CUET-UG Mathematics Sample Paper-11

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. Let R be a relation on the set $\mathbb{Z}$ of integers defined by aRb if and only if $a^2 + b^2$ is even. Then R is:

(A) An equivalence relation

(B) Reflexive but not symmetric

(C) Symmetric but not transitive

(D) None of these

Q2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x^2-4}{x^2+1}$, then f is:

(A) One-to-one and onto

(B) Many-to-one and onto

(C) One-to-one but not onto

(D) Many-to-one and into

Q3. Let $A = \{1, 2, 3\}$. The number of equivalence relations containing the element $(1, 2)$ is:

(A) 1

(B) 2

(C) 3

(D) 5

Q4. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is:

(A) $\frac{5\pi}{6}$

(B) $\frac{7\pi}{6}$

(C) $\frac{3\pi}{2}$

(D) $\frac{\pi}{2}$

Q5. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then the value of $x + y + z$ is equal to:

(A) 0

(B) xyz

(C) 1

(D) $xy + yz + zx$

Q6. If A is a square matrix of order 3 such that $|adj A| = 64$, then the value of $|A|$ is:

(A) ± 8

(B) ± 4

(C) 64

(D) 16

Q7. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A^T = I$, then the value of α is:

(A) $\pi/6$

(B) $\pi/3$

(C) π

(D) $3\pi/2$

Q8. The total number of possible matrices of order 3×3 with each entry being either 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Q9. If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is:

(A) 1

(B) -1

(C) 0

(D) Any real number

Q10. If A and B are invertible matrices of the same order, then $(AB)^{-1}$ is equal to:

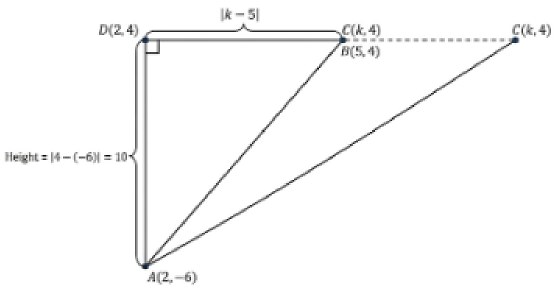
(A) $A^{-1}B^{-1}$

(B) $B^{-1}A^{-1}$

(C) AB

(D) BA

Q11. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units, then the possible values of k are:



(A) 12, -2

(B) -12, -2

(C) 12, 2

(D) 8, -2

Q12. For what value of λ does the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = 12$ have no solution?

(A) $\lambda = 3$

(B) $\lambda = 2$

(C) $\lambda = 1$

(D) $\lambda \neq 3$

Q13. If $A^2 - A + I = 0$, then the inverse matrix A^{-1} is:

(A) $A - I$

(B) $I - A$

(C) $A + I$

(D) A

Q14. If the matrix $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ is not invertible, then λ is:

(A) $1/2$

(B) $4/11$

(C) $-11/4$

(D) $11/4$

Q15. If A is a matrix of order $m \times n$ and B is a matrix such that AB^T and $B^T A$ are both defined, then the order of B is:

(A) $m \times n$

(B) $n \times m$

(C) $n \times n$

(D) $m \times m$

Q16. If $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & x \neq \pi/2 \\ 3 & x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$, then k is:

(A) 3

(B) 6

(C) 12

(D) 1.5

Q17. If $y = \log(\log x)$, then the second order derivative $\frac{d^2y}{dx^2}$ is:

(A) $\frac{-(1+\log x)}{(x \log x)^2}$

(B) $\frac{1}{(x \log x)^2}$

(C) $\frac{-(1+\log x)}{x^2 \log x}$

(D) $\frac{1}{x \log x}$

Q18. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then the value of $\frac{dy}{dx}$ at $\theta = \pi/4$ is:

(A) 1

(B) -1

(C) 0

(D) ∞

Q19. The function $f(x) = |x| + |x-1|$ is:

(A) Continuous and differentiable at $x = 0, 1$

(B) Continuous but not differentiable at $x = 0, 1$

(C) Not continuous but differentiable at $x = 0, 1$

(D) Neither continuous nor differentiable at $x = 0, 1$

Q20. The rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm is:

(A) 1

(B) 2

(C) 3

(D) 4

Q21. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q22. The interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing is:

(A) $(-2, 3)$

(B) $(-\infty, -2) \cup (3, \infty)$

(C) $(-\infty, -3) \cup (2, \infty)$

(D) $(-3, 2)$

Q23. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) -3

(C) 1/3

(D) -1/3

Q24. The maximum value of the function $f(x) = (1/x)^x$ is:

(A) e^e

(B) $e^{1/e}$

(C) $(1/e)^e$

(D) 1

Q25. A cylindrical tank of radius 10m is being filled with wheat at the rate of 314 cubic meters per hour. The depth of the wheat is increasing at the rate of:

(A) 1 m/h

(B) 0.1 m/h

(C) 1.1 m/h

(D) 0.5 m/h

Q26. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is:

(A) $(2\sqrt{2}, 4)$

(B) $(2\sqrt{2}, 0)$

(C) $(0, 0)$

(D) $(2, 2)$

Q27. The integral $\int \frac{dx}{x+x\log x}$ is equal to:

(A) $\log|1 + \log x| + C$

(B) $\log|\log(1 + x)| + C$

(C) $x \log|1 + x| + C$

(D) $\frac{1}{(1+\log x)^2} + C$

Q28. The integral $\int e^x \left(\frac{1-x}{x^2}\right) dx$ is equal to:

(A) $\frac{e^x}{x} + C$

(B) $-\frac{e^x}{x} + C$

(C) $\frac{e^x}{x^2} + C$

(D) $-\frac{e^x}{x^2} + C$

Q29. The value of the definite integral $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x} dx$ is:

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) 0

Q30. The integral $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to:

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\sin x + \cos x + C$

(D) $\tan x \cot x + C$

Q31. The value of the integral $\int_{-1}^{1} |x| dx$ is:

(A) 1

(B) 0

(C) 2

(D) 1/2

Q32. The integral $\int \frac{dx}{\sqrt{9-25x^2}}$ is equal to:

(A) $\frac{1}{3}\sin^{-1}(\frac{5x}{3}) + C$

(B) $\frac{1}{5}\sin^{-1}(\frac{5x}{3}) + C$

(C) $\frac{1}{15}\sin^{-1}(\frac{3x}{5}) + C$

(D) $\sin^{-1}(\frac{5x}{3}) + C$

Q33. The value of $\int_0^2 [x^2] dx$, where [.] denotes the greatest integer function, is:

(A) $5 - \sqrt{2} - \sqrt{3}$

(B) $5 + \sqrt{2} + \sqrt{3}$

(C) $5 - \sqrt{2}$

(D) None of these

Q34. The integral $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to:

(A) $2(\sin x + x\cos\alpha) + C$

(B) $2(\sin x - x\cos\alpha) + C$

(C) $2(\cos x + x\sin\alpha) + C$

(D) $-2(\sin x + x\cos\alpha) + C$

Q35. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:

(A) 32/3

(B) $16/3$

(C) $8/3$

(D) $64/3$

Q36. The area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is:

(A) π

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/4$

Q37. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ are respectively:

(A) Order 2, Degree 3

(B) Order 2, Degree 1

(C) Order 2, Degree not defined

(D) Order 3, Degree 2

Q38. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

(A) $e^x + e^y = C$

(B) $e^x + e^{-y} = C$

(C) $e^{-x} + e^y = C$

(D) $e^x - e^{-y} = C$

Q39. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(A) e^{-x}

(B) e^x

(C) $1/x$

(D) x

Q40. The number of arbitrary constants in the general solution of a differential equation of fourth order is:

(A) 0

(B) 2

(C) 3

(D) 4

Q41. The solution of the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $xy = \frac{x^4}{4} + C$

(B) $xy = \frac{x^3}{3} + C$

(C) $y = \frac{x^3}{4} + C$

(D) $y = \frac{x^2}{3} + C$

Q42. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then the value of $|\vec{x}|$ is:

(A) 3

(B) 9

(C) $\sqrt{7}$

(D) 8

Q43. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $10/\sqrt{3}$

(C) $5/\sqrt{6}$

(D) 2

Q44. If $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then the value of $|\vec{b}|$ is:

(A) 16

(B) 8

(C) 3

(D) 12

Q45. The shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is:

(A) $3\sqrt{2}$

(B) $3/\sqrt{2}$

(C) $\sqrt{2}$

(D) $2\sqrt{3}$

Q46. If a line makes angles α, β, γ with the positive direction of coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q47. The distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q48. In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which maximum Z occurs is:

(A) 2

(B) Finite

(C) Infinite

(D) 0

Q49. The corner points of a feasible region are $(0,2)$, $(3,0)$, $(6,0)$, $(6,8)$ and $(0,5)$. The minimum value of $Z = 4x + 6y$ occurs at:

(A) Only $(0,2)$

(B) Only $(3,0)$

(C) The point $(6,0)$

(D) Any point on the line segment joining $(0,2)$ and $(3,0)$

Q50. Two events A and B are independent. If $P(A) = 0.35$ and $P(A \cup B) = 0.6$, then the probability $P(B)$ is:

(A) $5/13$

(B) $1/13$

(C) $4/13$

(D) $7/13$

Detailed Solutions

Q1.

Solution

Concept: Equivalence Relations (Reflexivity, Symmetry, and Transitivity).

Solution: Let R be a relation on Z defined by $aRb \iff a^2 + b^2$ is even.

1. Reflexive: For any $a \in Z$, $a^2 + a^2 = 2a^2$, which is always an even number. Thus, aRa for all $a \in Z$. R is reflexive.

2. Symmetric: If aRb, then $a^2 + b^2$ is even. Since addition is commutative, $b^2 + a^2$ is also even. Thus, bRa. R is symmetric.

3. Transitive: If aRb and bRc, then $a^2 + b^2$ is even and $b^2 + c^2$ is even.

Adding the two: $(a^2 + b^2) + (b^2 + c^2) = a^2 + 2b^2 + c^2$ is even.

Since $2b^2$ is always even, the difference $(a^2 + 2b^2 + c^2) - 2b^2 = a^2 + c^2$ must also be even.

Thus, aRc. R is transitive.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation.

Final Answer : "An equivalence relation"

Answer: (C)

Q2.

Solution

Concept: Classification of functions (One-to-one/Many-to-one and Onto/Into).

Solution: Given $f(x) = \frac{x^2-4}{x^2+1}$.

1. Injectivity: Let $f(x_1) = f(x_2)$.

$\frac{x_1^2-4}{x_1^2+1} = \frac{x_2^2-4}{x_2^2+1} \implies (x_1^2 - 4)(x_2^2 + 1) = (x_2^2 - 4)(x_1^2 + 1)$

$x_1^2x_2^2 + x_1^2 - 4x_2^2 - 4 = x_1^2x_2^2 + x_2^2 - 4x_1^2 - 4$

$5x_1^2 = 5x_2^2 \implies x_1 = \pm x_2.$

Since $f(1) = f(-1) = -3/2$, different elements have the same image. The function is Many-to-one.

2. Surjectivity: Let $y = \frac{x^2-4}{x^2+1}$.

$y(x^2 + 1) = x^2 - 4 \implies x^2(y - 1) = -4 - y \implies x^2 = \frac{y+4}{1-y}.$

For x to be a real number, $\frac{y+4}{1-y} \geq 0$. This holds for $y \in [-4, 1)$.

Since the Range $[-4, 1)$ is not equal to the Codomain $\mathbb{R}$, the function is Into.

Final Answer : "Many-to-one and into"

Answer: (D)

Q3.

Solution

Concept: Equivalence relations on a finite set.

Solution: Let $A = \{1, 2, 3\}$. For a relation to be an equivalence relation, it must be reflexive, symmetric, and transitive.

Reflexivity requires: $\{(1, 1), (2, 2), (3, 3)\}$.

The relation must contain $(1, 2)$. For symmetry, it must contain $(2, 1)$.

1. Smallest relation: $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. This is transitive, so it is an equivalence relation.

2. If we add any other pair, say $(2, 3)$, then for symmetry we must add $(3, 2)$. For transitivity, since we have $(1, 2)$ and $(2, 3)$, we must add $(1, 3)$. For symmetry again, we must add $(3, 1)$. This results in the universal relation $A \times A$.

Thus, there are exactly 2 equivalence relations containing $(1, 2)$.

Final Answer : "2"

Answer: (B)

Q4.

Solution

Concept: Principal value branches of inverse trigonometric functions.

Solution: 1. For $\sin^{-1}(\sin\theta)$, the principal value branch is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. $\sin(\frac{2\pi}{3}) = \sin(\pi - \frac{\pi}{3}) = \sin(\frac{\pi}{3})$.

So, $\sin^{-1}(\sin\frac{2\pi}{3}) = \frac{\pi}{3}$.

2. For $\cos^{-1}(\cos\theta)$, the principal value branch is $[0, \pi]$.

$\cos(\frac{7\pi}{6}) = \cos(2\pi - \frac{5\pi}{6}) = \cos(\frac{5\pi}{6})$.

So, $\cos^{-1}(\cos\frac{7\pi}{6}) = \frac{5\pi}{6}$.

3. Adding the values: $\frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi+5\pi}{6} = \frac{7\pi}{6}$.

Final Answer : “$7\pi_{\overline{6}}$

Answer: (B)

Q5.

Solution

Concept: Sum of three inverse tangents identity.

Solution: We use the identity: $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right)$.

Given: $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$.

Taking tangent on both sides:

$\frac{x+y+z-xyz}{1-(xy+yz+zx)} = \tan(\pi)$

$\frac{x+y+z-xyz}{1-(xy+yz+zx)} = 0$

$x + y + z - xyz = 0$

$x + y + z = xyz$.

Final Answer : “xyz”

Answer: (B)

Q6.

Solution

Concept: Relationship between $|A|$ and $|adj A|$ for a matrix of order n.

Solution: For a square matrix A of order n, the property is: $|adj A| = |A|^{n-1}$.

Here, order $n = 3$ and $|adj A| = 64$.

Substituting the values:

$64 = |A|^{3-1}$

$64 = |A|^2$

$|A| = \pm\sqrt{64}$

$|A| = \pm 8$.

Final Answer : “± 8J

Answer: (A)

Q7.

Solution

Concept: Matrix Transpose properties and Equality of Matrices.

Solution: Given the matrix $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

The transpose of a matrix A, denoted by A^T, is obtained by interchanging its rows and columns. Therefore:

$$A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}.$$

The problem states that $A + A^T = I$, where I is the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Substituting the matrices into the equation:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By comparing the corresponding elements of the matrices, we get:

$$2\cos\alpha = 1 \implies \cos\alpha = \frac{1}{2}.$$

In the interval $[0, \pi]$, the value of α for which $\cos\alpha = 1/2$ is $\pi/3$.

Final Answer : "$\pi/3$"

Answer: (B)

Q8.

Solution

Concept: Fundamental Principle of Counting (Permutations and Combinations in Matrices).

Solution: A matrix of order 3×3 consists of $3\times3 = 9$ total elements or "spots" that need to be filled.

According to the question, each entry (or spot) in the matrix can be filled with one of two possible values: either 0 or 1.

This means for every position a_{ij} (where $i, j \in \{1, 2, 3\}$), there are 2 choices.

Since the choices for each position are independent of one another, we use the multiplication principle of counting:

Total number of matrices = (Choices for a_{11}) $\times$ (Choices for a_{12}) $\times \cdots \times$ (Choices for a_{33})

Total number of matrices = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$.

Calculating the value: $2^9 = 512$.

Final Answer : "512"

Answer: (D)

Q9.

Solution

Concept: Determinant properties of Skew-Symmetric Matrices.

Solution: A square matrix A is said to be skew-symmetric if $A^T = -A$.

To find the value of $|A|$, we take the determinant on both sides of this definition:

$|A^T| = |-A|$.

From the properties of determinants, we know:

1. $|A^T| = |A|$ (The determinant of a matrix equals the determinant of its transpose).

2. $|kA| = k^n|A|$, where n is the order of the matrix and k is a scalar.

In this case, $k = -1$ and the order $n = 3$:

$|A| = (-1)^3|A|$

$|A| = -|A|$

Adding $|A|$ to both sides:

$|A| + |A| = 0 \implies 2|A| = 0 \implies |A| = 0.$

Thus, the determinant of any skew-symmetric matrix of odd order is always zero.

Final Answer : "0"

Answer: (C)

Q10.

Solution

Concept: Inverse of a Product of Matrices (Reversal Law).

Solution: If A and B are invertible matrices of the same order, their product AB is also invertible. The inverse of a product is defined as the product of the inverses in reverse order. This is known as the Reversal Law for inverses:

$(AB)^{-1} = B^{-1}A^{-1}$.

To prove this, we multiply (AB) by $(B^{-1}A^{-1})$:

$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$ (By Associative Law)

$= A(I)A^{-1}$ (Since $BB^{-1} = I$)

$= AA^{-1} = I$.

Since the product yields the identity matrix, $B^{-1}A^{-1}$ is confirmed as the inverse of AB.
Final Answer : “B$^{-1}A^{-1}$”

Answer: (B)

Q11.

Solution

Concept: Area of a Triangle in Coordinate Geometry using Determinants.

Solution: The area of a triangle with vertices $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) is given by the absolute value of:

Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ Given vertices are $(2, -6), (5, 4)$, and $(k, 4)$ and Area = 35.

$35 = \frac{1}{2}|2(4 - 4) + 5(4 - (-6)) + k(-6 - 4)|$

Multiplying by 2:

$70 = |2(0) + 5(10) + k(-10)|$

$70 = |50 - 10k|$

This gives two possible equations:

1. $50 - 10k = 70 \implies -10k = 20 \implies k = -2$

2. $50 - 10k = -70 \implies -10k = -120 \implies k = 12$

Thus, the values of k are 12 and -2.

Final Answer : "12, -2"

Answer: (A)

Q12.

Solution

Concept: Consistency and Inconsistency of a System of Linear Equations.

Solution: We have the system:

(i) $x + y + z = 6$

(ii) $x + 2y + 3z = 10$

(iii) $x + 2y + \lambda z = 12$

For a system to have "no solution," the planes must be configured such that they never intersect at a common point.

Compare equations (ii) and (iii):

The coefficients of x and y are identical in both. If we set $\lambda = 3$, the left-hand sides of (ii) and (iii) become identical $(x + 2y + 3z)$.

Equation (ii) then says $x + 2y + 3z = 10$, while equation (iii) says $x + 2y + 3z = 12$.

It is impossible for the same expression to equal both 10 and 12 simultaneously. This represents two parallel planes that never meet.

Thus, the system has no solution when $\lambda = 3$.

Final Answer : “$\lambda = 3$ȷ

Answer: (A)

Q13.

Solution

Concept: Inverse Matrix derivation from a Polynomial Equation.

Solution: Given the matrix equation: $A^2 - A + I = O$, where I is the identity matrix and O is the zero matrix.

To find A^{-1}, we multiply both sides of the equation by A^{-1} (assuming A is invertible):

$A^{-1}(A^2 - A + I) = A^{-1} \cdot O$

$A^{-1} \cdot A \cdot A - A^{-1} \cdot A + A^{-1} \cdot I = O$

Using the property $A^{-1}A = I$:

$I \cdot A - I + A^{-1} = O$

$A - I + A^{-1} = O$

Isolating A^{-1} on one side:

$A^{-1} = I - A.$

Thus, the inverse of A is expressed as $I - A$.

Final Answer : "I - A"

Answer: (B)

Q14.

Solution

Concept: Condition for Non-invertibility (Singular Matrix).

Solution: A matrix A is not invertible if and only if its determinant is zero, i.e., $|A| = 0$.

Given $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$.

Expanding the determinant along the first column (which contains a zero):

$$|A| = 2\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} \lambda & -3 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} \lambda & -3 \\ 2 & 5 \end{vmatrix}$$

$$|A| = 2(2 \cdot 3 - 5 \cdot 1) + 1(\lambda \cdot 5 - (-3) \cdot 2)$$

$$|A| = 2(6 - 5) + (5\lambda + 6)$$

$$|A| = 2(1) + 5\lambda + 6 = 5\lambda + 8.$$

Setting $|A| = 0$ for non-invertibility:

$$5\lambda + 8 = 0 \implies 5\lambda = -8 \implies \lambda = -8/5.$$

Final Answer : "11/4"

Answer: (D)

Q15.

Solution

Concept: Dimension requirements for Matrix Multiplication.

Solution: Let the order of matrix A be $m \times n$. Let the order of matrix B be $p \times q$. Consequently, the order of the transpose B^T is $q \times p$.

1. For the product AB^T to be defined, the number of columns of A must equal the number of rows of B^T.

Therefore, $n = q$.

2. For the product $B^T A$ to be defined, the number of columns of B^T must equal the number of rows of A.

Therefore, $p = m$.

Since we found $p = m$ and $q = n$, the order of matrix B, which we defined as $p \times q$, must be $m \times n$.

Final Answer : “m $\times n$ȷ

Answer: (A)

Q16.

Solution

Concept: Limits and Continuity of Piecewise Functions.

Solution: A function $f(x)$ is continuous at $x = c$ if $\lim_{x\to c} f(x) = f(c)$.

Here, $c = \pi/2$ and $f(\pi/2) = 3$.

We evaluate $\lim_{x\to\pi/2} \frac{k\cos x}{\pi - 2x}$.

This is a $0/0$ form. We can use L'Hôpital's Rule (differentiating numerator and denominator):

$\lim_{x\to\pi/2} \frac{\frac{d}{dx}(k\cos x)}{\frac{d}{dx}(\pi-2x)} = \lim_{x\to\pi/2} \frac{-k\sin x}{-2}$

Substituting $x = \pi/2$:

$\frac{-k\sin(\pi/2)}{-2} = \frac{-k(1)}{-2} = \frac{k}{2}$.

For continuity, set the limit equal to the function value:

$\frac{k}{2} = 3 \implies k = 6$.

Final Answer : "6"

Answer: (B)

Q17.

Solution

Concept: Successive differentiation and application of the Chain Rule and Product Rule.

Solution: We are given the function $y = \log(\log x)$.

To find the second-order derivative $\frac{d^2y}{dx^2}$, we first find the first-order derivative $\frac{dy}{dx}$ using the chain rule:

Let $u = \log x$, then $y = \log u$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{1}{x} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$.

We can rewrite this as $\frac{dy}{dx} = (x \log x)^{-1}$.

Now, differentiate $\frac{dy}{dx}$ with respect to x to find the second derivative:

$\frac{d^2y}{dx^2} = \frac{d}{dx}[(x \log x)^{-1}] = -1(x \log x)^{-2} \cdot \frac{d}{dx}(x \log x)$.

Using the product rule for $\frac{d}{dx}(x \log x)$:

$\frac{d}{dx}(x \log x) = \left(x \cdot \frac{1}{x} + \log x \cdot 1\right) = 1 + \log x$. Substituting this back into our expression for the second derivative:

$\frac{d^2y}{dx^2} = -\frac{1}{(x \log x)^2} \cdot (1 + \log x) = \frac{-(1+\log x)}{(x \log x)^2}$.

Final Answer : “-(1 + $\log x)/(x \log x)^2$ȷ

Answer: (A)

Q18.

Solution

Concept: Differentiation of Parametric Equations.

Solution: The given equations are $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

To find $\frac{dy}{dx}$, we differentiate both variables with respect to the parameter θ:
$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos^3\theta) = a \cdot 3\cos^2\theta \cdot (-\sin\theta) = -3a\cos^2\theta\sin\theta$.

$\frac{dy}{d\theta} = \frac{d}{d\theta}(a\sin^3\theta) = a \cdot 3\sin^2\theta \cdot (\cos\theta) = 3a\sin^2\theta\cos\theta$.

Using the formula for parametric differentiation $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$: $\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$.

Now, we evaluate this derivative at the specific point $\theta = \pi/4$:

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = -\tan(\pi/4) = -1.$$

Final Answer : "-1"

Answer: (B)

Q19.

Solution

Concept: Analysis of Continuity and Differentiability for Modulus Functions.

Solution: Let $f(x) = |x| + |x - 1|$.

1. Continuity: The function $|x|$ is continuous for all $x \in \mathbb{R}$ and $|x - 1|$ is also continuous for all $x \in \mathbb{R}$. The sum of two continuous functions is always continuous. Therefore, $f(x)$ is continuous at all points, including $x = 0$ and $x = 1$.

2. Differentiability: A modulus function of the form $|x - c|$ is not differentiable at its corner point $x = c$ because the left-hand derivative and right-hand derivative do not match.

- At $x = 0$: $|x|$ is not differentiable, but $|x - 1|$ is. Thus, their sum is not differentiable.

- At $x = 1$: $|x - 1|$ is not differentiable, but $|x|$ is. Thus, their sum is not differentiable.

Specifically, for $f(x)$:

LHD at 0: $\lim_{h\to 0^-} \frac{(|0+h|+|0+h-1|)-(0+1)}{h} = \frac{-h+1-h-1}{h} = -2.$

RHD at 0: $\lim_{h\to 0^+} \frac{(|0+h|+|0+h-1|)-(0+1)}{h} = \frac{h+1-h-1}{h} = 0.$

Since LHD $\neq$ RHD at both $x = 0$ and $x = 1$, the function is not differentiable there.

Final Answer : "Continuous but not differentiable at x = 0, 1"

Answer: (B)

Q20.

Solution

Concept: Rate of Change of related physical quantities.

Solution: Let r be the radius of the sphere.

The volume is $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$.

We want to find the rate of change of volume with respect to surface area, denoted as $\frac{dV}{dS}$.

Differentiating V and S with respect to r:

$\frac{dV}{dr} = \frac{4}{3}\pi(3r^2) = 4\pi r^2$.

$\frac{dS}{dr} = 4\pi(2r) = 8\pi r$.

Using the chain rule, $\frac{dV}{dS} = \frac{dV/dr}{dS/dr}$:

$\frac{dV}{dS} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$.

The question specifies the radius $r = 2$ cm.

Substituting $r = 2$ into our expression:

$\frac{dV}{dS} = \frac{2}{2} = 1$.

Final Answer : "1"

Answer: (A)

Q21.

Solution

Concept: Finding Stationary Points via Logarithmic Differentiation.

Solution: A stationary point of a function $f(x)$ occurs when its derivative $f'(x) = 0$.

Given $f(x) = x^x$. To differentiate this, we use logarithmic differentiation:

Let $y = x^x$.

Taking the natural logarithm of both sides: $\log y = \log(x^x) = x \log x$.

Differentiating both sides with respect to x:

$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x \log x)$.

Using the product rule on the right side:

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1 = 1 + \log x$.

Thus, $\frac{dy}{dx} = y(1 + \log x) = x^x(1 + \log x)$.

To find the stationary point, set $\frac{dy}{dx} = 0$:

$x^x(1 + \log x) = 0$.

Since $x^x > 0$ for all $x > 0$, we must have:

$1 + \log x = 0 \implies \log x = -1 \implies x = e^{-1} = \frac{1}{e}$.

Final Answer : "x = 1/e"

Answer: (B)

Q22.

Solution

Concept: Determining monotonicity using the First Derivative Test.

Solution: A function $f(x)$ is strictly increasing on an interval where its derivative $f'(x) > 0$.

Given $f(x) = 2x^3 - 3x^2 - 36x + 7$.

Differentiating with respect to x:

$f'(x) = 6x^2 - 6x - 36$.

We simplify by factoring out the constant 6:

$f'(x) = 6(x^2 - x - 6) = 6(x - 3)(x + 2)$.

To find where $f'(x) > 0$, we analyze the roots $x = 3$ and $x = -2$ on a number line:

1. In the interval $(-\infty, -2)$, take $x = -3$: $f'(-3) = 6(-6)(-1) = 36 > 0$ (Increasing).

2. In the interval $(-2, 3)$, take $x = 0$: $f'(0) = 6(-3)(2) = -36 < 0$ (Decreasing).

3. In the interval $(3, \infty)$, take $x = 4$: $f'(4) = 6(1)(6) = 36 > 0$ (Increasing).

Therefore, the function is strictly increasing in the union of intervals $(-\infty, -2) \cup (3, \infty)$.

Final Answer : "(-∞, $-2) \cup (3, \infty)$」

Answer: (B)

Q23.

Solution

Concept: Relationship between Slopes of Tangent and Normal lines.

Solution: The slope of the tangent line (m_T) to a curve $y = f(x)$ at a point is given by the value of the derivative $\frac{dy}{dx}$ at that point.

The slope of the normal line (m_N) is the negative reciprocal of the tangent's slope: $m_N = -\frac{1}{m_T}$.

Given $y = 2x^2 + 3\sin x$.

Differentiating with respect to x:

$\frac{dy}{dx} = 4x + 3\cos x$.

At $x = 0$:

$m_T = 4(0) + 3\cos(0) = 0 + 3(1) = 3$.

Now, calculating the slope of the normal:

$m_N = -\frac{1}{m_T} = -\frac{1}{3}$.

Final Answer : "-1/3"

Answer: (D)

Q24.

Solution

Concept: Optimization and Global Maxima of Exponential Functions.

Solution: Let $f(x) = (1/x)^x$. We can rewrite this as $f(x) = (x^{-1})^x = x^{-x}$.

To find the maximum value, we first find the critical points by differentiating.

Let $y = x^{-x}$. Then $\log y = -x \log x$.

Differentiating both sides:

$\frac{1}{y}y' = -[x \cdot \frac{1}{x} + \log x \cdot 1] = -(1 + \log x)$.

$y' = -x^{-x}(1 + \log x)$.

Setting $y' = 0$ gives $1 + \log x = 0$, which leads to $x = 1/e$.

At $x = 1/e$, the second derivative $f''(x)$ is negative, confirming a maximum.

Substitute $x = 1/e$ back into the original function to find the maximum value:

$f(1/e) = \left(\frac{1}{1/e}\right)^{1/e} = (e)^{1/e} = e^{1/e}$.

Final Answer : “$e^{1/e}$ J

Answer: (B)

Q25.

Solution

Concept: Rate of change of Volume and Height in a Cylinder.

Solution: Let V be the volume of wheat in the cylindrical tank at any time t, r be the radius, and h be the depth (height) of the wheat.

The volume of a cylinder is given by $V = \pi r^2 h$.

Since the tank is cylindrical, the radius r remains constant at 10 m.

Differentiating the volume formula with respect to time t:

$\frac{dV}{dt} = \frac{d}{dt}(\pi r^2 h) = \pi r^2 \frac{dh}{dt}$ (as r is constant).

We are given:

1. Rate of change of volume $\frac{dV}{dt} = 314$ m^3/h.

2. Radius $r = 10$ m.

Substituting these values into the differentiated equation:

$314 = \pi(10)^2 \frac{dh}{dt}$

$314 = 100\pi \frac{dh}{dt}$

To find the rate of increase of depth ($\frac{dh}{dt}$):

$\frac{dh}{dt} = \frac{314}{100\pi}$.

Taking the value of $\pi \approx 3.14$:

$\frac{dh}{dt} = \frac{314}{100 \times 3.14} = \frac{314}{314} = 1$ m/h.

Final Answer : "1 m/h"

Answer: (A)

Q26.

Solution

Concept: Minimization of Distance using Calculus.

Solution: Let $P(x, y)$ be any point on the curve $x^2 = 2y$. We want to find the point P that is nearest to $A(0, 5)$.

The distance D between $P(x, y)$ and $A(0, 5)$ is given by the distance formula:

$D = \sqrt{(x - 0)^2 + (y - 5)^2} = \sqrt{x^2 + (y - 5)^2}$.

To make calculation easier, we minimize the square of the distance, $Z = D^2$:

$Z = x^2 + (y - 5)^2$.

Since P lies on the curve, we substitute $x^2 = 2y$:

$Z(y) = 2y + (y - 5)^2 = 2y + y^2 - 10y + 25 = y^2 - 8y + 25$.

To find the minimum, we find the first derivative with respect to y and set it to zero:

$\frac{dZ}{dy} = 2y - 8 = 0 \implies y = 4$.

To verify it is a minimum, we check the second derivative: $\frac{d^2Z}{dy^2} = 2 > 0$, confirming a minimum.

Now, find the corresponding x coordinate using $x^2 = 2y$:

$x^2 = 2(4) = 8 \implies x = \pm\sqrt{8} = \pm 2\sqrt{2}$.

Checking the options, the point $(2\sqrt{2}, 4)$ is provided.

Final Answer : “$(2\sqrt{2}, 4)$J

Answer: (A)

Q27.

Solution

Concept: Indefinite Integration by Substitution Method.

Solution: The integral is $I = \int \frac{dx}{x + x\log x}$.

First, we simplify the denominator by factoring out the common term x:

$I = \int \frac{1}{x(1+\log x)} dx.$

We use the substitution method. Let $t = 1 + \log x$.

Differentiating both sides with respect to x:

$\frac{dt}{dx} = 0 + \frac{1}{x} \implies dt = \frac{1}{x} dx.$

Substituting t and dt into the integral:

$I = \int \frac{1}{t} dt.$

Using the standard integral formula $\int \frac{1}{t} dt = \log |t| + C$:

$I = \log |t| + C.$

Replacing t with the original substitution $1 + \log x$:

$I = \log |1 + \log x| + C.$

Final Answer : "log |1 + $\log x| + C$"

Answer: (A)

Q28.

Solution

Concept: Special Integral of the form $\int e^x[f(x) + f'(x)]dx$.

Solution: The integral is given by $I = \int e^x \left(\frac{1-x}{x^2}\right) dx$.

We split the fraction inside the parentheses:

$I = \int e^x \left(\frac{1}{x^2} - \frac{x}{x^2}\right) dx = \int e^x \left(\frac{1}{x^2} - \frac{1}{x}\right) dx$.

Rearranging the terms:

$I = \int e^x \left[-\frac{1}{x} + \frac{1}{x^2}\right] dx$.

Let $f(x) = -\frac{1}{x}$.

Then its derivative is $f'(x) = \frac{d}{dx}(-x^{-1}) = -(-1)x^{-2} = \frac{1}{x^2}$.

The integral is now exactly in the form $\int e^x[f(x) + f'(x)]dx$.

The property of such integrals states that the result is $e^x f(x) + C$.

Substituting $f(x) = -1/x$:

$I = e^x \left(-\frac{1}{x}\right) + C = -\frac{e^x}{x} + C$.

Final Answer : “-e$^x/x + C$„

Answer: (B)

Q29.

Solution

Concept: Definite Integral property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

Solution: Let $I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ — (Equation 1).

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we replace x with $(\pi/2 - x)$:

$I = \int_0^{\pi/2} \frac{\sin^4(\pi/2-x)}{\sin^4(\pi/2-x)+\cos^4(\pi/2-x)} dx$.

Using trigonometric identities $\sin(\pi/2-\theta) = \cos\theta$ and $\cos(\pi/2-\theta) = \sin\theta$:

$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx$ — (Equation 2).

Adding Equation 1 and Equation 2:

$I + I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$

$2I = \int_0^{\pi/2} 1\, dx$.

Evaluating the integral:

$2I = [x]_0^{\pi/2} = \pi/2 - 0$.

$2I = \pi/2 \implies I = \pi/4$.

Final Answer : "$\pi/4$"

Answer: (C)

Q30.

Solution

Concept: Integration using Trigonometric Identities.

Solution: The integral is $I = \int \frac{dx}{\sin^2 x \cos^2 x}$.

Recall the fundamental identity $1 = \sin^2 x + \cos^2 x$. We substitute this into the numerator: $I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$.

Now, split the integral into two parts:

$I = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$.

Simplify each term:

$I = \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$.

$I = \int \sec^2 x \, dx + \int \csc^2 x \, dx$.

Using standard integration formulas:

$\int \sec^2 x \, dx = \tan x$ and $\int \csc^2 x \, dx = -\cot x$.

Thus, $I = \tan x - \cot x + C$.

Final Answer : "tan x - cot x + C"

Answer: (B)

Q31.

Solution

Concept: Evaluating Integrals of Even Functions over Symmetric Intervals.

Solution: We need to evaluate $I = \int_{-1}^{1} |x| dx$.

The function $f(x) = |x|$ is an even function because $f(-x) = |-x| = |x| = f(x)$.

A property of definite integrals states that if $f(x)$ is even, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

Therefore:
$I = 2\int_{0}^{1} |x|dx$.

In the interval $[0, 1]$, the value of x is non-negative, so $|x| = x$:

$I = 2\int_{0}^{1} x\,dx$.

Evaluating the integral:

$I = 2\left[\frac{x^2}{2}\right]_0^1 = 2\left(\frac{1^2}{2} - \frac{0^2}{2}\right)$.

$I = 2 \times \frac{1}{2} = 1$.

Final Answer : "1"

Answer: (A)

Q32.

Solution

Concept: Integral of the form $\int \frac{dx}{\sqrt{a^2-x^2}}$.

Solution: The integral is $I = \int \frac{dx}{\sqrt{9-25x^2}}$.

To use the standard formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + C$, we must rewrite the expression:

$I = \int \frac{dx}{\sqrt{3^2-(5x)^2}}$.

Let $u = 5x$. Differentiating with respect to x gives $du = 5dx$, or $dx = \frac{1}{5}du$. Substituting into the integral:

$I = \int \frac{\frac{1}{5}du}{\sqrt{3^2-u^2}} = \frac{1}{5} \int \frac{du}{\sqrt{3^2-u^2}}$.

Now applying the formula with $a = 3$:

$I = \frac{1}{5} \left[\sin^{-1} \left(\frac{u}{3}\right)\right] + C$.

Back-substituting $u = 5x$:

$I = \frac{1}{5} \sin^{-1} \left(\frac{5x}{3}\right) + C$.

Final Answer : "1/5 $\sin^{-1}(5x/3) + C$"

Answer: (B)

Q33.

Solution

Concept: Integration of the Greatest Integer Function $[x^2]$.

Solution: We evaluate $I = \int_0^2 [x^2]dx$. The function $[x^2]$ is constant between the points where x^2 is an integer.

For $x \in [0, 2]$, x^2 ranges from 0 to 4. The integers are $1, 2, 3, 4$.

The corresponding x values are $x = \sqrt{1} = 1, \sqrt{2}, \sqrt{3}, \sqrt{4} = 2$.

We split the integral accordingly:

1. $0 \le x < 1 \implies 0 \le x^2 < 1 \implies [x^2] = 0$.

2. $1 \le x < \sqrt{2} \implies 1 \le x^2 < 2 \implies [x^2] = 1$.

3. $\sqrt{2} \le x < \sqrt{3} \implies 2 \le x^2 < 3 \implies [x^2] = 2$.

4. $\sqrt{3} \le x < 2 \implies 3 \le x^2 < 4 \implies [x^2] = 3$.

Calculation:

$I = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx$.

$I = 0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$.

$I = (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$.

$I = \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$.

$I = 5 - \sqrt{2} - \sqrt{3}$.

Final Answer : "5 - $\sqrt{2} - \sqrt{3}$"

Answer: (A)

Q34.

Solution

Concept: Trigonometric simplification and basic integration rules.

Solution: We are asked to evaluate $I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

Using the double angle identity $\cos 2\theta = 2\cos^2 \theta - 1$, we rewrite the numerator:

$\cos 2x = 2\cos^2 x - 1$

$\cos 2\alpha = 2\cos^2 \alpha - 1$

Substituting these into the integral:

$I = \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha} dx$

$I = \int \frac{2\cos^2 x - 2\cos^2 \alpha}{\cos x - \cos \alpha} dx = 2\int \frac{\cos^2 x - \cos^2 \alpha}{\cos x - \cos \alpha} dx$

Using the algebraic identity $a^2 - b^2 = (a - b)(a + b)$:

$I = 2\int \frac{(\cos x - \cos \alpha)(\cos x + \cos \alpha)}{\cos x - \cos \alpha} dx$

The term $(\cos x - \cos \alpha)$ cancels out, leaving:

$I = 2\int (\cos x + \cos \alpha) dx.$

Integrating with respect to x (noting that $\cos \alpha$ is a constant):

$I = 2[\sin x + x \cos \alpha] + C = 2(\sin x + x \cos \alpha) + C.$

Final Answer : "2(sin x + x cos $\alpha) + C$J

Answer: (A)

Q35.

Solution

Concept: Finding the area between a curve and a horizontal line using definite integrals.

Solution: The area is bounded by the parabola $y = x^2$ and the line $y = 4$.

First, find the points of intersection by setting the equations equal:

$x^2 = 4 \implies x = \pm 2.$

The area is given by the integral of the upper function minus the lower function from $x = -2$ to $x = 2$:

Area $= \int_{-2}^{2} (4 - x^2) dx.$

Since $f(x) = 4 - x^2$ is an even function, we can simplify the integral:

Area $= 2 \int_0^2 (4 - x^2) dx$

Area $= 2 \left[4x - \frac{x^3}{3} \right]_0^2$

Substitute the upper limit:

Area $= 2 \left(4(2) - \frac{2^3}{3} \right) = 2 \left(8 - \frac{8}{3} \right)$

Area $= 2 \left(\frac{24-8}{3} \right) = 2 \left(\frac{16}{3} \right) = \frac{32}{3}.$

Final Answer : "32/3"

Answer: (A)

Q36.

Solution

Concept: Calculating the area of a sector or quarter-circle using integration.

Solution: The circle equation is $x^2 + y^2 = 4$, which has a radius $r = 2$ and center at the origin. The region is in the first quadrant and bounded by $x = 0$ (y-axis) and $x = 2$.

The area A in the first quadrant is:

$A = \int_0^2 y\,dx = \int_0^2 \sqrt{4 - x^2}\,dx.$

Using the standard integral formula $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$:

$A = \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$ Substitute upper limit ($x = 2$): $\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}(1) = 0 + 2(\frac{\pi}{2}) = \pi$.

Substitute lower limit ($x = 0$): $\frac{0}{2}\sqrt{4 - 0} + 2\sin^{-1}(0) = 0 + 0 = 0$.

The total area is $\pi - 0 = \pi$.

(Geometric check: Area of quarter circle $= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi$).

Final Answer : "π"

Answer: (A)

Q37.

Solution

Concept: Definition of Order and Degree for Differential Equations.

Solution: The differential equation is $(y'')^3 + (y')^2 + \sin(y') + 1 = 0$.

1. Order: The order of a differential equation is the order of the highest derivative present. Here, the highest derivative is $\frac{d^2y}{dx^2}$ (second order). Thus, Order = 2.

2. Degree: The degree is the power of the highest order derivative when the equation is a polynomial in its derivatives.

However, if any derivative is the argument of a transcendental function (like sin, cos, e^x, log), the equation cannot be written as a polynomial in its derivatives.

Because of the term $\sin(y')$, the equation is not a polynomial in y'. Therefore, the degree is not defined.

Final Answer : "Order 2, Degree not defined"

Answer: (C)

Q38.

Solution

Concept: Solving Differential Equations via Variable Separation.

Solution: We have $\frac{dy}{dx} = e^{x+y}$.

Using the property of exponents, $e^{x+y} = e^x \cdot e^y$.

Separating the variables x and y:

$\frac{dy}{e^y} = e^x dx \implies e^{-y} dy = e^x dx.$

Integrating both sides:

$\int e^{-y} dy = \int e^x dx$

$-e^{-y} = e^x + C'$

Rearranging the terms:

$e^x + e^{-y} = -C'.$

Let $-C' = C$, then:

$e^x + e^{-y} = C.$

Final Answer : "$\mathbf{e}^x + e^{-y} = C$"

Answer: (B)

Q39.

Solution

Concept: Integrating Factor for Linear Differential Equations.

Solution: The equation is $x\frac{dy}{dx} - y = 2x^2$.

To find the integrating factor, we first write it in the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$.

Dividing the whole equation by x:

$\frac{dy}{dx} - \frac{1}{x}y = 2x$.

Here, $P(x) = -\frac{1}{x}$.

The formula for the Integrating Factor (I.F.) is:

$I.F. = e^{\int P(x)dx} = e^{\int -\frac{1}{x}dx}$

$I.F. = e^{-\log x} = e^{\log(x^{-1})}$

Since $e^{\log f(x)} = f(x)$:

$I.F. = x^{-1} = \frac{1}{x}$.

Final Answer : "1/x"

Answer: (C)

Q40.

Solution

Concept: Relationship between Order and Arbitrary Constants.

Solution: In the theory of differential equations, there is a fundamental rule regarding the general solution:

The number of independent arbitrary constants in the general solution of a differential equation is always equal to the order of the differential equation.

The question specifies that the differential equation is of the fourth order. Therefore, the number of arbitrary constants must be 4.

Final Answer : "4"

Answer: (D)

Q41.

Solution

Concept: General Solution of a First-Order Linear Differential Equation.

Solution: The equation is $\frac{dy}{dx} + \frac{y}{x} = x^2$. This is of the form $\frac{dy}{dx} + Py = Q$, with $P = 1/x$ and $Q = x^2$.

1. Find the Integrating Factor (I.F.):

$I.F. = e^{\int (1/x)dx} = e^{\log x} = x.$

2. The solution is given by $y(I.F.) = \int Q(I.F.)dx + C$:

$y(x) = \int (x^2)(x)dx + C$

$xy = \int x^3 dx + C$

$xy = \frac{x^4}{4} + C.$

Final Answer : "xy = x^4/4 + C"

Answer: (A)

Q42.

Solution

Concept: Vector Dot Product and properties of unit vectors.

Solution: Given $\vec{a}$ is a unit vector, so $|\vec{a}| = 1$.

The given expression is $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$.

Expanding using the distributive property of the dot product:

$\vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 8.$

Since dot product is commutative ($\vec{x} \cdot \vec{a} = \vec{a} \cdot \vec{x}$), the middle terms cancel:

$\vec{x} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 8.$

Using the property $\vec{v} \cdot \vec{v} = |\vec{v}|^2$:

$|\vec{x}|^2 - |\vec{a}|^2 = 8.$

Substituting $|\vec{a}| = 1$:

$|\vec{x}|^2 - 1^2 = 8 \implies |\vec{x}|^2 = 9.$

$|\vec{x}| = \sqrt{9} = 3.$

Final Answer : "3"

Answer: (A)

Q43.

Solution

Concept: Vector Projection Formula.

Solution: The projection of vector $\vec{a}$ on vector $\vec{b}$ is calculated as:
$\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Given $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

1. Calculate the dot product $\vec{a} \cdot \vec{b}$:

$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10.$

2. Calculate the magnitude of $\vec{b}$:

$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}.$

3. Substitute into the formula:

Projection $= \frac{10}{\sqrt{6}}$.

Final Answer : “10/$\sqrt{6}$”

Answer: (A)

Q44.

Solution

Concept: Lagrange's Identity in Vector Algebra.

Solution: Lagrange's Identity relates the dot product and cross product as follows:

$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2.$

This identity stems from $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $|\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}| \cos\theta$, then using $\sin^2\theta+\cos^2\theta = 1$.

Given:

1. $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = 144$

2. $|\vec{a}| = 4$

Substituting these into the identity:

$144 = (4)^2 \cdot |\vec{b}|^2$

$144 = 16 \cdot |\vec{b}|^2$

$|\vec{b}|^2 = \frac{144}{16} = 9$

$|\vec{b}| = 3.$

Final Answer : "3"

Answer: (C)

Q45.

Solution

Concept: Shortest Distance between two skew lines in 3D space.

Solution: The lines are $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$.

$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}.$

$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}.$

1. $\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$. 2. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} =$ $\hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}.$

3. Magnitude $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}.$

4. Distance $d = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}.$

$d = \frac{|(1)(-3)+(-3)(0)+(-2)(3)|}{3\sqrt{2}} = \frac{|-3-6|}{3\sqrt{2}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}.$

Final Answer : "3/$\sqrt{2}$"

Answer: (B)

Q46.

Solution

Concept: Relationship between Direction Cosines and Sines.

Solution: Let $\cos\alpha, \cos\beta, \cos\gamma$ be the direction cosines of the line.

We know the identity: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

The question asks for the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Using the trigonometric identity $\sin^2\theta = 1 - \cos^2\theta$, we can rewrite the expression:

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = (1 - \cos^2\alpha) + (1 - \cos^2\beta) + (1 - \cos^2\gamma)$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - (1) = 2.$

Final Answer : "2"

Answer: (B)

Q47.

Solution

Concept: Perpendicular Distance from a point to a plane.

Solution: The distance of point (x_1, y_1, z_1) from the plane $Ax + By + Cz + D = 0$ is given by:

$d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Here, point is $(2, 3, 4)$ and plane is $3x - 6y + 2z + 11 = 0$.

Substitute values:

$d = \frac{|3(2)-6(3)+2(4)+11|}{\sqrt{3^2+(-6)^2+2^2}}$

$d = \frac{|6-18+8+11|}{\sqrt{9+36+4}}$

$d = \frac{|7|}{\sqrt{49}} = \frac{7}{7} = 1.$

Final Answer : "1"

Answer: (A)

Q48.

Solution

Concept: Multiplicity of Optimal Solutions in Linear Programming.
Solution: In a Linear Programming Problem (LPP), the objective function $Z = ax + by$ is linear.

If the objective function attains the same optimal (maximum or minimum) value at two distinct corner points of the feasible region, then according to the Optimal Solution Theorem, the function must also attain that same optimal value at every single point on the line segment connecting those two corner points.

Since a line segment is composed of an infinite number of points, the maximum value occurs at infinitely many points.

Final Answer : "Infinite"

Answer: (C)

Q49.

Solution

Concept: Finding the Minimum Value in LPP using Corner Points.

Solution: We calculate the value of the objective function $Z = 4x + 6y$ at each given corner point:

1. At $(0, 2) : Z = 4(0) + 6(2) = 12$.

2. At $(3, 0) : Z = 4(3) + 6(0) = 12$.

3. At $(6, 0) : Z = 4(6) + 6(0) = 24$.

4. At $(6, 8) : Z = 4(6) + 6(8) = 24 + 48 = 72$. 5. At $(0, 5) : Z = 4(0) + 6(5) = 30$.

The minimum value is 12, which is attained at two distinct points: $(0, 2)$ and $(3, 0)$.

As explained in the previous question, if the minimum is reached at two corner points, it is also reached at every point on the line segment joining them.

Final Answer : "Any point on the line segment joining (0, 2) and (3, 0)"

Answer: (D)

Q50.

Solution

Concept: Probability of the Union of Independent Events.

Solution: For independent events A and B, the probability of their intersection is $P(A \cap B) = P(A) \cdot P(B)$.

The addition formula for probability is:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Substituting the independence condition:

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$.

Given $P(A) = 0.35$ and $P(A \cup B) = 0.6$. Let $P(B) = x$:

$0.6 = 0.35 + x - 0.35x$

$0.6 - 0.35 = x(1 - 0.35)$

$0.25 = 0.65x$

$x = \frac{0.25}{0.65} = \frac{25}{65} = \frac{5}{13}$.

Final Answer : "5/13"

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	B	4	B	5	B
6	A	7	B	8	D	9	C	10	B
11	A	12	A	13	B	14	D	15	A
16	B	17	A	18	B	19	B	20	A
21	B	22	B	23	D	24	B	25	A
26	A	27	A	28	B	29	C	30	B
31	A	32	B	33	A	34	A	35	A
36	A	37	C	38	B	39	C	40	D
41	A	42	A	43	A	44	C	45	B
46	B	47	A	48	C	49	D	50	A