# CUET UG Maths Sample Paper - 13

**Duration: 1 Hour** **Maximum Marks: 250**

## Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

**Q1.** If $A$ is a square matrix of order 3 such that $A^2 = I$, then the value of $|A|$ is:

(A) 1

(B) -1

(C) $\pm 1$

(D) 0

**Q2.** The function $f(x) = |x - 1| + |x - 2|$ is not differentiable at:

(A) $x = 1$ only

(B) $x = 2$ only

(C) $x = 1$ and $x = 2$

(D) All real values of $x$

**Q3.** The area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:

(A) $12\pi$ sq. units

(B) $144\pi$ sq. units

(C) $7\pi$ sq. units

(D) $25\pi$ sq. units

**Q4.** The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ is:

(A) 2

(B) 1

(C) 0

(D) Not defined

**Q5.** If $P(A) = 0.6, P(B) = 0.3$ and $P(A \cap B) = 0.2$, then $P(A|B)$ is:

(A) 2/3

(B) 1/3

(C) 1/2

(D) 3/4

**Q6.** The value of $\int \frac{dx}{x\sqrt{x^2-1}}$ is:

(A) $\sin^{-1} x + C$

(B) $\sec^{-1} x + C$

(C) $\tan^{-1} x + C$

(D) $\cos^{-1} x + C$

**Q7.** The direction ratios of the line joining points $P(2, 3, 5)$ and $Q(-1, 2, 4)$ are:

(A) $(1, 5, 9)$

(B) $(3, 1, 1)$

(C) $(-3, -1, -1)$

(D) $(2, -6, 20)$

**Q8.** If the set $A$ contains 5 elements and set $B$ contains 6 elements, then the number of one-to-one functions from $A$ to $B$ is:

(A) $6^5$

(B) $5^6$

(C) $6!$

(D) ${}^6P_5$

**Q9.** The derivative of $\log(\sin x)$ with respect to $x$ is:

(A) $\tan x$

(B) $\cot x$

(C) $-\cot x$

(D) $\sec x$

**Q10.** The value of the determinant $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ is:

(A) 0

(B) $a+b+c$

(C) $(a+b+c)^2$

(D) $abc$

**Q11.** The slope of the tangent to the curve $y = x^3 - x$ at $x = 2$ is:

(A) 11

(B) 12

(C) 10

(D) 6

**Q12.** The vector projection of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ on $\hat{j}$ is:

(A) $\hat{i}$

(B) $\hat{j}$

(C) $\hat{k}$

(D) 1

**Q13.** The identity element for the binary operation $*$ defined on $\mathbb{Q}$ by $a * b = \frac{ab}{4}$ is:

(A) 1

(B) 0

(C) 4

(D) 1/4

**Q14.** The value of $\int_{-1}^{1} |x| dx$ is:

(A) 1

(B) 2

(C) 0

(D) 1/2

**Q15.** The order of the differential equation of all circles of fixed radius $r$ is:

(A) 1

(B) 2

(C) 3

(D) 4

**Q16.** If $\cos^{-1} x = y$, then:

(A) $0 \leq y \leq \pi$

(B) $-\pi/2 \leq y \leq \pi/2$

(C) $0 < y < \pi$

(D) $y \in [-\pi/2, \pi/2] - \{0\}$

**Q17.** The maximum value of $\sin x \cdot \cos x$ is:

(A) 1

(B) 2

(C) 1/2

(D) $\sqrt{2}$

**Q18.** If $A$ and $B$ are square matrices of the same order, then $(AB)^{-1}$ is:

(A) $A^{-1}B^{-1}$

(B) $B^{-1}A^{-1}$

(C) $AB^{-1}$

(D) $BA^{-1}$

**Q19.** The distance of the plane $2x - 3y + 6z + 7 = 0$ from the origin is:

(A) 1

(B) 7

(C) 1/7

(D) 2

**Q20.** The function $f(x) = x^2 - 4x + 5$ is increasing in the interval:

(A) $(-\infty, 2)$

(B) $(2, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, 4)$

**Q21.** If $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $0^\circ$

(B) 45°

(C) 90°

(D) 180°

**Q22.** In an LPP, if the objective function $Z = ax + by$ has the same maximum value at two corner points, then the number of optimal solutions is:

(A) 2

(B) 1

(C) Infinite

(D) 0

**Q23.** The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\pi/4$

(B) $\pi/2$

(C) $\pi$

(D) 1

**Q24.** If $A$ is a matrix of order $m \times n$ and $B$ is a matrix such that $AB$ and $BA$ are both defined, then the order of $B$ is:

(A) $m \times n$

(B) $n \times m$

(C) $n \times n$

(D) $m \times m$

**Q25.** The probability of obtaining an even prime number on each die when a pair of dice is rolled is:

(A) 0

(B) 1/3

(C) 1/12

(D) 1/36

**Q26.** If $f(x) = \sin^{-1} x + \cos^{-1} x$, then $f'(x)$ is:

(A) $\frac{2}{\sqrt{1-x^2}}$

(B) 0

(C) $\pi/2$

(D) 1

**Q27.** The position vector of the midpoint of the line segment joining $P(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $Q(4\hat{i} + \hat{j} - 2\hat{k})$ is:

(A) $3\hat{i} + 2\hat{j} + \hat{k}$

(B) $6\hat{i} + 4\hat{j} + 2\hat{k}$

(C) $2\hat{i} - 2\hat{j} - 6\hat{k}$

(D) $3\hat{i} + 2\hat{j} - \hat{k}$

**Q28.** The solution of $\frac{dy}{dx} = \frac{y}{x}$ is:

(A) $y = x + C$

(B) $xy = C$

(C) $y = Cx$

(D) $\log y = \log x$

**Q29.** The value of $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$ is:

(A) $\pi/2$

(B) $3\pi/4$

(C) $2\pi/3$

(D) $\pi$

**Q30.** If $A$ is a square matrix of order 3 and $|A| = 2$, then $|2A|$ is:

(A) 4

(B) 8

(C) 16

(D) 6

**Q31.** The area bounded by $y = x^2$, the x-axis and the lines $x = 1, x = 2$ is:

(A) 7/3 sq. units

(B) 3/7 sq. units

(C) 1 sq. unit

(D) 8/3 sq. units

**Q32.** Two vectors $\vec{a}$ and $\vec{b}$ are parallel if:

(A) $\vec{a} \cdot \vec{b} = 0$

(B) $\vec{a} \times \vec{b} = \vec{0}$

(C) $\vec{a} + \vec{b} = \vec{0}$

(D) $\vec{a} \cdot \vec{b} = 1$

**Q33.** The Integrating Factor of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is:

(A) $\tan x$

(B) $\sec x$

(C) $\log(\sec x)$

(D) $e^{\tan x}$

**Q34.** If $P(A) = 1/2, P(B) = 1/3$ and $A, B$ are independent, then $P(A \cup B)$ is:

(A) 5/6

(B) 2/3

(C) 1/6

(D) 1/3

**Q35.** The corner points of the feasible region for an LPP are $(0,0)$, $(5,0)$, $(3,4)$ and $(0,5)$. The maximum value of $Z = 4x + 3y$ is:

(A) 20

(B) 24

(C) 15

(D) 12

**Q36.** The derivative of $e^{\cos x}$ is:

(A) $e^{\cos x}$

(B) $-\sin x \cdot e^{\cos x}$

(C) $\cos x \cdot e^{\cos x - 1}$

(D) $\sin x \cdot e^{\cos x}$

**Q37.** The value of $\int e^x(\tan x + \log \sec x)dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \log \sec x + C$

(C) $e^x \sec x + C$

(D) $e^x(\tan x + \sec x) + C$

**Q38.** The equation of the line passing through $(1, 2, 3)$ with direction ratios $(2, 3, 4)$ is:

(A) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$

(B) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

(C) $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$

(D) $2x + 3y + 4z = 0$

**Q39.** If $A$ is a skew-symmetric matrix of order 3, then $|A|$ is:

(A) 1

(B) -1

(C) 0

(D) 3

**Q40.** The range of $\sin^{-1} x$ is:

(A) $[0, \pi]$

(B) $(-\pi/2, \pi/2)$

(C) $[-\pi/2, \pi/2]$

(D) $[-\pi, \pi]$

**Q41.** The rate of change of volume of a sphere with respect to its radius $r$ is:

(A) $4\pi r^2$

(B) $\frac{4}{3}\pi r^2$

(C) $8\pi r$

(D) $2\pi r^2$

**Q42.** The value of $\int_0^{\pi/2} \cos^2 x dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi$

(D) 1

**Q43.** The projection of $\vec{a} = \hat{i} - \hat{j}$ on $\vec{b} = \hat{i} + \hat{j}$ is:

(A) 1

(B) 0

(C) $\sqrt{2}$

(D) -1

**Q44.** If $A$ is a square matrix, then $A + A^T$ is always:

(A) Skew-symmetric

(B) Symmetric

(C) Identity matrix

(D) Zero matrix

**Q45.** The number of arbitrary constants in the particular solution of a differential equation of order 3 is:

(A) 3

(B) 2

(C) 1

(D) 0

**Q46.** The function $f(x) = x^3$ is:

(A) One-to-one and onto

(B) One-to-one but not onto

(C) Onto but not one-to-one

(D) Neither

**Q47.** The distance of the point $(1, 2, 3)$ from the y-axis is:

(A) $\sqrt{14}$

(B) $\sqrt{10}$

(C) 2

(D) $\sqrt{13}$

**Q48.** The value of $\sin(\tan^{-1} x)$ for $|x| < 1$ is:

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1+x^2}}$

(C) $\frac{x}{\sqrt{1+x^2}}$

(D) $\frac{1}{\sqrt{1-x^2}}$

**Q49.** If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then $|\vec{a} - \vec{b}|$ is:

(A) $\sqrt{5}$

(B) 5

(C) $\sqrt{13}$

(D) 1

**Q50.** If $A$ is a square matrix such that $A^2 = A$, then $|A|$ can be:

(A) 0 only

(B) 1 only

(C) 0 or 1

(D) Any real number

# Detailed Solutions

## Q1.

### Solution

**Concept:**
A square matrix $A$ satisfies the equation $A^2 = I$, where $I$ is the identity matrix. According to the properties of determinants, the determinant of a product of matrices is equal to the product of their determinants, i.e., $|AB| = |A||B|$. Applying this to the given equation:

$$|A^2| = |I|$$

$$|A| \cdot |A| = 1$$

$$|A|^2 = 1$$

**Solution:**
1. Given condition: - Matrix $A$ is of order 3. - $A^2 = I$.
2. Take the determinant on both sides:

$$\det(A^2) = \det(I)$$

3. Use the property $|A^n| = |A|^n$ and the fact that $|I| = 1$:

$$|A|^2 = 1$$

4. Solve for $|A|$:

$$|A| = \pm\sqrt{1}$$

$$|A| = \pm 1$$

5. Conclusion: The determinant of the matrix can be either 1 or -1.

**Answer: (C)**

## Q2.

### Solution

**Concept:**
A function is not differentiable at points where its graph has a "sharp corner" or "kink." For absolute value functions of the form $f(x) = |x - a|$, the function is continuous everywhere but not differentiable at $x = a$ because the left-hand derivative and right-hand derivative do not match at that specific point. For a sum of absolute value functions, the points of non-differentiability are the roots of the expressions inside the absolute value bars.

**Solution:**
1. Given function:

$$f(x) = |x - 1| + |x - 2|$$

2. Analyze the first term $|x - 1|$: This term is not differentiable where $x - 1 = 0$, which is $x = 1$.
3. Analyze the second term $|x - 2|$: This term is not differentiable where $x - 2 = 0$, which is $x = 2$.
4. Overall Differentiability: Since the sum of a differentiable function and a non-differentiable function is non-differentiable, $f(x)$ fails to be differentiable at both critical points. - At $x = 1$, $|x - 2|$ is smooth but $|x - 1|$ is sharp. - At $x = 2$, $|x - 1|$ is smooth but $|x - 2|$ is sharp.
5. Conclusion: The function is not differentiable at $x = 1$ and $x = 2$.

**Answer: (C)**

## Q3.

### Solution

**Concept:**
The standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a$ and $b$ are the lengths of the semi-major and semi-minor axes. The area of an ellipse can be derived using integration or the standard geometric formula:

$$\text{Area} = \pi ab$$

Using integration, we solve for $y = \frac{b}{a}\sqrt{a^2 - x^2}$ and integrate from $-a$ to $a$, then double the result (or integrate from 0 to $a$ and multiply by 4).

**Solution:**
1. Given equation:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

2. Identify the semi-axes $a$ and $b$: - $a^2 = 16 \implies a = 4$ - $b^2 = 9 \implies b = 3$
3. Apply the area formula:

$$\text{Area} = \pi \times a \times b$$

$$\text{Area} = \pi \times 4 \times 3$$

4. Final Calculation:

$$\text{Area} = 12\pi \text{ sq. units}$$

**Answer: (A)**

## Q4.

### Solution

**Concept:**

The **degree** of a differential equation is the highest power (exponent) of the highest-order derivative, provided the differential equation is a polynomial equation in derivatives. If any derivative is an argument of a transcendental function (like sin, cos, $e^x$, or log) and cannot be removed, the equation is not a polynomial in derivatives, and the degree is not defined.

**Solution:**

1. Given equation:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

2. Identify the order: The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

3. Check for polynomial form in derivatives: The term $\cos\left(\frac{dy}{dx}\right)$ involves a derivative inside a trigonometric function. This prevents the equation from being written as a polynomial in its derivatives ($\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots$).

4. Conclusion: Since the equation cannot be expressed as a polynomial in its derivatives, the degree is not defined.

**Answer: (D)**

## Q5.

### Solution

**Concept:**

**Conditional Probability** $P(A|B)$ is the probability of event $A$ occurring given that event $B$ has already occurred. It is calculated as the ratio of the probability of the intersection of $A$ and $B$ to the probability of the conditioning event $B$. The formula is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This formula assumes that $P(B) > 0$.

**Solution:**

1. Given data: - $P(A) = 0.6$ - $P(B) = 0.3$ - $P(A \cap B) = 0.2$

2. Substitute the values into the conditional probability formula:

$$P(A|B) = \frac{0.2}{0.3}$$

3. Simplify the fraction:

$$P(A|B) = \frac{2}{3}$$

4. Conclusion: The probability of $A$ given $B$ is $2/3$.

**Answer: (A)**

## Q6.

### Solution

**Concept:**

The integral $\int \frac{dx}{x\sqrt{x^2-1}}$ is a standard integral in calculus that corresponds to the derivative of the inverse secant function. Specifically, the derivative of $\sec^{-1} x$ is $\frac{1}{|x|\sqrt{x^2-1}}$. Alternatively, this can be solved using the trigonometric substitution $x = \sec\theta$, which simplifies the radical using the identity $\sec^2\theta - 1 = \tan^2\theta$.

**Solution:**

1. Recognize the standard form: The integral is $\int \frac{1}{x\sqrt{x^2-1}} dx$.
2. Verification via substitution: Let $x = \sec\theta$. Then $dx = \sec\theta\tan\theta\, d\theta$. Substituting these into the integral:

$$\int \frac{\sec\theta\tan\theta}{\sec\theta\sqrt{\sec^2\theta - 1}}\, d\theta$$

3. Simplify using $\sqrt{\sec^2\theta - 1} = \tan\theta$:

$$\int \frac{\sec\theta\tan\theta}{\sec\theta\tan\theta}\, d\theta = \int 1\, d\theta$$

4. Integrate:

$$\theta + C$$

5. Substitute back $\theta = \sec^{-1} x$:

$$\sec^{-1} x + C$$

**Answer: (B)**

## Q7.

### Solution

**Concept:**

The **direction ratios** of a line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are proportional to the components of the vector $\vec{PQ}$. The direction ratios $(a, b, c)$ are calculated as:

$$a = x_2 - x_1, \quad b = y_2 - y_1, \quad c = z_2 - z_1$$

Note that direction ratios are not unique; any scalar multiple of these values also serves as valid direction ratios for the same line.

**Solution:**

1. Given points: - $P(2, 3, 5)$ - $Q(-1, 2, 4)$
2. Calculate the differences in coordinates: - $a = -1 - 2 = -3$ - $b = 2 - 3 = -1$ - $c = 4 - 5 = -1$
3. Identify the direction ratios: The calculated ratios are $(-3, -1, -1)$.
4. Alternative form: Multiplying by $-1$, we get $(3, 1, 1)$. Both represent the same line direction. In the options provided, $(-3, -1, -1)$ is present.

**Answer: (C)**

# Q8.

## Solution

**Concept:**

A **one-to-one function** (injection) from set $A$ to set $B$ requires that each element in $A$ is mapped to a unique element in $B$. If $n(A) = r$ and $n(B) = n$, the number of one-to-one functions is given by the number of ways to arrange $n$ elements taken $r$ at a time:

$$P(n, r) = \frac{n!}{(n-r)!}$$

If $r > n$, no one-to-one function is possible (the number is 0).

**Solution:**

1. Given data: - Number of elements in domain $A$ ($r$) = 5 - Number of elements in codomain $B$ ($n$) = 6

2. Apply the permutation formula: The first element of $A$ has 6 choices. The second element of $A$ has 5 choices (to stay one-to-one). The third has 4, the fourth has 3, and the fifth has 2.

3. Calculate:

$$\text{Total functions} = 6 \times 5 \times 4 \times 3 \times 2$$

This is equivalent to ${}^6P_5$.

4. Final value:

$${}^6P_5 = \frac{6!}{(6-5)!} = \frac{720}{1} = 720$$

**Answer: (D)**

# Q9.

## Solution

**Concept:**

To find the derivative of a logarithmic composite function $\log(f(x))$, we use the **Chain Rule**. The general formula is:

$$\frac{d}{dx}[\log(u)] = \frac{1}{u} \cdot \frac{du}{dx}$$

In this case, the inner function is $u = \sin x$. The derivative of $\sin x$ is $\cos x$.

**Solution:**

1. Let $y = \log(\sin x)$.
2. Apply the Chain Rule:

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)$$

3. Substitute the derivative of $\sin x$:

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x$$

4. Simplify using trigonometric identities: Since $\frac{\cos x}{\sin x} = \cot x$:

$$\frac{dy}{dx} = \cot x$$

**Answer: (B)**

## Q10.

### Solution

**Concept:**

The value of a determinant can often be found by applying **Row or Column operations**. For a cyclic or symmetric determinant, adding all rows to the first row often reveals a common factor or a simplification. A key property is that if all elements of a row or column are zero, the determinant is zero.

**Solution:**

1. Given determinant:

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

2. Apply the operation $R_1 \rightarrow R_1 + R_2 + R_3$: The new first row elements become: - $(a-b)+(b-c)+(c-a)=0$ - $(b-c)+(c-a)+(a-b)=0$ - $(c-a)+(a-b)+(b-c)=0$

3. Rewrite the determinant:

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

4. Conclusion: Since all elements in the first row are zero, the value of the determinant is 0.

**Answer: (A)**

## Q11.

### Solution

**Concept:**

The **slope of the tangent** to a curve $y = f(x)$ at a specific point $(x_0, y_0)$ is equal to the value of the first derivative of the function evaluated at that point. Mathematically:

$$m = \left.\frac{dy}{dx}\right|_{x=x_0}$$

If the slope is positive, the function is increasing at that point; if negative, it is decreasing.

**Solution:**

1. Given curve: $y = x^3 - x$.

2. Differentiate $y$ with respect to $x$:

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = 3x^2 - 1$$

3. Substitute the given point $x = 2$:

$$m = 3(2)^2 - 1$$

$$m = 3(4) - 1$$

$$m = 12 - 1 = 11$$

4. Conclusion: The slope of the tangent at $x = 2$ is 11.

**Answer: (A)**

# Q12.

## Solution

**Concept:**

The **vector projection** of a vector $\vec{a}$ on another vector $\vec{b}$ is the vector component of $\vec{a}$ that lies in the direction of $\vec{b}$. The formula is:

$$\text{Proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$$

If $\vec{b}$ is a unit vector (like $\hat{i}, \hat{j},$ or $\hat{k}$), the formula simplifies to $(\vec{a}\cdot\vec{b})\vec{b}$.

**Solution:**

1. Given vectors: - $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ - $\vec{b} = \hat{j}$
2. Calculate the dot product $\vec{a}\cdot\vec{b}$:

$$\vec{a}\cdot\hat{j} = (1\hat{i} + 1\hat{j} + 1\hat{k})\cdot(0\hat{i} + 1\hat{j} + 0\hat{k})$$

$$= (1\times 0) + (1\times 1) + (1\times 0) = 1$$

3. Calculate the magnitude squared of $\vec{b}$ (where $\vec{b} = \hat{j}$):

$$|\hat{j}|^2 = 1^2 = 1$$

4. Apply the projection formula:

$$\text{Proj}_{\hat{j}}\vec{a} = \left(\frac{1}{1}\right)\hat{j} = \hat{j}$$

**Answer: (B)**

# Q13.

## Solution

**Concept:**

For a binary operation $*$ defined on a set $S$, an element $e \in S$ is called the **identity element** if for every $a \in S$:

$$a * e = e * a = a$$

To find the identity, we set up the equation $a * e = a$ using the specific rule of the operation and solve for $e$.

**Solution:**

1. Given operation on $\mathbb{Q}$:

$$a * b = \frac{ab}{4}$$

2. Let $e$ be the identity element. Then:

$$a * e = a$$

3. Substitute the operation rule:

$$\frac{a \cdot e}{4} = a$$

4. Solve for $e$ (assuming $a \neq 0$): Multiply both sides by 4:

$$ae = 4a$$

Divide by $a$:

$$e = 4$$

5. Verify: $4 \in \mathbb{Q}$ and $e * a = \frac{4a}{4} = a$. Thus, 4 is the identity.

**Answer: (C)**

## Q14.

### Solution

**Concept:**
The absolute value function $f(x) = |x|$ is defined as $x$ for $x \geq 0$ and $-x$ for $x < 0$. To integrate $|x|$ over an interval that spans across zero, we must split the integral at $x = 0$ into two parts where the function behaves linearly. Alternatively, geometrically, this represents the area of two identical triangles formed between the graph and the x-axis.

**Solution:**

1. Split the integral at the boundary $x = 0$:

$$\int_{-1}^{1} |x|\, dx = \int_{-1}^{0} (-x)\, dx + \int_{0}^{1} x\, dx$$

2. Integrate the first part:

$$\int_{-1}^{0} -x\, dx = \left[-\frac{x^2}{2}\right]_{-1}^{0} = 0 - \left(-\frac{(-1)^2}{2}\right) = \frac{1}{2}$$

3. Integrate the second part:

$$\int_{0}^{1} x\, dx = \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{2} - 0 = \frac{1}{2}$$

4. Sum the parts:

$$\text{Total Value} = \frac{1}{2} + \frac{1}{2} = 1$$

**Answer: (A)**

## Q15.

### Solution

**Concept:**

The **order of a differential equation** representing a family of curves is equal to the number of **independent arbitrary constants** in the general equation of that family. For a circle with a fixed radius $r$, the equation is:

$$(x-h)^2 + (y-k)^2 = r^2$$

Here, $h$ and $k$ are the coordinates of the center, which can vary, while $r$ is a given constant.

**Solution:**

1. General equation of a circle with fixed radius $r$:

$$(x-h)^2 + (y-k)^2 = r^2$$

2. Identify the arbitrary constants: - $h$ (x-coordinate of the center) - $k$ (y-coordinate of the center) Note: $r$ is **not** an arbitrary constant because the question states it is "fixed."
3. Count the constants: There are exactly 2 independent arbitrary constants ($h$ and $k$).
4. Conclusion: Since there are two arbitrary constants, the differential equation formed by eliminating them will be of the **second order**.

**Answer: (B)**

## Q16.

### Solution

**Concept:**

The notation $\cos^{-1} x = y$ represents the **inverse cosine function**, where $x$ is the value (ratio) and $y$ is the angle in radians. The **principal value branch** of $\cos^{-1} x$ is defined as the range of angles for which the cosine function is one-to-one, allowing for a unique inverse. For cosine, this interval starts at the maximum value (1 at 0 radians) and ends at the minimum value ($-1$ at $\pi$ radians).

**Solution:**

1. Define the function: The function $y = \cos^{-1} x$ is the inverse of $x = \cos y$.
2. Identify Domain and Range: - The domain of $\cos^{-1} x$ is $[-1, 1]$. - The range (principal value branch) is $[0, \pi]$.
3. Verify the interval: In the interval $[0, \pi]$, the cosine function decreases monotonically from 1 to $-1$, covering all possible values of $x$ exactly once.
4. Conclusion: Therefore, the value of $y$ must satisfy $0 \le y \le \pi$.

**Answer: (A)**

## Q17.

### Solution

**Concept:**

To find the maximum value of a trigonometric product like $\sin x \cos x$, we can use the **double angle identity**:

$$\sin 2x = 2 \sin x \cos x \implies \sin x \cos x = \frac{1}{2} \sin 2x$$

Since the maximum value of the sine function for any argument (including $2x$) is 1, the maximum value of the overall expression is easily determined.

**Solution:**

1. Rewrite the expression using the identity:

$$f(x) = \sin x \cos x = \frac{1}{2}(2 \sin x \cos x)$$

$$f(x) = \frac{1}{2} \sin 2x$$

2. Determine the range of $\sin 2x$: We know that $-1 \leq \sin 2x \leq 1$.
3. Calculate the maximum value: The maximum occurs when $\sin 2x = 1$:

$$f(x)_{\max} = \frac{1}{2}(1) = \frac{1}{2}$$

4. Conclusion: The maximum value of $\sin x \cos x$ is $1/2$.

**Answer: (C)**

## Q18.

### Solution

**Concept:**

The **Reversal Law of Inverses** states that the inverse of a product of two invertible matrices $A$ and $B$ is the product of their inverses in the reverse order. Mathematically:

$$(AB)^{-1} = B^{-1}A^{-1}$$

This property is fundamental in matrix algebra and is derived from the requirement that $(AB)(AB)^{-1} = I$. Substituting $B^{-1}A^{-1}$ for $(AB)^{-1}$ gives $A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$.

**Solution:**

1. Let $C = AB$. We are looking for $C^{-1}$. 2. By the property of inverses:

$$(AB) \cdot (B^{-1}A^{-1}) = A(B \cdot B^{-1})A^{-1}$$

3. Simplify the inner product: Since $B \cdot B^{-1} = I$:

$$A \cdot I \cdot A^{-1} = A \cdot A^{-1} = I$$

4. Similarly, check the reverse order:

$$(B^{-1}A^{-1}) \cdot (AB) = B^{-1}(A^{-1} \cdot A)B = B^{-1} \cdot I \cdot B = B^{-1} \cdot B = I$$

5. Conclusion: Since $(AB)(B^{-1}A^{-1}) = I$ and $(B^{-1}A^{-1})(AB) = I$, the inverse of $AB$ is $B^{-1}A^{-1}$.

**Answer: (B)**

# Q19.

## Solution

**Concept:**

The perpendicular distance $d$ of a point $(x_1, y_1, z_1)$ from a plane $ax + by + cz + d' = 0$ is given by the formula:

$$d = \frac{|ax_1 + by_1 + cz_1 + d'|}{\sqrt{a^2 + b^2 + c^2}}$$

When calculating the distance from the **origin**, the coordinates $(x_1, y_1, z_1)$ are $(0, 0, 0)$, which simplifies the numerator to just the absolute value of the constant term $|d'|$.

**Solution:**

1. Identify the coefficients and the point: - Plane: $2x-3y+6z+7 = 0$ - $a = 2, b = -3, c = 6, d' = 7$ - Point (Origin): $(0, 0, 0)$

2. Apply the distance formula:

$$d = \frac{|2(0) - 3(0) + 6(0) + 7|}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

3. Simplify the denominator:

$$\sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

4. Calculate the distance:

$$d = \frac{|7|}{7} = \frac{7}{7} = 1$$

**Answer: (A)**

## Q20.

### Solution

**Concept:**

A function $f(x)$ is **increasing** on an interval if its first derivative $f'(x)$ is greater than or equal to zero for all $x$ in that interval ($f'(x) \geq 0$). To find this interval: 1. Find $f'(x)$. 2. Set $f'(x) > 0$ and solve the inequality for $x$. The point where $f'(x) = 0$ is the turning point of the parabola.

**Solution:**

1. Given function: $f(x) = x^2 - 4x + 5$.

2. Find the first derivative:

$$f'(x) = 2x - 4$$

3. Determine where the function is increasing: Set $f'(x) > 0$:

$$2x - 4 > 0$$

$$2x > 4$$

$$x > 2$$

4. Conclusion: The function is increasing for all values of $x$ greater than 2, which corresponds to the interval $(2, \infty)$.

**Answer: (B)**

# Q21.

## Solution

**Concept:**

The relationship between the **dot product** and the **cross product** of two vectors $\vec{a}$ and $\vec{b}$ involves the angle $\theta$ between them. - Dot Product: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ - Magnitude of Cross Product: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ When these two values are equal, the trigonometric functions $\sin\theta$ and $\cos\theta$ must also be equal.

**Solution:**

1. Given the condition:

$$\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$$

2. Substitute the formulas for both products:

$$|\vec{a}||\vec{b}| \cos\theta = |\vec{a}||\vec{b}| \sin\theta$$

3. Simplify by dividing both sides by $|\vec{a}||\vec{b}|$ (assuming they are non-zero vectors):

$$\cos\theta = \sin\theta$$

4. Divide by $\cos\theta$ to express as a single trigonometric ratio:

$$\frac{\sin\theta}{\cos\theta} = 1 \implies \tan\theta = 1$$

5. Solve for $\theta$:

$$\theta = \tan^{-1}(1) = 45^\circ \text{ or } \pi/4 \text{ radians}$$

**Answer: (B)**

# Q22.

## Solution

**Concept:**

In a **Linear Programming Problem (LPP)**, the optimal (maximum or minimum) value of the objective function $Z = ax + by$ always occurs at the corner points of the feasible region. However, if two distinct corner points yield the same optimal value, then every point on the line segment joining those two corner points will also yield that same optimal value. Since a line segment consists of an infinite number of points, the problem has multiple optimal solutions.

**Solution:**

1. Property of LPP: If the maximum value of $Z$ is achieved at corner point $P_1$ and corner point $P_2$, then any point $P$ on the segment $P_1P_2$ can be represented as:

$$P = kP_1 + (1-k)P_2, \text{ where } 0 \le k \le 1$$

2. Evaluate $Z$ at $P$:

$$Z(P) = Z(kP_1 + (1-k)P_2) = kZ(P_1) + (1-k)Z(P_2)$$

3. Since $Z(P_1) = Z(P_2) = M$:

$$Z(P) = kM + (1-k)M = M$$

4. Conclusion: Every point on the segment is an optimal solution. Therefore, there are infinite optimal solutions.

**Answer: (C)**

## Q23.

### Solution

**Concept:**
The integral $\int \frac{1}{1+x^2}\,dx$ is a standard integral whose antiderivative is the inverse tangent function, $\tan^{-1} x$. According to the **Fundamental Theorem of Calculus**, to evaluate the definite integral from $a$ to $b$:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is the antiderivative. The limits are then substituted to find the numerical result.

**Solution:**
1. Identify the antiderivative:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x$$

2. Apply the limits from 0 to 1:

$$[\tan^{-1} x]_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$$

3. Substitute the principal values: - $\tan^{-1}(1) = \pi/4$ (since $\tan(\pi/4) = 1$) - $\tan^{-1}(0) = 0$ (since $\tan(0) = 0$)
4. Calculate the result:

$$\pi/4 - 0 = \pi/4$$

**Answer: (A)**

## Q24.

### Solution

**Concept:**
Matrix multiplication $AB$ is defined if and only if the number of columns in $A$ equals the number of rows in $B$. Similarly, $BA$ is defined if and only if the number of columns in $B$ equals the number of rows in $A$. For both $AB$ and $BA$ to be defined, the dimensions must "cycle" correctly.

**Solution:**
1. Let the order of $A$ be $m \times n$. 2. Let the order of $B$ be $p \times q$.
3. For $AB$ to be defined: Columns of $A$ must equal Rows of $B \implies n = p$.
4. For $BA$ to be defined: Columns of $B$ must equal Rows of $A \implies q = m$.
5. Combine the dimensions for $B$: Since $p = n$ and $q = m$, the order of $B$ must be $n \times m$.

**Answer: (B)**

## Q25.

### Solution

**Concept:**

The **Classical Definition of Probability** is the ratio of favorable outcomes to the total number of equally likely outcomes. When two dice are rolled, the total number of outcomes is $6 \times 6 = 36$. An **even prime number** is a specific category. In the set of prime numbers $\{2, 3, 5, 7, \ldots\}$, the number 2 is the only even prime number.

**Solution:**

1. Total number of outcomes (Sample Space $S$):

$$n(S) = 6 \times 6 = 36$$

2. Identify the favorable outcome (Event $E$): The condition is "an even prime number on each die." The only even prime number is 2. So, the only favorable outcome is the pair $(2, 2)$.

3. Count favorable outcomes:

$$n(E) = 1$$

4. Calculate probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{36}$$

**Answer: (D)**

# Q26.

## Solution

**Concept:**

This problem involves the derivatives of inverse trigonometric functions. The standard derivatives are:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \text{and} \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

Alternatively, one can use the inverse trigonometric identity:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

Since $\pi/2$ is a constant, its derivative with respect to $x$ will be zero.

**Solution:**

1. Given function: $f(x) = \sin^{-1} x + \cos^{-1} x$.
2. Method 1 (Using Identity): Recognize that $\sin^{-1} x + \cos^{-1} x = \pi/2$ for all $x \in [-1, 1]$.

$$f(x) = \frac{\pi}{2}$$

$$f'(x) = \frac{d}{dx}\left(\frac{\pi}{2}\right) = 0$$

3. Method 2 (Direct Differentiation):

$$f'(x) = \frac{d}{dx}(\sin^{-1} x) + \frac{d}{dx}(\cos^{-1} x)$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} + \left(-\frac{1}{\sqrt{1-x^2}}\right) = 0$$

**Answer: (B)**

# Q27.

## Solution

**Concept:**

The **midpoint** of a line segment joining two points with position vectors $\vec{a}$ and $\vec{b}$ is given by the average of the two vectors:

$$\vec{M} = \frac{\vec{a}+\vec{b}}{2}$$

The position vector of a point $(x, y, z)$ is represented as $x\hat{i} + y\hat{j} + z\hat{k}$. To find the midpoint, we sum the corresponding components of $\hat{i}, \hat{j},$ and $\hat{k}$ and divide each by 2.

**Solution:**

1. Given position vectors: - $\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ - $\vec{q} = 4\hat{i} + \hat{j} - 2\hat{k}$
2. Apply the midpoint formula:

$$\vec{M} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2}$$

3. Add the components:

$$\vec{M} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2}$$

$$\vec{M} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2}$$

4. Divide by 2:

$$\vec{M} = 3\hat{i} + 2\hat{j} + \hat{k}$$

**Answer: (A)**

## Q28.

### Solution

**Concept:**
This is a **first-order differential equation** that can be solved using the **Variable Separable** method. We rearrange the equation to group all terms containing $y$ with $dy$ and all terms containing $x$ with $dx$. After separating the variables, we integrate both sides. The integral of $1/u$ is $\log|u|$.

**Solution:**
1. Given equation: $\frac{dy}{dx} = \frac{y}{x}$.
2. Separate the variables:

$$\frac{1}{y}dy = \frac{1}{x}dx$$

3. Integrate both sides:

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\log|y| = \log|x| + \log|C|$$

(Note: We use $\log|C|$ as the constant of integration to simplify the expression).
4. Combine logarithmic terms:

$$\log|y| = \log|Cx|$$

5. Exponentiate both sides:

$$y = Cx$$

**Answer: (C)**

## Q29.

### Solution

**Concept:**
To evaluate the sum of inverse trigonometric functions, we find the **principal value** for each term. The principal value ranges are: - $\tan^{-1} x \in (-\pi/2, \pi/2)$ - $\cos^{-1} x \in [0, \pi]$ - $\sin^{-1} x \in [-\pi/2, \pi/2]$ For negative arguments, we use the identities $\cos^{-1}(-x) = \pi - \cos^{-1} x$ and $\sin^{-1}(-x) = -\sin^{-1} x$.

**Solution:**
1. Evaluate each term: - $\tan^{-1}(1) = \pi/4$ (since $\tan(\pi/4) = 1$) - $\cos^{-1}(-1/2) = \pi - \cos^{-1}(1/2) = \pi - \pi/3 = 2\pi/3$ - $\sin^{-1}(-1/2) = -\sin^{-1}(1/2) = -\pi/6$
2. Sum the values:

$$\text{Sum} = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

3. Find a common denominator (12):

$$\text{Sum} = \frac{3\pi + 8\pi - 2\pi}{12}$$

$$\text{Sum} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

**Answer: (B)**

## Q30.

### Solution

**Concept:**

According to the properties of determinants, if $A$ is a square matrix of order $n$, then for any scalar $k$:

$$|kA| = k^n|A|$$

This is because multiplying a matrix by a scalar $k$ multiplies every element of the matrix. When taking the determinant, the factor $k$ can be factored out from each of the $n$ rows.

**Solution:**

1. Given data: - Order of matrix $A$ $(n) = 3$ - Determinant of $A$ $(|A|) = 2$ - Scalar $(k) = 2$
2. Apply the formula:

$$|2A| = 2^3 \cdot |A|$$

3. Calculate:

$$|2A| = 8 \cdot 2 = 16$$

**Answer: (C)**

# Q31.

## Solution

**Concept:**

The area bounded by a curve $y = f(x)$, the $x$-axis, and vertical lines $x = a$ and $x = b$ is given by the definite integral:

$$\text{Area} = \int_a^b f(x)\,dx$$

For the parabola $y = x^2$, the area represents the space under the curve in the first quadrant between the specified vertical boundaries.

**Solution:**

1. Given function and boundaries: - $y = x^2$ - $x = 1, x = 2$
2. Set up the definite integral:

$$\text{Area} = \int_1^2 x^2\,dx$$

3. Integrate using the power rule $\int x^n\,dx = \frac{x^{n+1}}{n+1}$:

$$\text{Area} = \left[\frac{x^3}{3}\right]_1^2$$

4. Apply the upper and lower limits:

$$\text{Area} = \frac{2^3}{3} - \frac{1^3}{3}$$

$$\text{Area} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

5. Conclusion: The area of the region is $7/3$ square units.

**Answer: (A)**

# Q32.

## Solution

**Concept:**

The relationship between two vectors $\vec{a}$ and $\vec{b}$ is determined by their **cross product** and **dot product**. - If $\vec{a} \cdot \vec{b} = 0$, the vectors are perpendicular (orthogonal). - If $\vec{a} \times \vec{b} = \vec{0}$, the vectors are parallel (collinear), meaning one is a scalar multiple of the other ($\vec{a} = \lambda \vec{b}$). The magnitude of the cross product is $|\vec{a}||\vec{b}| \sin\theta$. For parallel vectors, the angle $\theta$ is $0^\circ$ or $180^\circ$, making $\sin\theta = 0$.

**Solution:**

1. Definition of Parallel Vectors: Two vectors are parallel if they lie along the same line or parallel lines.

2. Using Cross Product:

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n}$$

If $\theta = 0^\circ$ or $180^\circ$, then $\sin\theta = 0$. Therefore, $\vec{a} \times \vec{b} = \vec{0}$.

3. Checking Options: - Option (A) $\vec{a} \cdot \vec{b} = 0$ implies perpendicularity. - Option (B) $\vec{a} \times \vec{b} = \vec{0}$ implies they are parallel.

**Answer: (B)**

## Q33.

### Solution

**Concept:**

A first-order linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where $P$ and $Q$ are functions of $x$. The **Integrating Factor (I.F.)** is used to transform the left side of the equation into the derivative of a product. The formula for I.F. is:

$$\text{I.F.} = e^{\int P\,dx}$$

For the function $\tan x$, the integral is $\log(\sec x)$.

**Solution:**

1. Given differential equation:

$$\frac{dy}{dx} + y\tan x = \sec x$$

2. Identify $P(x)$: Comparing with $\frac{dy}{dx} + Py = Q$, we find $P = \tan x$.

3. Calculate the integral of $P$:

$$\int P\,dx = \int \tan x\,dx = \log(\sec x)$$

4. Calculate the Integrating Factor:

$$\text{I.F.} = e^{\int \tan x\,dx}$$

$$\text{I.F.} = e^{\log(\sec x)}$$

5. Use the identity $e^{\log f(x)} = f(x)$:

$$\text{I.F.} = \sec x$$

**Answer: (B)**

## Q34.

### Solution

**Concept:**

For any two events $A$ and $B$, the **Addition Theorem of Probability** states:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If events $A$ and $B$ are **independent**, the probability of their intersection is the product of their individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

**Solution:**

1. Given data: - $P(A) = 1/2$ - $P(B) = 1/3$ - $A$ and $B$ are independent.
2. Find the intersection $P(A \cap B)$:

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

3. Find the union $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

4. Use a common denominator (6):

$$P(A \cup B) = \frac{3 + 2 - 1}{6} = \frac{4}{6} = \frac{2}{3}$$

**Answer: (B)**

## Q35.

### Solution

**Concept:**

In Linear Programming, the **Optimal Value Theorem** states that if an optimal solution exists, it must occur at one of the corner points (vertices) of the feasible region. To find the maximum value of the objective function $Z = ax + by$, we evaluate $Z$ at each given corner point and select the largest result.

**Solution:**

1. Given objective function: $Z = 4x + 3y$.
2. Corner points provided: - $(0, 0)$ - $(5, 0)$ - $(3, 4)$ - $(0, 5)$
3. Evaluate $Z$ at each point: - At $(0, 0) : Z = 4(0) + 3(0) = 0$ - At $(5, 0) : Z = 4(5) + 3(0) = 20$ - At $(3, 4) : Z = 4(3) + 3(4) = 12 + 12 = 24$ - At $(0, 5) : Z = 4(0) + 3(5) = 15$
4. Comparison: The values are 0, 20, 24, and 15. The highest value is 24.

**Answer: (B)**

# Q36.

## Solution

**Concept:**

To differentiate an exponential function where the exponent is another function, we use the **Chain Rule**. The general formula is:

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

In this case, the inner function is $u = \cos x$. We must multiply the original exponential function by the derivative of $\cos x$, which is $-\sin x$.

**Solution:**

1. Let $y = e^{\cos x}$.

2. Apply the Chain Rule:

$$\frac{dy}{dx} = e^{\cos x} \cdot \frac{d}{dx}(\cos x)$$

3. Substitute the derivative of the cosine function:

$$\frac{dy}{dx} = e^{\cos x} \cdot (-\sin x)$$

4. Rearrange the terms:

$$\frac{dy}{dx} = -\sin x \cdot e^{\cos x}$$

**Answer: (B)**

# Q37.

## Solution

**Concept:**

The integral of the form $\int e^x [f(x) + f'(x)] dx$ is a standard result in calculus, equal to $e^x f(x) + C$. To use this shortcut, we identify one part of the trigonometric expression as $f(x)$ and check if the other part is its derivative. For the natural logarithm of a trigonometric function, $\frac{d}{dx}(\log \sec x) = \frac{1}{\sec x} \cdot (\sec x \tan x) = \tan x$.

**Solution:**

1. Given integral: $\int e^x (\tan x + \log \sec x)\, dx$.
2. Let $f(x) = \log \sec x$.
3. Find the derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(\log \sec x)$$

$$f'(x) = \frac{1}{\sec x} \cdot \frac{d}{dx}(\sec x)$$

$$f'(x) = \frac{1}{\sec x} \cdot (\sec x \tan x) = \tan x$$

4. Apply the formula: The integrand is in the form $e^x [f'(x) + f(x)]$.

$$\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$$

5. Substitute back $f(x)$:

$$\text{Result} = e^x \log \sec x + C$$

**Answer: (B)**

## Q38.

### Solution

**Concept:**

The **Cartesian equation of a line** in 3D space passing through a point $(x_1, y_1, z_1)$ with direction ratios $(a, b, c)$ is given by the symmetric form:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Direction ratios represent the vector components parallel to the line. To find the equation, we simply substitute the point's coordinates and the direction ratios into this template.

**Solution:**

1. Identify the given point: - $(x_1, y_1, z_1) = (1, 2, 3)$
2. Identify the direction ratios: - $(a, b, c) = (2, 3, 4)$
3. Substitute into the standard formula:

$$\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$$

4. Compare with options: The result matches Option (B).

**Answer: (B)**

## Q39.

### Solution

**Concept:**

A **skew-symmetric matrix** is a square matrix $A$ such that $A^T = -A$. One of the key properties of a skew-symmetric matrix is that all its diagonal elements must be zero. Another property involves the determinant: for any skew-symmetric matrix of **odd order** $(3 \times 3, 5 \times 5, \dots)$, the determinant is always zero. This can be proven using the property $|A^T| = |A|$ and $|-A| = (-1)^n|A|$.

**Solution:**

1. Given: $A$ is a skew-symmetric matrix of order $n = 3$. - $A^T = -A$
2. Take the determinant on both sides:

$$|A^T| = |-A|$$

3. Apply determinant properties: - Since $|A^T| = |A|$ - And $|-A| = (-1)^3|A| = -|A|$ (because $n = 3$ is odd)

$$|A| = -|A|$$

4. Solve for $|A|$:

$$|A| + |A| = 0$$

$$2|A| = 0 \implies |A| = 0$$

**Answer: (C)**

## Q40.

### Solution

**Concept:**

The **range** of an inverse trigonometric function (also known as the **principal value branch**) is the set of output angles for which the function is defined as a one-to-one mapping. For $\sin^{-1} x$, we find the angles $y$ such that $\sin y = x$. To ensure $y$ is unique, the range is restricted to the interval where the sine function is strictly increasing from its minimum $(-1)$ to its maximum $(1)$.

**Solution:**

1. Define the inverse sine function: $y = \sin^{-1} x \iff \sin y = x$, where $x \in [-1, 1]$.

2. Identify the quadrant behavior: - For $x > 0$, the angle is in the first quadrant ($0$ to $\pi/2$). - For $x < 0$, the angle is in the fourth quadrant ($-\pi/2$ to $0$).

3. Determine the full interval: The continuous branch that covers all values in the domain exactly once is the closed interval from $-\pi/2$ to $\pi/2$.

4. Conclusion: The range of $\sin^{-1} x$ is $[-\pi/2, \pi/2]$.

**Answer: (C)**

# Q41.

## Solution

**Concept:**

The **rate of change** of a geometric quantity with respect to a variable is found by differentiating its formula with respect to that variable. For a sphere, the volume $V$ is given by:

$$V = \frac{4}{3}\pi r^3$$

The rate of change of volume with respect to the radius $r$ is represented by the derivative $\frac{dV}{dr}$.

**Solution:**

1. Write the formula for the volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

2. Differentiate $V$ with respect to $r$:

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)$$

3. Apply the power rule ($\frac{d}{dr}r^n = nr^{n-1}$):

$$\frac{dV}{dr} = \frac{4}{3}\pi \cdot (3r^2)$$

4. Simplify the expression: The 3 in the numerator and denominator cancels out.

$$\frac{dV}{dr} = 4\pi r^2$$

5. Conclusion: The rate of change of volume with respect to the radius is $4\pi r^2$, which is also the formula for the surface area of the sphere.

**Answer: (A)**

# Q42.

## Solution

**Concept:**

To evaluate the definite integral of $\cos^2 x$, we use the **power-reduction identity** to convert the squared term into a linear trigonometric function. The identity is:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

This transformation makes the integration straightforward. After integrating, we apply the limits from 0 to $\pi/2$.

**Solution:**

1. Rewrite the integrand using the identity:

$$\int_0^{\pi/2} \cos^2 x \, dx = \int_0^{\pi/2} \frac{1 + \cos 2x}{2} \, dx$$

2. Take the constant factor $1/2$ outside:

$$\frac{1}{2} \int_0^{\pi/2} (1 + \cos 2x) \, dx$$

3. Integrate each term:

$$\frac{1}{2} \left[ x + \frac{\sin 2x}{2} \right]_0^{\pi/2}$$

4. Substitute the upper limit $(\pi/2)$:

$$\frac{1}{2} \left[ \frac{\pi}{2} + \frac{\sin(\pi)}{2} \right] = \frac{1}{2} \left[ \frac{\pi}{2} + 0 \right] = \frac{\pi}{4}$$

5. Substitute the lower limit (0):

$$\frac{1}{2} \left[ 0 + \frac{\sin(0)}{2} \right] = 0$$

6. Final result:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

**Answer: (B)**

# Q43.

## Solution

**Concept:**

The **scalar projection** of a vector $\vec{a}$ onto a vector $\vec{b}$ is the magnitude of the component of $\vec{a}$ in the direction of $\vec{b}$. The formula is:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

The dot product $\vec{a} \cdot \vec{b}$ measures the alignment of the two vectors. If the dot product is zero, the vectors are perpendicular, and the projection is zero.

**Solution:**

1. Identify the vectors: - $\vec{a} = \hat{i} - \hat{j}$ - $\vec{b} = \hat{i} + \hat{j}$

2. Calculate the dot product $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (1)(1) + (-1)(1)$$

$$\vec{a} \cdot \vec{b} = 1 - 1 = 0$$

3. Calculate the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

4. Apply the formula:

$$\text{Projection} = \frac{0}{\sqrt{2}} = 0$$

5. Conclusion: Since the dot product is zero, the vectors are orthogonal, and there is no projection of $\vec{a}$ onto $\vec{b}$.

**Answer: (B)**

## Q44.

### Solution

**Concept:**

A matrix $M$ is **symmetric** if $M^T = M$. A matrix is **skew-symmetric** if $M^T = -M$. For any square matrix $A$, we can analyze the transpose of the sum $A + A^T$. We use the property that the transpose of a sum is the sum of the transposes:

$$(A + B)^T = A^T + B^T$$

and that taking the transpose twice returns the original matrix: $(A^T)^T = A$.

**Solution:**

1. Let $X = A + A^T$.
2. Take the transpose of $X$:

$$X^T = (A + A^T)^T$$

3. Apply the property of transposes over addition:

$$X^T = A^T + (A^T)^T$$

4. Simplify using $(A^T)^T = A$:

$$X^T = A^T + A$$

5. Observe the result: Since addition of matrices is commutative ($A^T + A = A + A^T$), we have:

$$X^T = X$$

6. Conclusion: Since $X^T = X$, the matrix $A + A^T$ is always symmetric.

**Answer: (B)**

## Q45.

### Solution

**Concept:**

- **General Solution:** Contains a number of arbitrary constants equal to the order of the differential equation. - **Particular Solution:** Obtained from the general solution by assigning specific values to the arbitrary constants (usually based on initial or boundary conditions). By definition, a particular solution contains **no arbitrary constants**.

**Solution:**

1. Type of solution: Particular Solution. 2. Rule: A particular solution is a specific case where the general constants $C_1, C_2, \ldots$ have been replaced by fixed numbers. 3. Conclusion: Regardless of the order of the differential equation (whether it is 1st, 2nd, or 3rd order), the number of arbitrary constants in a particular solution is always **zero**.

**Answer: (D)**

## Q46.

### Solution

**Concept:**

A function $f : A \to B$ is: 1. **One-to-one (Injective):** If $f(x_1) = f(x_2)$ implies $x_1 = x_2$. For $f(x) = x^3$, this holds for all real numbers because every number has a unique cube. 2. **Onto (Surjective):** If for every $y$ in the codomain $\mathbb{R}$, there exists an $x$ in the domain $\mathbb{R}$ such that $f(x) = y$. The function $f(x) = x^3$ maps the set of all real numbers to the set of all real numbers without missing any values.

**Solution:**

1. **Check for One-to-one:** Let $f(x_1) = f(x_2)$.

$$x_1^3 = x_2^3$$

Taking the cube root of both sides:

$$x_1 = x_2$$

Since the cube root is unique for real numbers, the function is **one-to-one**.

2. **Check for Onto:** Let $y$ be any real number in the codomain. We need to find $x$ such that $x^3 = y$.

$$x = \sqrt[3]{y}$$

Since the cube root of any real number (positive, negative, or zero) is also a real number, every $y$ has a pre-image. Thus, the function is **onto**.

3. **Conclusion:** The function is both one-to-one and onto (Bijective).

**Answer: (A)**

## Q47.

### Solution

**Concept:**

The distance of a point $P(x, y, z)$ from a coordinate axis is found using the Pythagorean theorem in the plane perpendicular to that axis. - Distance from $x$-axis = $\sqrt{y^2 + z^2}$ - Distance from $y$-axis = $\sqrt{x^2 + z^2}$ - Distance from $z$-axis = $\sqrt{x^2 + y^2}$ To find the distance from the $y$-axis, we essentially find the length of the segment from $(x, y, z)$ to $(0, y, 0)$.

**Solution:**

1. Given point: $(1, 2, 3)$. Here $x = 1, y = 2, z = 3$.
2. Identify the target axis: $y$-axis. 3. Apply the distance formula from the $y$-axis:

$$d = \sqrt{x^2 + z^2}$$

4. Substitute the values:

$$d = \sqrt{1^2 + 3^2}$$

$$d = \sqrt{1 + 9} = \sqrt{10}$$

5. Conclusion: The distance from the $y$-axis is $\sqrt{10}$ units.

**Answer: (B)**

## Q48.

### Solution

**Concept:**

To express $\sin(\tan^{-1} x)$ as an algebraic expression, we use **Right-Triangle Trigonometry**. Let $\theta = \tan^{-1} x$. This implies $\tan\theta = \frac{x}{1}$. In a right-angled triangle, if the opposite side is $x$ and the adjacent side is 1, the hypotenuse is found using $a^2 + b^2 = c^2$. Then, $\sin\theta$ is defined as the ratio of the opposite side to the hypotenuse.

**Solution:**

1. Let $\theta = \tan^{-1} x \implies \tan\theta = \frac{x}{1}$. 2. In a right triangle: - Opposite side = $x$ - Adjacent side = 1
3. Calculate the Hypotenuse:

$$\text{Hypotenuse} = \sqrt{(x)^2 + (1)^2} = \sqrt{x^2 + 1}$$

4. Find $\sin\theta$:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{\sqrt{x^2 + 1}}$$

5. Conclusion: $\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$.

**Answer: (C)**

# Q49.

## Solution

**Concept:**

The magnitude of the difference between two vectors, $|\vec{a} - \vec{b}|$, can be found using the square of the magnitude property:

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

Expanding this using the distributive property of the dot product gives:

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$$

We substitute the known magnitudes and the dot product to find the squared result, then take the square root.

**Solution:**

1. Given data: - $|\vec{a}| = 2$ - $|\vec{b}| = 3$ - $\vec{a} \cdot \vec{b} = 4$
2. Set up the expansion formula:

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$$

3. Substitute the values:

$$|\vec{a} - \vec{b}|^2 = (2)^2 + (3)^2 - 2(4)$$

$$|\vec{a} - \vec{b}|^2 = 4 + 9 - 8$$

4. Simplify:

$$|\vec{a} - \vec{b}|^2 = 5$$

5. Solve for magnitude:

$$|\vec{a} - \vec{b}| = \sqrt{5}$$

**Answer: (A)**

## Q50.

### Solution

**Concept:**

An **idempotent matrix** is a matrix $A$ such that $A^2 = A$. To find the possible values of the determinant $|A|$, we apply the determinant property $|AB| = |A||B|$. Applying this to the condition $A^2 = A$:

$$|A^2| = |A|$$

$$|A|^2 = |A|$$

This is a quadratic equation in terms of the determinant value.

**Solution:**

1. Start with the given condition:

$$A^2 = A$$

2. Take the determinant of both sides:

$$\det(A^2) = \det(A)$$

3. Use the property $\det(A^2) = (\det A)^2$:

$$|A|^2 = |A|$$

4. Rearrange into standard quadratic form:

$$|A|^2 - |A| = 0$$

5. Factor the equation:

$$|A|(|A| - 1) = 0$$

6. Solve for $|A|$: - $|A| = 0$ - $|A| - 1 = 0 \implies |A| = 1$

7. Conclusion: The determinant of an idempotent matrix must be either 0 or 1.

**Answer: (C)**

## Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | C | 2 | C | 3 | A | 4 | D | 5 | A |
| 6 | B | 7 | C | 8 | D | 9 | B | 10 | A |
| 11 | A | 12 | B | 13 | C | 14 | A | 15 | B |
| 16 | A | 17 | C | 18 | B | 19 | A | 20 | B |
| 21 | B | 22 | C | 23 | A | 24 | B | 25 | D |
| 26 | B | 27 | A | 28 | C | 29 | B | 30 | C |
| 31 | A | 32 | B | 33 | B | 34 | B | 35 | B |
| 36 | B | 37 | B | 38 | B | 39 | C | 40 | C |
| 41 | A | 42 | B | 43 | B | 44 | B | 45 | D |
| 46 | A | 47 | B | 48 | C | 49 | A | 50 | C |