CUET-UG Mathematics Sample Paper-4

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to:

(A) A

(B) I

(C) 0

(D) $2A$

Q2. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, then $|3AB|$ is:

(A) -9

(B) -81

(C) -27

(D) 81

Q3. The number of all possible symmetric matrices of order 3×3 with each entry 0 or 1 is:

(A) 2^9

(B) 2^6

(C) 2^3

(D) 2^{27}

Q4. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then $A^k - (k-1)I$ is:

(A) $\begin{bmatrix} 1 & ka \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & ka \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 2-k & ka \\ 0 & 2-k \end{bmatrix}$

Q5. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units, then k is:

(A) 12

(B) -2

(C) $12, -2$

(D) $12, 2$

Q6. If A is a non-singular matrix, then $det(A^{-1})$ is:

(A) $1/det(A)$

(B) $det(A)$

(C) 1

(D) 0

Q7. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Then $|A \cdot adjA|$ is:

(A) 11

(B) 121

(C) 0

(D) 18

Q8. For what value of x is the matrix $\begin{bmatrix} x+1 & -3 \\ 2 & x+6 \end{bmatrix}$ singular?

(A) $0, -1$

(B) $-3, -4$

(C) $3, 4$

(D) $2, 3$

Q9. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then A^4 (where $i = \sqrt{-1}$) is:

(A) I

(B) $-I$

(C) A

(D) 0

Q10. Let $f : A \to B$ be an onto function where $n(A) = 5$ and $n(B) = 4$. The total number of such functions is:

(A) 4^5

(B) 240

(C) 120

(D) 60

Q11. The relation R in the set of real numbers R defined as $R = \{(a, b) : a \leq b^3\}$ is:

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) None of these

Q12. If $\sin^{-1} x = y$, then:

(A) $0 \leq y \leq \pi$

(B) $-\pi/2 \leq y \leq \pi/2$

(C) $0 < y < \pi$

(D) $-\pi/2 < y < \pi/2$

Q13. The value of $\cos^{-1}(\cos \frac{7\pi}{6})$ is:

(A) $7\pi/6$

(B) $5\pi/6$

(C) $\pi/6$

(D) $-\pi/6$

Q14. If $f(x) = \log(\frac{1+x}{1-x})$, then $f(\frac{2x}{1+x^2})$ is:

(A) $2f(x)$

(B) $[f(x)]^2$

(C) $3f(x)$

(D) $f(x)$

Q15. The function $f(x) = x^3 - 3x^2 + 3x - 100$ is:

(A) Always increasing

(B) Always decreasing

(C) Increasing for $x > 1$ only

(D) Decreasing for $x < 1$ only

Q16. If $y = \log_x e$, then dy/dx is:

(A) $1/x$

(B) $-1/(x(\log_e x)^2)$

(C) $1/(x \log_e x)$

(D) e^x

Q17. If $f(x) = |x|^3$, then $f''(0)$ is:

(A) 0

(B) 6

(C) Does not exist

(D) 1

Q18. The derivative of $\cos^{-1}(2x^2 - 1)$ with respect to $\cos^{-1} x$ is:

(A) 2

(B) $2/x$

(C) $1/2$

(D) -2

Q19. The function $f(x) = x + 1/x$ has a local minimum at:

(A) $x = 1$

(B) $x = -1$

(C) $x = 0$

(D) $x = 2$

Q20. The normal to the curve $x^2 = 4y$ passing through $(1, 2)$ is:

(A) $x + y = 3$

(B) $x - y = 3$

(C) $x + y = 1$

(D) $y - x = 1$

Q21. The rate of change of the volume of a cube with respect to its edge x when $x = 5$ cm is:

(A) 25

(B) 50

(C) 75

(D) 125

Q22. The function $f(x) = \tan x - x$:

(A) Always increases

(B) Always decreases

(C) Increases in $(0, \pi/2)$

(D) Decreases in $(0, \pi/2)$

Q23. The maximum value of $x^{1/x}$ is at $x =$:

(A) e

(B) $1/e$

(C) 1

(D) 2

Q24. If the sub-tangent at any point on the curve $y = a^x$ is of constant length, then the length is:

(A) $1/\log a$

(B) $\log a$

(C) a

(D) $1/a$

Q25. If $y = \sin^{-1} x$, then $(1 - x^2)y_2 - xy_1$ is:

(A) 0

(B) 1

(C) -1

(D) y

Q26. If $\int \frac{dx}{x+x\log x}$ is:

(A) $\log|1 + \log x| + C$

(B) $\log|\log x| + C$

(C) $\frac{1}{(1+\log x)^2} + C$

(D) $\tan^{-1}(\log x) + C$

Q27. If $\int_0^{\pi} |\cos x| dx$ is:

(A) 0

(B) 1

(C) 2

(D) 4

Q28. If $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}} dx$ is:

(A) $\frac{x^3}{3} + C$

(B) $\frac{x^2}{2} + C$

(C) $\frac{x}{2} + C$

(D) $x^2 + C$

Q29. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is:

(A) $\frac{\pi}{8}\log 2$

(B) $\frac{\pi}{4}\log 2$

(C) $\frac{\pi}{2}\log 2$

(D) 0

Q30. The area bounded by the curve $y^2 = 8x$ and its latus rectum is:

(A) $\frac{8}{3}$

(B) $\frac{16}{3}$

(C) $\frac{32}{3}$

(D) $\frac{64}{3}$

Q31. $\int e^x(1 + \tan x + \tan^2 x) dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \sec x + C$

(C) $e^x \tan^2 x + C$

(D) $e^x(1 + \tan x) + C$

Q32. The value of $\int_{-1}^{1} x|x|dx$ is:

(A) 2

(B) 1

(C) 0

(D) $\frac{2}{3}$

Q33. $\int \frac{dx}{e^x+e^{-x}}$ is:

(A) $\tan^{-1}(e^x) + C$

(B) $\log(e^x + e^{-x}) + C$

(C) $e^x - e^{-x} + C$

(D) $\tan^{-1}(e^{-x}) + C$

Q34. The area of the region bounded by $y = \sin x$, the x-axis and the lines $x = 0, x = \pi$ is:

(A) 1

(B) 2

(C) 0

(D) 4

Q35. $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x} dx$ is:

(A) $\pi/4$

(B) $\pi/2$

(C) $n\pi/4$

(D) 1

Q36. The general solution of $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^x = e^y + \frac{x^3}{3} + C$

(C) $e^y = e^x + x^3 + C$

(D) $y = x + \log x + C$

Q37. The order of the differential equation of all conics whose center is at origin is:

(A) 2

(B) 3

(C) 4

(D) 5

Q38. If $I.F. = \frac{1}{x^2}$, the differential equation could be:

(A) $\frac{dy}{dx} - \frac{2}{x}y = x$

(B) $\frac{dy}{dx} + \frac{2}{x}y = x$

(C) $\frac{dy}{dx} - \frac{1}{x}y = x$

(D) $\frac{dy}{dx} + \frac{1}{x}y = x$

Q39. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$ is:

(A) 2

(B) 1

(C) 3

(D) Not defined

Q40. The differential equation representing the family of curves $y = e^{mx}$ (m is arbitrary constant) is:

(A) $\frac{dy}{dx} = \frac{y}{x}\log y$

(B) $\frac{dy}{dx} = \frac{x}{y}\log y$

(C) $y\frac{dy}{dx} = \log y$

(D) $\frac{dy}{dx} = y\log y$

Q41. If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0

(B) $\pi/2$

(C) π

(D) $\pi/4$

Q42. The position vector of the point which divides the join of $(1, 2, 1)$ and $(-1, 1, 1)$ in the ratio $1 : 1$ is:

(A) $\hat{j} + \hat{k}$

(B) $\frac{3}{2}\hat{j} + \hat{k}$

(C) $\hat{i} + \frac{3}{2}\hat{j}$

(D) $0\hat{i} + 0\hat{j} + 0\hat{k}$

Q43. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then $|\vec{a} \times \vec{b}|$ is:

(A) 12

(B) 16

(C) 14

(D) 20

Q44. The coordinates of the foot of the perpendicular from the origin to the plane $2x - 3y + 4z - 6 = 0$ are:

(A) $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$

(B) $\left(\frac{2}{29}, \frac{-3}{29}, \frac{4}{29}\right)$

(C) $(2, -3, 4)$

(D) $(1, 1, 1)$

Q45. If the direction cosines of a line are k, k, k, then:

(A) $k = 1$

(B) $k = 1/3$

(C) $k = \pm 1/\sqrt{3}$

(D) $k = \pm 1/\sqrt{2}$

Q46. The distance of the point $(2, 5, -3)$ from the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$ is:

(A) $13/7$

(B) $11/7$

(C) $9/7$

(D) 7

Q47. The corner points of the feasible region for an LPP are $(0, 0)$, $(5, 0)$, $(3, 4)$ and $(0, 5)$. The maximum value of $Z = 4x + 3y$ is:

(A) 20

(B) 24

(C) 15

(D) 18

Q48. If $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, then $P(A'/B')$ is:

(A) $1/3$

(B) $3/7$

(C) $4/7$

(D) $1/7$

Q49. If A and B are two events such that $P(A|B) = P(B|A)$, then:

(A) $A \subset B$

(B) $B \subset A$

(C) $P(A) = P(B)$

(D) $A \cap B = \emptyset$

Q50. The probability of drawing a king or a heart from a deck of 52 cards is:

(A) 4/13

(B) 17/52

(C) 1/13

(D) 16/52

Detailed Solutions

Q1.

Solution

Concept: We are given that A is a square matrix and $A^2 = I$, where I is the identity matrix. This means that A is its own inverse.

Solution: We can use the property of the inverse of a matrix:

$$A \cdot A = I \quad \text{implies} \quad A^{-1} = A$$

Answer: (A)

Q2.

Solution

Concept: We are given two square matrices A and B with orders 3, and the determinants $|A| = -1$ and $|B| = 3$. We need to calculate $|3AB|$.

Solution: We use the property of determinants:

$$|3AB| = |3| \cdot |A| \cdot |B| = 3^3 \cdot |A| \cdot |B| = 27 \cdot (-1) \cdot 3 = -81$$

Answer: (B)

Q3.

Solution

Concept: The number of all possible symmetric matrices of order 3×3 with each entry being either 0 or 1.

Solution: A symmetric matrix is one where $a_{ij} = a_{ji}$. Therefore, for a 3×3 matrix, the diagonal elements can be chosen independently as 0 or 1 (2 choices for each). The off-diagonal elements are paired, so each pair can also be chosen independently (2 choices for each pair). Thus, the total number of symmetric matrices is:

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$$

Answer: (B)

Q4.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and asked to find $A^k - (k-1)I$, where I is the identity matrix.

Solution: We first calculate A^k:

$$A^k = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & ka \\ 0 & 1 \end{bmatrix}$$

Now, subtract $(k-1)I$:

$$A^k - (k-1)I = \begin{bmatrix} 1 & ka \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} k-1 & 0 \\ 0 & k-1 \end{bmatrix} = \begin{bmatrix} 1-(k-1) & ka \\ 0 & 1-(k-1) \end{bmatrix} = \begin{bmatrix} 2-k & ka \\ 0 & 2-k \end{bmatrix}$$

Answer: (D)

Q5.

Solution

Concept: We are given the vertices of a triangle and asked to find the value of k such that the area is 35 square units.

Solution: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substitute the given points $(2, -6)$, $(5, 4)$, $(k, 4)$:

$$35 = \frac{1}{2} |2(4-4) + 5(4-(-6)) + k((-6)-4)|$$

$$35 = \frac{1}{2} |0 + 50 - 10k|$$
$$70 = |50 - 10k|$$

Solving $50 - 10k = 70$ or $50 - 10k = -70$:

$$k = -2 \quad \text{or} \quad k = 12$$

Answer: (C)

Q6.

Solution

Concept: We are given that A is a non-singular matrix, and we need to find $\det(A^{-1})$.
Solution: The property of determinants states that $\det(A^{-1}) = \frac{1}{\det(A)}$. Since A is non-singular, $\det(A) \neq 0$.

Answer: (A)

Q7.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, and we are asked to find $\det(A \cdot \text{adj}(A))$.
Solution: We know that for any matrix A, $A \cdot \text{adj}(A) = \det(A) \cdot I$, where I is the identity matrix. Therefore:

$$\det(A \cdot \text{adj}(A)) = \det(\det(A) \cdot I) = \det(A)^2$$

First, calculate $\det(A)$:

$$\det(A) = 2 \times 4 - (-1) \times 3 = 8 + 3 = 11$$

Thus, $\det(A \cdot \text{adj}(A)) = 11^2 = 121$.

Answer: (B)

Q8.

Solution

Concept: We are given a matrix $\begin{bmatrix} x+1 & -3 \\ 2 & x+6 \end{bmatrix}$, and we need to find the value of x for which the matrix is singular, i.e., $\det(A) = 0$.
Solution: The determinant of the given matrix is:

$$\det \begin{bmatrix} x+1 & -3 \\ 2 & x+6 \end{bmatrix} = (x+1)(x+6) - (-3)(2) = (x+1)(x+6) + 6$$

$$= x^2 + 7x + 6 + 6 = x^2 + 7x + 12$$

Set the determinant equal to 0:

$$x^2 + 7x + 12 = 0$$

Solving the quadratic equation:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(12)}}{2(1)} = \frac{-7 \pm \sqrt{49 - 48}}{2} = \frac{-7 \pm 1}{2}$$

So, $x = -3$ or $x = -4$.

Answer: (B)

Q9.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ and asked to find A^4, where $i = \sqrt{-1}$.

Solution: First, we compute powers of A:

$$A^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}^2 = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$$A^4 = (A^2)^2 = (-I)^2 = I$$

Answer: (A)

Q10.

Solution

Concept: We are given that $f : A \to B$ is an onto function, where $n(A) = 5$ and $n(B) = 4$. We need to find the total number of such functions.

Solution: The total number of onto functions from a set of $n(A)$ elements to a set of $n(B)$ elements is given by the formula $n(B)^{n(A)}$, but since the function is onto, we must consider the number of surjective functions:

$$\text{Number of onto functions} = 4^5 = 1024$$

Answer: (D)

Q11.

Solution

Concept: We are given the relation $R = \{(a, b) : a \leq b^3\}$ on the set of real numbers. We need to determine whether the relation is reflexive, symmetric, or transitive.

Solution: 1. A relation R is reflexive if $(a, a) \in R$ for all $a \in R$. For $a \leq a^3$, this is true because $a^3 \geq a$ for all real numbers a. Therefore, the relation is reflexive. 2. A relation R is symmetric if $(a, b) \in R$ implies $(b, a) \in R$. This is not true because if $a \leq b^3$, it does not necessarily imply $b \leq a^3$. Therefore, the relation is not symmetric. 3. A relation R is transitive if $(a, b) \in R$ and $(b, c) \in R$ imply $(a, c) \in R$. This is true because if $a \leq b^3$ and $b \leq c^3$, then $a \leq (c^3)^3 = c^9$, which satisfies the transitivity condition.

Thus, the correct answer is: Reflexive.

Answer: (A)

Q12.

Solution

Concept: We are given $\sin^{-1} x = y$, and we need to determine the range of y.

Solution: The range of $\sin^{-1} x$ is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, which means y lies in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Answer: (B)

Q13.

Solution

Concept: We are asked to evaluate $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

Solution: The principal value of $\cos^{-1}(x)$ lies in the interval $[0, \pi]$. Since $\frac{7\pi}{6}$ is outside this range, we adjust it to $\frac{5\pi}{6}$. Thus:

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \frac{5\pi}{6}$$

Answer: (B)

Q14.

Solution

Concept: We are asked to find the value of $f\left(\frac{2x}{1+x^2}\right)$ given $f(x) = \log\left(\frac{1+x}{1-x}\right)$.

Solution: Using the property of logarithms, we can express $f\left(\frac{2x}{1+x^2}\right)$ as:

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1 + \frac{2x}{1+x^2}}{1 - \frac{2x}{1+x^2}}\right)$$

Simplifying this expression results in:

$$f\left(\frac{2x}{1+x^2}\right) = 2f(x)$$

Answer: (A)

Q15.

Solution

Concept: We are given the function $f(x) = x^3 - 3x^2 + 3x - 100$, and we need to determine whether the function is always increasing, decreasing, or has specific intervals of increase or decrease.
Solution: First, we compute the first derivative:

$$f'(x) = 3x^2 - 6x + 3$$

The critical points are found by setting $f'(x) = 0$:

$$3x^2 - 6x + 3 = 0 \quad \Rightarrow \quad x^2 - 2x + 1 = 0 \quad \Rightarrow \quad (x-1)^2 = 0$$

Thus, $x = 1$ is the only critical point. Checking the second derivative at $x = 1$:

$$f''(x) = 6x - 6 \quad \Rightarrow \quad f''(1) = 0$$

This tells us that the function changes its behavior at $x = 1$, so it increases for $x > 1$.

Answer: (C)

Q16.

Solution

Concept: We are asked to find $\frac{dy}{dx}$ for the function $y = \log_x e$.
Solution: The derivative of $\log_x e$ is given by:

$$\frac{dy}{dx} = \frac{1}{x \log_e x}$$

Answer: (C)

Q17.

Solution

Concept: We are asked to find $f''(0)$ for the function $f(x) = |x|^3$.
Solution: The function $f(x) = |x|^3$ is not differentiable at $x = 0$ because the left-hand derivative and the right-hand derivative are different at this point.
Thus, $f''(0)$ does not exist.

Answer: (C)

Q18.

Solution

Concept: We are asked to find the derivative of $\cos^{-1}(2x^2 - 1)$ with respect to $\cos^{-1} x$.

Solution: Using the chain rule:

$$\frac{d}{dx}\cos^{-1}(2x^2 - 1) = \frac{-1}{\sqrt{1 - (2x^2 - 1)^2}} \cdot 4x$$

Thus, the derivative with respect to $\cos^{-1} x$ is -2.

Answer: (D)

Q19.

Solution

Concept: We are given the function $f(x) = x + \frac{1}{x}$, and we need to determine where it has a local minimum.

Solution: First, we compute the first derivative:

$$f'(x) = 1 - \frac{1}{x^2}$$

Setting $f'(x) = 0$, we solve for x:

$$1 - \frac{1}{x^2} = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm 1$$

Checking the second derivative at $x = 1$:

$$f''(x) = \frac{2}{x^3}$$

At $x = 1$, $f''(1) = 2$, which is positive, indicating a local minimum at $x = 1$.

Answer: (A)

Q20.

Solution

Concept: We are given the curve $x^2 = 4y$ and need to find the normal to the curve passing through the point $(1, 2)$.

Solution: The derivative of $x^2 = 4y$ is:

$$2x = 4\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{x}{2}$$

At the point $(1, 2)$, the slope of the tangent is $\frac{dy}{dx} = \frac{1}{2}$. The slope of the normal is the negative reciprocal:

$$\text{slope of normal} = -2$$

Using the point-slope form of the equation of the normal:

$$y - 2 = -2(x - 1)$$

Simplifying:

$$y = -2x + 4$$

Thus, the equation of the normal is $x + y = 3$.

Answer: (A)

Q21.

Solution

Concept: We are given that the volume of a cube is $V = x^3$, where x is the edge of the cube. We need to find the rate of change of the volume with respect to x when $x = 5$.

Solution: The rate of change of the volume is given by the derivative of $V = x^3$ with respect to x:

$$\frac{dV}{dx} = 3x^2$$

Substituting $x = 5$:

$$\frac{dV}{dx} = 3(5)^2 = 3 \times 25 = 75$$

Thus, the rate of change of the volume is 75 cm^3 per cm.

Answer: (C)

Q22.

Solution

Concept: We are given $f(x) = \tan x - x$ and asked to determine whether this function always increases or decreases, or whether it increases or decreases in a specific interval.

Solution: The first derivative of $f(x) = \tan x - x$ is:

$$f'(x) = \sec^2 x - 1$$

Since $\sec^2 x - 1 = \tan^2 x$, we have:

$$f'(x) = \tan^2 x$$

For $0 < x < \frac{\pi}{2}$, $\tan^2 x > 0$, which means $f'(x) > 0$. Thus, $f(x)$ is increasing in the interval $(0, \pi/2)$.

Answer: (C)

Q23.

Solution

Concept: We are asked to find the maximum of $x^{1/x}$.

Solution: To find the maximum of $x^{1/x}$, we take the derivative of $f(x) = x^{1/x}$:

$$f'(x) = \frac{d}{dx}\left(x^{1/x}\right) = x^{1/x}\left(\frac{1 - \log x}{x^2}\right)$$

Setting $f'(x) = 0$, we solve for x to find the maximum point:

$$1 - \log x = 0 \quad \Rightarrow \quad x = e$$

Thus, the maximum value occurs at $x = e$.

Answer: (A)

Q24.

Solution

Concept: We are asked to find the sub-tangent for the curve $y = a^x$.

Solution: The equation of the sub-tangent is given by:

$$\text{sub-tangent} = \frac{y}{\frac{dy}{dx}}$$

For the curve $y = a^x$, we have:

$$\frac{dy}{dx} = a^x \ln a$$

Thus, the sub-tangent is:

$$\text{sub-tangent} = \frac{a^x}{a^x \ln a} = \frac{1}{\ln a}$$

So, the length of the sub-tangent is $\frac{1}{\log a}$.

Answer: (A)

Q25.

Solution

Concept: We are given the function $y = \sin^{-1} x$, and we need to find the expression for $(1 - x^2)y_2 - xy_1$.

Solution: Using the chain rule, the first derivative of $y = \sin^{-1} x$ is:

$$y_1 = \frac{1}{\sqrt{1 - x^2}}$$

The second derivative is:

$$y_2 = \frac{x}{(1 - x^2)^{3/2}}$$

Thus, we calculate:

$$(1 - x^2)y_2 - xy_1 = (1 - x^2) \cdot \frac{x}{(1 - x^2)^{3/2}} - x \cdot \frac{1}{\sqrt{1 - x^2}} = 0$$

Therefore, the value is 0.

Answer: (A)

Q26.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{x+x\log x}$.
Solution: We simplify the integral:

$$\int \frac{dx}{x + x\log x} = \int \frac{dx}{x(1+\log x)}$$

Using the substitution $u = 1 + \log x$, we get:

$$du = \frac{dx}{x}$$

Thus, the integral becomes:

$$\int \frac{du}{u} = \log|u| + C = \log|1+\log x| + C$$

So, the final answer is:

Answer: (A)

Q27.

Solution

Concept: We are asked to evaluate $\int_0^\pi |\cos x|dx$.
Solution: Since $\cos x$ is non-negative in the interval $[0, \pi/2]$ and negative in the interval $[\pi/2, \pi]$, we split the integral:

$$\int_0^\pi |\cos x|dx = \int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx$$

Evaluating both integrals:

$$\int_0^{\pi/2} \cos x\, dx = \sin x\Big|_0^{\pi/2} = 1$$

$$\int_{\pi/2}^{\pi} \cos x\, dx = -\sin x\Big|_{\pi/2}^{\pi} = 1$$

Thus, the total area is:

$$1 + 1 = 2$$

Answer: (C)

Q28.

Solution

Concept: We are asked to evaluate $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}} dx$.
Solution: First, simplify the expression using properties of logarithms:

$$e^{6\log x} = x^6, \quad e^{5\log x} = x^5, \quad e^{4\log x} = x^4, \quad e^{3\log x} = x^3$$

Thus, the integral becomes:

$$\int \frac{x^6 - x^5}{x^4 - x^3} dx = \int \frac{x^5(x-1)}{x^3(x-1)} dx = \int \frac{x^5}{x^3} dx = \int x^2 dx$$

The integral of x^2 is:

$$\frac{x^3}{3} + C$$

So, the final answer is:

Answer: (A)

Q29.

Solution

Concept: We are asked to evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$.
Solution: This integral can be computed numerically or using a table of integrals. The value is:

$$\frac{\pi}{8} \log 2$$

Answer: (A)

Q30.

Solution

Concept: We are asked to find the area bounded by the curve $y^2 = 8x$ and its latus rectum.
Solution: The latus rectum for the parabola $y^2 = 8x$ is given by the distance between the focus and the directrix. The area can be calculated using the integral:

$$\int_0^4 \sqrt{8x}\, dx = \frac{8x^{3/2}}{3}\Big|_0^4 = \frac{8 \times 8}{3} = \frac{64}{3}$$

Thus, the area is:

Answer: (C)

Q31.

Solution

Concept: We are given the integral $\int e^x(1+\tan x+\tan^2 x)dx$.

Solution: This is a straightforward integral involving the exponential function and trigonometric terms. We apply standard integration techniques for such functions, and the result is:

$$\int e^x(1+\tan x+\tan^2 x)dx = e^x \tan x + C$$

Thus, the correct answer is:

Answer: (A)

Q32.

Solution

Concept: We are asked to evaluate the integral $\int_{-1}^{1} x|x|dx$.

Solution: Since $|x| = -x$ for $x < 0$ and $|x| = x$ for $x > 0$, we split the integral as follows:

$$\int_{-1}^{1} x|x|dx = \int_{-1}^{0} x(-x)dx + \int_{0}^{1} xxdx$$

The first integral is:

$$\int_{-1}^{0} -x^2dx = \left[-\frac{x^3}{3}\right]_{-1}^{0} = 0 - \left(-\frac{(-1)^3}{3}\right) = \frac{1}{3}$$

The second integral is:

$$\int_{0}^{1} x^2dx = \left[\frac{x^3}{3}\right]_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

Adding both results gives:

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Thus, the correct answer is:

Answer: (D)

Q33.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{e^x+e^{-x}}$.

Solution: We can simplify the expression $e^x + e^{-x}$ as $2\cosh(x)$, where $\cosh(x)$ is the hyperbolic cosine function. The integral becomes:

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{2\cosh(x)}$$

This is a standard integral, and its result is:

$$\frac{1}{2}\log(e^x + e^{-x}) + C$$

Thus, the correct answer is:

Answer: (B)

Q34.

Solution

Concept: We are asked to find the area of the region bounded by the curve $y = \sin x$, the x-axis, and the lines $x = 0$ and $x = \pi$.

Solution: The area under the curve from $x = 0$ to $x = \pi$ is given by the integral:

$$\int_0^{\pi} \sin x \, dx$$

We compute this integral:

$$\int_0^{\pi} \sin x \, dx = -\cos x\Big|_0^{\pi} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$$

Thus, the correct answer is:

Answer: (B)

Q35.

Solution

Concept: We are asked to evaluate $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x} dx$.

Solution: This is a standard integral, and it is known that the value of this integral is $\pi/4$ for all n.

Thus, the correct answer is:

Answer: (A)

Q36.

Solution

Concept: We are given the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ and are asked to find its general solution.

Solution: This is a separable differential equation. To solve it, we separate the variables and integrate both sides. After solving the equation, we get the general solution:

$$e^y = e^x + \frac{x^3}{3} + C$$

Thus, the correct answer is:

Answer: (A)

Q37.

Solution

Concept: We are asked to find the order of the differential equation of all conics whose center is at the origin.

Solution: The equation of a conic can be written as $Ax^2 + By^2 = C$, and the order of this differential equation is 2. Thus, the correct answer is:

Answer: (A)

Q38.

Solution

Concept: We are given the integrating factor $I.F. = \frac{1}{x^2}$ and are asked to find the corresponding differential equation.

Solution: The differential equation with this integrating factor is:

$$\frac{dy}{dx} - \frac{2}{x}y = x$$

Thus, the correct answer is:

Answer: (A)

Q39.

Solution

Concept: We are asked to find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$.
Solution: The degree of this differential equation is not defined because it involves nonlinear terms. Thus, the correct answer is:

Answer: (D)

Q40.

Solution

Concept: We are asked to find the differential equation representing the family of curves $y = e^{mx}$, where m is an arbitrary constant.
Solution: The differential equation representing this family of curves is:

$$\frac{dy}{dx} = y \log y$$

Thus, the correct answer is:

Answer: (D)

Q41.

Solution

Concept: We are given that $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, which implies that the angle between $\vec{a}$ and $\vec{b}$ is π, i.e., 180°.
Solution: From the formula for the dot product, we know:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

Given $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, it follows that:

$$\cos\theta = -1 \quad \Rightarrow \quad \theta = \pi$$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is π radians.

Answer: (C)

Q42.

Solution

Concept: We need to find the position vector of the point dividing the line segment joining the points $(1, 2, 1)$ and $(-1, 1, 1)$ in the ratio 1:1.

Solution: The position vector of the point dividing the line segment in the ratio 1:1 (midpoint) is given by:

$$\left(\frac{1 + (-1)}{2}, \frac{2 + 1}{2}, \frac{1 + 1}{2}\right) = (0, \frac{3}{2}, 1)$$

Thus, the position vector is $\frac{3}{2}\hat{j} + \hat{k}$, which corresponds to option B.

Answer: (B)

Q43.

Solution

Concept: We are given that $|\vec{a}| = 10$, $|\vec{b}| = 2$, and $\vec{a} \cdot \vec{b} = 12$, and we need to find $|\vec{a} \times \vec{b}|$.

Solution: The magnitude of the cross product is given by:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

We are also given that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, so:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{12}{10 \times 2} = 0.6$$

Thus, $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$.

Now, we can calculate the magnitude of the cross product:

$$|\vec{a} \times \vec{b}| = 10 \times 2 \times 0.8 = 16$$

Thus, the correct answer is:

Answer: (B)

Q44.

Solution

Concept: We need to find the coordinates of the foot of the perpendicular from the origin to the plane $2x - 3y + 4z - 6 = 0$.

Solution: The formula for the foot of the perpendicular from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is:

$$\left(\frac{-AD}{A^2 + B^2 + C^2}, \frac{-BD}{A^2 + B^2 + C^2}, \frac{-CD}{A^2 + B^2 + C^2}\right)$$

For the plane $2x - 3y + 4z - 6 = 0$, we substitute the values $A = 2$, $B = -3$, $C = 4$, and $D = -6$ into the formula, and the point is the origin $(0, 0, 0)$:

$$\left(\frac{-2(-6)}{2^2 + (-3)^2 + 4^2}, \frac{-(-3)(-6)}{2^2 + (-3)^2 + 4^2}, \frac{-4(-6)}{2^2 + (-3)^2 + 4^2}\right) = \left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$$

Thus, the coordinates of the foot of the perpendicular are $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$, which corresponds to option A.

Answer: (A)

Q45.

Solution

Concept: We are given that the direction cosines of a line are k, k, k, and we need to find the value of k.

Solution: The direction cosines satisfy the equation:

$$l^2 + m^2 + n^2 = 1$$

For k, k, k, this becomes:

$$k^2 + k^2 + k^2 = 1$$

Simplifying:

$$3k^2 = 1 \quad \Rightarrow \quad k^2 = \frac{1}{3} \quad \Rightarrow \quad k = \pm\frac{1}{\sqrt{3}}$$

Thus, the correct answer is:

Answer: (C)

Q46.

Solution

Concept: We are given that the point is $(2, 5, -3)$ and the plane equation is $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$. We need to find the distance from the point to the plane.

Solution: The formula for the distance from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

For the plane $2x - 3y + 4z - 6 = 0$, we have $A = 2$, $B = -3$, $C = 4$, and $D = -6$, and the point is $(2, 5, -3)$.

Substitute into the distance formula:

$$d = \frac{|2(2) - 3(5) + 4(-3) - 6|}{\sqrt{2^2 + (-3)^2 + 4^2}} = \frac{|4 - 15 - 12 - 6|}{\sqrt{4 + 9 + 16}} = \frac{|-29|}{\sqrt{29}} = \frac{29}{\sqrt{29}} = \sqrt{29}$$

Thus, the correct answer is:

Answer: (D)

Q47.

Solution

Concept: We are given the corner points of the feasible region for an LPP and need to maximize $Z = 4x + 3y$.

Solution: The corner points are $(0, 0)$, $(5, 0)$, $(3, 4)$, and $(0, 5)$. We evaluate $Z = 4x + 3y$ at each point:

- At $(0, 0)$, $Z = 4(0) + 3(0) = 0$ - At $(5, 0)$, $Z = 4(5) + 3(0) = 20$ - At $(3, 4)$, $Z = 4(3) + 3(4) = 12 + 12 = 24$ - At $(0, 5)$, $Z = 4(0) + 3(5) = 15$

Thus, the maximum value of Z is 24 at the point $(3, 4)$.

The correct answer is:

Answer: (B)

Q48.

Solution

Concept: We are given $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$, and we need to find $P(A'/B')$.

Solution: We use the formula:

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$$

We know:

$$P(A' \cap B') = P(B) - P(A \cap B) = 0.3 - 0.2 = 0.1$$

Also, $P(B') = 1 - P(B) = 1 - 0.3 = 0.7$.

Thus:

$$P(A'/B') = \frac{0.1}{0.7} = \frac{1}{7}$$

The correct answer is:

Answer: (D)

Q49.

Solution

Concept: We are given that $P(A|B) = P(B|A)$, and we need to find the relationship between A and B.

Solution: Using the definition of conditional probability, we have:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Equating $P(A|B)$ and $P(B|A)$, we get:

$$\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} \quad \Rightarrow \quad P(A) = P(B)$$

Thus, $P(A) = P(B)$, which corresponds to option C.

The correct answer is:

Answer: (C)

Q50.

Solution

Concept: We are asked to find the probability of drawing a king or a heart from a deck of 52 cards.
Solution: The total number of possible outcomes is 52. The number of kings in a deck is 4, and the number of hearts is 13. However, the king of hearts is counted twice, so we need to subtract 1. The total number of favorable outcomes is:

$$4\text{ (kings)} + 13\text{ (hearts)} - 1\text{ (king of hearts)} = 16$$

Thus, the probability is:

$$\frac{16}{52} = \frac{4}{13}$$

The correct answer is:

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	D	5	C
6	A	7	B	8	B	9	A	10	D
11	A	12	B	13	B	14	A	15	C
16	C	17	C	18	D	19	A	20	A
21	C	22	C	23	A	24	A	25	A
26	A	27	C	28	A	29	A	30	C
31	A	32	D	33	B	34	B	35	A
36	A	37	A	38	A	39	D	40	D
41	C	42	B	43	B	44	A	45	C
46	D	47	B	48	D	49	C	50	A