CUET-UG Mathematics Sample Paper-18

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix of order 3 such that $|A| = 3$, then the value of $|\text{adj}(\text{adj } A)|$ is:

(A) 9

(B) 27

(C) 81

(D) 243

Q2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) I

(D) O

Q3. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Q4. If A is a symmetric matrix and B is a skew-symmetric matrix of the same order, then $(AB + BA)$ is:

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Null matrix

(D) Identity matrix

Q5. The function $f(x) = \frac{x}{\ln x}$ is increasing in the interval:

(A) $(0, 1)$

(B) $(0, e)$

(C) (e, ∞)

(D) $(-\infty, e)$

Q6. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\sqrt{3}$

Q7. The interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly decreasing is:

(A) $(-2, 3)$

(B) $(-\infty, -2)$

(C) $(3, \infty)$

(D) $(-\infty, -2) \cup (3, \infty)$

Q8. The point on the curve $y^2 = x$ where the tangent makes an angle of $\pi/4$ with the x-axis is:

(A) $(1/2, 1/4)$

(B) $(1/4, 1/2)$

(C) $(4, 2)$

(D) $(1, 1)$

Q9. $\int \frac{dx}{x + x \ln x}$ is equal to:

(A) $\ln|1 + \ln x| + C$

(B) $\ln|\ln x| + C$

(C) $x \ln(1 + \ln x) + C$

(D) $\frac{1}{(1+\ln x)^2} + C$

Q10. The value of $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 0

(D) π

Q11. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:

(A) $32/3$ sq. units

(B) $16/3$ sq. units

(C) $8/3$ sq. units

(D) $64/3$ sq. units

Q12. The order and degree of the differential equation $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^{1/3} + x^{1/4} = 0$ are:

(A) 2, 3

(B) 3, 2

(C) 2, 1

(D) 2, not defined

Q13. The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $\tan^{-1} y - \tan^{-1} x = C$

(C) $y - x = C(1 + xy)$

(D) $y + x = C(1 - xy)$

Q14. If a fair die is rolled, the probability that the number shown is a prime number is:

(A) $1/3$

(B) $1/2$

(C) $2/3$

(D) $1/6$

Q15. The maximum value of $Z = 3x + 4y$ subject to $x + y \leq 4, x \geq 0, y \geq 0$ is:

(A) 12

(B) 16

(C) 0

(D) 10

Q16. Let $A = \{1, 2, 3\}$. The number of equivalence relations containing $(1, 2)$ is:

(A) 1

(B) 2

(C) 3

(D) 8

Q17. If $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = (3 - x^3)^{1/3}$, then $f(f(x))$ is:

(A) $x^{1/3}$

(B) x^3

(C) x

(D) $3 - x^3$

Q18. Let $f : A \to B$ be an onto function where $n(A) = 5$ and $n(B) = 4$. The total number of such functions is:

(A) 240

(B) 120

(C) 1024

(D) 625

Q19. The value of $\cos^{-1}(\cos \frac{7\pi}{6})$ is:

(A) $7\pi/6$

(B) $5\pi/6$

(C) $\pi/6$

(D) $-\pi/6$

Q20. The domain of $\sin^{-1}(2x)$ is:

(A) $[-1, 1]$

(B) $[-1/2, 1/2]$

(C) $[-2, 2]$

(D) $(-\infty, \infty)$

Q21. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^4 is:

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

Q22. A square matrix A is called idempotent if $A^2 = A$. If A is idempotent, then $(I + A)^3 - 7A$ is:

(A) I

(B) A

(C) $I - A$

(D) $3A$

Q23. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactor of a_{ij}, then value of Δ is:

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Q24. If x, y, z are all different and $\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then $1 + xyz$ is:

(A) 0

(B) 1

(C) -1

(D) $x + y + z$

Q25. The area of a triangle with vertices $(-3, 0), (3, 0), (0, k)$ is 9 sq. units. The value of k is:

(A) 9

(B) 3

(C) -3

(D) 3 or -3

Q26. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$ is continuous at $x = 0$, then a is:

(A) 4

(B) 8

(C) 16

(D) 2

Q27. If $y = \tan^{-1}(\frac{\sin x+\cos x}{\cos x-\sin x})$, then $\frac{dy}{dx}$ is:

(A) 1

(B) 0

(C) 1/2

(D) $\pi/4$

Q28. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is:

(A) $\frac{\ln x}{(1+\ln x)^2}$

(B) $\frac{1}{(1+\ln x)^2}$

(C) $\frac{x-y}{1+\ln x}$

(D) $\frac{\ln x}{1+\ln x}$

Q29. If $x = a\cos^3 t, y = a\sin^3 t$, then $\frac{d^2y}{dx^2}$ at $t = \pi/4$ is:

(A) $3/a$

(B) $3\sqrt{2}/a$

(C) $4\sqrt{2}/(3a)$

(D) $8/(3a)$

Q30. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. The rate at which the radius of the balloon increases when the radius is 15 cm is:

(A) $1/\pi$ cm/s

(B) $2/\pi$ cm/s

(C) $1/(2\pi)$ cm/s

(D) $900/\pi$ cm/s

Q31. The maximum value of $x^{1/x}$ is:

(A) e

(B) $e^{1/e}$

(C) $1/e$

(D) $(1/e)^e$

Q32. The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ if the value of m is:

(A) 1

(B) 2

(C) 3

(D) 1/2

Q33. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $\sin(xe^x) + C$

(D) $\tan(e^x) + C$

Q34. $\int \frac{dx}{x^2+2x+2}$ equals:

(A) $x \tan^{-1}(x + 1) + C$

(B) $\tan^{-1}(x + 1) + C$

(C) $(x + 1) \tan^{-1}(x + 1) + C$

(D) $\tan^{-1} x + C$

Q35. The value of $\int_{1/e}^{e} |\ln x| dx$ is:

(A) $1 - 1/e$

(B) $2(1 - 1/e)$

(C) $e - 1/e$

(D) 0

Q36. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ is:

(A) $\pi^2/4$

(B) $\pi/4$

(C) $\pi^2/2$

(D) $\pi/2$

Q37. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is:

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

Q38. The area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$ is:

(A) $3/8$ sq. units

(B) $5/8$ sq. units

(C) $9/8$ sq. units

(D) $7/8$ sq. units

Q39. The area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and the y-axis in the first quadrant is:

(A) $\sqrt{2} - 1$

(B) $\sqrt{2} + 1$

(C) $\sqrt{2}$

(D) $2-\sqrt{2}$

Q40. The general solution of the differential equation $\frac{dy}{dx}+y=e^{-x}$ is:

(A) $y=(x+C)e^{x}$

(B) $y=(x+C)e^{-x}$

(C) $y=xe^{-x}+C$

(D) $y=e^{-x}+Cx$

Q41. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ is:

(A) $y=\frac{x^2}{4}+\frac{C}{x^2}$

(B) $y=\frac{x^4}{4}+C$

(C) $y=\frac{x^2}{4}+Cx^2$

(D) $y=x^2+C$

Q42. The integrating factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay$ is:

(A) $\frac{1}{y^2-1}$

(B) $\frac{1}{\sqrt{1-y^2}}$

(C) $\frac{1}{1-y^2}$

(D) $\sqrt{1-y^2}$

Q43. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=2$, $|\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\pi/6$

(B) $\pi/3$

(C) $\pi/4$

(D) $\pi/2$

Q44. The scalar projection of the vector $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ on the vector $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ is:

(A) $1/\sqrt{3}$

(B) $\sqrt{3}$

(C) 0

(D) 1

Q45. The value of λ for which the vectors $\hat{i} - \hat{j} + \hat{k}, 2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar is:

(A) 1

(B) 2

(C) 3

(D) 0

Q46. The distance of the point $(2, 3, -5)$ from the plane $x + 2y - 2z = 9$ is:

(A) 3

(B) 2

(C) 1

(D) 0

Q47. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is:

(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/6$

(D) 0

Q48. The shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is:

(A) $3\sqrt{2}$

(B) $3/\sqrt{2}$

(C) $\sqrt{6}$

(D) 0

Q49. In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which maximum Z occurs is:

(A) 0

(B) 2

(C) Finite

(D) Infinite

Q50. If A and B are two independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B')$ is:

(A) 0.12

(B) 0.18

(C) 0.28

(D) 0.7

Detailed Solutions

Q1.

Solution

Concept: For a square matrix A of order n, the determinant of the adjoint is $|\text{adj } A| = |A|^{n-1}$. Consequently, the determinant of the adjoint of the adjoint is given by $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$.

Solution: Given $|A| = 3$ and the order $n = 3$. Using the formula:

$$|\text{adj}(\text{adj } A)| = |A|^{(3-1)^2} = |A|^{2^2} = |A|^4$$

Substituting the given value of $|A|$:

$$|\text{adj}(\text{adj } A)| = 3^4 = 81$$

Final Answer: 81

Answer: (C)

Q2.

Solution

Concept: We can solve this by direct matrix computation or by using the Cayley-Hamilton Theorem, which states that every square matrix satisfies its own characteristic equation $|A - \lambda I| = 0$.

Solution: **Method 1: Direct Calculation**

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$A^2 - 5A = \begin{bmatrix} 7-5 & 10-10 \\ 15-15 & 22-20 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$$

Method 2: Cayley-Hamilton Theorem The characteristic equation is $|A - \lambda I| = 0$:

$$(1-\lambda)(4-\lambda) - 6 = 0 \implies \lambda^2 - 5\lambda + 4 - 6 = 0 \implies \lambda^2 - 5\lambda - 2 = 0$$

Replacing λ with A:

$$A^2 - 5A - 2I = O \implies A^2 - 5A = 2I$$

Final Answer: $2I$

Answer: (A)

Q3.

Solution

Concept: The total number of possible matrices is determined by the number of elements in the matrix and the number of choices available for each element. For an $m \times n$ matrix, there are $m \times n$ elements.

Solution: A 3×3 matrix has $3 \times 3 = 9$ entries (positions). Each entry can be either 0 or 1, providing 2 choices for each of the 9 positions. Total number of possible matrices = $2 \times 2 \times 2 \times \dots$ (9 times)

$$\text{Total matrices} = 2^9 = 512$$

Final Answer: 512

Answer: (D)

Q4.

Solution

Concept: Recall that a matrix A is symmetric if $A^T = A$ and a matrix B is skew-symmetric if $B^T = -B$. The transpose of a sum is the sum of transposes, and $(XY)^T = Y^T X^T$.

Solution: Given $A^T = A$ and $B^T = -B$. Let $X = AB + BA$. Find the transpose of X:

$$X^T = (AB + BA)^T = (AB)^T + (BA)^T$$

$$X^T = B^T A^T + A^T B^T$$

Substitute the given conditions:

$$X^T = (-B)(A) + (A)(-B)$$

$$X^T = -BA - AB = -(AB + BA)$$

Since $X^T = -X$, the matrix $AB + BA$ is skew-symmetric.

Final Answer: Skew-symmetric matrix

Answer: (B)

Q5.

Solution

Concept: A function $f(x)$ is increasing in an interval where its first derivative $f'(x) > 0$. We use the quotient rule: $(\frac{u}{v})' = \frac{vu'-uv'}{v^2}$.
Solution: Given $f(x) = \frac{x}{\ln x}$. Differentiate with respect to x:

$$f'(x) = \frac{(\ln x)(1) - (x)(\frac{1}{x})}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

For the function to be increasing, $f'(x) > 0$:

$$\frac{\ln x - 1}{(\ln x)^2} > 0$$

Since $(\ln x)^2$ is always positive (for $x > 0, x \neq 1$), we only need:

$$\ln x - 1 > 0 \implies \ln x > 1 \implies x > e$$

Thus, the interval is (e, ∞).
Final Answer: (e, ∞)

Answer: (C)

Q6.

Solution

Concept: The expression $a \sin x + b \cos x$ can be rewritten as $\sqrt{a^2 + b^2} \sin(x + \phi)$. The maximum value of such an expression is $\sqrt{a^2 + b^2}$.
Solution: Given $f(x) = \sin x + \cos x$. Here $a = 1$ and $b = 1$. Maximum value = $\sqrt{1^2 + 1^2} = \sqrt{2}$.
Final Answer: $\sqrt{2}$

Answer: (C)

Q7.

Solution

Concept: A function is strictly decreasing in the interval where its first derivative $f'(x) < 0$.
Solution: Given $f(x) = 2x^3 - 3x^2 - 36x + 7$.

$$f'(x) = 6x^2 - 6x - 36$$

Set $f'(x) < 0$:

$$6(x^2 - x - 6) < 0 \implies (x - 3)(x + 2) < 0$$

The roots are $x = -2$ and $x = 3$. The quadratic expression is negative between the roots. Therefore, the function is strictly decreasing in the interval $(-2, 3)$.
Final Answer: $(-2, 3)$

Answer: (A)

Q8.

Solution

Concept: The slope of the tangent to a curve at a point is given by dy/dx. If the tangent makes an angle θ with the x-axis, then $dy/dx = \tan\theta$.
Solution: Given $y^2 = x$. Differentiating with respect to x:

$$2y\frac{dy}{dx} = 1 \implies \frac{dy}{dx} = \frac{1}{2y}$$

The slope is $\tan(\pi/4) = 1$.

$$\frac{1}{2y} = 1 \implies y = \frac{1}{2}$$

Substitute $y = 1/2$ into the curve equation $y^2 = x$:

$$(1/2)^2 = x \implies x = \frac{1}{4}$$

The point is $(1/4, 1/2)$.
Final Answer: $(1/4, 1/2)$

Answer: (B)

Q9.

Solution

Concept: Integration by substitution (also known as u-substitution) is used when the integrand contains a function and its derivative. This technique simplifies the integral by changing the variable of integration.

Solution: The given integral is $I = \int \frac{dx}{x+x\ln x}$. First, factor out x from the denominator:

$$I = \int \frac{1}{x(1+\ln x)}\,dx$$

Now, let $u = 1 + \ln x$. Differentiating with respect to x gives:

$$\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}\,dx$$

Substitute u and du into the integral:

$$I = \int \frac{1}{u}\,du$$

The integral of $1/u$ is $\ln|u| + C$:

$$I = \ln|u| + C$$

Substituting back the original variable $u = 1 + \ln x$:

$$I = \ln|1 + \ln x| + C$$

Final Answer: $\ln|1 + \ln x| + C$

Answer: (A)

Q10.

Solution

Concept: This definite integral is solved using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. This property helps in simplifying trigonometric fractions where the denominator remains invariant under the transformation.

Solution: Let $I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx$ — (1) Using the property $x \to \pi/2 - x$:

$$I = \int_0^{\pi/2} \frac{\sin^{3/2}(\pi/2 - x)}{\sin^{3/2}(\pi/2 - x) + \cos^{3/2}(\pi/2 - x)}\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x}\,dx \text{ — (2)}$$

Adding (1) and (2):

$$2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2} = \pi/2 \implies I = \pi/4$$

Final Answer: $\pi/4$

Answer: (B)

Q11.

Solution

Concept: The area bounded by a curve $y = f(x)$ and a line $y = k$ is calculated by finding the points of intersection and integrating the difference between the upper and lower boundaries.

Solution: The curve is $y = x^2$ and the line is $y = 4$. Intersection: $x^2 = 4 \implies x = \pm 2$. Since the curve is symmetric about the y-axis, the total area is:

$$\text{Area} = 2\int_0^2 (4 - x^2)\,dx$$

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2 = 2\left(8 - \frac{8}{3}\right)$$

$$\text{Area} = 2\left(\frac{24 - 8}{3}\right) = \frac{32}{3} \text{ sq. units}$$

Final Answer: $32/3$ sq. units

Answer: (A)

Q12.

Solution

Concept: The **order** is the highest derivative present. The **degree** is the power of the highest derivative after the equation is made a polynomial in its derivatives (i.e., eliminating radical or fractional powers of derivatives).

Solution: Given: $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^{1/3} + x^{1/4} = 0$ 1. The highest order derivative is $\frac{d^2y}{dx^2}$, so the **order is 2**. 2. To find the degree, isolate the term with the fractional power of the derivative:

$$\frac{d^2y}{dx^2} + x^{1/4} = -(\frac{dy}{dx})^{1/3}$$

Cubing both sides to remove the $1/3$ power:

$$(\frac{d^2y}{dx^2} + x^{1/4})^3 = -\frac{dy}{dx}$$

The power of the highest order derivative $(\frac{d^2y}{dx^2})$ is 3. Thus, the **degree is 3**.

Final Answer: 2, 3

Answer: (A)

Q13.

Solution

Concept: This is a variable separable differential equation. We group all y terms with dy and all x terms with dx, then integrate both sides.

Solution:

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \implies \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both sides:

$$\int \frac{1}{1+y^2}\,dy = \int \frac{1}{1+x^2}\,dx$$

$$\tan^{-1} y = \tan^{-1} x + C \implies \tan^{-1} y - \tan^{-1} x = C$$

Final Answer: $\tan^{-1} y - \tan^{-1} x = C$

Answer: (B)

Q14.

Solution

Concept: The probability of an event is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes in the sample space.

Solution: When a fair die is rolled, the sample space $S = \{1, 2, 3, 4, 5, 6\}$. Total outcomes $n(S) = 6$. The prime numbers in this set are $\{2, 3, 5\}$. Note that 1 is not a prime number. Favorable outcomes $E = \{2, 3, 5\}$, so $n(E) = 3$.

$$P(\text{Prime}) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Final Answer: $1/2$

Answer: (B)

Q15.

Solution

Concept: In a Linear Programming Problem, the maximum or minimum value of the objective function Z always occurs at one of the corner points (vertices) of the feasible region.

Solution: The constraints are $x + y \leq 4, x \geq 0, y \geq 0$. The feasible region is a triangle with vertices: 1. Origin $(0, 0)$ 2. Intersection of $x + y = 4$ with x-axis: $(4, 0)$ 3. Intersection of $x + y = 4$ with y-axis: $(0, 4)$ Evaluate $Z = 3x + 4y$ at these points: - At $(0, 0)$: $Z = 3(0) + 4(0) = 0$ - At $(4, 0)$: $Z = 3(4) + 4(0) = 12$ - At $(0, 4)$: $Z = 3(0) + 4(4) = 16$ The maximum value is 16.

Final Answer: 16

Answer: (B)

Q16.

Solution

Concept: An equivalence relation on a set A must be reflexive, symmetric, and transitive. We start with the smallest possible set of ordered pairs and add elements as required by these properties.

Solution: Given $A = \{1, 2, 3\}$. For reflexivity, the relation R must contain $\{(1, 1), (2, 2), (3, 3)\}$. We are told R contains $(1, 2)$. For symmetry, R must contain $(2, 1)$. 1. Smallest relation: $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. This is transitive, so it is an equivalence relation. 2. If we add $(1, 3)$, symmetry requires $(3, 1)$. Transitivity then requires $(2, 3)$ and $(3, 2)$ (because $(2, 1) \in R$ and $(1, 3) \in R \implies (2, 3) \in R$). This results in the universal relation $A \times A$. There are exactly 2 such relations.

Final Answer: 2

Answer: (B)

Q17.

Solution

Concept: The composition of functions $f(f(x))$ involves substituting the entire expression of $f(x)$ into itself wherever the variable x appears.

Solution: Given $f(x) = (3 - x^3)^{1/3}$.

$$f(f(x)) = [3 - (f(x))^3]^{1/3}$$

Substitute the expression for $f(x)$:

$$f(f(x)) = [3 - ((3 - x^3)^{1/3})^3]^{1/3}$$

Since $((a)^{1/3})^3 = a$:

$$f(f(x)) = [3 - (3 - x^3)]^{1/3}$$

$$f(f(x)) = [3 - 3 + x^3]^{1/3}$$

$$f(f(x)) = [x^3]^{1/3} = x$$

Final Answer: x

Answer: (C)

Q18.

Solution

Concept: The number of onto functions (surjective functions) from a set A with n elements to a set B with m elements ($n \geq m$) is given by the formula:

$$\sum_{r=0}^{m} (-1)^r \binom{m}{r} (m - r)^n$$

Solution: Given $n(A) = 5$ and $n(B) = 4$. Using the formula for $n = 5, m = 4$:

$$\text{Total} = \binom{4}{0}4^5 - \binom{4}{1}3^5 + \binom{4}{2}2^5 - \binom{4}{3}1^5 + \binom{4}{4}0^5$$

$$= (1 \times 1024) - (4 \times 243) + (6 \times 32) - (4 \times 1) + 0$$

$$= 1024 - 972 + 192 - 4 = 240$$

Final Answer: 240

Answer: (A)

Q19.

Solution

Concept: The range of the principal value branch of $\cos^{-1} x$ is $[0, \pi]$. If the angle θ is outside this range, we must use trigonometric identities to find an equivalent angle within $[0, \pi]$.
Solution: The given angle is $7\pi/6$, which is greater than π. We know that $\cos(2\pi - \theta) = \cos\theta$.

$$\cos(7\pi/6) = \cos(2\pi - 7\pi/6) = \cos(5\pi/6)$$

Since $5\pi/6 \in [0, \pi]$, we have:

$$\cos^{-1}(\cos\frac{7\pi}{6}) = \cos^{-1}(\cos\frac{5\pi}{6}) = \frac{5\pi}{6}$$

Final Answer: $5\pi/6$

Answer: (B)

Q20.

Solution

Concept: The domain of the function $\sin^{-1}(u)$ is defined for $-1 \le u \le 1$.
Solution: For $\sin^{-1}(2x)$ to be defined:

$$-1 \le 2x \le 1$$

Dividing the entire inequality by 2:

$$-\frac{1}{2} \le x \le \frac{1}{2}$$

So, the domain is $[-1/2, 1/2]$.
Final Answer: $[-1/2, 1/2]$

Answer: (B)

Q21.

Solution

Concept: Calculate powers of a matrix by repeated multiplication. Note if the matrix represents a specific transformation (like a reflection).

Solution: Given $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Since $A^2 = I$, then $A^4 = (A^2)^2 = I^2 = I$.

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Final Answer: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Answer: (B)

Q22.

Solution

Concept: Use the algebraic expansion $(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$. Since I is the identity matrix, $IA = AI = A$ and $I^n = I$. For an idempotent matrix, $A^2 = A$ and $A^3 = A^2 \cdot A = A \cdot A = A$.
Solution: Expand $(I + A)^3$:

$$(I + A)^3 = I + 3A + 3A^2 + A^3$$

Substitute $A^2 = A$ and $A^3 = A$:

$$(I + A)^3 = I + 3A + 3A + A = I + 7A$$

Now, calculate the full expression:

$$(I + A)^3 - 7A = (I + 7A) - 7A = I$$

Final Answer: I

Answer: (A)

Q23.

Solution

Concept: The determinant of a matrix is the sum of the products of elements of any row (or column) with their corresponding cofactors.

Solution: - Option A: Elements of row 1 with cofactors of row 3 (sums to 0). - Option B: Elements of row 1 with cofactors of column 1 (incorrect mix). - Option C: Elements of row 2 with cofactors of row 1 (sums to 0). - Option D: Elements of column 1 with their corresponding cofactors A_{11}, A_{21}, A_{31}. This correctly defines the expansion along the first column.

$$\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$$

Final Answer: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer: (D)

Q24.

Solution

Concept: A determinant containing a sum in a column can be split into two determinants. Properties of determinants (like taking out common factors and swapping rows) can then be used.

Solution: Split the determinant:

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

In the second determinant, take x, y, z common from R_1, R_2, R_3:

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

Rearranging the first determinant to match the second (two column swaps):

$$(1 + xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

Since x, y, z are distinct, the Vandermonde determinant is non-zero.

$$1 + xyz = 0$$

Final Answer: 0

Answer: (A)

Q25.

Solution

Concept: The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Solution: Vertices: $(-3, 0), (3, 0), (0, k)$.

$$\text{Area} = \frac{1}{2}| - 3(0 - k) + 3(k - 0) + 0(0 - 0)| = 9$$

$$\frac{1}{2}|3k + 3k| = 9 \implies \frac{1}{2}|6k| = 9$$

$$|3k| = 9 \implies 3|k| = 9 \implies |k| = 3$$

$$k = \pm 3$$

Final Answer: 3 or -3

Answer: (D)

Q26.

Solution

Concept: A function is continuous at $x = 0$ if the Left Hand Limit (LHL), Right Hand Limit (RHL), and the value of the function at that point are all equal: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$.

Solution: 1. **LHL ($x < 0$):**

$$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = \lim_{x \to 0} 2\left(\frac{\sin 2x}{2x} \cdot 2\right)^2 = 2 \cdot (1 \cdot 2)^2 = 8$$

2. **Value at $x = 0$:** $f(0) = a$. 3. **RHL ($x > 0$):** Rationalize the denominator.

$$\lim_{x \to 0} \frac{\sqrt{x}(\sqrt{16 + \sqrt{x}} + 4)}{(16 + \sqrt{x}) - 16} = \lim_{x \to 0} \frac{\sqrt{x}(\sqrt{16 + \sqrt{x}} + 4)}{\sqrt{x}} = \sqrt{16 + 0} + 4 = 8$$

Since LHL = RHL = $f(0)$, we have $a = 8$.

Final Answer: 8

Answer: (B)

Q27.

Solution

Concept: Simplify the trigonometric expression inside the inverse tangent function using the formula $\tan(\pi/4 + x) = \frac{1+\tan x}{1-\tan x}$.

Solution: Divide numerator and denominator by $\cos x$:

$$y = \tan^{-1}\left(\frac{\frac{\sin x}{\cos x} + 1}{1 - \frac{\sin x}{\cos x}}\right) = \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$y = \tan^{-1}(\tan(\pi/4 + x))$$

Assuming x is in the principal range:

$$y = \frac{\pi}{4} + x$$

Differentiating with respect to x:

$$\frac{dy}{dx} = 0 + 1 = 1$$

Final Answer: 1

Answer: (A)

Q28.

Solution

Concept: For equations where the variable is in the exponent, take the natural logarithm on both sides to simplify before differentiating.

Solution: Taking ln on both sides:

$$\ln(x^y) = \ln(e^{x-y}) \implies y \ln x = x - y$$

Rearrange to solve for y:

$$y \ln x + y = x \implies y(1 + \ln x) = x \implies y = \frac{x}{1 + \ln x}$$

Apply the quotient rule $\frac{d}{dx}(\frac{u}{v}) = \frac{vu' - uv'}{v^2}$:

$$\frac{dy}{dx} = \frac{(1 + \ln x)(1) - x(\frac{1}{x})}{(1 + \ln x)^2} = \frac{1 + \ln x - 1}{(1 + \ln x)^2} = \frac{\ln x}{(1 + \ln x)^2}$$

Final Answer: $\frac{\ln x}{(1+\ln x)^2}$

Answer: (A)

Q29.

Solution

Concept: For parametric equations $x = f(t)$, $y = g(t)$, the first derivative is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and the second derivative is $\frac{d^2y}{dx^2} = \frac{d}{dt}(\frac{dy}{dx}) \cdot \frac{dt}{dx}$.

Solution: 1. $\frac{dx}{dt} = -3a\cos^2 t \sin t$ and $\frac{dy}{dt} = 3a\sin^2 t\cos t$. 2. $\frac{dy}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\tan t$. 3. $\frac{d^2y}{dx^2} = \frac{d}{dt}(-\tan t) \cdot \frac{1}{dx/dt} = -\sec^2 t \cdot \frac{1}{-3a\cos^2 t\sin t} = \frac{1}{3a\cos^4 t\sin t}$. 4. At $t = \pi/4$: $\cos t = 1/\sqrt{2}$, $\sin t = 1/\sqrt{2}$.

$$\frac{d^2y}{dx^2} = \frac{1}{3a(1/\sqrt{2})^4(1/\sqrt{2})} = \frac{1}{3a(1/4)(1/\sqrt{2})} = \frac{4\sqrt{2}}{3a}$$

Final Answer: $4\sqrt{2}/(3a)$

Answer: (C)

Q30.

Solution

Concept: The rate of change of volume V of a sphere is $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$. Given $\frac{dV}{dt}$, we find $\frac{dr}{dt}$.

Solution: Volume $V = \frac{4}{3}\pi r^3$. Differentiating with respect to t:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Given $\frac{dV}{dt} = 900$ cm^3/s and $r = 15$ cm:

$$900 = 4\pi(15)^2 \frac{dr}{dt}$$

$$900 = 4\pi(225)\frac{dr}{dt}$$

$$900 = 900\pi\frac{dr}{dt} \implies \frac{dr}{dt} = \frac{1}{\pi} \text{ cm/s}$$

Final Answer: $1/\pi$ cm/s

Answer: (A)

Q31.

Solution

Concept: To find the maximum value of a function $f(x)$, we find its first derivative, set it to zero to locate critical points, and then determine the maximum value by substituting the critical point back into the function.

Solution: Let $y = x^{1/x}$. Taking the natural logarithm on both sides:

$$\ln y = \frac{1}{x} \ln x$$

Differentiating with respect to x using the quotient rule:

$$\frac{1}{y}\frac{dy}{dx} = \frac{x(\frac{1}{x}) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

Set $\frac{dy}{dx} = 0$ to find critical points:

$$1 - \ln x = 0 \implies \ln x = 1 \implies x = e$$

For $x \in (0, e)$, $y' > 0$ (increasing), and for $x > e$, $y' < 0$ (decreasing). Therefore, a local maximum occurs at $x = e$. The maximum value is:

$$y(e) = e^{1/e}$$

Final Answer: $e^{1/e}$

Answer: (B)

Q32.

Solution

Concept: A line is tangent to a parabola if it intersects the curve at exactly one point. This means that when we substitute the equation of the line into the equation of the parabola, the resulting quadratic equation must have equal roots (its discriminant Δ must be zero).

Solution: Given the line $y = mx + 1$ and the curve $y^2 = 4x$. Substitute y into the curve's equation:

$$(mx + 1)^2 = 4x$$

$$m^2x^2 + 2mx + 1 - 4x = 0$$

$$m^2x^2 + 2(m - 2)x + 1 = 0$$

For the line to be a tangent, the discriminant $\Delta = b^2 - 4ac$ must be zero:

$$\Delta = [2(m - 2)]^2 - 4(m^2)(1) = 0$$

$$4(m^2 - 4m + 4) - 4m^2 = 0$$

$$4m^2 - 16m + 16 - 4m^2 = 0$$

$$-16m + 16 = 0 \implies 16m = 16 \implies m = 1$$

Final Answer: 1

Answer: (A)

Q33.

Solution

Concept: This integral can be solved using integration by substitution. If we let $u = f(x)$, then $du = f'(x)dx$.

Solution: Given integral: $I = \int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ Let $u = xe^x$. Differentiating both sides with respect to x using the product rule:

$$\frac{du}{dx} = 1 \cdot e^x + x \cdot e^x = e^x(1+x)$$

$$du = e^x(1+x)dx$$

Substitute u and du into the original integral:

$$I = \int \frac{1}{\cos^2 u}du = \int \sec^2 u\, du$$

The standard integral of $\sec^2 u$ is $\tan u + C$:

$$I = \tan u + C$$

Substituting back $u = xe^x$:

$$I = \tan(xe^x) + C$$

Final Answer: $\tan(xe^x) + C$

Answer: (A)

Q34.

Solution

Concept: For integrals with a quadratic denominator of the form $x^2 + bx + c$, we complete the square to match the standard integral form $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$.

Solution: Given integral: $I = \int \frac{dx}{x^2+2x+2}$ Complete the square for the denominator:

$$x^2 + 2x + 2 = (x^2 + 2x + 1) + 1 = (x+1)^2 + 1^2$$

Rewrite the integral:

$$I = \int \frac{dx}{(x+1)^2 + 1^2}$$

Using the standard integration formula with variable $(x + 1)$ and $a = 1$:

$$I = \frac{1}{1}\tan^{-1}\left(\frac{x+1}{1}\right) + C$$

$$I = \tan^{-1}(x+1) + C$$

Final Answer: $\tan^{-1}(x+1) + C$

Answer: (B)

Q35.

Solution

Concept: When integrating an absolute value function $|f(x)|$, split the definite integral into intervals where $f(x)$ is positive and where $f(x)$ is negative. Also, recall that $\int \ln x \, dx = x \ln x - x$.

Solution: Given integral: $I = \int_{1/e}^{e} |\ln x| dx$. The function $\ln x$ is negative for $x \in [1/e, 1)$ and positive for $x \in (1, e]$. We split the integral at $x = 1$:

$$I = \int_{1/e}^{1} (-\ln x) dx + \int_{1}^{e} (\ln x) dx$$

Using the antiderivative $x \ln x - x$: For the first part:

$$-[x \ln x - x]_{1/e}^{1} = -\left[(1 \ln 1 - 1) - \left(\frac{1}{e} \ln \frac{1}{e} - \frac{1}{e}\right)\right]$$

$$= -\left[(0 - 1) - \left(\frac{1}{e}(-1) - \frac{1}{e}\right)\right] = -\left[-1 - \left(-\frac{2}{e}\right)\right] = 1 - \frac{2}{e}$$

For the second part:

$$[x \ln x - x]_{1}^{e} = (e \ln e - e) - (1 \ln 1 - 1)$$

$$= (e(1) - e) - (0 - 1) = 0 - (-1) = 1$$

Add both parts together:

$$I = \left(1 - \frac{2}{e}\right) + 1 = 2 - \frac{2}{e} = 2\left(1 - \frac{1}{e}\right)$$

Final Answer: $2(1 - 1/e)$

Answer: (B)

Q36.

Solution

Concept: We use the definite integral property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ to eliminate the x multiplier in the numerator.

Solution: Let $I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$ — (1) Using the property $x \to \pi - x$:

$$I = \int_0^\pi \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\,dx$$

Since $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$:

$$I = \int_0^\pi \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx \text{ — (2)}$$

Add (1) and (2):

$$2I = \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x}\,dx = \pi \int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx$$

Let $u = \cos x$, then $du = -\sin x\,dx$. The limits change from $x = 0 \to u = 1$ to $x = \pi \to u = -1$:

$$2I = \pi \int_1^{-1} \frac{-du}{1+u^2} = \pi \int_{-1}^{1} \frac{du}{1+u^2}$$

$$2I = \pi[\tan^{-1} u]_{-1}^{1} = \pi\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \pi\left(\frac{\pi}{2}\right) = \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$

Final Answer: $\pi^2/4$

Answer: (A)

Q37.

Solution

Concept: Use the fundamental trigonometric identity $1 = \sin^2 x + \cos^2 x$ in the numerator to split the integral into standard forms.

Solution: Given integral: $I = \int \frac{dx}{\sin^2 x \cos^2 x}$ Rewrite the numerator ($1 = \sin^2 x + \cos^2 x$):

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

Split the fraction:

$$I = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$I = \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

$$I = \int (\sec^2 x + \csc^2 x)\, dx$$

Integrating term by term:

$$I = \tan x - \cot x + C$$

Final Answer: $\tan x - \cot x + C$

Answer: (B)

Q38.

Solution

Concept: To find the area bounded by two curves, determine their points of intersection to establish the limits of integration, then integrate the difference between the upper curve and the lower curve.

Solution: The curves are $y = \frac{x^2}{4}$ (upward parabola) and $y = \frac{x+2}{4}$ (straight line). Find intersection points:

$$\frac{x^2}{4} = \frac{x+2}{4} \implies x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0 \implies x = -1, x = 2$$

In the interval $[-1, 2]$, the line is above the parabola.

$$\text{Area} = \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{1}{4} \int_{-1}^{2} (x + 2 - x^2)\, dx$$

$$\text{Area} = \frac{1}{4} \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \frac{1}{4} \left[\left(\frac{4}{2} + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) \right]$$

$$= \frac{1}{4} \left[\left(6 - \frac{8}{3} \right) - \left(-\frac{3}{2} + \frac{1}{3} \right) \right] = \frac{1}{4} \left[\frac{10}{3} - \left(-\frac{7}{6} \right) \right]$$

$$= \frac{1}{4} \left(\frac{20+7}{6} \right) = \frac{27}{24} = \frac{9}{8} \text{ sq. units}$$

Final Answer: $9/8$ sq. units

Answer: (C)

Q39.

Solution

Concept: Identify the upper and lower curves in the given region (first quadrant, bounded by the y-axis). The intersection point sets the upper limit.
Solution: Find the intersection in the first quadrant:

$$\sin x = \cos x \implies \tan x = 1 \implies x = \frac{\pi}{4}$$

In the interval $0 \le x \le \frac{\pi}{4}$, the curve $y = \cos x$ is above $y = \sin x$. The y-axis is $x = 0$.

$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\, dx$$

$$\text{Area} = [\sin x + \cos x]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1) = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

Final Answer: $\sqrt{2} - 1$

Answer: (A)

Q40.

Solution

Concept: This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$. The solution is found by computing the Integrating Factor (IF) $= e^{\int P\,dx}$.
Solution: Given equation: $\frac{dy}{dx} + y = e^{-x}$. Here, $P = 1$ and $Q = e^{-x}$. Calculate the Integrating Factor:

$$\text{IF} = e^{\int 1\,dx} = e^x$$

The general solution is given by $y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$:

$$y \cdot e^x = \int (e^{-x} \cdot e^x)\, dx + C$$

$$ye^x = \int 1\, dx + C$$

$$ye^x = x + C$$

Divide by e^x:

$$y = (x + C)e^{-x}$$

Final Answer: $y = (x + C)e^{-x}$

Answer: (B)

Q41.

Solution

Concept: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$. To solve it, we find the Integrating Factor (IF) $= e^{\int P\,dx}$.

Solution: Standardize the equation by dividing by x:

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

Here, $P = \frac{2}{x}$ and $Q = x$. Calculate the Integrating Factor:

$$\text{IF} = e^{\int \frac{2}{x}\,dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

The general solution is $y \cdot \text{IF} = \int (Q \cdot \text{IF})\,dx + C$:

$$y \cdot x^2 = \int (x \cdot x^2)\,dx + C$$

$$yx^2 = \int x^3\,dx + C = \frac{x^4}{4} + C$$

Divide by x^2:

$$y = \frac{x^2}{4} + \frac{C}{x^2}$$

Final Answer: $y = \frac{x^2}{4} + \frac{C}{x^2}$

Answer: (A)

Q42.

Solution

Concept: For a linear differential equation in x of the form $\frac{dx}{dy} + Px = Q$, the integrating factor is IF $= e^{\int P\,dy}$.

Solution: Standardize the equation by dividing by $(1 - y^2)$:

$$\frac{dx}{dy} + \frac{y}{1-y^2}x = \frac{ay}{1-y^2}$$

Here, $P = \frac{y}{1-y^2}$. Calculate the Integrating Factor:

$$\text{IF} = e^{\int \frac{y}{1-y^2}\,dy}$$

To integrate $\frac{y}{1-y^2}$, let $u = 1 - y^2$, then $du = -2y\,dy \implies y\,dy = -\frac{1}{2}du$.

$$\int \frac{y}{1-y^2}\,dy = -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln(1-y^2) = \ln((1-y^2)^{-1/2})$$

$$\text{IF} = e^{\ln(1-y^2)^{-1/2}} = \frac{1}{\sqrt{1-y^2}}$$

Final Answer: $\frac{1}{\sqrt{1-y^2}}$

Answer: (B)

Q43.

Solution

Concept: The magnitude of the cross product of two vectors is given by $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, where θ is the angle between them.

Solution: Given $|\vec{a}| = 2$ and $|\vec{b}| = 7$. The cross product $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$. Find its magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

Use the sine formula:

$$7 = (2)(7)\sin\theta \implies \sin\theta = \frac{7}{14} = \frac{1}{2}$$

$$\theta = \sin^{-1}(1/2) = \pi/6$$

Final Answer: $\pi/6$

Answer: (A)

Q44.

Solution

Concept: The scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is given by the formula $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution: Given $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. 1. Calculate the dot product $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (1)(1) + (-1)(1) + (1)(1) = 1 - 1 + 1 = 1$$

2. Calculate the magnitude $|\vec{b}|$:

$$|\vec{b}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

3. Scalar projection:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{1}{\sqrt{3}}$$

Final Answer: $1/\sqrt{3}$

Answer: (A)

Q45.

Solution

Concept: Three vectors are coplanar if their scalar triple product is zero, which is equivalent to the determinant of the matrix formed by their components being zero.

Solution: Set the determinant to zero:

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$$

Expand along the first row:

$$1(1(\lambda) - (-1)(-1)) - (-1)(2(\lambda) - (-1)(\lambda)) + 1(2(-1) - (1)(\lambda)) = 0$$

$$1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda) = 0$$

$$\lambda - 1 + 3\lambda - 2 - \lambda = 0$$

$$3\lambda - 3 = 0 \implies 3\lambda = 3 \implies \lambda = 1$$

Final Answer: 1

Answer: (A)

Q46.

Solution

Concept: The perpendicular distance d from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Solution: Point: $(2, 3, -5)$. Plane: $x + 2y - 2z - 9 = 0$. Substitute values into the distance formula:

$$d = \frac{|(1)(2) + (2)(3) + (-2)(-5) - 9|}{\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$d = \frac{|2 + 6 + 10 - 9|}{\sqrt{1 + 4 + 4}} = \frac{|9|}{\sqrt{9}} = \frac{9}{3} = 3$$

Final Answer: 3

Answer: (A)

Q47.

Solution

Concept: The angle θ between two lines with direction vectors $\vec{b_1}$ and $\vec{b_2}$ is found using $\cos\theta = \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}$.

Solution: 1. **Line 1:** $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$. The $z = 2$ part means the z-component of direction is 0. Direction vector $\vec{b_1} = 3\hat{i} - 2\hat{j} + 0\hat{k}$. 2. **Line 2:** $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2} \implies \frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$. Direction vector $\vec{b_2} = 1\hat{i} + \frac{3}{2}\hat{j} + 2\hat{k}$ (or $2\hat{i} + 3\hat{j} + 4\hat{k}$ by scaling). Calculate dot product: $\vec{b_1} \cdot \vec{b_2} = (3)(1) + (-2)(3/2) + (0)(2) = 3 - 3 + 0 = 0$. Since the dot product is zero, the lines are perpendicular.

Final Answer: $\pi/2$

Answer: (A)

Q48.

Solution

Concept: The shortest distance SD between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is:

$$SD = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Solution: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k} \implies \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$. $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$.
Magnitude $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$. Dot product: $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -3 - 6 = -9$. $SD = |-9/3\sqrt{2}| = 3/\sqrt{2}$.
Final Answer: $3/\sqrt{2}$

Answer: (B)

Q49.

Solution

Concept: If a Linear Programming objective function attains its maximum value at two distinct corner points of a convex feasible region, then it must also attain the same maximum value at every point on the line segment connecting those two points.
Solution: The set of points on a line segment between two distinct points (x_1, y_1) and (x_2, y_2) is infinite. Therefore, the objective function Z will have the same maximum value at infinitely many points.
Final Answer: Infinite

Answer: (D)

Q50.

Solution

Concept: Two events A and B are independent if $P(A \cap B) = P(A)P(B)$. Similarly, A and B' are also independent, so $P(A \cap B') = P(A)P(B')$.
Solution: Given $P(A) = 0.3$ and $P(B) = 0.4$. First, find $P(B')$:

$$P(B') = 1 - P(B) = 1 - 0.4 = 0.6$$

Since A and B are independent, A and B' are also independent:

$$P(A \cap B') = P(A)\cdot P(B') = 0.3 \cdot 0.6 = 0.18$$

Final Answer: 0.18

Answer: (B)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	D	4	B	5	C
6	C	7	A	8	B	9	A	10	B
11	A	12	A	13	B	14	B	15	B
16	B	17	C	18	A	19	B	20	B
21	B	22	A	23	D	24	A	25	D
26	B	27	A	28	A	29	C	30	A
31	B	32	A	33	A	34	B	35	B
36	A	37	B	38	C	39	A	40	B
41	A	42	B	43	A	44	A	45	A
46	A	47	A	48	B	49	D	50	B