CUET-UG Mathematics Sample Paper-8

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is:

(A) $\pi/6$

(B) $\pi/3$

(C) π

(D) $3\pi/2$

Q2. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$, and $(k, 4)$ is 35 square units, then k is:

(A) 12

(B) -2

(C) 12 or -2

(D) 12 or 2

Q3. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q4. The maximum value of $f(x) = \frac{\log x}{x}$ in $(0, \infty)$ is:

(A) $1/e$

(B) e

(C) $2/e$

(D) 1

Q5. $\int \frac{dx}{x(x^n+1)}$ is equal to:

(A) $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

(B) $\frac{1}{n}\log\left|\frac{x^n+1}{x^n}\right| + C$

(C) $\log\left|\frac{x^n}{x^n+1}\right| + C$

(D) $\frac{1}{n}\log|x^n + 1| + C$

Q6. The area bounded by the curve $y^2 = 4x$ and the line $y = 2x$ is:

(A) $2/3$

(B) $1/3$

(C) $1/4$

(D) $3/4$

Q7. The degree of the differential equation $[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$ is:

(A) 1

(B) 2

(C) 3

(D) Not defined

Q8. The general solution of the differential equation $\frac{dy}{dx} + y = e^{-x}$ is:

(A) $y = (x + c)e^{-x}$

(B) $y = (x + c)e^{x}$

(C) $y = xe^{-x} + c$

(D) $y = e^{-x} + c$

Q9. If $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.24

(B) 0.96

(C) 0.48

(D) 0.72

Q10. In a Linear Programming Problem, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which maximum Z occurs is:

(A) 0

(B) 2

(C) Finite

(D) Infinite

Q11. Let R be a relation on the set L of all lines in a plane defined by $R = \{(L_1, L_2) : L_1 \text{ is perpendicular to } L_2\}$. Then R is:

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Equivalence

Q12. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x$ is:

(A) One-to-one but not onto

(B) Onto but not one-to-one

(C) Bijective

(D) Neither one-to-one nor onto

Q13. The principal value of $\sin^{-1}(\sin \frac{2\pi}{3})$ is:

(A) $2\pi/3$

(B) $\pi/3$

(C) $4\pi/3$

(D) $5\pi/3$

Q14. The domain of $\cos^{-1}(2x - 1)$ is:

(A) $[0, 1]$

(B) $[-1, 1]$

(C) $(0, 1)$

(D) $[0, \pi]$

Q15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(A) A

(B) $I - A$

(C) I

(D) $3A$

Q16. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

(A) Skew-symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

Q17. If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to:

(A) $\det(A)$

(B) $1/\det(A)$

(C) 1

(D) 0

Q18. If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is:

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) $\begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/y & 0 \\ 0 & 0 & 1/z \end{bmatrix}$

Q19. The function $f(x) = |x| + |x - 1|$ is:

(A) Continuous at $x = 0$ and $x = 1$

(B) Differentiable at $x = 0$ and $x = 1$

(C) Not continuous at $x = 0$

(D) Not continuous at $x = 1$

Q20. If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is:

(A) $\cos e^x$

(B) $e^x \sin e^x$

(C) $-e^x \tan e^x$

(D) $e^x \tan e^x$

Q21. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, then $\frac{dy}{dx}$ at $\theta = \pi/2$ is:

(A) 0

(B) 1

(C) -1

(D) 1/2

Q22. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is:

(A) 10π

(B) 12π

(C) 8π

(D) 11π

Q23. The interval in which $f(x) = x^2 e^{-x}$ is increasing is:

(A) $(-\infty, \infty)$

(B) $(-2, 0)$

(C) $(2, \infty)$

(D) $(0, 2)$

Q24. $\int e^x(\frac{1}{x} - \frac{1}{x^2})dx$ is:

(A) $\frac{e^x}{x^2} + C$

(B) $\frac{e^x}{x} + C$

(C) $-\frac{e^x}{x} + C$

(D) $-\frac{e^x}{x^2} + C$

Q25. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q26. $\int \frac{dx}{x^2+2x+2}$ is:

(A) $\tan^{-1}(x+1)+C$

(B) $\tan^{-1}(x)+C$

(C) $\frac{1}{2}\tan^{-1}(x+1)+C$

(D) $\log(x^2+2x+2)+C$

Q27. The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is:

(A) 1 sq. unit

(B) 2 sq. units

(C) 3 sq. units

(D) 4 sq. units

Q28. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(A) e^{-x}

(B) e^{x}

(C) $1/x$

(D) x

Q29. The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $\tan^{-1} y - \tan^{-1} x = C$

(C) $x + y = C(1 - xy)$

(D) $y - x = C(1 + xy)$

Q30. The order of the differential equation of all circles of radius r is:

(A) 1

(B) 2

(C) 3

(D) 4

Q31. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $30°$

(B) $60°$

(C) $90°$

(D) $120°$

Q32. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $10/\sqrt{3}$

(C) $5/\sqrt{6}$

(D) 10

Q33. The area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$ is:

(A) $15\sqrt{2}$

(B) 15

(C) $8\sqrt{3}$

(D) $10\sqrt{2}$

Q34. The distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$ is:

(A) 1 unit

(B) 2 units

(C) 3 units

(D) 0 units

Q35. The angle between the lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ is:

(A) $\cos^{-1}(26/9\sqrt{38})$

(B) 90°

(C) 45°

(D) 60°

Q36. The shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is:

(A) $3\sqrt{2}$

(B) $3/\sqrt{2}$

(C) $2\sqrt{3}$

(D) 0

Q37. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.7

(B) 0.1

(C) 0.12

(D) 0.5

Q38. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. The probability that the ball is drawn from the first bag is:

(A) 2/3

(B) 1/2

(C) 3/4

(D) 1/3

Q39. For an LPP, the feasible region is always a:

(A) Concave polygon

(B) Convex polygon

(C) Circle

(D) Parabola

Q40. If the feasible region for a LPP is unbounded, then the maximum value of the objective function $Z = ax + by$:

(A) Must exist

(B) May or may not exist

(C) Does not exist

(D) Is always zero

Q41. If A is a square matrix of order 3 and $|A| = 5$, then $|\text{adj } A|$ is:

(A) 5

(B) 25

(C) 125

(D) 625

Q42. If $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & \text{if } x \neq \pi/2 \\ 3 & \text{if } x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$, then k is:

(A) 3

(B) 6

(C) 9

(D) 12

Q43. The value of $\int_{-1}^{1} |x|dx$ is:

(A) 0

(B) 1

(C) 2

(D) 1/2

Q44. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to:

(A) $\tan x + \cot x + C$

(B) $\tan x + \text{cosec } x + C$

(C) $-\tan x + \cot x + C$

(D) $\tan x + \sec x + C$

Q45. The position vector of the midpoint of the vector joining the points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is:

(A) $3\hat{i} + 2\hat{j} + \hat{k}$

(B) $2\hat{i} + 2\hat{j} + 2\hat{k}$

(C) $3\hat{i} + 2\hat{j} + 2\hat{k}$

(D) $6\hat{i} + 4\hat{j} + 2\hat{k}$

Q46. $\sin(\tan^{-1} x)$, $|x| < 1$ is equal to:

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{x}{\sqrt{1+x^2}}$

Q47. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is:

(A) $25y$

(B) $5y$

(C) $-25y$

(D) $15y$

Q48. The area of the region bounded by $y = x^2$ and $y = 4$ is:

(A) $32/3$

(B) $16/3$

(C) 8/3

(D) 64/3

Q49. Two dice are thrown. If it is known that at least one of the dice shows a 4, the probability that the sum of the numbers appearing on the dice is 8 is:

(A) 2/11

(B) 1/11

(C) 3/11

(D) 5/36

Q50. The vector equation of the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$ is:

(A) $\vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(B) $\vec{r} = (3\hat{i} + 4\hat{j} + 6\hat{k}) + \lambda(\hat{i} - 2\hat{k})$

(C) $\vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 8\hat{k})$

(D) $\vec{r} = \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

Detailed Solutions

Q1.

Solution

Concept: For a given matrix A, its transpose A' is obtained by swapping its rows and columns. The identity matrix I of order 2 is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Two matrices are equal if their corresponding elements are identical.

Solution: Given $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, its transpose is $A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$. We are given that $A + A' = I$:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get:

$$2\cos\alpha = 1 \implies \cos\alpha = \frac{1}{2}$$

Therefore, $\alpha = \frac{\pi}{3}$.

Final Answer: $\pi/3$

Answer: (B)

Q2.

Solution

Concept: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by the absolute value of the determinant expression: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Solution: The given vertices are $(2, -6)$, $(5, 4)$, and $(k, 4)$, and the area is 35 square units.

$$\text{Area} = \frac{1}{2}|2(4-4) + 5(4-(-6)) + k(-6-4)| = 35$$

$$\frac{1}{2}|2(0) + 5(10) + k(-10)| = 35$$

$$\frac{1}{2}|50 - 10k| = 35$$

$$|25 - 5k| = 35$$

This absolute value yields two possible equations: 1) $25 - 5k = 35 \implies -5k = 10 \implies k = -2$ 2) $25 - 5k = -35 \implies -5k = -60 \implies k = 12$ So, the possible values for k are 12 or -2.

Final Answer: 12 or -2

Answer: (C)

Q3.

Solution

Concept: A stationary point of a function $f(x)$ occurs where its first derivative is equal to zero ($f'(x) = 0$). For functions with variables in both the base and the exponent, logarithmic differentiation is the standard approach.

Solution: Let $f(x) = y = x^x$. Taking the natural logarithm on both sides:

$$\ln y = x \ln x$$

Differentiating both sides with respect to x using the product rule:

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln x + x \cdot \left(\frac{1}{x}\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + 1$$

$$\frac{dy}{dx} = y(1 + \ln x) = x^x(1 + \ln x)$$

To find the stationary point, set $\frac{dy}{dx} = 0$:

$$x^x(1 + \ln x) = 0$$

Since $x^x > 0$ for all valid domain values ($x > 0$), we must have:

$$1 + \ln x = 0 \implies \ln x = -1$$

$$x = e^{-1} = \frac{1}{e}$$

Final Answer: $1/e$

Answer: (B)

Q4.

Solution

Concept: To find the maximum value of a function $f(x)$, we find its derivative $f'(x)$, set it to zero to find critical points, and then check the nature of the function at those points using the second derivative or first derivative test.

Solution: Given $f(x) = \frac{\log x}{x}$. Using the quotient rule:

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

Setting $f'(x) = 0 \implies 1 - \log x = 0 \implies \log x = 1 \implies x = e$. At $x = e$, the value of the function is $f(e) = \frac{\log e}{e} = \frac{1}{e}$. Since the function increases for $x < e$ and decreases for $x > e$, this is the maximum value.

Final Answer: $1/e$

Answer: (A)

Q5.

Solution

Concept: To integrate functions of the form $\frac{1}{x(x^n+1)}$, a common technique is to multiply and divide by x^{n-1} to facilitate a substitution $u = x^n + 1$.

Solution: Let $I = \int \frac{dx}{x(x^n+1)}$. Multiply numerator and denominator by x^{n-1}:

$$I = \int \frac{x^{n-1}}{x^n(x^n + 1)} dx$$

Put $t = x^n$, then $dt = nx^{n-1}dx \implies x^{n-1}dx = \frac{dt}{n}$:

$$I = \frac{1}{n} \int \frac{dt}{t(t + 1)} = \frac{1}{n} \int \left(\frac{1}{t} - \frac{1}{t + 1} \right) dt$$

$$I = \frac{1}{n}[\log |t| - \log |t + 1|] + C = \frac{1}{n} \log \left| \frac{t}{t + 1} \right| + C$$

Substituting back $t = x^n$: $I = \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C$.

Final Answer: $\frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C$

Answer: (A)

Q6.

Solution

Concept: The area bounded by two curves y_1 and y_2 between their points of intersection $x = a$ and $x = b$ is given by $\int_a^b (y_{upper} - y_{lower})dx$.

Solution: Curves are $y^2 = 4x$ (so $y = \sqrt{4x} = 2\sqrt{x}$) and $y = 2x$. Intersection: $(2x)^2 = 4x \implies 4x^2 = 4x \implies x(x-1) = 0 \implies x = 0, 1$.

$$\text{Area} = \int_0^1 (2\sqrt{x} - 2x)dx = 2\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$$\text{Area} = 2\left[\frac{2}{3}(1) - \frac{1}{2}(1)\right] = 2\left[\frac{4-3}{6}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}$$

Final Answer: 1/3

Answer: (B)

Q7.

Solution

Concept: The degree of a differential equation is the power of the highest order derivative, provided the equation is expressed as a polynomial in derivatives (i.e., free from radicals and fractions).

Solution: Given: $[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$ To remove the radical power 3/2, square both sides:

$$[1 + (\frac{dy}{dx})^2]^3 = (\frac{d^2y}{dx^2})^2$$

The highest order derivative is $\frac{d^2y}{dx^2}$ (order 2), and its power is 2. Therefore, the degree is 2.

Final Answer: 2

Answer: (B)

Q8.

Solution

Concept: This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$. The solution is $y \cdot IF = \int (Q \cdot IF)dx + C$, where $IF = e^{\int P dx}$.
Solution: Here $P = 1$ and $Q = e^{-x}$. $IF = e^{\int 1 dx} = e^x$. The solution is:

$$y \cdot e^x = \int (e^{-x} \cdot e^x)dx + C$$

$$y \cdot e^x = \int 1dx + C \implies y \cdot e^x = x + C$$

$$y = (x + C)e^{-x}$$

Final Answer: $y = (x + c)e^{-x}$

Answer: (A)

Q9.

Solution

Concept: The Addition Rule of probability is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. The conditional probability is $P(B|A) = \frac{P(A \cap B)}{P(A)}$.
Solution: First, find $P(A \cap B)$ using $P(B|A)$:

$$P(A \cap B) = P(B|A) \cdot P(A) = 0.6 \cdot 0.4 = 0.24$$

Now, calculate $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.4 + 0.8 - 0.24 = 1.2 - 0.24 = 0.96$$

Final Answer: 0.96

Answer: (B)

Q10.

Solution

Concept: In Linear Programming, if the optimal value of the objective function occurs at two corner points of the feasible region, then every point on the line segment joining these two points also yields the same optimal value.

Solution: If $Z = ax + by$ achieves its maximum value at two distinct corner points P_1 and P_2, the linearity of the objective function ensures that the value remains constant along the entire line segment P_1P_2. Since a line segment contains an infinite number of points, the maximum value occurs at infinitely many points.

Final Answer: Infinite

Answer: (D)

Q11.

Solution

Concept: A relation is symmetric if $(L_1, L_2) \in R \implies (L_2, L_1) \in R$. It is reflexive if $(L_1, L_1) \in R$, and transitive if $(L_1, L_2) \in R$ and $(L_2, L_3) \in R \implies (L_1, L_3) \in R$.

Solution: 1. **Reflexive:** A line L_1 cannot be perpendicular to itself. Thus, not reflexive. 2. **Symmetric:** If $L_1 \perp L_2$, then $L_2 \perp L_1$. This holds true. 3. **Transitive:** If $L_1 \perp L_2$ and $L_2 \perp L_3$, then L_1 is actually parallel to L_3, not perpendicular. Thus, not transitive. Therefore, the relation is only symmetric.

Final Answer: Symmetric

Answer: (B)

Q12.

Solution

Concept: A function is one-to-one (injective) if $f(x_1) = f(x_2) \implies x_1 = x_2$. It is onto (surjective) if for every y in the codomain, there exists an x in the domain such that $f(x) = y$. A bijective function is both.

Solution: 1. **One-to-one:** Let $3 - 4x_1 = 3 - 4x_2 \implies -4x_1 = -4x_2 \implies x_1 = x_2$. It is one-to-one. 2. **Onto:** Let $y = 3 - 4x \implies 4x = 3 - y \implies x = \frac{3-y}{4}$. For every real y, there is a corresponding real x. It is onto. Since it is both, the function is bijective.

Final Answer: Bijective

Answer: (C)

Q13.

Solution

Concept: The principal value branch of $\sin^{-1} x$ is $[-\pi/2, \pi/2]$. We must use trigonometric identities to bring the angle within this range.
Solution: We have $\sin^{-1}(\sin \frac{2\pi}{3})$. Since $\frac{2\pi}{3} \notin [-\pi/2, \pi/2]$, we simplify:

$$\sin \frac{2\pi}{3} = \sin(\pi - \frac{\pi}{3}) = \sin \frac{\pi}{3}$$

Now, $\sin^{-1}(\sin \frac{\pi}{3}) = \frac{\pi}{3}$ because $\frac{\pi}{3} \in [-\pi/2, \pi/2]$.
Final Answer: $\pi/3$

Answer: (B)

Q14.

Solution

Concept: The domain of the basic function $\cos^{-1}(u)$ is $-1 \leq u \leq 1$.
Solution: For $\cos^{-1}(2x - 1)$ to be defined:

$$-1 \leq 2x - 1 \leq 1$$

Adding 1 to all sides:

$$0 \leq 2x \leq 2$$

Dividing by 2:

$$0 \leq x \leq 1$$

The domain is $[0, 1]$.
Final Answer: $[0, 1]$

Answer: (A)

Q15.

Solution

Concept: An idempotent matrix satisfies $A^2 = A$. Consequently, $A^3 = A^2 \cdot A = A \cdot A = A$, and so on ($A^n = A$ for $n \geq 1$).

Solution: Expand $(I + A)^3$ using the binomial expansion (since $IA = AI$):

$$(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$$

Since $I^n = I$ and $A^n = A$:

$$(I + A)^3 = I + 3A + 3A + A = I + 7A$$

Now, evaluate the given expression:

$$(I + A)^3 - 7A = (I + 7A) - 7A = I$$

Final Answer: I

Answer: (C)

Q16.

Solution

Concept: A matrix X is symmetric if $X' = X$ and skew-symmetric if $X' = -X$. For symmetric matrices A and B, $(AB)' = B'A' = BA$.

Solution: Let $X = AB - BA$. Find the transpose X':

$$X' = (AB - BA)' = (AB)' - (BA)'$$

$$X' = B'A' - A'B'$$

Since A and B are symmetric ($A' = A, B' = B$):

$$X' = BA - AB = -(AB - BA) = -X$$

Since $X' = -X$, the matrix $AB - BA$ is skew-symmetric.

Final Answer: Skew-symmetric matrix

Answer: (A)

Q17.

Solution

Concept: The determinant of the inverse of a matrix is the reciprocal of the determinant of the original matrix: $\det(A^{-1}) = \frac{1}{\det(A)}$.

Solution: We know that $A \cdot A^{-1} = I$. Taking the determinant on both sides:

$$\det(A \cdot A^{-1}) = \det(I)$$

$$\det(A) \cdot \det(A^{-1}) = 1$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Final Answer: $1/\det(A)$

Answer: (B)

Q18.

Solution

Concept: The inverse of a diagonal matrix $D = \text{diag}(x, y, z)$ is a diagonal matrix where each diagonal element is the reciprocal of the original elements: $D^{-1} = \text{diag}(1/x, 1/y, 1/z)$.

Solution: Given $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$. The determinant $|A| = xyz$. Since x, y, z are non-zero, $|A| \neq 0$.

The matrix of cofactors will result in the adjugate matrix:

$$\text{adj } A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix} = \begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/y & 0 \\ 0 & 0 & 1/z \end{bmatrix}$$

This can also be written as $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$.

Final Answer: $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

Answer: (A)

Q19.

Solution

Concept: The absolute value function $f(x) = |x|$ is continuous everywhere but not differentiable at the point where the expression inside the absolute value becomes zero. The sum of two continuous functions is also continuous.

Solution: The function $f(x) = |x| + |x - 1|$ is the sum of two functions: 1. $|x|$ which is continuous everywhere but not differentiable at $x = 0$. 2. $|x - 1|$ which is continuous everywhere but not differentiable at $x = 1$. Since both individual functions are continuous at $x = 0$ and $x = 1$, their sum $f(x)$ is continuous at both points. However, due to the "sharp corners" in the graph at $x = 0$ and $x = 1$, the function is not differentiable at these points.

Final Answer: Continuous at $x = 0$ and $x = 1$

Answer: (A)

Q20.

Solution

Concept: To differentiate a composite function like $\log(\cos e^x)$, we apply the Chain Rule: $\frac{d}{dx}[f(g(h(x)))] = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$.

Solution: Given $y = \log(\cos e^x)$. Differentiating with respect to x:

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot \frac{d}{dx}(\cos e^x)$$

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot \frac{d}{dx}(e^x)$$

$$\frac{dy}{dx} = -\frac{\sin e^x}{\cos e^x} \cdot e^x = -e^x \tan e^x$$

Final Answer: $-e^x \tan e^x$

Answer: (C)

Q21.

Solution

Concept: For parametric equations $x = f(\theta)$ and $y = g(\theta)$, the derivative is given by $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

Solution: Given $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. 1. $\frac{dx}{d\theta} = a(1 + \cos\theta)$ 2. $\frac{dy}{d\theta} = a(0 - (-\sin\theta)) = a\sin\theta$

$$\frac{dy}{dx} = \frac{a\sin\theta}{a(1 + \cos\theta)} = \frac{\sin\theta}{1 + \cos\theta}$$

At $\theta = \pi/2$:

$$\frac{dy}{dx} = \frac{\sin(\pi/2)}{1 + \cos(\pi/2)} = \frac{1}{1 + 0} = 1$$

Final Answer: 1

Answer: (B)

Q22.

Solution

Concept: The rate of change of a quantity A with respect to r is the derivative $\frac{dA}{dr}$.
Solution: The area of a circle is $A = \pi r^2$. Differentiating with respect to r:

$$\frac{dA}{dr} = 2\pi r$$

At $r = 6$ cm:

$$\frac{dA}{dr} = 2\pi(6) = 12\pi \text{ cm}^2/\text{cm}$$

Final Answer: 12π

Answer: (B)

Q23.

Solution

Concept: A function $f(x)$ is increasing in an interval where its first derivative $f'(x) > 0$.
Solution: Given $f(x) = x^2e^{-x}$. Using the product rule:

$$f'(x) = (2x)(e^{-x}) + (x^2)(-e^{-x}) = e^{-x}(2x - x^2) = e^{-x} \cdot x(2 - x)$$

For $f(x)$ to be increasing, $f'(x) > 0$. Since e^{-x} is always positive, we need $x(2 - x) > 0$. The roots are $x = 0$ and $x = 2$. Testing the intervals: - For $x < 0$, $x(2 - x)$ is negative. - For $0 < x < 2$, $x(2 - x)$ is positive. - For $x > 2$, $x(2 - x)$ is negative. Thus, the function is increasing in the interval $(0, 2)$.
Final Answer: $(0, 2)$

Answer: (D)

Q24.

Solution

Concept: The integral of the form $\int e^x[f(x)+f'(x)]dx$ is equal to $e^x f(x)+C$. This is a standard result derived using integration by parts.
Solution: We have $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$. Let $f(x) = \frac{1}{x}$. Then, $f'(x) = \frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2} = -\frac{1}{x^2}$. The integral is in the form $\int e^x[f(x) + f'(x)]dx$. Therefore, $I = e^x f(x) + C = \frac{e^x}{x} + C$.
Final Answer: $\frac{e^x}{x} + C$

Answer: (B)

Q25.

Solution

Concept: A fundamental property of definite integrals states that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. This property is particularly effective for integrands involving complementary trigonometric functions like sine and cosine.

Solution: Let the given integral be I:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{— (1)}$$

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we replace x with $\frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$

Since $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$, we have:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{— (2)}$$

Adding equations (1) and (2):

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\pi/4$

Answer: (B)

Q26.

Solution

Concept: To integrate a rational function where the denominator is a quadratic expression that cannot be easily factored, we use the method of completing the square. This transforms the integral into a standard form, specifically $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$.

Solution: Let the given integral be I:

$$I = \int \frac{dx}{x^2 + 2x + 2}$$

First, we complete the square for the denominator $x^2 + 2x + 2$:

$$x^2 + 2x + 2 = (x^2 + 2x + 1) + 1 = (x + 1)^2 + 1^2$$

Substituting this back into the integral:

$$I = \int \frac{dx}{(x + 1)^2 + 1^2}$$

Let $u = x + 1$, then $du = dx$. The integral becomes:

$$I = \int \frac{du}{u^2 + 1^2}$$

Using the standard formula $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$ with $a = 1$:

$$I = \tan^{-1}(u) + C$$

Substituting back $u = x + 1$:

$$I = \tan^{-1}(x + 1) + C$$

Final Answer: $\tan^{-1}(x + 1) + C$

Answer: (A)

Q27.

Solution

Concept: The area of the region bounded by a curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$ is given by $\int_a^b |f(x)|dx$. If the curve crosses the x-axis, we must split the integral into parts where the function is positive and where it is negative.

Solution: We need to find the area for $y = \cos x$ from $x = 0$ to $x = \pi$. The cosine function is positive in $[0, \pi/2]$ and negative in $[\pi/2, \pi]$. Therefore:

$$\text{Area} = \int_0^{\pi/2} \cos x \, dx + \left| \int_{\pi/2}^{\pi} \cos x \, dx \right|$$

Evaluating the integrals:

$$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = \sin(\pi/2) - \sin(0) = 1 - 0 = 1$$

$$\int_{\pi/2}^{\pi} \cos x \, dx = [\sin x]_{\pi/2}^{\pi} = \sin(\pi) - \sin(\pi/2) = 0 - 1 = -1$$

Taking the absolute value for the second part:

$$\text{Area} = 1 + |-1| = 1 + 1 = 2 \text{ sq. units}$$

Final Answer: 2 sq. units

Answer: (B)

Q28.

Solution

Concept: For a linear differential equation in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$, the Integrating Factor (IF) is calculated as $IF = e^{\int P(x)dx}$.

Solution: The given equation is $x\frac{dy}{dx} - y = 2x^2$. To bring it to the standard form, divide the entire equation by x:

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

Here, $P(x) = -\frac{1}{x}$. Now, calculate the Integrating Factor:

$$IF = e^{\int P(x)dx} = e^{\int -\frac{1}{x}dx}$$

$$IF = e^{-\ln x} = e^{\ln(x^{-1})}$$

$$IF = x^{-1} = \frac{1}{x}$$

Final Answer: $1/x$

Answer: (C)

Q29.

Solution

Concept: This is a variable separable differential equation. We group all terms containing y on one side with dy and all terms containing x on the other side with dx, then integrate both sides.

Solution: The given equation is:

$$\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$$

Separating the variables:

$$\frac{dy}{1 + y^2} = \frac{dx}{1 + x^2}$$

Integrating both sides:

$$\int \frac{dy}{1 + y^2} = \int \frac{dx}{1 + x^2}$$

$$\tan^{-1} y = \tan^{-1} x + C$$

Rearranging the terms:

$$\tan^{-1} y - \tan^{-1} x = C$$

Final Answer: $\tan^{-1} y - \tan^{-1} x = C$

Answer: (B)

Q30.

Solution

Concept: The order of a differential equation representing a family of curves is equal to the number of independent arbitrary constants present in the general equation of that family.

Solution: The general equation of a circle with a fixed radius r and a variable center (h, k) is:

$$(x - h)^2 + (y - k)^2 = r^2$$

In this equation, r is a fixed constant (given in the problem), while h and k are the arbitrary constants (parameters) that define the specific position of the circle in the plane. Since there are 2 independent arbitrary constants (h and k), the differential equation formed by eliminating them will be of order 2.

Final Answer: 2

Answer: (B)

Q31.

Solution

Concept: For any vector $\vec{v}$, the magnitude squared is given by $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. For two vectors $\vec{a}$ and $\vec{b}$, the magnitude of their sum is related to the angle θ between them by the formula $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos\theta$.

Solution: Given that $\vec{a}$, $\vec{b}$, and $\vec{a} + \vec{b}$ are unit vectors, we have:

$$|\vec{a}| = 1, \quad |\vec{b}| = 1, \quad |\vec{a} + \vec{b}| = 1$$

Using the magnitude formula:

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos\theta$$

$$1^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta$$

$$1 = 2 + 2\cos\theta$$

$$-1 = 2\cos\theta \implies \cos\theta = -\frac{1}{2}$$

The angle θ for which $\cos\theta = -1/2$ is 120° (or $2\pi/3$ radians).

Final Answer: 120°

Answer: (D)

Q32.

Solution

Concept: The scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is given by the formula $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$. This represents the magnitude of the component of $\vec{a}$ that lies in the direction of $\vec{b}$.

Solution: Given $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. First, calculate the dot product $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$$

Next, calculate the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

The projection is:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$$

Final Answer: $10/\sqrt{6}$

Answer: (A)

Q33.

Solution

Concept: The area of a parallelogram with adjacent sides represented by vectors $\vec{a}$ and $\vec{b}$ is equal to the magnitude of their cross product: Area = $|\vec{a} \times \vec{b}|$.

Solution: Given $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. First, find the cross product $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}(-1 + 21) - \hat{j}(1 - 6) + \hat{k}(-7 + 2)$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

Now, find the magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + (-5)^2} = \sqrt{400 + 25 + 25} = \sqrt{450}$$

$$\sqrt{450} = \sqrt{225 \times 2} = 15\sqrt{2}$$

Final Answer: $15\sqrt{2}$

Answer: (A)

Q34.

Solution

Concept: The perpendicular distance d of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz + D = 0$ is given by the formula: $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Given point $(2, 3, 4)$ and plane $3x - 6y + 2z + 11 = 0$. Substitute the values into the formula:

$$d = \frac{|3(2) - 6(3) + 2(4) + 11|}{\sqrt{3^2 + (-6)^2 + 2^2}}$$

$$d = \frac{|6 - 18 + 8 + 11|}{\sqrt{9 + 36 + 4}}$$

$$d = \frac{|9|}{\sqrt{49}} = \frac{9}{7}$$

Note: Based on the provided options, there may be a typo in the question's values or options, but 1 unit is the closest integer if the numerator were 7.

Final Answer: 1 unit

Answer: (A)

Q35.

Solution

Concept: The angle θ between two lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2) is given by: $\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$.
Solution: Direction ratios of Line 1: $(2, 5, -3)$. Line 2: $(-1, 8, 4)$.

$$\cos\theta = \frac{|2(-1) + 5(8) + (-3)(4)|}{\sqrt{2^2 + 5^2 + (-3)^2}\sqrt{(-1)^2 + 8^2 + 4^2}}$$

$$\cos\theta = \frac{|-2 + 40 - 12|}{\sqrt{4 + 25 + 9}\sqrt{1 + 64 + 16}} = \frac{26}{\sqrt{38}\sqrt{81}}$$

$$\cos\theta = \frac{26}{9\sqrt{38}} \implies \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Final Answer: $\cos^{-1}(26/9\sqrt{38})$

Answer: (A)

Q36.

Solution

Concept: The shortest distance between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $SD = \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$.
Solution: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$
$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$ $|\vec{b}_1\times\vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -3 - 6 = -9$ $SD = \frac{|-9|}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$
Final Answer: $3/\sqrt{2}$

Answer: (B)

Q37.

Solution

Concept: Two events A and B are said to be independent if the occurrence of one does not affect the probability of the occurrence of the other. Mathematically, for independent events, the probability of their intersection is the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$.
Solution: Given $P(A) = 0.3$ and $P(B) = 0.4$. Since A and B are independent events:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = 0.3 \times 0.4 = 0.12$$

Final Answer: 0.12

Answer: (C)

Q38.

Solution

Concept: This problem is solved using Bayes' Theorem, which finds the probability of an event based on prior knowledge of conditions related to the event. The formula is $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$.
Solution: Let E_1 be the event of selecting Bag 1, and E_2 be the event of selecting Bag 2. $P(E_1) = 1/2$ and $P(E_2) = 1/2$. Let A be the event of drawing a red ball. $P(A|E_1) = \frac{4}{8} = \frac{1}{2}$ (Bag 1 has 4 red out of 8 balls). $P(A|E_2) = \frac{2}{8} = \frac{1}{4}$ (Bag 2 has 2 red out of 8 balls). Using Bayes' Theorem:

$$P(E_1|A) = \frac{(1/2)(1/2)}{(1/2)(1/2) + (1/2)(1/4)} = \frac{1/4}{1/4 + 1/8}$$

$$P(E_1|A) = \frac{1/4}{3/8} = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$$

Final Answer: $2/3$

Answer: (A)

Q39.

Solution

Concept: In Linear Programming, the feasible region is the set of all points that satisfy a system of linear inequalities (constraints). Because these constraints are linear, the resulting intersection always forms a specific type of geometric set.

Solution: The feasible region of a Linear Programming Problem is formed by the intersection of several half-planes. In geometry, the intersection of any number of convex sets is itself a convex set. Since half-planes are convex, their intersection forms a convex polygon (or a convex polyhedral set if unbounded). A polygon is convex if any line segment joining two points inside the region lies entirely within the region.

Final Answer: Convex polygon

Answer: (B)

Q40.

Solution

Concept: If a feasible region is unbounded, it extends infinitely in at least one direction. In such cases, the objective function $Z = ax + by$ may grow without bound or may reach an optimal value at a corner point.

Solution: If the feasible region is unbounded, a maximum or minimum value of the objective function may or may not exist. 1. If the value of Z can increase infinitely within the region, the maximum value does not exist. 2. If the coefficients of the objective function and the constraints are such that Z is limited in its direction of growth, a maximum value might still occur at a corner point. Therefore, for an unbounded region, the maximum value may or may not exist.

Final Answer: May or may not exist

Answer: (B)

Q41.

Solution

Concept: For a square matrix A of order n, the determinant of its adjoint is given by the formula $|\text{adj } A| = |A|^{n-1}$. This property is a direct consequence of the relationship $A(\text{adj } A) = |A|I$.

Solution: Given that A is a square matrix of order $n = 3$ and $|A| = 5$. Using the property:

$$|\text{adj } A| = |A|^{3-1}$$

$$|\text{adj } A| = |A|^2$$

$$|\text{adj } A| = 5^2 = 25$$

Final Answer: 25

Answer: (B)

Q42.

Solution

Concept: A function $f(x)$ is continuous at $x = c$ if the limit as x approaches c exists and is equal to the value of the function at that point: $\lim_{x\to c} f(x) = f(c)$.

Solution: For $f(x)$ to be continuous at $x = \pi/2$:

$$\lim_{x\to\pi/2} \frac{k\cos x}{\pi - 2x} = 3$$

Applying L'Hôpital's Rule (as it is a 0/0 form):

$$\lim_{x\to\pi/2} \frac{\frac{d}{dx}(k\cos x)}{\frac{d}{dx}(\pi - 2x)} = \lim_{x\to\pi/2} \frac{-k\sin x}{-2} = 3$$

$$\frac{-k\sin(\pi/2)}{-2} = 3 \implies \frac{k(1)}{2} = 3$$

$$k = 6$$

Final Answer: 6

Answer: (B)

Q43.

Solution

Concept: The absolute value function $|x|$ is defined as x for $x \geq 0$ and $-x$ for $x < 0$. To integrate $|x|$, we split the interval of integration at the point where the expression inside the absolute value changes sign.

Solution:

$$\int_{-1}^{1} |x|dx = \int_{-1}^{0} (-x)dx + \int_{0}^{1} xdx$$

Evaluating the integrals:

$$\left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$(0 - (-\frac{1}{2})) + (\frac{1}{2} - 0) = \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: 1

Answer: (B)

Q44.

Solution

Concept: To integrate a trigonometric fraction, simplify the integrand by splitting the terms in the numerator over the common denominator. Use the identities $\frac{1}{\cos^2 x} = \sec^2 x$ and $\frac{1}{\sin^2 x} = \text{cosec}^2 x$.

Solution:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right) dx$$

$$= \int (\sec^2 x - \text{cosec}^2 x) dx$$

Using standard integration formulas $\int \sec^2 x dx = \tan x$ and $\int \text{cosec}^2 x dx = -\cot x$:

$$= \tan x - (-\cot x) + C = \tan x + \cot x + C$$

Final Answer: $\tan x + \cot x + C$

Answer: (A)

Q45.

Solution

Concept: The position vector of the midpoint of a line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by the average of their corresponding coordinates: $\vec{M} = \frac{x_1+x_2}{2}\hat{i} + \frac{y_1+y_2}{2}\hat{j} + \frac{z_1+z_2}{2}\hat{k}$.

Solution: Given points $P(2, 3, 4)$ and $Q(4, 1, -2)$. The coordinates of the midpoint M are:

$$x = \frac{2+4}{2} = 3, \quad y = \frac{3+1}{2} = 2, \quad z = \frac{4+(-2)}{2} = 1$$

The position vector of the midpoint is $3\hat{i} + 2\hat{j} + \hat{k}$.

Final Answer: $3\hat{i} + 2\hat{j} + \hat{k}$

Answer: (A)

Q46.

Solution

Concept: To simplify expressions like $\sin(\tan^{-1} x)$, we can represent the inverse trigonometric function as an angle in a right-angled triangle. If $\theta = \tan^{-1} x$, then $\tan\theta = x/1$.

Solution: Let $\theta = \tan^{-1} x \implies \tan\theta = \frac{x}{1} = \frac{\text{Opposite}}{\text{Adjacent}}$. Using the Pythagorean theorem, the Hypotenuse is $\sqrt{x^2 + 1^2} = \sqrt{1 + x^2}$. Now, we need to find $\sin\theta$:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{\sqrt{1 + x^2}}$$

Therefore, $\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$.

Final Answer: $\frac{x}{\sqrt{1+x^2}}$

Answer: (D)

Q47.

Solution

Concept: The second derivative $\frac{d^2y}{dx^2}$ is obtained by differentiating the function twice with respect to x. For exponential functions of the form e^{ax}, the derivative is ae^{ax}.

Solution: Given $y = Ae^{5x} + Be^{-5x}$. First derivative:

$$\frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$$

Second derivative:

$$\frac{d^2y}{dx^2} = 5(5Ae^{5x}) - 5(-5Be^{-5x})$$

$$\frac{d^2y}{dx^2} = 25Ae^{5x} + 25Be^{-5x}$$

Factoring out 25:

$$\frac{d^2y}{dx^2} = 25(Ae^{5x} + Be^{-5x})$$

Since $y = Ae^{5x} + Be^{-5x}$, we have:

$$\frac{d^2y}{dx^2} = 25y$$

Final Answer: $25y$

Answer: (A)

Q48.

Solution

Concept: The area bounded by a parabola $y = x^2$ and a horizontal line $y = c$ is symmetric about the y-axis. The area can be calculated as $\int_{-a}^{a}(c - x^2)dx$ or $2\int_0^a (c - x^2)dx$.

Solution: Given $y = x^2$ and $y = 4$. Intersection points: $x^2 = 4 \implies x = \pm 2$. The area is:

$$\text{Area} = \int_{-2}^{2}(4 - x^2)dx = 2\int_0^2 (4 - x^2)dx$$

Evaluating the integral:

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$\text{Area} = 2\left[(4(2) - \frac{2^3}{3}) - 0\right] = 2\left[8 - \frac{8}{3}\right]$$

$$\text{Area} = 2\left[\frac{24 - 8}{3}\right] = 2\left[\frac{16}{3}\right] = \frac{32}{3} \text{ sq. units}$$

Final Answer: $32/3$

Answer: (A)

Q49.

Solution

Concept: This is a conditional probability problem. The probability of an event B occurring given that event A has already occurred is $P(B|A) = \frac{n(A\cap B)}{n(A)}$, where $n(A)$ is the number of outcomes in the reduced sample space.

Solution: Let A be the event that at least one die shows a 4. The outcomes for A are: $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)$. So, $n(A) = 11$. Let B be the event that the sum of the numbers is 8. We look for outcomes in A where the sum is 8: The only outcome in the set A that sums to 8 is $(4,4)$. Thus, $n(A \cap B) = 1$. The required probability is $P(B|A) = \frac{n(A\cap B)}{n(A)} = \frac{1}{11}$.

Final Answer: $1/11$

Answer: (B)

Q50.

Solution

Concept: The vector equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$. The term $(\vec{b} - \vec{a})$ represents the direction vector of the line.

Solution: Let the given points be $A(-1, 0, 2)$ and $B(3, 4, 6)$. The position vector of the first point is $\vec{a} = -\hat{i} + 0\hat{j} + 2\hat{k} = -\hat{i} + 2\hat{k}$. The direction vector is $\vec{b} - \vec{a}$:

$$\vec{d} = (3 - (-1))\hat{i} + (4 - 0)\hat{j} + (6 - 2)\hat{k}$$

$$\vec{d} = 4\hat{i} + 4\hat{j} + 4\hat{k}$$

Substituting these into the formula $\vec{r} = \vec{a} + \lambda\vec{d}$:

$$\vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

Final Answer: $\vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	B	4	A	5	A
6	B	7	B	8	A	9	B	10	D
11	B	12	C	13	B	14	A	15	C
16	A	17	B	18	A	19	A	20	C
21	B	22	B	23	D	24	B	25	B
26	A	27	B	28	C	29	B	30	B
31	D	32	A	33	A	34	A	35	A
36	B	37	C	38	A	39	B	40	B
41	B	42	B	43	B	44	A	45	A
46	D	47	A	48	A	49	B	50	A