CUET-UG Mathematics Sample Paper-3

Duration: 1 Hour | **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is equal to:

(A) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} n & 2n \\ 0 & n \end{bmatrix}$

(D) $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

Q2. If $f(x) = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$, then $f(x)$ is divisible by:

(A) x^3

(B) x^2

(C) $(x+3)$

(D) Both B and C

Q3. If A is a square matrix of order 3 and $|A| = k$, then $|\text{adj}(\text{adj}A)|$ is:

(A) k^4

(B) k^9

(C) k^3

(D) k^2

Q4. The value of $\begin{vmatrix} \sin^2 A & \sin A & \cos^2 A \\ \sin^2 B & \sin B & \cos^2 B \\ \sin^2 C & \sin C & \cos^2 C \end{vmatrix}$ is:

(A) 1

(B) 0

(C) $\sin A \sin B \sin C$

(D) None of these

Q5. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are:

(A) $-6, -4, -9$

(B) $-6, -4, 9$

(C) $-6, 4, 9$

(D) $-6, 12, -18$

Q6. If $A = [a_{ij}]$ is a 3×3 matrix where $a_{ij} = i^2 - j^2$, then A is a:

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Diagonal matrix

(D) Identity matrix

Q7. The inverse of $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is:

(A) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(B) $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

(C) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

(D) Does not exist

Q8. If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to:

(A) $\det(A)$

(B) $\frac{1}{\det(A)}$

(C) 1

(D) 0

Q9. If x, y, z are non-zero real numbers, the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is:

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) $\begin{bmatrix} 1/x & 1/y & 1/z \end{bmatrix}$

Q10. The number of reflexive relations on a set containing n elements is:

(A) 2^n

(B) 2^{n^2-n}

(C) 2^{n^2}

(D) n^2

Q11. If $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = (3 - x^3)^{1/3}$, then $f(f(x))$ is:

(A) $x^{1/3}$

(B) x^3

(C) x

(D) $(3 - x^3)$

Q12. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is:

(A) Increasing in $(0, \pi/4)$

(B) Decreasing in $(0, \pi/4)$

(C) Increasing in $(\pi/4, \pi/2)$

(D) Constant

Q13. The value of $\sin^{-1}\left(\cos \frac{33\pi}{5}\right)$ is:

(A) $\frac{3\pi}{5}$

(B) $\frac{-7\pi}{10}$

(C) $\frac{\pi}{10}$

(D) $\frac{-\pi}{10}$

Q14. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) π

(D) $\frac{\pi}{6}$

Q15. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is:

(A) $\frac{\cos x}{2y-1}$

(B) $\frac{\cos x}{1-2y}$

(C) $\frac{\sin x}{2y-1}$

(D) $\frac{\sin x}{1-2y}$

Q16. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x = 0$, then k is:

(A) 1

(B) 2.5

(C) 5

(D) 0

Q17. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is:

(A) $\frac{3}{2}$

(B) $\frac{3}{4t}$

(C) $\frac{3}{2t}$

(D) $\frac{3}{4}$

Q18. The derivative of $\log_{10} x$ with respect to x is:

(A) $\frac{1}{x}$

(B) $\frac{\log_{10} e}{x}$

(C) $\frac{1}{x \log_e 10}$

(D) Both B and C

Q19. The function $f(x) = [x]$ (greatest integer function) is NOT differentiable at:

(A) All real numbers

(B) All integers

(C) $x = 0.5$

(D) Only $x = 0$

Q20. The angle of intersection of the curves $y^2 = x$ and $x^2 = y$ at $(1, 1)$ is:

(A) $\tan^{-1}\left(\frac{3}{4}\right)$

(B) $\tan^{-1}\left(\frac{4}{3}\right)$

(C) $\frac{\pi}{2}$

(D) 0

Q21. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in $[0, 9]$ is:

(A) 128

(B) 0

(C) 160

(D) 135

Q22. The point on the curve $y = (x - 3)^2$ where the tangent is parallel to the chord joining $(3, 0)$ and $(4, 1)$ is:

(A) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(B) $\left(\frac{5}{2}, \frac{1}{4}\right)$

(C) $\left(\frac{1}{2}, \frac{1}{4}\right)$

(D) $(2, 1)$

Q23. A circular plate of metal is heated so that its radius increases by 2%. The percentage increase in its area is:

(A) 2%

(B) 4%

(C) 8%

(D) 20%

Q24. If $f(x) = x^3 - 6x^2 + 9x + 15$ has a local maximum at $x = \alpha$ and a local minimum at $x = \beta$, then (α, β) is:

(A) $(1, 3)$

(B) $(3, 1)$

(C) $(1, 2)$

(D) $(2, 3)$

Q25. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

(A) $(1, 2)$

(B) $(2, 1)$

(C) $(1, -2)$

(D) $(-1, 2)$

Q26. If $\int \frac{1}{\cos(x-a)\cos(x-b)}\, dx$ is:

(A) $\frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C$

(B) $\frac{1}{\sin(b-a)} \log\left|\frac{\cos(x-b)}{\cos(x-a)}\right| + C$

(C) $\frac{1}{\sin(a-b)} \log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

(D) None of these

Q27. If $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$ is:

(A) $2\sqrt{\tan x} + C$

(B) $\sqrt{\tan x} + C$

(C) $2\sqrt{\cot x} + C$

(D) $\log|\tan x| + C$

Q28. The value of $\int_0^{\pi/2} \log(\tan x)\, dx$ is:

(A) $\pi/2$

(B) $\pi \log 2$

(C) 0

(D) $-\pi/2 \log 2$

Q29. If $\int e^x \frac{x-1}{x^2}\,dx$ is:

(A) $\frac{e^x}{x} + C$

(B) $\frac{e^x}{x^2} + C$

(C) $\frac{e^x-1}{x} + C$

(D) $xe^x + C$

Q30. The area of the region bounded by $y^2 = x$ and $y = |x|$ is:

(A) $1/3$

(B) $1/6$

(C) $2/3$

(D) 1

Q31. The area bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \pi/2$ is:

(A) $2\sqrt{2} - 2$

(B) $\sqrt{2} - 1$

(C) $2(\sqrt{2} - 1)$

(D) $2\sqrt{2}$

Q32. The value of $\int_0^{\pi/4} \tan^3 x\,dx$ is:

(A) $\frac{1-\log 2}{2}$

(B) $\frac{1+\log 2}{2}$

(C) $1 - \log\sqrt{2}$

(D) $1/2$

Q33. If $\int_0^{a} \frac{1}{1+4x^2}\,dx = \pi/8$, then a is:

(A) $1/2$

(B) 1

(C) $\pi/4$

(D) 2

Q34. If $\int \frac{dx}{x(x^5+1)}$ is:

(A) $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$

(B) $\log\left|\frac{x^5}{x^5+1}\right| + C$

(C) $5\log\left|\frac{x^5+1}{x^5}\right| + C$

(D) $\frac{1}{5}\log\left|\frac{x^5+1}{x^5}\right| + C$

Q35. The area of the region bounded by $x^2 = 4y$ and $x = 4y - 2$ is:

(A) $3/8$

(B) $5/8$

(C) $9/8$

(D) $7/8$

Q36. The degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is:

(A) 1

(B) 2

(C) $1/2$

(D) Not defined

Q37. The general solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is:

(A) $\sin^{-1} x + \sin^{-1} y = C$

(B) $\sin^{-1} y - \sin^{-1} x = C$

(C) $\sin^{-1} x \cdot \sin^{-1} y = C$

(D) $x + y = C$

Q38. The solution of $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying $y(1) = 1$ is:

(A) $y = x \log x + x$

(B) $y = x \log x$

(C) $y = \log x + 1$

(D) $y = xe^{x-1}$

Q39. The number of arbitrary constants in the particular solution of a differential equation of order 3 is:

(A) 3

(B) 2

(C) 1

(D) 0

Q40. The integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$ is:

(A) $\cos x$

(B) $\sec x$

(C) $\tan x$

(D) $\sin x$

Q41. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $0°$

(B) $45°$

(C) $90°$

(D) $180°$

Q42. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is:

(A) 0

(B) -1

(C) 1

(D) 3

Q43. If a line makes an angle $\pi/4$ with both x and y axes, then the angle it makes with the z-axis is:

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi/6$

(D) 0

Q44. The equation of the plane passing through $(1, 1, 1)$ and parallel to the plane $2x + 3y - z + 5 = 0$ is:

(A) $2x + 3y - z - 4 = 0$

(B) $2x + 3y - z + 4 = 0$

(C) $x + y + z - 3 = 0$

(D) $2x + 3y - z = 0$

Q45. The direction ratios of the line perpendicular to the lines with direction ratios $(1, -2, 1)$ and $(0, 2, -3)$ are:

(A) $(4, 3, 2)$

(B) $(4, 3, 1)$

(C) $(4, 2, 2)$

(D) $(1, 0, 1)$

Q46. The distance between the parallel planes $2x-y+2z+3 = 0$ and $4x-2y+4z+21 = 0$ is:

(A) $5/2$

(B) $3/2$

(C) $7/2$

(D) $9/2$

Q47. In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which maximum Z occurs is:

(A) 2

(B) 0

(C) Infinite

(D) 1

Q48. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cap B)$ is:

(A) 0.24

(B) 0.32

(C) 0.48

(D) 0.96

Q49. If A and B are two events such that $A \subset B$ and $P(A) \neq 0$, then:

(A) $P(A|B) = 1$

(B) $P(A|B) < P(A)$

(C) $P(A|B) \geq P(A)$

(D) $P(A|B) = \frac{P(B)}{P(A)}$

Q50. A box contains 100 tickets numbered 1 to 100. One ticket is drawn. If it is known that the number is divisible by 2, the probability that it is also divisible by 5 is:

(A) $1/10$

(B) $1/5$

(C) $1/2$

(D) $1/4$

Detailed Solutions

Q1.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, and we need to find the value of A^n.

Solution: For the matrix A, the pattern for A^n can be derived by observation or using induction. We observe that:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}.$$

This suggests that $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$, since the upper-right element is increasing linearly with n.

Answer: (A)

Q2.

Solution

Concept: We are given the matrix $f(x) = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$, and we need to determine when $f(x)$ is divisible by x^2 or $(x+3)$.

Solution: We first compute the determinant $f(x)$:

$$f(x) = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}.$$

Using cofactor expansion or directly calculating the determinant, we find that $f(x)$ is divisible by x^2 and $(x+3)$.

Answer: (D)

Q3.

Solution

Concept: We are asked to determine $|\text{adj}(\text{adj}A)|$ when $|A| = k$.
Solution: The property of the adjugate matrix states that for any matrix A:

$$|\text{adj}A| = |A|^{n-1},$$

where n is the order of the matrix. Thus for a 3x3 matrix, $|\text{adj}A| = |A|^2$. Since $|A| = k$, we have:

$$|\text{adj}(\text{adj}A)| = |A|^{3-1} = k^3.$$

Answer: (C)

Q4.

Solution

Concept: We are asked to find the value of $\begin{vmatrix} \sin^2 A & \sin A & \cos^2 A \\ \sin^2 B & \sin B & \cos^2 B \\ \sin^2 C & \sin C & \cos^2 C \end{vmatrix}$.
Solution: We observe that the matrix has a pattern, and a determinant of such a structure is equal to 0, as the rows are linearly dependent.

Answer: (B)

Q5.

Solution

Concept: We are given $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, and we need to find the values of k, a, b.
Solution: From the matrix equality, we equate the corresponding elements:

$$k \times 0 = 0, \quad k \times 2 = 3a, \quad k \times 3 = 2b, \quad k \times (-4) = 24.$$

Solving the last equation $k \times (-4) = 24$ gives $k = -6$. Substituting $k = -6$ into the second and third equations, we find $a = -4$ and $b = 9$.

Answer: (B)

Q6.

Solution

Concept: We are given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, and we need to find A^2.

Solution: We compute A^2 by multiplying A by itself:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}.$$

Performing the matrix multiplication, we get:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

Answer: (C)

Q7.

Solution

Concept: We are asked to find the total number of possible square matrices of order 3 with each entry as $1, -1$ or 0.

Solution: Each element of the matrix can take one of three values: 1, -1, or 0. Since there are 9 elements in a 3×3 matrix, the total number of possible matrices is:

$$3^9.$$

Answer: (C)

Q8.

Solution

Concept: We are asked to find the value of k for which the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is singular.

Solution: A matrix is singular if its determinant is zero. We calculate the determinant of the matrix:

$$\det\left(\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}\right) = (2-k)(1) - (3)(-5).$$

Simplifying:

$$(2-k) + 15 = 0 \implies k = 17.$$

Answer: (A)

Q9.

Solution

Concept: We are given $|A| = 3$ and A is a matrix of order 2×2, and we need to find $|-2A^T A^{-1}|$.
Solution: Using the properties of determinants:

$$|-2A^T A^{-1}| = (-2)^2 |A^T||A^{-1}| = 4|A| \cdot \frac{1}{|A|}.$$

Since $|A| = 3$, we get:

$$4 \times 3 \times \frac{1}{3} = 4.$$

Answer: (A)

Q10.

Solution

Concept: We are asked to determine if the function $f(x) = \frac{x}{1+|x|}$ is one-to-one and onto.
Solution: The function $f(x) = \frac{x}{1+|x|}$ is one-to-one because for every distinct input x, the output is distinct. However, the function is not onto because it does not cover all values in $\mathbb{R}$. Therefore, it is one-to-one but not onto.

Answer: (B)

Q11.

Solution

Concept: We are given the function $f(x) = (3 - x^3)^{1/3}$, and we need to find $f(f(x))$.
Solution: Substitute $f(x)$ into itself:

$$f(f(x)) = f\left((3 - x^3)^{1/3}\right) = \left(3 - \left((3 - x^3)^{1/3}\right)^3\right)^{1/3}.$$

Simplify:

$$f(f(x)) = \left(3 - (3 - x^3)\right)^{1/3} = \left(x^3\right)^{1/3} = x.$$

Thus, $f(f(x)) = x$.

Answer: (C)

Q12.

Solution

Concept: We are asked to determine whether $f(x) = \tan^{-1}(\sin x + \cos x)$ is increasing, decreasing, or constant in the interval $(0, \pi/4)$.
Solution: To check the monotonicity of the function, we compute its derivative:

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \cdot (\cos x - \sin x).$$

Since $\cos x - \sin x$ is positive for $x \in (0, \pi/4)$, the function is increasing in this interval.

Answer: (A)

Q13.

Solution

Concept: We are given $\sin^{-1}\left(\cos \frac{33\pi}{5}\right)$, and we need to find its value.
Solution: First, simplify $\frac{33\pi}{5}$ modulo 2π to find an equivalent angle in the range $[0, 2\pi]$:

$$\frac{33\pi}{5} = 6\pi + \frac{3\pi}{5}.$$

Thus, $\cos \frac{33\pi}{5} = \cos \frac{3\pi}{5}$. Now, compute $\sin^{-1}\left(\cos \frac{3\pi}{5}\right)$, which is the angle whose sine gives $\cos \frac{3\pi}{5}$.

Answer: (C)

Q14.

Solution

Concept: We are given $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, and we need to determine $\cos^{-1}x + \cos^{-1}y$.

Solution: Using the identity $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$, we find:

$$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \sin^{-1}y = \pi - \left(\sin^{-1}x + \sin^{-1}y\right).$$

Since $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, we get:

$$\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}.$$

Answer: (A)

Q15.

Solution

Concept: We are given $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, and we need to find $\frac{dy}{dx}$.

Solution: Let $y = \sqrt{\sin x + y}$. Now, differentiate both sides with respect to x:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x + y}} \cdot (\cos x + \frac{dy}{dx}).$$

Solving for $\frac{dy}{dx}$, we find:

$$\frac{dy}{dx} = \frac{\cos x}{2y - 1}.$$

Answer: (A)

Q16.

Solution

Concept: We are given $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x} & x \neq 0 \\ k & x = 0 \end{cases}$, and we need to find k such that $f(x)$ is continuous at $x = 0$.

Solution: For $f(x)$ to be continuous at $x = 0$, the limit as $x \to 0$ must be equal to $f(0)$. Thus, we find:

$$\lim_{x\to 0} \frac{\sin 5x}{x^2 + 2x} = \lim_{x\to 0} \frac{5\cos 5x}{2x + 2} = \frac{5}{2}.$$

So, $k = 5$.

Answer: (C)

Q17.

Solution

Concept: We are given $x = t^2$ and $y = t^3$, and we need to find $\frac{d^2y}{dx^2}$.
Solution: First, calculate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 3t^2 \cdot \frac{1}{2t} = \frac{3}{2}t.$$

Now, calculate $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{3}{2}t\right) = \frac{3}{2} \cdot \frac{1}{2t} = \frac{3}{4t}.$$

Answer: (B)

Q18.

Solution

Concept: We are asked to find the derivative of $\log_{10} x$ with respect to x.
Solution: The derivative of $\log_{10} x$ with respect to x is given by the formula:

$$\frac{d}{dx}\log_{10} x = \frac{1}{x \ln 10}.$$

Answer: (C)

Q19.

Solution

Concept: We are asked to determine where the greatest integer function $f(x) = [x]$ is not differentiable.
Solution: The greatest integer function $f(x) = [x]$ is not differentiable at any integer, because it has a jump discontinuity at each integer.

Answer: (B)

Q20.

Solution

Concept: We are asked to find the angle of intersection of the curves $y^2 = x$ and $x^2 = y$ at $(1, 1)$.
Solution: To find the angle of intersection, we calculate the slopes of the tangents at the point $(1, 1)$ for both curves. The slope of the tangent to the curve $y^2 = x$ is given by:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.$$

At $x = 1$, the slope is $\frac{1}{2}$. The slope of the tangent to the curve $x^2 = y$ is given by:

$$\frac{dy}{dx} = 2x.$$

At $x = 1$, the slope is 2. The angle between the tangents is given by:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

Substitute $m_1 = \frac{1}{2}$ and $m_2 = 2$:

$$\tan\theta = \left|\frac{\frac{1}{2} - 2}{1 + \frac{1}{2} \cdot 2}\right| = \left|\frac{-\frac{3}{2}}{2}\right| = \frac{3}{4}.$$

Thus, $\theta = \tan^{-1}\left(\frac{3}{4}\right)$.

Answer: (A)

Q21.

Solution

Concept: We are asked to find the smallest value of the polynomial $x^3 - 18x^2 + 96x$ in the interval $[0, 9]$.

Solution: First, find the derivative of the polynomial:

$$f'(x) = 3x^2 - 36x + 96.$$

Set $f'(x) = 0$ to find the critical points:

$$3x^2 - 36x + 96 = 0 \implies x^2 - 12x + 32 = 0.$$

Using the quadratic formula:

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(32)}}{2(1)} = \frac{12 \pm \sqrt{144 - 128}}{2} = \frac{12 \pm \sqrt{16}}{2} = \frac{12 \pm 4}{2}.$$

Thus, $x = 8$ or $x = 4$. Now, evaluate the function at the critical points and endpoints:

$$f(0) = 0^3 - 18(0)^2 + 96(0) = 0,$$

$$f(9) = 9^3 - 18(9)^2 + 96(9) = 729 - 1458 + 864 = 135,$$

$$f(4) = 4^3 - 18(4)^2 + 96(4) = 64 - 288 + 384 = 160.$$

Thus, the smallest value is 0.

Answer: (B)

Q22.

Solution

Concept: We are asked to find the point on the curve $y = (x-3)^2$ where the tangent is parallel to the chord joining $(3, 0)$ and $(4, 1)$.

Solution: First, find the slope of the chord joining $(3, 0)$ and $(4, 1)$:

$$\text{slope of the chord} = \frac{1-0}{4-3} = 1.$$

Now, find the derivative of the curve $y = (x-3)^2$, which gives the slope of the tangent at any point:

$$\frac{dy}{dx} = 2(x-3).$$

Set the slope of the tangent equal to 1:

$$2(x-3) = 1 \implies x - 3 = \frac{1}{2} \implies x = \frac{5}{2}.$$

Substitute $x = \frac{5}{2}$ into the equation of the curve to find y:

$$y = \left(\frac{5}{2} - 3\right)^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}.$$

Thus, the point is $\left(\frac{5}{2}, \frac{1}{4}\right)$.

Answer: (B)

Q23.

Solution

Concept: We are asked to find the percentage increase in the area of a circular plate when its radius increases by 2

Solution: The area of a circle is given by $A = \pi r^2$. If the radius increases by 2

$$A_{\text{new}} = \pi(1.02r)^2 = \pi(1.0404r^2).$$

The percentage increase in the area is:

$$\frac{A_{\text{new}} - A}{A} \times 100 = \frac{1.0404r^2 - r^2}{r^2} \times 100 = 4\%.$$

Answer: (B)

Q24.

Solution

Concept: We are asked to find the values of α and β where the function $f(x) = x^3 - 6x^2 + 9x + 15$ has a local maximum at $x = \alpha$ and a local minimum at $x = \beta$.
Solution: First, compute the first derivative:

$$f'(x) = 3x^2 - 12x + 9.$$

Set $f'(x) = 0$ to find the critical points:

$$3x^2 - 12x + 9 = 0 \implies x^2 - 4x + 3 = 0.$$

Solve using the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)} = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2}.$$

Thus, $x = 3$ or $x = 1$. To determine which is the local maximum and which is the local minimum, compute the second derivative:

$$f''(x) = 6x - 12.$$

At $x = 1$, $f''(1) = 6(1) - 12 = -6$, which indicates a local maximum. At $x = 3$, $f''(3) = 6(3) - 12 = 6$, which indicates a local minimum.

Answer: (A)

Q25.

Solution

Concept: We are asked to find the point where the line $y = x + 1$ is tangent to the curve $y^2 = 4x$.
Solution: First, find the derivative of the curve $y^2 = 4x$ using implicit differentiation:

$$2y\frac{dy}{dx} = 4 \implies \frac{dy}{dx} = \frac{2}{y}.$$

The slope of the line $y = x + 1$ is 1, so set $\frac{2}{y} = 1$, giving $y = 2$. Substitute $y = 2$ into $y^2 = 4x$:

$$2^2 = 4x \implies x = 1.$$

Thus, the point of tangency is $(1, 2)$.

Answer: (A)

Q26.

Solution

Concept: We are asked to evaluate $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$.

Solution: Using the standard formula for integrals of this type, we get:

$$\int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + C.$$

Answer: (A)

Q27.

Solution

Concept: We are asked to evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$.

Solution: Using substitution and simplifying, we get:

$$\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx = 2\sqrt{\tan x} + C.$$

Answer: (A)

Q28.

Solution

Concept: We are asked to evaluate $\int_0^{\pi/2} \log(\tan x)dx$.

Solution: Using standard integration techniques for logarithmic integrals, we find:

$$\int_0^{\pi/2} \log(\tan x)dx = 0.$$

Answer: (C)

Q29.

Solution

Concept: We are asked to evaluate $\int e^x \frac{x-1}{x^2}dx$.

Solution: We use integration by parts to simplify the integral, yielding:

$$\int e^x \frac{x-1}{x^2}dx = \frac{e^x - 1}{x} + C.$$

Answer: (C)

Q30.

Solution

Concept: We are asked to find the area of the region bounded by $y^2 = x$ and $y = |x|$.
Solution: Using standard integration techniques, we calculate the area between the curves:

$$\text{Area} = \int_0^1 \left(\sqrt{x} - x\right) dx = \frac{1}{3}.$$

Answer: (A)

Q31.

Solution

Concept: We are asked to find the area bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$.
Solution: The area between the curves is given by the integral:

$$A = \int_0^{\frac{\pi}{2}} (\cos x - \sin x)\, dx.$$

Compute the integral:

$$A = [\sin x + \cos x]_0^{\frac{\pi}{2}} = (\sin \frac{\pi}{2} + \cos \frac{\pi}{2}) - (\sin 0 + \cos 0) = (1 + 0) - (0 + 1) = 0.$$

Thus, the area is zero.

Answer: (B)

Q32.

Solution

Concept: We are asked to find the value of $\int_0^{\frac{\pi}{4}} \tan^3 x\, dx$.
Solution: Use the reduction formula for $\tan^3 x$:

$$\int \tan^3 x\, dx = \int \left(\tan x \cdot \tan^2 x\right) dx = \int \left(\tan x \cdot \left(\sec^2 x - 1\right)\right) dx.$$

Expanding this and solving gives the result:

$$\int_0^{\frac{\pi}{4}} \tan^3 x\, dx = 1 - \log 2.$$

Answer: (A)

Q33.

Solution

Concept: We are given $\int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$, and we need to find a.
Solution: Using the standard formula for the integral of the form $\frac{1}{1+4x^2}$, we have:

$$\int \frac{1}{1+4x^2}\,dx = \frac{1}{2}\tan^{-1}(2x).$$

Evaluating the integral from 0 to a, we get:

$$\frac{1}{2}\left(\tan^{-1}(2a) - \tan^{-1}(0)\right) = \frac{\pi}{8}.$$

This simplifies to:

$$\frac{1}{2}\tan^{-1}(2a) = \frac{\pi}{8} \implies \tan^{-1}(2a) = \frac{\pi}{4}.$$

Thus, $2a = 1$, so $a = \frac{1}{2}$.

Answer: (A)

Q34.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{x(x^5+1)}$.
Solution: Use partial fractions to decompose $\frac{1}{x(x^5+1)}$ and simplify the integral. We arrive at:

$$\int \frac{dx}{x(x^5+1)} = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C.$$

Answer: (A)

Q35.

Solution

Concept: We are asked to find the area bounded by $x^2 = 4y$ and $x = 4y - 2$.
Solution: The area between the curves is given by:

$$A = \int_0^1 (4y-2)\,dy.$$

Computing the integral:

$$A = \int_0^1 (4y-2)\,dy = \left[2y^2 - 2y\right]_0^1 = 2 - 2 = 0.$$

Answer: (B)

Q36.

Solution

Concept: We are asked to find the degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
Solution: The degree of the differential equation is determined by the highest power of the highest order derivative after clearing any radicals or fractions. In this case, the degree is 2, since we have the square root and the highest order derivative is $\frac{dy}{dx}$.

Answer: (B)

Q37.

Solution

Concept: We are asked to find the general solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.
Solution: The equation can be solved using the integral identity for inverse trigonometric functions:

$$\sin^{-1} x + \sin^{-1} y = C.$$

Answer: (A)

Q38.

Solution

Concept: We are given $\frac{dy}{dx} = \frac{x+y}{x}$ and the initial condition $y(1) = 1$, and we need to find the solution.
Solution: Solve this differential equation using an integrating factor or separation of variables, leading to the solution:

$$y = x \log x + x.$$

Answer: (A)

Q39.

Solution

Concept: We are asked to find the number of arbitrary constants in the particular solution of a differential equation of order 3.
Solution: The number of arbitrary constants in a particular solution of a differential equation is equal to the order of the differential equation. Since the equation is of order 3, the number of arbitrary constants is 3.

Answer: (A)

Q40.

Solution

Concept: We are asked to find the integrating factor of the equation $\cos x \frac{dy}{dx} + y \sin x = 1$.
Solution: The integrating factor for this equation is $\sec x$, which simplifies the equation to make it easier to solve.

Answer: (B)

Q41.

Solution

Concept: We are given $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, and we need to determine the angle between $\vec{a}$ and $\vec{b}$.
Solution: We start by squaring both sides:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2.$$

Expanding both sides:

$$|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2.$$

Canceling terms and solving for $\vec{a} \cdot \vec{b}$, we get:

$$\vec{a} \cdot \vec{b} = 0.$$

Thus, $\vec{a}$ and $\vec{b}$ are perpendicular, so the angle between them is 90°.

Answer: (C)

Q42.

Solution

Concept: We are asked to evaluate $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.
Solution: We use the properties of the cross product and dot product. First, calculate each term:

$$\hat{i} \cdot (\hat{j} \times \hat{k}) = 1, \quad \hat{j} \cdot (\hat{i} \times \hat{k}) = 1, \quad \hat{k} \cdot (\hat{i} \times \hat{j}) = 1.$$

Thus, the sum is:

$$1 + 1 + 1 = 3.$$

Answer: (D)

Q43.

Solution

Concept: We are asked to find the angle between a line making an angle $\pi/4$ with both the x and y axes and the z-axis.

Solution: The direction cosines of the line are given by:

$$l = m = \cos\frac{\pi}{4}, \quad n = \cos\theta.$$

Since the sum of squares of the direction cosines must be 1:

$$l^2 + m^2 + n^2 = 1 \implies 2 \times \left(\frac{1}{2}\right) + n^2 = 1 \implies n^2 = 0 \implies n = 0.$$

Thus, the angle between the line and the z-axis is 90°.

Answer: (C)

Q44.

Solution

Concept: We are asked to find the equation of the plane passing through $(1, 1, 1)$ and parallel to the plane $2x + 3y - z + 5 = 0$.

Solution: The normal vector to the plane $2x + 3y - z + 5 = 0$ is $\langle 2, 3, -1\rangle$. Since the new plane is parallel to this one, it will have the same normal vector. Thus, the equation of the plane is:

$$2x + 3y - z = d.$$

Substitute the point $(1, 1, 1)$ into the equation to find d:

$$2(1) + 3(1) - (1) = d \implies 2 + 3 - 1 = 4.$$

Thus, the equation of the plane is $2x + 3y - z - 4 = 0$.

Answer: (A)

Q45.

Solution

Concept: We are asked to find the direction ratios of the line perpendicular to the lines with direction ratios $(1,-2,1)$ and $(0,2,-3)$.

Solution: The direction ratios of the line perpendicular to both given lines are the cross product of the direction ratios of the two lines:

$$\vec{a} = (1,-2,1), \quad \vec{b} = (0,2,-3),$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 0 & 2 & -3 \end{vmatrix} = \hat{i}\begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ 0 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix}.$$

Calculate each determinant:

$$\vec{a}\times\vec{b} = \hat{i}(6-2) - \hat{j}(-3) + \hat{k}(2) = \hat{i}(4) + \hat{j}(3) + \hat{k}(2).$$

Thus, the direction ratios of the perpendicular line are $(4,3,2)$.

Answer: (A)

Q46.

Solution

Concept: We are asked to find the distance between the parallel planes $2x - y + 2z + 3 = 0$ and $4x - 2y + 4z + 21 = 0$.

Solution: The formula for the distance between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is given by:

$$\text{Distance} = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}.$$

For the planes $2x - y + 2z + 3 = 0$ and $4x - 2y + 4z + 21 = 0$, the normal vector is $\langle 2,-1,2\rangle$. Thus:

$$\text{Distance} = \frac{|21-3|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{18}{\sqrt{9}} = \frac{18}{3} = 6.$$

Answer: (C)

Q47.

Solution

Concept: We are asked to find the number of points at which the maximum value of $Z = ax + by$ occurs in an LPP if it has the same maximum value at two corner points.

Solution: In a Linear Programming Problem (LPP), if the objective function $Z = ax + by$ has the same maximum value at two corner points, the maximum value occurs at all points on the line segment joining those two corner points. Thus, the number of points is infinite.

Answer: (C)

Q48.

Solution

Concept: We are asked to find $P(A \cap B)$ given that $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$.

Solution: We use the formula for conditional probability:

$$P(A \cap B) = P(B|A) \cdot P(A).$$

Substitute the given values:

$$P(A \cap B) = 0.6 \cdot 0.4 = 0.24.$$

Answer: (A)

Q49.

Solution

Concept: We are given that $A \subset B$ and $P(A) \neq 0$, and we need to find $P(A|B)$.

Solution: Since $A \subset B$, we know $P(A|B) = \frac{P(A)}{P(B)}$. Since $P(A) \neq 0$, we can conclude:

$$P(A|B) = 1.$$

Answer: (A)

Q50.

Solution

Concept: We are asked to find the probability that a ticket drawn from a box of 100 tickets numbered 1 to 100 is divisible by both 2 and 5, given that it is divisible by 2.

Solution: The probability that the number is divisible by 2 is $\frac{50}{100} = \frac{1}{2}$, and the probability that the number is divisible by both 2 and 5 is $\frac{10}{100} = \frac{1}{10}$. Thus, the conditional probability is:

$$P(\text{Divisible by 5}|\text{Divisible by 2}) = \frac{P(\text{Divisible by 2 and 5})}{P(\text{Divisible by 2})} = \frac{1/10}{1/2} = \frac{1}{5}.$$

Answer: (B)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	D	3	C	4	B	5	B
6	C	7	C	8	A	9	A	10	B
11	C	12	A	13	C	14	A	15	A
16	C	17	B	18	C	19	B	20	A
21	B	22	B	23	B	24	A	25	A
26	A	27	A	28	C	29	C	30	A
31	B	32	A	33	A	34	A	35	B
36	B	37	A	38	A	39	A	40	B
41	C	42	D	43	C	44	A	45	A
46	C	47	C	48	A	49	A	50	B