{Time}} = \frac{36}{6} = 6$ km/h. Equation: $u - v = 6$. 2. Calculate Downstream Speed (s_d): Distance = 60 km, Time = 5 hours. $s_d = \frac{\text{Distance}}{\text{Time}} = \frac{60}{5} = 12$ km/h. Equation: $u + v = 12$. 3. Solve for u (Still Water Speed): Adding the two equations: $(u - v) + (u + v) = 6 + 12$. $2u = 18$. $u = 9$ km/h. **Final Answer:** 9 km/h

Answer: (A)

Q3.

Solution

Concept: In a race, if person X beats person Y by d meters in a race of length L, the ratio of their speeds (or the ratio of distances covered in the same time) is: $\frac{\text{Distance covered by X}}{\text{Distance covered by Y}} = \frac{L}{L-d}$. **Solution:** 1. Ratio of A and B: In a 1000m race, A beats B by 100m. When A covers 1000m, B covers $1000 - 100 = 900$m. $\frac{A}{B} = \frac{1000}{900} = \frac{10}{9}$. 2. Ratio of B and C: In a 1000m race, B beats C by 100m. When B covers 1000m, C covers $1000 - 100 = 900$m. $\frac{B}{C} = \frac{1000}{900} = \frac{10}{9}$. 3. Ratio of A and C: To find how far C has gone when A reaches the finish line (1000m), we multiply the ratios: $\frac{A}{C} = \frac{A}{B} \times \frac{B}{C} = \frac{10}{9} \times \frac{10}{9} = \frac{100}{81}$. 4. Calculate Distance for C: When $A = 1000$m, we find C: $\frac{1000}{C} = \frac{100}{81} \implies C = \frac{1000\times 81}{100} = 810$m. 5. Distance A beats C: Distance = Distance of A − Distance of C Distance = 1000m − 810m = 190m. **Final Answer:** 190m

Answer: (B)

Q4.

Solution

Concept: Modular arithmetic allows us to substitute congruent values into algebraic expressions: 1. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$. 2. If $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$ for any positive integer k. **Solution:** 1. Given Congruences: $x \equiv 3 \pmod 7$ $y \equiv 4 \pmod 7$ 2. Calculate the sum $(x + y) \pmod 7$: $x + y \equiv 3 + 4 \pmod 7$ $x + y \equiv 7 \pmod 7$ Since 7 is exactly divisible by 7, $7 \equiv 0 \pmod 7$. 3. Calculate the square $(x + y)^2 \pmod 7$: $(x + y)^2 \equiv 0^2 \pmod 7$ $(x + y)^2 \equiv 0 \pmod 7$ **Final Answer:** 0

Answer: (A)

Q5.

Solution

Concept: For a square matrix A of order n, the determinant of the adjoint of A is related to the determinant of A by the formula: $|adj(A)| = |A|^{n-1}$ **Solution:** 1. Identify Given Values: Order of matrix A $(n) = 3$. Determinant of A $(|A|) = 5$. 2. Apply the Formula: $|adj(A)| = |A|^{3-1}$ $|adj(A)| = |A|^2$ 3. Calculate the Result: $|adj(A)| = 5^2 = 25$. **Final Answer:** 25

Answer: (C)

Q6.

Solution

Concept: A system of linear equations has infinitely many solutions if the determinant of the coefficient matrix (Δ) is zero and the determinants $\Delta_x, \Delta_y, \Delta_z$ are also zero (Cramer's Rule for dependent systems). Alternatively, the third equation must be a linear combination of the first two. **Solution:** 1. Set up the Coefficient Determinant (Δ): $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & k \end{vmatrix}$ 2. Expand the Determinant: $\Delta = 1(2k-9) - 1(k-3) + 1(3-2)$ $\Delta = 2k - 9 - k + 3 + 1$ $\Delta = k - 5$ 3. Condition for Infinite Solutions: For the system to have infinitely many solutions, we must have $\Delta = 0$. $k - 5 = 0 \implies k = 5$. (Checking consistency: $R_3 = 2R_2 - R_1$. Indeed, $1(2) - 1 = 1, 2(2) - 1 = 3$, $3(2) - 1 = 5$. Since $k = 5$, the RHS also follows: $5(2) - 2 = 8$. The system is consistent and dependent). **Final Answer:** 5

Answer: (B)

Q7.

Solution

Concept: 1. The transpose of a matrix A, denoted A^T, is obtained by interchanging rows and columns. 2. I is the identity matrix $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. 3. Matrix addition is performed element-wise. **Solution:** 1. Find A^T: $A = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} \implies A^T = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ 2. Calculate

$A + A^T$:** $A + A^T = \begin{bmatrix}\cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha\\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha\end{bmatrix}$ $A + A^T = \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}$ 3. Equate to Identity Matrix I: $\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $2\cos\alpha = 1 \implies \cos\alpha = \frac{1}{2}$ 4. Solve for α: $\cos\alpha = \frac{1}{2} \implies \alpha = \frac{\pi}{3}$ (or 60°). **Final Answer:** $\pi/3$

Answer: (B)

Q8.

Solution

Concept: 1. Marginal Cost (MC): The derivative of the Total Cost function, $MC = C'(x)$. 2. Minimizing MC: To find the minimum of the MC function, find its derivative (MC') and set it to zero, then verify with the second derivative ($MC'' > 0$). **Solution:** 1. Find Marginal Cost (MC): $C(x) = \frac{1}{3}x^3 + 5x^2 - 12x + 10$ $MC = \frac{d}{dx}C(x) = x^2 + 10x - 12$ 2. Differentiate MC to find the critical point: $\frac{d}{dx}(MC) = 2x + 10$ 3. Set the derivative to zero: $2x + 10 = 0 \implies 2x = -10 \implies x = -5$ 4. Verify Minima: $\frac{d^2}{dx^2}(MC) = 2$, which is > 0. Thus, MC is minimum at $x = -5$. (Note: While $x = -5$ is the mathematical solution, in real-world economic contexts, output x cannot be negative. However, based on the provided options, -5 is the required answer.) **Final Answer:** x = -5

Answer: (B)

Q9.

Solution

Concept: A function $f(x)$ is strictly decreasing in an interval where its first derivative is negative, i.e., $f'(x) < 0$. **Solution:** 1. Find the first derivative $f'(x)$: $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$ 2. Set the condition for strictly decreasing: $6x^2 - 30x + 36 < 0$ 3. Simplify

and factor the inequality: Divide by 6: $x^2 - 5x + 6 < 0$ $(x-2)(x-3) < 0$ 4. Determine the

interval: The expression $(x-2)(x-3)$ is negative when x lies between the roots. Therefore, $2 < x < 3$, or $x \in (2, 3)$. **Final Answer:** $(2, 3)$

Answer: (A)

Q10.

Solution

Concept: For a function of the form $f(x) = a\sin x + b\cos x$, the maximum value is given by $\sqrt{a^2+b^2}$ and the minimum value is $-\sqrt{a^2+b^2}$. **Solution:** 1. Identify Coefficients: $f(x) =$

$1\sin x + 1\cos x$ Here, $a = 1$ and $b = 1$. 2. Apply the Formula: Maximum value $= \sqrt{(1)^2 + (1)^2}$

Maximum value $= \sqrt{1+1} = \sqrt{2}$. 3. Alternative Method (Calculus): $f'(x) = \cos x - \sin x =$

$0 \implies \tan x = 1 \implies x = \pi/4$. $f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$. **Final**

Answer: $\sqrt{2}$

Answer: (C)

Q11.

Solution

Concept: 1. Consumer Surplus (CS):** Given by the formula $CS = \int_0^{x_0} f(x)\,dx - p_0 x_0$, where $p = f(x)$ is the demand function, p_0 is the equilibrium price, and x_0 is the equilibrium quantity. **Solution:** 1. Find equilibrium quantity x_0: Given $p_0 = 9$. $9 = 25 - x^2 \implies x^2 = 16 \implies x_0 = 4$

(as quantity $x > 0$). 2. Set up the integral: $CS = \int_0^4 (25 - x^2)\,dx - (9 \times 4)$ $CS = [25x - \frac{x^3}{3}]_0^4 - 36$

3. Evaluate: $CS = (25(4) - \frac{4^3}{3}) - 36$ $CS = (100 - \frac{64}{3}) - 36 = 64 - \frac{64}{3}$ $CS = \frac{192-64}{3} = \frac{128}{3}$ **Final**

Answer: 128/3

Answer: (C)

Q12.

Solution

Concept: 1. The curve $y^2 = 4x$ is a right-opening parabola symmetric about the x-axis. 2. The area bounded by a curve $y = f(x)$ from $x = a$ to $x = b$ is $\int_a^b y\,dx$. 3. Since the parabola is symmetric, the total area is twice the area above the x-axis: $A = 2\int_0^3 y\,dx$. **Solution:** 1. Express y in terms of x: $y^2 = 4x \implies y = \pm 2\sqrt{x}$. We take $y = 2\sqrt{x}$ for the upper half. 2. Set up the integral: Total Area $= 2\int_0^3 2\sqrt{x}\,dx = 4\int_0^3 x^{1/2}\,dx$ 3. Integrate: Area $= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = 4 \times \frac{2}{3}[x^{3/2}]_0^3$ Area $= \frac{8}{3}[3^{3/2}] = \frac{8}{3}[3\sqrt{3}]$ Area $= 8\sqrt{3}$ **Final Answer:** $8\sqrt{3}$

Answer: (B)

Q13.

Solution

Concept: To solve $\int \frac{1}{x(x^n+1)}\,dx$, multiply the numerator and denominator by x^{n-1} to facilitate substitution. **Solution:** 1. Rearrange the integral: $\int \frac{x^4}{x^5(x^5+1)}\,dx$ 2. Substitution: Let $t = x^5$. Then $dt = 5x^4\,dx \implies x^4\,dx = \frac{dt}{5}$. The integral becomes: $\frac{1}{5}\int \frac{dt}{t(t+1)}$ 3. Partial Fractions: $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$ $\frac{1}{5}\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt = \frac{1}{5}[\log|t| - \log|t+1|] + C$ 4. Simplify and Substitute back: $\frac{1}{5}\log\left|\frac{t}{t+1}\right| + C = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$ **Final Answer:** $1\frac{}{5\log\left|\frac{x^5}{x^5+1}\right|+C}$

Answer: (A)

Q14.

Solution

Concept: 1. Order: The order of a differential equation is the order of the highest-order derivative present in the equation. 2. Degree: The degree is the power of the highest-order derivative, provided the equation is a polynomial in its derivatives. **Solution:** 1. Identify the highest derivative: In the equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$, the highest order derivative is $\frac{d^2y}{dx^2}$. Therefore, Order = 2. 2. Identify the power of the highest derivative: The term $\frac{d^2y}{dx^2}$ is raised to the power of 1. Note that the power of 3 belongs to the first derivative, which does not determine the degree. Therefore, Degree = 1. **Final Answer:** Order 2, Degree 1

Answer: (A)

Q15.

Solution

Concept: For population growth proportional to current population P, the governing differential equation is $\frac{dP}{dt} = kP$. The general solution is $P(t) = P_0 e^{kt}$, where P_0 is the initial population and k is the growth constant. **Solution:** 1. Set up the doubling condition: At $t = 10$, $P(10) = 2P_0$.

Substitute into the solution: $2P_0 = P_0 e^{k(10)}$. 2. Solve for k: $2 = e^{10k}$ Taking natural logarithm

(log or ln) on both sides: $\log 2 = 10k$ $k = \frac{\log 2}{10}$ **Final Answer:** $\log 2_{\overline{10}}$

Answer: (A)

Q16.

Solution

Concept: In a Poisson distribution, a unique property is that the Mean (λ) is equal to the Variance (σ^2). The Standard Deviation (σ) is the square root of the Variance. **Solution:** 1. Identify the

Mean: Mean (λ) = 4. 2. Determine Variance: Since Variance = Mean in Poisson distribution,

Variance (σ^2) = 4. 3. Calculate Standard Deviation: $\sigma = \sqrt{\text{Variance}}$ $\sigma = \sqrt{4} = 2$. **Final Answer:**

2

Answer: (C)

Q17.

Solution

Concept: A Standard Normal Distribution (Z-distribution) is perfectly symmetrical about the mean ($Z = 0$). 1. The total area under the curve is 1.0. 2. Because it is symmetrical, exactly 50% of the area lies to the left of the mean and 50% to the right. **Solution:** 1. Total Area: 1.0. 2.

Symmetry Property: The mean ($Z = 0$) divides the distribution into two equal halves. 3. Area Calculation: The area from $Z = -\infty$ to $Z = 0$ represents the left half of the distribution. Area = $1.0/2 = 0.5$. **Final Answer:** 0.5

Answer: (B)

Q18.

Solution

Concept: The probability mass function for a Poisson distribution is $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$, where λ is the mean. **Solution:** 1. Use the given condition $P(X = 1) = P(X = 2)$: $\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$ $\lambda = \frac{\lambda^2}{2}$

(Dividing both sides by $e^{-\lambda}$ and λ, as $\lambda \neq 0$) $1 = \frac{\lambda}{2} \implies \lambda = 2$. 2. Calculate $P(X = 0)$: Substitute

$\lambda = 2$ and $k = 0$ into the formula: $P(X = 0) = \frac{e^{-2} \cdot 2^0}{0!}$ Since $2^0 = 1$ and $0! = 1$: $P(X = 0) = e^{-2}$. **Final Answer:** e^{-2}

Answer: (A)

Q19.

Solution

Concept: The present value (PV) of a perpetuity is calculated using the formula: $PV = \frac{R}{i}$ where R is the periodic payment and i is the interest rate per period. **Solution:** 1. Identify

Variables: Monthly payment (R) = 500. Annual interest rate = 12%. Monthly interest rate (i) = $\frac{12\%}{12} = 1\% = 0.01$. 2. Calculate Present Value: $PV = \frac{500}{0.01}$ $PV = 500 \times 100 = 50,000$. **Final**

Answer: 50,000

Answer: (A)

Q20.

Solution

Concept: To find the periodic investment (R) for a sinking fund to reach a future value (A), we use the formula: $R = \frac{A \times i}{(1+i)^n - 1}$ where A is the target amount, i is the interest rate, and n is the number of periods. **Solution:** 1. Identify Variables: Future Value (A) = 5,00,000. Annual interest

rate (i) = 8% = 0.08. Time (n) = 10 years. $(1.08)^{10} = 2.158$. 2. Substitute into the Formula:

$R = \frac{5,00,000 \times 0.08}{2.158 - 1}$ $R = \frac{40,000}{1.158}$ 3. Calculate the Result: $R \approx 34,542.31$. Based on the options, the

value is 34,542. **Final Answer:** 34,542

Answer: (A)

Q21.

Solution

Concept: The Equated Monthly Installment (EMI) is calculated using the reducing balance method formula: $EMI = P \times r \times \frac{(1+r)^n}{(1+r)^n-1}$ where P is the principal loan amount, r is the monthly interest rate, and n is the number of monthly installments. **Solution:** 1. Analyze the Formula

Components: P: Principal (1,00,000). r: Periodic rate (1% or 0.01). n: Number of periods (12). The standard formula for an annuity certain (present value) rearranged for the payment R (EMI) matches option A. 2. Verify Options: Option A is the standard EMI formula. Option B represents

a single future value calculation. Option C is the Simple Interest formula. Option D is the Present Value of a single future sum. **Final Answer:** P $\times r \times \frac{(1+r)^n}{(1+r)^n-1}$

Answer: (A)

Q22.

Solution

Concept: The p-value is the probability of obtaining test results at least as extreme as the results actually observed, under the assumption that the null hypothesis (H_0) is correct. 1. The Significance Level (α) is the threshold for "rarity" set by the researcher (commonly 0.05). 2. If p-value $\leq \alpha$, the result is statistically significant, and we reject H_0. **Solution:** 1. Decision Rule: In

hypothesis testing, the standard rule is: If p-value $< \alpha$: Reject the Null Hypothesis. If p-value $\geq \alpha$: Fail to reject the Null Hypothesis. 2. Interpretation: A small p-value indicates that the observed data is very unlikely under the null hypothesis, providing enough evidence to favor the alternative hypothesis. **Final Answer:** Less than the significance level (α)

Answer: (B)

Q23.

Solution

Concept: The t-test (specifically Student's t-test) is a statistical test used to compare means when certain conditions about the sample and population are met. **Solution:** 1. Sample Size Condition:

While the Z-test is used for large samples ($n \geq 30$), the t-test was specifically developed for small sample sizes ($n < 30$). 2. Standard Deviation Condition: The t-test is necessary when the

population standard deviation (σ) is unknown and must be estimated using the sample standard deviation (s). 3. Distribution Assumption: It assumes the underlying population from which the

sample is drawn follows a normal distribution. **Final Answer:** Small ($n < 30$) and population SD

is unknown

Answer: (B)

Q24.

Solution

Concept: According to the Fundamental Theorem of Linear Programming, if an optimal solution (maximum or minimum) for an objective function exists, it must occur at one of the vertices or corner points of the feasible region defined by the constraints. **Solution:** 1. The Feasible Region:

This is the set of all points that satisfy the given linear constraints. 2. The Objective Function: $Z = ax + by$ represents a family of parallel lines. As we move these lines across the feasible region, the extreme value will be hit at the "edges" of the region. 3. Corner Point Method: By evaluating the value of Z at each vertex (corner point) of the feasible region, we can determine the maximum or minimum value. 4. Conclusion: While any point in the region is a "possible" solution, the "optimal" solution is found at the corner points. **Final Answer:** The corner points of

the feasible region

Answer: (C)

Q25.

Solution

Concept: In Linear Programming, variables usually represent physical quantities (like number of units produced, hours worked, etc.). These quantities cannot be negative in a real-world context. **Solution:** 1. Definition: Non-negativity constraints ensure that the decision variables do not take

on negative values. 2. Mathematical Representation: For variables x and y, this is expressed as $x \geq 0$ and $y \geq 0$. 3. Graphical Impact: These constraints restrict the feasible region to the first quadrant of the Cartesian plane. 4. Analyzing Options: $x, y < 0$: Strictly negative (Incorrect). $x, y \leq 0$: Negative or zero (Incorrect). $x, y \geq 0$: Zero or positive (Correct). $x + y \geq 0$: This allows individual variables to be negative (e.g., $x = 5, y = -2$) (Incorrect). **Final Answer:** x, y ≥ 0

Answer: (C)

Q26.

Solution

Concept: In the method of Least Squares, the normal equations for a straight line $Y = a + bX$ are: 1. $\sum Y = na + b\sum X$ 2. $\sum XY = a\sum X + b\sum X^2$ **Solution:** 1. Apply the Condition: We are

given that $\sum X = 0$. This typically occurs when the time variable X is shifted so that the middle of the time period is the origin. 2. Substitute into the first normal equation: $\sum Y = na + b(0)$

$\sum Y = na$ 3. Solve for a: $a = \frac{\sum Y}{n}$ (Similarly, from the second equation, $b = \frac{\sum XY}{\sum X^2}$ when $\sum X = 0$).

Final Answer: $\sum Y/n$

Answer: (A)

Q27.

Solution

Concept: A moving average is a technique used in time series analysis to analyze data points by creating a series of averages of different subsets of the full data set. **Solution:** 1. Purpose: The

primary goal of a moving average (whether 3-yearly, 5-yearly, etc.) is to reduce the "noise" in a data set. 2. Smoothing: By taking the average of consecutive points, the impact of random,

erratic, or short-term fluctuations is minimized. 3. Trend Identification: This process helps

in revealing the underlying long-term trend that might be hidden by day-to-day or year-to-year volatility. 4. Analyzing Options: Option A: Seasonal variations are usually handled by specific

seasonal indices or centered moving averages of 4 or 12 periods. Option C: Moving averages provide estimates but cannot find "exact" future values. Option D: Cyclical movements are larger patterns; moving averages primarily address short-term "irregular" movements. **Final Answer:**

Smooth out short-term fluctuations

Answer: (B)

Q28.

Solution

Concept: 1. Downstream Speed (s_d): Speed in still water + Speed of current. 2. Upstream Speed (s_u): Speed in still water - Speed of current. 3. Time-Distance Relation: Time = $\frac{\text{Distance}}{\text{Speed}}$. 4. Total time for a round trip of distance D is $\frac{D}{s_d} + \frac{D}{s_u}$ = Total Time. **Solution:** 1. Identify Given Values:

Speed in still water (u) = 6 km/h. Speed of current (v) = 2 km/h. Total time (T) = 3 hours. 2.

Calculate Relative Speeds: $s_d = u + v = 6 + 2 = 8$ km/h. $s_u = u - v = 6 - 2 = 4$ km/h. 3. Set up

the Equation: Let the distance to the place be D. $\frac{D}{8} + \frac{D}{4} = 3$ 4. Solve for D: Find a common

denominator (8): $\frac{D}{8} + \frac{2D}{8} = 3$ $\frac{3D}{8} = 3$ $3D = 24 \implies D = 8$ km. **Final Answer:** 8 km

Answer: (A)

Q29.

Solution

Concept: 1.Matrix Multiplication: $(A^2)_{ij} = \sum a_{ik}b_{kj}$. 2. Scalar Multiplication: kA multiplies every element of A by k. 3. Cayley-Hamilton Theorem: Every square matrix satisfies its own characteristic equation $|A - \lambda I| = 0$. **Solution:** 1. Calculate A^2: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^2 =$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (1+6) & (2+8) \\ (3+12) & (6+16) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$ 2. Calculate $5A$: $5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$ 3. Subtract

$5A$ from A^2: $A^2 - 5A = \begin{bmatrix} 7-5 & 10-10 \\ 15-15 & 22-20 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $A^2 - 5A = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$

Final Answer: 2I

Answer: (A)

Q30.

Solution

Concept: A square matrix is singular if its determinant is equal to zero ($|A| = 0$). **Solution:** 1. Set the Determinant to Zero: $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ 2. Use Row/Column Operations: Apply $C_1 \to C_1 + C_2 + C_3$: $\begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0$ Take $(x+1)$ common: $(x+1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

3. Simplify the Determinant: Apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$: $(x+1)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0$

Expanding along C_1: $(x+1)(x-2)(x-2) = 0$ 4. Solve for x: $x + 1 = 0 \implies x = -1$ $x - 2 = 0 \implies x = 2$ The values are -1 and 2. **Final Answer:** -1, 2

Answer: (C)

Q31.

Solution

Concept: 1. Total Revenue (TR):Product of price and quantity, $TR = p \times x$. 2. Maximization: TR is maximized when its first derivative (Marginal Revenue) is zero, $\frac{d(TR)}{dx} = 0$, and the second derivative is negative. **Solution:** 1. Find the TR function: $p = 100 - 2x$ $TR = (100 - 2x)x = 100x - 2x^2$ 2. Differentiate TR to find MR: $MR = \frac{d}{dx}(100x - 2x^2) = 100 - 4x$ 3. Set MR to zero: $100 - 4x = 0$ $4x = 100 \implies x = 25$ 4. Verify Maxima: $\frac{d^2(TR)}{dx^2} = -4$, which is < 0. Thus, TR is maximized at $x = 25$. **Final Answer:** 25

Answer: (A)

Q32.

Solution

Concept: The relationship between Marginal Revenue (MR), Price (p), and Price Elasticity of Demand (e_d) is given by the formula: $MR = p\left(1 - \frac{1}{e_d}\right)$ Note: Here e_d is typically taken as the absolute value. **Solution:** 1. Identify Given Values: $MR = 25$ $e_d = 5$ 2. Substitute into the Formula: $25 = p\left(1 - \frac{1}{5}\right)$ $25 = p\left(\frac{5-1}{5}\right)$ $25 = p\left(\frac{4}{5}\right)$ 3.Solve for p: $p = \frac{25\times5}{4}$ $p = \frac{125}{4} = 31.25$

Final Answer: 31.25

Answer: (A)

Q33.

Solution

Concept: 1. Equilibrium Price (p_0): Found by substituting the equilibrium quantity x_0 into the supply function. 2. Producer Surplus (PS): The formula is $PS = p_0x_0 - \int_0^{x_0} g(x)\,dx$, where $p = g(x)$ is the supply function. **Solution:** 1. Find Equilibrium Price (p_0): Given $x_0 = 3$. $p_0 = 2 + (3)^2 = 2 + 9 = 11$. 2. Calculate p_0x_0: $p_0x_0 = 11 \times 3 = 33$. 3. Set up the Integral: $\int_0^3 (2+x^2)\,dx = \left[2x + \frac{x^3}{3}\right]_0^3 = [2(3) + \frac{3^3}{3}] - [0] = 6 + 9 = 15$. 4. Calculate PS: $PS = 33 - 15 = 18$.

Final Answer: 18

Answer: (B)

Q34.

Solution

Concept: To evaluate an integral of the form $\int \frac{f'(x)}{f(x)}\,dx$, we use the substitution $u = f(x)$, which results in $\log|f(x)| + C$. **Solution:** 1. Substitution: Let $u = 1 + e^x$. Then $du = e^x\,dx$. 2. Change Limits: When $x = 0$, $u = 1 + e^0 = 1 + 1 = 2$. When $x = 1$, $u = 1 + e^1 = 1 + e$. 3. Evaluate the Integral: $\int_0^1 \frac{e^x}{1+e^x}\,dx = \int_2^{1+e} \frac{1}{u}\,du = [\log u]_2^{1+e} = \log(1+e) - \log(2)$ 4. Simplify using Log Properties: $\log A - \log B = \log(\frac{A}{B})$ Result $= \log\left(\frac{1+e}{2}\right)$. **Final Answer:** $\log(\frac{1+e}{2})$

Answer: (B)

Q35.

Solution

Concept: The market value of a bond is determined by the relationship between its Coupon Rate and the Required Yield (Market Interest Rate): 1. If Coupon Rate = Yield, Bond sells at Par (Face Value). 2. If Coupon Rate > Yield, Bond sells at a Premium (> Face Value). 3. If Coupon Rate < Yield, Bond sells at a Discount (< Face Value). **Solution:** 1. Identify Given Rates: Coupon Rate = 7% Required Yield = 8% 2. Compare Rates: Since the Coupon Rate (7%) is less than the Required Yield (8%), investors will not pay the full face value for a lower-than-market return. 3. Conclusion: The bond must sell at a discount to compensate for the lower coupon payments. Therefore, the market value will be Less than 1,000. **Final Answer:** Less than 1,000

Answer: (C)

Q36.

Solution

Concept: To replace an asset, the sinking fund must accumulate the Depreciable Amount, which is the Cost minus the Scrap Value. The formula for annual installment R is: $R = (\text{Cost} - \text{Scrap Value}) \times \frac{1}{s_{\overline{n}|i}}$ **Solution:** 1. Calculate Target Amount (A): Cost = 10,00,000 Scrap Value = 1,00,000 $A = 10,00,000 - 1,00,000 = 9,00,000$ 2. Identify Sinking Fund Factor: Given $\frac{1}{s_{\overline{10}|0.05}} = 0.0795$ 3. Calculate Annual Contribution (R): $R = 9,00,000 \times 0.0795$ $R = 71,550$

Final Answer: 71,550

Answer: (A)

Q37.

Solution

Concept: The principal amount (P) of a loan is the Present Value of an Ordinary Annuity: $P = R \times \left[\frac{1-(1+i)^{-n}}{i}\right]$ where R is the EMI, i is the periodic interest rate, and n is the total number of periods. **Solution:** 1. Identify Variables: R = 1,200 i = 1% = 0.01 n = 2 years × 12 months = 24 2. Set up Calculation: $P = 1,200 \times \left[\frac{1-(1.01)^{-24}}{0.01}\right]$ Using $(1.01)^{-24} \approx 0.787566$ $P = 1,200 \times \left[\frac{1-0.787566}{0.01}\right]$ $P = 1,200 \times 21.24338 \approx 25,492.06$ **Final Answer:** 25,492

Answer: (B)

Q38.

Solution

Concept: The present value of a deferred perpetuity is calculated in two steps: 1. Find the value of the perpetuity at the time it starts ($V = R/i$). 2. Discount that value back to the present time ($PV = V \times (1+i)^{-k}$), where k is the number of years of deferment. **Solution:** 1. Value at start of payment (End of Year 4): $V = \frac{10,000}{0.10} = 1,00,000$ 2. Discount to Present (Year 0): The first payment is 5 years from now, meaning the perpetuity is deferred for 4 years (it behaves like a standard perpetuity starting at $t = 4$). $PV = \frac{1,00,000}{(1.10)^4}$ $PV = \frac{1,00,000}{1.4641} \approx 68,301.35$
Final Answer: 68,301

Answer: (B)

Q39.

Solution

Concept: In hypothesis testing, we face two types of errors: 1. Type I Error (α): Rejecting the null hypothesis (H_0) when it is actually true. This is also known as a "False Positive." 2. Type II Error (β): Failing to reject the null hypothesis (H_0) when it is actually false. **Solution:** 1. Definition of α: The Level of Significance (α) is the maximum probability of committing a Type I error that a researcher is willing to risk. 2. Standard Values:It is typically set at 0.05 (5%) or 0.01 (1%) before the test is conducted. 3. Relation to Confidence: A significance level of 0.05 corresponds to a 95% confidence level ($1 - \alpha$). **Final Answer:** The probability of Type I error

Answer: (B)

Q40.

Solution

Concept: The Standard Error (SE) of the mean measures how much the sample mean is expected to vary from the true population mean. It is calculated as: $SE = \frac{\sigma}{\sqrt{n}}$ where σ is the population standard deviation and n is the sample size. **Solution:** 1. Identify Given Values: Variance (σ^2) = 25. Sample size (n) = 16. 2. Calculate Standard Deviation (σ): $\sigma = \sqrt{\text{Variance}} = \sqrt{25} = 5$. 3. Calculate Standard Error: $SE = \frac{5}{\sqrt{16}}$ $SE = \frac{5}{4} = 1.25$. **Final Answer:** 1.25

Answer: (A)

Q41.

Solution

Concept: The choice of distribution for testing a population mean depends on the sample size and knowledge of the population standard deviation (σ). **Solution:** 1. Z-Distribution: Used when $n \geq 30$ (due to the Central Limit Theorem) OR when σ is known. 2. t-Distribution: Used specifically when: The sample size is small ($n < 30$). The population standard deviation (σ) is unknown. The population is approximately normally distributed. 3. Application: In this case, since the sample standard deviation (s) must be used as an estimate for σ and n is small, the Student's t-test is appropriate. **Final Answer:** t-Distribution

Answer: (C)

Q42.

Solution

Concept: For a Binomial distribution $B(n, p)$: 1. Mean: $\mu = np$ 2. Variance: $\sigma^2 = npq$ (where $q = 1 - p$ is the probability of failure). **Solution:** 1. Set up the equations: $np = 4$ $npq = 3$ 2.Find q: Divide the Variance by the Mean: $\frac{npq}{np} = \frac{3}{4}$ $q = 0.75$ 3. Find p: $p = 1 - q = 1 - 0.75 = 0.25$ 4. Solve for n: Using $np = 4$: $n(0.25) = 4$ $n = \frac{4}{0.25} = 16$. **Final Answer:** 16

Answer: (B)

Q43.

Solution

Concept: The Empirical Rule (or 68-95-99.7 rule) describes the percentage of data within standard deviations of the mean for a Normal Distribution: Mean $\pm 1\sigma$: $\approx 68.2\%$ Mean $\pm 2\sigma$: $\approx 95.4\%$ Mean $\pm 3\sigma$: $\approx 99.7\%$ **Solution:** 1. The question asks for the percentage within ± 2 standard deviations. 2. According to the standard normal table and the empirical rule, the area between $Z = -2$ and $Z = 2$ is approximately 0.9544. 3. This corresponds to 95.4%.

Final Answer: 95.4%

Answer: (C)

Q44.

Solution

Concept: When using an even-period moving average(like 4-yearly), the average of the first four values falls between Year 2 and Year 3. To align the trend value with a specific time period, we calculate a centered moving average by taking the mean of two successive moving averages.
Solution: 1. First 4-yearly average: This value is positioned at the midpoint of the first four years, which is $t = 2.5$ (between Year 2 and Year 3). 2. Second 4-yearly average: This value is positioned at the midpoint of years 2 through 5, which is $t = 3.5$ (between Year 3 and Year 4). 3. Centering: The average of these two values is calculated to "center" the data. The midpoint between $t = 2.5$ and $t = 3.5$ is exactly $t = 3$. 4. Conclusion: Therefore, the first centered moving average value is assigned to Year 3 (or the middle of Year 3).

Final Answer: The middle of Year 3

Answer: (B)

Q45.

Solution

Concept: The trend line $Y = a + bX$ allows us to estimate future values by substituting the number of time units (X) that have elapsed from the origin.

Solution: 1. Identify the Equation and Origin: Equation: $Y = 20 + 1.5X$ Origin: 2015 (At 2015, $X = 0$) X units: 1 year

2. Calculate X for the target year (2020): $X = \text{Target Year} - \text{Origin Year}$ $X = 2020 - 2015 = 5$

3. Substitute X into the trend equation: $Y = 20 + 1.5(5)$ $Y = 20 + 7.5$ $Y = 27.5$

Final Answer: 27.5

Answer: (B)

Q46.

Solution

Concept: Corner points of a feasible region are found at the intersection of the boundary lines of the constraints and the axes. To find them, we solve the equations $x + y = 6$ and $2x + y = 8$ simultaneously and check the intercepts.

Solution: 1. Find Intercepts: For $x + y = 6$: $(6, 0)$ and $(0, 6)$. For $2x + y = 8$: $(4, 0)$ and $(0, 8)$.

2. Intersection of the two lines: Subtract first from second: $(2x + y) - (x + y) = 8 - 6 \implies x = 2$. Substitute $x = 2$ in $x + y = 6 \implies 2 + y = 6 \implies y = 4$. Intersection point is $(2, 4)$.

3. Determine the Feasible Corner Points: The constraints are "$\leq$", so we look for points that satisfy all conditions. Testing $(4, 0)$: $4 + 0 \leq 6$ (True) and $2(4) + 0 \leq 8$ (True). This is a corner point. Testing $(6, 0)$: $2(6) + 0 \leq 8$ (False). Testing $(0, 8)$: $0 + 8 \leq 6$ (False). Testing $(4, 4)$: $4 + 4 \leq 6$ (False).

Final Answer: (4, 0)

Answer: (A)

Q47.

Solution

Concept: This refers to the Multiple Optimal Solutions (or Alternative Optima) property of Linear Programming. **Solution:** 1. The Theorem: If an objective function $Z = ax + by$ assumes the

same maximum (or minimum) value at two distinct corner points of the feasible region, then the objective function will have that same value at every point on the line segment connecting those two points. 2. Infinite Solutions: Since a line segment contains infinitely many points, there are

infinitely many optimal solutions in this scenario. 3. Geometric Intuition:This happens when the

objective function line is parallel to one of the binding constraint lines.

Final Answer: Every point on the line segment joining these two points is optimal.

Answer: (C)

Q48.

Solution

Concept: This is a first-order ordinary differential equation that can be solved using the **Variable Separable Method**, where we group all terms involving y on one side and all terms involving x on the other.

Solution: 1. Separate the variables: $\frac{dy}{y} = \frac{dx}{x}$

2. Integrate both sides: $\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$ $\log|y| = \log|x| + \log C$ (where $\log C$ is the constant of integration) 3. Simplify using logarithm properties: $\log|y| = \log|Cx|$ $y = Cx$ **Final Answer:** y =

Cx

Answer: (B)

Q49.

Solution

Concept: This problem follows an exponential growth pattern. If a population triples every T hours, the population after t hours is given by $P(t) = N \times 3^{(t/T)}$. **Solution:** 1. Identify Given

Values: Initial count = N Tripling period (T) = 2 hours Total time (t) = 6 hours 2. Calculate

the number of periods: Number of 2-hour intervals in 6 hours = $6/2 = 3$ periods. 3. Apply

the growth factor: After 2 hours: $3N$ After 4 hours: $3(3N) = 9N$ After 6 hours: $3(9N) = 27N$ Mathematically: $N \times 3^3 = 27N$. **Final Answer:** 27N

Answer: (C)

Q50.

Solution

Concept: 1. Average Cost (AC): Total Cost divided by output, $AC = \frac{C(x)}{x}$. 2. Marginal Cost (MC): The derivative of Total Cost, $MC = C'(x)$. 3. Economic Property: AC is equal to MC at the level of output where Average Cost is at its minimum. **Solution:** 1. Find AC and MC:

$AC = \frac{5x^2+20x+500}{x} = 5x + 20 + \frac{500}{x}$ $MC = \frac{d}{dx}(5x^2 + 20x + 500) = 10x + 20$ 2. Equate AC and MC:

$5x + 20 + \frac{500}{x} = 10x + 20$ 3. Solve for x: Subtract 20 from both sides: $5x + \frac{500}{x} = 10x$ Subtract

$5x$ from both sides: $\frac{500}{x} = 5x$ Multiply by x: $500 = 5x^2$ $x^2 = 100 \implies x = 10$ (output must be positive). **Final Answer:** x = 10

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	B	4	A	5	C
6	B	7	B	8	B	9	A	10	C
11	C	12	B	13	A	14	A	15	A
16	C	17	B	18	A	19	A	20	A
21	A	22	B	23	B	24	C	25	C
26	A	27	B	28	A	29	A	30	C
31	A	32	A	33	B	34	B	35	C
36	A	37	B	38	B	39	B	40	A
41	C	42	B	43	C	44	B	45	B
46	A	47	C	48	B	49	C	50	A