CUET-UG Mathematics Sample Paper-1

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A

(B) $I - A$

(C) I

(D) $3A$

Q2. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Q3. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ is a non-singular matrix, then λ cannot be:

(A) $-\frac{7}{5}$

(B) $\frac{5}{7}$

(C) $\frac{8}{5}$

(D) 1

Q4. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

(A) Skew-symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

Q5. If A is a matrix of order 3×3, then $|3A|$ is equal to:

(A) $3|A|$

(B) $9|A|$

(C) $27|A|$

(D) $81|A|$

Q6. If A is a square matrix of order 3 and $|A| = 5$, then $|adj A|$ is:

(A) 5

(B) 25

(C) 125

(D) 625

Q7. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) I

(D) 0

Q8. The area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units. The value of k is:

(A) 9

(B) 3

(C) -9

(D) 6

Q9. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to:

(A) A

(B) I

(C) 0

(D) $-I$

Q10. Let R be a relation on set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. This relation is:

(A) Reflexive but not symmetric

(B) Transitive

(C) Symmetric

(D) Equivalence

Q11. The function $f : R \to R$ defined by $f(x) = 3 - 4x$ is:

(A) One-to-one but not onto

(B) Onto but not one-to-one

(C) One-to-one and onto

(D) Neither one-to-one nor onto

Q12. If $f(x) = \frac{x-1}{x+1}$, then $f(f(x))$ is equal to:

(A) x

(B) $\frac{1}{x}$

(C) $\frac{-1}{x}$

(D) $-x$

Q13. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) $-\frac{\pi}{3}$

Q14. The domain of $\sin^{-1}(2x)$ is:

(A) $[-1, 1]$

(B) $[-\frac{1}{2}, \frac{1}{2}]$

(C) $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(D) $\mathbb{R}$

Q15. If $f(x) = |x-1| + |x-2|$, then $f(x)$ is not differentiable at:

(A) $x = 1$ only

(B) $x = 2$ only

(C) $x = 1$ and $x = 2$

(D) Everywhere differentiable

Q16. If $y = \log(\sin x)$, then $\frac{d^2y}{dx^2}$ is:

(A) $-\csc^2 x$

(B) $\sec^2 x$

(C) $-\cot x$

(D) $\cos x$

Q17. If $x = a\cos\theta$, $y = b\sin\theta$, then $\frac{dy}{dx}$ is:

(A) $\frac{b}{a}\tan\theta$

(B) $-\frac{b}{a}\cot\theta$

(C) $\frac{a}{b}\cot\theta$

(D) $-\frac{a}{b}\tan\theta$

Q18. The derivative of e^{x^3} with respect to x is:

(A) $3x^2e^{x^3}$

(B) e^{x^3}

(C) $3xe^{x^3}$

(D) $x^3e^{x^2}$

Q19. The function $f(x) = x^2 - 2x$ is decreasing in the interval:

(A) $(1, \infty)$

(B) $(-\infty, 1)$

(C) $(0, 2)$

(D) $(-1, 1)$

Q20. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) $-\frac{1}{3}$

Q21. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\sqrt{3}$

Q22. The rate of change of the volume of a sphere with respect to its radius r when $r = 3$ cm is:

(A) 12π

(B) 36π

(C) 24π

(D) 9π

Q23. If $y = \tan^{-1} x$, then y_2 (second derivative) is:

(A) $\frac{-2x}{(1+x^2)^2}$

(B) $\frac{1}{1+x^2}$

(C) $\frac{2x}{(1+x^2)^2}$

(D) $\frac{-1}{(1+x^2)^2}$

Q24. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = \frac{1}{e}$

(C) $x = 1$

(D) $x = 0$

Q25. If $f(x) = kx - \sin x$ is monotonically increasing for all $x \in \mathbb{R}$, then:

(A) $k < 1$

(B) $k > 1$

(C) $k < -1$

(D) $k = 0$

Q26. If $\int \frac{dx}{x^2-a^2}$ is equal to:

(A) $\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

(B) $\frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$

(C) $\frac{1}{2a} \log\left|\frac{a-x}{a+x}\right| + C$

(D) $\log\left|x + \sqrt{x^2 - a^2}\right| + C$

Q27. If $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ is:

(A) $\frac{e^x}{x} + C$

(B) $-\frac{e^x}{x^2} + C$

(C) $e^x \log x + C$

(D) $xe^x + C$

Q28. The value of $\int_0^{\pi/2} \sin^3 x \, dx$ is:

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{4}{3}$

(D) $\frac{\pi}{4}$

Q29. If $\int \tan^2 x \, dx$ is equal to:

(A) $\tan x + x + C$

(B) $\tan x - x + C$

(C) $\sec^2 x + C$

(D) $\sec x \tan x + C$

Q30. The area bounded by $y = x^2$, the x-axis, and the lines $x = 1, x = 2$ is:

(A) $\frac{7}{3}$

(B) $\frac{8}{3}$

(C) 1

(D) 2

Q31. The area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

(A) πa^2

(B) πb^2

(C) πab

(D) $2\pi ab$

Q32. If $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\,dx$ (where $x \in (0, \pi/4)$) is:

(A) $x + C$

(B) $\sin x - \cos x + C$

(C) $\log|\sin x + \cos x| + C$

(D) 1 + C

Q33. The value of $\int_{-a}^{a} f(x)\,dx = 0$ if $f(x)$ is:

(A) Even function

(B) Odd function

(C) Constant function

(D) Periodic function

Q34. If $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) 0

(D) 1

Q35. Area of the region bounded by $y^2 = 4x$ and $x = 3$ is:

(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $16\sqrt{3}$

(D) $2\sqrt{3}$

Q36. The degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y = 0$ is:

(A) 1

(B) 2

(C) 3

(D) Not defined

Q37. The general solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $\tan^{-1} y - \tan^{-1} x = C$

(C) $x + y = C(1 - xy)$

(D) $xy = C$

Q38. The Integrating Factor (I.F.) for $\frac{dy}{dx} + y \sec x = \tan x$ is:

(A) $\sec x + \tan x$

(B) $\log(\sec x + \tan x)$

(C) $e^{\sec x}$

(D) $\sec x \tan x$

Q39. The order of the differential equation of all circles of given radius r is:

(A) 1

(B) 2

(C) 3

(D) 4

Q40. The solution of $x\,dy - y\,dx = 0$ represents:

(A) Circles

(B) Parabolas

(C) Straight lines passing through origin

(D) Hyperbolas

Q41. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, then $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$ is:

(A) 15

(B) -18

(C) -15

(D) 0

Q42. The unit vector perpendicular to both $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is:

(A) $\hat{i} - \hat{j} + \hat{k}$

(B) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

(C) $\hat{i} + \hat{j} + \hat{k}$

(D) $\hat{k}$

Q43. The distance of the point (a, b, c) from the x-axis is:

(A) a

(B) $\sqrt{b^2 + c^2}$

(C) $\sqrt{a^2 + b^2}$

(D) $\sqrt{a^2 + c^2}$

Q44. The direction cosines of the z-axis are:

(A) $(1, 0, 0)$

(B) $(0, 1, 0)$

(C) $(0, 0, 1)$

(D) $(1, 1, 1)$

Q45. The shortest distance between lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is:

(A) $\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$

(B) $\frac{|(\vec{a}_2-\vec{a}_1)\cdot\vec{b}|}{|\vec{b}|}$

(C) $|\vec{a}_2 - \vec{a}_1|$

(D) 0

Q46. If a line makes angles α, β, γ with the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q47. Objective function of a LPP is $Z = 3x + 4y$. If the corner points are $(0,0), (4,0), (0,4)$ and $(2,3)$, the maximum value of Z is:

(A) 12

(B) 16

(C) 18

(D) 20

Q48. In LPP, the feasible region for the constraints $x + y \leq 1, x \geq 0, y \geq 0$ is:

(A) A triangle

(B) A rectangle

(C) Unbounded

(D) A circle

Q49. If $P(A) = \frac{1}{2}, P(B) = 0$, then $P(A|B)$ is:

(A) 0

(B) $\frac{1}{2}$

(C) Not defined

(D) 1

Q50. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.7

(B) 0.12

(C) 0.1

(D) 0.5

Detailed Solutions

Q1.

Solution

Concept: We are given that $A^2 = A$, meaning A is a projection matrix. We need to find $(I + A)^3 - 7A$.

Solution: First, expand $(I + A)^3$:

$$(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3 = I + 3A + 3A + A = I + 7A.$$

Now, subtract $7A$ from both sides:

$$(I + A)^3 - 7A = I + 7A - 7A = I.$$

Answer: (C)

Q2.

Solution

Concept: The number of all possible matrices of order 3×3 with each entry 0 or 1 is calculated by noting that each of the 9 elements can independently be either 0 or 1.

Solution: Since there are 9 elements and each can be either 0 or 1, the total number of possible matrices is:

$$2^9 = 512.$$

Answer: (D)

Q3.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ and we need to find the value of λ that makes the matrix singular.

Solution: The determinant of A is:

$$\det(A) = 2\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} - \lambda\begin{vmatrix} 0 & 5 \\ 1 & 3 \end{vmatrix} + (-3)\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}.$$

Calculating the 2x2 determinants:

$$\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1, \quad \begin{vmatrix} 0 & 5 \\ 1 & 3 \end{vmatrix} = 0 - 5 = -5, \quad \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = 0 - 2 = -2.$$

Thus, the determinant is:

$$\det(A) = 2(1) - \lambda(-5) + (-3)(-2) = 2 + 5\lambda + 6 = 8 + 5\lambda.$$

For A to be singular, $\det(A) = 0$, so:

$$8 + 5\lambda = 0 \implies \lambda = -\frac{8}{5}.$$

Thus, λ cannot be $-\frac{8}{5}$.

Answer: (A)

Q4.

Solution

Concept: For symmetric matrices A and B, we know that $AB - BA$ is always a skew-symmetric matrix.

Solution: Since A and B are symmetric, we have:

$$A^T = A \quad \text{and} \quad B^T = B.$$

Taking the transpose of $AB - BA$:

$$(AB - BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA),$$

which shows that $AB - BA$ is skew-symmetric.

Answer: (A)

Q5.

Solution

Concept: For a matrix A of order 3×3, we know that:

$$|cA| = c^n |A|,$$

where n is the order of the matrix. For A of order 3×3 and scalar $c = 3$, we get:

$$|3A| = 3^3 |A| = 27|A|.$$

Answer: (C)

Q6.

Solution

Concept: We are given that $|A| = 5$ and we need to find $|adjA|$. We know that for a matrix A of order 3, the determinant of the adjugate matrix is related to the determinant of A by the formula:

$$|adjA| = |A|^{n-1} = |A|^2,$$

where n is the order of the matrix. Since $|A| = 5$, we have:

$$|adjA| = 5^2 = 25.$$

Answer: (B)

Q7.

Solution

Concept: We are given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and need to find $A^2 - 5A$.

Solution: First, compute A^2:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}.$$

Now, calculate $5A$:

$$5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}.$$

Thus,

$$A^2 - 5A = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I.$$

Answer: (A)

Q8.

Solution

Concept: We are given a triangle with vertices $(-3, 0)$, $(3, 0)$, and $(0, k)$, and we need to find the value of k such that the area is 9 square units.

Solution: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

Substituting the coordinates $(-3, 0)$, $(3, 0)$, $(0, k)$:

$$\text{Area} = \frac{1}{2}\left|(-3)(0 - k) + 3(k - 0) + 0(0 - 0)\right| = \frac{1}{2}\left|3k + 3k\right| = \frac{1}{2} \times 6k = 3k.$$

We are given that the area is 9, so:

$$3k = 9 \implies k = 3.$$

Answer: (B)

Q9.

Solution

Concept: We are given $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and need to find A^2.

Solution: We compute A^2:

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

Answer: (B)

Q10.

Solution

Concept: We are given the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ on the set $A = \{1, 2, 3\}$, and we need to determine the type of relation.

Solution: For the relation R to be reflexive, it must contain $(1, 1)$, $(2, 2)$, $(3, 3)$, which it does. For the relation to be symmetric, for each (a, b) in R, the pair (b, a) must also be in R. But $(1, 2) \in R$ but $(2, 1) \notin R$, so R is not symmetric. For the relation to be transitive, if (a, b) and (b, c) are in R, then (a, c) must also be in R. We have $(1, 2)$ and $(2, 3)$ in R, so $(1, 3)$ must be in R, but it is not, so the relation is not transitive.

Thus, the relation is reflexive but not symmetric.

Answer: (A)

Q11.

Solution

Concept: The function $f(x) = 3 - 4x$ is a linear function, and we need to determine its properties.
Solution: The function $f(x) = 3 - 4x$ is one-to-one because it has a unique output for every input. It is also onto because for any real number y, we can find an x such that $f(x) = y$. Specifically, $x = \frac{3-y}{4}$.
Thus, the function is both one-to-one and onto.

Answer: (C)

Q12.

Solution

Concept: We are given that $f(x) = \frac{x-1}{x+1}$, and we need to find $f(f(x))$.
Solution: First, compute $f(f(x))$:

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{\frac{x-1-(x+1)}{x+1}}{\frac{x-1+(x+1)}{x+1}} = \frac{\frac{-2}{x+1}}{\frac{2x}{x+1}} = -\frac{1}{x}.$$

Answer: (C)

Q13.

Solution

Concept: We are given $\cos^{-1}\left(-\frac{1}{2}\right)$, and we need to find its principal value.

Solution: The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is the angle whose cosine is $-\frac{1}{2}$ and lies between 0 and π. We know that $\cos\frac{2\pi}{3} = -\frac{1}{2}$, so the principal value is $\frac{2\pi}{3}$.

Answer: (B)

Q14.

Solution

Concept: We are given $\sin^{-1}(2x)$, and we need to find its domain.
Solution: The domain of $\sin^{-1}(y)$ is $-1 \le y \le 1$. For $\sin^{-1}(2x)$, we must have:

$$-1 \le 2x \le 1 \implies -\frac{1}{2} \le x \le \frac{1}{2}.$$

Thus, the domain is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Answer: (B)

Q15.

Solution

Concept: We are given that $f(x) = |x-1| + |x-2|$, and we need to determine where it is not differentiable.

Solution: The function $f(x)$ is not differentiable at points where the absolute value terms change slope. The points where this happens are $x = 1$ and $x = 2$.

Thus, $f(x)$ is not differentiable at $x = 1$ and $x = 2$.

Answer: (C)

Q16.

Solution

Concept: We are given that $y = \log(\sin x)$, and we need to find $\frac{d^2y}{dx^2}$.

Solution: First, compute the first derivative of y:

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x.$$

Now, compute the second derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cot x) = -\csc^2 x.$$

Answer: (A)

Q17.

Solution

Concept: We are given that $x = a\cos\theta$ and $y = b\sin\theta$, and we need to find $\frac{dy}{dx}$.

Solution: To compute $\frac{dy}{dx}$, we use the chain rule:

$$\frac{dx}{d\theta} = -a\sin\theta, \quad \frac{dy}{d\theta} = b\cos\theta.$$

Thus,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta.$$

Answer: (B)

Q18.

Solution

Concept: We are asked to find the derivative of e^{x^3} with respect to x.

Solution: Using the chain rule:

$$\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3} \cdot \frac{d}{dx}(x^3) = e^{x^3} \cdot 3x^2.$$

Answer: (A)

Q19.

Solution

Concept: We are given the function $f(x) = x^2 - 2x$, and we need to find where it is decreasing.

Solution: The first derivative of $f(x)$ is:

$$\frac{df}{dx} = 2x - 2.$$

To find where $f(x)$ is decreasing, we solve $\frac{df}{dx} < 0$:

$$2x - 2 < 0 \implies x < 1.$$

Thus, the function is decreasing on the interval $(-\infty, 1)$.

Answer: (B)

Q20.

Solution

Concept: We are given the curve $y = 2x^2 + 3\sin x$ and need to find the slope of the normal at $x = 0$.

Solution: First, find the derivative of y:

$$\frac{dy}{dx} = 4x + 3\cos x.$$

At $x = 0$,

$$\frac{dy}{dx} = 4(0) + 3\cos(0) = 3.$$

The slope of the normal is the negative reciprocal of the slope of the tangent:

$$\text{slope of normal} = -\frac{1}{3}.$$

Answer: (D)

Q21.

Solution

Concept: We are given the function $f(x) = \sin x + \cos x$, and we need to find its maximum value.
Solution: The first derivative of $f(x)$ is:

$$\frac{df}{dx} = \cos x - \sin x.$$

Set $\frac{df}{dx} = 0$ to find the critical points:

$$\cos x = \sin x \implies x = \frac{\pi}{4}.$$

To confirm that this is a maximum, compute the second derivative:

$$\frac{d^2f}{dx^2} = -\sin x - \cos x.$$

At $x = \frac{\pi}{4}$:

$$\frac{d^2f}{dx^2} = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2} < 0,$$

which confirms a maximum. The maximum value of $f(x)$ is:

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}.$$

Answer: (C)

Q22.

Solution

Concept: We are asked to find the rate of change of the volume of a sphere with respect to its radius r when $r = 3$ cm.
Solution: The volume V of a sphere is given by:

$$V = \frac{4}{3}\pi r^3.$$

The rate of change of volume with respect to r is:

$$\frac{dV}{dr} = 4\pi r^2.$$

At $r = 3$, we have:

$$\frac{dV}{dr} = 4\pi(3)^2 = 36\pi.$$

Answer: (B)

Q23.

Solution

Concept: We are given $y = \tan^{-1} x$, and we need to find the second derivative y_2.

Solution: The first derivative of y is:

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Now, compute the second derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{-2x}{(1+x^2)^2}.$$

Answer: (A)

Q24.

Solution

Concept: We are asked to find the stationary point of the function $f(x) = x^x$.

Solution: First, take the natural logarithm of $f(x)$:

$$\log f(x) = x \log x.$$

Differentiate both sides with respect to x:

$$\frac{d}{dx} \log f(x) = \log x + 1.$$

For the stationary point, set the derivative equal to zero:

$$\log x + 1 = 0 \implies \log x = -1 \implies x = \frac{1}{e}.$$

Answer: (B)

Q25.

Solution

Concept: We are given $f(x) = kx - \sin x$ and we need to determine the value of k such that $f(x)$ is monotonically increasing for all $x \in \mathbb{R}$.
Solution: The first derivative of $f(x)$ is:

$$\frac{df}{dx} = k - \cos x.$$

For $f(x)$ to be monotonically increasing, $\frac{df}{dx} \geq 0$ for all x. Since $\cos x$ ranges from -1 to 1, we need:

$$k - \cos x \geq 0 \implies k \geq 1.$$

Answer: (B)

Q26.

Solution

Concept: We are given $\int \frac{dx}{x^2-a^2}$ and need to solve it using the standard integral form.
Solution: We know that:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C.$$

Thus, the answer is $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$.

Answer: (B)

Q27.

Solution

Concept: We are given $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$ and need to solve it.
Solution: We separate the terms:

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx = e^x \int \frac{1}{x} dx - e^x \int \frac{1}{x^2} dx.$$

We know that:

$$\int \frac{1}{x} dx = \log |x|, \quad \int \frac{1}{x^2} dx = -\frac{1}{x}.$$

Thus, the solution is:

$$e^x \log |x| - \frac{e^x}{x} + C.$$

Answer: (D)

Q28.

Solution

Concept: We are asked to compute $\int_0^{\pi/2} \sin^3 x \, dx$.

Solution: Use the reduction formula for powers of sine:

$$\int \sin^3 x \, dx = \int \sin x(1 - \cos^2 x) \, dx.$$

Substitute $u = \cos x$, then $du = -\sin x \, dx$, changing the limits of integration:

$$\int_0^{\pi/2} \sin^3 x \, dx = \int_1^0 (1 - u^2)(-du) = \int_0^1 (1 - u^2) \, du.$$

Solving:

$$\int_0^1 (1 - u^2) \, du = \left[u - \frac{u^3}{3} \right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}.$$

Answer: (A)

Q29.

Solution

Concept: We are given $\int \tan^2 x \, dx$ and need to solve it.

Solution: Using the identity $\tan^2 x = \sec^2 x - 1$, we rewrite the integral:

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \int \sec^2 x \, dx - \int 1 \, dx.$$

We know:

$$\int \sec^2 x \, dx = \tan x, \quad \int 1 \, dx = x.$$

Thus, the solution is:

$$\tan x - x + C.$$

Answer: (B)

Q30.

Solution

Concept: We are asked to find the area bounded by $y = x^2$, the x-axis, and the lines $x = 1$, $x = 2$.
Solution: The area is given by the integral:

$$\text{Area} = \int_1^2 x^2\, dx.$$

We compute the integral:

$$\int x^2\, dx = \frac{x^3}{3},$$

so the area is:

$$\left[\frac{x^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}.$$

Answer: (A)

Q31.

Solution

Concept: The area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the formula πab.
Solution: For the given ellipse, the area is:

$$\text{Area} = \pi ab.$$

Answer: (C)

Q32.

Solution

Concept: We are asked to compute $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\, dx$ for $x \in (0, \pi/4)$.
Solution: The integral simplifies as:

$$\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\, dx = \int \frac{\sin x + \cos x}{\sqrt{1 + 2\sin x \cos x}}\, dx.$$

Using the identity $\sin 2x = 2\sin x \cos x$, the integrand becomes simpler and can be reduced to:

$$\int (\sin x + \cos x)\, dx = \sin x - \cos x + C.$$

Answer: (B)

Q33.

Solution

Concept: We are given that $\int_{-a}^{a} f(x)\,dx = 0$ and need to find when this holds.
Solution: The integral of $f(x)$ from $-a$ to a is zero if $f(x)$ is an odd function. An odd function satisfies $f(-x) = -f(x)$, and the integral of any odd function over a symmetric interval around zero is zero.

Answer: (B)

Q34.

Solution

Concept: We are asked to compute $\int_0^1 \frac{dx}{1+x^2}$.
Solution: This is a standard integral:

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x).$$

Thus, the value of the integral is:

$$\left[\tan^{-1}(x)\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$$

Answer: (A)

Q35.

Solution

Concept: We are given $y^2 = 4x$ and $x = 3$, and we need to compute the area.
Solution: The area is given by:

$$\text{Area} = \int_0^3 \sqrt{4x}\,dx = 2\int_0^3 \sqrt{x}\,dx.$$

We compute the integral:

$$\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2},$$

so the area is:

$$2\left[\frac{2}{3}x^{3/2}\right]_0^3 = 2 \times \frac{2}{3}\left(3^{3/2} - 0\right) = \frac{4}{3} \times 3\sqrt{3} = 4\sqrt{3}.$$

Answer: (A)

Q36.

Solution

Concept: We are given the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y = 0$, and we need to find its degree.

Solution: The degree of a differential equation is the highest power of the highest-order derivative. In this equation, the highest-order derivative is $\frac{d^2y}{dx^2}$, and its degree is 1 because there is no power greater than 1 for $\frac{d^2y}{dx^2}$.

Answer: (A)

Q37.

Solution

Concept: We are given the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, and we need to find its general solution.

Solution: We can separate the variables:

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}.$$

Integrating both sides:

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2},$$

which gives:

$$\tan^{-1}(y) = \tan^{-1}(x) + C.$$

Thus, the general solution is:

$$\tan^{-1}(y) - \tan^{-1}(x) = C.$$

Answer: (A)

Q38.

Solution

Concept: We are given $\frac{dy}{dx} + y\sec x = \tan x$, and we need to find the integrating factor.

Solution: The standard form is $\frac{dy}{dx} + P(x)y = Q(x)$. Here, $P(x) = \sec x$ and $Q(x) = \tan x$. The integrating factor is:

$$I.F. = e^{\int P(x)\,dx} = e^{\int \sec x\,dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x.$$

Answer: (A)

Q39.

Solution

Concept: The order of the differential equation for all circles of given radius r is related to the equation of a circle.

Solution: The equation of a circle is $x^2 + y^2 = r^2$, which is a second-order equation. Therefore, the order of the differential equation is 2.

Answer: (B)

Q40.

Solution

Concept: We are given the equation $x\,dy - y\,dx = 0$, and we need to determine what it represents.

Solution: This equation represents the equation of a straight line passing through the origin. To see this, rearrange the equation:

$$\frac{dy}{dx} = \frac{y}{x}.$$

This is the equation of a straight line passing through the origin, i.e., $y = mx$, where m is the slope.

Answer: (C)

Q41.

Solution

Concept: We are given vectors $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, and we need to compute $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$.

Solution: First, calculate $\vec{a} + 3\vec{b}$ and $2\vec{a} - \vec{b}$:

$$\vec{a} + 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k} + 9\hat{i} + 6\hat{j} - 3\hat{k} = 10\hat{i} + 7\hat{j} - \hat{k},$$

$$2\vec{a} - \vec{b} = 2(\hat{i} + \hat{j} + 2\hat{k}) - (3\hat{i} + 2\hat{j} - \hat{k}) = 2\hat{i} + 2\hat{j} + 4\hat{k} - 3\hat{i} - 2\hat{j} + \hat{k} = -\hat{i} + 5\hat{k}.$$

Now, compute the dot product:

$$(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b}) = (10\hat{i} + 7\hat{j} - \hat{k}) \cdot (-\hat{i} + 5\hat{k}).$$

We calculate the dot product term by term:

$$(10\hat{i}) \cdot (-\hat{i}) = -10, \quad (7\hat{j}) \cdot (5\hat{k}) = 0, \quad (-\hat{k}) \cdot (-\hat{k}) = 1,$$

Thus, the dot product is:

$$-10 + 0 + 1 = -9.$$

So the result is -9.

Answer: (B)

Q42.

Solution

Concept: We are given $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$, and we need to find the unit vector perpendicular to both.
Solution: The cross product of $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is:

$$(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) = \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{j} + \hat{j} \times \hat{k}.$$

We know that $\hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{j} = 0, \hat{j} \times \hat{k} = \hat{i}$, so:

$$(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) = \hat{k} - \hat{j} + \hat{i}.$$

The magnitude of this vector is:

$$|\hat{i} - \hat{j} + \hat{k}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}.$$

Thus, the unit vector is:

$$\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}.$$

Answer: (B)

Q43.

Solution

Concept: We are asked to find the distance of the point (a, b, c) from the x-axis.
Solution: The distance from the x-axis is the distance between the point (a, b, c) and the line $y = 0, z = 0$. This is simply the distance from (a, b, c) to $(a, 0, 0)$, which is:

$$\text{Distance} = \sqrt{b^2 + c^2}.$$

Answer: (B)

Q44.

Solution

Concept: The direction cosines of the z-axis are the cosines of the angles the z-axis makes with the coordinate axes.
Solution: The z-axis is parallel to $\hat{k}$, and the direction cosines of the z-axis are $(0, 0, 1)$, which correspond to the angles $\alpha = 0°, \beta = 0°$, and $\gamma = 90°$.

Answer: (C)

Q45.

Solution

Concept: The shortest distance between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is given by:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}.$$

Solution: The formula for the shortest distance between two skew lines is:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}.$$

Answer: (A)

Q46.

Solution

Concept: We are asked to find the sum $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ for the direction cosines of a line.
Solution: The sum of the squares of the direction cosines for any line is always equal to 1. Thus:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1.$$

Answer: (A)

Q47.

Solution

Concept: We are given the objective function $Z = 3x + 4y$ and the corner points $(0,0), (4,0), (0,4), (2,3)$. We need to find the maximum value of Z.
Solution: We evaluate Z at each corner point: - At $(0,0) : Z = 3(0) + 4(0) = 0$, - At $(4,0) : Z = 3(4) + 4(0) = 12$, - At $(0,4) : Z = 3(0) + 4(4) = 16$, - At $(2,3) : Z = 3(2) + 4(3) = 6 + 12 = 18$.
Thus, the maximum value of Z is 18.

Answer: (C)

Q48.

Solution

Concept: We are given the constraints $x + y \leq 1, x \geq 0, y \geq 0$, and need to determine the feasible region.

Solution: The feasible region is the triangular region bounded by the lines $x + y = 1$, $x = 0$, and $y = 0$. Thus, the feasible region is a triangle.

Answer: (A)

Q49.

Solution

Concept: We are given $P(A) = \frac{1}{2}$ and $P(B) = 0$, and need to find $P(A|B)$.

Solution: The conditional probability $P(A|B) = \frac{P(A\cap B)}{P(B)}$. Since $P(B) = 0$, $P(A|B)$ is not defined.

Answer: (C)

Q50.

Solution

Concept: We are given that A and B are independent events, and we need to find $P(A \cap B)$.

Solution: For independent events, $P(A \cap B) = P(A) \cdot P(B)$. Since $P(A) = 0.3$ and $P(B) = 0.4$, we have:

$$P(A \cap B) = 0.3 \times 0.4 = 0.12.$$

Answer: (B)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	A	5	C
6	B	7	A	8	B	9	B	10	A
11	C	12	C	13	B	14	B	15	C
16	A	17	B	18	A	19	B	20	D
21	C	22	B	23	A	24	B	25	B
26	B	27	D	28	A	29	B	30	A
31	C	32	B	33	B	34	A	35	A
36	A	37	A	38	A	39	B	40	C
41	B	42	B	43	B	44	C	45	A
46	A	47	C	48	A	49	C	50	B