CUET-UG Mathematics Test Sample Paper-20

Duration: 1 Hour | **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. Let R be a relation on the set $\mathbb{N}$ of natural numbers defined by nRm if n is a factor of m. Then R is:

(A) Reflexive and symmetric

(B) Transitive and symmetric

(C) Equivalence

(D) Reflexive, transitive but not symmetric

Q2. The principal value of $\sin^{-1}(\sin \frac{2\pi}{3})$ is:

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) $-\frac{\pi}{3}$

Q3. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A

(B) $I - A$

(C) I

(D) $3A$

Q4. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units, then k is:

(A) 12

(B) -2

(C) -12, -2

(D) 12, -2

Q5. The function $f(x) = |x| + |x - 1|$ is:

(A) Continuous at $x = 0$ and $x = 1$

(B) Differentiable at $x = 0$ and $x = 1$

(C) Not continuous at $x = 0$

(D) None of these

Q6. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is:

(A) 10π

(B) 12π

(C) 8π

(D) 11π

Q7. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}} dx$ is equal to:

(A) $\log|\tan x + \sqrt{\tan^2 x + 4}| + C$

(B) $\frac{1}{2}\log|\tan x + \sqrt{\tan^2 x + 4}| + C$

(C) $\tan^{-1}(\frac{\tan x}{2}) + C$

(D) None of these

Q8. The area bounded by the curve $y^2 = 4x$ and the line $x = 3$ is:

(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $16\sqrt{3}$

(D) $2\sqrt{3}$

Q9. The order and degree of the differential equation $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^{1/3} + x^{1/4} = 0$ are:

(A) 2, 3

(B) 3, 2

(C) 2, not defined

(D) 2, 1

Q10. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/2$

(D) $2\pi/3$

Q11. The distance between the planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is:

(A) 2 units

(B) $4/\sqrt{29}$ units

(C) $2/\sqrt{29}$ units

(D) 8 units

Q12. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.7

(B) 0.12

(C) 0.1

(D) 0.5

Q13. If A is a 3×3 matrix and $|A| = 5$, then $|adjA|$ is:

(A) 5

(B) 25

(C) 125

(D) 10

Q14. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $1/\sqrt{2}$

Q15. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 0

(D) π

Q16. In a Linear Programming Problem, the objective function is always:

(A) Cubic

(B) Quadratic

(C) Linear

(D) Constant

Q17. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One bag is selected at random and a ball is drawn. If the ball is red, the probability it came from the first bag is:

(A) 2/3

(B) 1/3

(C) 1/2

(D) 4/7

Q18. The vector equation of a line passing through $(1, 2, 3)$ and parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ is:

(A) $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

(B) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

(C) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} + \hat{\jmath} + \hat{k})$

(D) None of these

Q19. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then A^{-1} is:

(A) $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$

(D) Does not exist

Q20. For what value of k is $f(x) = \begin{cases} kx^2 & x \leq 2 \\ 3 & x > 2 \end{cases}$ continuous at $x = 2$?

(A) 3/4

(B) 4/3

(C) 3

(D) 2

Q21. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and the angle between them is 60°, then $|\vec{a} - \vec{b}|$ is:

(A) $\sqrt{13}$

(B) $\sqrt{37}$

(C) 5

(D) $\sqrt{7}$

Q22. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $yx = \frac{x^4}{4} + C$

(B) $y = \frac{x^3}{4} + C$

(C) $xy = \frac{x^3}{3} + C$

(D) $y = x^4 + Cx$

Q23. The shortest distance between the lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ is:

(A) 0 (Lines intersect)

(B) $1/\sqrt{3}$

(C) $10/\sqrt{59}$

(D) 9

Q24. The value of $\tan(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3})$ is:

(A) 6/17

(B) 17/6

(C) 7/16

(D) 16/7

Q25. If A is a symmetric matrix and B is a skew-symmetric matrix, then $AB - BA$ is:

(A) Symmetric

(B) Skew-symmetric

(C) Identity matrix

(D) Zero matrix

Q26. If $x = a\cos^3\theta, y = a\sin^3\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is:

(A) 1

(B) -1

(C) 0

(D) ∞

Q27. The value of $\int \frac{dx}{x(x^2+1)}$ is:

(A) $\log|x| - \frac{1}{2}\log(x^2+1) + C$

(B) $\log|x| + \log(x^2+1) + C$

(C) $\tan^{-1} x + C$

(D) $\frac{1}{2}\log\left|\frac{x^2}{x^2+1}\right| + C$

Q28. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is:

(A) 2 sq. units

(B) 1 sq. unit

(C) 4 sq. units

(D) 0 sq. units

Q29. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$, then $P(A|B)$ is:

(A) 0.3

(B) 0.5

(C) 0.6

(D) 0.5

Q30. The feasible region for an LPP is shown in a graph. If the corner points are $(0,0)$, $(5,0)$, $(3,4)$ and $(0,5)$, the maximum value of $Z = 4x + 3y$ is:

(A) 20

(B) 24

(C) 25

(D) 15

Q31. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 4$, then $f^{-1}(x)$ is:

(A) $\frac{x+4}{3}$

(B) $\frac{x-4}{3}$

(C) $3x + 4$

(D) None of these

Q32. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$ if the value of α is:

(A) $\pi/6$

(B) $\pi/3$

(C) π

(D) $3\pi/2$

Q33. The interval in which $y = x^2 e^{-x}$ is increasing is:

(A) $(-\infty, \infty)$

(B) $(-2, 0)$

(C) $(0, 2)$

(D) $(2, \infty)$

Q34. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) $(a + b + c)$

(B) 0

(C) $(a - b)(b - c)(c - a)$

(D) 1

Q35. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm/s. At the instant when the radius of the circular wave is 10cm, how fast is the enclosed area increasing?

(A) 80π cm^2/s

(B) 40π cm^2/s

(C) 100π cm^2/s

(D) 20π cm^2/s

Q36. $\int e^x(\frac{1}{x} - \frac{1}{x^2})dx$ is:

(A) $\frac{e^x}{x^2} + C$

(B) $e^x \log x + C$

(C) $\frac{e^x}{x} + C$

(D) $-\frac{e^x}{x} + C$

Q37. The integrating factor (I.F.) of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay$ is:

(A) $\frac{1}{y^2-1}$

(B) $\frac{1}{\sqrt{1-y^2}}$

(C) $\frac{1}{1-y^2}$

(D) $\sqrt{1 - y^2}$

Q38. The angle between the lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+1}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ is:

(A) $\cos^{-1}(\frac{26}{9\sqrt{38}})$

(B) $90°$

(C) $45°$

(D) $\cos^{-1}(\frac{19}{9\sqrt{38}})$

Q39. If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0

(B) $\pi/2$

(C) $\pi/4$

(D) π

Q40. If x, y, z are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is:

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) None of these

Q41. The derivative of $\log(\sin x)$ with respect to $\sqrt{x}$ is:

(A) $2\sqrt{x} \cot x$

(B) $\frac{\cot x}{2\sqrt{x}}$

(C) $\frac{2 \cot x}{\sqrt{x}}$

(D) $\sqrt{x} \cot x$

Q42. $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q43. If $P(A) = 4/5$ and $P(A \cap B) = 7/10$, then $P(B|A)$ is:

(A) 1/10

(B) 1/8

(C) 7/8

(D) 17/20

Q44. The objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region. Then:

(A) Max Z occurs at only these two points.

(B) Max Z occurs at every point on the line segment joining these two points.

(C) The problem has no solution.

(D) The maximum value is 0.

Q45. A function $f : X \to Y$ is onto if and only if:

(A) Range of $f = Y$

(B) Range of $f \subset Y$

(C) f is one-to-one

(D) $X = Y$

Q46. If A is a matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then the order of matrix B is:

(A) $m \times n$

(B) $n \times m$

(C) $n \times n$

(D) $m \times m$

Q47. The area of the region bounded by the circle $x^2 + y^2 = 4$ is:

(A) 2π

(B) 4π

(C) 8π

(D) 16π

Q48. Which of the following is a homogeneous differential equation?

(A) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(B) $(xy)dx - (x^3 + y^3)dy = 0$

(C) $(x^2 + y^2)dx + 2xydy = 0$

(D) $y^2dx + (x^2 - xy - y^2)dy = 0$

Q49. $\sin(\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2}))$ is equal to:

(A) 1/2

(B) 1/3

(C) 1/4

(D) 1

Q50. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$ is continuous at $x = 0$, then a is:

(A) 4

(B) 8

(C) 16

(D) 0

Detailed Solutions

Q1.

Solution

Concept: A relation R on a set A is:

- **Reflexive** if aRa for all $a \in A$.
- **Transitive** if aRb and $bRc \implies aRc$.
- **Symmetric** if $aRb \implies bRa$.

Solution: 1. **Reflexivity:** For any $n \in \mathbb{N}$, n is a factor of n ($n = n \times 1$). Thus, nRn is true for all $n \in \mathbb{N}$. R is reflexive.
2. **Transitivity:** Let nRm and mRp. This means n is a factor of m and m is a factor of p. * $m = n \cdot k_1$ and $p = m \cdot k_2$ for some $k_1, k_2 \in \mathbb{N}$. * Substituting m: $p = (n \cdot k_1) \cdot k_2 = n(k_1k_2)$. * Since $k_1k_2 \in \mathbb{N}$, n is a factor of p. Thus, nRp is true. R is transitive.
3. **Symmetry:** Let $n = 2$ and $m = 4$. 2 is a factor of 4 ($2R4$), but 4 is not a factor of 2 ($4R2$). Thus, nRm does not imply mRn. R is not symmetric.
Final Answer: Reflexive, transitive but not symmetric

Answer: (D)

Q2.

Solution

Concept: The principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The identity $\sin^{-1}(\sin\theta) = \theta$ is valid only when θ lies within this range.
Solution: 1. **Analyze the input:** The given angle is $\frac{2\pi}{3}$. Note that $\frac{2\pi}{3} > \frac{\pi}{2}$, so it lies outside the principal value branch.
2. **Use Trigonometric Identities:** We use the supplementary angle identity $\sin(\pi - \theta) = \sin\theta$:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

3. **Calculate Principal Value:**

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

Since $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

$$\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Final Answer: $\frac{\pi}{3}$

Answer: (B)

Q3.

Solution

Concept: For a square matrix A where $A^2 = A$ (idempotent matrix), any higher power A^n also equals A. Additionally, the Identity matrix I commutes with A ($IA = AI = A$), allowing the use of standard binomial expansion.

Solution: 1. **Expand the binomial term:** Using $(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$: Since $I^n = I$ and $IA = A$, we get:

$$(I + A)^3 = I + 3A + 3A^2 + A^3$$

2. **Substitute $A^2 = A$:** If $A^2 = A$, then $A^3 = A^2 \cdot A = A \cdot A = A^2 = A$. Replacing A^2 and A^3 with A:

$$(I + A)^3 = I + 3A + 3A + A$$

$$(I + A)^3 = I + 7A$$

3. **Final Calculation:** Now, substitute this back into the original expression:

$$(I + A)^3 - 7A = (I + 7A) - 7A = I$$

Final Answer: I

Answer: (C)

Q4.

Solution

Concept: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is calculated using the determinant formula:

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

Since area is a magnitude, we take ± of the given area value to solve for the unknown variable.
Solution: 1. **Substitute given values:**

$$35 = \frac{1}{2}\left|\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}\right|$$

$$70 = |2(4-4) - (-6)(5-k) + 1(20-4k)|$$

2. **Simplify the determinant:**

$$70 = |0 + 6(5-k) + 20 - 4k|$$

$$70 = |30 - 6k + 20 - 4k|$$

$$70 = |50 - 10k|$$

3. **Solving for k:** * **Positive case:** $50 - 10k = 70 \implies -10k = 20 \implies k = -2$ * **Negative case:** $50 - 10k = -70 \implies -10k = -120 \implies k = 12$
Final Answer: 12, -2

Answer: (D)

Q5.

Solution

Concept: The function $f(x) = |x - a|$ is continuous everywhere on $\mathbb{R}$, but it is not differentiable at $x = a$ because the graph has a sharp corner (v-shape) at that point. The sum of two continuous functions is always continuous.

Solution: 1. **Continuity:** The function $f(x) = |x| + |x - 1|$ is the sum of two modulus functions. * $|x|$ is continuous for all $x \in \mathbb{R}$. * $|x - 1|$ is continuous for all $x \in \mathbb{R}$. Thus, $f(x)$ is continuous at $x = 0$ and $x = 1$.

2. **Differentiability:** * At $x = 0$, the term $|x|$ is not differentiable, while $|x - 1|$ is differentiable. Therefore, their sum is not differentiable at $x = 0$. * At $x = 1$, the term $|x - 1|$ is not differentiable, while $|x|$ is differentiable. Therefore, their sum is not differentiable at $x = 1$.

3. **Verification:** We can redefine the function as:

$$f(x) = \begin{cases} -2x + 1 & x < 0 \\ 1 & 0 \le x < 1 \\ 2x - 1 & x \ge 1 \end{cases}$$

$\lim_{x \to 0^-} f(x) = 1$ and $\lim_{x \to 0^+} f(x) = 1$. Similarly for $x = 1$. The function is continuous at both points.

Final Answer: Continuous at $x = 0$ and $x = 1$

Answer: (A)

Q6.

Solution

Concept: The rate of change of a quantity y with respect to x is given by the derivative $\frac{dy}{dx}$. For a circle, the area A is a function of its radius r.

Solution: 1. **Formula:** The area of a circle is $A = \pi r^2$.

2. **Differentiation:** Differentiating A with respect to r:

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2)$$

$$\frac{dA}{dr} = 2\pi r$$

3. **Substitution:** At $r = 6$ cm:

$$\frac{dA}{dr} = 2\pi(6)$$

$$\frac{dA}{dr} = 12\pi$$

Final Answer: 12π

Answer: (B)

Q7.

Solution

Concept: This integral can be solved using the substitution method and the standard integration formula:

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log|x + \sqrt{x^2 + a^2}| + C$$

Solution: 1. **Substitution:** Let $t = \tan x$. Then, $\frac{dt}{dx} = \sec^2 x \implies dt = \sec^2 x\, dx$.
2. **Transform the integral:** Substituting these into the given integral:

$$I = \int \frac{dt}{\sqrt{t^2 + 4}} = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

3. **Applying the formula:** Comparing with the standard form $\int \frac{dt}{\sqrt{t^2+a^2}}$ where $a = 2$:

$$I = \log|t + \sqrt{t^2 + 4}| + C$$

4. **Resubstituting $t = \tan x$:**

$$I = \log|\tan x + \sqrt{\tan^2 x + 4}| + C$$

Final Answer: $\log|\tan x + \sqrt{\tan^2 x + 4}| + C$

Answer: (A)

Q8.

Solution

Concept: The area bounded by the curve $y^2 = 4ax$ and the line $x = h$ is symmetric about the x-axis. The total area is calculated as:

$$\text{Area} = 2\int_0^h y\,dx$$

Solution: 1. **Identify the curve and limits:** The curve is $y^2 = 4x$, which gives $y = 2\sqrt{x}$ for the upper branch. The vertical line is $x = 3$. The region lies between $x = 0$ and $x = 3$.
2. **Set up the integral:** Since the parabola is symmetric about the x-axis:

$$\text{Total Area} = 2\int_0^3 2\sqrt{x}\,dx$$

$$\text{Total Area} = 4\int_0^3 x^{1/2}\,dx$$

3. **Integration:**

$$\text{Total Area} = 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = 4\cdot\frac{2}{3}\left[x^{3/2}\right]_0^3$$

$$\text{Total Area} = \frac{8}{3}[3^{3/2} - 0]$$

$$\text{Total Area} = \frac{8}{3}[3\sqrt{3}] = 8\sqrt{3}$$

Final Answer: $8\sqrt{3}$

Answer: (B)

Q9.

Solution

Concept: The **order** of a differential equation is the order of the highest order derivative appearing in the equation. The **degree** is the power of the highest order derivative when the equation is expressed as a polynomial in derivatives (i.e., after removing radicals and fractional powers from the derivatives).

Solution: 1. **Identify Order:** In the equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$, the highest derivative is $\frac{d^2y}{dx^2}$. Thus, **Order = 2**.

2. **Identify Degree:** The term $\left(\frac{dy}{dx}\right)^{1/3}$ has a fractional power. To find the degree, we rewrite the equation:

$$\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3}$$

Cubing both sides to rationalize the derivative:

$$\left(\frac{d^2y}{dx^2} + x^{1/4}\right)^3 = -\frac{dy}{dx}$$

In the polynomial expansion, the term involving the highest order derivative is $\left(\frac{d^2y}{dx^2}\right)^3$. Thus, **Degree = 3**.

Final Answer: 2, 3

Answer: (A)

Q10.

Solution

Concept: For any two vectors $\vec{a}$ and $\vec{b}$, the magnitude of their sum is given by:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

where θ is the angle between the vectors.

Solution: 1. **Given Data:** * $\vec{a}$ and $\vec{b}$ are unit vectors: $|\vec{a}| = 1, |\vec{b}| = 1$. * $\vec{a}+\vec{b}$ is a unit vector: $|\vec{a}+\vec{b}| = 1$.

2. **Apply the formula:**

$$1^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta$$

$$1 = 2 + 2\cos\theta$$

3. **Solve for θ:**

$$-1 = 2\cos\theta \implies \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Final Answer: $2\pi/3$

Answer: (D)

Q11.

Solution

Concept: The distance d between two parallel planes $Ax+By+Cz = D_1$ and $Ax+By+Cz = D_2$ is:

$$d = \frac{|D_2 - D_1|}{\sqrt{A^2+B^2+C^2}}$$

Solution: 1. **Identify the planes:** * Plane 1: $2x+3y+4z = 4$ * Plane 2: $4x+6y+8z = 12 \implies 2x+3y+4z = 6$ (dividing by 2)

2. **Compare coefficients:** $A = 2, B = 3, C = 4$. $D_1 = 4$ and $D_2 = 6$.

3. **Calculate distance:**

$$d = \frac{|6-4|}{\sqrt{2^2+3^2+4^2}} = \frac{2}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}}$$

Final Answer: $2/\sqrt{29}$ units

Answer: (C)

Q12.

Solution

Concept: Two events A and B are said to be independent if and only if:

$$P(A \cap B) = P(A) \times P(B)$$

Solution: 1. **Given Data:** * $P(A) = 0.3$ * $P(B) = 0.4$
2. **Calculate Intersection:** Since A and B are independent:

$$P(A \cap B) = 0.3 \times 0.4 = 0.12$$

Final Answer: 0.12

Answer: (B)

Q13.

Solution

Concept: For a square matrix A of order n, the determinant of its adjoint is given by:

$$|\text{adj } A| = |A|^{n-1}$$

Solution: 1. **Given Data:** * Order $n = 3$ * $|A| = 5$
2. **Apply Property:**

$$|\text{adj } A| = |A|^{3-1}$$

$$|\text{adj } A| = 5^2 = 25$$

Final Answer: 25

Answer: (B)

Q14.

Solution

Concept: A function of the form $f(x) = a\sin x + b\cos x$ has a maximum value given by:

$$\text{Max Value} = \sqrt{a^2 + b^2}$$

Alternatively, this can be solved by finding the derivative and setting it to zero.

Solution: 1. **Identify coefficients:** For $f(x) = \sin x + \cos x$, we have $a = 1$ and $b = 1$.
2. **Apply formula:**

$$\text{Maximum Value} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

3. **Verification using derivatives:** $f'(x) = \cos x - \sin x$. Setting $f'(x) = 0 \implies \tan x = 1 \implies x = \pi/4$. $f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

Final Answer: $\sqrt{2}$

Answer: (C)

Q15.

Solution

Concept: We use the definite integral property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. This is often referred to as the "King's Property".

Solution: 1. **Let the integral be I:**

$$I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx \quad \ldots(1)$$

2. **Apply property $\int_0^{\pi/2} f(\frac{\pi}{2} - x)\,dx$:** Since $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}\,dx \quad \ldots(2)$$

3. **Add (1) and (2):**

$$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\pi/4$

Answer: (B)

Q16.

Solution

Concept: In Linear Programming, the term "Linear" implies that all mathematical relationships (both the objective function and the constraints) are represented by linear equations or inequalities.
Solution: 1. **Definition:** An objective function in LPP is a function $Z = ax + by$ which is to be maximized or minimized. 2. **Nature of the function:** Since x and y are raised to the power of 1 and are not multiplied together, the function is strictly linear. 3. **Conclusion:** It cannot be quadratic, cubic, or constant in a standard LPP framework.
Final Answer: Linear

Answer: (C)

Q17.

Solution

Concept: Bayes' Theorem states: $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$
Solution: 1. **Events:** E_1: Selecting Bag 1, E_2: Selecting Bag 2. $P(E_1) = P(E_2) = 1/2$. A: Drawing a red ball.
2. **Conditional Probabilities:** $P(A|E_1) = \frac{4}{8} = \frac{1}{2}$ $P(A|E_2) = \frac{2}{8} = \frac{1}{4}$
3. **Applying Bayes' Theorem:**

$$P(E_1|A) = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\left(\frac{1}{2} \cdot \frac{1}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{4}\right)} = \frac{1/4}{1/4 + 1/8} = \frac{1/4}{3/8} = \frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3}$$

Final Answer: 2/3

Answer: (A)

Q18.

Solution

Concept: The vector equation of a straight line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is represented as:

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

where λ is a scalar constant.

Solution: 1. **Determine the position vector ($\vec{a}$):** The line passes through the point $(1, 2, 3)$. Therefore, the position vector of the point is:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

2. **Determine the direction vector ($\vec{b}$):** The line is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. Thus:

$$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$$

3. **Construct the equation:** Substituting these into the formula $\vec{r} = \vec{a} + \lambda\vec{b}$:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

Final Answer: $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

Answer: (B)

Q19.

Solution

Concept: For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse exists if $|A| = ad - bc \neq 0$ and is given by:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solution: 1. **Find the determinant:**

$$|A| = (1)(4) - (2)(3) = 4 - 6 = -2$$

2. **Calculate the inverse:**

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

3. **Simplify the matrix elements:**

$$A^{-1} = \begin{bmatrix} \frac{4}{-2} & \frac{-2}{-2} \\ \frac{-3}{-2} & \frac{1}{-2} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

Final Answer: $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

Answer: (A)

Q20.

Solution

Concept: A piecewise function $f(x)$ is continuous at a point $x = a$ if the left-hand limit, the right-hand limit, and the function value at that point are equal:

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

Solution: 1. **Find the limit from the left and $f(2)$:** For $x \le 2$, $f(x) = kx^2$.

$$\lim_{x\to 2^-} f(x) = k(2)^2 = 4k$$

2. **Find the limit from the right:** For $x > 2$, $f(x) = 3$.

$$\lim_{x\to 2^+} f(x) = 3$$

3. **Equate the limits for continuity:** Set the left-hand limit equal to the right-hand limit:

$$4k = 3$$

$$k = \frac{3}{4}$$

Final Answer: 3/4

Answer: (A)

Q21.

Solution

Concept: The magnitude of the difference of two vectors $\vec{a}$ and $\vec{b}$ is given by the formula:

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

where θ is the angle between the two vectors.
Solution: 1. **Given Data:** * $|\vec{a}| = 3$ * $|\vec{b}| = 4$ * $\theta = 60^\circ$
2. **Substitute into the formula:**

$$|\vec{a} - \vec{b}|^2 = (3)^2 + (4)^2 - 2(3)(4)\cos 60^\circ$$

$$|\vec{a} - \vec{b}|^2 = 9 + 16 - 24\left(\frac{1}{2}\right)$$

3. **Calculate:**

$$|\vec{a} - \vec{b}|^2 = 25 - 12 = 13$$

$$|\vec{a} - \vec{b}| = \sqrt{13}$$

Final Answer: $\sqrt{13}$

Answer: (A)

Q22.

Solution

Concept: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$. The general solution is found using the integrating factor (I.F.):

$$\text{I.F.} = e^{\int P\,dx}$$

The solution is given by: $y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + C$.
Solution: 1. **Identify P and Q:** Comparing $\frac{dy}{dx} + \frac{1}{x}y = x^2$ with the standard form: $P = \frac{1}{x}$ and $Q = x^2$.
2. **Calculate I.F.:**

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

3. **Solve for y:**

$$y(x) = \int (x^2 \cdot x)\,dx + C$$

$$yx = \int x^3\,dx + C$$

$$yx = \frac{x^4}{4} + C$$

Final Answer: $yx = \frac{x^4}{4} + C$

Answer: (A)

Q23.

Solution

Concept: The shortest distance between two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is:

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Solution: 1. **Identify parameters:** Line 1: $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$ Line 2: $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$
2. **Vector subtractions and products:** $\vec{a}_2 - \vec{a}_1 = 0\hat{i} + \hat{j} - 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k}$ $|\vec{b}_1 \times \vec{b}_2| = \sqrt{4+16+9} = \sqrt{29}$
3. **Compute distance:** Numerator: $|(0)(2) + (1)(-4) + (-4)(-3)| = |-4+12| = 8$ $d = \frac{8}{\sqrt{29}}$
Final Answer: $8/\sqrt{29}$

Answer: (A)

Q24.

Solution

Concept: To evaluate $\tan(A+B)$, we use the identity $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$. We can convert $\cos^{-1} x$ to $\tan^{-1} y$ using a right-angled triangle relationship.

Solution: 1. **Convert $\cos^{-1}$ to $\tan^{-1}$:** Let $A = \cos^{-1}\frac{4}{5} \implies \cos A = \frac{4}{5}$. Using the identity $\tan A = \frac{\sqrt{1-\cos^2 A}}{\cos A}$:

$$\tan A = \frac{\sqrt{1-(4/5)^2}}{4/5} = \frac{\sqrt{9/25}}{4/5} = \frac{3/5}{4/5} = \frac{3}{4}$$

So, $A = \tan^{-1}\frac{3}{4}$.

2. **Apply the tangent sum formula:** Let $B = \tan^{-1}\frac{2}{3}$, so $\tan B = \frac{2}{3}$.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\frac{3}{4} + \frac{2}{3}}{1 - (\frac{3}{4}\cdot\frac{2}{3})}$$

3. **Simplify:**

$$\tan(A+B) = \frac{\frac{9+8}{12}}{1 - \frac{6}{12}} = \frac{\frac{17}{12}}{\frac{6}{12}} = \frac{17}{6}$$

Final Answer: 17/6

Answer: (B)

Q25.

Solution

Concept: A matrix M is **symmetric** if $M^T = M$ and **skew-symmetric** if $M^T = -M$. We use the transpose property $(AB)^T = B^T A^T$ and $(A \pm B)^T = A^T \pm B^T$.

Solution: 1. **Given Data:** $A^T = A$ and $B^T = -B$. 2. **Find the transpose of $AB - BA$:**

$$(AB - BA)^T = (AB)^T - (BA)^T$$

$$(AB - BA)^T = B^T A^T - A^T B^T$$

3. **Substitute given properties:**

$$(AB - BA)^T = (-B)(A) - (A)(-B)$$

$$(AB - BA)^T = -BA + AB = AB - BA$$

4. **Conclusion:** Since the transpose of the matrix is equal to the matrix itself, $(AB - BA)$ is symmetric.

Final Answer: Symmetric

Answer: (A)

Q26.

Solution

Concept: For parametric equations $x = f(\theta)$ and $y = g(\theta)$, the derivative is given by:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Solution: 1. **Differentiate x with respect to θ:** $x = a\cos^3\theta \implies \frac{dx}{d\theta} = a \cdot 3\cos^2\theta \cdot (-\sin\theta) = -3a\cos^2\theta\sin\theta$

2. **Differentiate y with respect to θ:** $y = a\sin^3\theta \implies \frac{dy}{d\theta} = a \cdot 3\sin^2\theta \cdot (\cos\theta) = 3a\sin^2\theta\cos\theta$

3. **Find $\frac{dy}{dx}$:**

$$\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

4. **Substitute $\theta = \frac{\pi}{4}$:**

$$\frac{dy}{dx} = -\tan\left(\frac{\pi}{4}\right) = -1$$

Final Answer: -1

Answer: (B)

Q27.

Solution

Concept: To integrate a rational function where the denominator can be factored, we use the method of **Partial Fractions**. For the term $x(x^2 + 1)$, the decomposition is:

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

Solution: 1. **Partial Fraction Decomposition:**

$$1 = A(x^2 + 1) + (Bx + C)x$$

Setting $x = 0$: $1 = A(1) \implies A = 1$. Comparing coefficients of x^2: $0 = A + B \implies B = -1$. Comparing coefficients of x: $0 = C \implies C = 0$. Thus, $\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$.
2. **Integration:**

$$\int \frac{dx}{x(x^2+1)} = \int \frac{1}{x}dx - \int \frac{x}{x^2+1}dx$$

The first part is $\log|x|$. For the second part, use substitution $u = x^2 + 1$, $du = 2x\,dx$:

$$\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\log(x^2+1)$$

3. **Final Result:**

$$I = \log|x| - \frac{1}{2}\log(x^2+1) + C$$

Final Answer: $\log|x| - \frac{1}{2}\log(x^2+1) + C$

Answer: (A)

Q28.

Solution

Concept: The area bounded by a curve $y = f(x)$ and the x-axis between $x = a$ and $x = b$ is given by $\int_a^b |f(x)|dx$. Since $\cos x$ changes sign at $x = \pi/2$, we must split the integral to ensure the area remains positive.

Solution: 1. **Identify the intervals:** * On $[0, \pi/2]$, $\cos x \geq 0$. * On $[\pi/2, \pi]$, $\cos x \leq 0$.
2. **Set up the Area integral:**

$$\text{Area} = \int_0^{\pi/2} \cos x \, dx + \left| \int_{\pi/2}^{\pi} \cos x \, dx \right|$$

3. **Evaluate the integrals:** * $\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = \sin(\pi/2) - \sin(0) = 1 - 0 = 1$ * $\int_{\pi/2}^{\pi} \cos x \, dx = [\sin x]_{\pi/2}^{\pi} = \sin(\pi) - \sin(\pi/2) = 0 - 1 = -1$
4. **Calculate Total Area:**

$$\text{Area} = 1 + |-1| = 1 + 1 = 2 \text{ sq. units}$$

Final Answer: 2 sq. units

Answer: (A)

Q29.

Solution

Concept: To find the conditional probability $P(A|B)$, we use the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The intersection $P(A \cap B)$ is found using the Addition Theorem: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Solution: 1. **Find $P(A \cap B)$:**

$$0.8 = 0.5 + 0.6 - P(A \cap B)$$

$$0.8 = 1.1 - P(A \cap B)$$

$$P(A \cap B) = 1.1 - 0.8 = 0.3$$

2. **Calculate $P(A|B)$:**

$$P(A|B) = \frac{0.3}{0.6} = \frac{1}{2} = 0.5$$

Final Answer: 0.5

Answer: (B)

Q30.

Solution

Concept: According to the **Fundamental Theorem of Linear Programming**, the optimal value (maximum or minimum) of the objective function Z occurs at one of the corner points (vertices) of the feasible region.

Solution: Evaluate the objective function $Z = 4x + 3y$ at each given corner point:

1. **At (0, 0):** $Z = 4(0) + 3(0) = 0$ 2. **At (5, 0):** $Z = 4(5) + 3(0) = 20$ 3. **At (3, 4):** $Z = 4(3) + 3(4) = 12 + 12 = 24$ 4. **At (0, 5):** $Z = 4(0) + 3(5) = 15$

Comparing the values $\{0, 20, 24, 15\}$, the maximum value is 24.

Final Answer: 24

Answer: (B)

Q31.

Solution

Concept: To find the inverse of a function $f(x) = y$, we express x in terms of y. If $y = f(x)$, then $x = f^{-1}(y)$. Finally, we replace y with x to get the expression for $f^{-1}(x)$.

Solution: 1. **Set the function equal to y:**

$$y = 3x - 4$$

2. **Solve for x:**

$$y + 4 = 3x$$

$$x = \frac{y + 4}{3}$$

3. **Replace y with x to find the inverse function:**

$$f^{-1}(x) = \frac{x + 4}{3}$$

Final Answer: $\frac{x+4}{3}$

Answer: (A)

Q32.

Solution

Concept: The transpose A' is obtained by interchanging rows and columns. The identity matrix I for a 2×2 system is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Solution: 1. **Find A':** If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$.
2. **Set up the equation $A + A' = I$:**

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. **Solve for α:** Equating corresponding elements:

$$2\cos\alpha = 1 \implies \cos\alpha = \frac{1}{2}$$

The principal value of α is $\frac{\pi}{3}$.
Final Answer: $\pi/3$

Answer: (B)

Q33.

Solution

Concept: A function $y = f(x)$ is increasing if $f'(x) > 0$. We use the product rule for differentiation: $\frac{d}{dx}(uv) = u'v + uv'$.
Solution: 1. **Differentiate the function:** $y = x^2e^{-x}$ $\frac{dy}{dx} = (2x)e^{-x} + x^2(-e^{-x})$ $\frac{dy}{dx} = e^{-x}(2x - x^2) = xe^{-x}(2 - x)$
2. **Set the derivative greater than zero:** For the function to be increasing: $xe^{-x}(2 - x) > 0$. Since e^{-x} is always positive for all real x, we only need to consider: $x(2 - x) > 0$
3. **Find the interval:** The roots of $x(2 - x) = 0$ are $x = 0$ and $x = 2$. Testing intervals: - For $x < 0$: $(-)(+) =$ negative - For $0 < x < 2$: $(+)(+) =$ positive - For $x > 2$: $(+)(-) =$ negative Thus, the function is increasing in the interval $(0, 2)$.
Final Answer: $(0, 2)$

Answer: (C)

Q34.

Solution

Concept: If any two rows or columns of a determinant are identical or proportional, the value of the determinant is zero. We can use the operation $C_3 \to C_3 + C_2$ to simplify the expression.

Solution: 1. **Apply Column Operation:** Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$. Perform $C_3 \to C_3 + C_2$:

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

2. **Take out the common factor:** Taking $(a+b+c)$ common from C_3:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

3. **Check for identical columns:** Now, C_1 and C_3 are identical. When two columns are identical, the determinant value is 0. $\Delta = (a+b+c) \times 0 = 0$.

Final Answer: 0

Answer: (B)

Q35.

Solution

Concept: The area A of a circle is $A = \pi r^2$. Using the chain rule, the rate of change of area with respect to time t is:

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$$

Solution: 1. **Given Data:** * Speed of the wave (rate of change of radius): $\frac{dr}{dt} = 4$ cm/s * Radius at the specific instant: $r = 10$ cm
2. **Substitute into the rate formula:**

$$\frac{dA}{dt} = 2\pi(10)(4)$$

$$\frac{dA}{dt} = 80\pi \text{ cm}^2/\text{s}$$

Final Answer: $80\pi \ cm^2/s$

Answer: (A)

Q36.

Solution

Concept: The integral of the form $\int e^x [f(x) + f'(x)]\, dx$ is always:

$$e^x f(x) + C$$

Solution: 1. **Identify $f(x)$ and $f'(x)$:** The given integral is $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$. Let $f(x) = \frac{1}{x}$.
2. **Verify the derivative:** Using the power rule: $f'(x) = \frac{d}{dx}(x^{-1}) = -1x^{-2} = -\frac{1}{x^2}$. The expression matches the form $f(x) + f'(x)$.
3. **Apply the formula:**

$$\int e^x \left(\frac{1}{x} + \left(-\frac{1}{x^2}\right)\right) dx = e^x \left(\frac{1}{x}\right) + C$$

Final Answer: $\frac{e^x}{x} + C$

Answer: (C)

Q37.

Solution

Concept: For a differential equation of the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only, the Integrating Factor is:

$$\text{I.F.} = e^{\int P\, dy}$$

Solution: 1. **Rearrange to standard form:** Divide the entire equation $(1 - y^2)\frac{dx}{dy} + yx = ay$ by $(1 - y^2)$:

$$\frac{dx}{dy} + \left(\frac{y}{1 - y^2}\right) x = \frac{ay}{1 - y^2}$$

Here, $P = \frac{y}{1-y^2}$.
2. **Calculate the Integral of P:**

$$\int P\, dy = \int \frac{y}{1 - y^2}\, dy$$

Let $u = 1 - y^2 \implies du = -2y\, dy \implies y\, dy = -\frac{1}{2}du$.

$$\int P\, dy = -\frac{1}{2} \int \frac{1}{u}\, du = -\frac{1}{2} \log(1 - y^2) = \log(1 - y^2)^{-1/2}$$

3. **Find I.F.:**

$$\text{I.F.} = e^{\log(1-y^2)^{-1/2}} = (1 - y^2)^{-1/2} = \frac{1}{\sqrt{1 - y^2}}$$

Final Answer: $\frac{1}{\sqrt{1-y^2}}$

Answer: (B)

Q38.

Solution

Concept: If two lines have direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2), the angle θ between them is:

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Solution: 1. **Identify direction ratios:** Line 1: $a_1 = 2, b_1 = 5, c_1 = -3$ Line 2: $a_2 = -1, b_2 = 8, c_2 = 4$
2. **Calculate the numerator (dot product):**

$$|(2)(-1) + (5)(8) + (-3)(4)| = |-2 + 40 - 12| = 26$$

3. **Calculate the denominators (magnitudes):** $\sqrt{2^2 + 5^2 + (-3)^2} = \sqrt{4 + 25 + 9} = \sqrt{38}$ $\sqrt{(-1)^2 + 8^2 + 4^2} = \sqrt{1 + 64 + 16} = \sqrt{81} = 9$
4. **Find θ:**

$$\cos\theta = \frac{26}{9\sqrt{38}} \implies \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Final Answer: $\cos^{-1}(\frac{26}{9\sqrt{38}})$

Answer: (A)

Q39.

Solution

Concept: For any two vectors $\vec{a}$ and $\vec{b}$ with an angle θ between them: 1. The magnitude of the cross product is $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$. 2. The dot product is $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.
Solution: 1. **Set up the equation:** Given $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, we substitute the formulas:

$$|\vec{a}||\vec{b}| \sin\theta = |\vec{a}||\vec{b}| \cos\theta$$

2. **Simplify the expression:** Assuming $\vec{a}$ and $\vec{b}$ are non-zero vectors ($|\vec{a}|, |\vec{b}| \neq 0$):

$$\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \implies \tan\theta = 1$$

3. **Find the angle:** The value of θ for which $\tan\theta = 1$ is:

$$\theta = \frac{\pi}{4}$$

Final Answer: $\pi/4$

Answer: (C)

Q40.

Solution

Concept: For a diagonal matrix A where all diagonal elements are non-zero, the inverse A^{-1} is also a diagonal matrix where each diagonal element is the reciprocal of the corresponding element in A. Specifically, if $A = \text{diag}(d_1, d_2, d_3)$, then $A^{-1} = \text{diag}(1/d_1, 1/d_2, 1/d_3)$.

Solution: 1. **Identify the matrix type:** The matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a diagonal matrix.

2. **Apply the inverse property:** Given x, y, z are non-zero, the inverse is:

$$A^{-1} = \begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/y & 0 \\ 0 & 0 & 1/z \end{bmatrix}$$

3. **Match with options:** The result can be written as:

$$A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

Final Answer: $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

Answer: (A)

Q41.

Solution

Concept: The derivative of $f(x)$ with respect to $g(x)$ is calculated as:

$$\frac{d(f(x))}{d(g(x))} = \frac{f'(x)}{g'(x)}$$

Solution: 1. **Let $u = \log(\sin x)$ and $v = \sqrt{x}$:**
2. **Differentiate u with respect to x:**

$$\frac{du}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$$

3. **Differentiate v with respect to x:**

$$\frac{dv}{dx} = \frac{1}{2\sqrt{x}}$$

4. **Calculate the final derivative:**

$$\frac{du}{dv} = \frac{\cot x}{\frac{1}{2\sqrt{x}}} = 2\sqrt{x}\cot x$$

Final Answer: $2\sqrt{x}\cot x$

Answer: (A)

Q42.

Solution

Concept: The basic integration formula for this rational function is:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

Solution: 1. **Evaluate the definite integral:**

$$\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]_0^1$$

2. **Substitute the limits:**

$$\tan^{-1}(1) - \tan^{-1}(0)$$

3. **Calculate the values:** Since $\tan(\pi/4) = 1$ and $\tan(0) = 0$:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: $\pi/4$

Answer: (B)

Q43.

Solution

Concept: The formula for conditional probability is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Solution: 1. **Given Data:** * $P(A) = 4/5$ * $P(A \cap B) = 7/10$
2. **Substitute into the formula:**

$$P(B|A) = \frac{7/10}{4/5}$$

3. **Simplify the fraction:**

$$P(B|A) = \frac{7}{10} \times \frac{5}{4} = \frac{7}{2 \times 4} = \frac{7}{8}$$

Final Answer: 7/8

Answer: (C)

Q44.

Solution

Concept: In an LPP, if the objective function $Z = ax + by$ assumes the same maximum (or minimum) value at two distinct corner points of the feasible region, then the function will yield that same maximum value at every point on the line segment connecting those two corner points. This is known as having **infinitely many optimal solutions**.

Solution: 1. Let the two corner points be P_1 and P_2. 2. If $Z(P_1) = Z(P_2) = M$ (where M is the maximum value), any point P on the segment P_1P_2 can be represented as a convex combination: $P = \lambda P_1 + (1 - \lambda)P_2$ for $0 \leq \lambda \leq 1$. 3. Because Z is a linear function: $Z(P) = \lambda Z(P_1) + (1 - \lambda)Z(P_2) = \lambda M + (1 - \lambda)M = M$. 4. Therefore, every point on the line segment joining these two points is an optimal solution.

Final Answer: Max Z occurs at every point on the line segment joining these two points.

Answer: (B)

Q45.

Solution

Concept: A function $f : X \to Y$ is called **onto** (or **surjective**) if every element in the codomain Y is the image of at least one element in the domain X. In terms of sets, this means the actual set of outputs (the Range) must match the target set provided (the Codomain).

Solution: 1. **Definition of Range:** The Range is the set $\{f(x) : x \in X\}$. 2. **Definition of Codomain:** The Codomain is the set Y specified in the function mapping. 3. **Surjectivity Condition:** For f to be onto, for every $y \in Y$, there exists an $x \in X$ such that $f(x) = y$. 4. **Conclusion:** This condition is satisfied if and only if the Range of f is equal to Y.

Final Answer: Range of $f = Y$

Answer: (A)

Q46.

Solution

Concept: Let the order of matrix A be $m \times n$. Let the order of matrix B be $p \times q$. The order of the transpose B' is $q \times p$. Multiplication XY is defined if (columns of X) = (rows of Y).

Solution: 1. **Analyze AB':** A is $(m \times n)$ and B' is $(q \times p)$. For AB' to be defined: $n = q$.
2. **Analyze $B'A$:** B' is $(q \times p)$ and A is $(m \times n)$. For $B'A$ to be defined: $p = m$.
3. **Determine order of B:** Since B is $(p \times q)$, and we found $p = m$ and $q = n$: The order of B must be $m \times n$.

Final Answer: $m \times n$

Answer: (A)

Q47.

Solution

Concept: The equation $x^2 + y^2 = r^2$ represents a circle centered at the origin with radius r. The area of a circle is given by:

$$\text{Area} = \pi r^2$$

Solution: 1. **Identify the radius:** The given equation is $x^2 + y^2 = 4$. Comparing this with $x^2 + y^2 = r^2$, we get $r^2 = 4$, which means $r = 2$.
2. **Calculate the area:**

$$\text{Area} = \pi(2)^2 = 4\pi \text{ sq. units}$$

3. **Using Integration (Verification):** The area in the first quadrant is $\int_0^2 \sqrt{4 - x^2}\, dx$. Using the formula $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$:

$$\text{Area} = 4 \times \left[\frac{x}{2}\sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2}\right]_0^2 = 4 \times (2 \cdot \frac{\pi}{2}) = 4\pi$$

Final Answer: 4π

Answer: (B)

Q48.

Solution

Concept: A function $f(x, y)$ is homogeneous of degree n if $f(\lambda x, \lambda y) = \lambda^n f(x, y)$. In a homogeneous differential equation, every term in the coefficients of dx and dy must have the same total degree.

Solution: 1. **Option A:** $(4x + 6y + 5)$ and $(3y + 2x + 4)$. The constants (5 and 4) have degree 0, while x and y have degree 1. **Not homogeneous.** 2. **Option B:** (xy) has degree 2, but $(x^3 + y^3)$ has degree 3. **Not homogeneous.** 3. **Option C:** $(x^2 + y^2)$ has degree 2. $(2xy)$ also has degree 2 $(1 + 1 = 2)$. **Homogeneous.** 4. **Option D:** (y^2) has degree 2. $(x^2 - xy - y^2)$ has degree 2. **Homogeneous.**

Note: Both C and D are technically homogeneous. However, in many standard curricula, Option C is the textbook example of a homogeneous equation where M and N are clearly polynomials of the same degree.

Final Answer: $(\mathrm{x}^2 + y^2)dx + 2xydy = 0$

Answer: (C)

Q49.

Solution

Concept: The principal value branch of $\sin^{-1} x$ is $[-\pi/2, \pi/2]$. We use the property $\sin^{-1}(-x) = -\sin^{-1} x$.

Solution: 1. **Evaluate the inner term:** $\sin^{-1}(-1/2) = -\sin^{-1}(1/2) = -\frac{\pi}{6}$

2. **Substitute into the expression:**

$$\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right)$$

$$\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{2\pi + \pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right)$$

3. **Calculate the final value:** $\sin(\pi/2) = 1$

Final Answer: 1

Answer: (D)

Q50.

Solution

Concept: $f(0) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$. We use the standard limit $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$ and rationalization for the RHL.

Solution: 1. **Find LHL ($x < 0$):**

$$\lim_{x\to 0^-} \frac{1-\cos 4x}{x^2} = \lim_{x\to 0^-} \frac{2\sin^2 2x}{x^2}$$

Multiply and divide by 4:

$$\lim_{x\to 0^-} \frac{2\sin^2 2x}{(2x)^2} \cdot 4 = 2 \cdot (1)^2 \cdot 4 = 8$$

2. **Find RHL ($x > 0$):** Rationalize the denominator of $\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$:

$$\lim_{x\to 0^+} \frac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{(16+\sqrt{x})-16} = \lim_{x\to 0^+} \frac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{\sqrt{x}}$$

$$\lim_{x\to 0^+} (\sqrt{16+\sqrt{x}}+4) = \sqrt{16+0}+4 = 4+4 = 8$$

3. **Determine a:** Since LHL = RHL = 8, for continuity at $x = 0$, $f(0)$ must be 8.

$$a = 8$$

Final Answer: 8

Answer: (B)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	B	3	C	4	D	5	A
6	B	7	A	8	B	9	A	10	D
11	C	12	B	13	B	14	C	15	B
16	C	17	A	18	B	19	A	20	A
21	A	22	A	23	A	24	B	25	A
26	B	27	A	28	A	29	B	30	B
31	A	32	B	33	C	34	B	35	A
36	C	37	B	38	A	39	C	40	A
41	A	42	B	43	C	44	B	45	A
46	A	47	B	48	C	49	D	50	B