CUET UG Applied Mathematics Sample Paper - 9

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. A motorboat whose speed is 15 km/h in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream is:

(A) 4 km/h

(B) 5 km/h

(C) 6 km/h

(D) 10 km/h

Q2. If A is a square matrix of order 3 such that $|adj A| = 64$, then the value of $|A|$ is:

(A) ± 4

(B) ± 8

(C) 64

(D) 16

Q3. The value of $\int_1^e \frac{\ln x}{x} dx$ is:

(A) $1/2$

(B) 1

(C) e

(D) $1/e$

Q4. For a binomial distribution, if the mean is 4 and the variance is 3, then the number of trials n is:

(A) 10

(B) 12

(C) 16

(D) 20

Q5. An annuity where payments are made at the beginning of each period is called:

(A) Ordinary Annuity

(B) Annuity Due

(C) Perpetuity

(D) Deferred Annuity

Q6. The solution to the congruence $5x \equiv 2 \pmod 7$ is:

(A) $x \equiv 3 \pmod 7$

(B) $x \equiv 4 \pmod 7$

(C) $x \equiv 6 \pmod 7$

(D) $x \equiv 1 \pmod 7$

Q7. The marginal cost function is given by $MC = 4 + 0.5x$. If the fixed cost is ₹ 100, the total cost function $C(x)$ is:

(A) $4x + 0.25x^2$

(B) $4x + 0.5x^2 + 100$

(C) $4x + 0.25x^2 + 100$

(D) $100x + 4$

Q8. In a 500m race, A can beat B by 50m and in a 400m race, B can beat C by 40m. In a 500m race, A will beat C by:

(A) 90m

(B) 95m

(C) 100m

(D) 110m

Q9. Which of the following is NOT a property of a Normal Curve?

(A) It is bell-shaped

(B) Mean = Median = Mode

(C) Total area is 1

(D) It is skewed to the right

Q10. A sinking fund is created to pay off a debt of ₹ 5,00,000 after 10 years. If the interest is 10% p.a. compounded annually, and given $(1.1)^{10} = 2.5937$, the annual installment is approximately:

(A) ₹ 31,372

(B) ₹ 50,000

(C) ₹ 25,400

(D) ₹ 42,100

Q11. If x is the number of units produced, the average cost AC is minimum when:

(A) $MC = AC$

(B) $MC = 0$

(C) $AC = 0$

(D) $TR = TC$

Q12. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is:

(A) 3

(B) 2

(C) 1

(D) Not defined

Q13. The present value of a perpetuity of ₹ 1,200 payable at the end of every year at 8% p.a. is:

(A) ₹ 12,000

(B) ₹ 15,000

(C) ₹ 18,000

(D) ₹ 20,000

Q14. For a Poisson distribution, if $P(X = 0) = P(X = 1)$, then the mean of the distribution is:

(A) 0

(B) 1

(C) 2

(D) 0.5

Q15. In a LPP, if the objective function is $Z = 3x + 4y$ and the corner points are $(0, 0)$, $(4, 0)$, $(0, 5)$ and $(2, 4)$, the maximum value of Z is:

(A) 12

(B) 20

(C) 22

(D) 18

Q16. The time series component that accounts for a "fire in a factory" is:

(A) Secular Trend
(B) Seasonal Variation
(C) Irregular Variation
(D) Cyclical Variation

Q17. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then $A + A^T$ is:

(A) Symmetric
(B) Skew-symmetric
(C) Identity
(D) Diagonal

Q18. A pipe can empty a tank in 15 hours and another pipe can empty it in 10 hours. If both pipes are opened together, the full tank will be emptied in:

(A) 5 hours
(B) 6 hours
(C) 7.5 hours
(D) 12 hours

Q19. The value of $\int xe^x dx$ is:

(A) $e^x(x + 1) + C$
(B) $e^x(x - 1) + C$
(C) $x^2e^x + C$
(D) $e^x/x + C$

Q20. A bond with a face value of ₹ 1,000 is purchased for ₹ 950. The bond is said to be purchased at:

(A) Par

(B) Premium

(C) Discount

(D) Zero-coupon

Q21. In a sample of 100 people, 40 are smokers. The sample proportion is:

(A) 0.4

(B) 40

(C) 0.6

(D) 4

Q22. A man can row 10 km/h in still water. The speed of the stream is 2 km/h. The time taken to row 24 km upstream is:

(A) 2 hours

(B) 3 hours

(C) 2.4 hours

(D) 4 hours

Q23. If the population standard deviation is unknown and the sample size is 20, which test is used for the mean?

(A) Z-test

(B) t-test

(C) Chi-square test

(D) F-test

Q24. The nominal rate of interest is 12% p.a. What is the effective rate if interest is compounded monthly?

(A) 12%

(B) 12.68%

(C) 12.55%

(D) 13%

Q25. The solution of $\frac{dy}{dx} = \frac{y}{x}$ is:

(A) $y = x + C$

(B) $y = Cx$

(C) $xy = C$

(D) $x + y = C$

Q26. $3^{50} \pmod 5$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q27. In a LPP, the region represented by $x \geq 0, y \geq 0$ is:

(A) First Quadrant

(B) Second Quadrant

(C) Third Quadrant

(D) Fourth Quadrant

Q28. Consumer Surplus is the area:

(A) Above the price line and below the demand curve

(B) Below the price line and above the supply curve

(C) Below the price line and below the demand curve

(D) Above the price line and above the supply curve

Q29. The trace of an identity matrix of order 4 is:

(A) 1

(B) 0

(C) 4

(D) 16

Q30. In a t-distribution, the total area under the curve is:

(A) 1/2

(B) 1

(C) Depends on degrees of freedom

(D) 0

Q31. A person wants to have ₹ 1,00,000 after 5 years. How much should he invest now at 8% p.a. compounded continuously? ($e^{0.4} = 1.4918$)

(A) ₹ 67,033

(B) ₹ 75,000

(C) ₹ 80,000

(D) ₹ 50,000

Q32. The integrating factor ($I.F.$) for the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) e^x

(B) $\ln x$

(C) x

(D) $1/x$

Q33. Which index number is called the "Ideal Index Number"?

(A) Laspeyres'

(B) Paasche's

(C) Fisher's

(D) Marshall-Edgeworth

Q34. The probability of getting a sum of 7 when two dice are thrown is:

(A) 1/6

(B) 5/36

(C) 1/12

(D) 1/36

Q35. The cost of a machine is ₹ 10,000. If it depreciates by 10% every year, its value after 2 years will be:

(A) ₹ 8,000

(B) ₹ 8,100

(C) ₹ 9,000

(D) ₹ 7,900

Q36. In a two-tailed test, if $\alpha = 0.05$, the area in each tail is:

(A) 0.05

(B) 0.025

(C) 0.10

(D) 0.95

Q37. If $x \equiv 3 \pmod 5$, then $2x + 1 \pmod 5$ is:

(A) 1
(B) 2
(C) 3
(D) 0

Q38. The point where the total revenue equals total cost is the:

(A) Equilibrium point
(B) Break-even point
(C) Optimal point
(D) Satiety point

Q39. A 4×4 matrix has 16 elements. How many elements does a 4×4 skew-symmetric matrix have on its diagonal?

(A) 4
(B) 0
(C) 16
(D) 8

Q40. An EMI is ₹ 10,000. If the principal part is ₹ 6,500, the interest part is:

(A) ₹ 16,500
(B) ₹ 3,500
(C) ₹ 10,000
(D) ₹ 6,500

Q41. The area bounded by $y = \sqrt{x}$ and $y = x$ from $x = 0$ to $x = 1$ is:

(A) 1/3

(B) 1/2

(C) 1/6

(D) 1/4

Q42. For a normal distribution, approximately what percentage of values lie within $\mu \pm 3\sigma$?

(A) 68%

(B) 95%

(C) 99.7%

(D) 50%

Q43. Moving average method is used to measure:

(A) Seasonal variations

(B) Secular trend

(C) Cyclical variations

(D) Irregular variations

Q44. If A and B are square matrices of the same order, then $(AB)^{-1}$ is:

(A) $A^{-1}B^{-1}$

(B) $B^{-1}A^{-1}$

(C) AB

(D) BA

Q45. The value of $\int_0^2 (x^2 + 1)dx$ is:

(A) 4

(B) 14/3

(C) 8/3

(D) 10/3

Q46. If the Null Hypothesis is $H_0 : \mu = 50$, then the Alternative Hypothesis $H_1 : \mu > 50$ indicates a:

(A) Two-tailed test

(B) Left-tailed test

(C) Right-tailed test

(D) No-tailed test

Q47. A boat goes 8 km upstream and 12 km downstream in 7 hours. In another trip it goes 10 km upstream and 15 km downstream in 8 hours 45 minutes. The speed of the boat in still water is:

(A) 3 km/h

(B) 4 km/h

(C) 2 km/h

(D) 5 km/h

Q48. The face value of a treasury bill is ₹ 1,00,000. It is issued for ₹ 96,000 for 91 days. The yield (annual interest rate) is approx:

(A) 4%

(B) 16%

(C) 16.5%

(D) 12%

Q49. If $|A| = 5$ and A is of order 2×2, then $|3A|$ is:

(A) 15

(B) 45

(C) 5

(D) 9

Q50. The value of $\int_1^2 \frac{1}{x}dx$ is:

(A) $\ln 2$

(B) 1

(C) $1/2$

(D) e

Detailed Solutions

Q1.

Solution

Concept:
The speed of a boat in still water (u) and the speed of the stream (v) determine the speed of the boat downstream ($u + v$) and upstream ($u - v$). Time taken for a journey is given by the formula:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

The total time is the sum of time taken to go downstream and the time taken to return upstream.
Solution:
1. Let the speed of the boat in still water be $u = 15$ km/h and the speed of the stream be v km/h. 2. The speed of the boat downstream is $(15 + v)$ km/h. 3. The speed of the boat upstream is $(15 - v)$ km/h. 4. The time taken to go 30 km downstream is:

$$T_1 = \frac{30}{15 + v}$$

5. The time taken to return 30 km upstream is:

$$T_2 = \frac{30}{15 - v}$$

6. The total time given is 4 hours 30 minutes, which is 4.5 or 9/2 hours.

$$\frac{30}{15 + v} + \frac{30}{15 - v} = \frac{9}{2}$$

7. Dividing the entire equation by 3:

$$\frac{10}{15 + v} + \frac{10}{15 - v} = \frac{3}{2}$$

8. Taking the LCM and solving for v:

$$10\left(\frac{15 - v + 15 + v}{225 - v^2}\right) = \frac{3}{2}$$

$$10\left(\frac{30}{225 - v^2}\right) = \frac{3}{2} \implies \frac{300}{225 - v^2} = \frac{3}{2}$$

9. Cross-multiplying:

$$600 = 3(225 - v^2) \implies 200 = 225 - v^2$$

$$v^2 = 25 \implies v = 5 \text{ km/h}$$

Final Answer: The speed of the stream is 5 km/h.

Answer: (B)

Q2.

Solution

Concept:
For a square matrix A of order n, the determinant of the adjoint of A is related to the determinant of A by the property:

$$|adj A| = |A|^{n-1}$$

This property allows us to find the value of the determinant of the original matrix if the determinant of its adjoint is known.

Solution:
1. Given that A is a square matrix of order $n = 3$. 2. The determinant of the adjoint of A is given as:

$$|adj A| = 64$$

3. Using the property $|adj A| = |A|^{n-1}$, we substitute the values:

$$64 = |A|^{3-1}$$

4. Simplify the exponent:

$$64 = |A|^2$$

5. To find $|A|$, take the square root of both sides:

$$|A| = \pm\sqrt{64}$$

6. Therefore:

$$|A| = \pm 8$$

Final Answer: The value of $|A|$ is ± 8.

Answer: (B)

Q3.

Solution

Concept:

To evaluate the definite integral $\int_1^e \frac{\ln x}{x} dx$, we can use the method of substitution. If we let $u = \ln x$, then its derivative $du = \frac{1}{x}dx$ is present in the integral, making the integration straightforward.

Solution:

1. Let $u = \ln x$. Then, differentiating both sides gives:

$$du = \frac{1}{x}dx$$

2. Change the limits of integration according to u: - When $x = 1$, $u = \ln(1) = 0$. - When $x = e$, $u = \ln(e) = 1$. 3. Substitute these into the integral:

$$\int_1^e \frac{\ln x}{x} dx = \int_0^1 u\, du$$

4. Integrate u using the power rule:

$$\left[\frac{u^2}{2}\right]_0^1$$

5. Evaluate at the upper and lower limits:

$$\frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

Final Answer: The value of the integral is $1/2$.

Answer: (A)

Q4.

Solution

Concept:

In a Binomial Distribution $B(n, p)$, where n is the number of trials and p is the probability of success, the Mean and Variance are defined as:

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

where $q = 1 - p$ is the probability of failure.

Solution:

1. Given Mean $np = 4$. 2. Given Variance $npq = 3$. 3. To find q, divide the Variance by the Mean:

$$q = \frac{npq}{np} = \frac{3}{4}$$

4. Since $p = 1 - q$:

$$p = 1 - \frac{3}{4} = \frac{1}{4}$$

5. Now, use the Mean equation to find n:

$$np = 4 \implies n\left(\frac{1}{4}\right) = 4$$

6. Multiply both sides by 4:

$$n = 16$$

Final Answer: The number of trials n is 16.

Answer: (C)

Q5.

Solution

Concept:
An annuity is a series of equal payments made at regular intervals. Annuities are classified based on when the payment occurs within the period: - **Ordinary Annuity:** Payments are made at the end of each period. - **Annuity Due:** Payments are made at the beginning of each period. - **Perpetuity:** Payments continue indefinitely.

Solution:
1. The question asks for the name of an annuity where payments are made at the beginning of each period. 2. By definition, if the first payment is made immediately (at time $t = 0$) or at the start of every subsequent interval, it is categorized as an Annuity Due. 3. This is in contrast to an Ordinary Annuity (or Annuity Immediate), where the first payment occurs one period after the start.

Final Answer: An annuity where payments are made at the beginning of each period is called an Annuity Due.

Answer: (B)

Q6.

Solution

Concept:
A linear congruence of the form $ax \equiv b \pmod{m}$ can be solved by finding the modular multiplicative inverse of a modulo m, or by testing values of x such that $(ax - b)$ is divisible by m. If $\gcd(a, m) = 1$, a unique solution exists within the modulus.

Solution:
1. Given the congruence: $5x \equiv 2 \pmod{7}$. 2. We need to find an integer x such that $5x - 2$ is a multiple of 7. 3. Test small values for x: - If $x = 1$, $5(1) = 5 \equiv 5 \pmod{7}$. - If $x = 2$, $5(2) = 10 \equiv 3 \pmod{7}$. - If $x = 3$, $5(3) = 15 \equiv 1 \pmod{7}$. (This shows 3 is the modular inverse of 5 mod 7). - If $x = 4$, $5(4) = 20 \equiv 6 \pmod{7}$. - If $x = 5$, $5(5) = 25 \equiv 4 \pmod{7}$. - If $x = 6$, $5(6) = 30 \equiv 2 \pmod{7}$. 4. We find that for $x = 6$:

$$5 \times 6 = 30$$

$$30 = (4 \times 7) + 2 \implies 30 \equiv 2 \pmod{7}$$

5. Alternatively, since $5 \times 3 \equiv 1 \pmod{7}$, multiply both sides of $5x \equiv 2$ by 3:

$$15x \equiv 6 \pmod{7} \implies x \equiv 6 \pmod{7}$$

Final Answer: The solution is $x \equiv 6 \pmod{7}$.

Answer: (C)

Q7.

Solution

Concept:

The Total Cost function $C(x)$ can be obtained by integrating the Marginal Cost (MC) function with respect to the quantity x:

$$C(x) = \int MC\,dx + FC$$

where FC represents the Fixed Cost, which is the constant of integration in this context.

Solution:

1. Given the Marginal Cost function: $MC = 4 + 0.5x$. 2. Integrate MC to find the variable part of the cost:

$$\int (4 + 0.5x)\,dx = 4x + \frac{0.5x^2}{2}$$

3. Simplify the expression:

$$4x + 0.25x^2$$

4. Add the Fixed Cost ($FC = 100$) to get the Total Cost function $C(x)$:

$$C(x) = 4x + 0.25x^2 + 100$$

5. This function represents the sum of variable costs ($4x + 0.25x^2$) and the fixed overhead cost (100).

Final Answer: The total cost function is $C(x) = 4x + 0.25x^2 + 100$.

Answer: (C)

Q8.

Solution

Concept:
When comparing racers, we look at the ratio of distances covered in the same amount of time. If A beats B by d meters in a race of L meters, the ratio of their speeds (and distances) is $L : (L - d)$.
Solution:
1. In a 500m race, A beats B by 50m. This means when A covers 500m, B covers $500 - 50 = 450$m. The ratio $\frac{\text{Distance}_A}{\text{Distance}_B} = \frac{500}{450} = \frac{10}{9}$. 2. In a 400m race, B beats C by 40m. This means when B covers 400m, C covers $400 - 40 = 360$m. The ratio $\frac{\text{Distance}_B}{\text{Distance}_C} = \frac{400}{360} = \frac{10}{9}$. 3. We need to find how much A beats C by in a 500m race. Find the ratio $\frac{\text{Distance}_A}{\text{Distance}_C}$:

$$\frac{\text{Distance}_A}{\text{Distance}_C} = \frac{\text{Distance}_A}{\text{Distance}_B} \times \frac{\text{Distance}_B}{\text{Distance}_C} = \frac{10}{9} \times \frac{10}{9} = \frac{100}{81}$$

4. When A covers 100m, C covers 81m. 5. Scaling this to a 500m race: When A covers 500m (100×5), C covers $81 \times 5 = 405$m. 6. The distance by which A beats C is:

$$500 - 405 = 95\text{m}$$

Final Answer: A will beat C by 95m.

Answer: (B)

Q9.

Solution

Concept:
The Normal Distribution is a continuous probability distribution characterized by a symmetric, bell-shaped curve. Its properties include: - Symmetry about the mean (μ). - Mean = Median = Mode. - The total area under the curve is exactly 1. - It is asymptotic to the x-axis.
Solution:
1. Option (A) states it is bell-shaped, which is the fundamental visual description of a normal curve. 2. Option (B) states Mean = Median = Mode, which is true for any perfectly symmetric unimodal distribution like the Normal curve. 3. Option (C) states the total area is 1, which is a requirement for all probability density functions. 4. Option (D) states it is skewed to the right. Skewness implies asymmetry. Since the normal distribution is perfectly symmetric, its skewness is 0. 5. Therefore, being "skewed to the right" is not a property of the normal distribution.
Final Answer: The property "It is skewed to the right" is NOT a property of a Normal Curve.

Answer: (D)

Q10.

Solution

Concept:

The annual installment (R) for a sinking fund to accumulate an amount (A) over n periods at an interest rate i per period is given by the formula:

$$R = \frac{A \cdot i}{(1+i)^n - 1}$$

Solution:

1. Identify the given values: Target Amount (A) = ₹ 5,00,000 Rate (i) = 10% = 0.10 Time (n) = 10 years Given $(1.1)^{10} = 2.5937$ 2. Substitute the values into the formula:

$$R = \frac{5,00,000 \times 0.10}{2.5937 - 1}$$

3. Simplify the numerator and denominator:

$$R = \frac{50,000}{1.5937}$$

4. Perform the division:

$$R \approx 31,372.27$$

5. The annual installment required to accumulate the debt amount is approximately ₹ 31,372.

Final Answer: The annual installment is approximately ₹ 31,372.

Answer: (A)

Q11.

Solution

Concept:
The Average Cost (AC) is defined as the total cost per unit of output ($AC = C(x)/x$). To find the minimum value of any function, we take its derivative and set it to zero. For AC, the derivative $\frac{d(AC)}{dx} = 0$ leads to a specific relationship where the rate of change of the total cost (Marginal Cost) must equal the average cost.

Solution:
1. Let C be the total cost. Then $AC = \frac{C}{x}$. 2. To find the minimum AC, differentiate AC with respect to x:

$$\frac{d(AC)}{dx} = \frac{d}{dx}\left(\frac{C}{x}\right)$$

3. Using the quotient rule:

$$\frac{d(AC)}{dx} = \frac{x \cdot \frac{dC}{dx} - C \cdot 1}{x^2}$$

4. For a minimum, set the derivative to zero:

$$\frac{x \cdot MC - C}{x^2} = 0 \implies x \cdot MC - C = 0$$

5. Rearranging the terms:

$$x \cdot MC = C \implies MC = \frac{C}{x}$$

6. Since $\frac{C}{x} = AC$, we conclude that AC is at its minimum when $MC = AC$.
Final Answer: The average cost is minimum when $MC = AC$.

Answer: (A)

Q12.

Solution

Concept:
The order of a differential equation is the order of the highest derivative present. The degree is the power of the highest-order derivative, provided the equation can be expressed as a polynomial in its derivatives. If a derivative is an argument of a transcendental function (like sin, cos, e, log), the equation cannot be expressed as a polynomial in derivatives.

Solution:
1. Identify the derivatives in the equation: $\frac{d^2y}{dx^2}$ (second order) and $\frac{dy}{dx}$ (first order). 2. The highest order is 2, so the order of the differential equation is 2. 3. Observe the term $\sin\left(\frac{dy}{dx}\right)$. 4. Because the first derivative is inside a sine function, the differential equation cannot be written as a polynomial in its derivatives. 5. According to the rules of differential equations, if such a transcendental form exists, the degree of the differential equation is "Not defined."
Final Answer: The degree of the differential equation is Not defined.

Answer: (D)

Q13.

Solution

Concept:

A perpetuity is an annuity that pays a constant sum of money forever. The present value (PV) of a perpetuity is calculated by dividing the periodic payment (R) by the periodic interest rate (i).

$$PV = \frac{R}{i}$$

Solution:

1. Identify the periodic payment (R): ₹ 1,200. 2. Identify the annual interest rate (i): 8% = 0.08. 3. Since the payment is made annually, we use the annual rate directly. 4. Apply the perpetuity formula:

$$PV = \frac{1,200}{0.08}$$

5. To simplify the division, multiply both numerator and denominator by 100:

$$PV = \frac{1,20,000}{8}$$

6. Perform the calculation:

$$PV = 15,000$$

Final Answer: The present value is ₹ 15,000.

Answer: (B)

Q14.

Solution

Concept:

The Poisson probability mass function is given by:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

where λ is the mean (and also the variance) of the distribution.

Solution:

1. Given that $P(X = 0) = P(X = 1)$. 2. Write the formula for $k = 0$:

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda} \cdot 1}{1} = e^{-\lambda}$$

3. Write the formula for $k = 1$:

$$P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-\lambda} \cdot \lambda$$

4. Equate the two probabilities:

$$e^{-\lambda} = e^{-\lambda} \cdot \lambda$$

5. Since $e^{-\lambda}$ is never zero, we can divide both sides by $e^{-\lambda}$:

$$1 = \lambda$$

6. Therefore, the mean (λ) of the distribution is 1.

Final Answer: The mean of the distribution is 1.

Answer: (B)

Q15.

Solution

Concept:

According to the Corner Point Theorem in Linear Programming (LPP), the optimal value (maximum or minimum) of the objective function Z occurs at one of the vertices (corner points) of the feasible region. To find the maximum Z, we calculate the value of Z at each given corner point.

Solution:

1. Given objective function: $Z = 3x + 4y$. 2. List the corner points and calculate Z for each: - At $(0, 0)$: $Z = 3(0) + 4(0) = 0$ - At $(4, 0)$: $Z = 3(4) + 4(0) = 12$ - At $(0, 5)$: $Z = 3(0) + 4(5) = 20$ - At $(2, 4)$: $Z = 3(2) + 4(4) = 6 + 16 = 22$ 3. Compare the calculated values: 0, 12, 20, 22. 4. The highest value is 22, which occurs at the point $(2, 4)$.

Final Answer: The maximum value of Z is 22.

Answer: (C)

Q16.

Solution

Concept:
In a time series, variations are classified based on their nature and duration. - **Secular Trend:** Long-term direction. - **Seasonal Variation:** Regular periodic fluctuations within a year. - **Cyclical Variation:** Periodic fluctuations lasting more than a year. - **Irregular Variation:** Sudden, unpredictable, and random events like floods, strikes, or fires.

Solution:
1. A "fire in a factory" is an event that is not predictable and does not follow any specific rhythmic pattern or long-term trend. 2. It happens suddenly and randomly due to chance factors. 3. Such unpredictable and non-recurring events are classified under the "Irregular" or "Random" component of a time series. 4. Unlike seasonal changes (holidays) or cyclical changes (recessions), a fire is an isolated incident.

Final Answer: The component is Irregular Variation.

Answer: (C)

Q17.

Solution

Concept:
For any square matrix A, the transpose A^T is obtained by interchanging its rows and columns. A matrix M is called **Symmetric** if $M^T = M$. A fundamental property of matrices is that the sum of a square matrix and its transpose is always symmetric.

Solution:
1. Given $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. 2. Find the transpose A^T:

$$A^T = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

3. Calculate $M = A + A^T$:

$$M = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 8 \end{bmatrix}$$

4. Check for symmetry by finding M^T:

$$M^T = \begin{bmatrix} 4 & 4 \\ 4 & 8 \end{bmatrix}$$

5. Since $M = M^T$, the resulting matrix is symmetric. This is a general property: $(A + A^T)^T = A^T + (A^T)^T = A^T + A$, which equals the original sum.

Final Answer: The matrix $A + A^T$ is Symmetric.

Answer: (A)

Q18.

Solution

Concept:
When multiple pipes work together to empty or fill a tank, their individual rates of work are added. If a pipe empties a tank in h hours, its rate of work is $1/h$ of the tank per hour.

Solution:
1. Rate of the first pipe = $1/15$ tank per hour. 2. Rate of the second pipe = $1/10$ tank per hour. 3. Combined rate of both pipes = $\frac{1}{15} + \frac{1}{10}$. 4. Find a common denominator (30):

$$\text{Combined Rate} = \frac{2}{30} + \frac{3}{30} = \frac{5}{30} = \frac{1}{6} \text{ tank per hour}$$

5. The time taken to empty the full tank is the reciprocal of the combined rate:

$$\text{Time} = \frac{1}{1/6} = 6 \text{ hours}$$

Final Answer: The tank will be emptied in 6 hours.

Answer: (B)

Q19.

Solution

Concept:
To integrate the product of two functions where one is algebraic (x) and one is exponential (e^x), we use the Integration by Parts formula:

$$\int u\,dv = uv - \int v\,du$$

According to the ILATE rule, we choose $u = x$ and $dv = e^x dx$.

Solution:
1. Let $u = x \implies du = dx$. 2. Let $dv = e^x dx \implies v = e^x$. 3. Apply the formula:

$$\int xe^x dx = xe^x - \int e^x dx$$

4. Integrate the remaining term:

$$xe^x - e^x + C$$

5. Factor out e^x:

$$e^x(x-1) + C$$

Final Answer: The value is $e^x(x-1) + C$.

Answer: (B)

Q20.

Solution

Concept:

In financial markets, a bond is compared by its Market Price and its Face Value (Par Value). - **At Par:** Market Price = Face Value. - **At Premium:** Market Price > Face Value. - **At Discount:** Market Price < Face Value.

Solution:

1. Given Face Value = ₹ 1,000. 2. Given Purchase Price (Market Price) = ₹ 950. 3. Since ₹ 950 < ₹ 1,000, the bond is being sold for less than its face value. 4. By definition, a security trading below its face value is said to be trading at a discount.

Final Answer: The bond is purchased at a Discount.

Answer: (C)

Q21.

Solution

Concept:

The sample proportion (p) is a point estimate of the population proportion. It is calculated by dividing the number of individuals in the sample who possess a certain characteristic (x) by the total number of individuals in the sample (n).

$$p = \frac{x}{n}$$

Solution:

1. Identify the given values: Total sample size (n) = 100. Number of smokers (favorable outcomes, x) = 40. 2. Apply the sample proportion formula:

$$p = \frac{40}{100}$$

3. Convert the fraction to a decimal:

$$p = 0.4$$

4. This value represents that 40% of the sampled population are smokers.

Final Answer: The sample proportion is 0.4.

Answer: (A)

Q22.

Solution

Concept:
The effective speed of a boat traveling upstream is the speed of the boat in still water (u) minus the speed of the stream (v). The time taken for the journey is the distance divided by this effective upstream speed.

$$\text{Time} = \frac{\text{Distance}}{u - v}$$

Solution:
1. Identify the speed in still water: $u = 10$ km/h. 2. Identify the speed of the stream: $v = 2$ km/h. 3. Calculate the upstream speed:

$$\text{Upstream Speed} = u - v = 10 - 2 = 8 \text{ km/h}$$

4. Identify the distance to be covered: 24 km. 5. Calculate the time taken:

$$\text{Time} = \frac{24}{8} = 3 \text{ hours}$$

Final Answer: The time taken is 3 hours.

Answer: (B)

Q23.

Solution

Concept:
In hypothesis testing for the population mean, the choice of test depends on the sample size (n) and whether the population standard deviation (σ) is known. - If σ is known and n is large, the **Z-test** is used. - If σ is unknown and n is small (typically $n < 30$), the **t-test** (Student's t-test) is used.

Solution:
1. The population standard deviation (σ) is stated to be unknown. 2. The sample size (n) is given as 20. 3. Since $n = 20$, it is considered a small sample ($n < 30$). 4. Under these specific conditions—unknown population variance and a small sample size—the t-test is the appropriate statistical tool to estimate the mean.

Final Answer: The t-test is used for the mean.

Answer: (B)

Q24.

Solution

Concept:

The effective rate of interest (r_e) is the actual interest earned or paid in a year when compounding occurs more than once. The formula is:

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

where r is the nominal annual rate and m is the number of compounding periods per year.

Solution:

1. Identify the nominal rate (r): $12\% = 0.12$. 2. Identify the compounding frequency: Monthly ($m = 12$). 3. Substitute the values into the formula:

$$r_e = \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

4. Simplify the periodic rate:

$$r_e = (1 + 0.01)^{12} - 1 = (1.01)^{12} - 1$$

5. Calculate $(1.01)^{12} \approx 1.126825$. 6. Find the effective rate:

$$r_e = 1.126825 - 1 = 0.126825 \approx 12.68\%$$

Final Answer: The effective rate is 12.68%.

Answer: (B)

Q25.

Solution

Concept:
The differential equation $\frac{dy}{dx} = \frac{y}{x}$ is a first-order separable differential equation. We can solve it by grouping terms with y on one side and terms with x on the other, then integrating.

Solution:

1. Rearrange the variables:

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

2. Integrate both sides:

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

3. Apply the logarithmic integration rule:

$$\ln|y| = \ln|x| + C_1$$

4. Express the constant C_1 as $\ln C$ (where $C > 0$):

$$\ln|y| = \ln|x| + \ln C$$

5. Use the properties of logarithms ($\ln a + \ln b = \ln ab$):

$$\ln|y| = \ln|Cx|$$

6. Remove the logarithms from both sides:

$$y = Cx$$

Final Answer: The solution is $y = Cx$.

Answer: (B)

Q26.

Solution

Concept:

To find $a^n \pmod{m}$, we can use the property of modular exponentiation. We observe the pattern of powers or use Fermat's Little Theorem if the modulus is prime.

Solution:

1. We need to find $3^{50} \pmod 5$. 2. Observe the powers of 3 modulo 5: - $3^1 \equiv 3 \pmod 5$ - $3^2 = 9 \equiv 4 \equiv -1 \pmod 5$ 3. Use the property $(a^k)^n \equiv (a^k \pmod{m})^n \pmod{m}$: - $3^{50} = (3^2)^{25}$ - Since $3^2 \equiv -1 \pmod 5$, we have $(3^2)^{25} \equiv (-1)^{25} \pmod 5$. 4. Calculate the power of -1: - $(-1)^{25} = -1$ (since the exponent is odd). 5. Convert the negative remainder to a positive one within modulus 5: - $-1 + 5 = 4$. 6. Therefore, $3^{50} \equiv 4 \pmod 5$.

Final Answer: The value is 4.

Answer: (D)

Q27.

Solution

Concept:

In a Cartesian coordinate system, the two axes divide the plane into four regions called quadrants. The signs of the coordinates (x, y) determine the quadrant.

Solution:

1. In the first quadrant, both x and y are positive $(x > 0, y > 0)$. 2. The constraints $x \geq 0$ and $y \geq 0$ include the positive axes and the region where both variables are non-negative. 3. In the second quadrant, $x < 0, y > 0$. 4. In the third quadrant, $x < 0, y < 0$. 5. In the fourth quadrant, $x > 0, y < 0$. 6. Therefore, the region represented by $x \geq 0, y \geq 0$ is the First Quadrant.

Final Answer: The region is the First Quadrant.

Answer: (A)

Q28.

Solution

Concept:

Consumer Surplus (CS) is an economic measure of consumer benefit. It is calculated as the difference between the maximum price a consumer is willing to pay and the actual price they pay.

Solution:

1. On a graph where the vertical axis is Price and the horizontal axis is Quantity, the Demand Curve represents the willingness to pay. 2. The Equilibrium Price (or Price Line) is a horizontal line representing the market price. 3. The area above the price line reflects the "savings" for consumers who were willing to pay more than the market price. 4. This area is bounded above by the Demand Curve and below by the Price Line. 5. Thus, Consumer Surplus is the area above the price line and below the demand curve.

Final Answer: Consumer Surplus is the area above the price line and below the demand curve.

Answer: (A)

Q29.

Solution

Concept:

The trace of a square matrix is defined as the sum of the elements on the main diagonal (from top-left to bottom-right). An identity matrix (I_n) of order n is a square matrix in which all diagonal elements are 1 and all other elements are 0.

Solution:

1. An identity matrix of order 4 (I_4) is represented as:

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Identify the elements on the main diagonal: 1, 1, 1, 1. 3. Calculate the sum of these elements:

$$\text{Trace}(I_4) = 1 + 1 + 1 + 1 = 4$$

4. In general, the trace of an identity matrix of order n is always n.

Final Answer: The trace is 4.

Answer: (C)

Q30.

Solution

Concept:
The t-distribution is a continuous probability distribution. Like all probability density functions (PDFs), the total area under its curve represents the total probability of all possible outcomes.

Solution:
1. By the definition of a probability distribution, the integral of the probability density function over its entire range (from $-\infty$ to $+\infty$) must be equal to 1. 2. This rule applies to the Normal distribution, the t-distribution, the Chi-square distribution, and all other valid probability distributions. 3. While the shape of the t-distribution changes depending on the degrees of freedom (it becomes more "peaked" as degrees of freedom increase), the area enclosed between the curve and the x-axis remains constant. 4. Therefore, the total area is exactly 1.

Final Answer: The total area under the curve is 1.

Answer: (B)

Q31.

Solution

Concept:
The present value (PV) of a future sum (FV) under continuous compounding is given by the formula:

$$PV = FV \cdot e^{-rt}$$

where r is the annual interest rate as a decimal and t is the time in years. This formula accounts for interest being calculated and added to the principal at every infinitesimally small moment in time.

Solution:
1. Identify the given variables: Future Value (FV) = ₹ 1,00,000 Time (t) = 5 years Annual interest rate (r) = 8% = 0.08 2. Substitute the values into the formula for present value:

$$PV = 1,00,000 \cdot e^{-(0.08)(5)}$$

3. Simplify the exponent:

$$PV = 1,00,000 \cdot e^{-0.4}$$

4. We are given $e^{0.4} = 1.4918$. Therefore, $e^{-0.4} = \frac{1}{e^{0.4}}$:

$$PV = \frac{1,00,000}{1.4918}$$

5. Perform the division:

$$PV \approx 67,033.11$$

6. Rounding to the nearest rupee, we get ₹ 67,033.

Final Answer: The person should invest ₹ 67,033.

Answer: (A)

Q32.

Solution

Concept:

A first-order linear differential equation is of the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

The Integrating Factor ($I.F.$) is used to transform the left side into the derivative of a product. It is calculated as:

$$I.F. = e^{\int P(x)\,dx}$$

Solution:

1. Identify $P(x)$ from the given differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$: The coefficient of y is $P(x) = \frac{1}{x}$.
2. Substitute $P(x)$ into the Integrating Factor formula:

$$I.F. = e^{\int \frac{1}{x}\,dx}$$

3. Integrate the exponent:

$$\int \frac{1}{x}\,dx = \ln|x|$$

4. Substitute back into the expression:

$$I.F. = e^{\ln x}$$

5. Using the property $e^{\ln f(x)} = f(x)$:

$$I.F. = x$$

Final Answer: The integrating factor is x.

Answer: (C)

Q33.

Solution

Concept:
Index numbers measure changes in price or quantity over time. Various economists proposed different formulas. An "Ideal" index number is expected to satisfy certain criteria, primarily the Time Reversal Test and the Factor Reversal Test.
Solution:
1. **Laspeyres' Index:** Uses base year quantities as weights ($L = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$). It fails both the Time and Factor Reversal tests. 2. **Paasche's Index:** Uses current year quantities as weights ($P = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$). It also fails both tests. 3. **Fisher's Ideal Index:** This is defined as the geometric mean of Laspeyres' and Paasche's indices:

$$F = \sqrt{L \times P}$$

4. Fisher's formula is called "Ideal" because it satisfies both the Time Reversal and Factor Reversal tests, making it mathematically superior for measuring economic shifts.
Final Answer: Fisher's Index is known as the "Ideal Index Number."

Answer: (C)

Q34.

Solution

Concept:
When two dice are thrown, the total number of outcomes in the sample space is $6 \times 6 = 36$. To find the probability of a specific sum, we must identify all ordered pairs (d_1, d_2) that add up to that sum.
Solution:
1. Total outcomes $(n(S)) = 36$. 2. Identify pairs where the sum is 7: - $(1, 6)$ - $(2, 5)$ - $(3, 4)$ - $(4, 3)$ - $(5, 2)$ - $(6, 1)$ 3. Count the number of favorable outcomes $(n(E))$: There are exactly 6 pairs. 4. Calculate the probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

5. Simplify the fraction:

$$P(E) = \frac{1}{6}$$

Final Answer: The probability of getting a sum of 7 is 1/6.

Answer: (A)

Q35.

Solution

Concept:

Depreciation refers to the reduction in the value of an asset over time. When a constant percentage rate is applied annually, it follows the Reducing Balance Method (Compound Interest format with a negative rate). The formula for value after n years is:

$$A = P(1 - r)^n$$

Solution:

1. Identify the given values: Original cost (P) = ₹ 10,000 Rate of depreciation (r) = 10% = 0.10 Time (n) = 2 years 2. Calculate the value after 1 year:

$$V_1 = 10,000 - (10\% \text{ of } 10,000) = 10,000 - 1,000 = 9,000$$

3. Calculate the value after the 2nd year (applying 10% to the new value):

$$V_2 = 9,000 - (10\% \text{ of } 9,000) = 9,000 - 900 = 8,100$$

4. Alternatively, using the formula:

$$A = 10,000(1 - 0.10)^2 = 10,000(0.9)^2$$

$$A = 10,000 \times 0.81 = 8,100$$

Final Answer: The value after 2 years will be ₹ 8,100.

Answer: (B)

Q36.

Solution

Concept:
The level of significance (α) represents the total probability of committing a Type I error (rejecting a true null hypothesis). In a two-tailed test, the critical region is split equally between the two extreme ends (tails) of the distribution.

Solution:
1. Identify the given significance level: $\alpha = 0.05$. 2. In a two-tailed test, we test for differences in both directions (greater than or less than). 3. The total area of rejection (α) must be divided equally into two parts, one for each tail. 4. Calculation:

$$\text{Area in each tail} = \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

5. This means there is a 2.5% probability in the left tail and a 2.5% probability in the right tail, totaling 5%.

Final Answer: The area in each tail is 0.025.

Answer: (B)

Q37.

Solution

Concept:
Modular arithmetic follows specific rules for addition and multiplication. If $a \equiv b \pmod{m}$, then:
- $ka \equiv kb \pmod{m}$ - $a + c \equiv b + c \pmod{m}$

Solution:
1. Given the congruence: $x \equiv 3 \pmod{5}$. 2. We need to find the value of $2x + 1 \pmod{5}$. 3. First, multiply the entire congruence by 2:

$$2x \equiv 2(3) \pmod{5}$$

$$2x \equiv 6 \pmod{5}$$

4. Simplify $6 \pmod{5}$:

$$6 = 5(1) + 1 \implies 6 \equiv 1 \pmod{5}$$

So, $2x \equiv 1 \pmod{5}$. 5. Now, add 1 to both sides:

$$2x + 1 \equiv 1 + 1 \pmod{5}$$

$$2x + 1 \equiv 2 \pmod{5}$$

Final Answer: The value is $2 \pmod{5}$.

Answer: (B)

Q38.

Solution

Concept:
The Break-even point (BEP) is the level of production or sales at which a business neither makes a profit nor incurs a loss. At this point, the total revenue (TR) generated is exactly equal to the total costs (TC) incurred.
Solution:
1. Profit (P) is defined as $P = TR - TC$. 2. When a company "breaks even," its profit is zero: $0 = TR - TC$. 3. This implies $TR = TC$. 4. At this point, the contribution margin has covered all fixed costs. 5. Any production beyond this point will result in a profit, and any production below this point results in a loss.
Final Answer: The point is known as the Break-even point.

Answer: (B)

Q39.

Solution

Concept:
A square matrix A is called skew-symmetric if $A^T = -A$. This property imposes a strict requirement on the elements of the main diagonal (a_{ii}): they must all be zero because $a_{ii} = -a_{ii}$ only if $a_{ii} = 0$.
Solution:
1. Let A be a 4×4 skew-symmetric matrix. 2. The elements of the main diagonal are $a_{11}, a_{22}, a_{33}, a_{44}$. 3. By the property of skew-symmetry:

$$a_{11} = 0, a_{22} = 0, a_{33} = 0, a_{44} = 0$$

4. Therefore, the number of non-zero elements on the main diagonal is zero. 5. While there are 4 diagonal positions, the question asks for the value/nature of the elements or can be interpreted as asking how many non-zero diagonal elements exist; in skew-symmetric matrices, all diagonal entries are specifically 0.
Final Answer: The diagonal elements of a skew-symmetric matrix are all 0.

Answer: (B)

Q40.

Solution

Concept:
An Equated Monthly Installment (EMI) consists of two parts: the interest on the current outstanding balance and the principal repayment.

$$\text{EMI} = \text{Principal Repayment} + \text{Interest Component}$$

Solution:
1. Identify the given variables: Total EMI = ₹ 10,000 Principal component = ₹ 6,500 2. Use the EMI formula to isolate the interest part:

$$\text{Interest Component} = \text{EMI} - \text{Principal Repayment}$$

3. Substitute the values:

$$\text{Interest Component} = 10,000 - 6,500$$

4. Calculate the result:

$$\text{Interest Component} = 3,500$$

5. Thus, ₹ 3,500 of the payment goes toward paying the bank's interest, while the remaining ₹ 6,500 reduces the actual loan balance.
Final Answer: The interest part is ₹ 3,500.

Answer: (B)

Q41.

Solution

Concept:
The area bounded between two curves $f(x)$ and $g(x)$ over an interval $[a, b]$ is given by:

$$\text{Area} = \int_a^b |f(x) - g(x)|\, dx$$

To solve this, we first find the intersection points to determine the limits of integration and then identify which function is greater in that interval.
Solution:
1. Find the intersection points of $y = \sqrt{x}$ and $y = x$:

$$\sqrt{x} = x \implies x = x^2 \implies x^2 - x = 0 \implies x(x-1) = 0$$

The points are $x = 0$ and $x = 1$. 2. Between $x = 0$ and $x = 1$, test a point (e.g., $x = 0.25$): $\sqrt{0.25} = 0.5$ and $x = 0.25$. Since $0.5 > 0.25$, $\sqrt{x} \geq x$ in this interval. 3. Set up the integral:

$$\text{Area} = \int_0^1 (\sqrt{x} - x)\, dx = \int_0^1 (x^{1/2} - x)\, dx$$

4. Integrate:

$$\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1 = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$$

5. Evaluate at limits:

$$\left(\frac{2}{3}(1) - \frac{1}{2}(1)\right) - (0) = \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

Final Answer: The area is 1/6.

Answer: (C)

Q42.

Solution

Concept:
The Empirical Rule (or 68-95-99.7 rule) for a Normal Distribution describes the percentage of data falling within specific standard deviations from the mean (μ): - $\mu \pm 1\sigma \approx 68.27\%$ - $\mu \pm 2\sigma \approx 95.45\%$ - $\mu \pm 3\sigma \approx 99.73\%$
Solution:
1. The Normal Distribution is symmetric and asymptotic. While it extends to infinity, almost all values lie near the mean. 2. The question asks for the percentage within ±3 standard deviations. 3. According to the standardized properties of the normal curve, the area under the curve between $Z = -3$ and $Z = +3$ is 0.9973. 4. Converting this to a percentage: $0.9973 \times 100 = 99.73\%$. 5. This implies that only 0.27% of the data (or about 3 in 1,000) falls outside this range.
Final Answer: Approximately 99.7% of values lie within $\mu \pm 3\sigma$.

Answer: (C)

Q43.

Solution

Concept:
In Time Series analysis, the "Secular Trend" represents the smooth, long-term direction of the data. To isolate this trend from short-term fluctuations (seasonal or irregular), smoothing techniques are used.

Solution:
1. **Moving Average Method:** This involves calculating the average of data points over a specific period (e.g., 3-year or 5-year) and shifting the period forward. 2. By averaging, the short-term ups and downs "cancel out," leaving behind the underlying long-term direction. 3. This is a common non-mathematical (or semi-mathematical) way to determine the secular trend. 4. Other methods for measuring trend include the Method of Least Squares and the Free-hand Curve method.

Final Answer: The moving average method is used to measure the Secular trend.

Answer: (B)

Q44.

Solution

Concept:
The Reversal Property of Inverses states that the inverse of a product of two invertible matrices A and B is equal to the product of their inverses taken in the reverse order.

Solution:
1. Let $C = AB$. We want to find C^{-1} such that $C \cdot C^{-1} = I$. 2. Suppose we try the reverse order product $B^{-1}A^{-1}$:

$$(AB)(B^{-1}A^{-1})$$

3. Using the associative property of matrix multiplication:

$$A(B \cdot B^{-1})A^{-1}$$

4. Since $B \cdot B^{-1} = I$:

$$A(I)A^{-1} = A \cdot A^{-1} = I$$

5. Since the product results in the Identity matrix, $B^{-1}A^{-1}$ is indeed the inverse of AB. 6. Note: The order $(A^{-1}B^{-1})$ generally does not work because matrix multiplication is not commutative.

Final Answer: $(AB)^{-1} = B^{-1}A^{-1}$.

Answer: (B)

Q45.

Solution

Concept:
The Definite Integral $\int_a^b f(x)\,dx$ calculates the net area under the curve $f(x)$ from $x = a$ to $x = b$. This is solved by finding the antiderivative $F(x)$ and evaluating $F(b) - F(a)$.
Solution:
1. Given integral: $\int_0^2 (x^2 + 1)\,dx$. 2. Find the antiderivative using the power rule:

$$\int (x^2 + 1)\,dx = \frac{x^3}{3} + x$$

3. Apply the limits from 0 to 2:

$$\left[\frac{x^3}{3} + x\right]_0^2$$

4. Evaluate at the upper limit ($x = 2$):

$$\frac{2^3}{3} + 2 = \frac{8}{3} + 2 = \frac{8+6}{3} = \frac{14}{3}$$

5. Evaluate at the lower limit ($x = 0$):

$$\frac{0^3}{3} + 0 = 0$$

6. Final calculation: $\frac{14}{3} - 0 = \frac{14}{3}$.
Final Answer: The value of the integral is 14/3.

Answer: (B)

Q46.

Solution

Concept:
Hypothesis testing involves a Null Hypothesis (H_0) and an Alternative Hypothesis (H_1). The direction of the test is determined by the inequality sign in H_1: - $H_1 : \mu \neq \mu_0$ is a **Two-tailed test** (looks for any difference). - $H_1 : \mu < \mu_0$ is a **Left-tailed test** (looks for a decrease). - $H_1 : \mu > \mu_0$ is a **Right-tailed test** (looks for an increase).
Solution:
1. The Null Hypothesis is $H_0 : \mu = 50$, which assumes the mean is exactly 50. 2. The Alternative Hypothesis is $H_1 : \mu > 50$. 3. Because the Alternative Hypothesis uses the "greater than" symbol (>), we are specifically testing if the population parameter is significantly larger than the hypothesized value. 4. The rejection region (critical region) for this test lies entirely in the right tail of the sampling distribution. 5. Therefore, this is a one-tailed test, specifically a right-tailed test.
Final Answer: The hypothesis indicates a Right-tailed test.

Answer: (C)

Q47.

Solution

Concept:

This problem involves two variables: speed of boat in still water (u) and speed of stream (v). - Downstream Speed (D) = $u + v$ - Upstream Speed (U) = $u - v$ The time equation is: $\frac{\text{Distance Up}}{\text{Up Speed}} + \frac{\text{Distance Down}}{\text{Down Speed}} = \text{Total Time}$.

Solution:

1. Let $1/U = x$ and $1/D = y$. 2. From the first trip: $8x + 12y = 7$. 3. From the second trip: $10x + 15y = 8.75$ (since 8 hours 45 min = $8\frac{3}{4} = 35/4$). 4. Multiply the first equation by 5 and the second by 4: - $40x + 60y = 35$ - $40x + 60y = 35$ 5. These equations are dependent, indicating multiple solutions or a specific ratio. Let's simplify the ratio: $8x + 12y = 7 \implies 4(2x + 3y) = 7 \implies 2x + 3y = 1.75$. 6. Standard boat problems usually have distinct integer or simple fractional results. Testing standard values: If $u = 3, v = 1$: $U = 2, D = 4$. Then $8/2 + 12/4 = 4 + 3 = 7$. (Matches Trip 1). Check Trip 2: $10/2 + 15/4 = 5 + 3.75 = 8.75$. (Matches Trip 2). 7. Thus, $u - v = 2$ and $u + v = 4$. 8. Adding the two: $2u = 6 \implies u = 3$ km/h.

Final Answer: The speed of the boat in still water is 3 km/h.

Answer: (A)

Q48.

Solution

Concept:

A Treasury Bill (T-Bill) is a short-term debt instrument issued at a discount to its face value. The return (yield) is the difference between the face value and the purchase price, annualized over the holding period.

$$\text{Yield} = \frac{F - P}{P} \times \frac{365}{d} \times 100$$

Solution:

1. Identify the values: Face Value (F) = ₹ 1,00,000 Purchase Price (P) = ₹ 96,000 Discount ($F - P$) = ₹ 4,000 Days (d) = 91 2. Calculate the return for the 91-day period:

$$\text{Period Return} = \frac{4,000}{96,000} = \frac{1}{24} \approx 0.04167 \text{ (or } 4.167\%)$$

3. Annualize the rate:

$$\text{Annual Yield} = \frac{1}{24} \times \frac{365}{91} \times 100$$

4. Since $365/91 \approx 4.01$:

$$\text{Annual Yield} \approx 4.167\% \times 4.01 \approx 16.7\%$$

5. Looking at the options, 16.5% is the closest standard yield calculation.

Final Answer: The yield is approximately 16.5%.

Answer: (C)

Q49.

Solution

Concept:
For a square matrix A of order n and a scalar k, the property of determinants states:

$$|kA| = k^n|A|$$

This is because each of the n rows is multiplied by k, and the determinant is a multilinear function of the rows.
Solution:
1. Identify the order of the matrix (n): 2 (since it is 2×2). 2. Identify the scalar (k): 3. 3. Identify the determinant of A ($|A|$): 5. 4. Apply the property formula:

$$|3A| = 3^n|A|$$

5. Substitute $n = 2$:

$$|3A| = 3^2 \times 5$$

6. Calculate the power:

$$3^2 = 9$$

7. Perform the final multiplication:

$$9 \times 5 = 45$$

Final Answer: The value of $|3A|$ is 45.

Answer: (B)

Q50.

Solution

Concept:

The Fundamental Theorem of Calculus is used to evaluate the definite integral of $1/x$. The antiderivative of $1/x$ is $\ln|x|$.

$$\int_a^b \frac{1}{x}\,dx = [\ln|x|]_a^b = \ln|b| - \ln|a|$$

Solution:

1. Set up the integration:

$$\int_1^2 \frac{1}{x}\,dx$$

2. Find the antiderivative:

$$\ln|x|$$

3. Apply the limits from 1 to 2:

$$\ln(2) - \ln(1)$$

4. Use the property that the natural log of 1 is 0 ($\ln 1 = 0$):

$$\ln(2) - 0 = \ln 2$$

5. Therefore, the definite integral evaluates to the natural logarithm of 2.

Final Answer: The value is $\ln 2$.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	C	5	B
6	C	7	C	8	B	9	D	10	A
11	A	12	D	13	B	14	B	15	C
16	C	17	A	18	B	19	B	20	C
21	A	22	B	23	B	24	B	25	B
26	D	27	A	28	A	29	C	30	B
31	A	32	C	33	C	34	A	35	B
36	B	37	B	38	B	39	B	40	B
41	C	42	C	43	B	44	B	45	B
46	C	47	A	48	C	49	B	50	A