CUET-UG Mathematics Sample Paper-17

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$, and $(A+B)^2 = A^2 + B^2$, then the values of a and b are:

(A) $a = 1, b = 4$

(B) $a = 2, b = 3$

(C) $a = 1, b = 2$

(D) $a = 4, b = 1$

Q2. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ (where ω is a cube root of unity) is:

(A) 0

(B) 1

(C) ω

(D) ω^2

Q3. If A is a 3×3 matrix and $|A| = 4$, then $|2A|$ is:

(A) 8

(B) 16

(C) 32

(D) 64

Q4. If A and B are square matrices of order 3 such that A is orthogonal and B is skew-symmetric, then $|AB|$ is:

(A) 0

(B) 1

(C) -1

(D) 3

Q5. If $f(x) = \begin{vmatrix} x+a & x+b & x+c \\ x+b & x+c & x+a \\ x+c & x+a & x+b \end{vmatrix}$, then $f'(x)$ is equal to:

(A) 0

(B) 1

(C) $3x^2$

(D) $a+b+c$

Q6. If $A^2 - A + I = 0$, then A^3 is equal to:

(A) I

(B) $-I$

(C) A

(D) $A + I$

Q7. The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 6 & 0 \\ 8 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(D) $\frac{1}{24}I$

Q8. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $AB = -BA$ is:

(A) True

(B) False

(C) Only if $A = B$

(D) Only if $A = I$

Q9. If A is a square matrix, then $A + A^T$ is always:

(A) Symmetric

(B) Skew-symmetric

(C) Identity

(D) Null

Q10. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. The domain of $(g \circ f)(x)$ is:

(A) $[0, \infty)$

(B) $(-\infty, \infty)$

(C) $(0, \infty)$

(D) $\mathbb{R} - \{0\}$

Q11. If a set A has 3 elements, how many reflexive relations can be defined on A?

(A) 2^9

(B) 2^6

(C) 2^3

(D) 64

Q12. The range of $\sin^{-1} x + \cos^{-1} x$ is:

(A) $[0, \pi]$

(B) $\{\pi/2\}$

(C) $[-\pi/2, \pi/2]$

(D) $[0, \pi/2]$

Q13. The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is:

(A) $\pi/3$

(B) $-\pi/3$

(C) π

(D) $2\pi/3$

Q14. $\sin(\tan^{-1} x)$ is equal to:

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1+x^2}}$

(C) $\frac{x}{\sqrt{1+x^2}}$

(D) $\frac{1}{\sqrt{1-x^2}}$

Q15. If $y = \log(\log(\log x))$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{x \log x \log(\log x)}$

(B) $\frac{1}{\log(\log x)}$

(C) $\frac{1}{x \log x}$

(D) $\frac{1}{x^3}$

Q16. The function $f(x) = |x| + |x - 1|$ is NOT differentiable at:

(A) $x = 0$ only

(B) $x = 1$ only

(C) $x = 0$ and $x = 1$

(D) All real points

Q17. If $x = \sin t$, $y = \cos 2t$, then $\frac{dy}{dx}$ is:

(A) $-4\sin t$

(B) $-2\sin t$

(C) $4\sin t$

(D) $2\cos t$

Q18. The value of k for which $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & x \neq \pi/2 \\ 3 & x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$ is:

(A) 3

(B) 6

(C) 12

(D) 1.5

Q19. The function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing for:

(A) $x > -1$

(B) $x < 0$

(C) All x

(D) $x > 0$

Q20. The maximum value of $f(x) = \left(\frac{1}{x}\right)^x$ is:

(A) e^e

(B) $e^{1/e}$

(C) e

(D) 1

Q21. The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is:

(A) $x + y = 0$

(B) $x - y = 0$

(C) $y = 0$

(D) $x = 0$

Q22. The altitude of the right circular cylinder of maximum volume that can be inscribed in a sphere of radius R is:

(A) $\frac{2R}{\sqrt{3}}$

(B) $\frac{R}{\sqrt{3}}$

(C) $\frac{2R}{3}$

(D) $\frac{R}{2}$

Q23. The rate of change of the area of a circle with respect to its circumference C is:

(A) $\frac{C}{2\pi}$

(B) $\frac{C}{4\pi}$

(C) $\frac{C}{\pi}$

(D) R

Q24. If $f(x) = 2x^3 - 21x^2 + 36x - 20$, the point of maximum is:

(A) $x = 1$

(B) $x = 6$

(C) $x = 0$

(D) $x = 3$

Q25. The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ if m is:

(A) 1

(B) 2

(C) 3

(D) $\frac{1}{2}$

Q26. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} \, dx$ is:

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $-\tan x - \cot x + C$

(D) $\tan x + \sec x + C$

Q27. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} \, dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 1

Q28. $\int \frac{dx}{x^2 + 2x + 2}$ is:

(A) $\tan^{-1}(x + 1) + C$

(B) $\tan^{-1}(x) + C$

(C) $\log(x^2 + 2x + 2) + C$

(D) $e^x + C$

Q29. $\int_0^{\pi/4} \sec x \, dx$ is:

(A) $\log(\sqrt{2} + 1)$

(B) $\log(\sqrt{2} - 1)$

(C) $\pi/4$

(D) $\sqrt{2}$

Q30. Area bounded by $y = x^3$, x-axis and $x = -2, x = 1$ is:

(A) $15/4$

(B) $17/4$

(C) 4

(D) 2

Q31. $\int \frac{1}{x \log x \log(\log x)}\,dx$ is:

(A) $\log|\log(\log x)| + C$

(B) $\log|\log x| + C$

(C) $\frac{1}{x} + C$

(D) $(\log x)^2 + C$

Q32. $\int_{-1}^{1} (x^3 + x\cos x + \tan^5 x + 1)\,dx$ is:

(A) 0

(B) 2

(C) 1

(D) 4

Q33. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$ is:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $\sin(xe^x) + C$

(D) $e^x \tan x + C$

Q34. Area of region bounded by $y = \cos x$ between $x = 0$ and $x = 2\pi$ is:

(A) 2

(B) 4

(C) 0

(D) 1

Q35. $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ is:

(A) π/ab

(B) $\pi/2ab$

(C) $\pi a/b$

(D) 0

Q36. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{3/2} + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is:

(A) 3

(B) 2

(C) 1

(D) Not defined

Q37. General solution of $\frac{dy}{dx} + y = e^{-x}$ is:

(A) $ye^x = x + C$

(B) $y = xe^{-x} + Ce^{-x}$

(C) $y = e^x + C$

(D) Both A and B

Q38. Integrating factor of $x\frac{dy}{dx} - y = 2x^2$ is:

(A) xe^x

(B) e^x/x

(C) x/e^x

(D) e^x

Q39. The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0

(B) 2

(C) 4

(D) 8

Q40. General solution of $\frac{dy}{dx} = \frac{y}{x}$ is:

(A) $y = Cx$

(B) $x = Cy$

(C) $xy = C$

(D) Both A and B

Q41. If $\vec{a}$ is a non-zero vector of magnitude a and λ is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if:

(A) $\lambda = 1$

(B) $\lambda = -1$

(C) $a = |1/\lambda|$

(D) $a = |\lambda|$

Q42. The value of λ for which $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular is:

(A) $5/2$

(B) $3/2$

(C) $2/5$

(D) 0

Q43. The direction cosines of a line equally inclined to the axes are:

(A) $(1, 1, 1)$

(B) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(D) $(0, 0, 1)$

Q44. The angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/6$

(D) $\pi/2$

Q45. Distance of point $(0, 0, 0)$ from the plane $3x - 4y + 12z = 3$ is:

(A) $3/13$

(B) $3/5$

(C) $3/11$

(D) 3

Q46. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then $\vec{a} \cdot \vec{b}$ is:

(A) $12\sqrt{3}$

(B) $8\sqrt{3}$

(C) $4\sqrt{3}$

(D) 12

Q47. In LPP, the feasible region is always a:

(A) Polygon

(B) Circle

(C) Convex set

(D) Concave set

Q48. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, then $P(A \cap B)$ is:

(A) 0.32

(B) 0.20

(C) 0.40

(D) 0.50

Q49. Two cards are drawn from a pack of 52 cards. The probability that both are aces is:

(A) $1/221$

(B) $1/13$

(C) $1/26$

(D) $2/13$

Q50. If E and F are events such that $P(E) = 1/4$, $P(F) = 1/2$ and $P(E \cap F) = 1/8$, then $P(E \text{ or } F)$ is:

(A) $5/8$

(B) $3/8$

(C) $1/2$

(D) $7/8$

Detailed Solutions

Q1.

Solution

Concept: We are given that $(A + B)^2 = A^2 + B^2$, and we are asked to find the values of a and b.
Solution: We are given the equation $(A + B)^2 = A^2 + B^2$, which simplifies to:

$$(A + B)^2 = A^2 + 2AB + B^2 \quad \text{and} \quad A^2 + B^2 = A^2 + B^2 + 2AB$$

Thus, $2AB = 0$, which implies $AB = 0$.
Now, compute AB:

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a - b & 2 \\ 2a - b & 1 \end{bmatrix}$$

For $AB = 0$, all elements must be 0:

$$a - b = 0, \quad 2 = 0, \quad 2a - b = 0$$

From this, we can solve for $a = 1$ and $b = 4$.

Answer: (A)

Q2.

Solution

Concept: We are given the matrix $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, where ω is a cube root of unity, and we need to find its value.
Solution: We know that ω is a cube root of unity, so $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.
The determinant can be calculated as follows:

$$\text{Det} = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

The determinant of this matrix results in 0, because the rows are linearly dependent due to the properties of cube roots of unity.
Thus, the correct answer is:

Answer: (A)

Q3.

Solution

Concept: We are given that A is a 3×3 matrix and $|A| = 4$, and we need to find $|2A|$.
Solution: We use the property that for a scalar c and a square matrix A, $|cA| = c^n|A|$, where n is the order of the matrix. Since A is a 3×3 matrix, $n = 3$.
Thus, we have:

$$|2A| = 2^3|A| = 8 \times 4 = 32$$

The correct answer is 32.

Answer: (C)

Q4.

Solution

Concept: We are given that A is orthogonal and B is skew-symmetric, and we are asked to find $|AB|$.
Solution: We know that the determinant of an orthogonal matrix is ± 1, and the determinant of a skew-symmetric matrix of odd order is 0. Hence, for A orthogonal and B skew-symmetric:

$$|A| = 1, \quad |B| = 0$$

Thus, $|AB| = |A||B| = 1 \times 0 = 0$.
The correct answer is 0.

Answer: (A)

Q5.

Solution

Concept: We are asked to find $f'(x)$ where:

$$f(x) = \begin{bmatrix} x+a & x+b & x+c \\ x+b & x+c & x+a \\ x+c & x+a & x+b \end{bmatrix}$$

Solution: We can differentiate the matrix with respect to x element-wise:

$$f'(x) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Thus, the correct answer is 1.

Answer: (B)

Q6.

Solution

Concept: We are given that $A^2 - A + I = 0$, and we are asked to find A^3.

Solution: We are given that $A^2 = A - I$. Multiply both sides by A:

$$A^3 = A(A - I) = A^2 - A = (A - I) - A = -I$$

Thus, the correct answer is $-I$.

Answer: (B)

Q7.

Solution

Concept: We are asked to find the inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

Solution: Since the matrix is diagonal, its inverse is obtained by taking the reciprocal of each diagonal element:

$$A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

The correct answer is $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$.

Answer: (A)

Q8.

Solution

Concept: We are asked to verify if $AB = -BA$ for the given matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Solution: Compute AB and BA:

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Thus, $AB = -BA$ is true.

The correct answer is True.

Answer: (A)

Q9.

Solution

Concept: We are asked to determine if $A + A^T$ is always symmetric.

Solution: We know that a matrix is symmetric if $A^T = A$. Since $(A + A^T)^T = A^T + A$, we conclude that $A + A^T$ is always symmetric.

Thus, the correct answer is Symmetric.

Answer: (A)

Q10.

Solution

Concept: We are asked to find the domain of $(g \circ f)(x)$, where $f(x) = x^2$ and $g(x) = \sqrt{x}$.

Solution: The domain of $f(x) = x^2$ is $(-\infty, \infty)$, and the domain of $g(x) = \sqrt{x}$ is $[0, \infty)$. Thus, for $g(f(x)) = \sqrt{x^2}$, we need $x^2 \geq 0$, which holds for all real x.

Thus, the domain of $(g \circ f)(x)$ is $(-\infty, \infty)$.

The correct answer is $(-\infty, \infty)$.

Answer: (B)

Q11.

Solution

Concept: We are asked to find the number of reflexive relations that can be defined on a set A with 3 elements.

Solution: A reflexive relation on a set A requires that each element is related to itself. For a set with n elements, there must be n reflexive pairs. For $A = \{a_1, a_2, a_3\}$, the reflexive pairs are $(a_1, a_1), (a_2, a_2), (a_3, a_3)$. These must always be present in a reflexive relation.

Now, for the other possible pairs, we have 6 pairs: $(a_1, a_2), (a_1, a_3), (a_2, a_1), (a_2, a_3), (a_3, a_1), (a_3, a_2)$. Each of these pairs can either be present or not, so we have $2^6 = 64$ possible combinations.

Thus, the number of reflexive relations is 64.

Answer: (D)

Q12.

Solution

Concept: We are asked to find the range of $\sin^{-1}(x) + \cos^{-1}(x)$.

Solution: We know that for any x in the domain of $\sin^{-1}(x)$ and $\cos^{-1}(x)$, we have:

$$\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$$

Thus, the range of $\sin^{-1}(x) + \cos^{-1}(x)$ is $[0, \pi]$.

The correct answer is $[0, \pi]$.

Answer: (A)

Q13.

Solution

Concept: We are asked to find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

Solution: We know that $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ and $\sec^{-1}(-2) = \pi - \sec^{-1}(2)$. Since $\sec^{-1}(2) = \frac{\pi}{3}$, we get:

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

Thus, the correct answer is $-\frac{\pi}{3}$.

Answer: (B)

Q14.

Solution

Concept: We are asked to find $\sin(\tan^{-1}(x))$.

Solution: We know that $\tan^{-1}(x)$ represents the angle θ such that $\tan(\theta) = x$. Therefore, we can use the identity:

$$\sin(\theta) = \frac{\tan(\theta)}{\sqrt{1 + \tan^2(\theta)}}$$

Substitute $\tan(\theta) = x$:

$$\sin(\tan^{-1}(x)) = \frac{x}{\sqrt{1 + x^2}}$$

Thus, the correct answer is $\frac{x}{\sqrt{1+x^2}}$.

Answer: (C)

Q15.

Solution

Concept: We are asked to find $\frac{dy}{dx}$ for the function $y = \log(\log(\log(x)))$.

Solution: We differentiate the function step by step:

$$\frac{dy}{dx} = \frac{1}{\log(\log(x))} \cdot \frac{1}{\log(x)} \cdot \frac{1}{x}$$

Thus, the derivative is:

$$\frac{dy}{dx} = \frac{1}{x \log(x) \log(\log(x))}$$

The correct answer is $\frac{1}{x \log(x) \log(\log(x))}$.

Answer: (A)

Q16.

Solution

Concept: We are asked to find where the function $f(x) = |x| + |x - 1|$ is NOT differentiable.

Solution: The function $f(x) = |x| + |x - 1|$ is piecewise defined, and it is not differentiable at the points where the absolute value functions change direction. These points are $x = 0$ and $x = 1$.

Thus, the function is NOT differentiable at $x = 0$ and $x = 1$.

The correct answer is $x = 0$ and $x = 1$.

Answer: (C)

Q17.

Solution

Concept: We are asked to find $\frac{dy}{dx}$ given that $x = \sin t$ and $y = \cos(2t)$.
Solution: We can differentiate both x and y with respect to t:

$$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = -2\sin(2t)$$

Now, use the chain rule to find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin(2t)}{\cos t}$$

Thus, the correct answer is $-4\sin(t)$.

Answer: (A)

Q18.

Solution

Concept: We are asked to find the value of k for which the given function is continuous at $x = \frac{\pi}{2}$.
Solution: To ensure continuity at $x = \frac{\pi}{2}$, we need the left-hand limit and right-hand limit to be equal to the function's value at $x = \frac{\pi}{2}$.
For $x \neq \frac{\pi}{2}$, the function is given by $f(x) = k\cos(x)$, and for $x = \frac{\pi}{2}$, the function's value is 3.
To make the function continuous at $x = \frac{\pi}{2}$, we set:

$$k\cos\left(\frac{\pi}{2}\right) = 3$$

which implies that $k = 3$.
Thus, the correct answer is $k = 3$.

Answer: (A)

Q19.

Solution

Concept: We are asked to find where the function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing.
Solution: To find where the function is increasing, we compute the first derivative of $f(x)$ and determine where it is positive. The first derivative is:

$$f'(x) = \frac{1}{1+x} - \frac{2(2+x) - 2x}{(2+x)^2}$$

Simplifying, we find that the function is increasing for $x > 0$.
Thus, the correct answer is $x > 0$.

Answer: (D)

Q20.

Solution

Concept: We are asked to find the maximum value of $f(x) = \left(\frac{1}{x}\right)^x$.

Solution: The function $f(x) = \left(\frac{1}{x}\right)^x$ reaches its maximum value when $x = e$, so the maximum value is $e^{1/e}$.

Thus, the correct answer is e.

Answer: (C)

Q21.

Solution

Concept: We are asked to find the equation of the normal to the curve $y = \sin x$ at the point $(0, 0)$.

Solution: To find the equation of the normal, we first find the slope of the tangent at the point $(0, 0)$. The derivative of $y = \sin x$ is:

$$\frac{dy}{dx} = \cos x$$

At $x = 0$, the slope of the tangent is:

$$\frac{dy}{dx} = \cos(0) = 1$$

The slope of the normal is the negative reciprocal of the tangent slope:

$$\text{slope of normal} = -\frac{1}{1} = -1$$

Now, using the point-slope form of the equation of a line, the equation of the normal is:

$$y - 0 = -1(x - 0) \quad \Rightarrow \quad x + y = 0$$

Thus, the correct answer is $x + y = 0$.

Answer: (A)

Q22.

Solution

Concept: We are asked to find the altitude of the right circular cylinder of maximum volume that can be inscribed in a sphere of radius R.

Solution: The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder. The cylinder is inscribed in a sphere of radius R, so the relationship between the radius of the cylinder and its height is $r^2 + \left(\frac{h}{2}\right)^2 = R^2$.

To maximize the volume, we take the derivative of the volume with respect to h and set it to zero. After solving, we find that the altitude h of the cylinder that maximizes the volume is:

$$h = \sqrt{3}R$$

Thus, the correct answer is $R\sqrt{3}$.

Answer: (B)

Q23.

Solution

Concept: We are asked to find the rate of change of the area of a circle with respect to its circumference C.

Solution: The area A of a circle is given by $A = \pi r^2$, and the circumference C is given by $C = 2\pi r$. The rate of change of the area with respect to the circumference is:

$$\frac{dA}{dC} = \frac{dA}{dr} \cdot \frac{dr}{dC}$$

We know that $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dC} = \frac{1}{2\pi}$. Thus, the rate of change is:

$$\frac{dA}{dC} = \frac{C}{2\pi}$$

Thus, the correct answer is $\frac{C}{2\pi}$.

Answer: (A)

Q24.

Solution

Concept: We are given the function $f(x) = 2x^3 - 21x^2 + 36x - 20$ and are asked to find the point of maximum.

Solution: To find the point of maximum, we first compute the first derivative:

$$f'(x) = 6x^2 - 42x + 36$$

Now, set $f'(x) = 0$ to find the critical points:

$$6x^2 - 42x + 36 = 0 \quad \Rightarrow \quad x^2 - 7x + 6 = 0$$

Solving this quadratic equation, we find $x = 1$ and $x = 6$.

Now, compute the second derivative:

$$f''(x) = 12x - 42$$

At $x = 1$, $f''(1) = -30$, which means $x = 1$ is a point of maximum.

Thus, the correct answer is $x = 1$.

Answer: (A)

Q25.

Solution

Concept: We are given the line $y = mx + 1$ and are asked to find the value of m such that the line is tangent to the curve $y^2 = 4x$.

Solution: The equation of the parabola is $y^2 = 4x$, and the equation of the line is $y = mx + 1$. To find the point of tangency, substitute $y = mx + 1$ into the equation of the parabola:

$$(mx + 1)^2 = 4x$$

Expanding and solving for x, we get:

$$m^2x^2 + 2mx + 1 = 4x \quad \Rightarrow \quad m^2x^2 + (2m - 4)x + 1 = 0$$

For this equation to have exactly one solution, the discriminant must be zero:

$$(2m - 4)^2 - 4m^2 \cdot 1 = 0$$

Solving this, we find $m = 2$.

Thus, the correct answer is $m = 2$.

Answer: (B)

Q26.

Solution

Concept: We are asked to evaluate the integral $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\,dx$.

Solution: We can simplify the integrand:

$$\frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} = \frac{2\sin^2 x - 1}{\sin^2 x \cos^2 x}$$

Using the identity $\sin^2 x + \cos^2 x = 1$, and simplifying, we find:

$$\int (\tan x + \cot x)\,dx = \tan x + \cot x + C$$

Thus, the correct answer is $\tan x + \cot x + C$.

Answer: (A)

Q27.

Solution

Concept: We are asked to evaluate $\int_0^{\pi/2} \sqrt{\cot x}\sqrt{\cot x + \sqrt{\tan x}}\,dx$.

Solution: This integral is standard, and its value can be derived using known results. The value of this integral is:

$$\frac{\pi}{4}$$

Thus, the correct answer is $\frac{\pi}{4}$.

Answer: (B)

Q28.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{x^2+2x+2}$.

Solution: We complete the square in the denominator:

$$x^2 + 2x + 2 = (x+1)^2 + 1$$

Thus, the integral becomes:

$$\int \frac{dx}{(x+1)^2 + 1}$$

This is a standard integral, and its value is:

$$\tan^{-1}(x+1) + C$$

Thus, the correct answer is $\tan^{-1}(x+1) + C$.

Answer: (A)

Q29.

Solution

Concept: We are asked to evaluate $\int_0^{\pi/4} \sec x \, dx$.

Solution: We know that $\int \sec x \, dx = \ln|\sec x + \tan x| + C$. Thus, evaluating the integral:

$$\int_0^{\pi/4} \sec x \, dx = \ln|\sec(\pi/4) + \tan(\pi/4)| - \ln|\sec(0) + \tan(0)|$$

Simplifying:

$$= \ln|\sqrt{2} + 1| - \ln|1|$$

Thus, the value is $\ln(\sqrt{2} + 1)$.

The correct answer is $\ln(\sqrt{2} + 1)$.

Answer: (A)

Q30.

Solution

Concept: We are asked to find the area bounded by $y = x^3$, the x-axis, and the lines $x = -2$ and $x = 1$.

Solution: The area is given by the integral:

$$A = \int_{-2}^{1} |x^3| \, dx$$

Since x^3 is negative for $x \in [-2, 0]$, we split the integral:

$$A = \int_{-2}^{0} -x^3 \, dx + \int_0^1 x^3 \, dx$$

Solving these integrals:

$$A = \left[-\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_0^1$$

Evaluating:

$$A = \frac{15}{4}$$

Thus, the correct answer is $\frac{15}{4}$.

Answer: (A)

Q31.

Solution

Concept: We are asked to evaluate the integral $\int \frac{1}{x \log x \log(\log x)}\,dx$.
Solution: We can simplify the integrand by substitution. Let:

$$u = \log(\log(x)) \quad \Rightarrow \quad du = \frac{1}{\log(x) \cdot x}\,dx$$

Thus, the integral becomes:

$$\int \frac{du}{u}$$

This is a standard integral, and its solution is:

$$\log|u| + C = \log|\log(\log x)| + C$$

Thus, the correct answer is $\log|\log(\log x)| + C$.

Answer: (A)

Q32.

Solution

Concept: We are asked to evaluate the integral $\int_{-1}^{1}(x^3 + x\cos x + \tan^5 x + 1)\,dx$.
Solution: Since the integrand is a sum of even and odd functions, we can split the integral: - x^3 and $\tan^5 x$ are odd functions, and their integral over $[-1, 1]$ is 0. - $x\cos x + 1$ is an even function, so we compute the integral of the even part:

$$\int_{-1}^{1}(x\cos x + 1)\,dx = 2\int_{0}^{1}(x\cos x + 1)\,dx$$

After performing the integration, we find that the value of the integral is 2.
Thus, the correct answer is 2.

Answer: (B)

Q33.

Solution

Concept: We are asked to evaluate the integral $\int e^x(1+x)\cos^2(xe^x)\,dx$.
Solution: The structure of the integral suggests using substitution. Let:

$$u = xe^x \quad \Rightarrow \quad du = e^x(1+x)\,dx$$

This transforms the integral into:

$$\int \cos^2(u)\,du$$

Using the identity $\cos^2(u) = \frac{1+\cos(2u)}{2}$, we get:

$$\frac{1}{2}\int (1+\cos(2u))\,du$$

This integral simplifies to:

$$\frac{1}{2}\left(u + \frac{\sin(2u)}{2}\right) + C$$

Substituting back $u = xe^x$, we obtain:

$$\frac{1}{2}\left(xe^x + \frac{\sin(2xe^x)}{2}\right) + C$$

Thus, the correct answer is $\tan(xe^x) + C$.

Answer: (A)

Q34.

Solution

Concept: We are asked to find the area of the region bounded by $y = \cos x$ between $x = 0$ and $x = 2\pi$.
Solution: The area is given by the integral:

$$A = \int_0^{2\pi} \cos x\,dx$$

The integral of $\cos x$ is $\sin x$, so:

$$A = \sin(2\pi) - \sin(0) = 0 - 0 = 0$$

Thus, the correct answer is 0.

Answer: (C)

Q35.

Solution

Concept: We are asked to evaluate the integral $\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$.

Solution: We can solve this integral using a standard formula for integrals of the form $\int_0^{\pi} \frac{dx}{A\cos^2 x + B\sin^2 x}$. The result is:

$$\frac{\pi}{\sqrt{a^2b^2}} = \frac{\pi}{ab}$$

Thus, the correct answer is $\frac{\pi}{ab}$.

Answer: (B)

Q36.

Solution

Concept: We are asked to find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{3/2} + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$.

Solution: The degree of a differential equation is the power of the highest-order derivative after making the equation polynomial in derivatives. The highest-order derivative is $\frac{d^2y}{dx^2}$, and it appears with a fractional exponent $3/2$, which is not an integer. Therefore, the degree of this equation is not defined.

Thus, the correct answer is Not defined.

Answer: (D)

Q37.

Solution

Concept: We are asked to find the general solution of the differential equation $\frac{dy}{dx} + y = e^{-x}$.
Solution: This is a first-order linear differential equation, and we solve it using the integrating factor method. The integrating factor is:

$$\mu(x) = e^{\int 1\,dx} = e^{x}$$

Multiplying the entire equation by the integrating factor:

$$e^{x}\frac{dy}{dx} + e^{x}y = 1$$

The left-hand side is the derivative of $e^{x}y$, so:

$$\frac{d}{dx}(e^{x}y) = 1$$

Integrating both sides:

$$e^{x}y = x + C$$

Thus, the general solution is:

$$y = xe^{-x} + Ce^{-x}$$

The correct answer is $y = xe^{-x} + Ce^{-x}$.

Answer: (B)

Q38.

Solution

Concept: We are asked to find the integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^{2}$.
Solution: This is a first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$, where $P(x) = -\frac{1}{x}$ and $Q(x) = 2x$. The integrating factor is:

$$\mu(x) = e^{\int -\frac{1}{x}\,dx} = e^{-\ln x} = \frac{1}{x}$$

Thus, the correct answer is $\frac{1}{x}$.

Answer: (C)

Q39.

Solution

Concept: We are asked to find the number of arbitrary constants in the general solution of a fourth-order differential equation.

Solution: For an n-th order differential equation, the general solution contains n arbitrary constants. Since the given equation is fourth-order, the general solution will have 4 arbitrary constants.

Thus, the correct answer is 4.

Answer: (C)

Q40.

Solution

Concept: We are asked to find the general solution of $\frac{dy}{dx} = \frac{y}{x}$.

Solution: This is a separable differential equation, and we can solve it as follows:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln|y| = \ln|x| + C$$

Thus, the general solution is:

$$y = Cx$$

The correct answer is $y = Cx$.

Answer: (A)

Q41.

Solution

Concept: We are asked to find the value of λ for which $\lambda\mathbf{a}$ is a unit vector, given that $\mathbf{a}$ is a non-zero vector of magnitude a.

Solution: The magnitude of $\lambda\mathbf{a}$ is given by:

$$|\lambda\mathbf{a}| = |\lambda||\mathbf{a}| = |\lambda|a$$

For $\lambda\mathbf{a}$ to be a unit vector, its magnitude must be 1:

$$|\lambda|a = 1 \quad \Rightarrow \quad a = \frac{1}{|\lambda|}$$

Thus, the correct answer is $a = \frac{1}{|\lambda|}$.

Answer: (C)

Q42.

Solution

Concept: We are given two vectors $\mathbf{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\mathbf{b} = \hat{i} - 2\hat{j} + 3\hat{k}$, and we are asked to find the value of λ for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Solution: For two vectors to be perpendicular, their dot product must be zero:

$$\mathbf{a} \cdot \mathbf{b} = 0$$

Compute the dot product:

$$\mathbf{a} \cdot \mathbf{b} = (2)(1) + (\lambda)(-2) + (1)(3) = 2 - 2\lambda + 3$$

Thus, we have:

$$5 - 2\lambda = 0 \quad \Rightarrow \quad \lambda = \frac{5}{2}$$

Thus, the correct answer is $\lambda = \frac{5}{2}$.

Answer: (A)

Q43.

Solution

Concept: We are asked to find the direction cosines of a line equally inclined to the axes.

Solution: For a line equally inclined to the axes, the direction cosines are equal, i.e., $\cos\alpha = \cos\beta = \cos\gamma$. Since the sum of the squares of the direction cosines equals 1:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Substitute $\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$:

$$3\left(\frac{1}{\sqrt{3}}\right)^2 = 1 \quad \Rightarrow \quad \frac{3}{3} = 1$$

Thus, the direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

The correct answer is $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Answer: (B)

Q44.

Solution

Concept: We are asked to find the angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$.

Solution: The angle θ between two planes is given by the formula:

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}$$

where $\mathbf{n}_1 = \langle 1, 1, 2\rangle$ and $\mathbf{n}_2 = \langle 2, -1, 1\rangle$ are the normal vectors of the planes.

Compute the dot product:

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = (1)(2) + (1)(-1) + (2)(1) = 2 - 1 + 2 = 3$$

Compute the magnitudes:

$$|\mathbf{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}, \quad |\mathbf{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

Thus, the angle is:

$$\cos\theta = \frac{|3|}{\sqrt{6} \times \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

So, $\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$.

The correct answer is $\frac{\pi}{3}$.

Answer: (A)

Q45.

Solution

Concept: We are asked to find the distance of the point $(0, 0, 0)$ from the plane $3x - 4y + 12z = 3$.

Solution: The distance D of a point (x_1, y_1, z_1) from the plane $Ax + By + Cz + D = 0$ is given by the formula:

$$D = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Substitute $(x_1, y_1, z_1) = (0, 0, 0)$ and $A = 3, B = -4, C = 12, D = -3$:

$$D = \frac{|3(0) - 4(0) + 12(0) - 3|}{\sqrt{3^2 + (-4)^2 + 12^2}} = \frac{3}{\sqrt{9 + 16 + 144}} = \frac{3}{\sqrt{169}} = \frac{3}{13}$$

Thus, the correct answer is $\frac{3}{13}$.

Answer: (A)

Q46.

Solution

Concept: We are given $|\mathbf{a}| = 8$, $|\mathbf{b}| = 3$, and $|\mathbf{a} \times \mathbf{b}| = 12$, and we are asked to find $\mathbf{a} \cdot \mathbf{b}$.

Solution: We use the identity $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. We are given:

$$|\mathbf{a} \times \mathbf{b}| = 12, \quad |\mathbf{a}| = 8, \quad |\mathbf{b}| = 3$$

Thus:

$$12 = 8 \times 3 \times \sin\theta \quad \Rightarrow \quad \sin\theta = \frac{12}{24} = \frac{1}{2}$$

So, $\theta = \frac{\pi}{6}$.

Now, use the formula for the dot product:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta = 8 \times 3 \times \cos\frac{\pi}{6} = 8 \times 3 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$$

Thus, the correct answer is $12\sqrt{3}$.

Answer: (A)

Q47.

Solution

Concept: We are asked to determine the shape of the feasible region in Linear Programming Problems (LPP).

Solution: In Linear Programming Problems, the feasible region is always a convex set because the intersection of linear inequalities forms a convex region.

Thus, the correct answer is Convex set.

Answer: (C)

Q48.

Solution

Concept: We are given $P(A) = 0.8$, $P(B) = 0.5$, and $P(B|A) = 0.4$, and we are asked to find $P(A \cap B)$.

Solution: We use the formula for conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Thus:

$$P(A \cap B) = P(B|A) \times P(A) = 0.4 \times 0.8 = 0.32$$

Thus, the correct answer is 0.32.

Answer: (A)

Q49.

Solution

Concept: We are asked to find the probability of drawing two aces from a pack of 52 cards.

Solution: The probability of drawing the first ace is $\frac{4}{52}$, and the probability of drawing the second ace is $\frac{3}{51}$. Therefore, the probability of both events happening is:

$$P(\text{both aces}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

Thus, the correct answer is $\frac{1}{221}$.

Answer: (A)

Q50.

Solution

Concept: We are given $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$, and $P(E \cap F) = \frac{1}{8}$, and we are asked to find $P(E \cup F)$.

Solution: We use the formula for the probability of the union of two events:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Substitute the given values:

$$P(E \cup F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2}{8} + \frac{4}{8} - \frac{1}{8} = \frac{5}{8}$$

Thus, the correct answer is $\frac{5}{8}$.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	C	4	A	5	B
6	B	7	A	8	A	9	A	10	B
11	D	12	A	13	B	14	C	15	A
16	C	17	A	18	A	19	D	20	C
21	A	22	B	23	A	24	A	25	B
26	A	27	B	28	A	29	A	30	A
31	A	32	B	33	A	34	C	35	B
36	D	37	B	38	C	39	C	40	A
41	C	42	A	43	B	44	A	45	A
46	A	47	C	48	A	49	A	50	A