CUET-UG Applied Mathematics Sample Paper-6

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a 3×3 matrix and $|3A| = k|A|$, then the value of k is:

(A) 3

(B) 9

(C) 27

(D) 81

Q2. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to:

(A) A

(B) $I - A$

(C) $I + A$

(D) $3A$

Q3. Let A be a non-singular matrix of order 3. If $|adj A| = 64$, then $|A|$ is:

(A) ± 4

(B) ± 8

(C) 64

(D) 16

Q4. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) $(a+b+c)$

(B) 0

(C) $(a-b)(b-c)(c-a)$

(D) 1

Q5. The function $f(x) = \frac{x}{\log x}$ increases in the interval:

(A) $(0, 1)$

(B) $(0, e)$

(C) (e, ∞)

(D) $(-\infty, e)$

Q6. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{\log x}{(1+\log x)^2}$

(B) $\frac{1}{(1+\log x)^2}$

(C) $\frac{\log x}{1+\log x}$

(D) $\frac{e^x}{x^y}$

Q7. The point on the curve $y = x^2 - 3x + 2$ where the tangent is perpendicular to the line $y = x$ is:

(A) $(1, 0)$

(B) $(0, 2)$

(C) $(2, 0)$

(D) $(1, 2)$

Q8. The minimum value of $2x^3 - 21x^2 + 36x - 20$ in the interval $[0, 2]$ is:

(A) -20

(B) -3

(C) -2

(D) -5

Q9. The value of $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ is:

(A) $\log|\sin x + \cos x| + C$

(B) $-\log|\sin x + \cos x| + C$

(C) $\log|\sin x - \cos x| + C$

(D) $\tan^{-1}(\sin x) + C$

Q10. The area of the region bounded by $y^2 = x$, the y-axis, and the lines $y = 2$ and $y = 3$ is:

(A) $19/3$

(B) $5/2$

(C) $7/3$

(D) $13/3$

Q11. The value of $\int_0^{\pi/4} \tan^2 x dx$ is:

(A) $1 - \frac{\pi}{4}$

(B) $\frac{\pi}{4} - 1$

(C) $1 + \frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Q12. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are:

(A) 2, 3

(B) 3, 2

(C) 2, 1

(D) 2, Not defined

Q13. The general solution of $\frac{dy}{dx} + y = e^{-x}$ is:

(A) $y = e^{-x}(x + C)$

(B) $ye^{x} = x + C$

(C) $y = xe^{x} + C$

(D) $y = e^{x}(x + C)$

Q14. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

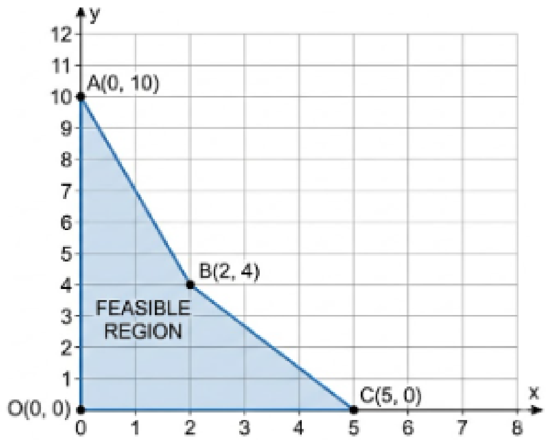
(A) 0.24

(B) 0.96

(C) 0.48

(D) 0.52

Q15. Which corner point minimizes the objective function $Z = 4x + 3y$ (excluding the origin)?



(A) (0, 10)

(B) (2, 4)

(C) (5, 0)

(D) All give same value

Q16. Find the smallest positive integer x such that $3x + 4 \equiv 2 \pmod 7$.

(A) 2

(B) 4

(C) 5

(D) 6

Q17. A man can row 6 km/h in still water. If the speed of the current is 2 km/h, it takes him 3 hours to row to a place and come back. How far is the place?

(A) 8 km

(B) 10 km

(C) 12 km

(D) 15 km

Q18. In a 100m race, A can beat B by 25m and B can beat C by 4m. In the same race, A can beat C by:

(A) 29m

(B) 28m

(C) 21m

(D) 26m

Q19. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively. If both are opened and A is closed after 3 minutes, how much further time will B take to fill the tank?

(A) 8 min 15 sec

(B) 7 min

(C) 9 min

(D) 5 min 45 sec

Q20. If A is a 2×2 matrix such that $A^2 - 5A + 7I = 0$, then A^{-1} is:

(A) $\frac{1}{7}(5I - A)$

(B) $\frac{1}{5}(7I - A)$

(C) $5I + A$

(D) $7I - 5A$

Q21. The system of equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if k is not equal to:

(A) 0

(B) 1

(C) -1

(D) 2

Q22. If the inverse of matrix A is $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then the inverse of matrix $2A$ is:

(A) $\begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 0.5 & 1 \\ 0 & 0.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0.5 & 0 \\ 1 & 0.5 \end{bmatrix}$

Q23. The total revenue function is given by $R(x) = 100x - 0.5x^2$. The actual revenue from selling the 101st unit is:

(A) 0

(B) -0.5

(C) 100

(D) 99.5

Q24. The cost function $C(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 10$. The level of output x at which Marginal Cost is minimum is:

(A) 3

(B) 2

(C) 0

(D) 9

Q25. If the demand function is $p = 20 - 2x$, the marginal revenue function is:

(A) $20 - 2x$

(B) $20 - 4x$

(C) $20x - 2x^2$

(D) 20

Q26. The demand function for a commodity is $p = 30 - 2x - x^2$. If the equilibrium quantity is $x_0 = 3$, the Consumer Surplus is:

(A) 18

(B) 27

(C) 36

(D) 45

Q27. The supply function is $p = 2x^2 + 4$. If the equilibrium price is $p_0 = 12$, the Producer Surplus is:

(A) 10.67

(B) 16.25

(C) 8.33

(D) 12.0

Q28. The Marginal Revenue function is $MR = 10 - 4x$. The total revenue function $R(x)$ (assuming $R(0) = 0$) is:

(A) $10x - 4x^2$

(B) $10x - 2x^2$

(C) $10 - 2x^2$

(D) $10x - x^2$

Q29. The area bounded by $y = \sqrt{x}$ and $y = x$ is:

(A) 1/2

(B) 1/3

(C) 1/6

(D) 1/4

Q30. A culture of bacteria grows at a rate proportional to the number present. If the number triples in 10 hours, the equation for $N(t)$ is:

(A) $N = N_0 e^{(t \log 3)/10}$

(B) $N = N_0 e^{3t/10}$

(C) $N = N_0 e^{10t}$

(D) $N = N_0 e^{30t}$

Q31. The differential equation governing the value of an asset V that depreciates at a constant percentage rate r is:

(A) $\frac{dV}{dt} = rV$

(B) $\frac{dV}{dt} = -rV$

(C) $\frac{dV}{dt} = -r$

(D) $\frac{d^2V}{dt^2} = -rV$

Q32. In a Poisson distribution, if the mean is λ, then the standard deviation is:

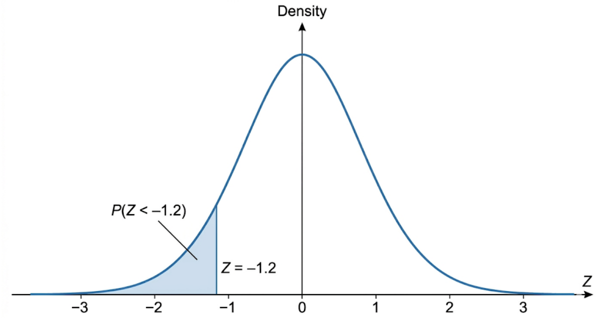
(A) λ^2

(B) λ

(C) $\sqrt{\lambda}$

(D) $1/\lambda$

Q33. If $P(0 < Z < 1.2) = 0.3849$, what is the shaded area $P(Z < -1.2)$?



(A) 0.1151

(B) 0.8849

(C) 0.3849

(D) 0.5

Q34. If 3% of the items manufactured by a company are defective, the probability of finding exactly 2 defective items in a sample of 100 is: (Use $e^{-3} = 0.0498$)

(A) 0.2241

(B) 0.1494

(C) 0.0498

(D) 0.2500

Q35. For a Normal distribution, the points of inflection are at:

(A) $\mu \pm \sigma$

(B) $\mu \pm 2\sigma$

(C) $\mu \pm 3\sigma$

(D) Mean and Median

Q36. What is the moving average for the year 2021?

Year	Sales	3-Point Moving Average
2018	50	-
2019	60	-
2020	55	(50+60+55)/3 = 55.0
2021	70	(60+55+70)/3 = 61.7
2022	75	(55+70+75)/3 = 66.7

(A) 60.0

(B) 61.67

(C) 66.67

(D) 65.0

Q37. In the method of least squares for a straight line $y = a + bx$, the normal equation for b is:

(A) $\sum y = na + b \sum x$

(B) $\sum xy = a \sum x + b \sum x^2$

(C) $\sum x^2 = a \sum x + b \sum y$

(D) $\sum y^2 = n \sum x$

Q38. Seasonal variations in a time series repeat themselves within a period of:

(A) 1 year

(B) 5 years

(C) 10 years

(D) 1 month only

Q39. The probability of committing a Type II error is denoted by:

(A) α

(B) β

(C) $1 - \alpha$

(D) $1 - \beta$

Q40. In a t-test, if the calculated value t is greater than the table value t_α at a given level of significance, we:

(A) Accept H_0

(B) Reject H_0

(C) Reject H_1

(D) Need more data

Q41. The hypothesis which is tested for possible rejection under the assumption that it is true is called:

(A) Alternative Hypothesis

(B) Null Hypothesis

(C) Statistical Hypothesis

(D) Simple Hypothesis

Q42. A sample of 10 students has a mean height of 160 cm. To test if the population mean height is 165 cm, the test statistic used is:

(A) Z-test

(B) t-test

(C) F-test

(D) Chi-square test

Q43. What is the present value of a perpetuity of ₹ 900 payable at the end of every year, if money is worth 9% per annum?

(A) ₹ 10,000

(B) ₹ 8,100

(C) ₹ 9,000

(D) ₹ 1,000

Q44. The effective rate of interest equivalent to a nominal rate of 8% per annum compounded quarterly is:

(A) 8.24%

(B) 8.16%

(C) 8.08%

(D) 8.30%

Q45. An annuity in which payments continue forever is called:

(A) Ordinary Annuity

(B) Annuity Due

(C) Perpetuity

(D) Deferred Annuity

Q46. A sinking fund is a fund created to:

(A) Pay off a liability in future

(B) Save tax

(C) Invest in stock market

(D) Distribute dividends

Q47. If the coupon rate of a bond is higher than the market interest rate, the bond will sell at:

(A) A discount

(B) Par value

(C) A premium

(D) Zero value

Q48. The value of a ₹ 1,000 zero-coupon bond maturing in 2 years with a required return of 10% compounded annually is:

(A) ₹ 800

(B) ₹ 826.45

(C) ₹ 900

(D) ₹ 1,210

Q49. Which of the following is NOT a requirement for a Linear Programming Problem?

(A) Objective function must be linear

(B) Constraints must be linear

(C) Decision variables must be non-negative

(D) Decision variables must be integers only

Q50. In LPP, the optimal solution of the objective function always occurs at:

(A) The origin

(B) Any point inside the feasible region

(C) Corner points of the feasible region

(D) Any point on the X-axis

Detailed Solutions

Q1.

Solution

Concept: The determinant property for a scalar multiple of a matrix states that if A is an $n \times n$ matrix and c is a scalar, then $|cA| = c^n|A|$.

Solution: Given that A is a 3×3 matrix, its order is $n = 3$.
We are given the relation $|3A| = k|A|$.
Using the property $|cA| = c^n|A|$, for $c = 3$ and $n = 3$, we have:
$|3A| = 3^3|A|$
$|3A| = 27|A|$
Comparing this with the given $|3A| = k|A|$, we can conclude that $k = 27$.

Final Answer : "27"

Answer: (C)

Q2.

Solution

Concept: Binomial expansion of $(X \pm Y)^3 = X^3 \pm 3X^2Y + 3XY^2 \pm Y^3$.
Properties of identity matrix I: $A \cdot I = I \cdot A = A$ and $I^n = I$ for any positive integer n.
Given condition $A^2 = I$. From this, $A^3 = A^2 \cdot A = I \cdot A = A$.

Solution: We need to evaluate $(A - I)^3 + (A + I)^3 - 7A$.
First, expand $(A - I)^3$:
$(A - I)^3 = A^3 - 3A^2I + 3AI^2 - I^3$
Using $A^2 = I$, $A^3 = A$, and $I^2 = I$, $I^3 = I$:
$(A - I)^3 = A - 3I \cdot I + 3A \cdot I - I$
$(A - I)^3 = A - 3I + 3A - I = 4A - 4I$

Next, expand $(A + I)^3$:
$(A + I)^3 = A^3 + 3A^2I + 3AI^2 + I^3$
Using $A^2 = I$, $A^3 = A$, and $I^2 = I$, $I^3 = I$:
$(A + I)^3 = A + 3I \cdot I + 3A \cdot I + I$
$(A + I)^3 = A + 3I + 3A + I = 4A + 4I$

Now substitute these back into the original expression:
$(A - I)^3 + (A + I)^3 - 7A = (4A - 4I) + (4A + 4I) - 7A$
$= 4A - 4I + 4A + 4I - 7A$
Combine like terms:
$= (4A + 4A - 7A) + (-4I + 4I)$
$= (8A - 7A) + 0I$
$= A$

Final Answer : "A"

Answer: (A)

Q3.

Solution

Concept: For a non-singular square matrix A of order n, the determinant of its adjoint is given by the formula $|adj A| = |A|^{n-1}$.

Solution: Given that A is a non-singular matrix of order 3, so $n = 3$.
We are given $|adj A| = 64$.
Using the formula $|adj A| = |A|^{n-1}$:
$|adj A| = |A|^{3-1}$
$|adj A| = |A|^2$
Substitute the given value:
$64 = |A|^2$
To find $|A|$, take the square root of both sides:
$|A| = \pm\sqrt{64}$
$|A| = \pm 8$

Final Answer : "±8"

Answer: (B)

Q4.

Solution

Concept: Properties of determinants:

1. If a scalar multiple of one column (or row) is added to another column (or row), the value of the determinant remains unchanged.
2. If any two columns (or rows) of a determinant are identical, the value of the determinant is zero.

Solution: Let the given determinant be Δ:

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Apply the column operation $C_3 \to C_3 + C_2$. This means we add the elements of the second column to the corresponding elements of the third column. The value of the determinant remains unchanged.

$$\Delta = \begin{vmatrix} 1 & a & a+(b+c) \\ 1 & b & b+(c+a) \\ 1 & c & c+(a+b) \end{vmatrix} \Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$ Now, we can take the common factor $(a+b+c)$ out of the third column (C_3):

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$ Observe the first column (C_1) and the third column (C_3). They are identical.

Since two columns of the determinant are identical, its value is 0.

So, $\Delta = (a+b+c) \cdot 0$

$\Delta = 0$

Final Answer : "0"

Answer: (B)

Q5.

Solution

Concept: A function $f(x)$ is increasing in an interval if its first derivative $f'(x) > 0$ for all x in that interval.

The domain of $f(x) = \frac{x}{\log x}$ requires $x > 0$ and $\log x \neq 0 \Rightarrow x \neq 1$.

So, the domain is $(0, 1) \cup (1, \infty)$.

Differentiation using the quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$.

Solution: Given the function $f(x) = \frac{x}{\log x}$.

Let $u = x$ and $v = \log x$.

Then $u' = \frac{d}{dx}(x) = 1$.

And $v' = \frac{d}{dx}(\log x) = \frac{1}{x}$.

Now, apply the quotient rule to find $f'(x)$:

$f'(x) = \frac{u'v - uv'}{v^2} = \frac{1 \cdot (\log x) - x \cdot \left(\frac{1}{x}\right)}{(\log x)^2}$ $f'(x) = \frac{\log x - 1}{(\log x)^2}$

For the function to be increasing, $f'(x) > 0$.

$\frac{\log x - 1}{(\log x)^2} > 0$

Since $(\log x)^2$ is always positive for $x \in (0, 1) \cup (1, \infty)$, the sign of $f'(x)$ depends only on the numerator.

So, we need $\log x - 1 > 0$.

$\log x > 1$

Since the base of $\log x$ is e (natural logarithm), we have:

$x > e^1$

$x > e$

Therefore, the function $f(x)$ increases in the interval (e, ∞).

Final Answer : "(e, ∞)"

Answer: (C)

Q6.

Solution

Concept: This problem requires logarithmic differentiation because the variable appears in the base and exponent, and implicit differentiation.
Properties of logarithms: $\log(a^b) = b\log a$ and $\log_e e = 1$.
Quotient rule for differentiation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$.

Solution: Given the equation $x^y = e^{x-y}$.
To simplify, take the natural logarithm (ln) on both sides:
$\ln(x^y) = \ln(e^{x-y})$
Using the logarithm property $\ln(a^b) = b\ln a$:
$y\ln x = (x-y)\ln e$
Since $\ln e = 1$:
$y\ln x = x - y$
Now, rearrange the equation to express y explicitly in terms of x:
$y\ln x + y = x$
Factor out y:
$y(\ln x + 1) = x$
$y = \frac{x}{1+\ln x}$
Now, differentiate y with respect to x using the quotient rule. Let $u = x$ and $v = 1 + \ln x$.
Then $u' = \frac{d}{dx}(x) = 1$.
And $v' = \frac{d}{dx}(1+\ln x) = 0 + \frac{1}{x} = \frac{1}{x}$.
Applying the quotient rule $\frac{dy}{dx} = \frac{u'v-uv'}{v^2}$:
$\frac{dy}{dx} = \frac{1\cdot(1+\ln x)-x\cdot(\frac{1}{x})}{(1+\ln x)^2}$
$\frac{dy}{dx} = \frac{1+\ln x-1}{(1+\ln x)^2}$
$\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

Final Answer : "$\frac{\log x}{(1+\log x)^2}$"

Answer: (A)

Q7.

Solution

Concept: The slope of the tangent to a curve $y = f(x)$ at any point is given by its derivative $\frac{dy}{dx}$. If two lines are perpendicular, the product of their slopes is -1.

Solution: Given curve: $y = x^2 - 3x + 2$.
First, find the slope of the tangent to the curve by differentiating y with respect to x:
$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 3x + 2) = 2x - 3$.
Let m_t be the slope of the tangent. So, $m_t = 2x - 3$.

Given line: $y = x$.
The slope of this line, let's call it m_l, is the coefficient of x, which is $m_l = 1$.

The tangent to the curve is perpendicular to the line $y = x$.
For perpendicular lines, the product of their slopes is -1:
$m_t \cdot m_l = -1$
$(2x - 3) \cdot 1 = -1$
$2x - 3 = -1$
$2x = -1 + 3$
$2x = 2$
$x = 1$.

Now, substitute $x = 1$ back into the curve's equation to find the corresponding y-coordinate:
$y = (1)^2 - 3(1) + 2$
$y = 1 - 3 + 2$
$y = 0$.
So, the point on the curve where the tangent is perpendicular to the line $y = x$ is $(1, 0)$.

Final Answer : "$(1, 0)$**"**

Answer: (A)

Q8.

Solution

Concept: To find the absolute extrema of a continuous function $f(x)$ on a closed interval $[a, b]$:
1. Find the derivative $f'(x)$ and identify critical points where $f'(x) = 0$.
2. Keep only the critical points that lie within the interval $[a, b]$.
3. Evaluate $f(x)$ at the critical points and the endpoints $x = a$ and $x = b$.
4. The smallest value is the absolute minimum and the largest is the absolute maximum.

Solution: Given $f(x) = 2x^3 - 21x^2 + 36x - 20$ on the interval $[0, 2]$.

Step 1: Find critical points. Differentiating the function:
$f'(x) = 6x^2 - 42x + 36$ Set $f'(x) = 0$ and simplify:
$6(x^2 - 7x + 6) = 0 \implies 6(x-1)(x-6) = 0$ The critical points are $x = 1$ and $x = 6$. Since our interval is $[0, 2]$, we only consider $x = 1$.

Step 2: Evaluate the function at the relevant points. We test the endpoints ($x = 0, x = 2$) and the valid critical point ($x = 1$):

- **At $x = 0$:** $f(0) = 2(0)^3 - 21(0)^2 + 36(0) - 20 = -20$
- **At $x = 1$:** $f(1) = 2(1)^3 - 21(1)^2 + 36(1) - 20 = 2 - 21 + 36 - 20 = -3$
- **At $x = 2$:** $f(2) = 2(2)^3 - 21(2)^2 + 36(2) - 20 = 16 - 84 + 72 - 20 = -16$

Step 3: Conclusion. Comparing the values $\{-20, -3, -16\}$, the smallest value is -20.

Final Answer: "-20"

Answer: (A)

Q9.

Solution

Concept: Integration by the method of substitution and the logarithmic integration rule.

Solution: We are required to evaluate the following indefinite integral:

$$I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

By observing the structure of the integrand, we notice that the derivative of the denominator is very similar to the numerator. Let us substitute the denominator:
Let $u = \sin x + \cos x$.
Differentiating u with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos x) = \cos x - \sin x$$

This implies:

$$du = (\cos x - \sin x)dx$$

Multiplying both sides by -1 to match our numerator exactly:

$$-du = -(\cos x - \sin x)dx = (\sin x - \cos x)dx$$

Now, substitute u and $-du$ into the original integral:

$$I = \int \frac{-du}{u} = -\int \frac{1}{u} du$$

Applying the standard integration rule for reciprocal functions, $\int \frac{1}{u} du = \log |u| + C$:

$$I = -\log |u| + C$$

Replacing u with our original expression $\sin x + \cos x$:

$$I = -\log |\sin x + \cos x| + C$$

Final Answer : "$-\log |\sin x + \cos x| + C$"

Answer: (B)

Q10.

Solution

Concept: Area bounded by a curve, the y-axis, and specific horizontal lines using definite integration.

Solution: The area A of a region bounded by a curve $x = f(y)$, the y-axis, and the horizontal lines $y = c$ and $y = d$ is given by:

$$A = \int_c^d x\,dy$$

We are given the following information:

- Equation of the curve: $y^2 = x$, which can be written as $x = y^2$.
- The region is bounded by the y-axis and the lines $y = 2$ and $y = 3$.

Substituting the function and the limits into the area formula:

$$A = \int_2^3 y^2\,dy$$

Integrating y^2 using the power rule $\int y^n dy = \frac{y^{n+1}}{n+1}$:

$$A = \left[\frac{y^3}{3}\right]_2^3$$

Now, evaluate the definite integral by applying the upper and lower limits:

$$A = \frac{3^3}{3} - \frac{2^3}{3} = \frac{27}{3} - \frac{8}{3}$$

$$A = \frac{19}{3}$$

The area is $19/3$ square units.

Final Answer : "19/3"

Answer: (A)

Q11.

Solution

Concept: Definite integration of trigonometric functions using the identity $\tan^2 x = \sec^2 x - 1$.

Solution: We need to evaluate the definite integral:

$$I = \int_0^{\pi/4} \tan^2 x \, dx$$

There is no direct formula for the integral of $\tan^2 x$. We must use the Pythagorean identity $1 + \tan^2 x = \sec^2 x$, which gives:

$$\tan^2 x = \sec^2 x - 1$$

Substitute this into the integral:

$$I = \int_0^{\pi/4} (\sec^2 x - 1) \, dx$$

Integrate each term separately. The integral of $\sec^2 x$ is $\tan x$, and the integral of 1 is x:

$$I = [\tan x - x]_0^{\pi/4}$$

Evaluate the expression at the upper limit $(\pi/4)$ and lower limit (0):

$$I = \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$$

Knowing that $\tan(\pi/4) = 1$ and $\tan(0) = 0$:

$$I = (1 - \pi/4) - (0 - 0) = 1 - \frac{\pi}{4}$$

Final Answer : "$1 - \frac{\pi}{4}$"

Answer: (A)

Q12.

Solution

Concept: Determining the Order (highest derivative) and Degree (power of the highest derivative) of a differential equation.

Solution: The given differential equation is:

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$$

Step 1: Order. The order is defined as the order of the highest derivative present in the equation. In this case, the derivatives are $\frac{d^2y}{dx^2}$ (second order) and $\frac{dy}{dx}$ (first order). Thus, Order = 2.

Step 2: Degree. The degree is the exponent of the highest order derivative when the equation is a polynomial in terms of its derivatives. To check the degree, we must eliminate the fractional power $(1/3)$ from the first derivative.
Rearrange the equation:

$$\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3}$$

Cube both sides to clear the fraction:

$$\left(\frac{d^2y}{dx^2} + x^{1/4}\right)^3 = -\frac{dy}{dx}$$

Expanding the left side (using binomial expansion):

$$\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{d^2y}{dx^2}\right)^2 x^{1/4} + 3\left(\frac{d^2y}{dx^2}\right) x^{1/2} + x^{3/4} = -\frac{dy}{dx}$$

Now the equation is a polynomial in derivatives. The highest order derivative is $\frac{d^2y}{dx^2}$ and its highest power is 3. Thus, Degree = 3.

Final Answer : "2, 3"

Answer: (A)

Q13.

Solution

Concept: Solving first-order linear differential equations using the Integrating Factor (I.F.) method.

Solution: The given equation is:

$$\frac{dy}{dx} + y = e^{-x}$$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 1$ and $Q = e^{-x}$.
Step 1: Find the Integrating Factor (I.F.).

$$I.F. = e^{\int P dx} = e^{\int 1 dx} = e^{x}$$

Step 2: Apply the general solution formula.
The solution is given by:

$$y \cdot (I.F.) = \int (Q \cdot I.F.) dx + C$$

Substitute the values of $I.F.$ and Q:

$$y \cdot e^{x} = \int (e^{-x} \cdot e^{x}) dx + C$$

Since $e^{-x} \cdot e^{x} = e^{0} = 1$:

$$y \cdot e^{x} = \int 1 dx + C$$

$$y \cdot e^{x} = x + C$$

Step 3: Rearrange for y.
Multiply both sides by e^{-x}:

$$y = e^{-x}(x + C)$$

Final Answer : "$y = e^{-x}(x + C)$"

Answer: (A)

Q14.

Solution

Concept: Using the Conditional Probability formula and the Addition Theorem for probability.

Solution: We are given: $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$.

Step 1: Find the probability of intersection $P(A \cap B)$.

By the definition of conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies 0.6 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

Step 2: Find the probability of union $P(A \cup B)$.

Using the addition theorem:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute the values:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Final Answer : "0.96"

Answer: (B)

Q15.

Solution

Concept: Minimizing an objective function in Linear Programming using the Corner Point Method.

Solution: We are given the objective function $Z = 4x + 3y$. We need to evaluate its value at each of the corner points provided (excluding the origin O):

(a) Point A(0, 10): $Z = 4(0) + 3(10) = 0 + 30 = 30$

(b) Point B(2, 4): $Z = 4(2) + 3(4) = 8 + 12 = 20$

(c) Point C(5, 0): $Z = 4(5) + 3(0) = 20 + 0 = 20$

The minimum value of the objective function is 20. This value occurs at both points B(2, 4) and C(5, 0). According to the given multiple-choice options, Point B(2, 4) is listed as a primary corner point for the minimum.

Final Answer : "(2, 4)"

Answer: (B)

Q16.

Solution

Concept: Solving linear congruences in modular arithmetic.

Solution: We need to find the smallest positive integer x such that:

$$3x + 4 \equiv 2 \pmod{7}$$

Step 1: Simplify the congruence.
Subtract 4 from both sides:

$$3x \equiv 2 - 4 \pmod{7}$$

$$3x \equiv -2 \pmod{7}$$

To work with positive integers, add the modulus (7) to -2:

$$3x \equiv 5 \pmod{7}$$

Step 2: Find the modular inverse or test values. We check small positive integers for x:

- $x = 1 \implies 3(1) = 3 \not\equiv 5 \pmod{7}$
- $x = 2 \implies 3(2) = 6 \not\equiv 5 \pmod{7}$
- $x = 3 \implies 3(3) = 9 \equiv 2 \pmod{7} \neq 5$
- $x = 4 \implies 3(4) = 12$. Since $12 = 7 \times 1 + 5$, $12 \equiv 5 \pmod{7}$.

The smallest positive integer is $x = 4$.

Final Answer : "4"

Answer: (B)

Q17.

Solution

Concept: Relative speed in water: Downstream and Upstream motion.

Solution: Let:

Speed of the man in still water (u) = 6 km/h.

Speed of the current (v) = 2 km/h.

Total time taken (T) = 3 hours.

Step 1: Calculate speeds.

Downstream speed $D = u + v = 6 + 2 = 8$ km/h.

Upstream speed $U = u - v = 6 - 2 = 4$ km/h.

Step 2: Set up the time equation.

Let the distance to the place be d.

Time taken to go downstream + Time taken to return upstream = Total Time.

$$\frac{d}{8} + \frac{d}{4} = 3$$

Step 3: Solve for d.

Multiplying the equation by 8 (the LCM of 8 and 4):

$$d + 2d = 24$$

$$3d = 24 \implies d = 8 \text{ km}$$

The place is 8 km away.

Final Answer : "8 km"

Answer: (A)

Q18.

Solution

Concept: Distance ratios in competitive racing.

Solution: In a 100m race:

Step 1: Ratio between A and B.

A beats B by 25m. This means when A covers 100m, B covers $100 - 25 = 75$m.

$$\frac{\text{Distance A}}{\text{Distance B}} = \frac{100}{75} = \frac{4}{3}$$

Step 2: Ratio between B and C.

B beats C by 4m. This means when B covers 100m, C covers $100 - 4 = 96$m.

$$\frac{\text{Distance B}}{\text{Distance C}} = \frac{100}{96} = \frac{25}{24}$$

Step 3: Combine the ratios for A and C.

$$\frac{\text{Distance A}}{\text{Distance C}} = \frac{\text{Distance A}}{\text{Distance B}} \times \frac{\text{Distance B}}{\text{Distance C}} = \frac{4}{3} \times \frac{25}{24}$$

$$\frac{\text{Distance A}}{\text{Distance C}} = \frac{1 \times 25}{3 \times 6} = \frac{25}{18}$$

Step 4: Scale to 100m.

Multiply the ratio by 4 to set A's distance to 100:

$$\frac{\text{Distance A}}{\text{Distance C}} = \frac{25 \times 4}{18 \times 4} = \frac{100}{72}$$

When A covers 100m, C covers 72m.

A beats C by $100 - 72 = 28$m.

Final Answer : "28m"

Answer: (B)

Q19.

Solution

Concept: Work and time calculations for multiple pipes (filling a tank).

Solution: Pipe A fills the tank in 12 mins. Rate of A = $1/12$ per min.

Pipe B fills the tank in 15 mins. Rate of B = $1/15$ per min.

Step 1: Work done while both pipes are open.

Combined rate = $\frac{1}{12} + \frac{1}{15} = \frac{5+4}{60} = \frac{9}{60} = \frac{3}{20}$ per min.

In 3 minutes, work done = $3 \times \frac{3}{20} = \frac{9}{20}$.

Step 2: Calculate remaining work.

Remaining work = $1 - \frac{9}{20} = \frac{11}{20}$.

Step 3: Time taken by B to finish.

Time = $\frac{\text{Remaining Work}}{\text{Rate of B}} = \frac{11/20}{1/15} = \frac{11}{20} \times 15 = \frac{33}{4}$ minutes.

Step 4: Convert to minutes and seconds.

$\frac{33}{4} = 8\frac{1}{4}$ minutes.

$1/4$ of a minute = $0.25 \times 60 = 15$ seconds.

Total time = 8 min 15 sec.

Final Answer : "8 min 15 sec"

Answer: (A)

Q20.

Solution

Concept: Using the Cayley-Hamilton theorem / Matrix polynomial to find an inverse.

Solution: We are given the equation:

$$A^2 - 5A + 7I = 0$$

To find the inverse matrix A^{-1}, we multiply the entire equation by A^{-1} (assuming A is invertible):

$$A^{-1} \cdot (A^2 - 5A + 7I) = A^{-1} \cdot 0$$

Distributing the A^{-1}:

$$A^{-1}A \cdot A - 5A^{-1}A + 7A^{-1}I = 0$$

Since $A^{-1}A = I$ and $A^{-1}I = A^{-1}$:

$$I \cdot A - 5I + 7A^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

Rearranging to solve for $7A^{-1}$:

$$7A^{-1} = 5I - A$$

Dividing by 7:

$$A^{-1} = \frac{1}{7}(5I - A)$$

Final Answer : "$\frac{1}{7}(5I - A)$"

Answer: (A)

Q21.

Solution

Concept: Condition for a unique solution in a system of linear equations using determinants.

Solution: A system of equations has a unique solution if the determinant of the coefficient matrix Δ is not equal to zero ($\Delta \neq 0$).
The coefficient matrix is:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$$

Expanding along the first row:

$$\Delta = 1 \cdot \begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$\Delta = 1(k - (-2)) - 1(2k - (-3)) + 1(4 - 3)$$

$$\Delta = (k + 2) - (2k + 3) + 1$$

$$\Delta = k + 2 - 2k - 3 + 1 = -k$$

For a unique solution:

$$\Delta \neq 0 \implies -k \neq 0 \implies k \neq 0$$

Final Answer : "0"

Answer: (A)

Q22.

Solution

Concept: Matrix inversion property for scalar multiples: $(cA)^{-1} = \frac{1}{c}A^{-1}$.

Solution: We are given the inverse of matrix A:

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

We need to find the inverse of the matrix $2A$.
Using the property $(cA)^{-1} = \frac{1}{c}A^{-1}$, where $c = 2$:

$$(2A)^{-1} = \frac{1}{2}A^{-1}$$

Substituting the given A^{-1}:

$$(2A)^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Multiply each element of the matrix by 0.5:

$$(2A)^{-1} = \begin{bmatrix} 1 \times 0.5 & 2 \times 0.5 \\ 0 \times 0.5 & 1 \times 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.5 \end{bmatrix}$$

Final Answer : “$\begin{bmatrix} 0.5 & 1 \\ 0 & 0.5 \end{bmatrix}$”

Answer: (B)

Q23.

Solution

Concept: Revenue calculation for a discrete unit using the change in the Total Revenue function.

Solution: The actual revenue generated by the sale of the n^{th} unit is the difference between the total revenue from selling n units and the total revenue from selling $n - 1$ units.
This is expressed as:

$$\text{Revenue of } n^{\text{th}} \text{ unit} = R(n) - R(n-1)$$

Given the Total Revenue function: $R(x) = 100x - 0.5x^2$. We need to find the revenue for the 101st unit ($x = 101$).

Step 1: Calculate Total Revenue for 101 units ($R(101)$).

$$R(101) = 100(101) - 0.5(101)^2$$

$$R(101) = 10100 - 0.5(10201)$$

$$R(101) = 10100 - 5100.5 = 4999.5$$

Step 2: Calculate Total Revenue for 100 units ($R(100)$).

$$R(100) = 100(100) - 0.5(100)^2$$

$$R(100) = 10000 - 0.5(10000)$$

$$R(100) = 10000 - 5000 = 5000$$

Step 3: Calculate the difference.

$$\text{Actual Revenue} = R(101) - R(100) = 4999.5 - 5000 = -0.5$$

The revenue from the 101st unit is -0.5, which matches Option (B).
Final Answer : "-0.5"

Answer: (B)

Q24.

Solution

Concept: Finding the minimum of the Marginal Cost (MC) function using first and second derivatives.

Solution: Given the Total Cost function:

$$C(x) = \frac{1}{3}x^3 - 3x^2 + 9x + 10$$

Step 1: Determine the Marginal Cost (MC) function.
Marginal Cost is the rate of change of Total Cost with respect to output x:

$$MC = \frac{dC}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x^2 + 9x + 10\right)$$

$$MC = x^2 - 6x + 9$$

Step 2: Find the critical point for MC.
To find the level of output where MC is minimized, differentiate the MC function and set it to zero:

$$\frac{d(MC)}{dx} = 2x - 6$$

Setting the derivative to zero: $2x - 6 = 0 \implies 2x = 6 \implies x = 3$.

Step 3: Verify the minimum.
Check the second derivative of the MC function:

$$\frac{d^2(MC)}{dx^2} = 2$$

Since the second derivative is positive $(2 > 0)$, the function reaches a minimum at $x = 3$. This corresponds to Option (A).

Final Answer : "3"

Answer: (A)

Q25.

Solution

Concept: Derivation of the Marginal Revenue (MR) function from the demand (price) function.

Solution: The demand function provides the price per unit (p) as a function of quantity (x):

$$p = 20 - 2x$$

Step 1: Calculate the Total Revenue (R) function.
Total Revenue is the product of price and quantity sold:

$$R(x) = p \times x = (20 - 2x)x$$

$$R(x) = 20x - 2x^2$$

Step 2: Calculate the Marginal Revenue (MR) function.
Marginal Revenue is the derivative of the Total Revenue function with respect to x:

$$MR = \frac{dR}{dx} = \frac{d}{dx}(20x - 2x^2)$$

Using the power rule of differentiation:

$$MR = 20(1) - 2(2x) = 20 - 4x$$

Thus, the marginal revenue function is $20 - 4x$, which is Option (B).

Final Answer : "$20 - 4x$"

Answer: (B)

Q26.

Solution

Concept: Calculating Consumer Surplus (CS) as the area between the demand curve and the equilibrium price level.

Solution: The formula for Consumer Surplus is:

$$CS = \int_0^{x_0} f(x)\,dx - (p_0 \times x_0)$$

Given: Demand function $p = 30 - 2x - x^2$ and equilibrium quantity $x_0 = 3$.

Step 1: Find the equilibrium price (p_0).
Substitute $x_0 = 3$ into the demand function:

$$p_0 = 30 - 2(3) - (3)^2 = 30 - 6 - 9 = 15$$

Step 2: Integrate the demand function from 0 to 3.

$$\int_0^3 (30 - 2x - x^2)\,dx = \left[30x - x^2 - \frac{x^3}{3}\right]_0^3$$

$$= \left(30(3) - (3)^2 - \frac{3^3}{3}\right) - (0)$$

$$= (90 - 9 - 9) = 72$$

Step 3: Calculate Consumer Surplus.

$$CS = 72 - (p_0 \times x_0) = 72 - (15 \times 3)$$

$$CS = 72 - 45 = 27$$

The Consumer Surplus is 27, which matches Option (B).

Final Answer : "27"

Answer: (B)

Q27.

Solution

Concept: Calculating Producer Surplus (PS) as the area between the equilibrium price level and the supply curve.

Solution: The formula for Producer Surplus is:

$$PS = (p_0 \times x_0) - \int_0^{x_0} g(x)\, dx$$

Given: Supply function $p = 2x^2 + 4$ and equilibrium price $p_0 = 12$.

Step 1: Find the equilibrium quantity (x_0).
Set the supply function equal to the equilibrium price:

$$12 = 2x^2 + 4 \implies 2x^2 = 8 \implies x^2 = 4 \implies x_0 = 2$$

Step 2: Integrate the supply function from 0 to 2.

$$\int_0^2 (2x^2 + 4)\, dx = \left[\frac{2x^3}{3} + 4x\right]_0^2$$

$$= \left(\frac{2(8)}{3} + 4(2)\right) - 0 = \frac{16}{3} + 8 = \frac{16 + 24}{3} = \frac{40}{3} \approx 13.33$$

Step 3: Calculate Producer Surplus.

$$PS = (p_0 \times x_0) - 13.33 = (12 \times 2) - 13.33$$

$$PS = 24 - 13.33 = 10.67$$

This corresponds to Option (A).

Final Answer : "10.67"

Answer: (A)

Q28.

Solution

Concept: Integration of the Marginal Revenue (MR) function to obtain the Total Revenue (R) function.

Solution: Total Revenue is the antiderivative of Marginal Revenue:

$$R(x) = \int MR\,dx$$

Given: $MR = 10 - 4x$.

Step 1: Perform the integration.

$$R(x) = \int (10 - 4x)\,dx = 10x - 4 \cdot \frac{x^2}{2} + C$$

$$R(x) = 10x - 2x^2 + C$$

Step 2: Solve for the constant of integration (C).
It is assumed that if zero units are sold, the revenue is zero. Therefore, $R(0) = 0$.

$$0 = 10(0) - 2(0)^2 + C \implies C = 0$$

Step 3: State the Total Revenue function.

$$R(x) = 10x - 2x^2$$

This matches Option (B).

Final Answer : "$10x - 2x^2$"

Answer: (B)

Q29.

Solution

Concept: Area between two curves determined by the difference of their definite integrals.

Solution: We need to find the area bounded by $y = \sqrt{x}$ and $y = x$.

Step 1: Find intersection points.
Set the two equations equal: $\sqrt{x} = x$.
Squaring both sides: $x = x^2 \implies x^2 - x = 0 \implies x(x-1) = 0$.
The points of intersection are $x = 0$ and $x = 1$.

Step 2: Set up the integral.
In the interval $[0, 1]$, $\sqrt{x} \geq x$. The area is:

$$A = \int_0^1 (\sqrt{x} - x)\, dx = \int_0^1 (x^{1/2} - x)\, dx$$

Step 3: Evaluate the integral.

$$A = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1 = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$$

$$A = \left(\frac{2}{3}(1)^{3/2} - \frac{(1)^2}{2}\right) - (0)$$

$$A = \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

The area is $1/6$, which is Option (C).

Final Answer : "1/6"

Answer: (C)

Q30.

Solution

Concept: Modeling biological growth using first-order differential equations and exponential functions.

Solution: The growth rate is proportional to the number present: $\frac{dN}{dt} = kN$.
The general solution to this differential equation is:

$$N(t) = N_0 e^{kt}$$

where N_0 is the initial number of bacteria.

Step 1: Determine the growth constant (k).
We are told the population triples ($N = 3N_0$) when $t = 10$ hours.

$$3N_0 = N_0 e^{10k} \implies 3 = e^{10k}$$

Taking the natural logarithm of both sides:

$$\log 3 = 10k \implies k = \frac{\log 3}{10}$$

Step 2: Formulate the final equation.
Substitute k back into the general solution:

$$N(t) = N_0 e^{\left(\frac{\log 3}{10}\right)t} = N_0 e^{(t \log 3)/10}$$

This matches Option (A).

Final Answer : "$N = N_0 e^{(t \log 3)/10}$"

Answer: (A)

Q31.

Solution

Concept: Differential equation representation of exponential decay (depreciation).

Solution: Depreciation is the process where the value of an asset decreases over time.
1. The rate of change of the value V with respect to time t is written as $\frac{dV}{dt}$.
2. The term "constant percentage rate r" implies that the rate of change is proportional to the current value V.
3. Because the value is decreasing (depreciating), the rate of change must be negative.

Combining these facts, we get:

$$\text{Rate of change} = -(\text{rate}) \times (\text{current value})$$

$$\frac{dV}{dt} = -rV$$

This represents a standard first-order differential equation for decay, matching Option (B).

Final Answer : "$\frac{dV}{dt} = -rV$"

Answer: (B)

Q32.

Solution

Concept: Relationship between Mean, Variance, and Standard Deviation in a Poisson Distribution.

Solution: The Poisson distribution is a discrete probability distribution defined by the parameter λ, which represents the average number of events in a given interval.

Step 1: Understand the distribution parameters.

In a Poisson distribution with parameter λ:

- The Mean ($E[X]$) is equal to λ.
- The Variance ($Var(X)$) is also equal to λ.

Step 2: Relate Variance to Standard Deviation.

The standard deviation (σ) is mathematically defined as the positive square root of the variance. Therefore:

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

Substituting the Poisson variance (λ):

$$\sigma = \sqrt{\lambda}$$

Thus, the standard deviation of a Poisson distribution is the square root of its mean. This corresponds to Option (C).

Final Answer : "$\sqrt{\lambda}$"

Answer: (C)

Q33.

Solution

Concept: Symmetry of the Standard Normal Distribution (Z-curve) and Probability Areas.

Solution: The Standard Normal Distribution is a bell-shaped curve that is perfectly symmetrical about the mean $Z = 0$.

Step 1: Use the property of symmetry.

We are given the area from the mean to $Z = 1.2$ as $P(0 < Z < 1.2) = 0.3849$.

Due to symmetry, the area from the mean to $Z = -1.2$ is exactly the same:

$$P(-1.2 < Z < 0) = 0.3849$$

Step 2: Use the property of the total area.

The total area under the curve is 1.0. Because the curve is symmetrical, the area to the left of the mean ($Z < 0$) is exactly 0.5.

Step 3: Calculate the shaded tail area.

The area to the left of $Z = -1.2$ (the shaded area in the tail) is the total area of the left half minus the area between -1.2 and 0:

$$P(Z < -1.2) = P(Z < 0) - P(-1.2 < Z < 0)$$

$$P(Z < -1.2) = 0.5000 - 0.3849 = 0.1151$$

Therefore, the shaded area is 0.1151, which corresponds to Option (A).

Final Answer : "0.1151"

Answer: (A)

Q34.

Solution

Concept: Applying the Poisson Distribution to approximate probabilities in a large sample.

Solution: When the probability of success p is small and the sample size n is large, the number of successes follows a Poisson distribution with parameter $\lambda = np$.

Step 1: Calculate the mean (λ).

Given $n = 100$ items and a defect rate $p = 3\% = 0.03$:

$$\lambda = n \times p = 100 \times 0.03 = 3$$

Step 2: Use the Poisson Probability Mass Function.

The formula for exactly k occurrences is:

$$P(X = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

For exactly 2 defective items ($k = 2$):

$$P(X = 2) = \frac{e^{-3} \cdot 3^2}{2!}$$

Step 3: Substitute the given value of e^{-3}.

Given $e^{-3} = 0.0498$:

$$P(X = 2) = \frac{0.0498 \times 9}{2 \times 1} = \frac{0.4482}{2} = 0.2241$$

The probability is 0.2241, which matches Option (A).

Final Answer : "0.2241"

Answer: (A)

Q35.

Solution

Concept: Geometric characteristics and inflection points of the Normal Distribution curve.

Solution: The Normal Distribution curve is defined by the probability density function $f(x)$. The points of inflection are the specific values of x where the curve changes its curvature from concave down (near the mean) to concave up (as it moves toward the tails).

Step 1: Analyze the derivatives of the Normal Density Function.

The second derivative $f''(x)$ determines the points of inflection. For a normal curve with mean μ and standard deviation σ, setting $f''(x) = 0$ results in:

$$x = \mu - \sigma \quad \text{and} \quad x = \mu + \sigma$$

Step 2: Conclusion.
This means that the points where the curve changes direction are located exactly one standard deviation away from the mean on both sides. This is a fundamental property of the bell-shaped curve. Therefore, the points are at $\mu \pm \sigma$, which is Option (A).

Final Answer : "$\mu \pm \sigma$"

Answer: (A)

Q36.

Solution

Concept: Calculation of a 3-point Moving Average in Time Series analysis.

Solution: A 3-point moving average is a statistical calculation used to smooth out short-term fluctuations in time series data. It is the arithmetic mean of the value for the specific year, the year before, and the year after.

Step 1: Identify the required data points for the year 2021.

To find the average for 2021, we need the sales for 2020, 2021, and 2022:

- Sales in 2020 (Y_{2020}) = 55
- Sales in 2021 (Y_{2021}) = 70
- Sales in 2022 (Y_{2022}) = 75

Step 2: Calculate the mean.

$$\text{Moving Average for 2021} = \frac{Y_{2020} + Y_{2021} + Y_{2022}}{3}$$

$$\text{Moving Average for 2021} = \frac{55 + 70 + 75}{3} = \frac{200}{3}$$

Step 3: Solve the division.

$$\frac{200}{3} \approx 66.666\ldots$$

Rounding to two decimal places, we get 66.67. This corresponds to Option (C).

Final Answer : "66.67"

Answer: (C)

Q37.

Solution

Concept: Normal Equations for determining the coefficients in the Method of Least Squares.

Solution: In the method of least squares for a linear trend $y = a + bx$, we aim to find the constants a (intercept) and b (slope) that minimize the sum of the squares of the errors. This is achieved by solving a set of simultaneous equations known as the normal equations.

Step 1: Derive the equations.
For the equation $y = a + bx$, we sum all observations:
1. $\sum y = \sum(a + bx) \implies \sum y = na + b\sum x$
To find the equation related specifically to the coefficient b, we multiply the original linear equation by x and then sum:

2. $\sum(xy) = \sum(x(a + bx)) = \sum(ax + bx^2)$
This simplifies to:

$$\sum xy = a\sum x + b\sum x^2$$

Step 2: Compare with options.
The second normal equation, which is used to solve for b, is $\sum xy = a\sum x + b\sum x^2$.
This matches Option (B).

Final Answer : "$\sum xy = a\sum x + b\sum x^2$"

Answer: (B)

Q38.

Solution

Concept: Definition and duration of the Seasonal Variation component in Time Series.

Solution: A time series consists of four components: Trend, Seasonal, Cyclical, and Irregular variations.

Step 1: Understand Seasonal Variation.
Seasonal variations are periodic fluctuations that occur regularly due to recurring events such as weather patterns, social customs, or holidays. These patterns are characterized by their short-term nature and regularity.

Step 2: Determine the timeframe.
By definition in statistical time series analysis:

- Seasonal variations occur within a period of **one year** (e.g., quarterly, monthly, or weekly patterns that repeat every 12 months).
- Variations that take longer than a year to repeat (usually 2 to 10 years) are classified as *Cyclical Variations*.

Therefore, seasonal variations repeat themselves within a period of 1 year, which corresponds to Option (A).

Final Answer : "1 year"

Answer: (A)

Q39.

Solution

Concept: Study of Errors in Statistical Hypothesis Testing (Type I and Type II).
Solution: When performing a statistical test, we make a decision to either accept or reject a null hypothesis (H_0). This decision-making process can lead to two specific types of errors:

(a) **Type I Error (α):** This occurs when we reject the null hypothesis H_0 even though it is actually true. It is also known as a "False Positive" or the level of significance of the test.

(b) **Type II Error (β):** This occurs when we fail to reject (i.e., we accept) the null hypothesis H_0 even though it is actually false. It is also known as a "False Negative."

The power of a test is defined as $1 - \beta$, which is the probability of correctly rejecting a false null hypothesis. Based on these definitions, the probability of committing a Type II error is represented by the Greek letter β.
Final Answer : "β"

Answer: (B)

Q40.

Solution

Concept: Decision criteria for Hypothesis Testing using critical (table) values.
Solution: In statistical inference, after calculating a test statistic (like t, Z, or F), we compare it with a critical value obtained from statistical tables (the table value) based on the chosen level of significance (α) and the degrees of freedom.

- **Acceptance Region:** If the calculated value of t is less than or equal to the table value ($t \leq t_\alpha$), it means the observed difference is likely due to chance. In this case, we fail to reject (accept) the null hypothesis H_0.
- **Rejection Region:** If the calculated value of t is greater than the table value ($t > t_\alpha$), the difference is statistically significant. It indicates that the observed result is highly unlikely to occur under the assumption of the null hypothesis.

Therefore, if the calculated value t exceeds the table value t_α, we conclude that there is sufficient evidence to **Reject** H_0.
Final Answer : "Reject H_0"

Answer: (B)

Q41.

Solution

Concept: Foundational definitions of Statistical Hypotheses.
Solution: To investigate a population parameter, we formulate two competing hypotheses:

(a) **Null Hypothesis (H_0):** This is a statement of "no effect," "no change," or "no difference." It is the starting assumption that we maintain unless our data provides strong evidence to the contrary. Critically, we specifically set up the Null Hypothesis so that we can test it for possible rejection.

(b) **Alternative Hypothesis (H_1 or H_a):** This is the statement that contradicts the null hypothesis and is what we aim to support by rejecting H_0.

Because the test is structured to evaluate whether the data is consistent with H_0 or if H_0 should be rejected, the hypothesis tested for possible rejection under the assumption that it is true is the **Null Hypothesis**.
Final Answer : "Null Hypothesis"

Answer: (B)

Q42.

Solution

Concept: Selecting appropriate test statistics for population means based on sample size.
Solution: When testing a hypothesis regarding a population mean, we must consider the sample size (n) and the information available about the population variance.

(a) **Large Sample Test (Z-test):** Generally used when the sample size $n \geq 30$. In large samples, the sampling distribution of the mean is approximately normal.

(b) **Small Sample Test (t-test):** Specifically designed by William Sealy Gosset ("Student") for situations where the sample size is small ($n < 30$) and the population standard deviation (σ) is unknown, requiring the use of the sample standard deviation (s) instead.

In the given problem, the sample size is $n = 10$. Since 10 is less than 30, this is categorized as a small sample. Therefore, the correct statistical test to compare the sample mean height (160 cm) against the hypothesized population mean (165 cm) is the **t-test**.
Final Answer : "t-test"

Answer: (B)

Q43.

Solution

Concept: Valuation of infinite cash flow streams (Perpetuities).

Solution: A perpetuity is a special type of annuity where the periodic payments R continue forever. The present value (PV) represents the lump sum amount required today to generate those infinite payments at a given interest rate i. The formula for the present value of an ordinary perpetuity is:

$$PV = \frac{R}{i}$$

Given the following values from the problem:

- Annual payment (R) = ₹ 900
- Annual interest rate (i) = 9% = 0.09

Substituting these values into the formula:

$$PV = \frac{900}{0.09}$$

To simplify the calculation, multiply the numerator and denominator by 100:

$$PV = \frac{90000}{9} = 10,000$$

The present value of the perpetuity is ₹ 10,000.

Final Answer : "₹ 10,000"

Answer: (A)

Q44.

Solution

Concept: Conversion of Nominal Interest Rates to Effective Annual Rates.

Solution: The nominal interest rate is the stated annual rate, but when interest is compounded multiple times per year, the actual interest earned (the effective rate) is higher. The formula for the effective rate of interest (r_e) is:

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Where:

- r = Nominal annual rate = 8% = 0.08
- m = Number of compounding periods per year (Quarterly $\implies m = 4$)

Step 1: Calculate the periodic rate.

$$\frac{r}{m} = \frac{0.08}{4} = 0.02$$

Step 2: Substitute into the formula.

$$r_e = (1 + 0.02)^4 - 1 = (1.02)^4 - 1$$

Step 3: Calculate the power value.

$$(1.02)^2 = 1.0404$$

$$(1.02)^4 = 1.0404 \times 1.0404 = 1.08243216$$

Step 4: Subtract 1 and convert to percentage.

$$r_e = 1.08243216 - 1 = 0.08243216 \approx 8.24\%$$

The effective rate is 8.24%.

Final Answer : "8.24%"

Answer: (A)

Q45.

Solution

Concept: Terminology and classification of financial annuities.
Solution: Annuities are categorized based on the timing and duration of the payments:

- **Ordinary Annuity:** Payments are made at the end of each period for a fixed duration.
- **Annuity Due:** Payments are made at the beginning of each period for a fixed duration.
- **Deferred Annuity:** An annuity where payments start after a specific lapse of time.
- **Perpetuity:** This is a specific type of annuity where the periodic payments start at a fixed date and continue indefinitely into the future with no end date.

Since the question describes an annuity in which payments "continue forever," the correct term is **Perpetuity**.
Final Answer : "Perpetuity"

Answer: (C)

Q46.

Solution

Concept: Financial planning for future debt obligations (Sinking Funds).

Solution: A sinking fund is a fund established by a company or government entity to systematically save money over time to pay off a specific debt or to replace a specific asset at a future date.

Working mechanism: 1. The entity makes regular periodic contributions into an interest-bearing account.

2. The interest compounds over time.

3. At the end of the term, the total accumulated amount (future value) is used to retire a major liability, such as a bond issue that has reached maturity.

This method ensures that the entity does not face a sudden cash flow crisis when a large debt falls due. Therefore, its purpose is to **Pay off a liability in future**.

Final Answer : "Pay off a liability in future"

Answer: (A)

Q47.

Solution

Concept: The inverse relationship between Bond Prices and Interest Rates.

Solution: A bond's "Coupon Rate" is the fixed interest it pays based on its face value. The "Market Interest Rate" is the rate currently offered by new bonds of similar risk.

(a) If Coupon Rate > Market Rate: The bond pays more than what an investor can get elsewhere. Demand for this bond increases, driving its price above the face value. This bond is said to sell at a **Premium**.

(b) If Coupon Rate < Market Rate: The bond pays less than the market. Investors will only buy it if the price is lower than the face value to compensate for the lower interest. This bond sells at a **Discount**.

(c) If Coupon Rate = Market Rate: The bond sells at its face value, or **Par**.

Because the bond's coupon rate is higher than the market rate, it is more valuable than new issues and will sell at a **Premium**.

Final Answer : "A premium"

Answer: (C)

Q48.

Solution

Concept: Valuation of Zero-Coupon Bonds (Present Value of a single sum).

Solution: A zero-coupon bond is a bond that does not pay periodic interest. Instead, it is sold at a significant discount and pays its full face value at maturity. The current value of the bond is the present value of the face value.
The formula for Present Value (PV) is:

$$PV = \frac{F}{(1+r)^n}$$

Given:

- Face Value (F) = ₹ 1,000
- Required Return (r) = 10% = 0.10
- Time to maturity (n) = 2 years

Step 1: Set up the calculation.

$$PV = \frac{1000}{(1+0.10)^2} = \frac{1000}{(1.1)^2}$$

Step 2: Calculate the denominator.

$$(1.1) \times (1.1) = 1.21$$

Step 3: Perform the division.

$$PV = \frac{1000}{1.21} \approx 826.44628$$

Rounding to the nearest paisa, we get ₹ 826.45.

Final Answer : "₹ 826.45"

Answer: (B)

Q49.

Solution

Concept: Core characteristics and constraints of Linear Programming Problems (LPP).

Solution: Linear Programming is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships.
The standard requirements for an LPP are:

- **Linearity:** The objective function and all constraints must be linear (first-degree) equations or inequalities.
- **Non-negativity:** The decision variables must be zero or positive ($x_i \geq 0$).
- **Continuity:** In standard LPP, variables can take any fractional or decimal value within the feasible region.

Step 1: Evaluate the options.
Options A, B, and C are essential requirements for any LPP. Option (D), which states that variables must be *integers only*, is a requirement for "Integer Programming," which is a specialized subset of mathematical programming, but it is **not** a general requirement for all Linear Programming Problems.

Final Answer : "Decision variables must be integers only"

Answer: (D)

Q50.

Solution

Concept: The Corner Point Theorem and Feasible Regions in LPP.

Solution: In Linear Programming, the set of all possible solutions that satisfy the given constraints forms a convex polygon (or polyhedron in higher dimensions) known as the **Feasible Region**.
Step 1: Apply the fundamental theorem of LPP. The Objective Function $Z = ax + by$ represents a family of parallel lines. As we move these lines across the feasible region to maximize or minimize Z, the last point of contact between the line and the feasible region will be at one of the vertices.

Step 2: Conclusion. The Corner Point Theorem states that the maximum or minimum value of the objective function, if it exists, must occur at a **Corner point (vertex)** of the feasible region. If the optimal value occurs at two adjacent corner points, then every point on the line segment joining them also provides the same optimal value. However, the optimal solution *always* occurs at at least one corner point.

Final Answer : "Corner points of the feasible region"

Answer: (C)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	B	5	C
6	A	7	A	8	A	9	B	10	A
11	A	12	A	13	A	14	B	15	B
16	B	17	A	18	B	19	A	20	A
21	A	22	B	23	B	24	A	25	B
26	B	27	A	28	B	29	C	30	A
31	B	32	C	33	A	34	A	35	A
36	C	37	B	38	A	39	B	40	B
41	B	42	B	43	A	44	A	45	C
46	A	47	C	48	B	49	D	50	C