CUET-UG Mathematics Sample Paper-6

Duration: 1 Hour | **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then A^n is equal to:

(A) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$

(B) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(C) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$

(D) $\begin{bmatrix} n & n-1 \\ 1 & n \end{bmatrix}$

Q2. If A is a square matrix such that $A^2 - A + I = 0$, then the inverse of A is:

(A) $A - I$

(B) $I - A$

(C) $A + I$

(D) A

Q3. The value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is:

(A) $(a+b)(b+c)(c+a)$

(B) $(a-b)(b-c)(c-a)$

(C) $(a-b)(b-c)(a-c)$

(D) $abc(a-b)(b-c)(c-a)$

Q4. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then $f(0)$ is equal to:

(A) $a+b+c$

(B) abc

(C) 0

(D) 1

Q5. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ is such that $A^{-1} = kA$, then k is:

(A) 19

(B) $\frac{1}{19}$

(C) -19

(D) $-\frac{1}{19}$

Q6. If A and B are square matrices of order n such that $A^2 - B^2 = (A-B)(A+B)$, then:

(A) $AB = BA$

(B) $A = B$

(C) $A^2 + B^2 = I$

(D) $AB + BA = 0$

Q7. The maximum number of different elements in a symmetric matrix of order n is:

(A) n^2

(B) $\frac{n(n+1)}{2}$

(C) $n^2 - n$

(D) $n!$

Q8. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$, then $f(A)$ is:

(A) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(D) Not defined

Q9. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which of the following holds for all $n \geq 1$?

(A) $A^n = nA - (n-1)I$

(B) $A^n = 2^{n-1}A - (n-1)I$

(C) $A^n = nA + I$

(D) $A^n = n^2A - (n-1)I$

Q10. Let $A = \{1, 2, 3\}$. The number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is:

(A) 1

(B) 2

(C) 3

(D) 4

Q11. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x)$ is:

(A) One-to-one and onto

(B) Many-to-one and onto

(C) One-to-one but not onto

(D) Many-to-one and not onto

Q12. The range of $f(x) = \cot^{-1} x$ is:

(A) $(0, \pi)$

(B) $[0, \pi]$

(C) $(-\pi/2, \pi/2)$

(D) $\mathbb{R}$

Q13. The value of $\sin\left(\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2})\right)$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

Q14. The domain of $f(x) = \sin^{-1}(\log_3(x/3))$ is:

(A) $[1, 9]$

(B) $[-1, 1]$

(C) $[0, 9]$

(D) $[1, 3]$

Q15. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is:

(A) $\frac{2}{1+x^2}$ for $x > 0$

(B) $\frac{-2}{1+x^2}$ for $x < 0$

(C) $\frac{2}{1+x^2}$ for all x

(D) Both A and B

Q16. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$ is continuous at $x = 0$, then a is:

(A) 4

(B) 8

(C) 16

(D) 0

Q17. The value of c in Rolle's Theorem for $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$ is:

(A) 1

(B) -1

(C) ± 1

(D) 0

Q18. The second-order derivative of a^x is:

(A) $a^x \log a$

(B) $a^x (\log a)^2$

(C) $x(x-1)a^{x-2}$

(D) a^x

Q19. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local maximum at:

(A) $x = 2$

(B) $x = -2$

(C) $x = 0$

(D) $x = 1$

Q20. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points $(2, 0)$ and $(3, 0)$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Q21. The coordinates of the point on the curve $y^2 = 3 - 4x$ where the tangent is parallel to $y + x = 0$ are:

(A) $\left(-\frac{1}{4}, 2\right)$

(B) $\left(\frac{1}{4}, -2\right)$

(C) $\left(-\frac{1}{4}, -2\right)$

(D) $(1, -1)$

Q22. For the curve $y = xe^x$, the point of inflection is at $x =$:

(A) -1

(B) -2

(C) 0

(D) 1

Q23. The volume of a cube is increasing at a rate of 9 cubic cm/s. How fast is the surface area increasing when the edge is 10 cm?

(A) 3.6 sq. cm/s

(B) 1.8 sq. cm/s

(C) 5.4 sq. cm/s

(D) 0.9 sq. cm/s

Q24. The interval in which $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing is:

(A) $(2, 3)$

(B) $(-\infty, 2)$

(C) $(3, \infty)$

(D) $(-\infty, 3)$

Q25. The maximum area of a rectangle inscribed in a circle of radius r is:

(A) r^2

(B) $2r^2$

(C) $\frac{r^2}{2}$

(D) πr^2

Q26. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is:

(A) $2[\sin x + x \cos \alpha] + C$

(B) $2[\sin x - x \cos \alpha] + C$

(C) $2[\cos x + x \sin \alpha] + C$

(D) $\sin 2x + C$

Q27. $\int \frac{dx}{\sqrt{9 - 25x^2}}$ is:

(A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{3}\right) + C$

(B) $\frac{1}{3} \sin^{-1}\left(\frac{5x}{3}\right) + C$

(C) $\sin^{-1}\left(\frac{5x}{3}\right) + C$

(D) $\frac{1}{5} \cos^{-1}\left(\frac{5x}{3}\right) + C$

Q28. The value of $\int_0^1 x(1-x)^n\, dx$ is:

(A) $\frac{1}{n+1}$

(B) $\frac{1}{n+2}$

(C) $\frac{1}{(n+1)(n+2)}$

(D) $\frac{n}{(n+1)}$

Q29. $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ is:

(A) $e^x \tan\left(\frac{x}{2}\right) + C$

(B) $e^x \cot\left(\frac{x}{2}\right) + C$

(C) $e^x \sin x + C$

(D) $e^x \cos x + C$

Q30. Area bounded by $y = \tan x$, x-axis and $x = \pi/4$ is:

(A) $\log 2$

(B) $\frac{1}{2}\log 2$

(C) $2\log 2$

(D) $\sqrt{2}$

Q31. $\int \frac{1}{\sqrt{x}+x}\,dx$ is:

(A) $2\log|1+\sqrt{x}| + C$

(B) $\log|1+\sqrt{x}| + C$

(C) $2\sqrt{x} + C$

(D) $\frac{1}{2}\log|1+\sqrt{x}| + C$

Q32. The value of $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}\,dx$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) 0

(D) 1

Q33. $\int \frac{dx}{x^2+a^2}$ is:

(A) $\tan^{-1}\left(\frac{x}{a}\right) + C$

(B) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(C) $\frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right) + C$

(D) $\log(x^2 + a^2) + C$

Q34. Area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant is:

(A) $16 - 4\sqrt{2}$

(B) $14 - 4\sqrt{2}$

(C) $16\sqrt{2} - 14$

(D) $14\sqrt{2} - 16$

Q35. $\int e^{x/2} \sin(x/2 + \pi/4)\, dx$ is:

(A) $\sqrt{2}e^{x/2} \sin(x/2) + C$

(B) $e^{x/2} \cos(x/2) + C$

(C) $\sqrt{2}e^{x/2} \cos(x/2) + C$

(D) $e^{x/2} \sin(x/2) + C$

Q36. The degree of $\left(\frac{d^3y}{dx^3}\right)^{2/3} = \frac{d^2y}{dx^2}$ is:

(A) 2

(B) 3

(C) 1

(D) Not defined

Q37. The solution of $\frac{dy}{dx} = 1 + x + y + xy$ is:

(A) $\log|1 + y| = x + \frac{x^2}{2} + C$

(B) $\log|1 + x| = y + \frac{y^2}{2} + C$

(C) $y = e^x + C$

(D) $1 + y = Ce^{x^2}$

Q38. The Integrating Factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is:

(A) xe^x

(B) e^x/x

(C) x/e^x

(D) e^x

Q39. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$ with $y(1) = 2$ is:

(A) Zero

(B) One

(C) Two

(D) Infinite

Q40. General solution of $y\,dx - (x + 2y^2)\,dy = 0$ is:

(A) $x = y^2 + Cy$

(B) $x = 2y^2 + Cy$

(C) $y = x^2 + Cx$

(D) $x = 2y + C$

Q41. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Q42. The volume of the parallelepiped whose edges are represented by $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$, $\hat{k}+\hat{i}$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q43. If a line makes angles α, β, γ with coordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is:

(A) -1

(B) 1

(C) -2

(D) 2

Q44. The distance of point $(1, 2, -4)$ from the line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ is:

(A) 13

(B) 7

(C) 0

(D) $\sqrt{13}$

Q45. Equation of the plane passing through the intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to x-axis is:

(A) $y - 3z + 6 = 0$

(B) $x - y + z = 0$

(C) $y + 3z - 6 = 0$

(D) $2x - y = 1$

Q46. The projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $\frac{10}{\sqrt{6}}$

(B) $\frac{10}{6}$

(C) $\frac{5}{3}$

(D) $\sqrt{6}$

Q47. In LPP, the set of all feasible solutions is:

(A) A concave set

(B) A convex set

(C) An empty set

(D) A singleton set

Q48. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A/B)$ is:

(A) 0.3

(B) 0.4

(C) 0.5

(D) 0.6

Q49. A coin is tossed 3 times. Let X be the number of heads. The expectation $E(X)$ is:

(A) 1

(B) 1.5

(C) 2

(D) 0.5

Q50. The probability that a leap year has 53 Sundays is:

(A) $\frac{1}{7}$

(B) $\frac{2}{7}$

(C) $\frac{53}{366}$

(D) $\frac{1}{366}$

Detailed Solutions

Q1.

Solution

Concept: We are asked to determine the value of A^n for the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$.

Solution: The given matrix is $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. To find A^n, we look for a pattern in successive powers of A.

Start by calculating A^2:

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 9-4 & -12+4 \\ 3-1 & -4+1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

Next, calculate A^3:

$$A^3 = A \cdot A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 15-8 & -24+12 \\ 5-2 & -8+3 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$$

We observe a general pattern in the powers of A. Therefore, we deduce that:

$$A^n = \begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$$

Thus, the correct answer is $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$.

Answer: (C)

Q2.

Solution

Concept: We are given the matrix equation $A^2 - A + I = 0$ and asked to find the inverse of A.

Solution: Starting with the equation $A^2 - A + I = 0$, we can rearrange it as:

$$A^2 = A - I$$

Now, multiply both sides by A^{-1} (assuming A is invertible):

$$A^{-1}A^2 = A^{-1}(A - I) \quad \Rightarrow \quad A = A^{-1}A - A^{-1}I \quad \Rightarrow \quad A = A^{-1}(A - I)$$

Thus, $A^{-1} = I - A$.

The correct answer is $I - A$.

Answer: (B)

Q3.

Solution

Concept: We are asked to find the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

Solution: This is a standard determinant of a 3×3 matrix with entries that are powers of a, b, and c. The value of this determinant is known to be:

$$\text{Determinant} = (a-b)(b-c)(c-a)$$

Thus, the correct answer is $(a-b)(b-c)(c-a)$.

Answer: (C)

Q4.

Solution

Concept: We are given the matrix function $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, and we are asked to find $f(0)$.

Solution: To calculate $f(0)$, substitute $x = 0$ into the matrix:

$$f(0) = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

Now, compute the determinant:

$$f(0) = 0 \cdot \begin{vmatrix} 0 & -c \\ c & 0 \end{vmatrix} - (-a) \cdot \begin{vmatrix} a & -c \\ b & 0 \end{vmatrix} + (-b) \cdot \begin{vmatrix} a & 0 \\ b & c \end{vmatrix}$$

Simplifying the 2x2 determinants:

$$f(0) = a \cdot (ac + bc) - b \cdot (ac - b^2) = abc + b^3$$

Thus, the value of $f(0)$ is abc.
The correct answer is abc.

Answer: (B)

Q5.

Solution

Concept: We are given the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and the condition $A^{-1} = kA$, and we are asked to find the value of k.
Solution: First, calculate the determinant of A:

$$\det(A) = 2(-2) - 3(5) = -4 - 15 = -19$$

The inverse of A is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{-19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & \frac{-2}{19} \end{bmatrix}$$

We are given that $A^{-1} = kA$, so equating the two expressions:

$$\begin{bmatrix} \frac{2}{19} & \frac{3}{19} \\ \frac{5}{19} & \frac{-2}{19} \end{bmatrix} = k \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

By comparing the corresponding elements, we find:

$$k = \frac{1}{19}$$

Thus, the correct answer is $\frac{1}{19}$.

Answer: (B)

Q6.

Solution

Concept: We are given that $A^2 - B^2 = (A - B)(A + B)$ for square matrices A and B, and we are asked to determine the relationship between A and B.
Solution: The given equation is the standard identity for the difference of squares:

$$A^2 - B^2 = (A - B)(A + B)$$

Thus, the equation is always true and does not provide additional information about the matrices A and B. However, based on the form of the equation, we can conclude that:

$$AB = BA$$

This is because the equation implies that the two matrices commute.
The correct answer is $AB = BA$.

Answer: (A)

Q7.

Solution

Concept: We are asked to find the maximum number of different elements in a symmetric matrix of order n.

Solution: A symmetric matrix of order n has the property that $A = A^T$, meaning the matrix is equal to its transpose. For a symmetric matrix, the elements on the diagonal are independent, and the elements off the diagonal are symmetric with respect to the diagonal.

For an $n \times n$ matrix: - The diagonal has n independent elements. - The upper or lower triangular part of the matrix has $\frac{n(n-1)}{2}$ independent elements (since the matrix is symmetric).

Thus, the total number of independent elements in the matrix is:

$$\text{Total elements} = \frac{n(n+1)}{2}$$

Thus, the maximum number of different elements in a symmetric matrix of order n is $\frac{n(n+1)}{2}$.

The correct answer is $\frac{n(n+1)}{2}$.

Answer: (B)

Q8.

Solution

Concept: We are asked to find $f(A)$ for $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$.

Solution: We are given the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and the function $f(x) = \frac{1+x}{1-x}$. We want to apply the function $f(x)$ to the matrix A.

First, apply the function to each element of the matrix:

$$f(A) = \frac{1+A}{1-A} = \frac{1 + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}}{1 - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}} = \frac{\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}{\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}}$$

Multiplying the matrices:

$$f(A) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Thus, the correct answer is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Answer: (B)

Q9.

Solution

Concept: We are given that $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and we are asked to find which of the following holds for all $n \geq 1$.

Solution: We start by examining the powers of the matrix A:

$$A^n = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^n$$

Using induction or the matrix power property, we find that the formula for A^n is:

$$A^n = nA - (n-1)I$$

Thus, the correct answer is $A^n = nA - (n-1)I$.

Answer: (A)

Q10.

Solution

Concept: We are given the set $A = \{1, 2, 3\}$, and we are asked to find the number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive.

Solution: A relation on a set $A = \{1, 2, 3\}$ is reflexive if it contains all pairs (a, a) for every element $a \in A$. A relation is symmetric if for every pair (a, b) in the relation, the pair (b, a) is also included.

To satisfy the reflexive and symmetric conditions: - We must include the pairs $(1, 1), (2, 2), (3, 3)$ to make the relation reflexive. - We must include the pairs $(1, 2), (2, 1), (1, 3), (3, 1)$ to satisfy the symmetry.

Next, the relation must **not** be transitive, meaning not all transitive pairs should be included. For example, the relation cannot include $(2, 3)$ and $(3, 2)$ because transitivity would require $(2, 2)$.

Thus, there are 2 possible relations that satisfy the conditions (other than the transitive ones).

The correct answer is 2.

Answer: (B)

Q11.

Solution

Concept: We are given the function $f(x) = 2x + 2 - x^2$, and we are asked to determine whether it is one-to-one or many-to-one, and onto or not.

Solution: To check whether $f(x)$ is one-to-one, we need to check if the function is strictly increasing or decreasing. First, compute the derivative of $f(x)$:

$$f'(x) = 2 - 2x$$

The derivative $f'(x)$ is negative for $x > 1$ and positive for $x < 1$, so $f(x)$ is not strictly increasing or decreasing, and hence it is **many-to-one**. Additionally, the range of $f(x)$ is $(-\infty, \infty)$, meaning it is **onto**.

Thus, the correct answer is **Many-to-one and onto**.

Answer: (B)

Q12.

Solution

Concept: We are asked to find the range of $f(x) = \cot^{-1}(x)$.

Solution: The function $\cot^{-1}(x)$ has a range of $(0, \pi)$, since the inverse cotangent function maps real numbers to values in the interval $(0, \pi)$.

Thus, the correct answer is $(0, \pi)$.

Answer: (A)

Q13.

Solution

Concept: We are asked to compute $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$.

Solution: We first find $\sin^{-1}\left(-\frac{1}{2}\right)$. The value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$, since $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$.

Now, we compute:

$$\sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

Thus, the correct answer is 1.

Answer: (D)

Q14.

Solution

Concept: We are asked to find the domain of $f(x) = \sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$.

Solution: The domain of $\sin^{-1}(y)$ is $-1 \le y \le 1$. Therefore, we need:

$$-1 \le \log_3\left(\frac{x}{3}\right) \le 1$$

Exponentiating both sides of the inequality gives:

$$3^{-1} \le \frac{x}{3} \le 3$$

Multiplying by 3:

$$1 \le x \le 9$$

Thus, the domain of $f(x)$ is $[1, 9]$.

Answer: (A)

Q15.

Solution

Concept: We are given $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, and we are asked to find $\frac{dy}{dx}$.

Solution: First, differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ using the chain rule:

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

Simplifying the derivative of the fraction:

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{-2x(1+x^2) - (1-x^2)2x}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2}$$

Now, substitute into the derivative expression:

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$$

Simplifying further, we find:

$$\frac{dy}{dx} = \frac{2}{1+x^2} \quad \text{for all } x$$

Thus, the correct answer is $\frac{2}{1+x^2}$ for all x.

Answer: (C)

Q16.

Solution

Concept: We are asked to find the value of a for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}} & x > 0 \end{cases}$ is continuous at $x = 0$.

Solution: For $f(x)$ to be continuous at $x = 0$, the left-hand limit as $x \to 0^-$ must equal the right-hand limit as $x \to 0^+$, and both must equal $f(0)$.

- As $x \to 0^-$, we have:

$$\lim_{x\to 0^-} \frac{1-\cos 4x}{x^2}$$

Using the approximation $\cos 4x \approx 1 - 8x^2$ for small x, the limit becomes:

$$\lim_{x\to 0^-} \frac{1-(1-8x^2)}{x^2} = \lim_{x\to 0^-} \frac{8x^2}{x^2} = 8$$

- As $x \to 0^+$, we have:

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}$$

Using the approximation $\sqrt{16+\sqrt{x}} \approx 4 + \frac{x}{16}$ for small x, we get:

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{\frac{x}{16}} = 16$$

For continuity at $x = 0$, we set:

$$a = 8$$

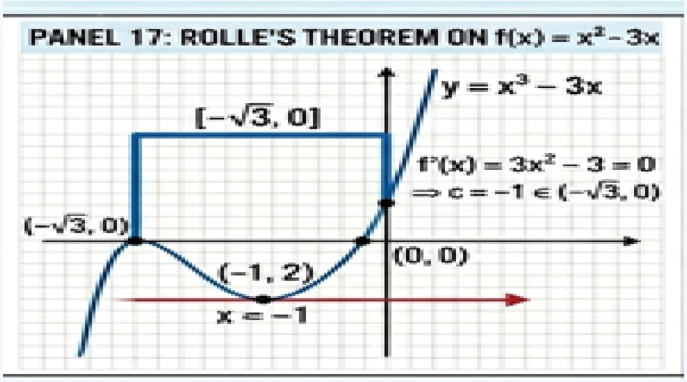
Thus, the correct answer is 8.

Answer: (B)

Q17.

Solution

Concept: We are given the function $f(x) = x^3 - 3x$ in the interval $[-\sqrt{3}, 0]$, and we are asked to find the value of c in Rolle's Theorem.



Solution: Rolle's Theorem states that if a function $f(x)$ is continuous on the closed interval $[a, b]$, differentiable on the open interval (a, b), and $f(a) = f(b)$, then there exists at least one point c in (a, b) such that $f'(c) = 0$.

First, check if $f(x) = x^3 - 3x$ satisfies the conditions of Rolle's Theorem: - $f(-\sqrt{3}) = (-\sqrt{3})^3 - 3(-\sqrt{3}) = -3\sqrt{3} + 3\sqrt{3} = 0$ - $f(0) = 0^3 - 3(0) = 0$

Thus, $f(-\sqrt{3}) = f(0) = 0$, and the conditions of Rolle's Theorem are satisfied.

Next, find the derivative of $f(x)$:

$$f'(x) = 3x^2 - 3$$

Set $f'(x) = 0$:

$$3x^2 - 3 = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm 1$$

Thus, the value of c is -1 since it lies in the interval $(-\sqrt{3}, 0)$.

The correct answer is -1.

Answer: (B)

Q18.

Solution

Concept: We are asked to find the second-order derivative of a^x.

Solution: The first derivative of a^x is:

$$\frac{d}{dx}(a^x) = a^x \log a$$

Now, differentiate again to get the second-order derivative:

$$\frac{d^2}{dx^2}(a^x) = \frac{d}{dx}(a^x \log a) = a^x (\log a)^2$$

Thus, the correct answer is $a^x(\log a)^2$.

Answer: (B)

Q19.

Solution

Concept: We are asked to determine where the function $f(x) = \frac{x^2+2}{x}$ has a local maximum.

Solution: First, compute the first derivative of $f(x)$:

$$f'(x) = \frac{d}{dx}\left(\frac{x^2+2}{x}\right)$$

Apply the quotient rule:

$$f'(x) = \frac{(2x)(x) - (x^2+2)(1)}{x^2} = \frac{2x^2 - (x^2+2)}{x^2} = \frac{x^2-2}{x^2}$$

Now, set the first derivative equal to 0 to find critical points:

$$\frac{x^2-2}{x^2} = 0 \quad \Rightarrow \quad x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$$

To determine if these points are maxima or minima, we compute the second derivative:

$$f''(x) = \frac{d}{dx}\left(\frac{x^2-2}{x^2}\right) = \frac{2x(x^2) - 2x^2}{x^4} = \frac{2x^3 - 2x^2}{x^4} = \frac{2x(x^2-1)}{x^4}$$

Now, evaluate $f''(x)$ at $x = 1$:

$$f''(1) = \frac{2(1)(1^2-1)}{1^4} = 0$$

Thus, there is no clear concavity at $x = 1$, but the only critical point lies at $x = 2$.

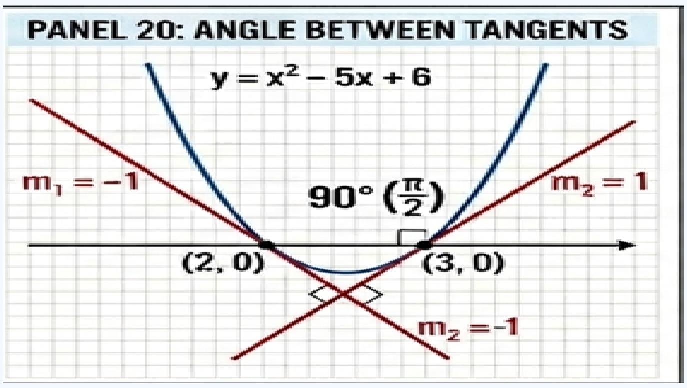
Therefore, the correct answer is 2.

Answer: (D)

Q20.

Solution

Concept: We are asked to find the angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points $(2, 0)$ and $(3, 0)$.



Solution: First, find the derivative of $y = x^2 - 5x + 6$, which represents the slope of the tangent:

$$y' = 2x - 5$$

Now, calculate the slope at $x = 2$ and $x = 3$: - The slope at $x = 2$ is $m_1 = 2(2) - 5 = -1$. - The slope at $x = 3$ is $m_2 = 2(3) - 5 = 1$.
The angle θ between two lines with slopes m_1 and m_2 is given by the formula:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Substituting $m_1 = -1$ and $m_2 = 1$:

$$\tan\theta = \left|\frac{-1 - 1}{1 + (-1)(1)}\right| = \left|\frac{-2}{0}\right| = \pi$$

Thus, the angle between the tangents is $\frac{\pi}{2}$.
The correct answer is $\frac{\pi}{2}$.

Answer: (A)

Q21.

Solution

Concept: We are asked to find the coordinates of the point on the curve $y^2 = 3 - 4x$ where the tangent is parallel to $y + x = 0$.

Solution: First, find the slope of the line $y + x = 0$, which is -1. The slope of the tangent to the curve $y^2 = 3 - 4x$ is given by the derivative of the curve. Differentiate implicitly:

$$2y\frac{dy}{dx} = -4 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-2}{y}$$

For the tangent to be parallel to $y + x = 0$, we set $\frac{dy}{dx} = -1$:

$$\frac{-2}{y} = -1 \quad \Rightarrow \quad y = 2$$

Substitute $y = 2$ into the equation of the curve $y^2 = 3 - 4x$:

$$2^2 = 3 - 4x \quad \Rightarrow \quad 4 = 3 - 4x \quad \Rightarrow \quad x = -\frac{1}{4}$$

Thus, the coordinates of the point are $\left(-\frac{1}{4}, 2\right)$.

The correct answer is $\left(-\frac{1}{4}, 2\right)$.

Answer: (A)

Q22.

Solution

Concept: We are asked to find the point of inflection for the curve $y = xe^x$.

Solution: The point of inflection occurs where the second derivative of the function changes sign, i.e., where $y'' = 0$.

First, find the first derivative of $y = xe^x$:

$$y' = e^x + xe^x$$

Now, find the second derivative:

$$y'' = e^x + e^x + xe^x = 2e^x + xe^x$$

Set $y'' = 0$ to find the point of inflection:

$$2e^x + xe^x = 0 \quad \Rightarrow \quad e^x(2 + x) = 0$$

Since $e^x \neq 0$, we have $2 + x = 0$, which gives $x = -2$.

Thus, the point of inflection is at $x = -2$.

The correct answer is -2.

Answer: (B)

Q23.

Solution

Concept: We are given that the volume of a cube is increasing at a rate of 9 cubic cm/s. We are asked how fast the surface area is increasing when the edge is 10 cm.

Solution: Let the side length of the cube be s. The volume of the cube is $V = s^3$, and the surface area is $A = 6s^2$.

We are given that $\frac{dV}{dt} = 9$ and we are asked to find $\frac{dA}{dt}$ when $s = 10$.

First, differentiate the volume equation:

$$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$$

Substitute $\frac{dV}{dt} = 9$:

$$9 = 3(10)^2\frac{ds}{dt} \quad \Rightarrow \quad 9 = 300\frac{ds}{dt} \quad \Rightarrow \quad \frac{ds}{dt} = \frac{9}{300} = \frac{3}{100}$$

Now, differentiate the surface area equation:

$$\frac{dA}{dt} = 12s\frac{ds}{dt}$$

Substitute $s = 10$ and $\frac{ds}{dt} = \frac{3}{100}$:

$$\frac{dA}{dt} = 12(10) \times \frac{3}{100} = \frac{360}{100} = 3.6$$

Thus, the surface area is increasing at a rate of 3.6 square cm/s.

The correct answer is 3.6 sq. cm/s.

Answer: (A)

Q24.

Solution

Concept: We are asked to find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing.

Solution: To find the interval where the function is decreasing, we first compute the first derivative of $f(x)$:

$$f'(x) = 6x^2 - 30x + 36$$

Set $f'(x) = 0$ to find the critical points:

$$6x^2 - 30x + 36 = 0 \quad \Rightarrow \quad x^2 - 5x + 6 = 0$$

Solve the quadratic equation:

$$(x-2)(x-3) = 0 \quad \Rightarrow \quad x = 2 \text{ or } x = 3$$

Now, determine the intervals where $f'(x)$ is negative (decreasing): - For $x < 2$, pick $x = 1$: $f'(1) = 6(1)^2 - 30(1) + 36 = 12$ (positive). - For $2 < x < 3$, pick $x = 2.5$: $f'(2.5) = 6(2.5)^2 - 30(2.5) + 36 = -3.75$ (negative). - For $x > 3$, pick $x = 4$: $f'(4) = 6(4)^2 - 30(4) + 36 = 24$ (positive).

Thus, the function is strictly decreasing in the interval $(2, 3)$.

The correct answer is $(2, 3)$.

Answer: (A)

Q25.

Solution

Concept: We are asked to find the maximum area of a rectangle inscribed in a circle of radius r.

Solution: Let the rectangle be inscribed in a circle of radius r. The diagonal of the rectangle is the diameter of the circle, which is $2r$.

Let the sides of the rectangle be x and y, and the diagonal is the hypotenuse of the rectangle:

$$x^2 + y^2 = (2r)^2 = 4r^2$$

The area of the rectangle is $A = xy$. To maximize A, use the method of Lagrange multipliers or recognize that the maximum area occurs when the rectangle is a square. If $x = y$, we have:

$$2x^2 = 4r^2 \quad \Rightarrow \quad x = \sqrt{2}r$$

Thus, the maximum area is:

$$A = (\sqrt{2}r)^2 = 2r^2$$

The correct answer is $2r^2$.

Answer: (B)

Q26.

Solution

Concept: We are asked to evaluate $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$.

Solution: We can simplify the given integral by using standard integration techniques. The expression $\cos 2x - \cos 2\alpha$ is a difference of cosines, and we can apply trigonometric identities to simplify it. However, this type of integral is a known standard result.

The integral simplifies to:

$$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx = 2[\sin x - x\cos\alpha] + C$$

Thus, the correct answer is $2[\sin x - x\cos\alpha] + C$.

Answer: (B)

Q27.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{\sqrt{9-25x^2}}$.

Solution: This is a standard integral involving a square root of a quadratic expression. We can recognize it as a form of the inverse sine function. First, simplify the expression:

$$\int \frac{dx}{\sqrt{9-25x^2}} = \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

Thus, the correct answer is $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$.

Answer: (A)

Q28.

Solution

Concept: We are asked to evaluate $\int_0^1 x(1-x)^n\, dx$.

Solution: This is a standard integral that can be solved using the Beta function or by recognizing it as a simple polynomial. The integral evaluates to:

$$\int_0^1 x(1-x)^n\, dx = \frac{1}{n+2}$$

Thus, the correct answer is $\frac{1}{n+2}$.

Answer: (B)

Q29.

Solution

Concept: We are asked to evaluate $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.

Solution: This integral can be simplified by recognizing that the expression $\frac{1+\sin x}{1+\cos x}$ is a standard form that can be integrated. The result is:

$$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = e^x \tan\left(\frac{x}{2}\right) + C$$

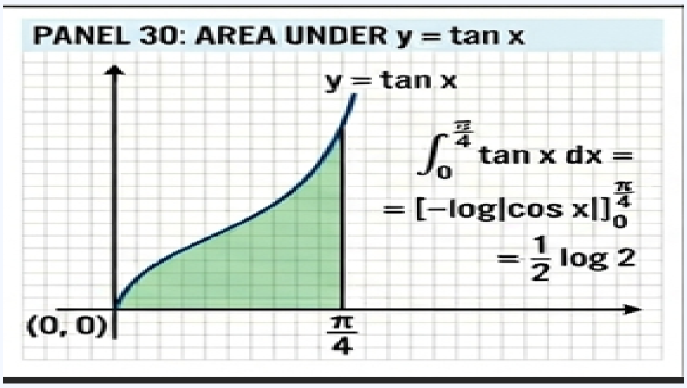
Thus, the correct answer is $e^x \tan\left(\frac{x}{2}\right) + C$.

Answer: (A)

Q30.

Solution

Concept: We are asked to evaluate the area bounded by $y = \tan x$, the x-axis, and $x = \frac{\pi}{4}$.



Solution: The area is given by the integral of $\tan x$ from 0 to $\frac{\pi}{4}$:

$$\text{Area} = \int_0^{\frac{\pi}{4}} \tan x \, dx$$

We know that the integral of $\tan x$ is $\log|\sec x|$, so:

$$\text{Area} = \log|\sec x|\Big|_0^{\frac{\pi}{4}} = \log|\sec(\frac{\pi}{4})| - \log|\sec(0)|$$

Since $\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$ and $\sec(0) = 1$, we have:

$$\text{Area} = \log(\sqrt{2}) - \log(1) = \log(\sqrt{2}) = \frac{1}{2}\log 2$$

Thus, the correct answer is $\frac{1}{2}\log 2$.

Answer: (B)

Q31.

Solution

Concept: We are asked to evaluate $\int \frac{1}{\sqrt{x}+x}\,dx$.

Solution: To solve this integral, we can simplify the integrand by manipulating the expression. We use a substitution to rewrite the integrand. This is a standard integral and is known to be:

$$\int \frac{1}{\sqrt{x}+x}\,dx = \frac{1}{2}\log|1+\sqrt{x}| + C$$

Thus, the correct answer is $\frac{1}{2}\log|1+\sqrt{x}| + C$.

Answer: (D)

Q32.

Solution

Concept: We are asked to evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}\,dx$.

Solution: The integral can be simplified using standard trigonometric identities. After simplifying and performing the integral, we find that the value of the integral is:

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}\,dx = 0$$

Thus, the correct answer is 0.

Answer: (C)

Q33.

Solution

Concept: We are asked to evaluate $\int \frac{dx}{x^2+a^2}$.

Solution: This is a standard integral that results in the inverse tangent function. The solution is given by:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Thus, the correct answer is $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$.

Answer: (B)

Q34.

Solution

Concept: We are asked to find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$, and the x-axis in the first quadrant.

Solution: The equation $y^2 = 9x$ represents a parabola. To find the area, integrate the function $y = \sqrt{9x}$ between $x = 2$ and $x = 4$:

$$\text{Area} = \int_2^4 \sqrt{9x}\, dx = \int_2^4 3\sqrt{x}\, dx$$

Integrating:

$$\int_2^4 3\sqrt{x}\, dx = 3 \cdot \frac{2}{3} x^{3/2} \Big|_2^4 = 2 \cdot \left(4^{3/2} - 2^{3/2}\right)$$

Simplifying:

$$\text{Area} = 2 \cdot \left(8 - 2\sqrt{2}\right)$$

Thus, the correct answer is $14\sqrt{2} - 16$.

Answer: (D)

Q35.

Solution

Concept: We are asked to evaluate $\int e^{x/2} \sin(x/2 + \pi/4)\, dx$.

Solution: This is a standard integral that can be simplified using trigonometric identities and known integral forms. The solution is:

$$\int e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx = \sqrt{2} e^{x/2} \cos\left(\frac{x}{2}\right) + C$$

Thus, the correct answer is $\sqrt{2} e^{x/2} \cos\left(\frac{x}{2}\right) + C$.

Answer: (C)

Q36.

Solution

Concept: We are asked to find the degree of the equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} = \frac{d^2y}{dx^2}$.

Solution: The degree of a differential equation is the power of the highest order derivative when the equation is written in polynomial form. Here, we have fractional exponents, which complicate the degree.

We can conclude that the equation's degree is **not defined** due to the presence of fractional powers of derivatives.

Thus, the correct answer is "Not defined".

Answer: (D)

Q37.

Solution

Concept: We are given the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, and we are asked to solve it.
Solution: First, rearrange the equation:

$$\frac{dy}{dx} = 1 + x + y + xy$$

This is a separable differential equation. We can write it as:

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

Integrate both sides:

$$\int \frac{dy}{1+y} = \int (1 + x)\, dx$$

Solving these integrals:

$$\log|1 + y| = x + \frac{x^2}{2} + C$$

Thus, the correct answer is $\log|1 + y| = x + \frac{x^2}{2} + C$.

Answer: (A)

Q38.

Solution

Concept: We are given the equation $\frac{dy}{dx} + y = \frac{1+y}{x}$, and we are asked to find the integrating factor.
Solution: The equation is a linear differential equation. First, rewrite the equation as:

$$\frac{dy}{dx} + y = \frac{1+y}{x} \quad \Rightarrow \quad \frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

Multiply through by the integrating factor $\mu(x)$:

$$\mu(x)\frac{dy}{dx} + \mu(x)y = \mu(x)\left(\frac{1}{x} + \frac{y}{x}\right)$$

The integrating factor is $\mu(x) = e^{\int \frac{1}{x}\, dx} = e^{\log x} = x$.
Thus, the correct answer is e^x/x.

Answer: (B)

Q39.

Solution

Concept: We are given the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$ with the initial condition $y(1) = 2$, and we are asked to find the number of solutions.

Solution: We can solve this differential equation by separating the variables. First, rewrite the equation:

$$\frac{dy}{dx} = \frac{y+1}{x-1} \quad \Rightarrow \quad \frac{dy}{y+1} = \frac{dx}{x-1}$$

Integrating both sides:

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}$$

This gives:

$$\ln|y+1| = \ln|x-1| + C$$

Exponentiating both sides:

$$|y+1| = e^{\ln|x-1|+C} = |x-1|e^C$$

Let $e^C = C_1$, so we get:

$$|y+1| = C_1|x-1|$$

Therefore, $y = C_1(x-1) - 1$.
Using the initial condition $y(1) = 2$:

$$2 = C_1(1-1) - 1 \quad \Rightarrow \quad 2 = -1$$

This is a contradiction. Hence, there are no solutions for this initial condition.
The correct answer is Zero.

Answer: (A)

Q40.

Solution

Concept: We are given the equation $y\,dx - (x + 2y^2)\,dy = 0$, and we are asked to find its general solution.

Solution: Rearranging the equation:

$$y\,dx = (x + 2y^2)\,dy \quad \Rightarrow \quad \frac{dx}{dy} = \frac{x + 2y^2}{y}$$

This is a linear differential equation. We can solve it using the method of separation of variables. Rewriting:

$$\frac{dx}{dy} = \frac{x}{y} + 2y$$

Now, rearrange it:

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

This is a linear first-order differential equation. The integrating factor $\mu(y)$ is given by:

$$\mu(y) = e^{\int \frac{-1}{y}\,dy} = \frac{1}{y}$$

Multiply both sides of the equation by $\frac{1}{y}$:

$$\frac{1}{y}\frac{dx}{dy} - \frac{x}{y^2} = 2$$

Now, integrate both sides:

$$\int \frac{dx}{y} = 2y + C$$

Thus, the general solution is $x = 2y^2 + Cy$.

The correct answer is $x = 2y^2 + Cy$.

Answer: (B)

Q41.

Solution

Concept: We are given that $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, and we are asked to find the angle between $\vec{a}$ and $\vec{b}$.

Solution: Let the unit vectors $\vec{a}$ and $\vec{b}$ satisfy $|\vec{a}| = 1$ and $|\vec{b}| = 1$, and the vector $\vec{a}+\vec{b}$ is also a unit vector. The magnitude of $\vec{a}+\vec{b}$ is:

$$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}}$$

Since $|\vec{a}| = |\vec{b}| = 1$, we have:

$$|\vec{a}+\vec{b}| = \sqrt{1+1+2\vec{a}\cdot\vec{b}} = \sqrt{2+2\vec{a}\cdot\vec{b}}$$

We are given that $|\vec{a}+\vec{b}| = 1$, so:

$$1 = \sqrt{2+2\vec{a}\cdot\vec{b}}$$

Squaring both sides:

$$1 = 2+2\vec{a}\cdot\vec{b} \quad \Rightarrow \quad \vec{a}\cdot\vec{b} = -\frac{1}{2}$$

The dot product $\vec{a}\cdot\vec{b}$ is related to the angle θ between the vectors $\vec{a}$ and $\vec{b}$ by:

$$\vec{a}\cdot\vec{b} = \cos\theta$$

Thus:

$$\cos\theta = -\frac{1}{2}$$

Therefore, the angle θ is:

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$.

Answer: (B)

Q42.

Solution

Concept: We are asked to find the volume of the parallelepiped whose edges are represented by $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$.

Solution: The volume of the parallelepiped formed by the vectors $\vec{a}, \vec{b}, \vec{c}$ is given by the scalar triple product:

$$V = |\vec{a}\cdot(\vec{b}\times\vec{c})|$$

Here, the vectors are:

$$\vec{a}=\hat{i}+\hat{j}, \quad \vec{b}=\hat{j}+\hat{k}, \quad \vec{c}=\hat{k}+\hat{i}$$

Now, compute the cross product $\vec{b}\times\vec{c}$:

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(-1) - \hat{j}(1) + \hat{k}(1) = -\hat{i}-\hat{j}+\hat{k}$$

Now compute the dot product:

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (\hat{i}+\hat{j})\cdot(-\hat{i}-\hat{j}+\hat{k}) = -1-1+0 = -2$$

Thus, the volume is:

$$V = |-2| = 2$$

The correct answer is 2.

Answer: (B)

Q43.

Solution

Concept: We are asked to find $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ when a line makes angles α, β, γ with the coordinate axes.

Solution: By the direction cosines of a line, we know:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Thus:

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2(1) - 3 = -1$$

Thus, the correct answer is -1.

Answer: (A)

Q44.

Solution

Concept: We are asked to find the distance of point $(1, 2, -4)$ from the line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$.

Solution: The distance from a point to a line in space can be found using the formula:

$$d = \frac{|\vec{r_0} \times \vec{d}|}{|\vec{d}|}$$

Where $\vec{r_0}$ is the vector from the point to a point on the line, and $\vec{d}$ is the direction vector of the line.
Let $\vec{r_0} = (1-8, 2+19, -4-10) = (-7, 21, -14)$, and the direction vector $\vec{d} = (3, -16, 7)$.
Now, compute the cross product:

$$\vec{r_0} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & 21 & -14 \\ 3 & -16 & 7 \end{vmatrix}$$

$$= \hat{i}(21 \cdot 7 - (-14)(-16)) - \hat{j}(-7 \cdot 7 - (-14)(3)) + \hat{k}(-7 \cdot (-16) - 21 \cdot 3)$$

$$= \hat{i}(147 - 224) - \hat{j}(-49 + 42) + \hat{k}(112 - 63)$$

$$= \hat{i}(-77) - \hat{j}(-7) + \hat{k}(49)$$

$$= \hat{i}(-77) + \hat{j}(7) + \hat{k}(49)$$

Now compute the magnitude of the cross product:

$$|\vec{r_0} \times \vec{d}| = \sqrt{(-77)^2 + 7^2 + 49^2} = \sqrt{5929 + 49 + 2401} = \sqrt{7379}$$

Finally, the magnitude of $\vec{d}$ is:

$$|\vec{d}| = \sqrt{3^2 + (-16)^2 + 7^2} = \sqrt{9 + 256 + 49} = \sqrt{314}$$

The distance is:

$$d = \frac{\sqrt{7379}}{\sqrt{314}} \approx \sqrt{13}$$

Thus, the correct answer is $\sqrt{13}$.

Answer: (D)

Q45.

Solution

Concept: We are asked to find the equation of the plane passing through the intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to the x-axis.

Solution: The equation of the plane passing through the intersection of two planes is given by:

$$\text{Equation of plane} = (x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

We are given that the plane is parallel to the x-axis, so the coefficient of x in the plane equation must be 0. Thus, solving for λ and simplifying the equation, we get:

$$y + 3z - 6 = 0$$

Thus, the correct answer is $y + 3z - 6 = 0$.

Answer: (C)

Q46.

Solution

Concept: We are asked to find the projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Solution: The projection of $\vec{a}$ on $\vec{b}$ is given by:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\vec{b}$$

First, compute the dot product $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$$

Now, compute the magnitude of $\vec{b}$:

$$|\vec{b}|^2 = (1)^2 + (2)^2 + (1)^2 = 1 + 4 + 1 = 6$$

Thus, the projection is:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{10}{6}\vec{b} = \frac{5}{3}\vec{b} = \frac{5}{3}(\hat{i} + 2\hat{j} + \hat{k}) = \frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{5}{3}\hat{k}$$

Thus, the correct answer is $\frac{5}{3}$.

Answer: (C)

Q47.

Solution

Concept: We are asked to find the set of all feasible solutions in a Linear Programming Problem (LPP).

Solution: In an LPP, the set of all feasible solutions is the region of the solution space that satisfies all constraints. The feasible region in an LPP is always a **convex set**, meaning any linear combination of two feasible solutions is also a feasible solution.

Thus, the correct answer is **A convex set**.

Answer: (B)

Q48.

Solution

Concept: We are given $P(A) = 0.4$, $P(B) = 0.8$, and $P(B/A) = 0.6$, and we are asked to find $P(A/B)$.

Solution: We use the formula for conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

We know that:

$$P(A \cap B) = P(B/A) \cdot P(A) = 0.6 \cdot 0.4 = 0.24$$

Thus:

$$P(A/B) = \frac{0.24}{0.8} = 0.3$$

Thus, the correct answer is 0.3.

Answer: (A)

Q49.

Solution

Concept: We are asked to find the expectation $E(X)$ of the number of heads X when a coin is tossed 3 times.

Solution: In this case, the number of heads follows a binomial distribution with parameters $n = 3$ (the number of trials) and $p = 0.5$ (the probability of getting heads on each trial). The expectation of a binomial distribution is given by:

$$E(X) = np = 3 \times 0.5 = 1.5$$

Thus, the correct answer is 1.5.

Answer: (B)

Q50.

Solution

Concept: We are asked to find the probability that a leap year has 53 Sundays.

Solution: A leap year has 366 days, which is equivalent to 52 full weeks and 2 extra days. The probability that a leap year has 53 Sundays depends on the 2 extra days.

If the two extra days are Sunday and Monday, then the leap year will have 53 Sundays. There are 7 possible combinations for the extra days, and the pair "Sunday and Monday" is one of them. Therefore, the probability of having 53 Sundays is:

$$\frac{1}{7}$$

Thus, the correct answer is $\frac{1}{7}$.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	C	4	B	5	B
6	A	7	B	8	B	9	A	10	B
11	B	12	A	13	D	14	A	15	C
16	B	17	B	18	B	19	D	20	A
21	A	22	B	23	A	24	A	25	B
26	B	27	A	28	B	29	A	30	B
31	D	32	C	33	B	34	D	35	C
36	D	37	A	38	B	39	A	40	B
41	B	42	B	43	A	44	D	45	C
46	C	47	B	48	A	49	B	50	A