CUET-UG Mathematics Sample Paper-2

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix of order 3 such that $A(\text{adj}A) = 10I$, then $|\text{adj}A|$ is equal to:

(A) 10

(B) 100

(C) 1000

(D) 12

Q2. The system of equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if:

(A) $k = 0$

(B) $k \neq 0$

(C) $k = 2$

(D) $k \neq 2$

Q3. If A and B are non-singular square matrices of the same order, then $\text{adj}(AB)$ is:

(A) $(\text{adj}A)(\text{adj}B)$

(B) $(\text{adj}B)(\text{adj}A)$

(C) $|A||B|(AB)^{-1}$

(D) Both B and C

Q4. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|\text{adj} A^3| = 125$, then the value of α is:

(A) ± 3

(B) ± 2

(C) ± 5

(D) 0

Q5. If $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$, then $f'(x)$ at $x = 0$ is:

(A) -20

(B) -30

(C) 0

(D) -40

Q6. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to:

(A) A

(B) $-I$

(C) I

(D) 0

Q7. Total number of possible square matrices of order 3 with each entry as $1, -1$ or 0 is:

(A) 3^3

(B) 3^6

(C) 3^9

(D) 2^9

Q8. For what value of k is the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ singular?

(A) $k = 17$

(B) $k = -13$

(C) $k = 13$

(D) $k = -17$

Q9. If $|A| = 3$ and A is a matrix of order 2×2, then $|-2A^T A^{-1}|$ is:

(A) 4

(B) -4

(C) 12

(D) -12

Q10. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+|x|}$. Then $f(x)$ is:

(A) One-to-one and onto

(B) One-to-one but not onto

(C) Onto but not one-to-one

(D) Neither one-to-one nor onto

Q11. If $n(A) = 3$ and $n(B) = 4$, the number of injection mappings (one-to-one) from A to B is:

(A) 12

(B) 24

(C) 64

(D) 81

Q12. The range of $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is:

(A) $[0, \pi]$

(B) $[\pi/4, 3\pi/4]$

(C) $(\pi/4, 3\pi/4)$

(D) $[-\pi/2, \pi/2]$

Q13. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is:

(A) $\frac{3-\sqrt{5}}{2}$

(B) $\frac{3+\sqrt{5}}{2}$

(C) $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$

(D) $\frac{2}{3+\sqrt{5}}$

Q14. A relation R on the set $\mathbb{N}$ of natural numbers is defined as xRy if $x + 2y = 10$. The domain of R is:

(A) $\{2, 4, 6, 8\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{2, 4, 6\}$

(D) $\{1, 3, 5, 7\}$

Q15. The function $f(x) = e^{|x|}$ is:

(A) Continuous and differentiable at $x = 0$

(B) Continuous but not differentiable at $x = 0$

(C) Discontinuous at $x = 0$

(D) None of these

Q16. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{2(1+x^2)}$

(B) $\frac{1}{1+x^2}$

(C) $\frac{2}{1+x^2}$

(D) $\frac{-1}{2(1+x^2)}$

Q17. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{\log x}{(1+\log x)^2}$

(B) $\frac{1}{(1+\log x)^2}$

(C) $\frac{\log x}{1+\log x}$

(D) $\frac{e^x}{x^y}$

Q18. The derivative of $\sin^2 x$ with respect to $e^{\cos x}$ is:

(A) $\frac{-2\cos x}{e^{\cos x}}$

(B) $\frac{2\cos x}{e^{\cos x}}$

(C) $\frac{\sin 2x}{e^{\cos x}}$

(D) $\frac{-\sin 2x}{e^{\cos x}}$

Q19. The point on the curve $y^2 = x$ where the tangent makes an angle of $\frac{\pi}{4}$ with the x-axis is:

(A) $\left(\frac{1}{2}, \frac{1}{4}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $(4, 2)$

(D) $(1, 1)$

Q20. The function $f(x) = \frac{x}{\log x}$ increases in the interval:

(A) $(0, e)$

(B) (e, ∞)

(C) $(0, 1)$

(D) $(1, e)$

Q21. The maximum value of $\left(\frac{1}{x}\right)^x$ is:

(A) e

(B) $e^{1/e}$

(C) $\left(\frac{1}{e}\right)^e$

(D) e^e

Q22. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/s. At the instant when the radius is 10 cm, how fast is the enclosed area increasing?

(A) 40π

(B) 80π

(C) 100π

(D) 20π

Q23. The absolute maximum value of $y = x^3 - 3x + 2$ in $[0, 2]$ is:

(A) 4

(B) 2

(C) 0

(D) 6

Q24. If $f(x) = x^2e^{-x}$, then the interval where it decreases is:

(A) $(-\infty, 0) \cup (2, \infty)$

(B) $(0, 2)$

(C) $(2, \infty)$

(D) $(-\infty, 2)$

Q25. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at $t = 2$ is:

(A) $\frac{7}{6}$

(B) $\frac{6}{7}$

(C) 1

(D) 0

Q26. If A is a square matrix of order 3 such that $A(\text{adj}A) = 10I$, then $|\text{adj}A|$ is equal to:

(A) 10

(B) 100

(C) 1000

(D) 12

Q27. The system of equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if:

(A) $k = 0$

(B) $k \neq 0$

(C) $k = 2$

(D) $k \neq 2$

Q28. If A and B are non-singular square matrices of the same order, then $\text{adj}(AB)$ is:

(A) $(\text{adj}A)(\text{adj}B)$

(B) $(\text{adj}B)(\text{adj}A)$

(C) $|A||B|(AB)^{-1}$

(D) Both B and C

Q29. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|\text{adj}A^3| = 125$, then the value of α is:

(A) ± 3

(B) ± 2

(C) ± 5

(D) 0

Q30. If $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$, then $f'(x)$ at $x = 0$ is:

(A) -20

(B) -30

(C) 0

(D) -40

Q31. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to:

(A) A

(B) $-I$

(C) I

(D) 0

Q32. Total number of possible square matrices of order 3 with each entry as $1, -1$ or 0 is:

(A) 3^3

(B) 3^6

(C) 3^9

(D) 2^9

Q33. For what value of k is the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ singular?

(A) $k = 17$

(B) $k = -13$

(C) $k = 13$

(D) $k = -17$

Q34. If $|A| = 3$ and A is a matrix of order 2×2, then $|-2A^T A^{-1}|$ is:

(A) 4

(B) -4

(C) 12

(D) -12

Q35. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+|x|}$. Then $f(x)$ is:

(A) One-to-one and onto

(B) One-to-one but not onto

(C) Onto but not one-to-one

(D) Neither one-to-one nor onto

Q36. If $n(A) = 3$ and $n(B) = 4$, the number of injection mappings (one-to-one) from A to B is:

(A) 12

(B) 24

(C) 64

(D) 81

Q37. The range of $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is:

(A) $[0, \pi]$

(B) $[\pi/4, 3\pi/4]$

(C) $(\pi/4, 3\pi/4)$

(D) $[-\pi/2, \pi/2]$

Q38. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is:

(A) $\frac{3-\sqrt{5}}{2}$

(B) $\frac{3+\sqrt{5}}{2}$

(C) $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$

(D) $\frac{2}{3+\sqrt{5}}$

Q39. A relation R on the set $\mathbb{N}$ of natural numbers is defined as xRy if $x + 2y = 10$. The domain of R is:

(A) $\{2, 4, 6, 8\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{2, 4, 6\}$

(D) $\{1, 3, 5, 7\}$

Q40. The function $f(x) = e^{|x|}$ is:

(A) Continuous and differentiable at $x = 0$

(B) Continuous but not differentiable at $x = 0$

(C) Discontinuous at $x = 0$

(D) None of these

Q41. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{2(1+x^2)}$

(B) $\frac{1}{1+x^2}$

(C) $\frac{2}{1+x^2}$

(D) $\frac{-1}{2(1+x^2)}$

Q42. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{\log x}{(1+\log x)^2}$

(B) $\frac{1}{(1+\log x)^2}$

(C) $\frac{\log x}{1+\log x}$

(D) $\frac{e^x}{x^y}$

Q43. The derivative of $\sin^2 x$ with respect to $e^{\cos x}$ is:

(A) $\frac{-2\cos x}{e^{\cos x}}$

(B) $\frac{2\cos x}{e^{\cos x}}$

(C) $\frac{\sin 2x}{e^{\cos x}}$

(D) $\frac{-\sin 2x}{e^{\cos x}}$

Q44. The point on the curve $y^2 = x$ where the tangent makes an angle of $\frac{\pi}{4}$ with the x-axis is:

(A) $\left(\frac{1}{2}, \frac{1}{4}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $(4, 2)$

(D) $(1, 1)$

Q45. The function $f(x) = \frac{x}{\log x}$ increases in the interval:

(A) $(0, e)$

(B) (e, ∞)

(C) $(0, 1)$

(D) $(1, e)$

Q46. The maximum value of $\left(\frac{1}{x}\right)^x$ is:

(A) e

(B) $e^{1/e}$

(C) $\left(\frac{1}{e}\right)^e$

(D) e^e

Q47. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/s. At the instant when the radius is 10 cm, how fast is the enclosed area increasing?

(A) 40π

(B) 80π

(C) 100π

(D) 20π

Q48. The absolute maximum value of $y = x^3 - 3x + 2$ in $[0, 2]$ is:

(A) 4

(B) 2

(C) 0

(D) 6

Q49. If $f(x) = x^2 e^{-x}$, then the interval where it decreases is:

(A) $(-\infty, 0) \cup (2, \infty)$

(B) $(0, 2)$

(C) $(2, \infty)$

(D) $(-\infty, 2)$

Q50. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at $t = 2$ is:

(A) $\frac{7}{6}$

(B) $\frac{6}{7}$

(C) 1

(D) 0

Detailed Solutions

Q1.

Solution

Concept: We are given that A is a square matrix of order 3 such that $A(\text{adj}A) = 10I$, and we need to find $|\text{adj}A|$.

Solution: Using the property of the adjugate matrix, we know that:

$$A \cdot (\text{adj}A) = |A| \cdot I.$$

We are given that $A \cdot (\text{adj}A) = 10I$, so we have:

$$|A| \cdot I = 10I.$$

This implies that:

$$|A| = 10.$$

Now, using the formula for the determinant of the adjugate matrix:

$$|\text{adj}A| = |A|^2,$$

where $n = 3$ (order of the matrix). Therefore:

$$|\text{adj}A| = 10^2 = 100.$$

Answer: (B)

Q2.

Solution

Concept: We are given the system of equations:

$$x + y + z = 2,$$

$$2x + y - z = 3,$$

$$3x + 2y + kz = 4.$$

We need to determine the value of k for which this system has a unique solution.

Solution: We can write the system of equations in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

The system will have a unique solution if the determinant of the coefficient matrix is non-zero. We compute the determinant of the coefficient matrix:

$$\det \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} = 1 \cdot \begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}.$$

Computing the 2x2 determinants:

$$\begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} = k + 2, \quad \begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} = 2k + 3, \quad \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1.$$

Thus, the determinant is:

$$\det = (k + 2) - (2k + 3) + 1 = k + 2 - 2k - 3 + 1 = -k.$$

For a unique solution, the determinant must be non-zero:

$$-k \neq 0 \implies k \neq 0.$$

Answer: (B)

Q3.

Solution

Concept: We are given that A and B are non-singular square matrices of the same order, and we need to find $\text{adj}(AB)$.

Solution: Using the property of the adjugate matrix, we know that:

$$\text{adj}(AB) = \text{adj}(A) \cdot \text{adj}(B).$$

Thus, the answer is:

$$\text{adj}(AB) = (\text{adj}A)(\text{adj}B).$$

Answer: (A)

Q4.

Solution

Concept: We are given $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|\text{adj}A^3| = 125$, and we need to find the value of α.

Solution: We know that:

$$|\text{adj}(A^3)| = |A^3|^{n-1} = |A|^3.$$

Thus, we have:

$$|\text{adj}(A^3)| = |A|^3 = 125.$$

This implies:

$$|A| = 5.$$

Now, we compute $|A|$:

$$|A| = \alpha^2 - 4.$$

Setting $|A| = 5$, we get:

$$\alpha^2 - 4 = 5 \implies \alpha^2 = 9 \implies \alpha = \pm 3.$$

Answer: (A)

Q5.

Solution

Concept: We are given $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$, and we need to find $f'(x)$ at $x = 0$.

Solution: We compute $f(x)$ as the determinant of the matrix:

$$f(x) = x\begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 7 & x \end{vmatrix} + 7\begin{vmatrix} 2 & x \\ 7 & 6 \end{vmatrix}.$$

Calculating the 2x2 determinants:

$$\begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} = x^2 - 12, \quad \begin{vmatrix} 2 & 2 \\ 7 & x \end{vmatrix} = 2x - 14, \quad \begin{vmatrix} 2 & x \\ 7 & 6 \end{vmatrix} = 12 - 7x.$$

Thus:

$$f(x) = x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x).$$

Simplifying:

$$f(x) = x^3 - 12x - 6x + 42 + 84 - 49x = x^3 - 67x + 126.$$

Now, differentiate $f(x)$:

$$f'(x) = 3x^2 - 67.$$

At $x = 0$:

$$f'(0) = 3(0)^2 - 67 = -67.$$

Answer: (B)

Q6.

Solution

Concept: We are given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, and we need to compute A^2.

Solution: First, compute A^2:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}.$$

Multiply the matrices:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2a & 2b & -2 \end{bmatrix}.$$

So:

$$A^2 = I.$$

Answer: (C)

Q7.

Solution

Concept: We are asked to compute the total number of possible square matrices of order 3 with each entry as 1, -1, or 0.

Solution: Each element of the matrix can be 1, -1, or 0. Since there are 9 elements in a 3×3 matrix, the total number of possible matrices is:

$$3^9.$$

Answer: (C)

Q8.

Solution

Concept: We are asked for the value of k such that the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is singular.

Solution: A matrix is singular if its determinant is zero. The determinant of the given matrix is:

$$\det = (2-k)(1) - (3)(-5) = 2 - k + 15 = 17 - k.$$

For the matrix to be singular, the determinant must be zero:

$$17 - k = 0 \implies k = 17.$$

Answer: (A)

Q9.

Solution

Concept: We are given $|A| = 3$ and A is a matrix of order 2×2, and we need to compute $|-2A^T A^{-1}|$.

Solution: First, use the property of the determinant for the transpose and inverse:

$$|A^T| = |A| \quad \text{and} \quad |A^{-1}| = \frac{1}{|A|}.$$

Thus:

$$|-2A^T A^{-1}| = |-2| \cdot |A^T| \cdot |A^{-1}| = 2 \cdot |A| \cdot \frac{1}{|A|} = 2.$$

Answer: (A)

Q10.

Solution

Concept: We are given the function $f(x) = \frac{x}{1+|x|}$ and need to determine its properties.

Solution: The function $f(x)$ is continuous but not differentiable at $x = 0$ because the absolute value function causes a cusp at $x = 0$. Moreover, it is not onto because the range of $f(x)$ does not cover all real numbers.

Answer: (B)

Q11.

Solution

Concept: We are given that $n(A) = 3$ and $n(B) = 4$, and we need to find the number of injection mappings (one-to-one) from A to B.

Solution: An injection (one-to-one mapping) from a set A to a set B is a function where every element of A is mapped to a unique element in B. The number of injections from a set of size m to a set of size n is given by the formula:

$$n^m,$$

where n is the size of the target set and m is the size of the domain set. Here, $n = 4$ and $m = 3$, so the number of injections is:

$$4^3 = 64.$$

Answer: (C)

Q12.

Solution

Concept: We are given the function $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$, and we need to find its range.

Solution: We know that:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},$$

for $x \in [-1, 1]$. Thus, the function becomes:

$$f(x) = \frac{\pi}{2} + \tan^{-1} x.$$

The range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Therefore, the range of $f(x)$ is:

$$[0, \pi].$$

Answer: (A)

Q13.

Solution

Concept: We are asked to compute $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$.

Solution: Let $\theta = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$, so:

$$\cos\theta = \frac{\sqrt{5}}{3}.$$

We need to compute $\tan\left(\frac{\theta}{2}\right)$. Using the half-angle identity for tangent:

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}.$$

First, compute $\sin\theta$ using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \left(\frac{\sqrt{5}}{3}\right)^2 = 1 - \frac{5}{9} = \frac{4}{9},$$

so:

$$\sin\theta = \frac{2}{3}.$$

Now, substitute into the half-angle identity:

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{3-\sqrt{5}}{2}.$$

Answer: (A)

Q14.

Solution

Concept: We are given a relation R on the set $\mathbb{N}$ of natural numbers, defined as xRy if $x + 2y = 10$, and we need to determine the domain of R.
Solution: For xRy, we have the equation:

$$x + 2y = 10.$$

We need to find the natural numbers x and y that satisfy this equation. Rearranging:

$$x = 10 - 2y.$$

For x to be a natural number, y must be such that $10 - 2y$ is a positive natural number. Therefore, $2y$ must be less than or equal to 10, implying:

$$y \in \{1, 2, 3, 4\}.$$

Thus, the domain of R is $\{2, 4, 6, 8\}$.

Answer: (A)

Q15.

Solution

Concept: We are given the function $f(x) = e^{|x|}$, and we need to determine its properties.
Solution: The function $f(x) = e^{|x|}$ is continuous for all $x \in \mathbb{R}$, but it is not differentiable at $x = 0$ because the absolute value function causes a sharp point at $x = 0$.
Thus, $f(x)$ is continuous but not differentiable at $x = 0$.

Answer: (B)

Q16.

Solution

Concept: We are given $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, and we need to find $\frac{dy}{dx}$.

Solution: Differentiate y with respect to x using the chain rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right).$$

Let $u = \frac{\sqrt{1+x^2}-1}{x}$, so:

$$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx}.$$

Now, compute $\frac{du}{dx}$:

$$u = \frac{\sqrt{1+x^2}-1}{x}, \quad \text{so differentiate with respect to } x.$$

Using differentiation rules, we find:

$$\frac{du}{dx} = \frac{1}{1+x^2}.$$

Thus, we have:

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{\sqrt{1+x^2}-1}{x}\right)^2}.$$

After simplifying, we find that the answer is $\frac{1}{1+x^2}$.

Answer: (B)

Q17.

Solution

Concept: We are given $x^y = e^{x-y}$, and we need to find $\frac{dy}{dx}$.
Solution: Take the natural logarithm of both sides of the equation:

$$\ln(x^y) = \ln(e^{x-y}),$$

$$y \ln x = x - y.$$

Now, differentiate both sides with respect to x:

$$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x - y),$$

$$\frac{dy}{dx} \ln x + \frac{y}{x} = 1 - \frac{dy}{dx}.$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx}(\ln x + 1) = 1 - \frac{y}{x},$$

$$\frac{dy}{dx} = \frac{\log x}{1 + \log x}.$$

Answer: (C)

Q18.

Solution

Concept: We are asked to find the derivative of $\sin^2 x$ with respect to $e^{\cos x}$.
Solution: We use the chain rule to differentiate $\sin^2 x$ with respect to $e^{\cos x}$:

$$\frac{d}{dx}\left(\sin^2 x\right) = 2 \sin x \cos x.$$

Now, differentiate $e^{\cos x}$:

$$\frac{d}{dx}(e^{\cos x}) = -e^{\cos x} \sin x.$$

Thus, the derivative is:

$$\frac{2 \sin x \cos x}{e^{\cos x}}.$$

Answer: (D)

Q19.

Solution

Concept: We are asked to find the point on the curve $y^2 = x$ where the tangent makes an angle of $\frac{\pi}{4}$ with the x-axis.

Solution: The slope of the tangent to the curve $y^2 = x$ is given by:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.$$

The tangent makes an angle $\frac{\pi}{4}$ with the x-axis, so the slope is 1. Thus:

$$\frac{1}{2\sqrt{x}} = 1 \implies \sqrt{x} = \frac{1}{2} \implies x = \frac{1}{4}.$$

Substitute $x = \frac{1}{4}$ into the equation $y^2 = x$:

$$y^2 = \frac{1}{4} \implies y = \pm\frac{1}{2}.$$

Thus, the point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

Answer: (B)

Q20.

Solution

Concept: We are given the function $f(x) = x \log x$, and we need to find the interval where it increases.

Solution: The first derivative of $f(x) = x \log x$ is:

$$\frac{df}{dx} = \log x + 1.$$

For the function to increase, $\frac{df}{dx} > 0$, so:

$$\log x + 1 > 0 \implies \log x > -1 \implies x > \frac{1}{e}.$$

Thus, the function increases in the interval $(1/e, \infty)$.

Answer: (B)

Q21.

Solution

Concept: We are given $\left(\frac{1}{x}\right)^x$, and we need to find the maximum value of this expression.

Solution: Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x}$. To find the maximum, we first take the natural logarithm of both sides:

$$\ln f(x) = -x \ln x.$$

Now, differentiate with respect to x:

$$\frac{d}{dx}(\ln f(x)) = -\ln x - 1.$$

Set the derivative equal to zero to find the critical points:

$$-\ln x - 1 = 0 \implies \ln x = -1 \implies x = \frac{1}{e}.$$

Substitute $x = \frac{1}{e}$ back into $f(x)$ to get the maximum value:

$$f\left(\frac{1}{e}\right) = \left(\frac{1}{\frac{1}{e}}\right)^{\frac{1}{e}} = e^{\frac{1}{e}}.$$

Answer: (B)

Q22.

Solution

Concept: We are given that a stone is dropped into a quiet lake, and waves move in circles at a speed of 4 cm/s. At the instant when the radius is 10 cm, we need to find how fast the enclosed area is increasing.

Solution: Let r be the radius of the wave. The rate of change of the area of the circle is given by the formula:

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}.$$

The area A of the circle is $A = \pi r^2$, so:

$$\frac{dA}{dr} = 2\pi r.$$

Now, substitute $r = 10$ cm and $\frac{dr}{dt} = 4$ cm/s:

$$\frac{dA}{dt} = 2\pi(10) \cdot 4 = 80\pi \text{ cm}^2/\text{s}.$$

Answer: (B)

Q23.

Solution

Concept: We are asked to find the absolute maximum value of $y = x^3 - 3x + 2$ in the interval $[0, 2]$.
Solution: The first derivative of $y = x^3 - 3x + 2$ is:

$$\frac{dy}{dx} = 3x^2 - 3.$$

Set $\frac{dy}{dx} = 0$ to find the critical points:

$$3x^2 - 3 = 0 \implies x^2 = 1 \implies x = \pm 1.$$

Since $x = -1$ is outside the interval $[0, 2]$, we check the values of y at the endpoints and at $x = 1$: - At $x = 0$, $y = 0^3 - 3(0) + 2 = 2$, - At $x = 1$, $y = 1^3 - 3(1) + 2 = 0$, - At $x = 2$, $y = 2^3 - 3(2) + 2 = 2$. Thus, the absolute maximum value of y is 2.

Answer: (A)

Q24.

Solution

Concept: We are given the function $f(x) = x^2 e^{-x}$, and we need to find the interval where it decreases.
Solution: The first derivative of $f(x) = x^2 e^{-x}$ is:

$$f'(x) = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2).$$

To determine where $f(x)$ is decreasing, set $f'(x) < 0$:

$$e^{-x}(2x - x^2) < 0.$$

Since $e^{-x} > 0$ for all x, we solve:

$$2x - x^2 < 0 \implies x(2 - x) < 0.$$

This inequality holds when $0 < x < 2$, so the function decreases in the interval $(0, 2)$.

Answer: (B)

Q25.

Solution

Concept: We are given the parametric equations $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$, and we need to find the slope of the tangent at $t = 2$.

Solution: The slope of the tangent is given by $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. First, compute the derivatives:

$$\frac{dx}{dt} = 2t + 3, \quad \frac{dy}{dt} = 4t - 2.$$

At $t = 2$:

$$\frac{dx}{dt} = 2(2) + 3 = 7, \quad \frac{dy}{dt} = 4(2) - 2 = 6.$$

Thus, the slope of the tangent is:

$$\frac{dy}{dx} = \frac{6}{7}.$$

Answer: (B)

Q26.

Solution

Concept: We are given A is a square matrix of order 3 such that $A(\text{adj}A) = 10I$, and we need to find $|\text{adj}A|$.

Solution: Using the property of the adjugate matrix, we know:

$$A \cdot (\text{adj}A) = |A| \cdot I.$$

We are given $A \cdot (\text{adj}A) = 10I$, so:

$$|A| \cdot I = 10I \implies |A| = 10.$$

The determinant of the adjugate matrix is given by:

$$|\text{adj}A| = |A|^2 = 10^2 = 100.$$

Answer: (B)

Q27.

Solution

Concept: We are given the system of equations:

$$x + y + z = 2,$$

$$2x + y - z = 3,$$

$$3x + 2y + kz = 4.$$

We need to determine the value of k for which this system has a unique solution.

Solution: The system will have a unique solution if the determinant of the coefficient matrix is non-zero. The coefficient matrix is:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}.$$

We compute the determinant of the matrix:

$$\det = 1 \cdot \begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}.$$

Calculating the 2x2 determinants:

$$\begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} = k + 2, \quad \begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} = 2k + 3, \quad \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1.$$

Thus, the determinant is:

$$\det = (k + 2) - (2k + 3) + 1 = k + 2 - 2k - 3 + 1 = -k.$$

For a unique solution, the determinant must be non-zero:

$$-k \neq 0 \implies k \neq 0.$$

Answer: (B)

Q28.

Solution

Concept: We are given A and B are non-singular square matrices of the same order, and we need to compute $\text{adj}(AB)$.

Solution: Using the property of the adjugate matrix, we know that:

$$\text{adj}(AB) = \text{adj}(A) \cdot \text{adj}(B).$$

Thus, the answer is:

$$\text{adj}(AB) = (\text{adj}A)(\text{adj}B).$$

Answer: (D)

Q29.

Solution

Concept: We are given $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|\text{adj}A^3| = 125$, and we need to find the value of α.

Solution: We know that:

$$|\text{adj}(A^3)| = |A^3|^{n-1} = |A|^3.$$

Thus, we have:

$$|\text{adj}(A^3)| = |A|^3 = 125.$$

This implies:

$$|A| = 5.$$

Now, we compute $|A|$:

$$|A| = \alpha^2 - 4.$$

Setting $|A| = 5$, we get:

$$\alpha^2 - 4 = 5 \implies \alpha^2 = 9 \implies \alpha = \pm 3.$$

Answer: (A)

Q30.

Solution

Concept: We are given $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$, and we need to find $f'(x)$ at $x = 0$.

Solution: We compute $f(x)$ as the determinant of the matrix:

$$f(x) = x\begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 7 & x \end{vmatrix} + 7\begin{vmatrix} 2 & x \\ 7 & 6 \end{vmatrix}.$$

Calculating the 2x2 determinants:

$$\begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} = x^2 - 12, \quad \begin{vmatrix} 2 & 2 \\ 7 & x \end{vmatrix} = 2x - 14, \quad \begin{vmatrix} 2 & x \\ 7 & 6 \end{vmatrix} = 12 - 7x.$$

Thus:

$$f(x) = x(x^2 - 12) - 3(2x - 14) + 7(12 - 7x).$$

Simplifying:

$$f(x) = x^3 - 12x - 6x + 42 + 84 - 49x = x^3 - 67x + 126.$$

Now, differentiate $f(x)$:

$$f'(x) = 3x^2 - 67.$$

At $x = 0$:

$$f'(0) = 3(0)^2 - 67 = -67.$$

Answer: (D)

Q31.

Solution

Concept: We are given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, and we need to compute A^2.

Solution: We compute A^2 by multiplying the matrix by itself:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}.$$

Performing the matrix multiplication:

$$A^2 = \begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 + 0 \cdot a & 1 \cdot 0 + 0 \cdot 1 + 0 \cdot b & 1 \cdot 0 + 0 \cdot 0 + 0 \cdot (-1) \\ 0 \cdot 1 + 1 \cdot 0 + 0 \cdot a & 0 \cdot 0 + 1 \cdot 1 + 0 \cdot b & 0 \cdot 0 + 1 \cdot 0 + 0 \cdot (-1) \\ a \cdot 1 + b \cdot 0 + (-1) \cdot a & a \cdot 0 + b \cdot 1 + (-1) \cdot b & a \cdot 0 + b \cdot 0 + (-1) \cdot (-1) \end{bmatrix}.$$

Simplifying each entry:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, $A^2 = I$.

Answer: (C)

Q32.

Solution

Concept: We are asked to compute the total number of possible square matrices of order 3 with each entry as $1, -1$ or 0.

Solution: Each element of the matrix can be one of three values: $1, -1, 0$. Since the matrix is of order 3, it has 9 entries, and for each entry, there are 3 choices. Therefore, the total number of possible matrices is:

$$3^9 = 19683.$$

Answer: (D)

Q33.

Solution

Concept: We are given the matrix $\begin{bmatrix} 2 & -k & 3 \\ -5 & 1 & 0 \end{bmatrix}$, and we need to determine for what value of k the matrix is singular.

Solution: A matrix is singular if its determinant is zero. We compute the determinant of the given matrix:

$$\det = \begin{vmatrix} 2 & -k & 3 \\ -5 & 1 & 0 \end{vmatrix}.$$

Using cofactor expansion along the first row:

$$\det = 2 \cdot \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} - (-k) \cdot \begin{vmatrix} -5 & 0 \\ 1 & 0 \end{vmatrix} + 3 \cdot \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix}.$$

Calculating the 2x2 determinants:

$$\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0, \quad \begin{vmatrix} -5 & 0 \\ 1 & 0 \end{vmatrix} = 0, \quad \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix} = (-5)(1) - (1)(1) = -6.$$

Thus:

$$\det = 2(0) + k(0) + 3(-6) = -18.$$

For this matrix to be singular, we require the determinant to be 0, but the determinant is always -18, so the matrix is non-singular.

Answer: (D)

Q34.

Solution

Concept: We are given $|A| = 3$ and A is a matrix of order 2×2, and we need to compute $|-2A^T A^{-1}|$.

Solution: Using the properties of determinants:

$$|A^T| = |A| \quad \text{and} \quad |A^{-1}| = \frac{1}{|A|}.$$

Thus, the determinant of $-2A^T A^{-1}$ is:

$$|-2A^T A^{-1}| = |-2| \cdot |A^T| \cdot |A^{-1}| = 2 \cdot |A| \cdot \frac{1}{|A|} = 2.$$

Answer: (A)

Q35.

Solution

Concept: We are asked to compute the range of the function $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$.

Solution: We know that:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},$$

for $x \in [-1, 1]$. Thus, the function becomes:

$$f(x) = \frac{\pi}{2} + \tan^{-1} x.$$

The range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Therefore, the range of $f(x)$ is:

$$[0, \pi].$$

Answer: (A)

Q36.

Solution

Concept: We are given that $n(A) = 3$ and $n(B) = 4$, and we need to find the number of injection mappings (one-to-one) from A to B.

Solution: An injection (one-to-one mapping) from a set A to a set B is a function where every element of A is mapped to a unique element in B. The number of injections from a set of size m to a set of size n is given by the formula:

$$n^m,$$

where n is the size of the target set and m is the size of the domain set. Here, $n = 4$ and $m = 3$, so the number of injections is:

$$4^3 = 64.$$

Answer: (C)

Q37.

Solution

Concept: We are given the function $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$, and we need to find its range.

Solution: We know that:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},$$

for $x \in [-1, 1]$. Thus, the function becomes:

$$f(x) = \frac{\pi}{2} + \tan^{-1} x.$$

The range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Therefore, the range of $f(x)$ is:

$$[0, \pi] .$$

Answer: (A)

Q38.

Solution

Concept: We are asked to compute $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$.

Solution: Let $\theta = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$, so:

$$\cos\theta = \frac{\sqrt{5}}{3}.$$

We need to compute $\tan\left(\frac{\theta}{2}\right)$. Using the half-angle identity for tangent:

$$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{\sin\theta}.$$

First, compute $\sin\theta$ using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \left(\frac{\sqrt{5}}{3}\right)^2 = 1 - \frac{5}{9} = \frac{4}{9},$$

so:

$$\sin\theta = \frac{2}{3}.$$

Now, substitute into the half-angle identity:

$$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{3 - \sqrt{5}}{2}.$$

Answer: (A)

Q39.

Solution

Concept: We are given a relation R on the set $\mathbb{N}$ of natural numbers, defined as xRy if $x + 2y = 10$, and we need to determine the domain of R.
Solution: For xRy, we have the equation:

$$x + 2y = 10.$$

We need to find the natural numbers x and y that satisfy this equation. Rearranging:

$$x = 10 - 2y.$$

For x to be a natural number, y must be such that $10 - 2y$ is a positive natural number. Therefore, $2y$ must be less than or equal to 10, implying:

$$y \in \{1, 2, 3, 4\}.$$

Thus, the domain of R is $\{2, 4, 6, 8\}$.

Answer: (A)

Q40.

Solution

Concept: We are given the function $f(x) = e^{|x|}$, and we need to determine its properties.
Solution: The function $f(x) = e^{|x|}$ is continuous for all $x \in \mathbb{R}$, but it is not differentiable at $x = 0$ because the absolute value function causes a sharp point at $x = 0$.
Thus, $f(x)$ is continuous but not differentiable at $x = 0$.

Answer: (B)

Q41.

Solution

Concept: We are given $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, and we need to find $\frac{dy}{dx}$.
Solution: We apply the chain rule to differentiate:

$$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{\sqrt{1+x^2}-1}{x}\right)^2} \cdot \frac{d}{dx}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right).$$

Simplifying the derivative and solving, we find:

$$\frac{dy}{dx} = \frac{1}{1 + x^2}.$$

Answer: (B)

Q42.

Solution

Concept: We are given $x^y = e^{x-y}$, and we need to find $\frac{dy}{dx}$.
Solution: Taking the natural logarithm of both sides:

$$\ln(x^y) = \ln(e^{x-y}),$$

$$y \ln x = x - y.$$

Differentiating both sides with respect to x:

$$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x - y),$$

$$\frac{dy}{dx}\ln x + \frac{y}{x} = 1 - \frac{dy}{dx}.$$

Solving for $\frac{dy}{dx}$, we get:

$$\frac{dy}{dx} = \frac{\log x}{1 + \log x}.$$

Answer: (C)

Q43.

Solution

Concept: We are given $\frac{d}{dx}\sin^2 x$ with respect to $e^{\cos x}$.
Solution: We first apply the chain rule for differentiation:

$$\frac{d}{dx}\left(\sin^2 x\right) = 2\sin x \cos x.$$

Now, applying the chain rule to $e^{\cos x}$, we differentiate and find:

$$\frac{d}{dx}(e^{\cos x}) = -e^{\cos x}\sin x.$$

Thus, we get:

$$\frac{d}{dx}\left(\sin^2 x\right) \text{ with respect to } e^{\cos x} = \frac{-2\cos x}{e^{\cos x}}.$$

Answer: (A)

Q44.

Solution

Concept: We are asked to find the point on the curve $y^2 = x$ where the tangent makes an angle of $\frac{\pi}{4}$ with the x-axis.

Solution: The slope of the tangent to the curve $y^2 = x$ is given by:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.$$

The tangent makes an angle $\frac{\pi}{4}$, so the slope is 1. Thus:

$$\frac{1}{2\sqrt{x}} = 1 \implies \sqrt{x} = \frac{1}{2} \implies x = \frac{1}{4}.$$

Substitute $x = \frac{1}{4}$ into the equation $y^2 = x$:

$$y^2 = \frac{1}{4} \implies y = \pm\frac{1}{2}.$$

Thus, the point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

Answer: (B)

Q45.

Solution

Concept: We are given the function $f(x) = x \log x$, and we need to find the interval where it increases.

Solution: The first derivative of $f(x) = x \log x$ is:

$$f'(x) = \log x + 1.$$

For the function to increase, $f'(x) > 0$, so:

$$\log x + 1 > 0 \implies \log x > -1 \implies x > \frac{1}{e}.$$

Thus, the function increases in the interval $(1/e, \infty)$.

Answer: (B)

Q46.

Solution

Concept: We are given $\left(\frac{1}{x}\right)^x$, and we need to find the maximum value of this expression.

Solution: Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x}$. To find the maximum, we first take the natural logarithm of both sides:

$$\ln f(x) = -x \ln x.$$

Now, differentiate with respect to x:

$$\frac{d}{dx}(\ln f(x)) = -\ln x - 1.$$

Set the derivative equal to zero to find the critical points:

$$-\ln x - 1 = 0 \implies \ln x = -1 \implies x = \frac{1}{e}.$$

Substitute $x = \frac{1}{e}$ back into $f(x)$ to get the maximum value:

$$f\left(\frac{1}{e}\right) = \left(\frac{1}{\frac{1}{e}}\right)^{\frac{1}{e}} = e^{\frac{1}{e}}.$$

Answer: (A)

Q47.

Solution

Concept: We are given that a stone is dropped into a quiet lake, and waves move in circles at a speed of 4 cm/s. At the instant when the radius is 10 cm, we need to find how fast the enclosed area is increasing.

Solution: Let r be the radius of the wave. The rate of change of the area of the circle is given by the formula:

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}.$$

The area A of the circle is $A = \pi r^2$, so:

$$\frac{dA}{dr} = 2\pi r.$$

Now, substitute $r = 10$ cm and $\frac{dr}{dt} = 4$ cm/s:

$$\frac{dA}{dt} = 2\pi(10) \cdot 4 = 80\pi \text{ cm}^2/\text{s}.$$

Answer: (B)

Q48.

Solution

Concept: We are asked to find the absolute maximum value of $y = x^3 - 3x + 2$ in the interval $[0, 2]$.

Solution: The first derivative of $y = x^3 - 3x + 2$ is:

$$\frac{dy}{dx} = 3x^2 - 3.$$

Set $\frac{dy}{dx} = 0$ to find the critical points:

$$3x^2 - 3 = 0 \implies x^2 = 1 \implies x = \pm 1.$$

Since $x = -1$ is outside the interval $[0, 2]$, we check the values of y at the endpoints and at $x = 1$: - At $x = 0$, $y = 0^3 - 3(0) + 2 = 2$, - At $x = 1$, $y = 1^3 - 3(1) + 2 = 0$, - At $x = 2$, $y = 2^3 - 3(2) + 2 = 2$. Thus, the absolute maximum value of y is 2.

Answer: (A)

Q49.

Solution

Concept: We are given $f(x) = x^2 e^{-x}$, and we need to find the interval where it decreases.

Solution: The first derivative of $f(x) = x^2 e^{-x}$ is:

$$f'(x) = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2).$$

To determine where $f(x)$ is decreasing, set $f'(x) < 0$:

$$e^{-x}(2x - x^2) < 0.$$

Since $e^{-x} > 0$ for all x, we solve:

$$2x - x^2 < 0 \implies x(2 - x) < 0.$$

This inequality holds when $0 < x < 2$, so the function decreases in the interval $(0, 2)$.

Answer: (B)

Q50.

Solution

Concept: We are given the parametric equations $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$, and we need to find the slope of the tangent at $t = 2$.

Solution: The slope of the tangent is given by $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. First, compute the derivatives:

$$\frac{dx}{dt} = 2t + 3, \quad \frac{dy}{dt} = 4t - 2.$$

At $t = 2$:

$$\frac{dx}{dt} = 2(2) + 3 = 7, \quad \frac{dy}{dt} = 4(2) - 2 = 6.$$

Thus, the slope of the tangent is:

$$\frac{dy}{dx} = \frac{6}{7}.$$

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	A	5	B
6	C	7	C	8	A	9	A	10	B
11	C	12	A	13	A	14	A	15	B
16	B	17	C	18	D	19	B	20	B
21	B	22	B	23	A	24	B	25	B
26	B	27	B	28	D	29	A	30	D
31	C	32	D	33	D	34	A	35	A
36	C	37	A	38	A	39	A	40	B
41	B	42	C	43	A	44	B	45	B
46	A	47	B	48	A	49	B	50	A