

CUET-UG Mathematics Test Sample Paper-19

Duration: 1 Hour

Maximum Marks: 250

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then BB^T equals:

- (A) I
- (B) A
- (C) B^{-1}
- (D) $(A^T)^{-1}$

Q2. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Then A^n is equal to:

- (A) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$
- (B) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$
- (C) $\begin{bmatrix} n & 2n \\ 0 & n \end{bmatrix}$
- (D) $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

Q3. If the value of a third-order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor is:

- (A) 12



- (B) 144
- (C) 1728
- (D) 0

Q4. If w is a complex cube root of unity and $\Delta = \begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$, then Δ is:

- (A) 0
- (B) 1
- (C) w
- (D) w^2

Q5. For a matrix A , if $A^2 - A + I = 0$, then the inverse of A is:

- (A) $A - I$
- (B) $I - A$
- (C) $A + I$
- (D) A

Q6. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

- (A) 27
- (B) 18
- (C) 81
- (D) 512

Q7. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc \neq 0$, then A^{-1} is:

- (A) $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- (B) $\frac{1}{ad-bc} \begin{bmatrix} d & b \\ c & a \end{bmatrix}$



$$(C) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(D) \frac{1}{ad-bc} \begin{bmatrix} a & -c \\ -b & d \end{bmatrix}$$

Q8. Let $f : R \rightarrow R$ be defined by $f(x) = \cos x$. Then f is:

- (A) One-to-one and onto
- (B) Neither one-to-one nor onto
- (C) One-to-one but not onto
- (D) Onto but not one-to-one

Q9. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is:

- (A) Reflexive
- (B) Symmetric
- (C) Transitive
- (D) Equivalence

Q10. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, then $f \circ g(x)$ is:

- (A) $8x$
- (B) $2x$
- (C) $8x^3$
- (D) x

Q11. The number of binary operations on the set $\{a, b\}$ is:

- (A) 10
- (B) 16
- (C) 20
- (D) 8

Q12. The value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ is:



- (A) π
- (B) $-\frac{\pi}{2}$
- (C) 0
- (D) $2\sqrt{3}$

Q13. $\sin(\tan^{-1} x)$, $|x| < 1$ is equal to:

- (A) $\frac{x}{\sqrt{1-x^2}}$
- (B) $\frac{1}{\sqrt{1-x^2}}$
- (C) $\frac{1}{\sqrt{1+x^2}}$
- (D) $\frac{x}{\sqrt{1+x^2}}$

Q14. If $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to:

- (A) $0, \frac{1}{2}$
- (B) $1, \frac{1}{2}$
- (C) 0
- (D) $\frac{1}{2}$

Q15. If $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k is:

- (A) 3
- (B) 6
- (C) 9
- (D) 12

Q16. If $y = \log(\tan e^x)$, then $\frac{dy}{dx}$ is:

- (A) $e^x \sec^2(e^x)$
- (B) $\frac{e^x \sec^2(e^x)}{\tan e^x}$
- (C) $e^x \tan e^x$
- (D) $\frac{1}{\tan e^x}$



Q17. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1} x$ is:

- (A) 2
- (B) 1
- (C) $\frac{1}{2}$
- (D) 0

Q18. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is:

- (A) $\frac{\log x}{(1+\log x)^2}$
- (B) $\frac{1}{(1+\log x)^2}$
- (C) $\frac{\log x}{1+\log x}$
- (D) $\frac{x^y}{e^{x-y}}$

Q19. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$ is:

- (A) 1
- (B) -1
- (C) 2
- (D) 0

Q20. The rate of change of the area of a circle with respect to its radius r when $r = 6$ cm is:

- (A) 10π
- (B) 12π
- (C) 8π
- (D) 11π

Q21. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

- (A) (1, 2)
- (B) (2, 1)



(C) $(1, -2)$

(D) $(-1, 2)$

Q22. The local minimum value of the function f given by $f(x) = 3 + |x|, x \in R$ is:

(A) 0

(B) 3

(C) -3

(D) Does not exist

Q23. The function $f(x) = \tan x - x$ is:

(A) Always increasing

(B) Always decreasing

(C) Increasing in $(0, \pi/2)$

(D) Decreasing in $(0, \pi/2)$

Q24. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals:

(A) $10^x - x^{10} + C$

(B) $10^x + x^{10} + C$

(C) $(x^{10} + 10^x)^{-1} + C$

(D) $\log(10^x + x^{10}) + C$

Q25. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals:

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

Q26. The value of $\int_0^{2\pi} \cos^5 x dx$ is:

(A) 0



- (B) 1
- (C) $5/4$
- (D) 2π

Q27. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ equals:

- (A) $\tan(xe^x) + C$
- (B) $\cot(xe^x) + C$
- (C) $\tan(e^x) + C$
- (D) $\tan x + C$

Q28. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is:

- (A) $\frac{\pi}{3}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{12}$
- (D) $\frac{2\pi}{3}$

Q29. $\int \frac{dx}{\sqrt{9-25x^2}}$ is:

- (A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + C$
- (B) $\frac{1}{3} \sin^{-1}\left(\frac{5x}{3}\right) + C$
- (C) $\sin^{-1}\left(\frac{5x}{3}\right) + C$
- (D) $\frac{1}{5} \cos^{-1}\left(\frac{5x}{3}\right) + C$

Q30. The area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is:

- (A) 2 sq units
- (B) 4 sq units
- (C) 0 sq units
- (D) 1 sq unit

Q31. Area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant is:



- (A) $16 - 4\sqrt{2}$
- (B) $14 - 4\sqrt{2}$
- (C) $16\sqrt{2} - 8$
- (D) $4\sqrt{2}$

Q32. The order and degree (if defined) of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\left(\frac{d^2y}{dx^2}\right)$ are:

- (A) Order 2, Degree 2
- (B) Order 2, Degree Not Defined
- (C) Order 1, Degree 2
- (D) Order 2, Degree 1

Q33. The solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

- (A) $yx = \frac{x^4}{4} + C$
- (B) $y = \frac{x^3}{4} + C$
- (C) $yx = \frac{x^3}{3} + C$
- (D) $y = \frac{x^4}{4} + C$

Q34. The general solution of $e^x dy + (ye^x + 2x)dx = 0$ is:

- (A) $ye^x + x^2 = C$
- (B) $ye^x + x = C$
- (C) $y + x^2 = C$
- (D) $y = x^2e^{-x} + C$

Q35. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then $|\vec{a} \times \vec{b}|$ is:

- (A) 12
- (B) 16
- (C) 20
- (D) 5



- Q36.** The direction cosines of a line which makes equal angles with the coordinate axes are:
- (A) $(1, 1, 1)$
 - (B) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
 - (C) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
 - (D) $(0, 0, 1)$
- Q37.** The angle between two vectors \vec{a} and \vec{b} with magnitudes $\sqrt{3}$ and 2 respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$ is:
- (A) $\pi/6$
 - (B) $\pi/3$
 - (C) $\pi/4$
 - (D) $\pi/2$
- Q38.** The vector equation of the line passing through $(1, 2, 3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$ is:
- (A) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$
 - (B) $\vec{r} = (3\hat{i} + 2\hat{j} - 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$
 - (C) $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$
 - (D) $\vec{r} = \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$
- Q39.** If a line has direction ratios $(2, -1, -2)$, its direction cosines are:
- (A) $(2/3, -1/3, -2/3)$
 - (B) $(2/9, -1/9, -2/9)$
 - (C) $(1/3, -1/3, -1/3)$
 - (D) $(2, -1, -2)$
- Q40.** The distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin is:
- (A) $6/\sqrt{29}$



- (B) $6/29$
- (C) $\sqrt{29}/6$
- (D) 6

Q41. The optimal value of the objective function is attained at the points:

- (A) Given by intersection of inequations with axes only
- (B) Given by corner points of the feasible region
- (C) Given by any point inside the feasible region
- (D) Given by origin only

Q42. Which of the following is a convex set?

- (A) $\{(x, y) : x^2 + y^2 \geq 1\}$
- (B) $\{(x, y) : y^2 \leq x\}$
- (C) $\{(x, y) : 3x^2 + 4y^2 \geq 12\}$
- (D) None of these

Q43. If A and B are events such that $P(A|B) = P(B|A)$, then:

- (A) $A \subset B$ but $A \neq B$
- (B) $A = B$
- (C) $P(A) = P(B)$
- (D) $A \cap B = \phi$

Q44. A black and a red die are rolled. The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5 is:

- (A) $1/3$
- (B) $1/6$
- (C) $1/2$
- (D) $1/4$



Q45. If $P(A) = 1/2$, $P(B) = 0$, then $P(A|B)$ is:

- (A) 0
- (B) $1/2$
- (C) Not defined
- (D) 1

Q46. Two events A and B are independent if:

- (A) A and B are mutually exclusive
- (B) $P(A'B') = [1 - P(A)][1 - P(B)]$
- (C) $P(A) = P(B)$
- (D) $P(A) + P(B) = 1$

Q47. Probability of solving a specific problem independently by A and B are $1/2$ and $1/3$ respectively. If both try, the probability that the problem is solved is:

- (A) $2/3$
- (B) $1/6$
- (C) $5/6$
- (D) $1/3$

Q48. The value of $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$ is:

- (A) 0
- (B) 1
- (C) 2
- (D) $1/2$

Q49. If \vec{a} , \vec{b} , \vec{c} are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

- (A) 1
- (B) 3



(C) $-3/2$

(D) 0

Q50. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is:

(A) $(2\sqrt{2}, 4)$

(B) $(2, 2)$

(C) $(0, 0)$

(D) $(2, 4)$



Detailed Solutions

Q1.

Solution

Concept: For any non-singular matrices A and B , the following properties apply: 1. $(AB)^T = B^T A^T$ (Reversal law of transpose) 2. $(A^{-1})^T = (A^T)^{-1}$ 3. If A is non-singular, $AA^{-1} = A^{-1}A = I$ 4. Given $AA^T = A^T A$ (A is a normal matrix).

Solution: 1. **Given Data:** A is a 3×3 non-singular matrix. $AA^T = A^T A$ $B = A^{-1}A^T$
 2. **Expressing B^T :** $B^T = (A^{-1}A^T)^T$ * Using $(XY)^T = Y^T X^T$, we get: $B^T = (A^T)^T (A^{-1})^T$
 * Since $(A^T)^T = A$, then $B^T = A(A^T)^{-1}$
 3. **Calculating BB^T :** $BB^T = (A^{-1}A^T)(A(A^T)^{-1})$ * By associativity: $BB^T = A^{-1}(A^T A)(A^T)^{-1}$
 4. **Substituting the Commuting Property:** * Since $A^T A = AA^T$, substitute this into the equation: $BB^T = A^{-1}(AA^T)(A^T)^{-1} = A^{-1}A(A^T(A^T)^{-1}) = BB^T = I \cdot I = I$

Final Answer: I

Answer: (A)

Q2.

Solution

Concept: For a square matrix A , A^n represents the product of the matrix A with itself n times. For matrices of the form $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, the powers follow a specific linear pattern in the upper-right element.

Solution: 1. **Given Data:** $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
 2. **Calculating A^2 :** $A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (1 \cdot 1 + 2 \cdot 0) & (1 \cdot 2 + 2 \cdot 1) \\ (0 \cdot 1 + 1 \cdot 0) & (0 \cdot 2 + 1 \cdot 1) \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ * Note that $4 = 2(2)$.
 3. **Calculating A^3 :** $A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (1 \cdot 1 + 4 \cdot 0) & (1 \cdot 2 + 4 \cdot 1) \\ (0 \cdot 1 + 1 \cdot 0) & (0 \cdot 2 + 1 \cdot 1) \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$ * Note that $6 = 2(3)$.
 4. **Generalizing for A^n :** * Observing the pattern, the element a_{12} becomes $2n$, while other elements remain constant. * Therefore, $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$.

Final Answer: $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

Answer: (A)

Q3.

Solution

Concept: For any square matrix A of order n , let C be the matrix of its cofactors. The determinant of the cofactor matrix, $|C|$, is related to the determinant of the original matrix, $|A|$, by the property:

$$|C| = |A|^{n-1}$$

This follows from the fact that $\text{adj}(A) = C^T$, and $|\text{adj}(A)| = |A|^{n-1}$.

Solution: 1. **Given Data:** * Order of the determinant (n) = 3 * Value of the determinant ($|A|$) = 12

2. **Applying the Property:** * We need to find the determinant of the cofactor matrix, $|C|$. * Using the formula $|C| = |A|^{n-1}$: * $|C| = 12^{3-1}$

3. **Calculation:** * $|C| = 12^2$ * $|C| = 144$

Final Answer: 144

Answer: (B)

Q4.

Solution

Concept: For a complex cube root of unity w , the two most important properties are: 1. $1 + w + w^2 = 0$ 2. $w^3 = 1$

Solution: 1. **Given Data:**

$$\Delta = \begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$$

2. **Applying Row/Column Operations:** * Perform the operation $C_1 \rightarrow C_1 + C_2 + C_3$ (adding all columns to the first column):

$$\Delta = \begin{vmatrix} 1 + w + w^2 & w & w^2 \\ w + w^2 + 1 & w^2 & 1 \\ w^2 + 1 + w & 1 & w \end{vmatrix}$$

3. **Using Properties of w :** * Since $1 + w + w^2 = 0$, the determinant becomes:

$$\Delta = \begin{vmatrix} 0 & w & w^2 \\ 0 & w^2 & 1 \\ 0 & 1 & w \end{vmatrix}$$

4. **Conclusion:** * If any row or column of a determinant consists entirely of zeros, the value of the determinant is 0. * Therefore, $\Delta = 0$.

Final Answer: 0

Answer: (A)



Q5.

Solution

Concept: For a non-singular matrix A , the inverse A^{-1} satisfies the property $A \cdot A^{-1} = A^{-1} \cdot A = I$. When given a polynomial equation in A , we can isolate the identity matrix I and multiply by A^{-1} to find the expression for the inverse.

Solution: 1. **Given Equation:**

$$A^2 - A + I = 0$$

2. **Isolating the Identity Matrix (I):** * Rearrange the equation to keep I on one side:

$$I = A - A^2$$

3. **Multiplying by A^{-1} :** * Pre-multiply both sides by A^{-1} :

$$A^{-1} \cdot I = A^{-1}(A - A^2)$$

* Distribute A^{-1} across the terms:

$$A^{-1} = A^{-1}A - A^{-1}A^2$$

4. **Simplifying the Expression:** * Since $A^{-1}A = I$ and $A^{-1}A^2 = A^{-1}AA = IA = A$:

$$A^{-1} = I - A$$

Final Answer: $I - A$

Answer: (B)



Q6.

Solution

Concept: For a matrix of order $m \times n$, the total number of elements is $m \times n$. If each element can be filled in k different ways, the total number of possible matrices is given by:

$$\text{Total Matrices} = k^{\text{number of elements}}$$

Solution: 1. ****Determine Number of Elements:**** * The matrix order is 3×3 . * Total number of elements = $3 \times 3 = 9$.

2. ****Identify Choices per Entry:**** * Each entry can be either 0 or 1. * Number of choices for each entry (k) = 2.

3. ****Applying the Counting Principle:**** * Since there are 9 positions and each position has 2 independent choices, we multiply the choices for each position: * Total possible matrices = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ * Total possible matrices = 2^9

4. ****Calculation:**** * $2^9 = 512$

Final Answer: 512

Answer: (D)



Q7.

Solution

Concept: For a square matrix A , the inverse A^{-1} exists if and only if the determinant $|A| \neq 0$. The formula for the inverse is:

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $|A| = ad - bc$ and the adjugate is obtained by swapping the diagonal elements and changing the signs of the off-diagonal elements:

$$\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solution: 1. ****Given Matrix:****

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2. ****Calculate the Determinant:**** $|A| = (a)(d) - (b)(c) = ad - bc$ * It is given that $ad - bc \neq 0$, so the inverse exists.

3. ****Find the Adjugate Matrix:**** * Swap a and d . * Multiply b and c by -1 . * $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4. ****Applying the Formula:**** * $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Final Answer: $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Answer: (A)

Q8.

Solution

Concept: A function $f : X \rightarrow Y$ is: 1. ****One-to-one (Injective)**** if $f(x_1) = f(x_2) \implies x_1 = x_2$.
2. ****Onto (Surjective)**** if the range of f is equal to the codomain Y .

Solution: 1. ****Checking One-to-one:**** * $f(x) = \cos x$. We know $\cos(0) = 1$ and $\cos(2\pi) = 1$.
* Since different inputs produce the same output, f is ****not one-to-one****.

2. ****Checking Onto:**** * The codomain is given as \mathbb{R} . * The range of $\cos x$ is $[-1, 1]$. * Since the range $[-1, 1] \neq \mathbb{R}$, f is ****not onto****.

Final Answer: Neither one-to-one nor onto

Answer: (B)



Q9.

Solution

Concept: For a relation R on set A : 1. **Reflexive:** $(a, a) \in R$ for all $a \in A$. 2. **Symmetric:** $(a, b) \in R \implies (b, a) \in R$. 3. **Transitive:** $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$.

Solution: 1. **Given:** $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 1)\}$. 2. **Reflexive:** $(1, 1), (2, 2), (3, 3)$ are not in R . Not reflexive. 3. **Symmetric:** We have $(1, 2) \in R$ and its reverse $(2, 1) \in R$. This holds for all elements. Thus, it is **symmetric**. 4. **Transitive:** We have $(1, 2) \in R$ and $(2, 1) \in R$, but $(1, 1) \notin R$. Not transitive.

Final Answer: Symmetric

Answer: (B)

Q10.

Solution

Concept: The composition of functions $(f \circ g)(x)$ is defined as $f(g(x))$. You substitute the entire expression of $g(x)$ into the variable x in function f .

Solution: 1. **Given:** $f(x) = 8x^3$ and $g(x) = x^{1/3}$. 2. **Applying Composition:** $(f \circ g)(x) = f(g(x))$ * Substitute $g(x)$ into f : $f(x^{1/3}) = 8(x^{1/3})^3$ 3. **Simplifying:** * Using the power rule $(a^m)^n = a^{mn}$: $(x^{1/3})^3 = x^{(1/3 \cdot 3)} = x^1$ * **Result:** $8x$

Final Answer: $8x$

Answer: (A)

Q11.

Solution

Concept: A binary operation on a set A is a function $*$: $A \times A \rightarrow A$. The number of binary operations is given by the formula $n^{(n^2)}$, where n is the number of elements in set A .

Solution: 1. **Given:** Set $\{a, b\}$, so the number of elements $n = 2$. 2. **Applying the Formula:** * Number of binary operations = $n^{(n^2)} = 2^{(2^2)} = 2^4$ 3. **Calculation:** * $2^4 = 16$

Final Answer: 16

Answer: (B)



Q12.

Solution

Concept: To find the value of inverse trigonometric expressions, we use the principal value branches: 1. $\tan^{-1} x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ 2. $\cot^{-1}(-x) = \pi - \cot^{-1} x$, where $\cot^{-1} x \in (0, \pi)$

Solution: 1. **Evaluate $\tan^{-1}(\sqrt{3})$:** Since $\tan(\frac{\pi}{3}) = \sqrt{3}$, then $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

2. **Evaluate $\cot^{-1}(-\sqrt{3})$:** Using the property $\cot^{-1}(-x) = \pi - \cot^{-1} x$: $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$ Since $\cot(\frac{\pi}{6}) = \sqrt{3}$, then $\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$. So, $\cot^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

3. **Subtract the values:** $\frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$.

Final Answer: $-\frac{\pi}{2}$

Answer: (B)

Q13.

Solution

Concept: To simplify $\sin(\tan^{-1} x)$, we represent the inverse tangent as an angle in a right-angled triangle. Let $\theta = \tan^{-1} x$, then $\tan \theta = \frac{x}{1}$.

Solution: 1. **Set up the Triangle:** Opposite side = x * Adjacent side = 1 * Hypotenuse = $\sqrt{(\text{opp})^2 + (\text{adj})^2} = \sqrt{x^2 + 1}$

2. **Find $\sin \theta$:** $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ * $\sin \theta = \frac{x}{\sqrt{x^2+1}}$

3. **Conclusion:** $\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$

Final Answer: $\frac{x}{\sqrt{1+x^2}}$

Answer: (D)

Q14.

Solution

Concept: For equations involving inverse trigonometric functions, substitution of options is often the most efficient verification method, or applying the property $\sin^{-1}(1-x) = \frac{\pi}{2} - \sin^{-1} x$ and taking the sine of both sides.

Solution: 1. **Check $x = 0$:** LHS: $\sin^{-1}(1-0) - 2 \sin^{-1}(0) = \sin^{-1}(1) - 0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$.

So, $x = 0$ is a solution.

2. **Check $x = \frac{1}{2}$:** LHS: $\sin^{-1}(1 - \frac{1}{2}) - 2 \sin^{-1}(\frac{1}{2}) = \sin^{-1}(\frac{1}{2}) - 2 \sin^{-1}(\frac{1}{2}) = -\sin^{-1}(\frac{1}{2}) = -\frac{\pi}{6} \neq \frac{\pi}{2}$. So, $x = \frac{1}{2}$ is not a solution.

3. **Conclusion:** Since $x = 1/2$ fails, options A, B, and D are incorrect. Only $x = 0$ satisfies the equation.

Final Answer: 0

Answer: (C)



Q15.

Solution

Concept: For a function to be continuous at $x = a$, the limit as $x \rightarrow a$ must exist and equal the functional value $f(a)$:

$$\lim_{x \rightarrow a} f(x) = f(a)$$

L'Hôpital's Rule can be used to evaluate limits of the form $0/0$.

Solution: 1. ****Given Data:**** $f(\pi/2) = 3$. 2. ****Setting up the Limit:****

$$\lim_{x \rightarrow \pi/2} \frac{k \cos x}{\pi - 2x} = 3$$

3. ****Applying L'Hôpital's Rule:**** * Differentiate numerator: $\frac{d}{dx}(k \cos x) = -k \sin x$ * Differentiate denominator: $\frac{d}{dx}(\pi - 2x) = -2$

$$\lim_{x \rightarrow \pi/2} \frac{-k \sin x}{-2} = \frac{k \sin(\pi/2)}{2} = \frac{k(1)}{2} = \frac{k}{2}$$

4. ****Solving for k :**

$$\frac{k}{2} = 3 \implies k = 6$$

Final Answer: 6

Answer: (B)

Q16.

Solution

Concept: To differentiate a composite function $y = \log(u)$ where $u = \tan(v)$ and $v = e^x$, we use the Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

Solution: 1. ****Differentiating Step-by-Step:**** * Outer function (log): $\frac{1}{\tan e^x}$ * Middle function (tan): $\sec^2(e^x)$ * Inner function (e^x): e^x 2. ****Combining terms:****

$$\frac{dy}{dx} = \frac{1}{\tan e^x} \cdot \sec^2(e^x) \cdot e^x$$

3. ****Result:****

$$\frac{dy}{dx} = \frac{e^x \sec^2(e^x)}{\tan e^x}$$

Final Answer: $\frac{e^x \sec^2(e^x)}{\tan e^x}$

Answer: (B)



Q17.

Solution

Concept: To find the derivative of u with respect to v , use $\frac{du}{dv} = \frac{du/dx}{dv/dx}$. We use the substitution $x = \sin \theta$ to simplify the inverse trigonometric expression.

Solution: 1. ****Simplify $u = \sin^{-1}(2x\sqrt{1-x^2})$:** ****** Let $x = \sin \theta$, then $\sqrt{1-x^2} = \cos \theta$. ***** $u = \sin^{-1}(2 \sin \theta \cos \theta) = \sin^{-1}(\sin 2\theta) = 2\theta$. ***** Since $\theta = \sin^{-1} x$, then $u = 2 \sin^{-1} x$. 2. ****Identify v :** ****** $v = \sin^{-1} x$. 3. ****Differentiate:** ****** $\frac{du}{dv} = \frac{d}{dv}(2v) = 2$.

Final Answer: 2

Answer: (A)

Q18.

Solution

Concept: When variables appear in exponents, take the natural logarithm on both sides to simplify the equation before differentiating implicitly.

Solution: 1. ****Taking Logarithms:** ******

$$\log(x^y) = \log(e^{x-y}) \implies y \log x = x - y$$

2. ****Isolating y :** ******

$$y \log x + y = x \implies y(1 + \log x) = x \implies y = \frac{x}{1 + \log x}$$

3. ****Applying Quotient Rule:** ******

$$\frac{dy}{dx} = \frac{(1 + \log x)(1) - x(\frac{1}{x})}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$$

Final Answer: $\frac{\log x}{(1 + \log x)^2}$

Answer: (A)



Q19.

Solution

Concept: For implicit differentiation of $\sqrt{x} + \sqrt{y} = k$, differentiate each term with respect to x using $\frac{d}{dx}(\sqrt{y}) = \frac{1}{2\sqrt{y}} \frac{dy}{dx}$.

Solution: 1. ****Differentiate w.r.t. x :****

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

2. ****Solving for $\frac{dy}{dx}$:****

$$\frac{1}{2\sqrt{y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \implies \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

3. ****Evaluate at $(1/4, 1/4)$:****

$$\frac{dy}{dx} = -\frac{\sqrt{1/4}}{\sqrt{1/4}} = -1$$

Final Answer: -1

Answer: (B)

Q20.

Solution

Concept: The area of a circle A is given by $A = \pi r^2$. The "rate of change of area with respect to radius" is the derivative $\frac{dA}{dr}$.

Solution: 1. ****Formula:**** $A = \pi r^2$ 2. ****Differentiating w.r.t. r :****

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = 2\pi r$$

3. ****Evaluating at $r = 6$:****

$$\frac{dA}{dr} = 2\pi(6) = 12\pi$$

Final Answer: 12π

Answer: (B)



Q21.

Solution

Concept: A line is tangent to a curve if they share a common point and the slope of the line equals the derivative of the curve at that point. Alternatively, we can substitute the line equation into the curve and check for a single point of intersection.

Solution: 1. **Given:** Line $y = x + 1$ (Slope $m = 1$) and Curve $y^2 = 4x$. 2. **Differentiating the curve:**

$$2y \frac{dy}{dx} = 4 \implies \frac{dy}{dx} = \frac{2}{y}$$

3. **Equating Slopes:** Set $\frac{dy}{dx} = 1$:

$$\frac{2}{y} = 1 \implies y = 2$$

4. **Finding x :** Substitute $y = 2$ into the line equation:

$$2 = x + 1 \implies x = 1$$

5. **Verification:** At $(1, 2)$, $y^2 = 2^2 = 4$ and $4x = 4(1) = 4$. The point satisfies both.

Final Answer: $(1, 2)$

Answer: (A)

Q22.

Solution

Concept: The absolute value function $|x|$ has its minimum value of 0 at $x = 0$. For a function $f(x) = c + |x|$, the minimum value occurs where the absolute term is zero.

Solution: 1. **Given Function:** $f(x) = 3 + |x|$ 2. **Analyzing $|x|$:** We know that $|x| \geq 0$ for all $x \in \mathbb{R}$. The minimum value of $|x|$ is 0, which occurs at $x = 0$. 3. **Calculating Minimum of $f(x)$:**

$$f(0) = 3 + |0| = 3$$

Since $|x|$ can only increase as x moves away from 0, 3 is the local (and absolute) minimum value.

Final Answer: 3

Answer: (B)



Q23.

Solution

Concept: A function is increasing in an interval if its derivative $f'(x) > 0$ for all x in that interval.

Solution: 1. **Given Function:** $f(x) = \tan x - x$ 2. **Differentiating w.r.t. x :**

$$f'(x) = \sec^2 x - 1$$

3. **Using Trigonometric Identity:** Since $1 + \tan^2 x = \sec^2 x$, then $\sec^2 x - 1 = \tan^2 x$.

$$f'(x) = \tan^2 x$$

4. **Analyzing the Derivative:** For any $x \in (0, \pi/2)$, $\tan x$ is defined and non-zero. Since $\tan^2 x$ is a square of a real number, $f'(x) > 0$ for all $x \in (0, \pi/2)$. Therefore, the function is increasing in $(0, \pi/2)$.

Final Answer: Increasing in $(0, \pi/2)$

Answer: (C)

Q24.

Solution

Concept: For integrals of the form $\int \frac{f'(x)}{f(x)} dx$, the result is $\log |f(x)| + C$. We check if the numerator is the derivative of the denominator.

Solution: 1. **Identify $f(x)$:** Let $f(x) = x^{10} + 10^x$. 2. **Differentiate $f(x)$:** $\frac{d}{dx}(x^{10}) = 10x^9$ * $\frac{d}{dx}(10^x) = 10^x \log_e 10$ * So, $f'(x) = 10x^9 + 10^x \log_e 10$. 3. **Integration:** * The integral is exactly $\int \frac{f'(x)}{f(x)} dx$. * **Result:** $\log(x^{10} + 10^x) + C$.

Final Answer: $\log(10^x + x^{10}) + C$

Answer: (D)



Q25.

Solution

Concept: To solve $\int \frac{dx}{\sin^2 x \cos^2 x}$, we can replace the 1 in the numerator with the identity $\sin^2 x + \cos^2 x$.

Solution: 1. ****Rewrite Numerator:****

$$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

2. ****Split the Integral:****

$$\int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$\int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx = \int (\sec^2 x + \csc^2 x) dx$$

3. ****Integrate:**** $\int \sec^2 x dx = \tan x$ * $\int \csc^2 x dx = -\cot x$ * **Result:** $\tan x - \cot x + C$.

Final Answer: $\tan x - \cot x + C$

Answer: (B)

Q26.

Solution

Concept: Properties of definite integrals can simplify symmetric intervals. Specifically, for $\int_0^{2a} f(x) dx$, if $f(2a - x) = -f(x)$, the integral is 0.

Solution: 1. ****Apply Property:**** Let $f(x) = \cos^5 x$ and the interval be $[0, 2\pi]$. 2. ****Check for Symmetry:**** $f(2\pi - x) = \cos^5(2\pi - x) = \cos^5 x$. * This doesn't directly make it 0. Use the property $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$ if $f(2a - x) = f(x)$. * $\int_0^{2\pi} \cos^5 x dx = 2 \int_0^\pi \cos^5 x dx$. 3. ****Check $[0, \pi]$:** * In the new integral, $2a = \pi$. * $f(\pi - x) = \cos^5(\pi - x) = (-\cos x)^5 = -\cos^5 x$. * Since $f(\pi - x) = -f(x)$, the integral over $[0, \pi]$ is 0. 4. ****Result:**** $2 \times 0 = 0$.

Final Answer: 0

Answer: (A)



Q27.

Solution

Concept: Use the substitution method when the derivative of an inner function is present in the numerator.

Solution: 1. **Substitution:** Let $u = xe^x$. 2. **Differentiate:** $du = (x \cdot e^x + e^x \cdot 1)dx = e^x(1 + x)dx$. 3. **Transform Integral:**

$$\int \frac{1}{\cos^2 u} du = \int \sec^2 u du$$

4. **Integrate:** $\tan u + C = \tan(xe^x) + C$.

Final Answer: $\tan(xe^x) + C$

Answer: (A)

Q28.

Solution

Concept: The standard integral $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$. We evaluate this at the given limits.

Solution: 1. **Integrate:** $[\tan^{-1} x]_1^{\sqrt{3}}$ 2. **Evaluate at Upper Limit:** $\tan^{-1}(\sqrt{3}) = \pi/3$. 3. **Evaluate at Lower Limit:** $\tan^{-1}(1) = \pi/4$. 4. **Calculation:**

$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$$

Final Answer: $\frac{\pi}{12}$

Answer: (C)

Q29.

Solution

Concept: Use the standard integral form $\int \frac{1}{\sqrt{a^2 - (mx)^2}} dx = \frac{1}{m} \sin^{-1}(\frac{mx}{a}) + C$.

Solution: 1. **Rewrite Integral:**

$$\int \frac{dx}{\sqrt{3^2 - (5x)^2}}$$

2. **Identify constants:** $a = 3, m = 5$. 3. **Apply Formula:**

$$\frac{1}{5} \sin^{-1} \left(\frac{5x}{3} \right) + C$$

Final Answer: $\frac{1}{5} \sin^{-1} \left(\frac{5x}{3} \right) + C$

Answer: (A)



Q30.

Solution

Concept: The area bounded by a curve $y = f(x)$ and the x -axis from $x = a$ to $x = b$ is given by $\int_a^b |f(x)|dx$. Since $\cos x$ changes sign over the interval $[0, 2\pi]$, we must integrate the absolute value or split the integral where the curve crosses the x -axis.

Solution: 1. **Identify Intervals:** $\cos x$ is positive in $[0, \pi/2]$ and $[3\pi/2, 2\pi]$. $\cos x$ is negative in $[\pi/2, 3\pi/2]$. 2. **Setup the Integral:** Area = $\int_0^{\pi/2} \cos x dx + |\int_{\pi/2}^{3\pi/2} \cos x dx| + \int_{3\pi/2}^{2\pi} \cos x dx$ 3. **Calculate:** $[\sin x]_0^{\pi/2} = 1 - 0 = 1$ $|[\sin x]_{\pi/2}^{3\pi/2}| = |-1 - 1| = 2$ $[\sin x]_{3\pi/2}^{2\pi} = 0 - (-1) = 1$ 4. **Total Area:** $1 + 2 + 1 = 4$ sq units.

Final Answer: 4 sq units

Answer: (B)

Q31.

Solution

Concept: The area in the first quadrant bounded by $y^2 = 4ax$ from $x = x_1$ to $x = x_2$ is given by $\int_{x_1}^{x_2} y dx$, where $y = \sqrt{4ax}$.

Solution: 1. **Express y in terms of x :** $y^2 = 9x \implies y = 3\sqrt{x}$ (taking positive root for the first quadrant). 2. **Setup the Integral:** Area = $\int_2^4 3\sqrt{x} dx = 3 \int_2^4 x^{1/2} dx$ 3. **Integrate:** $3 \left[\frac{x^{3/2}}{3/2} \right]_2^4 = 3 \cdot \frac{2}{3} [x^{3/2}]_2^4 = 2[x\sqrt{x}]_2^4$ 4. **Evaluate Limits:** $2[(4\sqrt{4}) - (2\sqrt{2})]$ $2[8 - 2\sqrt{2}] = 16 - 4\sqrt{2}$

Final Answer: $16 - 4\sqrt{2}$

Answer: (A)

Q32.

Solution

Concept: 1. **Order:** The order of a differential equation is the order of the highest derivative present. 2. **Degree:** The degree is the power of the highest order derivative, provided the equation is a polynomial in its derivatives. If a derivative is within a transcendental function (like \log , \sin , or e^x) and cannot be removed, the degree is not defined.

Solution: 1. **Identify Order:** The highest derivative present in the equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\left(\frac{d^2y}{dx^2}\right)$ is the second derivative. Thus, the **Order is 2**. 2. **Check for Degree:** The term $\log\left(\frac{d^2y}{dx^2}\right)$ involves a derivative inside a logarithmic function. 3. **Conclusion:** Since the equation cannot be expressed as a polynomial in $\frac{d^2y}{dx^2}$, the **Degree is Not Defined**.

Final Answer: Order 2, Degree Not Defined

Answer: (B)



Q33.

Solution

Concept: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$ and $Q = x^2$. The solution is found using the Integrating Factor (I.F.):

$$I.F. = e^{\int P dx}$$

The solution is: $y(I.F.) = \int Q(I.F.)dx + C$.

Solution: 1. ****Find I.F.:****

$$I.F. = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

2. ****Apply Solution Formula:****

$$y(x) = \int (x^2)(x)dx + C$$

$$yx = \int x^3 dx + C$$

3. ****Integrate:****

$$yx = \frac{x^4}{4} + C$$

Final Answer: $yx = \frac{x^4}{4} + C$

Answer: (A)

Q34.

Solution

Concept: The equation can be rearranged to check if it is an exact differential equation or a linear one. An equation of the form $Mdx + Ndy = 0$ is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Solution: 1. ****Rearrange the Equation:**** Divide by dx : $e^x \frac{dy}{dx} + ye^x + 2x = 0$ 2. ****Recognize the Derivative:**** Observe that $\frac{d}{dx}(ye^x) = ye^x + e^x \frac{dy}{dx}$. So, the equation becomes: $\frac{d}{dx}(ye^x) + 2x = 0$.

3. ****Integrate both sides with respect to x :**

$$\int \frac{d}{dx}(ye^x) dx + \int 2x dx = \int 0 dx$$

$$ye^x + x^2 = C$$

Final Answer: $ye^x + x^2 = C$

Answer: (A)



Q35.

Solution

Concept: The relationship between the dot product and the magnitude of the cross product is given by Lagrange's Identity:

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

Solution: 1. ****Given Data:**** $|\vec{a}| = 10, |\vec{b}| = 2, \vec{a} \cdot \vec{b} = 12$. 2. ****Apply Lagrange's Identity:**** $|\vec{a} \times \vec{b}|^2 = (10)^2(2)^2 - (12)^2$ $|\vec{a} \times \vec{b}|^2 = (100)(4) - 144$ 3. ****Calculation:**** $|\vec{a} \times \vec{b}|^2 = 400 - 144 = 256$ $|\vec{a} \times \vec{b}| = \sqrt{256} = 16$

Final Answer: 16

Answer: (B)

Q36.

Solution

Concept: Let the direction cosines of the line be l, m, n . These are defined as $l = \cos \alpha, m = \cos \beta, n = \cos \gamma$, where α, β, γ are the angles with the axes. A fundamental property is:

$$l^2 + m^2 + n^2 = 1$$

Solution: 1. ****Given Data:**** The line makes equal angles, so $\alpha = \beta = \gamma$. 2. ****Relationship:**** This implies $l = m = n$. 3. ****Substitute into Identity:**** $l^2 + l^2 + l^2 = 1 \implies 3l^2 = 1 \implies l^2 = \frac{1}{3}$ 4. ****Determine Cosines:**** $l = m = n = \pm \frac{1}{\sqrt{3}}$. Taking the positive values, we get $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.

Final Answer: $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

Answer: (B)

Q37.

Solution

Concept: The angle θ between two vectors \vec{a} and \vec{b} is found using the dot product formula:

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Solution: 1. ****Given Data:**** $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2, \vec{a} \cdot \vec{b} = \sqrt{6}$. 2. ****Apply Formula:**** $\cos \theta = \frac{\sqrt{6}}{\sqrt{3} \cdot 2} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{3} \cdot 2} = \frac{\sqrt{2}}{2}$ 3. ****Simplify:**** $\cos \theta = \frac{1}{\sqrt{2}}$ 4. ****Find Angle:**** $\theta = \cos^{-1}(\frac{1}{\sqrt{2}}) = \pi/4$.

Final Answer: $\pi/4$

Answer: (C)



Q38.

Solution

Concept: The vector equation of a line passing through a point with position vector \vec{a} and parallel to vector \vec{b} is:

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Solution: 1. ****Identify Position Vector (\vec{a}):**** The line passes through (1, 2, 3), so $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.
 2. ****Identify Parallel Vector (\vec{b}):**** The line is parallel to $3\hat{i} + 2\hat{j} - 2\hat{k}$, so $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$. 3. ****Form Equation:**** $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$.

Final Answer: $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

Answer: (A)

Q39.

Solution

Concept: If (a, b, c) are direction ratios, the direction cosines (l, m, n) are given by:

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Solution: 1. ****Given Ratios:**** (2, -1, -2). 2. ****Calculate Magnitude:**** $\sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$. 3. ****Calculate Cosines:**** $l = 2/3, m = -1/3, n = -2/3$. 4. ****Result:**** (2/3, -1/3, -2/3).

Final Answer: (2/3, -1/3, -2/3)

Answer: (A)

Q40.

Solution

Concept: The perpendicular distance d from the origin (0, 0, 0) to the plane $Ax + By + Cz + D = 0$ is:

$$d = \frac{|D|}{\sqrt{A^2 + B^2 + C^2}}$$

Solution: 1. ****Identify Coefficients:**** For $2x - 3y + 4z - 6 = 0$: $A = 2, B = -3, C = 4, D = -6$.
 2. ****Calculate Denominator:**** $\sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$. 3. ****Apply Formula:**** $d = \frac{|-6|}{\sqrt{29}} = \frac{6}{\sqrt{29}}$.

Final Answer: $6/\sqrt{29}$

Answer: (A)



Q41.

Solution

Concept: According to the **Corner Point Theorem** in Linear Programming, the optimal (maximum or minimum) value of the objective function, if it exists, must occur at one of the vertices (corner points) of the feasible region.

Solution: 1. **Feasible Region:** This is the set of all points that satisfy the given constraints. 2. **Objective Function:** Usually defined as $Z = ax + by$. 3. **Optimality:** The fundamental theorem of linear programming states that if the feasible region is bounded (a convex polygon), the objective function attains its optimal value at the **corner points** of this region.

Final Answer: Given by corner points of the feasible region

Answer: (B)

Q42.

Solution

Concept: A set is **convex** if for any two points in the set, the line segment joining them lies entirely within the set. Geometrically, sets like circles (interior), ellipses (interior), and parabolas (interior) are convex, whereas the exterior of these shapes is generally not convex.

Solution: 1. **Option A ($x^2 + y^2 \geq 1$):** This represents the region **outside** a circle. If you pick two points on opposite sides of the circle, the line joining them passes through the "hole" (the interior), which is not in the set. Not convex. 2. **Option B ($y^2 \leq x$):** This represents the region **inside** a parabola opening to the right. Any line segment between two points inside this region remains inside. This is a **convex set**. 3. **Option C ($3x^2 + 4y^2 \geq 12$):** Similar to Option A, this is the region **outside** an ellipse. Not convex.

Final Answer: $\{(x, y) : y^2 \leq x\}$

Answer: (B)



Q43.

Solution

Concept: The conditional probability $P(A|B)$ is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Similarly, $P(B|A) = \frac{P(B \cap A)}{P(A)}$.

Solution: 1. ****Given Equation:****

$$P(A|B) = P(B|A)$$

2. ****Substitute Definitions:****

$$\frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

3. ****Simplify:**** Since $P(A \cap B) = P(B \cap A)$, we can cancel the numerators (assuming $P(A \cap B) \neq 0$):

$$\frac{1}{P(B)} = \frac{1}{P(A)}$$

4. ****Conclusion:****

$$P(A) = P(B)$$

Final Answer: $P(A) = P(B)$

Answer: (C)

Q44.

Solution

Concept: Conditional probability $P(E|F)$ is the probability of event E occurring given that event F has already occurred. It is calculated as:

$$P(E|F) = \frac{n(E \cap F)}{n(F)}$$

where $n(F)$ is the number of outcomes in the reduced sample space.

Solution: 1. ****Define Event F (Given condition):**** Black die results in a 5. * Outcomes for F : $(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$ * $n(F) = 6$. 2. ****Define Event E (Target):**** Sum greater than 9. * We look for outcomes in F where the sum is > 9 . * $(5, 5)$ gives sum 10. * $(5, 6)$ gives sum 11. * $n(E \cap F) = 2$. 3. ****Calculate Probability:**** * $P(E|F) = \frac{2}{6} = \frac{1}{3}$.

Final Answer: $1/3$

Answer: (A)



Q45.

Solution

Concept: The conditional probability $P(A|B)$ is mathematically defined by the ratio:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This expression is only valid when the denominator $P(B) \neq 0$.

Solution: 1. **Given Data:** $P(A) = 1/2$ and $P(B) = 0$. 2. **Applying the Formula:** $P(A|B) = \frac{P(A \cap B)}{0}$. 3. **Conclusion:** Division by zero is undefined in mathematics. Therefore, $P(A|B)$ is **not defined** when the given event has a probability of 0.

Final Answer: Not defined

Answer: (C)

Q46.

Solution

Concept: Two events A and B are independent if $P(A \cap B) = P(A)P(B)$. This also implies that their complements A' and B' are independent:

$$P(A' \cap B') = P(A')P(B')$$

Solution: 1. **Option A:** Mutually exclusive means $P(A \cap B) = 0$, which contradicts independence unless one probability is 0. 2. **Option B:** $P(A'B')$ is notation for $P(A' \cap B')$. * Since A' and B' are independent if A and B are: * $P(A' \cap B') = P(A') \cdot P(B')$ * Using $P(E') = 1 - P(E)$: * $P(A'B') = [1 - P(A)][1 - P(B)]$. 3. **Conclusion:** This is a correct mathematical characterization of independence.

Final Answer: $P(A'B') = [1 - P(A)][1 - P(B)]$

Answer: (B)



Q47.

Solution

Concept: A problem is "solved" if at least one person solves it. It is easier to calculate the probability that the problem is **not** solved (both fail) and subtract it from 1:

$$P(\text{Solved}) = 1 - P(A' \cap B')$$

Solution: 1. **Given Probabilities:** $P(A) = 1/2, P(B) = 1/3$. 2. **Probabilities of Failure:** $P(A') = 1 - 1/2 = 1/2$ * $P(B') = 1 - 1/3 = 2/3$ 3. **Probability both fail (Independent events):** $P(A' \cap B') = P(A') \cdot P(B') = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$ 4. **Probability problem is solved:** $P = 1 - \frac{1}{3} = \frac{2}{3}$.

Final Answer: $2/3$

Answer: (A)

Q48.

Solution

Concept: For a definite integral with symmetric limits $\int_{-a}^a f(x)dx$, we check if the function is even or odd: 1. If $f(-x) = -f(x)$ (Odd function), the integral is 0. 2. If $f(-x) = f(x)$ (Even function), the integral is $2 \int_0^a f(x)dx$.

Solution: 1. **Identify the function:** $f(x) = \log\left(\frac{2-x}{2+x}\right)$. 2. **Test for Odd/Even:** $f(-x) = \log\left(\frac{2-(-x)}{2+(-x)}\right) = \log\left(\frac{2+x}{2-x}\right)$ * Using the property $\log\left(\frac{1}{m}\right) = -\log m$: $f(-x) = \log\left(\left(\frac{2-x}{2+x}\right)^{-1}\right) = -\log\left(\frac{2-x}{2+x}\right)$ 3. **Conclusion:** * Since $f(-x) = -f(x)$, the function is **odd**. * Therefore, $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx = 0$.

Final Answer: 0

Answer: (A)

Q49.

Solution

Concept: For any vector sum, the magnitude squared is given by $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. For the sum of three vectors, we use:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

Solution: 1. **Given Data:** * $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, so $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$. * $\vec{a} + \vec{b} + \vec{c} = 0$. 2. **Apply the Formula:** * $|0|^2 = (1)^2 + (1)^2 + (1)^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ * $0 = 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ 3. **Solve for the Required Value:** * $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$ * $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -3/2$

Final Answer: $-3/2$

Answer: (C)

Q50.

Solution

Concept: To find the nearest point, we minimize the distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Alternatively, minimize the square of the distance $S = D^2$ to simplify calculations.

Solution: 1. **Identify the Point and Curve:** * Target point: $(0, 5)$. Point on curve: (x, y) where $x^2 = 2y$. 2. **Setup Distance Squared (S):** * $S = (x - 0)^2 + (y - 5)^2$ * Substitute $x^2 = 2y$: $S = 2y + (y - 5)^2 = 2y + y^2 - 10y + 25 = y^2 - 8y + 25$. 3. **Minimize S with respect to y:** * $\frac{dS}{dy} = 2y - 8 = 0 \implies y = 4$. 4. **Find x:** * $x^2 = 2(4) = 8 \implies x = \pm\sqrt{8} = \pm 2\sqrt{2}$. 5. **Conclusion:** * The point is $(2\sqrt{2}, 4)$ or $(-2\sqrt{2}, 4)$. Comparing with options: $(2\sqrt{2}, 4)$.

Final Answer: $(2\sqrt{2}, 4)$

Answer: (A)



Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	B	4	A	5	B
6	D	7	A	8	B	9	B	10	A
11	B	12	B	13	D	14	C	15	B
16	B	17	A	18	A	19	B	20	B
21	A	22	B	23	C	24	D	25	B
26	A	27	A	28	C	29	A	30	B
31	A	32	B	33	A	34	A	35	B
36	B	37	C	38	A	39	A	40	A
41	B	42	B	43	C	44	A	45	C
46	B	47	A	48	A	49	C	50	A

