CUET UG Applied Mathematics Sample Paper - 8

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. The value of a machine depreciates at the rate of 10% per annum on its reducing balance. If the scrap value of the machine at the end of 3 years is ₹ 72,900, its initial value was:

(A) ₹ 80,000

(B) ₹ 1,00,000

(C) ₹ 90,000

(D) ₹ 1,10,000

Q2. For a sample of size $n = 10$, the calculated t-statistic is 2.26. At a 5% level of significance with $df = 9$, the critical value is 2.262. The decision for the Null Hypothesis H_0 is:

(A) Reject H_0

(B) Fail to reject H_0

(C) Insufficient data

(D) Accept the Alternative Hypothesis

Q3. A pipe can fill a tank in 12 hours, but due to a leak at the bottom, it takes 15 hours. If the tank is full, the leak can empty it in:

(A) 40 hours

(B) 50 hours

(C) 60 hours

(D) 30 hours

Q4. The demand function for a product is $p = 100 - 5x$. The marginal revenue (MR) when $x = 4$ is:

(A) 60

(B) 80

(C) 40

(D) 20

Q5. In a Normal Distribution, if the mean is 100 and the standard deviation is 15, the area to the right of $x = 130$ (where $Z = 2$ has area 0.4772 from mean) is:

(A) 0.0228

(B) 0.9772

(C) 0.0500

(D) 0.4772

Q6. A man rows 18 km downstream in 3 hours and 12 km upstream in 3 hours. The speed of the stream is:

(A) 1 km/h

(B) 2 km/h

(C) 3 km/h

(D) 1.5 km/h

Q7. The effective rate of interest corresponding to a nominal rate of 6% per annum compounded semi-annually is:

(A) 6.00%

(B) 6.09%

(C) 6.12%

(D) 6.15%

Q8. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is equal to:

(A) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} n & 2n \\ 0 & n \end{bmatrix}$

(D) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Q9. In a 100m race, A covers the distance in 36 seconds and B in 45 seconds. In this race, A beats B by:

(A) 20m

(B) 25m

(C) 9m

(D) 15m

Q10. The value of $\int e^x(x-1)dx$ is:

(A) $e^x(x-2)+C$

(B) $e^x(x-1)+C$

(C) xe^x+C

(D) $e^x(x-2)-e^x+C$

Q11. A perpetuity paying ₹ 500 monthly at an interest rate of 12% p.a. compounded monthly has a present value of:

(A) ₹ 50,000

(B) ₹ 41,666

(C) ₹ 60,000

(D) ₹ 5,000

Q12. If the trace of a 3×3 matrix A is 10, then the trace of $5A$ is:

(A) 10

(B) 15

(C) 50

(D) 25

Q13. $2^{100} \pmod 7$ is equal to:

(A) 1

(B) 2

(C) 4

(D) 6

Q14. The producer surplus (PS) for the supply function $p = 2 + x^2$ at $x = 3$ is:

(A) 9

(B) 18

(C) 27

(D) 33

Q15. The solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is:

(A) $e^y = e^x + C$

(B) $e^{-y} = e^x + C$

(C) $e^y = -e^x + C$

(D) $y = x + C$

Q16. A company installs a new machine for ₹ 2,00,000 which is expected to last for 10 years. If the scrap value is ₹ 20,000, the annual depreciation using the straight-line method is:

(A) ₹ 18,000

(B) ₹ 20,000

(C) ₹ 22,000

(D) ₹ 15,000

Q17. If a Null Hypothesis H_0 is "The coin is fair," the Alternative Hypothesis H_1 for a two-tailed test is:

(A) $P(H) > 0.5$

(B) $P(H) < 0.5$

(C) $P(H) \neq 0.5$

(D) $P(H) = 0.5$

Q18. In a LPP, the constraints $3x + 2y \leq 6$, $x + y \geq 1$, $x, y \geq 0$ define a region that is:

(A) Unbounded

(B) Bounded

(C) Empty

(D) A single point

Q19. A sinking fund is created to accumulate ₹ 10,00,000 in 5 years. If the bank offers 8% p.a. interest, the periodic payment is made at the:

(A) Start of each year

(B) End of each year

(C) Middle of each year

(D) Once at the end of 5 years

Q20. The mean of a Poisson distribution is 0.49. The variance is:

(A) 0.7

(B) 0.49

(C) 0.07

(D) 0.245

Q21. The value of $\int_0^1 \frac{1}{1+x^2}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 1

(D) 0

Q22. A bond has a face value of ₹ 1,000 and a semi-annual coupon of 5%. The annual coupon amount is:

(A) ₹ 50

(B) ₹ 100

(C) ₹ 25

(D) ₹ 10

Q23. If the secular trend line is $Y = 20 + 1.5X$ with origin at 2020, the predicted value for 2025 (where X is in years) is:

(A) 27.5

(B) 21.5

(C) 20

(D) 30.5

Q24. In an 800m race, A gives B a start of 100m and wins by 20 seconds. If A's speed is 8 m/s, B's speed is:

(A) 5 m/s

(B) 5.83 m/s

(C) 6 m/s

(D) 7 m/s

Q25. If $|A| = 0$, then the system of equations $AX = B$ has:

(A) Unique solution

(B) No solution or Infinite solutions

(C) Exactly two solutions

(D) Always no solution

Q26. The probability of getting at least one head in 3 flips of a fair coin is:

(A) 1/8

(B) 3/8

(C) 7/8

(D) 1/2

Q27. The order of the differential equation of all non-vertical lines in a plane is:

(A) 1

(B) 2

(C) 3

(D) 0

Q28. A perpetuity of ₹ 2,000 per year at 5% interest rate has a present value. If the rate increases to 10%, the present value:

(A) Doubles

(B) Is halved

(C) Remains same

(D) Increases by ₹ 1,000

Q29. The point of tangency between the objective function and the feasible region in a LPP is known as the:

(A) Feasible solution

(B) Optimal solution

(C) Basic solution

(D) Degenerate solution

Q30. If $x \equiv 2 \pmod 3$, then $x^2 \equiv$

(A) $1 \pmod 3$

(B) $2 \pmod 3$

(C) $0 \pmod 3$

(D) $4 \pmod 3$

Q31. The present value of ₹ 10,000 due in 2 years at 10% p.a. compounded continuously is:

(A) $10000e^{-0.2}$

(B) $10000e^{0.2}$

(C) $10000(1.1)^{-2}$

(D) $10000e^{-2}$

Q32. Which of the following is a measure of the "goodness of fit" for a regression line?

(A) Standard deviation

(B) Correlation coefficient

(C) Coefficient of determination (R^2)

(D) Mean absolute deviation

Q33. For a Normal distribution, the ratio of Mean Deviation to Standard Deviation is approximately:

(A) 2/3

(B) 4/5

(C) 1/2

(D) 1

Q34. A sinking fund earns 12% p.a. compounded quarterly. The periodic interest rate (i) used in the formula is:

(A) 12%

(B) 4%

(C) 3%

(D) 1%

Q35. The area between $y = x$ and $y = x^2$ in the first quadrant is:

(A) 1/2

(B) 1/3

(C) 1/6

(D) 1/4

Q36. In a t-test, the "tails" of the distribution are:

(A) Thinner than Normal distribution
(B) Thicker than Normal distribution
(C) Identical to Normal distribution
(D) Non-existent

Q37. A man can row 6 km/h in still water. If the speed of the current is 2 km/h, it takes him 3 hours to row to a place and back. How far is the place?

(A) 8 km
(B) 10 km
(C) 12 km
(D) 6 km

Q38. The inverse of the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$
(B) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(C) $\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$
(D) $\begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$

Q39. The degree of the differential equation $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)}$ is:

(A) 1

(B) 2

(C) 1/2

(D) Not defined

Q40. An EMI of ₹ 5,000 consists of ₹ 2,000 interest in the first month. The principal repayment in that month is:

(A) ₹ 5,000

(B) ₹ 3,000

(C) ₹ 2,000

(D) ₹ 7,000

Q41. In hypothesis testing, the probability of a Type II error is denoted by:

(A) α

(B) β

(C) $1 - \alpha$

(D) p-value

Q42. The total cost function is $C(x) = 0.1x^2 + 5x + 200$. The marginal cost at $x = 10$ is:

(A) ₹ 7

(B) ₹ 5

(C) ₹ 20

(D) ₹ 2

Q43. Which time series component is associated with "festivals and holidays"?

(A) Secular Trend

(B) Seasonal Variation

(C) Cyclical Variation

(D) Irregular Variation

Q44. The present value of an annuity due is always ______ than the present value of an ordinary annuity (all else equal).

(A) Lower

(B) Higher

(C) Equal

(D) Half

Q45. If a sample is taken from a population such that every member has an equal chance of being selected, it is called:

(A) Judgmental sampling

(B) Convenience sampling

(C) Random sampling

(D) Quota sampling

Q46. The integration of $\frac{1}{x \ln x}$ is:

(A) $\ln(\ln x) + C$

(B) $(\ln x)^2 + C$

(C) $1/x^2 + C$

(D) $\ln x + C$

Q47. In a binomial distribution, $P(X = 0) = (1/3)^5$. The number of trials n is:

(A) 3

(B) 5

(C) 1

(D) 15

Q48. A bond is said to be "at par" when:

(A) Market price = Face Value

(B) Market price > Face Value

(C) Market price < Face Value

(D) Coupon rate = 0

Q49. Solving $x + y = 5$ and $x - y = 1$ using matrices, the value of x is:

(A) 2

(B) 3

(C) 4

(D) 5

Q50. For a normal distribution, the point of inflection occurs at:

(A) $\mu \pm \sigma$

(B) $\mu \pm 2\sigma$

(C) μ

(D) σ

Detailed Solutions

Q1.

Solution

Concept:

The Reducing Balance Method (also known as the Written Down Value method) calculates depreciation as a percentage of the asset's book value at the beginning of each period. The relationship between the initial value (P), the scrap value (S), the rate of depreciation (r), and the time (n) is given by the formula:

$$S = P(1 - r)^n$$

where r is expressed as a decimal.

Solution:

1. Identify the given values from the problem: Scrap Value (S) = ₹ 72,900 Rate of depreciation (r) = 10% = 0.10 Time (n) = 3 years 2. Substitute the values into the reducing balance formula:

$$72,900 = P(1 - 0.10)^3$$

3. Simplify the expression inside the parentheses:

$$72,900 = P(0.9)^3$$

4. Calculate the cube of 0.9:

$$(0.9)^3 = 0.9 \times 0.9 \times 0.9 = 0.729$$

5. Rewrite the equation to solve for P:

$$72,900 = P \times 0.729$$

$$P = \frac{72,900}{0.729}$$

6. Perform the division:

$$P = 1,00,000$$

Final Answer: The initial value of the machine was ₹ 1,00,000.

Answer: (B)

Q2.

Solution

Concept:
In a t-test for a single mean, we compare the calculated t-statistic (t_{cal}) with the critical t-value (t_{crit}) obtained from t-distribution tables for a specific level of significance (α) and degrees of freedom (df). The decision rule is: - If $|t_{cal}| > t_{crit}$, we reject the Null Hypothesis (H_0). - If $|t_{cal}| \leq t_{crit}$, we fail to reject (accept) the Null Hypothesis (H_0).

Solution:
1. Identify the given statistical values: Calculated t-statistic (t_{cal}) = 2.26 Critical t-value (t_{crit}) = 2.262 Level of significance (α) = 5% (0.05) Degrees of freedom (df) = 9 2. Compare the magnitude of the calculated value with the critical value:

$$2.26 < 2.262$$

3. Since the calculated t-value is less than the critical value, the observed difference is not statistically significant at the 5% level. 4. This means there is not enough evidence to move the result into the "rejection region." 5. Therefore, we do not have sufficient grounds to throw out the Null Hypothesis.

Final Answer: Fail to reject H_0.

Answer: (B)

Q3.

Solution

Concept:

This problem is based on the principle of work and time. The net rate of filling a tank is the difference between the rate of the inlet pipe and the rate of the outlet (leak). If a pipe fills a tank in x hours and the net time with a leak is y hours, the rate of the leak is:

$$\text{Rate of Leak} = \frac{1}{\text{Inlet Rate}} - \frac{1}{\text{Net Rate}}$$

Solution:

1. Determine the rates of work per hour: Inlet pipe rate = $\frac{1}{12}$ of the tank per hour. Net rate (with leak) = $\frac{1}{15}$ of the tank per hour. 2. Let the time taken by the leak to empty the full tank be L hours. The rate of the leak is $\frac{1}{L}$. 3. Set up the equation for the combined work:

$$\frac{1}{12} - \frac{1}{L} = \frac{1}{15}$$

4. Rearrange the equation to isolate $\frac{1}{L}$:

$$\frac{1}{L} = \frac{1}{12} - \frac{1}{15}$$

5. Find a common denominator (which is 60):

$$\frac{1}{L} = \frac{5}{60} - \frac{4}{60}$$

$$\frac{1}{L} = \frac{1}{60}$$

6. Solve for L:

$$L = 60 \text{ hours}$$

Final Answer: The leak can empty the full tank in 60 hours.

Answer: (C)

Q4.

Solution

Concept:

Total Revenue (TR) is the product of price (p) and quantity (x). The Marginal Revenue (MR) is the rate of change of Total Revenue with respect to quantity, found by differentiating the TR function:

$$TR = p \cdot x$$

$$MR = \frac{d(TR)}{dx}$$

Solution:

1. Given the demand function:

$$p = 100 - 5x$$

2. Find the Total Revenue (TR) function:

$$TR = (100 - 5x) \cdot x$$

$$TR = 100x - 5x^2$$

3. Find the Marginal Revenue (MR) function by taking the derivative:

$$MR = \frac{d}{dx}(100x - 5x^2)$$

$$MR = 100 - 10x$$

4. Substitute the given quantity $x = 4$ into the MR equation:

$$MR = 100 - 10(4)$$

$$MR = 100 - 40$$

$$MR = 60$$

Final Answer: The marginal revenue when $x = 4$ is 60.

Answer: (A)

Q5.

Solution

Concept:

In a Normal Distribution, the total area under the curve is 1 (or 100%). The curve is symmetric about the mean μ, meaning 0.5 area lies to the left and 0.5 to the right. To find the area for a specific x, we convert it to a Z-score:

$$Z = \frac{x - \mu}{\sigma}$$

The area to the right of x is: $0.5 - \text{Area}(0 \text{ to } Z)$.

Solution:

1. Identify the given parameters: Mean (μ) = 100 Standard Deviation (σ) = 15 Target Value (x) = 130 2. Calculate the Z-score for $x = 130$:

$$Z = \frac{130 - 100}{15} = \frac{30}{15} = 2$$

3. The problem provides the area from the mean to $Z = 2$: Area from Mean to $Z = 0.4772$ 4. We need the area to the "right" of $x = 130$, which corresponds to the upper tail of the distribution. 5. Since the total area to the right of the mean is 0.5:

$$\text{Area to the right of } Z = 2 = 0.5000 - 0.4772$$

$$\text{Area} = 0.0228$$

Final Answer: The area to the right of $x = 130$ is 0.0228.

Answer: (A)

Q6.

Solution

Concept:

The relative speed of a boat changes based on the direction of the stream. - Downstream Speed (S_d) = Speed in still water (u) + Speed of stream (v) - Upstream Speed (S_u) = Speed in still water (u) - Speed of stream (v) Speed is calculated as Distance/Time. Once S_d and S_u are known, the speed of the stream can be found using:

$$v = \frac{S_d - S_u}{2}$$

Solution:

1. Calculate the Downstream Speed (S_d): Distance = 18 km, Time = 3 hours.

$$S_d = \frac{18}{3} = 6 \text{ km/h}$$

2. Calculate the Upstream Speed (S_u): Distance = 12 km, Time = 3 hours.

$$S_u = \frac{12}{3} = 4 \text{ km/h}$$

3. Use the formula for the speed of the stream (v):

$$v = \frac{S_d - S_u}{2}$$

4. Substitute the values:

$$v = \frac{6-4}{2} = \frac{2}{2} = 1 \text{ km/h}$$

5. Alternatively, let speed in still water be u. Then $u + v = 6$ and $u - v = 4$. Subtracting the second from the first gives $2v = 2$, so $v = 1$.

Final Answer: The speed of the stream is 1 km/h.

Answer: (A)

Q7.

Solution

Concept:

The effective rate of interest (r_e) is the actual interest rate earned or paid in a year due to the result of compounding over a given period. If the nominal rate is r per annum and it is compounded n times a year, the effective rate is:

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

Solution:

1. Identify the given values: Nominal rate $(r) = 6\% = 0.06$ Compounding frequency = semi-annually, so $n = 2$ 2. Calculate the interest rate per compounding period:

$$\frac{r}{n} = \frac{0.06}{2} = 0.03$$

3. Substitute the values into the effective rate formula:

$$r_e = (1 + 0.03)^2 - 1$$

4. Calculate the square of 1.03:

$$(1.03)^2 = 1.03 \times 1.03 = 1.0609$$

5. Subtract 1 to find the rate in decimal form:

$$r_e = 1.0609 - 1 = 0.0609$$

6. Convert the decimal to a percentage:

$$0.0609 \times 100 = 6.09\%$$

Final Answer: The effective rate of interest is 6.09%.

Answer: (B)

Q8.

Solution

Concept:

To find the power of a matrix A^n, we can observe the pattern by calculating A^2, A^3, etc., or use the property of a specific type of matrix. For an upper triangular matrix with 1s on the diagonal, the term in the upper right typically follows an arithmetic progression when raised to a power.

Solution:

1. Given matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. 2. Calculate A^2:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (1 \cdot 1 + 2 \cdot 0) & (1 \cdot 2 + 2 \cdot 1) \\ (0 \cdot 1 + 1 \cdot 0) & (0 \cdot 2 + 1 \cdot 1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

3. Calculate A^3:

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$$

4. Observe the pattern: - For $n = 1$, the $(1, 2)$ element is $2(1) = 2$. - For $n = 2$, the $(1, 2)$ element is $2(2) = 4$. - For $n = 3$, the $(1, 2)$ element is $2(3) = 6$. 5. Generalizing for power n, the matrix will be:

$$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$$

Final Answer: $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$.

Answer: (A)

Q9.

Solution

Concept:

When two runners finish a race at different times, the winner beats the other by a certain distance. This distance is the amount the slower runner still has to cover when the winner crosses the finish line.

$$\text{Distance beaten} = \text{Speed of slower runner} \times \text{Difference in time}$$

Solution:

1. Identify the given values: Race distance = 100m. Time taken by A (T_A) = 36 seconds. Time taken by B (T_B) = 45 seconds. 2. Since $T_A < T_B$, A is the winner. 3. Calculate the difference in time:

$$\Delta T = 45 - 36 = 9 \text{ seconds}$$

4. Calculate the speed of the slower runner (B):

$$\text{Speed}_B = \frac{\text{Distance}}{\text{Time}_B} = \frac{100}{45} = \frac{20}{9} \text{ m/s}$$

5. The distance A beats B by is the distance B travels in the 9 seconds after A finishes:

$$\text{Distance} = \text{Speed}_B \times \Delta T$$

$$\text{Distance} = \frac{20}{9} \times 9 = 20\text{m}$$

Final Answer: A beats B by 20m.

Answer: (A)

Q10.

Solution

Concept:
To integrate the product of an exponential function and a polynomial, we use Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Alternatively, for the specific form $\int e^x[f(x) + f'(x)]dx$, the result is $e^x f(x) + C$.
Solution:
1. Look at the integral: $\int e^x(x-1)dx$. 2. Let $u = x - 1$ and $dv = e^x dx$. 3. Differentiate u: $du = dx$. 4. Integrate dv: $v = e^x$. 5. Apply the Integration by Parts formula:

$$\int e^x(x-1)dx = (x-1)e^x - \int e^x dx$$

6. Perform the remaining integration:

$$(x-1)e^x - e^x + C$$

7. Factor out e^x:

$$e^x(x - 1 - 1) + C$$

$$e^x(x-2) + C$$

Final Answer: The value is $e^x(x-2) + C$.

Answer: (A)

Q11.

Solution

Concept:

A perpetuity is an annuity that continues indefinitely. The present value (PV) of a perpetuity is calculated by dividing the periodic payment (R) by the periodic interest rate (i):

$$PV = \frac{R}{i}$$

If the interest is compounded monthly, the annual rate must be converted to a monthly rate.

Solution:

1. Identify the given values: Periodic payment (R) = ₹ 500 per month. Annual interest rate = 12%.
2. Calculate the monthly interest rate (i):

$$i = \frac{12\%}{12} = 1\% = 0.01 \text{ per month}$$

3. Substitute the values into the perpetuity formula:

$$PV = \frac{500}{0.01}$$

4. Perform the division:

$$PV = 50,000$$

5. This value represents the lump sum amount that, if invested today at 12% per annum compounded monthly, would yield ₹ 500 every month forever.

Final Answer: The present value is ₹ 50,000.

Answer: (A)

Q12.

Solution

Concept:
The trace of a square matrix, denoted by $Tr(A)$, is the sum of the elements on its main diagonal. A key property of the trace is its linearity: for any scalar k and square matrix A:

$$Tr(kA) = k \cdot Tr(A)$$

Solution:
1. Identify the given values: Trace of matrix A ($Tr(A)$) = 10. Scalar multiplier (k) = 5. 2. Apply the property of the trace:

$$Tr(5A) = 5 \times Tr(A)$$

3. Substitute the known trace value:

$$Tr(5A) = 5 \times 10$$

4. Calculate the final result:

$$Tr(5A) = 50$$

5. This holds because if $A = [a_{ij}]$, then $5A = [5a_{ij}]$. The sum of the diagonal elements of $5A$ is $\sum 5a_{ii} = 5 \sum a_{ii}$.
Final Answer: The trace of $5A$ is 50.

Answer: (C)

Q13.

Solution

Concept:

To find the remainder of a large power using modular arithmetic, we use the property $(a^b)^c \equiv (a^b \pmod{n})^c \pmod{n}$ or look for a pattern/cycle in the powers.

Solution:

1. We need to find $2^{100} \pmod 7$. 2. Look at the powers of 2 modulo 7: - $2^1 \equiv 2 \pmod 7$ - $2^2 \equiv 4 \pmod 7$ - $2^3 \equiv 8 \equiv 1 \pmod 7$ 3. Since $2^3 \equiv 1 \pmod 7$, the powers follow a cycle of 3. 4. Express the exponent 100 in terms of the cycle length 3:

$$100 = 3 \times 33 + 1$$

5. Use the property of exponents:

$$2^{100} = (2^3)^{33} \times 2^1$$

6. Substitute the modular values:

$$2^{100} \equiv (1)^{33} \times 2 \pmod 7$$

$$2^{100} \equiv 1 \times 2 = 2 \pmod 7$$

Final Answer: $2^{100} \pmod 7$ is equal to 2.

Answer: (B)

Q14.

Solution

Concept:

Producer Surplus (PS) represents the benefit to sellers who are able to sell a product at a market price higher than the minimum they would be willing to accept. It is calculated as:

$$PS = (p_0 \times x_0) - \int_0^{x_0} S(x)\, dx$$

where $p = S(x)$ is the supply function, p_0 is the price, and x_0 is the quantity.

Solution:

1. Given supply function: $p = 2 + x^2$. 2. Given quantity $(x_0) = 3$. 3. Calculate the market price (p_0) at $x_0 = 3$:

$$p_0 = 2 + (3)^2 = 2 + 9 = 11$$

4. Find the total revenue: $p_0 \times x_0 = 11 \times 3 = 33$. 5. Evaluate the integral of the supply function from 0 to 3:

$$\int_0^3 (2 + x^2)\, dx = [2x + \frac{x^3}{3}]_0^3$$

6. Substitute the limits:

$$(2(3) + \frac{3^3}{3}) - (0) = (6 + 9) = 15$$

7. Calculate PS:

$$PS = 33 - 15 = 18$$

Final Answer: The producer surplus is 18.

Answer: (B)

Q15.

Solution

Concept:

A differential equation of the form $\frac{dy}{dx} = f(x)g(y)$ can be solved using the variable separable method. We rearrange the terms so that all y terms are on one side and x terms are on the other, then integrate both sides.

Solution:

1. Given equation: $\frac{dy}{dx} = e^{x-y}$. 2. Using properties of exponents ($e^{a-b} = e^a/e^b$):

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

3. Separate the variables:

$$e^y\, dy = e^x\, dx$$

4. Integrate both sides:

$$\int e^y\, dy = \int e^x\, dx$$

5. The integral of e^u is $e^u + C$:

$$e^y = e^x + C$$

6. This represents the general solution for the given differential equation.

Final Answer: The solution is $e^y = e^x + C$.

Answer: (A)

Q16.

Solution

Concept:

The straight-line method of depreciation assumes that the asset loses an equal amount of value every year throughout its useful life. The formula for annual depreciation is:

$$\text{Annual Depreciation} = \frac{\text{Cost Price} - \text{Scrap Value}}{\text{Useful Life (in years)}}$$

Solution:

1. Identify the given values: Cost Price of the machine = ₹ 2,00,000 Scrap Value (at the end of life) = ₹ 20,000 Estimated Useful Life = 10 years 2. Calculate the total depreciable amount:

$$\text{Total Depreciable Amount} = 2,00,000 - 20,000 = 1,80,000$$

3. Apply the straight-line method formula:

$$\text{Annual Depreciation} = \frac{1,80,000}{10}$$

4. Perform the division:

$$\text{Annual Depreciation} = 18,000$$

5. This means the book value of the machine decreases by ₹ 18,000 every single year for 10 years until it reaches its scrap value.

Final Answer: The annual depreciation is ₹ 18,000.

Answer: (A)

Q17.

Solution

Concept:
In statistical hypothesis testing, the Alternative Hypothesis (H_1) defines the direction of the test. - A **one-tailed test** looks for a change in one specific direction (greater than or less than). - A **two-tailed test** looks for any significant difference from the hypothesized value, regardless of direction. This is represented by the "not equal to" symbol ($\neq$).

Solution:
1. The Null Hypothesis is $H_0 : P(H) = 0.5$ (The coin is fair). 2. For a two-tailed test, we are interested in whether the probability of a head is significantly different from 0.5, meaning it could be either biased toward heads or biased toward tails. 3. Therefore, the Alternative Hypothesis must cover both possibilities: $P(H) < 0.5$ and $P(H) > 0.5$. 4. In mathematical notation, this combined condition is written as:

$$H_1 : P(H) \neq 0.5$$

5. Options (A) and (B) would represent one-tailed tests.

Final Answer: The Alternative Hypothesis is $P(H) \neq 0.5$.

Answer: (C)

Q18.

Solution

Concept:
A region in Linear Programming (LPP) is "bounded" if it can be enclosed within a circle of finite radius; otherwise, it is "unbounded." We determine this by plotting the constraints on a graph and finding the intersection area in the first quadrant (due to $x, y \geq 0$).

Solution:
1. Plot the first constraint $3x + 2y \leq 6$: - Intercepts are $(2, 0)$ and $(0, 3)$. Since it is $\leq$, the region is toward the origin. 2. Plot the second constraint $x + y \geq 1$: - Intercepts are $(1, 0)$ and $(0, 1)$. Since it is $\geq$, the region is away from the origin. 3. Apply non-negativity: $x \geq 0, y \geq 0$ (First Quadrant). 4. The feasible region is the area trapped between the line $x + y = 1$ and the line $3x + 2y = 6$ within the first quadrant. 5. This region is a closed polygon (specifically a quadrilateral with vertices $(1, 0), (2, 0), (0, 3), (0, 1)$). 6. Because the region is entirely enclosed and does not extend to infinity, it is classified as bounded.

Final Answer: The region is Bounded.

Answer: (B)

Q19.

Solution

Concept:
A sinking fund is a financial strategy where a sum of money is set aside periodically to accumulate a specific amount for a future liability (like debt repayment or asset replacement). By convention and in standard mathematical formulas for sinking funds, payments are treated as an **Ordinary Annuity**.

Solution:
1. Analyze the timing of payments in financial products: - If payments are at the start, it is an "Annuity Due." - If payments are at the end, it is an "Ordinary Annuity." 2. Sinking funds are typically structured such that the interest is earned on the balance throughout the period, and the contribution is added at the conclusion of that period. 3. This end-of-period payment structure is the defining characteristic of an ordinary annuity. 4. Therefore, for the mathematical calculation of periodic payments in a sinking fund, the payments are assumed to be made at the end of each year.
Final Answer: The periodic payment is made at the end of each year.

Answer: (B)

Q20.

Solution

Concept:
The Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval. A unique mathematical property of the Poisson distribution is that its mean (λ) and its variance (σ^2) are always equal.

$$\text{Mean} = \text{Variance} = \lambda$$

Solution:
1. Identify the given information: The distribution is Poisson. The mean of the distribution (λ) = 0.49. 2. Recall the property of the Poisson distribution:

$$\text{Variance} = \lambda$$

3. Substitute the value of the mean:

$$\text{Variance} = 0.49$$

4. Note: While the Standard Deviation would be $\sqrt{0.49} = 0.7$, the question specifically asks for the variance.
Final Answer: The variance is 0.49.

Answer: (B)

Q21.

Solution

Concept:

The integral $\int \frac{1}{1+x^2} dx$ is a standard integral in calculus that results in the inverse trigonometric function $\arctan(x)$ (also written as $\tan^{-1} x$). To evaluate the definite integral from a to b, we find the antiderivative and apply the Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Solution:

1. Identify the standard integral form:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

2. Apply the limits of integration from 0 to 1:

$$\left[\tan^{-1}(x)\right]_0^1$$

3. Evaluate at the upper limit (1):

$$\tan^{-1}(1)$$

Since $\tan(\pi/4) = 1$, we have $\tan^{-1}(1) = \pi/4$. 4. Evaluate at the lower limit (0):

$$\tan^{-1}(0)$$

Since $\tan(0) = 0$, we have $\tan^{-1}(0) = 0$. 5. Subtract the results:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: The value of the integral is $\pi/4$.

Answer: (B)

Q22.

Solution

Concept:

A bond's coupon is the interest payment made to the bondholder. The coupon rate is expressed as an annual percentage of the Face Value. If payments are semi-annual, the total annual payment remains the same, but it is split into two equal parts.

$$\text{Annual Coupon Amount} = \text{Face Value} \times \text{Annual Coupon Rate}$$

Solution:

1. Identify the given values: Face Value = ₹ 1,000 Semi-annual coupon rate = 5% (Note: Usually, if a rate is stated as "semi-annual coupon of 5%," it refers to the rate per period). 2. If the payment per period (6 months) is 5% of the face value:

$$\text{Semi-annual Payment} = 1,000 \times 0.05 = ₹\ 50$$

3. Since there are two such periods in a year:

$$\text{Annual Coupon Amount} = 50 \times 2 = ₹\ 100$$

4. Alternatively, if 10% was the annual rate, 5% would be the semi-annual rate. In either interpretation of standard bond terminology for this level, the annual cash flow is the total of the two payments.

Final Answer: The annual coupon amount is ₹ 100.

Answer: (B)

Q23.

Solution

Concept:

A secular trend line represents the long-term direction of time series data. In the linear equation $Y = a + bX$, Y is the predicted value, a is the intercept at the origin year, and b is the slope (change per unit of X). X represents the time deviation from the origin year.

Solution:

1. Given trend equation: $Y = 20 + 1.5X$. 2. Origin year: 2020 (This is the point where $X = 0$). 3. Target year: 2025. 4. Calculate the value of X for the year 2025:

$$X = 2025 - 2020 = 5 \text{ years}$$

5. Substitute $X = 5$ into the trend equation:

$$Y = 20 + 1.5(5)$$

6. Perform the multiplication:

$$1.5 \times 5 = 7.5$$

7. Add to the intercept:

$$Y = 20 + 7.5 = 27.5$$

Final Answer: The predicted value for 2025 is 27.5.

Answer: (A)

Q24.

Solution

Concept:

In a race where A gives B a start of x meters, A must run the full distance D, while B runs $(D - x)$. If A wins by t seconds, it means A finishes his distance, and then B takes an additional t seconds to finish his respective distance.

$$\text{Time taken by A} + \text{Winning Margin} = \text{Time taken by B}$$

Solution:

1. Calculate the time taken by A (T_A): Distance for A = 800m. Speed of A = 8 m/s.

$$T_A = \frac{800}{8} = 100 \text{ seconds}$$

2. Determine the time taken by B (T_B): A wins by 20 seconds, meaning B takes 20 seconds more than A to cover his distance.

$$T_B = T_A + 20 = 100 + 20 = 120 \text{ seconds}$$

3. Determine the distance covered by B (D_B): A gives B a start of 100m.

$$D_B = 800 - 100 = 700\text{m}$$

4. Calculate the speed of B (S_B):

$$S_B = \frac{D_B}{T_B} = \frac{700}{120}$$

5. Simplify the fraction:

$$S_B = \frac{70}{12} = \frac{35}{6} \approx 5.83 \text{ m/s}$$

Final Answer: The speed of B is 5.83 m/s.

Answer: (B)

Q25.

Solution

Concept:
Cramer's Rule and Matrix Inversion states that a system of linear equations $AX = B$ has a unique solution if and only if the matrix A is non-singular ($|A| \neq 0$). If $|A| = 0$, the matrix is singular, and the system does not have a unique solution.

Solution:
1. When the determinant of the coefficient matrix is zero ($|A| = 0$), the inverse A^{-1} does not exist. 2. In this scenario, we must look at the relationship between the matrices to determine consistency. 3. If $(adjA)B = O$ (Zero matrix), the system has **Infinitely many solutions**. 4. If $(adjA)B \neq O$, the system has **No solution**. 5. Therefore, the condition $|A| = 0$ implies that the system definitely does not have a single unique solution; it is either inconsistent or dependent.

Final Answer: The system has No solution or Infinite solutions.

Answer: (B)

Q26.

Solution

Concept:
When flipping a fair coin multiple times, the total number of outcomes is 2^n, where n is the number of flips. The probability of an event "at least one" is most easily calculated using the complement rule:

$$P(\text{At least one}) = 1 - P(\text{None})$$

In the context of coins, "none" means getting tails on every single flip.

Solution:
1. Identify the number of trials: $n = 3$. 2. Calculate the total number of possible outcomes:

$$2^3 = 2 \times 2 \times 2 = 8$$

(The sample space is: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT). 3. Identify the outcome for "no heads": This only happens in one case: (T, T, T). 4. Calculate the probability of getting no heads:

$$P(\text{No heads}) = \frac{1}{8}$$

5. Apply the complement rule to find the probability of at least one head:

$$P(\text{At least one head}) = 1 - P(\text{No heads})$$

$$P = 1 - \frac{1}{8} = \frac{7}{8}$$

Final Answer: The probability of getting at least one head is 7/8.

Answer: (C)

Q27.

Solution

Concept:
The order of a differential equation is the order of the highest derivative present in the equation. When forming a differential equation from a family of curves, the order is equal to the number of independent arbitrary constants present in the general equation of that family.

Solution:
1. Identify the general equation for the family of non-vertical lines in a plane:

$$y = mx + c$$

2. Identify the arbitrary constants in this equation: - m (the slope of the line) - c (the y-intercept) 3. There are exactly 2 independent arbitrary constants (m and c). 4. To eliminate two constants, we must differentiate the equation twice: - First derivative: $y' = m$ - Second derivative: $y'' = 0$ 5. The resulting differential equation $y'' = 0$ is of the second order. 6. Therefore, the order of the differential equation is 2.

Final Answer: The order of the differential equation is 2.

Answer: (B)

Q28.

Solution

Concept:
A perpetuity is a constant stream of identical cash flows with no end date. The present value (PV) of a perpetuity is inversely proportional to the interest rate (i):

$$PV = \frac{R}{i}$$

where R is the annual payment. If the interest rate increases, the denominator increases, causing the present value to decrease.

Solution:
1. Calculate the initial Present Value (PV_1) with $i = 5\% = 0.05$:

$$PV_1 = \frac{2,000}{0.05} = 40,000$$

2. Calculate the new Present Value (PV_2) with $i = 10\% = 0.10$:

$$PV_2 = \frac{2,000}{0.10} = 20,000$$

3. Compare the two values:

$$\frac{PV_2}{PV_1} = \frac{20,000}{40,000} = \frac{1}{2}$$

4. Since the new present value is half of the original value, the present value is halved. 5. Conceptually, because the interest rate doubled, you need only half as much principal today to generate the same annual payment.

Final Answer: The present value is halved.

Answer: (B)

Q29.

Solution

Concept:

In Linear Programming (LPP), the objective function is represented by a series of parallel lines (iso-profit or iso-cost lines). As we move these lines across the feasible region to maximize or minimize the objective, the "last" point(s) of contact within the feasible region identify the best possible result.

Solution:

1. The Feasible Region is the set of all possible points that satisfy the constraints. 2. The Objective Function $Z = ax + by$ is what we are trying to optimize. 3. According to the Corner Point Theorem, the optimal value (maximum or minimum) of the objective function must occur at one of the vertices (corners) of the feasible region. 4. When the objective function line is moved as far as possible while still touching the feasible region, it "taps" or is "tangent" to a corner point (or an edge). 5. This specific point of intersection/tangency that provides the best value for Z is defined as the Optimal Solution.

Final Answer: The point of tangency is known as the Optimal solution.

Answer: (B)

Q30.

Solution

Concept:

Modular arithmetic deals with remainders. The property of modular exponentiation states that if $a \equiv b \pmod{m}$, then $a^n \equiv b^n \pmod{m}$. We first find the remainder of the base and then raise that remainder to the required power, simplifying again if necessary.

Solution:

1. Identify the given congruence:

$$x \equiv 2 \pmod{3}$$

2. We need to find the value of $x^2 \pmod{3}$. 3. Square both sides of the congruence:

$$x^2 \equiv 2^2 \pmod{3}$$

4. Calculate the square:

$$x^2 \equiv 4 \pmod{3}$$

5. Simplify 4 within modulus 3 (divide 4 by 3 and find the remainder):

$$4 = 3(1) + 1$$

$$4 \equiv 1 \pmod{3}$$

6. Therefore, $x^2 \equiv 1 \pmod{3}$.

Final Answer: The value is $1 \pmod{3}$.

Answer: (A)

Q31.

Solution

Concept:

Continuous compounding occurs when the number of compounding periods per year approaches infinity. The present value (PV) of a future sum (FV) under continuous compounding is given by the formula:

$$PV = FV \cdot e^{-rt}$$

where r is the annual interest rate as a decimal, and t is the time in years.

Solution:

1. Identify the given variables: Future Value (FV) = ₹ 10,000 Time (t) = 2 years Annual interest rate (r) = 10% = 0.10 2. Substitute the values into the continuous compounding formula:

$$PV = 10,000 \cdot e^{-(0.10)(2)}$$

3. Simplify the exponent:

$$PV = 10,000 \cdot e^{-0.2}$$

4. This expression represents the amount that must be invested today to grow to ₹ 10,000 in 2 years at a 10% rate compounded every instant.

Final Answer: The present value is $10,000e^{-0.2}$.

Answer: (A)

Q32.

Solution

Concept:

The coefficient of determination, denoted as R^2, is a statistical measure that represents the proportion of the variance for a dependent variable that is explained by an independent variable in a regression model. It provides a numerical value between 0 and 1 indicating the "goodness of fit" of the model.

Solution:

1. Analyze the statistical tools: - **Standard deviation:** Measures the dispersion of data points around their mean. - **Correlation coefficient (r):** Measures the strength and direction of a linear relationship. - **Coefficient of determination (R^2):** Square of the correlation coefficient. It explains how much of the change in the output is due to the change in the input. 2. A value of $R^2 = 1.0$ indicates that the model explains all the variability of the response data around its mean (perfect fit). 3. A value of 0 indicates the model explains none of the variability. 4. Therefore, R^2 is the specific metric used to evaluate how well a regression line fits the observed data.

Final Answer: The coefficient of determination (R^2) is the measure.

Answer: (C)

Q33.

Solution

Concept:

In a perfectly Normal Distribution, there are fixed mathematical ratios between different measures of dispersion. The two most common measures are the Mean Deviation (MD) and the Standard Deviation (SD or σ). The approximate relationship is:

$$MD \approx \frac{4}{5}\sigma$$

Solution:

1. For a normal curve, the Mean Deviation about the mean is calculated using the integral of the probability density function. 2. The exact value is $MD = \sqrt{\frac{2}{\pi}}\sigma$. 3. Approximate the value of $\sqrt{\frac{2}{\pi}}$:

$$\sqrt{\frac{2}{3.14159}} \approx \sqrt{0.6366} \approx 0.7979$$

4. Convert this decimal to a fraction:

$$0.7979 \approx 0.80 = \frac{4}{5}$$

5. Therefore, the ratio $MD/SD \approx 4/5$. 6. Note: Another common ratio is $QD \approx \frac{2}{3}\sigma$ for Quartile Deviation.

Final Answer: The ratio is approximately 4/5.

Answer: (B)

Q34.

Solution

Concept:

When interest is compounded more than once a year, the nominal annual rate must be adjusted to a periodic rate (i) and the number of years must be adjusted to the total number of periods. The periodic rate is:

$$i = \frac{\text{Nominal Rate}}{\text{Compounding Periods per Year}}$$

Solution:

1. Identify the nominal annual rate: 12% per annum. 2. Identify the compounding frequency: Quarterly. 3. Determine the number of compounding periods in one year (m): Since there are 4 quarters in a year, $m = 4$. 4. Calculate the periodic interest rate (i):

$$i = \frac{12\%}{4}$$

5. Simplify the calculation:

$$i = 3\% = 0.03$$

6. This 3% is the rate applied to the balance at the end of every three-month period.

Final Answer: The periodic interest rate is 3%.

Answer: (C)

Q35.

Solution

Concept:

The area between two curves $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is given by:

$$\text{Area} = \int_a^b |f(x) - g(x)|\, dx$$

To find the limits, we set $f(x) = g(x)$ and solve for x.

Solution:

1. Find the intersection points of $y = x$ and $y = x^2$:

$$x = x^2 \implies x^2 - x = 0 \implies x(x-1) = 0$$

The points are $x = 0$ and $x = 1$. 2. Determine which curve is on top in the interval $(0, 1)$: Pick $x = 0.5$. Here $y = 0.5$ for the line and $y = 0.25$ for the parabola. So, $x > x^2$ in this region. 3. Set up the integral:

$$\text{Area} = \int_0^1 (x - x^2)\, dx$$

4. Integrate:

$$\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

5. Evaluate at the limits:

$$\left(\frac{1}{2} - \frac{1}{3}\right) - (0) = \frac{3-2}{6} = \frac{1}{6}$$

Final Answer: The area is 1/6.

Answer: (C)

Q36.

Solution

Concept:

The t-distribution (Student's t-distribution) is similar to the standard normal distribution (Z-distribution) in that it is symmetric and bell-shaped. However, the t-distribution is flatter and has "heavier" or "thicker" tails. This characteristic accounts for the additional uncertainty when the population standard deviation is unknown and the sample size is small.

Solution:

1. Compare the probability density functions of the Z and t distributions. 2. The standard normal distribution has less area in its tails compared to the t-distribution for any finite degrees of freedom. 3. As the degrees of freedom increase (as sample size n grows), the t-distribution approaches the normal distribution. 4. For small samples, the spread of the t-distribution is wider, meaning more extreme values are more likely than in a normal distribution. 5. In statistical terminology, this increased likelihood of extreme values is described as having "thicker" or "heavier" tails.

Final Answer: In a t-test, the tails of the distribution are thicker than the Normal distribution.

Answer: (B)

Q37.

Solution

Concept:

This is a problem of "Average Speed" or "Total Time" in boat and stream physics. - Downstream Speed $(S_d) = u + v$ - Upstream Speed $(S_u) = u - v$ If the distance to the place is D, the total time taken for the round trip is:

$$T_{total} = \frac{D}{u+v} + \frac{D}{u-v}$$

Solution:

1. Identify the given values: Speed in still water $(u) = 6$ km/h Speed of current $(v) = 2$ km/h Total time $(T) = 3$ hours 2. Calculate the speeds in both directions: Downstream speed $(S_d) = 6 + 2 = 8$ km/h Upstream speed $(S_u) = 6 - 2 = 4$ km/h 3. Set up the time equation:

$$\frac{D}{8} + \frac{D}{4} = 3$$

4. Find a common denominator (8):

$$\frac{D}{8} + \frac{2D}{8} = 3$$

$$\frac{3D}{8} = 3$$

5. Solve for D:

$$3D = 24 \implies D = 8 \text{ km}$$

Final Answer: The place is 8 km away.

Answer: (A)

Q38.

Solution

Concept:

A diagonal matrix is a square matrix where all entries outside the main diagonal are zero. For a 2×2 diagonal matrix $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, the inverse A^{-1} exists if $a, b \neq 0$ and is given by:

$$A^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix}$$

Solution:

1. Given matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. 2. Calculate the determinant:

$$|A| = (3 \times 2) - (0 \times 0) = 6$$

3. Find the Adjoint of A: For a 2×2 matrix, swap diagonal elements and change signs of off-diagonal elements.

$$adj(A) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

4. Apply the formula $A^{-1} = \frac{1}{|A|} adj(A)$:

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

5. Simplify the elements:

$$A^{-1} = \begin{bmatrix} 2/6 & 0 \\ 0 & 3/6 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$$

Final Answer: The inverse is $\begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$.

Answer: (A)

Q39.

Solution

Concept:
The degree of a differential equation is the power of the highest-order derivative, provided the equation is expressed as a polynomial in its derivatives (i.e., free from radicals/fractions involving the derivatives).

Solution:
1. Look at the given equation:

$$\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)}$$

2. Identify the highest-order derivative: $\frac{d^2y}{dx^2}$ (order 2). 3. To find the degree, we must remove the square root (radical). Square both sides:

$$\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)$$

4. Now the equation is in polynomial form regarding its derivatives. 5. The highest-order derivative is $\frac{d^2y}{dx^2}$ and its power in this polynomial form is 1. 6. Therefore, the degree of the differential equation is 1.

Final Answer: The degree is 1.

Answer: (A)

Q40.

Solution

Concept:
Equated Monthly Installment (EMI) consists of two components: the interest on the outstanding loan amount and the repayment of the principal. In any given period:

$$\text{EMI} = \text{Principal Repayment} + \text{Interest Component}$$

Solution:
1. Identify the given values for the first month: Total EMI amount = ₹ 5,000 Interest component = ₹ 2,000 2. Use the EMI component formula to find the principal repayment:

$$\text{Principal Repayment} = \text{EMI} - \text{Interest Component}$$

3. Substitute the values:

$$\text{Principal Repayment} = 5,000 - 2,000$$

4. Calculate the result:

$$\text{Principal Repayment} = 3,000$$

5. Note: As the loan progresses, the interest component decreases and the principal repayment component increases within the same fixed EMI.
Final Answer: The principal repayment is ₹ 3,000.

Answer: (B)

Q41.

Solution

Concept:
In statistical hypothesis testing, we make a decision about a population based on sample evidence. Two types of errors can occur: - **Type I Error (α):** Rejecting the Null Hypothesis when it is actually true. - **Type II Error (β):** Failing to reject the Null Hypothesis when it is actually false.
Solution:
1. Statistical tests aim to minimize both errors, but there is usually a trade-off between them. 2. The probability of committing a Type I error is denoted by the Greek letter alpha (α). 3. The probability of committing a Type II error is denoted by the Greek letter beta (β). 4. The "Power of the Test" is defined as $1 - \beta$, which is the probability of correctly rejecting a false Null Hypothesis. 5. In the context of the question, we are specifically looking for the notation used for the Type II error.
Final Answer: The probability of a Type II error is denoted by β.

Answer: (B)

Q42.

Solution

Concept:

Marginal Cost (MC) is defined as the additional cost incurred by producing one more unit of a good. Mathematically, it is the first derivative of the Total Cost (C) function with respect to the quantity (x):

$$MC = \frac{dC}{dx}$$

Solution:

1. Identify the given Total Cost function:

$$C(x) = 0.1x^2 + 5x + 200$$

2. Differentiate the function with respect to x: - The derivative of $0.1x^2$ is $0.1 \times 2x = 0.2x$. - The derivative of $5x$ is 5. - The derivative of the constant 200 is 0. 3. Write the Marginal Cost function:

$$MC(x) = 0.2x + 5$$

4. Substitute the given production level $x = 10$:

$$MC(10) = 0.2(10) + 5$$

$$MC(10) = 2 + 5 = 7$$

Final Answer: The marginal cost is ₹ 7.

Answer: (A)

Q43.

Solution

Concept:

Time series data can be decomposed into four main components: - **Secular Trend:** Long-term direction (years/decades). - **Seasonal Variation:** Rhythmic patterns that repeat within a year (seasons, months, holidays). - **Cyclical Variation:** Long-term oscillations (business cycles, 2-10 years). - **Irregular Variation:** Unpredictable, random events.

Solution:

1. Festivals and holidays occur at fixed intervals every year (e.g., Diwali, Christmas, Eid). 2. Business activity, sales, and travel often spike during these specific times annually. 3. Since these fluctuations repeat regularly within a twelve-month period due to social or climatic factors, they are classified under seasonal variations. 4. Irregular variations would be something like a sudden earthquake or strike, which is not what a holiday represents.

Final Answer: This is associated with Seasonal Variation.

Answer: (B)

Q44.

Solution

Concept:
The timing of payments significantly affects the present value of money. - **Ordinary Annuity:** Payments are made at the **end** of each period. - **Annuity Due:** Payments are made at the **beginning** of each period.

Solution:
1. In an Annuity Due, every payment is made one period earlier than in an Ordinary Annuity. 2. Because payments are received sooner, they have less time to be discounted. 3. Mathematically:

$$PV_{\text{due}} = PV_{\text{ordinary}} \times (1 + i)$$

4. Since $(1 + i)$ is greater than 1 for any positive interest rate, the present value of an annuity due will always be higher. 5. In simpler terms, getting money today is worth more than getting that same money a month from now.

Final Answer: The present value of an annuity due is always higher.

Answer: (B)

Q45.

Solution

Concept:
Sampling methods are divided into probability (random) and non-probability (non-random) sampling. In probability sampling, every element in the population has a known, non-zero probability of being selected.

Solution:
1. **Random Sampling:** This is the gold standard where every individual has an equal and independent chance of selection. This eliminates selection bias. 2. **Judgmental/Convenience/Quota Sampling:** These are non-probability methods where the researcher's bias, ease of access, or specific proportions dictate the selection. 3. The specific condition "every member has an equal chance" is the fundamental definition of Simple Random Sampling. 4. This ensures that the sample is representative of the population for statistical inference.

Final Answer: This is called Random sampling.

Answer: (C)

Q46.

Solution

Concept:

To integrate a function of the form $\int \frac{f'(x)}{f(x)}\,dx$, we use the method of substitution. The general result for such an integral is $\ln|f(x)| + C$. In this case, we look for a relationship between the numerator and the denominator.

Solution:

1. Identify the integral: $\int \frac{1}{x\ln x}\,dx$. 2. Let $u = \ln x$. 3. Differentiate u with respect to x:

$$\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}\,dx$$

4. Substitute u and du into the integral:

$$\int \frac{1}{\ln x} \cdot \left(\frac{1}{x}\,dx\right) = \int \frac{1}{u}\,du$$

5. Integrate with respect to u:

$$\int \frac{1}{u}\,du = \ln|u| + C$$

6. Substitute back $u = \ln x$:

$$\ln|\ln x| + C$$

Final Answer: The result is $\ln(\ln x) + C$.

Answer: (A)

Q47.

Solution

Concept:
The Binomial Distribution $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$ describes the probability of k successes in n independent trials. The probability of zero successes ($X = 0$) simplifies significantly because $\binom{n}{0} = 1$ and $p^0 = 1$.

$$P(X = 0) = (1 - p)^n$$

Solution:
1. Given the probability: $P(X = 0) = (1/3)^5$. 2. Match this with the theoretical formula:

$$(1 - p)^n = \left(\frac{1}{3}\right)^5$$

3. In this specific form, the base represents the probability of failure ($q = 1 - p$) and the exponent represents the number of trials (n). 4. Comparing the exponents directly:

$$n = 5$$

5. This implies the experiment was conducted 5 times, and for each trial, the probability of failure was $1/3$ (meaning the probability of success p was $2/3$).
Final Answer: The number of trials n is 5.

Answer: (B)

Q48.

Solution

Concept:
The "Par Value" or "Face Value" of a bond is the amount of money the bond issuer promises to pay the bondholder at the time of maturity. A bond can trade in the market at different prices depending on interest rate fluctuations.
Solution:
1. **Trading at a Premium:** When the market price of the bond is higher than its Face Value. 2. **Trading at a Discount:** When the market price of the bond is lower than its Face Value. 3. **Trading at Par:** When the market price is exactly equal to the Face Value. 4. This typically happens when the bond's coupon rate is equal to the current market interest rate (yield). 5. Therefore, a bond is "at par" if it is being sold for the same amount as its stated face value.
Final Answer: A bond is at par when Market price = Face Value.

Answer: (A)

Q49.

Solution

Concept:
A system of linear equations can be represented as $AX = B$. The solution can be found using the inverse matrix $X = A^{-1}B$ or by using Cramer's rule. For a 2×2 system, simple elimination is often faster, but the matrix approach is conceptually required here.

Solution:
1. Write the system as a matrix equation:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

2. Find the determinant of the coefficient matrix A:

$$|A| = (1 \times -1) - (1 \times 1) = -1 - 1 = -2$$

3. To find x using Cramer's rule, replace the first column with the constants:

$$|A_x| = \begin{vmatrix} 5 & 1 \\ 1 & -1 \end{vmatrix} = (5 \times -1) - (1 \times 1) = -5 - 1 = -6$$

4. Calculate x:

$$x = \frac{|A_x|}{|A|} = \frac{-6}{-2} = 3$$

5. Verification: $3 + 2 = 5$ and $3 - 2 = 1$. Both equations hold true when $y = 2$.

Final Answer: The value of x is 3.

Answer: (B)

Q50.

Solution

Concept:
In calculus, a point of inflection is a point on a curve at which the concavity changes (from concave up to concave down, or vice-versa). For a Normal Distribution curve, these points occur exactly at a distance of one standard deviation from the mean.

Solution:
1. The probability density function of a normal distribution is a bell-shaped curve. 2. The center of the curve is the mean (μ). 3. The "steepness" of the curve changes as you move away from the mean. 4. Mathematically, the second derivative of the normal distribution function is zero at the points $x = \mu + \sigma$ and $x = \mu - \sigma$. 5. At these specific points, the curve stops bending away from the mean and starts flattening out toward the horizontal axis. 6. These are collectively referred to as $\mu \pm \sigma$.

Final Answer: The points of inflection occur at $\mu \pm \sigma$.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	C	4	A	5	A
6	A	7	B	8	A	9	A	10	A
11	A	12	C	13	B	14	B	15	A
16	A	17	C	18	B	19	B	20	B
21	B	22	B	23	A	24	B	25	B
26	C	27	B	28	B	29	B	30	A
31	A	32	C	33	B	34	C	35	C
36	B	37	A	38	A	39	A	40	B
41	B	42	A	43	B	44	B	45	C
46	A	47	B	48	A	49	B	50	A