CUET UG Maths Sample Paper - 10

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix of order 3 and $|A| = 5$, then $|adj(adj A)|$ is:

(A) 25

(B) 125

(C) 625

(D) 3125

Q2. If $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is:

(A) 3

(B) 6

(C) 12

(D) 0

Q3. The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is:

(A) $8\sqrt{3}$ sq. units

(B) $4\sqrt{3}$ sq. units

(C) $12\sqrt{3}$ sq. units

(D) $16\sqrt{3}$ sq. units

Q4. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) 2, 2

(B) 2, 3

(C) 1, 2

(D) 2, 1

Q5. The corner points of the feasible region for an LPP are $(0,2)$, $(3,0)$, $(6,0)$, $(6,8)$ and $(0,5)$. Let $Z = 4x + 6y$ be the objective function. The minimum value of Z occurs at:

(A) $(0,2)$ only

(B) $(3,0)$ only

(C) The mid-point of the line segment joining $(0,2)$ and $(3,0)$

(D) Any point on the line segment joining $(0,2)$ and $(3,0)$

Q6. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) 0

Q7. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. The probability that the ball is drawn from the first bag is:

(A) 2/3

(B) 1/2

(C) 3/4

(D) 1/3

Q8. The shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is:

(A) $3\sqrt{2}$

(B) $3/\sqrt{2}$

(C) 0

(D) $1/\sqrt{3}$

Q9. The principal value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is:

(A) $2\pi/3$

(B) $\pi/3$

(C) $4\pi/3$

(D) $-\pi/3$

Q10. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A

(B) $I - A$

(C) I

(D) $3A$

Q11. Relation R on $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is:

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Equivalence

Q12. If $y = \log(\cos e^x)$, then dy/dx is:

(A) $-e^x \tan e^x$

(B) $e^x \tan e^x$

(C) $e^x \cos e^x$

(D) None of these

Q13. Integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(A) x

(B) $1/x$

(C) e^x

(D) $\log x$

Q14. The maximum value of the function $f(x) = \sin x + \cos x$ is:

(A) $\sqrt{2}$

(B) 2

(C) 1

(D) $1/\sqrt{2}$

Q15. Value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is:

(A) π

(B) $-\pi/3$

(C) $\pi/3$

(D) $2\pi/3$

Q16. Projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $5/\sqrt{6}$

(C) 10

(D) 6

Q17. If $P(A) = 0.4, P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.24

(B) 0.96

(C) 0.48

(D) 0.16

Q18. Derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sin^{-1} x$ is:

(A) 1

(B) 2

(C) 1/2

(D) 0

Q19. The integral $\int \frac{dx}{x^2+2x+2}$ is:

(A) $\tan^{-1}(x+1) + C$

(B) $\tan^{-1} x + 1$

(C) $\log(x^2 + 2x + 2)$

(D) None of these

Q20. Number of binary operations on the set $\{a, b\}$ is:

(A) 2

(B) 4

(C) 16

(D) 8

Q21. Matrix A is skew-symmetric, then the diagonal elements a_{ii} are:

(A) 1

(B) -1

(C) 0

(D) Any real number

Q22. Rate of change of area of circle with respect to radius r at $r = 5$ is:

(A) 10π

(B) 5π

(C) 20π

(D) π

Q23. The integral $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ is:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $e^x + C$

(D) None of these

Q24. Distance of point $(2, 3, 4)$ from x-axis is:

(A) 5

(B) $\sqrt{13}$

(C) $\sqrt{20}$

(D) $\sqrt{29}$

Q25. If A is 3×4 and B is 4×3, then the order of AB is:

(A) 4×4

(B) 3×3

(C) 3×4

(D) Not defined

Q26. The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is:

(A) 2 sq. units

(B) 4 sq. units

(C) 0 sq. units

(D) 1 sq. unit

Q27. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/2$

(D) $2\pi/3$

Q28. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^y = e^x + x^2 + C$

(C) $e^x = e^y + \frac{x^3}{3} + C$

(D) $e^{x+y} = x^2 + C$

Q29. The value of λ for which the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar is:

(A) 4

(B) -4

(C) 8

(D) -8

Q30. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B')$ is:

(A) 0.12

(B) 0.18

(C) 0.28

(D) 0.7

Q31. The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \leq 4, x \geq 0, y \geq 0$ is:

(A) 12

(B) 16

(C) 10

(D) 0

Q32. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q33. The integral $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to:

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\sin x + \cos x + C$

(D) $\tan x \cot x + C$

Q34. The degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is:

(A) 1

(B) 2

(C) 3

(D) Not defined

Q35. The feasible region for an LPP is shown in a graph. If the region is unbounded, then the objective function $Z = ax + by$:

(A) Must have a maximum

(B) Must have a minimum

(C) May or may not have an optimal value

(D) Never has a solution

Q36. If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, then dy/dx is:

(A) $\frac{3}{1+x^2}$

(B) $\frac{1}{1+x^2}$

(C) $\frac{3}{1+9x^2}$

(D) $\frac{3x^2}{1+x^2}$

Q37. The points of local minima for $f(x) = x^3 - 3x + 2$ is:

(A) $x = 1$

(B) $x = -1$

(C) $x = 0$

(D) $x = 2$

Q38. The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals:

(A) $\pi/3$

(B) $\pi/6$

(C) $\pi/12$

(D) $\pi/4$

Q39. The direction cosines of a line equally inclined to the axes are:

(A) $(1, 1, 1)$

(B) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(C) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

(D) $(0, 0, 1)$

Q40. If $P(A) = 1/2, P(B) = 0$, then $P(A|B)$ is:

(A) 0

(B) 1/2

(C) Not defined

(D) 1

Q41. Total number of possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Q42. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3 - 4x$, then f is:

(A) One-to-one but not onto

(B) Onto but not one-to-one

(C) Bijective

(D) Neither

Q43. The value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ is:

(A) $e^x/x + C$

(B) $-e^x/x^2 + C$

(C) $e^x \log x + C$

(D) $xe^x + C$

Q44. The projection of vector $\vec{a} = \hat{i} - \hat{j}$ on $\vec{b} = \hat{i} + \hat{j}$ is:

(A) 0

(B) 1

(C) 2

(D) -1

Q45. Two events A and B are independent if:

(A) A and B are mutually exclusive

(B) $P(A' \cap B') = [1 - P(A)][1 - P(B)]$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$

Q46. The area of the triangle with vertices $(1, 0), (6, 0), (4, 3)$ using determinants is:

(A) 7.5 sq. units

(B) 15 sq. units

(C) 10 sq. units

(D) 20 sq. units

Q47. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) 1/3

(C) -1/3

(D) -3

Q48. The integral $\int \frac{dx}{x+x\log x}$ is:

(A) $\log|1 + \log x| + C$

(B) $\log|\log x| + C$

(C) $x\log(1 + x) + C$

(D) None of these

Q49. Equation of the line passing through $(1, 2, 3)$ and parallel to x-axis is:

(A) $\frac{x-1}{1} = \frac{y-2}{0} = \frac{z-3}{0}$

(B) $\frac{x-1}{0} = \frac{y-2}{1} = \frac{z-3}{0}$

(C) $x = 1, y = 2, z = 3$

(D) None of these

Q50. If A is a square matrix of order 3, then $|kA|$ is:

(A) $k|A|$

(B) $k^2|A|$

(C) $k^3|A|$

(D) $3k|A|$

Detailed Solutions

Q1.

Solution

Concept:

For a square matrix A of order n, the property of the adjoint states that $adj(kA) = k^{n-1}adj(A)$. More importantly, the determinant of the adjoint is given by $|adj A| = |A|^{n-1}$. For the double adjoint, the property states:

$$adj(adj A) = |A|^{n-2}A$$

Taking the determinant on both sides:

$$|adj(adj A)| = |A|^{(n-1)^2}$$

This formula allows us to find the determinant of a nested adjoint without calculating the matrices themselves.

Solution:

1. Given data: - Order of the matrix $(n) = 3$ - Determinant of matrix A $(|A|) = 5$
2. Apply the formula for $|adj(adj A)|$:

$$|adj(adj A)| = |A|^{(n-1)^2}$$

3. Substitute the values: - $n - 1 = 3 - 1 = 2$ - $(n - 1)^2 = 2^2 = 4$
4. Final calculation:

$$|adj(adj A)| = 5^4$$

$$5 \times 5 \times 5 \times 5 = 625$$

Answer: (C)

Q2.

Solution

Concept:

A function $f(x)$ is said to be continuous at a point $x = a$ if the Left Hand Limit (LHL), Right Hand Limit (RHL), and the value of the function at that point $f(a)$ are all equal. Mathematically:

$$\lim_{x \to a} f(x) = f(a)$$

For trigonometric limits involving $x \to \frac{\pi}{2}$, we often use the substitution method where $x = \frac{\pi}{2} + h$ such that as $x \to \frac{\pi}{2}$, $h \to 0$.

Solution:

1. Given data: - $f(x) = \frac{k \cos x}{\pi - 2x}$ for $x \neq \frac{\pi}{2}$ - $f(\frac{\pi}{2}) = 3$
2. Set up the limit for continuity:

$$\lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = 3$$

3. Evaluate the limit using substitution $x = \frac{\pi}{2} + h$: - As $x \to \frac{\pi}{2}$, $h \to 0$. - $\cos x = \cos(\frac{\pi}{2} + h) = -\sin h$ - $\pi - 2x = \pi - 2(\frac{\pi}{2} + h) = \pi - \pi - 2h = -2h$
4. Substitute into the limit:

$$\lim_{h \to 0} \frac{k(-\sin h)}{-2h} = \lim_{h \to 0} \frac{k}{2} \cdot \frac{\sin h}{h}$$

Since $\lim_{h \to 0} \frac{\sin h}{h} = 1$:

$$\frac{k}{2} \times 1 = \frac{k}{2}$$

5. Equate to the function value:

$$\frac{k}{2} = 3 \implies k = 6$$

Answer: (B)

Q3.

Solution

Concept:

The area of a region bounded by a curve $y^2 = f(x)$, the x-axis, and vertical lines $x = a$ and $x = b$ is given by the definite integral:

$$\text{Area} = \int_a^b y\, dx$$

For a parabola $y^2 = 4ax$ which is symmetric about the x-axis, the total area bounded by a vertical line includes the regions both above and below the x-axis. Therefore, we calculate the area in the first quadrant and multiply by 2.

Solution:

1. Given equations: - Curve: $y^2 = 4x \implies y = \pm\sqrt{4x} = \pm 2\sqrt{x}$ - Line: $x = 3$ - Boundary: From $x = 0$ (vertex) to $x = 3$.

2. Set up the integral for the upper half (1st quadrant):

$$\text{Area}_1 = \int_0^3 2\sqrt{x}\, dx$$

3. Integrate:

$$2\int_0^3 x^{1/2}\, dx = 2\left[\frac{x^{3/2}}{3/2}\right]_0^3$$

$$= 2 \cdot \frac{2}{3}[x^{3/2}]_0^3 = \frac{4}{3}[3\sqrt{3} - 0]$$

$$= 4\sqrt{3} \text{ sq. units}$$

4. Total Area (symmetric parts):

$$\text{Total Area} = 2 \times 4\sqrt{3} = 8\sqrt{3} \text{ sq. units}$$

Answer: (A)

Q4.

Solution

Concept:

In a differential equation: - **Order:** The order of the highest order derivative appearing in the equation. - **Degree:** The power of the highest order derivative, provided the equation is expressed as a polynomial in derivatives. To find the degree, all fractional powers and radicals must be removed from the derivatives.

Solution:

1. Given equation:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$$

2. Identify the derivatives: - First derivative: $\frac{dy}{dx}$ - Second derivative: $\frac{d^2y}{dx^2}$ The highest order derivative is $\frac{d^2y}{dx^2}$, so the **Order = 2**.

3. Remove fractional powers to find the degree: Square both sides to eliminate the exponent $3/2$:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2=\left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3}=\left(\frac{d^2y}{dx^2}\right)^2$$

4. Identify the power of the highest order derivative: In the polynomial form, the power of $\frac{d^2y}{dx^2}$ is 2. So, the **Degree = 2**.

Answer: (A)

Q5.

Solution

Concept:

According to the **Corner Point Method** in Linear Programming, if the feasible region is bounded, the optimal value (maximum or minimum) of the objective function $Z = ax + by$ must occur at the corner points (vertices). If the objective function attains the same optimal value at two corner points, then it attains that same value at every point on the line segment joining those two points.

Solution:

1. Given Objective Function:

$$Z = 4x + 6y$$

2. Corner points provided: $(0, 2), (3, 0), (6, 0), (6, 8), (0, 5)$.

3. Calculate Z at each corner point: - At $(0, 2) : Z = 4(0) + 6(2) = 12$ - At $(3, 0) : Z = 4(3) + 6(0) = 12$ - At $(6, 0) : Z = 4(6) + 6(0) = 24$ - At $(6, 8) : Z = 4(6) + 6(8) = 24 + 48 = 72$ - At $(0, 5) : Z = 4(0) + 6(5) = 30$

4. Comparison: The minimum value of Z is 12. This occurs at both $(0, 2)$ and $(3, 0)$.

5. Conclusion: Since the minimum value is reached at two distinct vertices, any point on the line segment joining these two points will also yield $Z = 12$.

Answer: (D)

Q6.

Solution

Concept:

This integral is solved using the **King's Property** of definite integrals, which states:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

This property is particularly effective for trigonometric functions involving limits like 0 to $\pi/2$ because $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$. When the original and modified integrals are added, the terms usually simplify to 1.

Solution:

1. Let the given integral be I:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad \ldots(1)$$

2. Apply the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: Replace x with $(\pi/2 - x)$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx \quad \ldots(2)$$

3. Add equations (1) and (2):

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

4. Integrate:

$$2I = [x]_0^{\pi/2} = \pi/2 - 0$$

$$I = \pi/4$$

Answer: (C)

Q7.

Solution

Concept:

This problem is solved using **Bayes' Theorem**, which calculates the probability of an event based on prior knowledge of conditions related to the event. For two mutually exclusive events E_1 (Bag 1 selected) and E_2 (Bag 2 selected), and an event A (Red ball drawn):

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

This formula determines the "posterior probability" that the red ball came from Bag 1.

Solution:

1. Define the events: - E_1: Selecting Bag 1. $P(E_1) = 1/2$ - E_2: Selecting Bag 2. $P(E_2) = 1/2$ - A: Drawing a Red ball.

2. Calculate conditional probabilities $P(A|E_i)$: - Bag 1 (4R, 4B): Total = 8. $P(A|E_1) = 4/8 = 1/2$ - Bag 2 (2R, 6B): Total = 8. $P(A|E_2) = 2/8 = 1/4$

3. Apply Bayes' Theorem for $P(E_1|A)$:

$$P(E_1|A) = \frac{(1/2) \times (1/2)}{(1/2 \times 1/2) + (1/2 \times 1/4)}$$

$$P(E_1|A) = \frac{1/4}{1/4 + 1/8}$$

4. Simplify the denominator:

$$1/4 + 1/8 = 2/8 + 1/8 = 3/8$$

$$P(E_1|A) = \frac{1/4}{3/8} = \frac{1}{4} \times \frac{8}{3} = 2/3$$

Answer: (A)

Q8.

Solution

Concept:

The shortest distance (SD) between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is the magnitude of the projection of the vector $(\vec{a}_2 - \vec{a}_1)$ on the vector perpendicular to both lines $(\vec{b}_1 \times \vec{b}_2)$. The formula is:

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

If the scalar triple product in the numerator is zero, the lines intersect and the distance is zero.

Solution:

1. Identify vectors from the lines: - $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ - $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

2. Calculate $(\vec{a}_2 - \vec{a}_1)$:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$$

3. Calculate $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$$

Magnitude $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$.

4. Calculate the Dot Product:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -3 + 0 - 6 = -9$$

5. Final Shortest Distance:

$$d = \left| \frac{-9}{3\sqrt{2}} \right| = \frac{3}{\sqrt{2}}$$

Answer: (B)

Q9.

Solution

Concept:
The principal value of $\sin^{-1}(\sin\theta)$ is equal to θ only if θ lies within the principal value branch of $\sin^{-1}x$, which is $[-\pi/2, \pi/2]$. If the angle θ lies outside this interval, we must use trigonometric identities such as $\sin(\pi-\theta) = \sin\theta$ or $\sin(\theta-2\pi) = \sin\theta$ to bring the angle within the allowed range without changing the value of the sine function.

Solution:

1. Given expression:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

2. Check the interval: The angle is $\frac{2\pi}{3} = 120°$. The principal range is $[-\pi/2, \pi/2]$, which is $[-90°, 90°]$. Since $120° > 90°$, we cannot simply cancel the $\sin^{-1}$ and sin.

3. Use the identity $\sin(\pi-\theta) = \sin\theta$:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

4. Substitute back:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

5. Evaluate: Since $\frac{\pi}{3} \in [-\pi/2, \pi/2]$, the final value is $\frac{\pi}{3}$.

Answer: (B)

Q10.

Solution

Concept:
In matrix algebra, if A is an **idempotent matrix**, it satisfies the condition $A^2 = A$. This implies that any higher power of A (like $A^3, A^4, \dots$) will also be equal to A. Furthermore, when expanding binomial expressions like $(I + A)^n$, where I is the identity matrix, we can use the standard binomial expansion because I commutes with any matrix ($AI = IA$).

Solution:
1. Given condition: $A^2 = A$. Note: $A^3 = A^2 \cdot A = A \cdot A = A^2 = A$. In general, $A^n = A$ for $n \geq 1$.
2. Expand $(I + A)^3$:

$$(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$$

Using the properties $I^n = I$, $IA = A$, and $A^n = A$:

$$(I + A)^3 = I + 3A + 3A + A$$

$$(I + A)^3 = I + 7A$$

3. Evaluate the full expression:

$$(I + A)^3 - 7A = (I + 7A) - 7A$$

$$= I$$

Answer: (C)

Q11.

Solution

Concept:
A relation R on a set A is analyzed based on three properties: 1. **Reflexive:** If $(a, a) \in R$ for every $a \in A$. 2. **Symmetric:** If $(a, b) \in R \implies (b, a) \in R$. 3. **Transitive:** If $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$. If all three hold, it is an Equivalence Relation.

Solution:
1. Given set $A = \{1, 2, 3\}$ and relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$.
2. **Check Reflexivity:** Since $(1, 1)$, $(2, 2)$, and $(3, 3)$ are all in R, the relation is **Reflexive**.
3. **Check Symmetry:** $(1, 2) \in R$ but $(2, 1) \notin R$. Therefore, it is **not symmetric**.
4. **Check Transitivity:** $(1, 2) \in R$ and $(2, 3) \in R$. For transitivity, $(1, 3)$ must be in R. However, $(1, 3) \notin R$. Therefore, it is **not transitive**.
5. Conclusion: The only property satisfied among the standard definitions is reflexivity.

Answer: (A)

Q12.

Solution

Concept:

This problem requires the application of the **Chain Rule** for differentiation. For a composite function $y = f(g(h(x)))$, the derivative is:

$$\frac{dy}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

The basic derivatives used here are: $\frac{d}{dx}(\log u) = \frac{1}{u}\frac{du}{dx}$, $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$, and $\frac{d}{dx}(e^x) = e^x$.

Solution:

1. Given function: $y = \log(\cos e^x)$.
2. Apply Chain Rule (Step 1 - Outer log):

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot \frac{d}{dx}(\cos e^x)$$

3. Apply Chain Rule (Step 2 - Cosine):

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot \frac{d}{dx}(e^x)$$

4. Apply Chain Rule (Step 3 - Exponential):

$$\frac{dy}{dx} = \frac{-\sin e^x}{\cos e^x} \cdot e^x$$

5. Simplify: Since $\frac{\sin u}{\cos u} = \tan u$:

$$\frac{dy}{dx} = -e^x \tan e^x$$

Answer: (A)

Q13.

Solution

Concept:

For a first-order linear differential equation in the standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

The **Integrating Factor (I.F.)** is calculated using the formula:

$$I.F. = e^{\int P(x)\,dx}$$

The given equation must first be rearranged into this standard form by ensuring the coefficient of $\frac{dy}{dx}$ is 1.

Solution:

1. Given equation: $x\frac{dy}{dx} - y = 2x^2$.
2. Convert to standard form by dividing by x:

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

3. Identify $P(x)$: Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$, we get:

$$P(x) = -\frac{1}{x}$$

4. Calculate $I.F.$:

$$I.F. = e^{\int -\frac{1}{x}\,dx}$$

$$I.F. = e^{-\log x}$$

5. Simplify using logarithmic properties ($e^{\log u} = u$ and $n \log a = \log a^n$):

$$I.F. = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

Answer: (B)

Q14.

Solution

Concept:

To find the maximum value of a function $f(x) = a\sin x + b\cos x$, one can use calculus (differentiation) or the direct trigonometric identity: The range of $a\sin x + b\cos x$ is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$. Thus, the maximum value is $+\sqrt{a^2+b^2}$.

Solution:

1. Given function: $f(x) = \sin x + \cos x$.
2. Identify coefficients: Here, $a = 1$ and $b = 1$.
3. Apply the formula for maximum value:

$$\text{Max Value} = \sqrt{a^2+b^2}$$

$$\text{Max Value} = \sqrt{1^2+1^2} = \sqrt{2}$$

4. Verification via Calculus: $f'(x) = \cos x - \sin x$. Set $f'(x) = 0 \implies \tan x = 1 \implies x = \pi/4$. $f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

Answer: (A)

Q15.

Solution

Concept:

This problem involves finding the principal values of inverse trigonometric functions: - $\tan^{-1} x$: Range is $(-\pi/2, \pi/2)$. - $\sec^{-1} x$: Range is $[0, \pi] - \{\pi/2\}$. For negative values in $\sec^{-1}$, we use the identity: $\sec^{-1}(-x) = \pi - \sec^{-1} x$.

Solution:

1. Evaluate $\tan^{-1}\sqrt{3}$: Since $\tan(\pi/3) = \sqrt{3}$ and $\pi/3 \in (-\pi/2, \pi/2)$:

$$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

2. Evaluate $\sec^{-1}(-2)$: Using the identity $\sec^{-1}(-x) = \pi - \sec^{-1} x$:

$$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$$

Since $\sec(\pi/3) = 2$:

$$\sec^{-1}(-2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

3. Substitute into the original expression:

$$\frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

Answer: (B)

Q16.

Solution

Concept:

The **scalar projection** of a vector $\vec{a}$ on another vector $\vec{b}$ is the magnitude of the component of $\vec{a}$ in the direction of $\vec{b}$. It is calculated using the dot product formula:

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

This results in a scalar value. If the vector projection were required, we would multiply this result by the unit vector $\hat{b}$.

Solution:

1. Given vectors: - $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ - $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$
2. Calculate the Dot Product $(\vec{a} \cdot \vec{b})$:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1)$$

$$= 2 + 6 + 2 = 10$$

3. Calculate the magnitude of $\vec{b}$ $(|\vec{b}|)$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

4. Calculate the projection:

$$\text{Projection} = \frac{10}{\sqrt{6}}$$

Answer: (A)

Q17.

Solution

Concept:

This problem utilizes the **Addition Theorem of Probability** and the definition of **Conditional Probability**. The Addition Theorem states:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability is defined as:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies P(A \cap B) = P(A) \cdot P(B|A)$$

Solution:

1. Given data: - $P(A) = 0.4$ - $P(B) = 0.8$ - $P(B|A) = 0.6$
2. Find the intersection $P(A \cap B)$:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B) = 0.4 \times 0.6 = 0.24$$

3. Find the union $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Answer: (B)

Q18.

Solution

Concept:

To find the derivative of one function u with respect to another function v, we use the parameterization method:

$$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

Alternatively, we can simplify the functions using trigonometric substitutions. For $\sin^{-1}(2x\sqrt{1-x^2})$, the substitution $x = \sin\theta$ simplifies the expression significantly based on the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$.

Solution:

1. Let $u = \sin^{-1}(2x\sqrt{1-x^2})$ and $v = \sin^{-1} x$.
2. Simplify u using substitution $x = \sin\theta$:

$$u = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$$

$$u = \sin^{-1}(2\sin\theta\cos\theta)$$

$$u = \sin^{-1}(\sin 2\theta) = 2\theta$$

Since $x = \sin\theta \implies \theta = \sin^{-1} x$:

$$u = 2\sin^{-1} x$$

3. Express u in terms of v: Since $v = \sin^{-1} x$, we have:

$$u = 2v$$

4. Differentiate u with respect to v:

$$\frac{du}{dv} = \frac{d}{dv}(2v) = 2$$

Answer: (B)

Q19.

Solution

Concept:

To integrate a function with a quadratic denominator $ax^2 + bx + c$, we use the method of **completing the square**. Once the denominator is in the form $(x \pm \alpha)^2 + \beta^2$, the integral matches the standard form:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Solution:

1. Given integral: $\int \frac{dx}{x^2+2x+2}$.
2. Complete the square for $x^2 + 2x + 2$:

$$x^2 + 2x + 2 = (x^2 + 2x + 1) + 1$$

$$= (x + 1)^2 + 1^2$$

3. Substitute back into the integral:

$$\int \frac{dx}{(x + 1)^2 + 1^2}$$

4. Apply the standard formula where $u = x + 1$ and $a = 1$:

$$\tan^{-1}(x + 1) + C$$

Answer: (A)

Q20.

Solution

Concept:

A binary operation on a set A is a function $* : A \times A \to A$. The total number of binary operations on a set with n elements is determined by the total number of functions from $A \times A$ to A. - The number of elements in $A \times A$ is $n \times n = n^2$. - The number of elements in the codomain is n. The formula for the total number of functions is (size of codomain)$^{\text{size of domain}}$. Thus, Total Binary Operations $= n^{(n^2)}$.

Solution:

1. Given set $A = \{a, b\}$. Number of elements $(n) = 2$.
2. Calculate the size of the domain $(A \times A)$: $n^2 = 2^2 = 4$.
3. Apply the formula:

$$\text{Total Operations} = n^{(n^2)}$$

$$= 2^4$$

4. Final Calculation: $2 \times 2 \times 2 \times 2 = 16$.

Answer: (C)

Q21.

Solution

Concept:

A square matrix $A = [a_{ij}]$ is called **skew-symmetric** if $A^T = -A$. This means for all indices i and j, the relation $a_{ji} = -a_{ij}$ must hold. For the diagonal elements, where the row index and column index are the same ($i = j$), this property leads to a specific mathematical requirement: $a_{ii} = -a_{ii}$.

Solution:

1. Definition of skew-symmetry:

$$a_{ij} = -a_{ji} \text{ for all } i, j$$

2. For diagonal elements ($i = j$): Substitute $j = i$ into the definition:

$$a_{ii} = -a_{ii}$$

3. Solve the equation:

$$a_{ii} + a_{ii} = 0$$

$$2a_{ii} = 0$$

$$a_{ii} = 0$$

4. Conclusion: All diagonal elements of a skew-symmetric matrix must be zero.

Answer: (C)

Q22.

Solution

Concept:

The **rate of change** of one quantity with respect to another is found by differentiating the function relating the two variables. If $y = f(x)$, the rate of change of y with respect to x is dy/dx. In geometry, to find how the area A of a circle changes with respect to its radius r, we differentiate the area formula $A = \pi r^2$ with respect to r.

Solution:

1. Area of a circle formula:

$$A = \pi r^2$$

2. Differentiate A with respect to r:

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2)$$

$$\frac{dA}{dr} = 2\pi r$$

3. Substitute the given radius ($r = 5$):

$$\left.\frac{dA}{dr}\right|_{r=5} = 2\pi(5)$$

$$= 10\pi$$

4. Interpretation: The area is increasing at a rate of 10π square units per unit change in radius.

Answer: (A)

Q23.

Solution

Concept:
This integral is solved using the **method of substitution**. We look for a part of the integrand whose derivative is also present. The expression in the denominator, xe^x, has a derivative that appears in the numerator. The derivative of $u \cdot v$ (Product Rule) is $u'v + uv'$. Specifically, $\frac{d}{dx}(xe^x) = (1)e^x + x(e^x) = e^x(1 + x)$.

Solution:
1. Given integral: $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$.
2. Let $u = xe^x$. Then, $\frac{du}{dx} = e^x + xe^x = e^x(1 + x)$. This implies $du = e^x(1 + x)dx$.
3. Substitute u and du into the integral:

$$\int \frac{1}{\cos^2 u} du$$

4. Simplify the integrand: Since $\frac{1}{\cos^2 u} = \sec^2 u$:

$$\int \sec^2 u \, du$$

5. Integrate:

$$\int \sec^2 u \, du = \tan u + C$$

6. Substitute back $u = xe^x$:

$$\tan(xe^x) + C$$

Answer: (A)

Q24.

Solution

Concept:

The distance of a point $P(x, y, z)$ from the coordinate axes in 3D space can be found using the Pythagorean theorem in a plane perpendicular to the axis. - Distance from x-axis: $d = \sqrt{y^2 + z^2}$ - Distance from y-axis: $d = \sqrt{x^2 + z^2}$ - Distance from z-axis: $d = \sqrt{x^2 + y^2}$ For the x-axis, the foot of the perpendicular from (x, y, z) is $(x, 0, 0)$.

Solution:

1. Given point: $P(2, 3, 4)$. Here, $x = 2, y = 3, z = 4$.
2. To find the distance from the x-axis: The projection (foot of perpendicular) on the x-axis is $A(2, 0, 0)$.
3. Use the distance formula between $P(2, 3, 4)$ and $A(2, 0, 0)$:

$$d = \sqrt{(2-2)^2 + (3-0)^2 + (4-0)^2}$$

$$d = \sqrt{0^2 + 3^2 + 4^2}$$

$$d = \sqrt{9 + 16} = \sqrt{25} = 5$$

Answer: (A)

Q25.

Solution

Concept:

The multiplication of two matrices A and B is defined only if the number of columns in the first matrix equals the number of rows in the second matrix. If matrix A has dimensions $m \times n$ and matrix B has dimensions $n \times p$, then: 1. Matrix multiplication AB is possible. 2. The resulting matrix AB will have dimensions $m \times p$ (the number of rows of A and the number of columns of B).

Solution:

1. Identify the dimensions of matrix A: Order of $A = 3 \times 4$. (Rows = 3, Columns = 4)
2. Identify the dimensions of matrix B: Order of $B = 4 \times 3$. (Rows = 4, Columns = 3)
3. Check for compatibility: Columns of A (4) = Rows of B (4). Multiplication is defined.
4. Determine the order of AB: The order will be (Rows of A) $\times$ (Columns of B). Order of $AB = 3 \times 3$.

Answer: (B)

Q26.

Solution

Concept:

The area bounded by a trigonometric curve $y = f(x)$ over an interval is the sum of the absolute values of the areas of the individual loops. Since $y = \cos x$ is positive in some intervals and negative in others, a simple definite integral from 0 to 2π would yield zero (as the areas above and below the x-axis cancel out). To find the actual area, we must integrate the absolute value:

$$\text{Area} = \int_0^{2\pi} |\cos x|\, dx$$

This corresponds to four equal quadrants of the cosine wave, each having an area of 1 unit.

Solution:

1. Identify the roots of $\cos x = 0$ in $[0, 2\pi]$: The curve crosses the x-axis at $x = \pi/2$ and $x = 3\pi/2$.

2. Split the integral into three parts:

$$\text{Area} = \int_0^{\pi/2} \cos x\, dx + \left| \int_{\pi/2}^{3\pi/2} \cos x\, dx \right| + \int_{3\pi/2}^{2\pi} \cos x\, dx$$

3. Calculate each part: - Part 1: $[\sin x]_0^{\pi/2} = 1 - 0 = 1$ - Part 2: $|[\sin x]_{\pi/2}^{3\pi/2}| = |-1 - 1| = 2$ - Part 3: $[\sin x]_{3\pi/2}^{2\pi} = 0 - (-1) = 1$

4. Sum the areas:

$$\text{Total Area} = 1 + 2 + 1 = 4 \text{ sq. units}$$

Answer: (B)

Q27.

Solution

Concept:
For any two vectors $\vec{a}$ and $\vec{b}$, the magnitude of their sum is given by the formula:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

where θ is the angle between the two vectors. A **unit vector** has a magnitude of exactly 1. If $\vec{a}$, $\vec{b}$, and their sum $\vec{a}+\vec{b}$ are all unit vectors, we can substitute their magnitudes into this equation to solve for θ.

Solution:

1. Given data: - $|\vec{a}| = 1$ - $|\vec{b}| = 1$ - $|\vec{a}+\vec{b}| = 1$
2. Substitute into the magnitude squared formula:

$$1^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta$$

$$1 = 1 + 1 + 2\cos\theta$$

3. Solve for $\cos\theta$:

$$1 = 2 + 2\cos\theta$$

$$-1 = 2\cos\theta$$

$$\cos\theta = -1/2$$

4. Find the angle θ:

$$\theta = \cos^{-1}(-1/2) = 120^\circ \text{ or } 2\pi/3$$

Answer: (D)

Q28.

Solution

Concept:
This is a **Variable Separable** differential equation. The primary goal is to group all terms containing y with dy and all terms containing x with dx. Using the laws of exponents ($e^{a-b} = e^a \cdot e^{-b}$), we can factor out the common term e^{-y} to separate the variables. Once separated, we integrate both sides independently.

Solution:

1. Given equation:

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

2. Factor out e^{-y} on the right-hand side:

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

3. Separate the variables: Multiply both sides by e^y and dx:

$$e^y \, dy = (e^x + x^2) \, dx$$

4. Integrate both sides:

$$\int e^y \, dy = \int (e^x + x^2) \, dx$$

5. Perform the integration:

$$e^y = e^x + \frac{x^3}{3} + C$$

Answer: (A)

Q29.

Solution

Concept:

Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are **coplanar** (lie in the same plane) if their **Scalar Triple Product** is equal to zero. The Scalar Triple Product $\vec{a} \cdot (\vec{b} \times \vec{c})$ can be calculated using the determinant of a matrix formed by the components of the three vectors. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, etc., then:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

Solution:

1. Write the vectors in determinant form:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

2. Expand the determinant along the first row:

$$2[2(5) - (-3)(\lambda)] - (-1)[1(5) - (-3)(3)] + 1[1(\lambda) - 2(3)] = 0$$

3. Simplify the terms:

$$2(10 + 3\lambda) + 1(5 + 9) + (\lambda - 6) = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

4. Combine like terms:

$$7\lambda + 28 = 0$$

5. Solve for λ:

$$7\lambda = -28 \implies \lambda = -4$$

Answer: (B)

Q30.

Solution

Concept:

Two events A and B are **independent** if the occurrence of one does not affect the probability of the other. Mathematically, this implies $P(A \cap B) = P(A) \cdot P(B)$. Furthermore, if A and B are independent, then: - A and B' are independent. - A' and B are independent. - A' and B' are independent. The notation B' represents the complement of event B, where $P(B') = 1 - P(B)$.

Solution:

1. Given data: - $P(A) = 0.3$ - $P(B) = 0.4$
2. Find $P(B')$:

$$P(B') = 1 - P(B) = 1 - 0.4 = 0.6$$

3. Use the property of independence for A and B':

$$P(A \cap B') = P(A) \cdot P(B')$$

4. Substitute the values:

$$P(A \cap B') = 0.3 \times 0.6 = 0.18$$

Answer: (B)

Q31.

Solution

Concept:

To find the maximum value of an objective function $Z = ax + by$ in a **Linear Programming Problem (LPP)**, we identify the feasible region defined by the constraints. According to the **Fundamental Theorem of LPP**, the optimal value occurs at one of the corner points (vertices) of the feasible region. The constraints $x \geq 0$ and $y \geq 0$ restrict the region to the first quadrant.

Solution:

1. Identify the constraints: - $x + y \leq 4$ - $x \geq 0, y \geq 0$
2. Find the corner points of the feasible region: - The line $x + y = 4$ intersects the x-axis at $(4, 0)$. - The line $x + y = 4$ intersects the y-axis at $(0, 4)$. - The intersection of $x = 0$ and $y = 0$ is the origin $(0, 0)$. The corner points are $(0, 0)$, $(4, 0)$, and $(0, 4)$.
3. Evaluate the objective function $Z = 3x+4y$ at each corner point: - At $(0, 0) : Z = 3(0)+4(0) = 0$ - At $(4, 0) : Z = 3(4) + 4(0) = 12$ - At $(0, 4) : Z = 3(0) + 4(4) = 16$
4. Conclusion: Comparing the values, the maximum value of Z is 16.

Answer: (B)

Q32.

Solution

Concept:

A **stationary point** of a function $f(x)$ occurs where its first derivative is zero ($f'(x) = 0$). For functions of the form x^x, we use **Logarithmic Differentiation** because the variable is in both the base and the exponent. The derivative of $\log(f(x))$ is $\frac{1}{f(x)} f'(x)$, and we use the product rule to differentiate $x \log x$.

Solution:

1. Let $y = x^x$. Taking natural logs on both sides:

$$\log y = \log(x^x) = x \log x$$

2. Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x \log x)$$

Using the product rule:

$$\frac{1}{y}\frac{dy}{dx} = (1)\log x + x\left(\frac{1}{x}\right) = \log x + 1$$

3. Find the derivative $f'(x)$:

$$\frac{dy}{dx} = y(\log x + 1) = x^x(\log x + 1)$$

4. Set $f'(x) = 0$ to find stationary points: Since x^x is never zero for $x > 0$:

$$\log x + 1 = 0$$

$$\log x = -1$$

5. Solve for x:

$$x = e^{-1} = \frac{1}{e}$$

Answer: (B)

Q33.

Solution

Concept:

To integrate a trigonometric fraction where the denominator is a product of squared sine and cosine terms, we can use the fundamental trigonometric identity $\sin^2 x + \cos^2 x = 1$ to split the integral. By replacing the '1' in the numerator with this identity, the fraction can be divided into two simpler parts that match the standard derivatives of $\tan x$ and $-\cot x$.

Solution:

1. Rewrite the integral:

$$\int \frac{1}{\sin^2 x \cos^2 x}\, dx$$

2. Substitute $1 = \sin^2 x + \cos^2 x$:

$$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

3. Split the fraction into two terms:

$$\int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$\int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

4. Simplify using reciprocal identities:

$$\int (\sec^2 x + \csc^2 x)\, dx$$

5. Integrate term by term:

$$\int \sec^2 x\, dx + \int \csc^2 x\, dx = \tan x - \cot x + C$$

Answer: (B)

Q34.

Solution

Concept:

- **Order:** The highest derivative present in the equation. - **Degree:** The highest power of the highest-order derivative, provided the differential equation can be written as a **polynomial** in terms of its derivatives. If a derivative is the argument of a transcendental function (like sin, cos, e^x, log) and cannot be removed, the equation is not a polynomial in derivatives, and the degree is undefined.

Solution:

1. Given equation:

$$\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$$

2. Identify the Order: The highest order derivative is $\frac{d^2y}{dx^2}$, so the **Order is 2**.

3. Analyze for Degree: The term $\sin\left(\frac{dy}{dx}\right)$ indicates that the equation cannot be expressed as a polynomial in the derivatives (the derivative $\frac{dy}{dx}$ is inside a sine function).

4. Conclusion: Since the equation is not a polynomial in its derivatives, the **Degree is Not defined**.

Answer: (D)

Q35.

Solution

Concept:

In Linear Programming, the **Feasible Region** is the set of all points that satisfy the constraints.
- A **Bounded** region is enclosed in a circle (finite). - An **Unbounded** region extends infinitely in at least one direction. According to the properties of LPP, if the feasible region is unbounded, the objective function $Z = ax + by$ might not have a finite maximum or minimum value because the variables can increase indefinitely.

Solution:

1. Condition: The feasible region is unbounded.

2. Rule for Optimal Values: If the feasible region is unbounded, then a maximum or a minimum value of the objective function **may or may not exist**.

3. Testing for existence: To confirm a minimum value M exists in an unbounded region, we must check if the open half-plane $ax + by < M$ has no point in common with the feasible region. If it does have common points, the minimum does not exist. A similar check applies for the maximum.

4. Conclusion: In an unbounded region, the optimal value is not guaranteed.

Answer: (C)

Q36.

Solution

Concept:

To differentiate an inverse trigonometric function like $\tan^{-1} f(x)$, we use the formula:

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

However, when the argument is a complex algebraic fraction, it is more efficient to use **trigonometric substitution**. For the expression $\frac{3x-x^3}{1-3x^2}$, the substitution $x = \tan\theta$ is ideal because it matches the triple-angle formula for tangent:

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Solution:

1. Given function: $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.
2. Let $x = \tan\theta \implies \theta = \tan^{-1} x$. Substitute x into the expression:

$$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

3. Apply the triple-angle identity:

$$y = \tan^{-1}(\tan 3\theta)$$

$$y = 3\theta$$

4. Substitute back $\theta = \tan^{-1} x$:

$$y = 3\tan^{-1} x$$

5. Differentiate with respect to x:

$$\frac{dy}{dx} = 3 \cdot \frac{d}{dx}(\tan^{-1} x) = \frac{3}{1+x^2}$$

Answer: (A)

Q37.

Solution

Concept:

A point of **local minima** for a function $f(x)$ is found using the Second Derivative Test: 1. Find the first derivative $f'(x)$ and set it to zero to find critical points. 2. Find the second derivative $f''(x)$. 3. At a critical point $x = c$, if $f''(c) > 0$, then $f(x)$ has a local minimum at c. If $f''(c) < 0$, it is a local maximum.

Solution:

1. Given function: $f(x) = x^3 - 3x + 2$.

2. Find the first derivative:

$$f'(x) = 3x^2 - 3$$

3. Find critical points by setting $f'(x) = 0$:

$$3x^2 - 3 = 0 \implies x^2 = 1 \implies x = 1, -1$$

4. Find the second derivative:

$$f''(x) = 6x$$

5. Test the critical points: - For $x = -1$: $f''(-1) = 6(-1) = -6$. Since $-6 < 0$, $x = -1$ is a point of **local maxima**. - For $x = 1$: $f''(1) = 6(1) = 6$. Since $6 > 0$, $x = 1$ is a point of **local minima**.

Answer: (A)

Q38.

Solution

Concept:

The definite integral of the form $\int \frac{1}{1+x^2} dx$ results in $\tan^{-1} x$. According to the **Fundamental Theorem of Calculus**, once the antiderivative $F(x)$ is found, the value of the definite integral from a to b is:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

We then evaluate the inverse tangent at the specific limits using standard trigonometric values.

Solution:

1. Given integral: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$.
2. Find the antiderivative:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$$

3. Apply the limits from 1 to $\sqrt{3}$:

$$[\tan^{-1} x]_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

4. Substitute standard principal values: - $\tan^{-1}(\sqrt{3}) = \pi/3$ (since $\tan 60^\circ = \sqrt{3}$) - $\tan^{-1}(1) = \pi/4$ (since $\tan 45^\circ = 1$)
5. Calculate the final value:

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$$

Answer: (C)

Q39.

Solution

Concept:

The **direction cosines** (l, m, n) of a line are the cosines of the angles α, β, γ that the line makes with the x, y, and z axes respectively. A key property of direction cosines is:

$$l^2 + m^2 + n^2 = 1$$

If a line is **equally inclined** to the axes, then $\alpha = \beta = \gamma$, which implies $l = m = n$. We can solve for these values using the identity above.

Solution:

1. Since the line is equally inclined to the axes:

$$\cos\alpha = \cos\beta = \cos\gamma \implies l = m = n$$

2. Use the identity $l^2 + m^2 + n^2 = 1$:

$$l^2 + l^2 + l^2 = 1$$

$$3l^2 = 1$$

3. Solve for l:

$$l^2 = \frac{1}{3} \implies l = \pm\frac{1}{\sqrt{3}}$$

4. Conclusion: Since $l = m = n$, the direction cosines are $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ or $(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$.

Answer: (B)

Q40.

Solution

Concept:

The **conditional probability** of an event A given event B is defined by the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This formula is valid only when $P(B) \neq 0$. If the probability of the condition (B) is zero, it means the event B is an impossible event. Dividing by zero is mathematically undefined, therefore the conditional probability itself becomes undefined.

Solution:

1. Given data: - $P(A) = 1/2$ - $P(B) = 0$

2. Set up the conditional probability formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

3. Substitute the value of $P(B)$:

$$P(A|B) = \frac{P(A \cap B)}{0}$$

4. Conclusion: Since division by zero is not allowed in mathematics, the probability $P(A|B)$ is not defined.

Answer: (C)

Q41.

Solution

Concept:

A matrix of order $m \times n$ contains $m \times n$ total elements (positions). Each position in the matrix can be filled by any of the available entries. According to the **Fundamental Principle of Counting**, if there are n positions and each position can be filled in k ways, the total number of possible matrices is given by:

$$\text{Total Matrices} = (\text{Number of possible entries})^{\text{Total number of elements}}$$

Solution:

1. Identify the order of the matrix: Order $= 3 \times 3$. Total elements $= 3 \times 3 = 9$.
2. Identify the possible entries for each element: The entries can be 0 or 1. Number of choices for each position $= 2$.
3. Apply the formula:

$$\text{Total Matrices} = 2^9$$

4. Calculate the value:

$$2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$$

Answer: (D)

Q42.

Solution

Concept:

For a function $f : A \to B$: 1. **One-to-one (Injective):** Every distinct element in the domain has a distinct image in the codomain. For a linear function $f(x) = ax + b$, it is always one-to-one if $a \neq 0$. 2. **Onto (Surjective):** Every element in the codomain has at least one pre-image in the domain (Range = Codomain). A function that is both one-to-one and onto is called **Bijective**.

Solution:

1. Given function: $f(x) = 3 - 4x$ where $f : \mathbb{R} \to \mathbb{R}$.

2. **Check for One-to-one:** Let $f(x_1) = f(x_2)$

$$3 - 4x_1 = 3 - 4x_2 \implies -4x_1 = -4x_2 \implies x_1 = x_2$$

Since $x_1 = x_2$, the function is **one-to-one**.

3. **Check for Onto:** Let $y \in \mathbb{R}$ (Codomain). We need to find $x \in \mathbb{R}$ such that $f(x) = y$.

$$y = 3 - 4x \implies 4x = 3 - y \implies x = \frac{3 - y}{4}$$

Since for every real number y, there exists a real number x, the function is **onto**.

4. Conclusion: The function is both one-to-one and onto, making it **Bijective**.

Answer: (C)

Q43.

Solution

Concept:

The integral of the form $\int e^x[f(x) + f'(x)]dx$ is a standard integral in calculus. Using the property of integration by parts, it can be proven that:

$$\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$$

This is a very powerful shortcut for integrals where the integrand consists of an exponential function multiplied by the sum of a function and its derivative.

Solution:

1. Given integral: $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$.
2. Identify the function $f(x)$: Let $f(x) = \frac{1}{x} = x^{-1}$.
3. Find the derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

4. Compare with the standard form: The integrand is exactly in the form $e^x[f(x) + f'(x)]$.
5. Apply the formula:

$$\int e^x \left[\frac{1}{x} + \left(-\frac{1}{x^2}\right)\right] dx = e^x \cdot f(x) + C$$

$$= e^x \cdot \frac{1}{x} + C = \frac{e^x}{x} + C$$

Answer: (A)

Q44.

Solution

Concept:

The **scalar projection** of vector $\vec{a}$ on vector $\vec{b}$ is given by the formula:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

The dot product measures the overlap of the two vectors. If the vectors are perpendicular, the projection will be zero.

Solution:

1. Given vectors: - $\vec{a} = \hat{\imath} - \hat{\jmath}$ - $\vec{b} = \hat{\imath} + \hat{\jmath}$

2. Calculate the Dot Product ($\vec{a} \cdot \vec{b}$):

$$\vec{a} \cdot \vec{b} = (1)(1) + (-1)(1)$$

$$= 1 - 1 = 0$$

3. Calculate the magnitude of $\vec{b}$ ($|\vec{b}|$):

$$|\vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

4. Calculate the projection:

$$\text{Projection} = \frac{0}{\sqrt{2}} = 0$$

5. Reasoning: Since the dot product is zero, the vectors are orthogonal (perpendicular), meaning $\vec{a}$ has no component in the direction of $\vec{b}$.

Answer: (A)

Q45.

Solution

Concept:

Independence of two events A and B can be defined in multiple equivalent ways: 1. $P(A \cap B) = P(A)P(B)$ 2. $P(A|B) = P(A)$ 3. $P(B|A) = P(B)$ Additionally, by De Morgan's Laws and the definition of independent complements, if A and B are independent, then their complements A' and B' are also independent, meaning:

$$P(A' \cap B') = P(A')P(B')$$

Solution:

1. Definition of independence: A and B are independent if $P(A \cap B) = P(A)P(B)$.

2. Check the properties of complements: If A and B are independent, then A' and B' are also independent. Therefore, $P(A' \cap B') = P(A') \cdot P(B')$.

3. Substitute $P(E') = 1 - P(E)$:

$$P(A' \cap B') = [1 - P(A)][1 - P(B)]$$

4. Conclusion: Option (B) represents a valid necessary and sufficient condition for independence derived from the fundamental definition.

Answer: (B)

Q46.

Solution

Concept:

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be calculated using the determinant formula:

$$\text{Area} = \frac{1}{2}\left\|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right\|$$

Since area must always be a positive quantity, we take the absolute value of the determinant result. This method is often more efficient than geometric formulas when coordinates are given.

Solution:

1. Given vertices: $(1, 0), (6, 0), (4, 3)$.
2. Set up the determinant:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

3. Expand the determinant along the second column (it contains two zeros):

$$\text{Area} = \frac{1}{2}\left|-0 + 0 - 3\begin{vmatrix} 1 & 1 \\ 6 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}|-3(1-6)|$$

$$= \frac{1}{2}|-3(-5)|$$

4. Calculate the final value:

$$= \frac{1}{2}|15| = 7.5 \text{ sq. units}$$

Answer: (A)

Q47.

Solution

Concept:

The slope of the tangent to a curve $y = f(x)$ at a point $x = x_0$ is given by the derivative $f'(x_0)$. The **normal** to a curve at a point is the line perpendicular to the tangent at that point. If the slope of the tangent is m_t, then the slope of the normal (m_n) is given by:

$$m_n = -\frac{1}{m_t}$$

This relationship holds as long as the tangent is not horizontal ($m_t \neq 0$).

Solution:

1. Given curve: $y = 2x^2 + 3\sin x$.
2. Find the derivative $\frac{dy}{dx}$:

$$\frac{dy}{dx} = 4x + 3\cos x$$

3. Calculate the slope of the tangent (m_t) at $x = 0$:

$$m_t = \left.\frac{dy}{dx}\right|_{x=0} = 4(0) + 3\cos(0)$$

$$m_t = 0 + 3(1) = 3$$

4. Calculate the slope of the normal (m_n):

$$m_n = -\frac{1}{m_t} = -\frac{1}{3}$$

Answer: (C)

Q48.

Solution

Concept:

This integral can be solved using the **method of substitution**. We look for a function u whose derivative du is present in the expression. For the term $1 + \log x$, the derivative is $\frac{1}{x}$. By factoring out x from the denominator, we can clearly see this relationship and simplify the integral into a standard logarithmic form.

Solution:

1. Given integral: $\int \frac{dx}{x+x\log x}$.
2. Factor out x in the denominator:

$$\int \frac{dx}{x(1+\log x)}$$

3. Use substitution: Let $u = 1 + \log x$. Then $\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}dx$.
4. Substitute u and du into the integral:

$$\int \frac{1}{u}\, du$$

5. Integrate:

$$\int \frac{1}{u}\, du = \log|u| + C$$

6. Substitute back $u = 1 + \log x$:

$$\log|1 + \log x| + C$$

Answer: (A)

Q49.

Solution

Concept:

The Cartesian equation of a line passing through the point (x_1, y_1, z_1) with direction ratios (a, b, c) is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

If a line is **parallel to the x-axis**, its direction ratios are proportional to the unit vector along the x-axis, which is $\hat{i}$. Therefore, the direction ratios are $(1, 0, 0)$.

Solution:

1. Given point: $(x_1, y_1, z_1) = (1, 2, 3)$.

2. Direction ratios for a line parallel to the x-axis: $a = 1, b = 0, c = 0$.

3. Substitute into the standard equation:

$$\frac{x - 1}{1} = \frac{y - 2}{0} = \frac{z - 3}{0}$$

4. Note on zero denominators: In 3D geometry, denominators of zero in line equations indicate that the coordinate remains constant (i.e., $y = 2$ and $z = 3$). The notation above is the standard symmetric form.

Answer: (A)

Q50.

Solution

Concept:

The property of determinants states that if every element of a row or column of a matrix A is multiplied by a constant k, then the determinant of the resulting matrix is $k|A|$. For a square matrix A of order n, if we multiply the entire matrix by k (i.e., kA), every element in all n rows is multiplied by k. Therefore, the factor k comes out of the determinant n times. The formula is:

$$|kA| = k^n |A|$$

Solution:

1. Given information: Matrix A is a square matrix of order $n = 3$.

2. Apply the property of determinants: For a scalar k and matrix $A_{n \times n}$:

$$|kA| = k^n |A|$$

3. Substitute $n = 3$:

$$|kA| = k^3 |A|$$

Answer: (C)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	A	5	D
6	C	7	A	8	B	9	B	10	C
11	A	12	A	13	B	14	A	15	B
16	A	17	B	18	B	19	A	20	C
21	C	22	A	23	A	24	A	25	B
26	B	27	D	28	A	29	B	30	B
31	B	32	B	33	B	34	D	35	C
36	A	37	A	38	C	39	B	40	C
41	D	42	C	43	A	44	A	45	B
46	A	47	C	48	A	49	A	50	C