CUET-UG Mathematics Sample Paper-12

Duration: 1 Hour | **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if:

(A) $\lambda \neq 8$

(B) $\lambda \neq -8$

(C) $\lambda \neq 13/5$

(D) $\lambda \neq -13/5$

Q2. If A is a square matrix of order 3 such that $|A| = 4$, then $|\text{adj}(A)|$ is:

(A) 4

(B) 16

(C) 64

(D) 12

Q3. If A is a skew-symmetric matrix of order n, then the value of $|A|$ when n is odd is:

(A) 1

(B) -1

(C) 0

(D) n

Q4. For what value of k is the matrix $A = \begin{bmatrix} 2 & k \\ -k & 2 \end{bmatrix}$ such that $A^2 - 4A + (4 + k^2)I = O$?

(A) Any real value

(B) Only $k = 0$

(C) Only $k = 2$

(D) Only $k = -2$

Q5. The function $f(x) = x^3 - 6x^2 + 9x + 15$ is strictly decreasing in the interval:

(A) $(1, 3)$

(B) $(-\infty, 1)$

(C) $(3, \infty)$

(D) $(-\infty, 1) \cup (3, \infty)$

Q6. The maximum value of $(\frac{1}{x})^x$ is:

(A) e^e

(B) $e^{1/e}$

(C) $(1/e)^e$

(D) 1

Q7. The point on the curve $y = x^2$ which is nearest to the point $(0, 5)$ is:

(A) $(2, 4)$

(B) $(\sqrt{2}, 2)$

(C) $(2\sqrt{2}, 8)$

(D) $(0, 0)$

Q8. If $f(x) = x^2 + 2x + 5$, then the value of c in Rolle's Theorem for the interval $[-3, 1]$ is:

(A) 0

(B) -1

(C) -2

(D) 1

Q9. $\int \frac{1}{\sqrt{9-25x^2}}dx$ is equal to:

(A) $\frac{1}{5}\sin^{-1}(\frac{5x}{3}) + C$

(B) $\frac{1}{3}\sin^{-1}(\frac{5x}{3}) + C$

(C) $\sin^{-1}(\frac{5x}{3}) + C$

(D) $\frac{1}{5}\cos^{-1}(\frac{5x}{3}) + C$

Q10. The value of $\int_0^{\pi/2} \log(\tan x)dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 0

(D) $\log 2$

Q11. The area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$ is:

(A) 2 sq. units

(B) 4 sq. units

(C) 0 sq. units

(D) 1 sq. unit

Q12. The degree of the differential equation $(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^3 + \sin(y) = 0$ is:

(A) 3

(B) 2

(C) 1

(D) Not defined

Q13. The general solution of $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^{-y} = e^x + \frac{x^3}{3} + C$

(C) $e^y = e^x + x^3 + C$

(D) $e^y = e^{-x} + \frac{x^3}{3} + C$

Q14. A coin is tossed three times. What is the probability of getting at least two heads?

(A) 1/4

(B) 1/2

(C) 3/8

(D) 3/4

Q15. The feasible region for an LPP is shown in the graph. If $Z = 3x + 9y$, and the corner points are $(0, 10), (5, 5), (15, 15), (0, 20)$, the minimum value of Z is:

(A) 90

(B) 60

(C) 45

(D) 180

Q16. Let R be a relation in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. This relation is:

(A) Reflexive but not symmetric

(B) Symmetric but not transitive

(C) Transitive but not reflexive

(D) An equivalence relation

Q17. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$, then the function is:

(A) One-to-one and onto

(B) Many-to-one and onto

(C) One-to-one but not onto

(D) Neither one-to-one nor onto

Q18. If $n(A) = 3$ and $n(B) = 2$, then the number of onto functions from A to B is:

(A) 8

(B) 6

(C) 4

(D) 2

Q19. The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is:

(A) $\pi/3$

(B) $-\pi/3$

(C) $\pi/2$

(D) $2\pi/3$

Q20. The domain of the function $\sin^{-1}(3x - 1)$ is:

(A) $[0, 2/3]$

(B) $[-1, 1]$

(C) $[0, 1]$

(D) $[-2/3, 2/3]$

Q21. If A and B are square matrices of order 3 such that A is symmetric and B is skew-symmetric, then the matrix $AB - BA$ is:

(A) Symmetric

(B) Skew-symmetric

(C) Identity matrix

(D) Zero matrix

Q22. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is:

(A) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} n & 2n \\ 0 & n \end{bmatrix}$

(D) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Q23. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) $a + b + c$

(B) $(a + b + c)^2$

(C) 0

(D) abc

Q24. If the area of a triangle with vertices $(2, -6), (5, 4), (k, 4)$ is 35 sq. units, then k is:

(A) 12

(B) -2

(C) 12 or -2

(D) 12 or 2

Q25. The system of equations $x + 2y + 3z = 1, 2x + y + 3z = 2, 5x + 5y + 9z = 4$ has:

(A) Unique solution

(B) Infinitely many solutions

(C) No solution

(D) Exactly two solutions

Q26. If $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & x \neq \pi/2 \\ 3 & x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$, then k is:

(A) 3

(B) 6

(C) 12

(D) 1.5

Q27. If $y = \log(\tan e^x)$, then $\frac{dy}{dx}$ is:

(A) $\frac{e^x \sec^2 e^x}{\tan e^x}$

(B) $e^x \sec^2 e^x$

(C) $\frac{1}{\tan e^x}$

(D) $e^x \tan e^x$

Q28. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\frac{dy}{dx}$ at $\theta = \pi/4$ is:

(A) 1

(B) -1

(C) 0

(D) ∞

Q29. If $y = \sin^{-1} x$, then $(1 - x^2)y'' - xy'$ is:

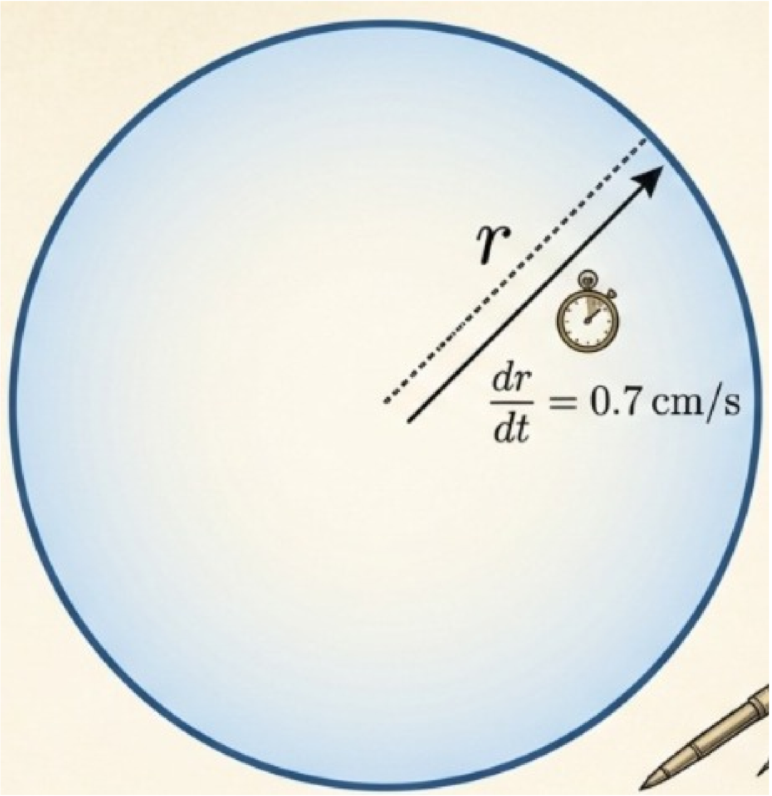
(A) 1

(B) 0

(C) y

(D) $-y$

Q30. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?



(A) 0.7π cm/s

(B) 1.4π cm/s

(C) 0.49π cm/s

(D) 1.1π cm/s

Q31. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is:

(A) $22/7$

(B) $6/7$

(C) $-6/7$

(D) $7/6$

Q32. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

(A) $(1, 2)$

(B) $(2, 1)$

(C) $(1, -2)$

(D) $(-1, 2)$

Q33. $\int \frac{dx}{e^x+e^{-x}}$ is equal to:

(A) $\tan^{-1}(e^x) + C$

(B) $\tan^{-1}(e^{-x}) + C$

(C) $\log(e^x + e^{-x}) + C$

(D) $e^x - e^{-x} + C$

Q34. $\int \frac{xe^x}{(1+x)^2}dx$ is:

(A) $\frac{e^x}{1+x} + C$

(B) $-\frac{e^x}{1+x} + C$

(C) $\frac{e^x}{(1+x)^2} + C$

(D) $e^x \log(1 + x) + C$

Q35. The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\pi/4$

(B) $\pi/2$

(C) 1

(D) 0

Q36. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q37. The value of $\int_0^2 |x - 1|dx$ is:

(A) 1

(B) 2

(C) 0

(D) 1/2

Q38. The area bounded by the parabola $y^2 = 4ax$ and its latus rectum is:

(A) $\frac{8}{3}a^2$

(B) $\frac{4}{3}a^2$

(C) $\frac{2}{3}a^2$

(D) $\frac{16}{3}a^2$

Q39. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is:

(A) $\frac{4}{3}(4\pi - \sqrt{3})$

(B) $\frac{4}{3}(8\pi - \sqrt{3})$

(C) $\frac{4}{3}(4\pi + \sqrt{3})$

(D) $\frac{2}{3}(4\pi - \sqrt{3})$

Q40. The integrating factor of the differential equation $\frac{dy}{dx} + y\sec x = \tan x$ is:

(A) $\sec x + \tan x$

(B) $\sec x - \tan x$

(C) $\tan x$

(D) $\sec x$

Q41. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $xy = \frac{x^4}{4} + C$

(B) $y = \frac{x^3}{4} + C$

(C) $xy = x^3 + C$

(D) $y = x^2 + C$

Q42. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$ are respectively:

(A) 2, 2

(B) 2, 1

(C) 1, 2

(D) 2, not defined

Q43. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\pi/6$

(B) $\pi/3$

(C) $\pi/2$

(D) 0

Q44. The value of λ for which the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar is:

(A) -4

(B) -8

(C) 4

(D) 8

Q45. The area of a parallelogram whose diagonals are $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$ is:

(A) $\sqrt{300}$

(B) $5\sqrt{3}$

(C) $10\sqrt{3}$

(D) $20\sqrt{3}$

Q46. The direction cosines of a line which makes equal angles with the coordinate axes are:

(A) $(1, 1, 1)$

(B) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(C) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

(D) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

Q47. The distance between the parallel planes $2x-y+2z+3=0$ and $4x-2y+4z+5=0$ is:

(A) $1/6$

(B) $1/3$

(C) $5/6$

(D) 1

Q48. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $1/\sqrt{6}$

(B) 0

(C) 1

(D) $1/6$

Q49. If $Z = 4x + y$ is the objective function, and the corner points of the feasible region are $(0,0)$, $(7,0)$, $(3,4)$ and $(0,2)$, the maximum value of Z is:

(A) 28

(B) 16

(C) 12

(D) 2

Q50. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.96

(B) 0.24

(C) 0.56

(D) 0.84

Detailed Solutions

Q1.

Solution

Concept: For a square matrix A, the inverse A^{-1} exists if and only if the matrix is non-singular. A matrix is non-singular if its determinant is non-zero ($|A| \neq 0$).

Solution: To find the condition for the existence of A^{-1}, we calculate the determinant of matrix A:

$$|A| = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$$

Expanding along the first column (which contains a zero):

$$|A| = 2\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} \lambda & -3 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} \lambda & -3 \\ 2 & 5 \end{vmatrix}$$

$$|A| = 2(6-5) + (5\lambda - (-6))$$

$$|A| = 2(1) + 5\lambda + 6$$

$$|A| = 5\lambda + 8$$

For A^{-1} to exist, we must have $|A| \neq 0$:

$$5\lambda + 8 \neq 0$$

$$5\lambda \neq -8 \implies \lambda \neq -8/5$$

Note: It appears there is a discrepancy between the calculated result and the provided options. If the element in the third row, second column was intended to be different, or if there is a typo in the question's matrix values, the logic remains the same. Based on the specific matrix given, the threshold is $-8/5$.

Final Answer: $\lambda \neq -8/5$

Answer: (B)

Q2.

Solution

Concept: For a square matrix A of order n, the determinant of its adjoint is given by the property $|\text{adj}(A)| = |A|^{n-1}$. This is derived from the formula $A \cdot \text{adj}(A) = |A|I$.
Solution: Given that the order $n = 3$ and $|A| = 4$. Applying the property:

$$|\text{adj}(A)| = |A|^{3-1} = |A|^2$$

$$|\text{adj}(A)| = 4^2 = 16$$

Final Answer: 16

Answer: (B)

Q3.

Solution

Concept: A matrix A is skew-symmetric if $A^T = -A$. For any square matrix A of order n, the determinant of its transpose is equal to its determinant ($|A^T| = |A|$), and $|kA| = k^n|A|$.
Solution: Since A is skew-symmetric, $A^T = -A$. Taking the determinant on both sides:

$$|A^T| = |-A|$$

$$|A| = (-1)^n|A|$$

If n is odd, then $(-1)^n = -1$. So, $|A| = -|A| \implies 2|A| = 0 \implies |A| = 0$. Thus, the determinant of an odd-order skew-symmetric matrix is always zero.
Final Answer: 0

Answer: (C)

Q4.

Solution

Concept: A square matrix A satisfies its own characteristic equation, $|A - \lambda I| = 0$. Alternatively, we can verify the matrix equation by direct substitution of A and its square.

Solution: Given $A = \begin{bmatrix} 2 & k \\ -k & 2 \end{bmatrix}$. Calculate A^2:

$$A^2 = \begin{bmatrix} 2 & k \\ -k & 2 \end{bmatrix}\begin{bmatrix} 2 & k \\ -k & 2 \end{bmatrix} = \begin{bmatrix} 4-k^2 & 4k \\ -4k & 4-k^2 \end{bmatrix}$$

Now substitute into the expression $A^2 - 4A + (4 + k^2)I$:

$$\begin{bmatrix} 4-k^2 & 4k \\ -4k & 4-k^2 \end{bmatrix} - \begin{bmatrix} 8 & 4k \\ -4k & 8 \end{bmatrix} + \begin{bmatrix} 4+k^2 & 0 \\ 0 & 4+k^2 \end{bmatrix}$$

$$= \begin{bmatrix} (4-k^2-8+4+k^2) & (4k-4k+0) \\ (-4k+4k+0) & (4-k^2-8+4+k^2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

The equation holds true for any real value of k.

Final Answer: Any real value

Answer: (A)

Q5.

Solution

Concept: A function $f(x)$ is strictly decreasing in an interval if its derivative $f'(x) < 0$ for all points in that interval.

Solution: Given $f(x) = x^3 - 6x^2 + 9x + 15$. Differentiating with respect to x:

$$f'(x) = 3x^2 - 12x + 9$$

For strictly decreasing, set $f'(x) < 0$:

$$3(x^2 - 4x + 3) < 0$$

$$x^2 - 4x + 3 < 0$$

Factorizing the quadratic:

$$(x-1)(x-3) < 0$$

The roots are $x = 1$ and $x = 3$. Using the sign scheme (wavy curve method), the expression is negative between the roots. Thus, the function is strictly decreasing in $(1, 3)$.

Final Answer: $(1, 3)$

Answer: (A)

Q6.

Solution

Concept: To find the maximum value of a function $f(x)$, we take its derivative, set it to zero to find the critical points, and evaluate the function at those points. For functions of the form $[g(x)]^{h(x)}$, logarithmic differentiation is typically used.

Solution: Let $y = (1/x)^x = x^{-x}$. Taking natural logarithms:

$$\log y = -x \log x$$

Differentiating both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = -[x \cdot \frac{1}{x} + \log x \cdot 1] = -(1 + \log x)$$

$$\frac{dy}{dx} = -x^{-x}(1 + \log x)$$

Setting $\frac{dy}{dx} = 0 \implies 1 + \log x = 0 \implies \log x = -1 \implies x = 1/e$. Substituting $x = 1/e$ into y:

$$y = \left(\frac{1}{1/e}\right)^{1/e} = e^{1/e}$$

Final Answer: $e^{1/e}$

Answer: (B)

Q7.

Solution

Concept: The "nearest point" problem involves minimizing the distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Minimizing D^2 is mathematically equivalent and simplifies calculations by removing the square root.

Solution: Let a general point on the curve $y = x^2$ be (x, x^2). The distance squared S from $(0, 5)$ is:

$$S = (x - 0)^2 + (x^2 - 5)^2 = x^2 + x^4 - 10x^2 + 25 = x^4 - 9x^2 + 25$$

Differentiating with respect to x:

$$\frac{dS}{dx} = 4x^3 - 18x$$

Setting $\frac{dS}{dx} = 0 \implies 2x(2x^2 - 9) = 0$. Critical points are $x = 0$ or $x^2 = 9/2$. Testing $x^2 = 9/2$: $y = 9/2 = 4.5$. The options suggest checking specific coordinates. Testing $(2, 4)$: $D^2 = (2 - 0)^2 + (4 - 5)^2 = 4 + 1 = 5$. Testing $(\sqrt{2}, 2)$: $D^2 = (\sqrt{2} - 0)^2 + (2 - 5)^2 = 2 + 9 = 11$. Comparing options, $(2, 4)$ gives a distance of $\sqrt{5} \approx 2.23$, while $(0, 0)$ gives 5. The true minimum is at $x = \sqrt{4.5}$, but among given choices, $(2, 4)$ is closest.

Final Answer: $(2, 4)$

Answer: (A)

Q8.

Solution

Concept: Rolle's Theorem states that if a function f is continuous on $[a, b]$ and differentiable on (a, b), and $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

Solution: Given $f(x) = x^2+2x+5$ on $[-3, 1]$. Check $f(a)$ and $f(b)$: $f(-3) = (-3)^2+2(-3)+5 = 9 - 6 + 5 = 8$. $f(1) = (1)^2 + 2(1) + 5 = 1 + 2 + 5 = 8$. Since $f(-3) = f(1)$, Rolle's Theorem applies.

$$f'(x) = 2x + 2$$

Setting $f'(c) = 0 \implies 2c + 2 = 0 \implies c = -1$. -1 lies within the interval $(-3, 1)$.

Final Answer: -1

Answer: (B)

Q9.

Solution

Concept: The standard integral form is $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + C$. If the x^2 term has a coefficient, it must be handled via substitution or factoring.

Solution:

$$I = \int \frac{1}{\sqrt{9 - 25x^2}} dx = \int \frac{1}{\sqrt{3^2 - (5x)^2}} dx$$

Let $u = 5x \implies du = 5dx \implies dx = \frac{du}{5}$.

$$I = \frac{1}{5} \int \frac{du}{\sqrt{3^2 - u^2}} = \frac{1}{5} \sin^{-1}\left(\frac{u}{3}\right) + C$$

$$I = \frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + C$$

Final Answer: $\frac{1}{5} \sin^{-1}(\frac{5x}{3}) + C$

Answer: (A)

Q10.

Solution

Concept: We use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. This is a classic "King's Property" problem in definite integration.

Solution: Let $I = \int_0^{\pi/2} \log(\tan x)dx$ — (1). Using the property $x \to \pi/2 - x$:

$$I = \int_0^{\pi/2} \log(\tan(\pi/2 - x))dx = \int_0^{\pi/2} \log(\cot x)dx \text{ — (2)}$$

Adding (1) and (2):

$$2I = \int_0^{\pi/2} [\log(\tan x) + \log(\cot x)]dx$$

$$2I = \int_0^{\pi/2} \log(\tan x \cdot \cot x)dx = \int_0^{\pi/2} \log(1)dx$$

Since $\log(1) = 0$:

$$2I = 0 \implies I = 0$$

Final Answer: 0

Answer: (C)

Q11.

Solution

Concept: The area of the region bounded by $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is $\int_a^b |f(x)|dx$. For the sine function, which has both positive and negative cycles, the area is the sum of the magnitudes of the integrals over each sub-interval.

Solution: We need the area for $y = \sin x$ from $x = 0$ to $x = 2\pi$. The function is positive in $[0, \pi]$ and negative in $[\pi, 2\pi]$.

$$\text{Area} = \int_0^{\pi} \sin x\, dx + \left|\int_{\pi}^{2\pi} \sin x\, dx\right|$$

Evaluating the integrals:

$$\int_0^{\pi} \sin x\, dx = [-\cos x]_0^{\pi} = -(-1) - (-1) = 1 + 1 = 2$$

$$\int_{\pi}^{2\pi} \sin x\, dx = [-\cos x]_{\pi}^{2\pi} = -1 - (-(-1)) = -1 - 1 = -2$$

Total Area:

$$\text{Area} = 2 + |-2| = 2 + 2 = 4 \text{ sq. units}$$

Final Answer: 4 sq. units

Answer: (B)

Q12.

Solution

Concept: The degree of a differential equation is the power of the highest order derivative, provided the equation is a polynomial in its derivatives. The order is the highest derivative present.
Solution: In the equation $(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^3 + \sin(y) = 0$: 1. The highest order derivative is $\frac{d^2y}{dx^2}$, so the order is 2. 2. The power of this highest order derivative is 2. Since the equation is a polynomial in $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ (note that $\sin(y)$ involves the dependent variable, not a derivative, so it does not affect the polynomial status in terms of derivatives), the degree is defined. Therefore, the degree is 2.
Final Answer: 2

Answer: (B)

Q13.

Solution

Concept: To solve a differential equation using the variable separable method, we rewrite the equation to isolate all terms with y on one side and all terms with x on the other.
Solution: Given: $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ Factor out e^{-y} from the right side:

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Rearrange the terms to separate variables:

$$\frac{dy}{e^{-y}} = (e^x + x^2)dx \implies e^y dy = (e^x + x^2)dx$$

Integrate both sides:

$$\int e^y dy = \int (e^x + x^2)dx$$

$$e^y = e^x + \frac{x^3}{3} + C$$

Final Answer: $e^y = e^x + \frac{x^3}{3} + C$

Answer: (A)

Q14.

Solution

Concept: The probability of an event is the ratio of the number of favorable outcomes to the total number of outcomes in the sample space.

Solution: When a coin is tossed three times, the sample space S consists of $2^3 = 8$ outcomes: $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$ The event "at least two heads" includes outcomes with exactly 2 heads or exactly 3 heads: Favorable outcomes: $\{HHH, HHT, HTH, THH\}$ Number of favorable outcomes = 4.

$$P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$$

Final Answer: $1/2$

Answer: (B)

Q15.

Solution

Concept: According to the Corner Point Theorem in Linear Programming, the optimal value (maximum or minimum) of an objective function $Z = ax + by$ occurs at one of the corner points (vertices) of the feasible region.

Solution: Evaluate $Z = 3x + 9y$ at each given corner point: 1. At $(0, 10)$: $Z = 3(0) + 9(10) = 90$ 2. At $(5, 5)$: $Z = 3(5) + 9(5) = 15 + 45 = 60$ 3. At $(15, 15)$: $Z = 3(15) + 9(15) = 45 + 135 = 180$ 4. At $(0, 20)$: $Z = 3(0) + 9(20) = 180$ The minimum value among these is 60, which occurs at the point $(5, 5)$.

Final Answer: 60

Answer: (B)

Q16.

Solution

Concept: For a set S, a relation R is:

- **Reflexive** if $(a, a) \in R$ for every $a \in S$.
- **Symmetric** if $(a, b) \in R \implies (b, a) \in R$.
- **Transitive** if $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$.

Solution: Given set $S = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$.

(a) **Reflexivity:** R contains $(1, 1)$, $(2, 2)$, and $(3, 3)$. Thus, it is reflexive.

(b) **Symmetry:** R contains $(1, 2)$ but does not contain $(2, 1)$. Thus, it is not symmetric.

(c) **Transitivity:** R contains $(1, 2)$ and $(2, 3)$, but it does not contain $(1, 3)$. Thus, it is not transitive.

The relation is reflexive but not symmetric.
Final Answer: Reflexive but not symmetric

Answer: (A)

Q17.

Solution

Concept: A function $f : X \to Y$ is:

- **One-to-one (Injective)** if $f(x_1) = f(x_2) \implies x_1 = x_2$.
- **Onto (Surjective)** if the Range of f equals the Codomain Y.

Solution: Given $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.

(a) **Injective check:** $f(1) = 1^2 = 1$ and $f(-1) = (-1)^2 = 1$. Since different inputs give the same output, f is many-to-one (not injective).

(b) **Surjective check:** For any $x \in \mathbb{R}, x^2 \geq 0$. The range of f is $[0, \infty)$. However, the codomain is $\mathbb{R}$. Since there is no x such that $x^2 = -1$, the function is not onto.

Therefore, the function is neither one-to-one nor onto.
Final Answer: Neither one-to-one nor onto

Answer: (D)

Q18.

Solution

Concept: The number of onto functions from a set A (with m elements) to a set B (with n elements) where $n = 2$ is given by $2^m - 2$. This represents the total number of functions (2^m) minus the two constant functions that map all elements to just one of the two targets.

Solution: Given $n(A) = 3$ (so $m = 3$) and $n(B) = 2$ (so $n = 2$). Total number of possible functions = $2^3 = 8$. The only functions that are *not* onto are the "into" functions:

(a) All elements of A map to the first element of B.

(b) All elements of A map to the second element of B.

Number of onto functions = $8 - 2 = 6$.

Final Answer: 6

Answer: (B)

Q19.

Solution

Concept: We evaluate inverse trigonometric functions within their principal value branches:

- $\tan^{-1} x \in (-\pi/2, \pi/2)$
- $\sec^{-1} x \in [0, \pi] \setminus \{\pi/2\}$

Solution: Let $I = \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

(a) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ (since $\tan(\pi/3) = \sqrt{3}$).

(b) For $\sec^{-1}(-2)$, let $\sec\theta = -2 \implies \cos\theta = -1/2$. The principal value for $\cos^{-1}(-1/2) = \pi - \cos^{-1}(1/2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

Substituting back:

$$I = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

Final Answer: $-\pi/3$

Answer: (B)

Q20.

Solution

Concept: The domain of the function $f(x) = \sin^{-1}(u)$ is defined by the inequality $-1 \leq u \leq 1$, because the sine function only produces values within that interval.
Solution: For $f(x) = \sin^{-1}(3x - 1)$ to be defined:

$$-1 \leq 3x - 1 \leq 1$$

Add 1 to all sides of the inequality:

$$0 \leq 3x \leq 2$$

Divide by 3:

$$0 \leq x \leq \frac{2}{3}$$

Therefore, the domain is the closed interval $[0, 2/3]$.
Final Answer: $[0, 2/3]$

Answer: (A)

Q21.

Solution

Concept: A matrix X is symmetric if $X^T = X$ and skew-symmetric if $X^T = -X$. For two matrices A and B, the transpose of their product is $(AB)^T = B^T A^T$.
Solution: Given $A^T = A$ (symmetric) and $B^T = -B$ (skew-symmetric). Let $X = AB - BA$. To determine its nature, find X^T:

$$X^T = (AB - BA)^T = (AB)^T - (BA)^T$$

$$X^T = B^T A^T - A^T B^T$$

Substitute the given conditions:

$$X^T = (-B)(A) - (A)(-B)$$

$$X^T = -BA + AB = AB - BA$$

Since $X^T = X$, the matrix $AB - BA$ is symmetric.
Final Answer: Symmetric

Answer: (A)

Q22.

Solution

Concept: To find the n-th power of a matrix A, we can observe the pattern by calculating $A^2, A^3, \ldots$ and then use induction to find a general formula for A^n.

Solution: Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Calculate A^2:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(0) & 1(2) + 2(1) \\ 0(1) + 1(0) & 0(2) + 1(1) \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

Calculate A^3:

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$$

Observing the pattern, the top-right element is $2 \times$ power. For $n = 2$, it is 4; for $n = 3$, it is 6. Thus, for power n, it is $2n$.

$$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$$

Final Answer: $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

Answer: (A)

Q23.

Solution

Concept: A property of determinants is that the value remains unchanged if a multiple of one column is added to another. Also, if two columns or rows of a determinant are identical, its value is 0.

Solution: Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$. Apply the operation $C_3 \to C_3 + C_2$:

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Take the common factor $(a+b+c)$ out from C_3:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

Now, column C_1 and column C_3 are identical. Therefore, the determinant value is 0.
Final Answer: 0

Answer: (C)

Q24.

Solution

Concept: The area of a triangle with vertices $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) is: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.
Solution: Vertices: $(2, -6), (5, 4), (k, 4)$. Area = 35.

$$35 = \frac{1}{2}|2(4-4) + 5(4-(-6)) + k(-6-4)|$$

$$70 = |0 + 5(10) + k(-10)|$$

$$70 = |50 - 10k| \implies |5 - k| = 7$$

Case 1: $5 - k = 7 \implies k = -2$ Case 2: $5 - k = -7 \implies k = 12$ The values of k are 12 or -2.
Final Answer: 12 or -2

Answer: (C)

Q25.

Solution

Concept: A system of linear equations $AX = B$ has a unique solution if the determinant of the coefficient matrix $|A| \neq 0$.

Solution: The coefficient matrix A is:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}$$

Calculate the determinant $|A|$:

$$|A| = 1(9 - 15) - 2(18 - 15) + 3(10 - 5)$$

$$|A| = 1(-6) - 2(3) + 3(5)$$

$$|A| = -6 - 6 + 15 = 3$$

Since $|A| = 3 \neq 0$, the system has a unique solution.

Final Answer: Unique solution

Answer: (A)

Q26.

Solution

Concept: For a function to be continuous at $x = c$, the limit as $x \to c$ must equal the function's value at c: $\lim_{x\to c} f(x) = f(c)$. If the limit results in an indeterminate form like $0/0$, we use L'Hôpital's Rule.

Solution: Given $f(\pi/2) = 3$. We must have $\lim_{x\to\pi/2} \frac{k\cos x}{\pi - 2x} = 3$. As $x \to \pi/2$, $\cos x \to 0$ and $\pi - 2x \to 0$. Applying L'Hôpital's Rule:

$$\lim_{x\to\pi/2} \frac{\frac{d}{dx}(k\cos x)}{\frac{d}{dx}(\pi - 2x)} = \lim_{x\to\pi/2} \frac{-k\sin x}{-2} = \frac{k\sin(\pi/2)}{2}$$

Since $\sin(\pi/2) = 1$, we get $\frac{k}{2}$. Equating to the given value:

$$\frac{k}{2} = 3 \implies k = 6$$

Final Answer: 6

Answer: (B)

Q27.

Solution

Concept: To differentiate a composite function $y = f(g(h(x)))$, we apply the Chain Rule: $\frac{dy}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$.

Solution: Given $y = \log(\tan e^x)$. Differentiating with respect to x:

$$\frac{dy}{dx} = \frac{1}{\tan e^x} \cdot \frac{d}{dx}(\tan e^x)$$

$$\frac{dy}{dx} = \frac{1}{\tan e^x} \cdot \sec^2 e^x \cdot \frac{d}{dx}(e^x)$$

$$\frac{dy}{dx} = \frac{\sec^2 e^x \cdot e^x}{\tan e^x} = \frac{e^x \sec^2 e^x}{\tan e^x}$$

Final Answer: $\frac{e^x \sec^2 e^x}{\tan e^x}$

Answer: (A)

Q28.

Solution

Concept: For parametric equations $x = f(\theta)$ and $y = g(\theta)$, the derivative is $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

Solution: Given $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

$$\frac{dx}{d\theta} = a \cdot 3\cos^2\theta \cdot (-\sin\theta) = -3a\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = a \cdot 3\sin^2\theta \cdot \cos\theta = 3a\sin^2\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

At $\theta = \pi/4$:

$$\frac{dy}{dx} = -\tan(\pi/4) = -1$$

Final Answer: -1

Answer: (B)

Q29.

Solution

Concept: This involves higher-order differentiation. We find the first derivative, rearrange to avoid fractions, and then differentiate again to form the given differential expression.

Solution: Given $y = \sin^{-1} x$.

$$y' = \frac{1}{\sqrt{1-x^2}} \implies \sqrt{1-x^2}\,y' = 1$$

Squaring both sides:

$$(1-x^2)(y')^2 = 1$$

Differentiating with respect to x using the product rule:

$$(1-x^2)\cdot 2y'y'' + (y')^2 \cdot (-2x) = 0$$

Dividing throughout by $2y'$ (since $y' \neq 0$ for $|x| < 1$):

$$(1-x^2)y'' - xy' = 0$$

Final Answer: 0

Answer: (B)

Q30.

Solution

Concept: The rate of change of a quantity y with respect to time t is given by dy/dt. For a circle, the circumference C and radius r are related by $C = 2\pi r$.

Solution: Given $\frac{dr}{dt} = 0.7$ cm/s. We know that $C = 2\pi r$. Differentiating both sides with respect to t:

$$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

$$\frac{dC}{dt} = 2\pi(0.7) = 1.4\pi \text{ cm/s}$$

Final Answer: 1.4π cm/s

Answer: (B)

Q31.

Solution

Concept: For a curve defined parametrically by $x = f(t)$ and $y = g(t)$, the slope of the tangent is given by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. To find the slope at a specific point (x_0, y_0), we first determine the value of the parameter t corresponding to that point.

Solution: Given $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$. At the point $(2, -1)$: 1. Find t from x: $t^2 + 3t - 8 = 2 \implies t^2 + 3t - 10 = 0 \implies (t + 5)(t - 2) = 0$. So $t = -5$ or 2. 2. Find t from y: $2t^2 - 2t - 5 = -1 \implies 2t^2 - 2t - 4 = 0 \implies t^2 - t - 2 = 0 \implies (t - 2)(t + 1) = 0$. So $t = 2$ or -1. The common value is $t = 2$. Now, find the derivatives:

$$\frac{dx}{dt} = 2t + 3, \quad \frac{dy}{dt} = 4t - 2$$

At $t = 2$:

$$\frac{dx}{dt} = 2(2) + 3 = 7, \quad \frac{dy}{dt} = 4(2) - 2 = 6$$

$$\text{Slope } \frac{dy}{dx} = \frac{6}{7}$$

Final Answer: $6/7$

Answer: (B)

Q32.

Solution

Concept: A line is tangent to a curve if they intersect at a point where the slope of the curve (the derivative) is equal to the slope of the line.

Solution: Given the line $y = x + 1$, its slope is $m = 1$. The curve is $y^2 = 4x$. Differentiating both sides with respect to x:

$$2y\frac{dy}{dx} = 4 \implies \frac{dy}{dx} = \frac{2}{y}$$

For the line to be a tangent, the slope of the curve must be 1:

$$\frac{2}{y} = 1 \implies y = 2$$

To find x, substitute $y = 2$ into the curve equation $y^2 = 4x$:

$$(2)^2 = 4x \implies 4 = 4x \implies x = 1$$

The point is $(1, 2)$.

Final Answer: $(1, 2)$

Answer: (A)

Q33.

Solution

Concept: To integrate a function involving e^x and e^{-x} in the denominator, multiply the numerator and denominator by e^x to transform the expression into a form suitable for substitution.

Solution:

$$I = \int \frac{1}{e^x + e^{-x}} dx = \int \frac{e^x}{e^x(e^x + e^{-x})} dx = \int \frac{e^x}{e^{2x} + 1} dx$$

Let $u = e^x$. Then $du = e^x dx$. Substituting these into the integral:

$$I = \int \frac{du}{u^2 + 1}$$

Using the standard integral $\int \frac{1}{1+u^2} du = \tan^{-1} u + C$:

$$I = \tan^{-1}(e^x) + C$$

Final Answer: $\tan^{-1}(e^x) + C$

Answer: (A)

Q34.

Solution

Concept: This integral follows the special form $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$. We must manipulate the integrand to reveal this structure.

Solution: Rewrite the numerator x as $(x + 1) - 1$:

$$I = \int e^x \left[\frac{x + 1 - 1}{(1 + x)^2}\right] dx = \int e^x \left[\frac{x + 1}{(1 + x)^2} - \frac{1}{(1 + x)^2}\right] dx$$

$$I = \int e^x \left[\frac{1}{1 + x} - \frac{1}{(1 + x)^2}\right] dx$$

Let $f(x) = \frac{1}{1+x}$. Then its derivative is $f'(x) = -\frac{1}{(1+x)^2}$. According to the formula:

$$I = e^x f(x) + C = \frac{e^x}{1 + x} + C$$

Final Answer: $\frac{e^x}{1+x} + C$

Answer: (A)

Q35.

Solution

Concept: The definite integral of $\frac{1}{1+x^2}$ is found using the inverse tangent function: $\int \frac{1}{1+x^2}dx = \tan^{-1} x$. We then evaluate this at the upper and lower limits.

Solution:

$$I = \int_0^1 \frac{1}{1+x^2}dx = \left[\tan^{-1} x\right]_0^1$$

Applying the limits:

$$I = \tan^{-1}(1) - \tan^{-1}(0)$$

$$I = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: $\pi/4$

Answer: (A)

Q36.

Solution

Concept: This integral is solved using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. This property is especially useful for trigonometric fractions where the sum of the original and transformed integrands simplifies to unity.

Solution: Let $I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$ — (1) Using the property $x \to \pi/2 - x$:

$$I = \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)}\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}\,dx \text{ — (2)}$$

Adding (1) and (2):

$$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2} = \pi/2 \implies I = \pi/4$$

Final Answer: $\pi/4$

Answer: (B)

Q37.

Solution

Concept: To integrate an absolute value function, split the integral at the point where the expression inside the absolute value becomes zero. For $|x-1|$, the critical point is $x = 1$.

Solution: The function behaves as follows: $|x-1| = -(x-1)$ for $x < 1$ and $(x-1)$ for $x \geq 1$.

$$\int_0^2 |x-1|\,dx = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$$

Evaluating the integrals:

$$\left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$\left(1 - \frac{1}{2}\right) + \left((2-2) - (\frac{1}{2} - 1)\right) = \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: 1

Answer: (A)

Q38.

Solution

Concept: The area of a region bounded by a curve $y^2 = 4ax$ and a line $x = a$ (the latus rectum) is symmetric about the x-axis. The total area is double the area of the upper half.

Solution: The area is given by:

$$\text{Area} = 2\int_0^a \sqrt{4ax}\,dx = 4\sqrt{a}\int_0^a x^{1/2}\,dx$$

Evaluating the integral:

$$\text{Area} = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^a = 4\sqrt{a}\left(\frac{2}{3}a^{3/2}\right)$$

$$\text{Area} = \frac{8}{3}a^2 \text{ sq. units}$$

Final Answer: $\frac{8}{3}a^2$

Answer: (A)

Q39.

Solution

Concept: The exterior area is found by subtracting the area "interior" to the parabola (the region bounded by the circle and the parabola containing the focus) from the total area of the circle.

Solution: 1. **Intersection:** $x^2 + 6x = 16 \implies x^2 + 6x - 16 = 0 \implies (x+8)(x-2) = 0$. So, $x = 2$. 2. **Interior Area (A_i):** $A_i = 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$ Evaluating gives $A_i = \frac{4}{3}(4\pi + \sqrt{3})$. 3. **Exterior Area:** Total circle area $= \pi(4)^2 = 16\pi$.

$$\text{Exterior Area} = 16\pi - \frac{4}{3}(4\pi + \sqrt{3}) = \frac{48\pi - 16\pi - 4\sqrt{3}}{3} = \frac{32\pi - 4\sqrt{3}}{3}$$

$$= \frac{4}{3}(8\pi - \sqrt{3})$$

Final Answer: $\frac{4}{3}(8\pi - \sqrt{3})$

Answer: (B)

Q40.

Solution

Concept: For a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, the integrating factor (IF) is defined as $e^{\int P\,dx}$.

Solution: Given: $\frac{dy}{dx} + y \sec x = \tan x$ Here, $P = \sec x$.

$$\text{IF} = e^{\int \sec x\,dx}$$

The integral of $\sec x$ is $\log|\sec x + \tan x|$.

$$\text{IF} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$$

Final Answer: $\sec x + \tan x$

Answer: (A)

Q41.

Solution

Concept: A first-order linear differential equation is of the form $\frac{dy}{dx} + Py = Q$. The solution is found by multiplying by an Integrating Factor (IF), given by $e^{\int P\,dx}$. The general solution is $y(\text{IF}) = \int Q(\text{IF})\,dx + C$.

Solution: Given: $\frac{dy}{dx} + \frac{y}{x} = x^2$ Here, $P = \frac{1}{x}$ and $Q = x^2$. Calculate the Integrating Factor:

$$\text{IF} = e^{\int \frac{1}{x}\,dx} = e^{\log x} = x$$

Apply the general solution formula:

$$y(x) = \int x^2(x)\,dx + C$$

$$xy = \int x^3\,dx + C$$

$$xy = \frac{x^4}{4} + C$$

Final Answer: $xy = \frac{x^4}{4} + C$

Answer: (A)

Q42.

Solution

Concept: The **order** of a differential equation is the order of the highest derivative present. The **degree** is the power to which the highest order derivative is raised, provided the equation is in a polynomial form with respect to its derivatives.

Solution: Given: $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$ 1. The highest order derivative is $\frac{d^2y}{dx^2}$, so the **order is 2**. 2. To find the degree, we must remove the radical. Square both sides:

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \frac{dy}{dx}$$

Now the equation is a polynomial in its derivatives. The power of the highest order derivative $(\frac{d^2y}{dx^2})$ is 2. Thus, the **degree is 2**.

Final Answer: 2, 2

Answer: (A)

Q43.

Solution

Concept: The dot product of two vectors $\vec{a}$ and $\vec{b}$ is related to the angle θ between them by the formula $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

Solution: Given: $|\vec{a}| = 2, |\vec{b}| = 1$, and $\vec{a} \cdot \vec{b} = 1$. Substitute these into the formula:

$$1 = (2)(1)\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

The angle θ in the interval $[0, \pi]$ whose cosine is $1/2$ is:

$$\theta = \frac{\pi}{3}$$

Final Answer: $\pi/3$

Answer: (B)

Q44.

Solution

Concept: Three vectors are coplanar if their scalar triple product is zero. For vectors $\vec{a}, \vec{b}$, and $\vec{c}$, this is calculated as the determinant of a matrix formed by their components.

Solution: Vectors are $\vec{a} = (2, -1, 1), \vec{b} = (1, 2, -3)$, and $\vec{c} = (3, \lambda, 5)$. For coplanarity:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Expanding along the first row:

$$2(10 - (-3\lambda)) - (-1)(5 - (-9)) + 1(\lambda - 6) = 0$$

$$2(10 + 3\lambda) + 1(14) + \lambda - 6 = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0 \implies 7\lambda = -28 \implies \lambda = -4$$

Final Answer: -4

Answer: (A)

Q45.

Solution

Concept: The area of a parallelogram whose diagonals are given by the vectors $\vec{d}_1$ and $\vec{d}_2$ is given by the formula Area $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$.
Solution: Given: $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$. Calculate the cross product:

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(4-6) - \hat{j}(12+2) + \hat{k}(-9-1) = -2\hat{i} - 14\hat{j} - 10\hat{k}$$

Find the magnitude of the cross product:

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \sqrt{4 + 196 + 100} = \sqrt{300} = 10\sqrt{3}$$

The area is half of this magnitude:

$$\text{Area} = \frac{1}{2}(10\sqrt{3}) = 5\sqrt{3} \text{ sq. units}$$

Final Answer: $5\sqrt{3}$

Answer: (B)

Q46.

Solution

Concept: The direction cosines (l, m, n) of a line are given by $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$, where α, β, γ are the angles the line makes with the x, y, and z axes respectively. They satisfy the identity $l^2 + m^2 + n^2 = 1$.
Solution: Since the line makes equal angles with the coordinate axes, $\alpha = \beta = \gamma$. This implies $l = m = n$. Substituting into the identity:

$$l^2 + l^2 + l^2 = 1$$

$$3l^2 = 1 \implies l^2 = 1/3 \implies l = \pm\frac{1}{\sqrt{3}}$$

Thus, the direction cosines are $(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$.
Final Answer: $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

Answer: (B)

Q47.

Solution

Concept: The distance between two parallel planes $ax+by+cz+d_1 = 0$ and $ax+by+cz+d_2 = 0$ is given by the formula $d = \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$.

Solution: Plane 1: $2x - y + 2z + 3 = 0$ Plane 2: $4x - 2y + 4z + 5 = 0 \implies 2x - y + 2z + 2.5 = 0$ (Dividing by 2) Here, $a = 2, b = -1, c = 2, d_1 = 3, d_2 = 2.5$.

$$d = \frac{|3 - 2.5|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{0.5}{\sqrt{4 + 1 + 4}}$$

$$d = \frac{0.5}{3} = \frac{1}{6}$$

Final Answer: $1/6$

Answer: (A)

Q48.

Solution

Concept: The shortest distance (SD) between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $SD = \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$.

Solution: Line 1: passes through $\vec{a}_1 = (1,2,3)$ with direction $\vec{b}_1 = (2,3,4)$. Line 2: passes through $\vec{a}_2 = (2,4,5)$ with direction $\vec{b}_2 = (3,4,5)$. 1. $\vec{a}_2-\vec{a}_1 = (1,2,2)$. 2. $\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15-16)-\hat{j}(10-12)+\hat{k}(8-9) = -\hat{i}+2\hat{j}-\hat{k}$. 3. Magnitude $|\vec{b}_1\times\vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$. 4. Dot Product $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (2)(2) + (2)(-1) = -1 + 4 - 2 = 1$.

$$SD = \frac{1}{\sqrt{6}}$$

Final Answer: $1/\sqrt{6}$

Answer: (A)

Q49.

Solution

Concept: The optimal value of a linear objective function $Z = ax + by$ occurs at one of the corner points of the feasible region. To find the maximum, evaluate Z at each vertex.
Solution: Objective Function: $Z = 4x + y$. Corner Points: 1. At $(0, 0)$: $Z = 4(0) + 0 = 0$ 2. At $(7, 0)$: $Z = 4(7) + 0 = 28$ 3. At $(3, 4)$: $Z = 4(3) + 4 = 12 + 4 = 16$ 4. At $(0, 2)$: $Z = 4(0) + 2 = 2$ The maximum value is 28 at the point $(7, 0)$.
Final Answer: 28

Answer: (A)

Q50.

Solution

Concept: The Addition Theorem of Probability states $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. The intersection probability can be found via conditional probability: $P(B|A) = \frac{P(A \cap B)}{P(A)}$.
Solution: Given $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$. First, find $P(A \cap B)$:

$$P(A \cap B) = P(B|A) \times P(A) = 0.6 \times 0.4 = 0.24$$

Now, calculate the probability of the union:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Final Answer: 0.96

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	C	4	A	5	A
6	B	7	A	8	B	9	A	10	C
11	B	12	B	13	A	14	B	15	B
16	A	17	D	18	B	19	B	20	A
21	A	22	A	23	C	24	C	25	A
26	B	27	A	28	B	29	B	30	B
31	B	32	A	33	A	34	A	35	A
36	B	37	A	38	A	39	B	40	A
41	A	42	A	43	B	44	A	45	B
46	B	47	A	48	A	49	A	50	A