CUET-UG Mathematics Sample Paper-15

Duration: 1 Hour **Maximum Marks: 250**

Instructions

- This paper contains a total of 50 Multiple Choice Questions.
- Each correct answer carries **+5 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. Let $A = \{1, 2, 3, ..., n\}$. The number of onto functions from A to $\{a, b\}$ is:

(A) $n^2 - 2$

(B) $2^n - 2$

(C) $2^{n-1} - 1$

(D) $n!$

Q2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. The inverse function $f^{-1}(x)$ is:

(A) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

(B) $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

(C) $\log\left(\frac{2+x}{2-x}\right)$

(D) $\log(\sqrt{x^2 + 1})$

Q3. If R is a relation on the set of natural numbers N defined by xRy if $x + 2y = 10$, then the domain of R is:

(A) $\{2, 4, 6, 8\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{2, 4, 6\}$

(D) $\{1, 3, 5, 7\}$

Q4. The value of $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]$ is:

(A) $\sqrt{\frac{x^2+1}{x^2+2}}$

(B) $\sqrt{\frac{x^2+2}{x^2+1}}$

(C) $\frac{x}{\sqrt{x^2+1}}$

(D) $\frac{1}{\sqrt{x^2+2}}$

Q5. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then the value of $\cos^{-1}x + \cos^{-1}y$ is:

(A) $\pi/3$

(B) $2\pi/3$

(C) $\pi/6$

(D) π

Q6. If A is a non-singular matrix of order 3 and $|A| = 3$, then $|adj(adjA)|$ is:

(A) 81

(B) 243

(C) 729

(D) 9

Q7. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Null matrix

(D) Identity matrix

Q8. If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$, then which of the following is true?

(A) $1 + a^2 + bc = 0$

(B) $1 - a^2 + bc = 0$

(C) $1 - a^2 - bc = 0$

(D) $a^2 + bc - 1 = 0$

Q9. If A is a square matrix of order n and k is a scalar, then $|kA|$ is equal to:

(A) $k|A|$

(B) $k^{n-1}|A|$

(C) $k^n|A|$

(D) $n^k|A|$

Q10. The matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is a:

(A) Diagonal matrix

(B) Upper triangular matrix

(C) Skew-symmetric matrix

(D) Symmetric matrix

Q11. If ω is a complex cube root of unity, then the value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is:

(A) 0

(B) 1

(C) ω

(D) ω^2

Q12. The system of linear equations $x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = 4$ has a unique solution if:

(A) $k \neq 0$

(B) $k = 0$

(C) $k \neq -1$

(D) $k = -1$

Q13. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) I

(D) O

Q14. If x, y, z are all non-zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then the value of $x^{-1} + y^{-1} + z^{-1}$ is:

(A) 1

(B) -1

(C) 0

(D) xyz

Q15. If A is a 3×3 matrix and $|A| = 4$, then the value of $|3A|$ is:

(A) 12

(B) 36

(C) 108

(D) 64

Q16. The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is continuous at $x = 0$. The value of $f(0)$ must be:

(A) $a - b$

(B) $a + b$

(C) $\log(a + b)$

(D) ab

Q17. If $x = e^t \sin t$ and $y = e^t \cos t$, then $\frac{dy}{dx}$ at $t = 0$ is:

(A) 1

(B) -1

(C) 0

(D) e

Q18. If $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx}$ is:

(A) 0

(B) 1

(C) 1/2

(D) -1

Q19. The function $f(x) = [x]$ (Greatest Integer Function) is NOT differentiable at:

(A) All real numbers

(B) Only at $x = 0$

(C) All integers

(D) All rational numbers

Q20. The maximum value of $\sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\sqrt{3}$

Q21. The curve $y = x^{1/5}$ has at the origin $(0, 0)$:

(A) A horizontal tangent

(B) A vertical tangent

(C) No tangent

(D) A slope of 1

Q22. The function $f(x) = \frac{x}{\log x}$ is strictly increasing in the interval:

(A) $(0, 1)$

(B) $(1, e)$

(C) (e, ∞)

(D) $(0, \infty)$

Q23. The side of an equilateral triangle is increasing at the rate of 2 cm/s. The rate at which the area increases when the side is 10 cm is:

(A) $10\sqrt{3}$ cm^2/s

(B) 20 cm^2/s

(C) 10 cm^2/s

(D) $5\sqrt{3}$ cm^2/s

Q24. The point on the curve $y = x^2$ where the tangent is parallel to the chord joining $(0, 0)$ and $(1, 1)$ is:

(A) $(1/4, 1/16)$

(B) $(1/2, 1/4)$

(C) $(3/4, 9/16)$

(D) $(1/8, 1/64)$

Q25. The minimum value of $2x + 3y$ subject to $xy = 6$ $(x, y > 0)$ is:

(A) 12

(B) 10

(C) 6

(D) 5

Q26. The interval in which $f(x) = \sin x - \cos x$ is strictly decreasing in $0 \le x \le 2\pi$ is:

(A) $(0, 3\pi/4)$

(B) $(3\pi/4, 7\pi/4)$

(C) $(\pi/4, 5\pi/4)$

(D) $(0, \pi/2)$

Q27. The integral $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $\sec(xe^x) + C$

(D) $\tan(x + e^x) + C$

Q28. The value of $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ is:

(A) $\pi^2/8$

(B) $\pi^2/32$

(C) $\pi/4$

(D) $\pi^2/16$

Q29. The integral $\int \frac{dx}{x^2+2x+2}$ is equal to:

(A) $\tan^{-1}(x + 1) + C$

(B) $\log(x^2 + 2x + 2) + C$

(C) $\frac{1}{2} \tan^{-1}(x + 1) + C$

(D) $\tan^{-1} x + 1 + C$

Q30. The value of $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$ is:

(A) 1

(B) 0

(C) $1/7$

(D) π

Q31. The integral $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ is:

(A) $\log|\sin x + \cos x| + C$

(B) $-\log|\sin x + \cos x| + C$

(C) $\log|\sin x - \cos x| + C$

(D) $\tan(x - \pi/4) + C$

Q32. If $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, then the value of $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$ is:

(A) $\pi^2/4$

(B) $\pi^2/2$

(C) $\pi/4$

(D) $\pi/2$

Q33. The value of $\int_1^e \log x dx$ is:

(A) e

(B) 1

(C) $e - 1$

(D) 0

Q34. The integral $\int \frac{dx}{\sqrt{1-e^{2x}}}$ is equal to:

(A) $\sin^{-1}(e^x) + C$

(B) $\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$

(C) $-\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$

(D) $e^x\sqrt{1 - e^{2x}} + C$

Q35. The area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q36. The area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x-axis in the first quadrant is:

(A) $16 - 4\sqrt{2}$

(B) $14 - 4\sqrt{2}$

(C) $14\sqrt{2}$

(D) 16

Q37. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is:

(A) 1

(B) 2

(C) 3

(D) Not defined

Q38. The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $\tan^{-1} y - \tan^{-1} x = C$

(C) $x + y = C(1 - xy)$

(D) $y - x = C(1 + xy)$

Q39. The integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$ is:

(A) $\cos x$

(B) $\sec x$

(C) $\tan x$

(D) $\sin x$

Q40. The general solution of $\frac{dy}{dx} + y \tan x = \sec x$ is:

(A) $y \sec x = \tan x + C$

(B) $y \tan x = \sec x + C$

(C) $y = \sin x + C \cos x$

(D) $y \cos x = \sin x + C$

Q41. The differential equation representing the family of curves $y = e^{mx}$ is:

(A) $x\frac{dy}{dx} = y \log y$

(B) $\frac{dy}{dx} = my$

(C) $y\frac{dy}{dx} = x$

(D) $\frac{dy}{dx} = \frac{y}{x} \log y$

Q42. If the angle between two unit vectors $\vec{a}$ and $\vec{b}$ is θ, then $|\vec{a} - \vec{b}|$ is:

(A) $2\cos(\theta/2)$

(B) $2\sin(\theta/2)$

(C) $\sqrt{2}\sin(\theta/2)$

(D) $\sqrt{2}\cos(\theta/2)$

Q43. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if λ is:

(A) -2

(B) 0

(C) 1

(D) -1

Q44. If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0

(B) $\pi/2$

(C) π

(D) $\pi/4$

Q45. The coordinates of the foot of the perpendicular from the point $(0, 0, 0)$ to the plane $2x - 3y + 4z - 6 = 0$ are:

(A) $(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29})$

(B) $(2, -3, 4)$

(C) $(\frac{6}{29}, -\frac{9}{29}, \frac{12}{29})$

(D) $(0, 0, 6)$

Q46. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is:

(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/6$

(D) 0

Q47. The distance between the parallel planes $2x-y+2z+3 = 0$ and $4x-2y+4z+5 = 0$ is:

(A) $1/6$

(B) $1/3$

(C) $1/2$

(D) 1

Q48. The region represented by the inequalities $x \geq 6, y \geq 2, 2x+y \leq 10, x \geq 0, y \geq 0$ is:

(A) Unbounded

(B) A polygon

(C) Empty set

(D) A triangle

Q49. The maximum value of $Z = 3x + 4y$ subject to $x + y \leq 4, x \geq 0, y \geq 0$ is:

(A) 12

(B) 16

(C) 10

(D) 0

Q50. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.96

(B) 0.24

(C) 0.56

(D) 0.64

Detailed Solutions

Q1.

Solution

Concept: Number of onto functions (surjections) from a set with n elements to a set with m elements.

Solution: Given $A = \{1, 2, 3, ..., n\}$ and $B = \{a, b\}$.

1. The total number of functions from A to B is 2^n, as each of the n elements in the domain has 2 choices in the codomain.

2. An onto function must cover all elements of the codomain. The only functions that are NOT onto are those where all elements of A map to a single element in B.

3. There are only two such cases:

- All elements map to a (Constant function $f(x) = a$).

- All elements map to b (Constant function $f(x) = b$).

4. Number of onto functions = Total functions - Non-onto functions = $2^n - 2$.

Final Answer : "$2^n - 1$"

Answer: (B)

Q2.

Solution

Concept: Finding the inverse of a function $y = f(x)$ by solving for x.

Solution: Let $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

Multiply both sides by $(e^x + e^{-x})$:

$y(e^x + e^{-x}) = e^x - e^{-x}$

$ye^x + ye^{-x} = e^x - e^{-x}$

Multiply the entire equation by e^x to eliminate negative exponents:

$ye^{2x} + y = e^{2x} - 1$

Rearrange to group e^{2x} terms:

$1 + y = e^{2x} - ye^{2x}$

$1 + y = e^{2x}(1 - y)$

$e^{2x} = \frac{1+y}{1-y}$

Taking the natural logarithm:

$2x = \log\left(\frac{1+y}{1-y}\right) \implies x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ Thus, $f^{-1}(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$.

Final Answer : "$\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$"

Answer: (A)

Q3.

Solution

Concept: The domain is the set of all possible input values (x) for which the relation is defined in the set of Natural numbers (N).

Solution: The relation is $x + 2y = 10$, where $x, y \in \{1, 2, 3, ...\}$.

Solving for x: $x = 10 - 2y$.

We test values of $y \in N$:

- If $y = 1, x = 10 - 2(1) = 8$

- If $y = 2, x = 10 - 2(2) = 6$

- If $y = 3, x = 10 - 2(3) = 4$

- If $y = 4, x = 10 - 2(4) = 2$

- If $y = 5, x = 10 - 2(5) = 0$ (But $0 \notin N$)

The valid values for x are $\{2, 4, 6, 8\}$.

Therefore, the domain is $\{2, 4, 6, 8\}$.

Final Answer : "2,4,6,8"

Answer: (A)

Q4.

Solution

Concept: Evaluating composite inverse trigonometric functions using triangle substitutions.

Solution: 1. Let $\cot^{-1} x = \theta$. Then $\cot\theta = \frac{x}{1}$. By Pythagoras, $\sin\theta = \frac{1}{\sqrt{x^2+1}}$.

2. The expression becomes $\cos[\tan^{-1}(\frac{1}{\sqrt{x^2+1}})]$.

3. Let $\tan^{-1}(\frac{1}{\sqrt{x^2+1}}) = \phi$. Then $\tan\phi = \frac{1}{\sqrt{x^2+1}}$.

4. In a right triangle with Opposite = 1 and Adjacent = $\sqrt{x^2+1}$, the Hypotenuse is $\sqrt{1^2 + (\sqrt{x^2+1})^2} = \sqrt{1 + x^2 + 1} = \sqrt{x^2+2}$.

5. Therefore, $\cos\phi = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}} = \sqrt{\frac{x^2+1}{x^2+2}}$.

Final Answer : “$\sqrt{\frac{x^2+1}{x^2+2}}$”

Answer: (A)

Q5.

Solution

Concept: Using the identity $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$.

Solution: We are given $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$.

Using the identity, we can write:

$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

$\sin^{-1}y = \frac{\pi}{2} - \cos^{-1}y$

Substitute these into the given equation:

$(\frac{\pi}{2} - \cos^{-1}x) + (\frac{\pi}{2} - \cos^{-1}y) = \frac{2\pi}{3}$

$\pi - (\cos^{-1}x + \cos^{-1}y) = \frac{2\pi}{3}$

$\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

Final Answer : "$\pi/3$"

Answer: (A)

Q6.

Solution

Concept: Determinant property of double adjoint: $|adj(adjA)| = |A|^{(n-1)^2}$.

Solution: Given A is a non-singular matrix of order $n = 3$ and $|A| = 3$.

The formula for the determinant of the adjoint of an adjoint is:

$|adj(adjA)| = |A|^{(n-1)^2}$

Substitute $n = 3$:

$|adj(adjA)| = |A|^{(3-1)^2} = |A|^{2^2} = |A|^4$

Substitute $|A| = 3$:

$|adj(adjA)| = 3^4 = 81.$

Final Answer : "81"

Answer: (A)

Q7.

Solution

Concept: Transposition properties: $(AB)^T = B^T A^T$ and skew-symmetry $M^T = -M$.

Solution: Given A and B are symmetric, so $A^T = A$ and $B^T = B$.

Let $X = AB - BA$. Find the transpose of X:

$X^T = (AB - BA)^T = (AB)^T - (BA)^T$

$X^T = B^T A^T - A^T B^T$

Substitute $A^T = A$ and $B^T = B$:

$X^T = BA - AB$

$X^T = -(AB - BA) = -X$

Since $X^T = -X$, the matrix $AB - BA$ is a skew-symmetric matrix.

Final Answer : "Skew-symmetric matrix"

Answer: (B)

Q8.

Solution

Concept: Matrix equality and algebraic expansion of matrix multiplication for a 2×2 matrix.

Solution: Given the matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ and the condition $A^2 = I$.

First, calculate A^2 (which is $A \times A$):

$$A^2 = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} (a)(a) + (b)(c) & (a)(b) + (b)(-a) \\ (c)(a) + (-a)(c) & (c)(b) + (-a)(-a) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix} = \begin{bmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{bmatrix}.$$

The Identity matrix I for a 2×2 system is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Equating $A^2 = I$, we get:

$a^2 + bc = 1.$

To match the options, we rearrange the equation by moving all terms to one side or rearranging: $1 - a^2 - bc = 0.$

Comparing this with the given options, option (C) is correct.

Final Answer : "$1 - a^2 - bc = 0$"

Answer: (C)

Q9.

Solution

Concept: Properties of determinants regarding scalar multiplication of matrices.

Solution: Let A be a square matrix of order $n \times n$. When we multiply a matrix A by a scalar k, every single entry in the matrix is multiplied by k.

According to the properties of determinants:

1. If every element of a row (or column) of a determinant is multiplied by a scalar k, then the value of the determinant is multiplied by k.

2. In the matrix kA, there are n rows, and every row has been multiplied by k.

3. Therefore, to find $|kA|$, we factor out k from the first row, then the second row, and so on, for all n rows.

Mathematically: $|kA| = k \times k \times \cdots \times k$ (n times) $\times |A| = k^n|A|$.

Final Answer : "$k^n|A|$"

Answer: (C)

Q10.

Solution

Concept: Definitions of symmetric and skew-symmetric matrices.

Solution: A matrix A is Skew-Symmetric if $A^T = -A$ and all its diagonal elements are zero.

Given $A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$.

1. Check the diagonal: All diagonal elements (a_{11}, a_{22}, a_{33}) are 0.

2. Find the transpose A^T (swap rows and columns):

$$A^T = \begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}.$$

3. Find $-A$ by multiplying every element by -1:

$$-A = \begin{bmatrix} -0 & -5 & -(-7) \\ -(-5) & -0 & -11 \\ -7 & -(-11) & -0 \end{bmatrix} = \begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}.$$

Since $A^T = -A$, the matrix is skew-symmetric.

Final Answer : "Skew-symmetric matrix"

Answer: (C)

Q11.

Solution

Concept: Properties of the complex cube roots of unity (ω) and determinant row/column operations.

Solution: We know the fundamental property of cube roots of unity: $1 + \omega + \omega^2 = 0$.

Given the determinant $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$.

Perform the column operation $C_1 \to C_1 + C_2 + C_3$:

$$\Delta = \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ \omega + \omega^2 + 1 & \omega^2 & 1 \\ \omega^2 + 1 + \omega & 1 & \omega \end{vmatrix}.$$

Substituting $1 + \omega + \omega^2 = 0$ into the first column:

$$\Delta = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}.$$

In any determinant, if all elements of a row or a column are zero, the total value of the determinant is zero.

Therefore, $\Delta = 0$.

Final Answer : "0"

Answer: (A)

Q12.

Solution

Concept: Conditions for a unique solution in a system of linear equations using the determinant of the coefficient matrix (Δ).

Solution: A system of linear equations has a unique solution if and only if the determinant of the coefficient matrix is non-zero ($\Delta \neq 0$).

The coefficient matrix for the given system is:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}.$$

Expand the determinant along the first row:

$$\Delta = 1[(1)(k) - (2)(-1)] - 1[(2)(k) - (3)(-1)] + 1[(2)(2) - (3)(1)]$$

$$\Delta = 1(k + 2) - 1(2k + 3) + 1(4 - 3)$$

$$\Delta = k + 2 - 2k - 3 + 1 = -k.$$

For a unique solution, we require $\Delta \neq 0$:

$$-k \neq 0 \implies k \neq 0.$$

Final Answer : "$k \neq 0$"

Answer: (A)

Q13.

Solution

Concept: Characteristic equations and the Cayley-Hamilton Theorem.

Solution: For matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the characteristic equation is given by $|A - \lambda I| = 0$.

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda) - (2)(3) = 0$$

$$\lambda^2 - 5\lambda + 4 - 6 = 0 \implies \lambda^2 - 5\lambda - 2 = 0.$$

According to the Cayley-Hamilton Theorem, a square matrix satisfies its own characteristic equation. Thus, we can replace λ with A and the constant with the identity matrix I:

$$A^2 - 5A - 2I = O$$

Rearranging the terms to find $A^2 - 5A$:

$$A^2 - 5A = 2I.$$

Final Answer : "$2I$"

Answer: (A)

Q14.

Solution

Concept: Simplification of determinants by factoring out variables from rows/columns.

Solution: Given $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0.$

Take x common from Row 1, y from Row 2, and z from Row 3:

$$xyz \begin{vmatrix} \frac{1}{x}+1 & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix} = 0.$$

Now, apply the operation $R_1 \to R_1 + R_2 + R_3$:

$$xyz \begin{vmatrix} 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix} = 0.$$

Factoring out $(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z})$ from Row 1:

$$xyz(1+\tfrac{1}{x}+\tfrac{1}{y}+\tfrac{1}{z}) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix} = 0.$$

Since x, y, z are non-zero, for the product to be zero, the term in the parenthesis must be zero:

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0 \implies x^{-1}+y^{-1}+z^{-1} = -1.$$

Final Answer : "-1"

Answer: (B)

Q15.

Solution

Concept: Scalar multiplication property for determinants of order n.

Solution: We use the established property $|kA| = k^n|A|$, where n is the order of the square matrix A and k is a scalar.

In this problem:

1. The matrix A is a 3×3 matrix, so the order $n = 3$.

2. The scalar $k = 3$.

3. The determinant of A is given as $|A| = 4$.

Now, substitute these values into the formula:

$|3A| = 3^3 \times |A|$

$|3A| = 27 \times 4$

$|3A| = 108.$

Final Answer : "108"

Answer: (C)

Q16.

Solution

Concept: For a function $f(x)$ to be continuous at $x = 0$, the value of the function $f(0)$ must be equal to the limit of the function as x approaches 0.

Solution: We need to find $f(0) = \lim_{x\to 0} \frac{\log(1+ax)-\log(1-bx)}{x}$.

Using the properties of limits, we can separate the terms:

$$f(0) = \lim_{x\to 0} \left[\frac{\log(1+ax)}{x} - \frac{\log(1-bx)}{x}\right]$$

To apply the standard limit formula $\lim_{u\to 0} \frac{\log(1+u)}{u} = 1$, we multiply and divide the first term by a and the second term by $-b$:

$$f(0) = \lim_{x\to 0} \left[a \cdot \frac{\log(1+ax)}{ax} - (-b) \cdot \frac{\log(1-bx)}{-bx}\right]$$

As $x \to 0$, $ax \to 0$ and $-bx \to 0$. Substituting the standard limits:

$$f(0) = a(1) - (-b)(1)$$

$$f(0) = a + b.$$

Thus, for the function to be continuous, $f(0)$ must be $a + b$.

Final Answer : $a + b$

Answer: (B)

Q17.

Solution

Concept: Parametric differentiation states that if x and y are functions of t, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Solution: Given $x = e^t \sin t$ and $y = e^t \cos t$.

Step 1: Differentiate x with respect to t using the product rule $(uv)' = u'v + uv'$: $\frac{dx}{dt} = \frac{d}{dt}(e^t) \cdot \sin t + e^t \cdot \frac{d}{dt}(\sin t) = e^t \sin t + e^t \cos t = e^t(\sin t + \cos t)$.

Step 2: Differentiate y with respect to t using the product rule:

$\frac{dy}{dt} = \frac{d}{dt}(e^t) \cdot \cos t + e^t \cdot \frac{d}{dt}(\cos t) = e^t \cos t - e^t \sin t = e^t(\cos t - \sin t)$.

Step 3: Find $\frac{dy}{dx}$:

$\frac{dy}{dx} = \frac{e^t(\cos t - \sin t)}{e^t(\sin t + \cos t)} = \frac{\cos t - \sin t}{\sin t + \cos t}$.

Step 4: Evaluate at $t = 0$:

$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\cos 0 - \sin 0}{\sin 0 + \cos 0} = \frac{1-0}{0+1} = 1$.

Final Answer : 1

Answer: (A)

Q18.

Solution

Concept: Simplify the trigonometric expression using the identity $\tan(\frac{\pi}{4}+x) = \frac{1+\tan x}{1-\tan x}$ before differentiating.

Solution: Given $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$.

Divide both numerator and denominator by $\cos x$:

$$y = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}\right) = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right).$$

Using the identity $\tan(\frac{\pi}{4}+x) = \frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x} = \frac{1+\tan x}{1-\tan x}$:

$$y = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+x\right)\right].$$

This simplifies to:

$$y = \frac{\pi}{4}+x.$$

Differentiating both sides with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}(x) = 0+1 = 1.$$

Final Answer : 1

Answer: (B)

Q19.

Solution

Concept: Differentiability requires continuity. If a function is discontinuous at a point, it cannot have a derivative at that point.

Solution: The Greatest Integer Function $f(x) = [x]$ maps x to the largest integer $\leq x$. 1. Let n be an arbitrary integer. To check for continuity at $x = n$:

- $\lim_{x \to n^-} [x] = n - 1$

- $\lim_{x \to n^+} [x] = n$

- $f(n) = n$

2. Since the left-hand limit $(n - 1)$ is not equal to the right-hand limit (n), the function has a jump discontinuity at every integer n.

3. Because the function is discontinuous at all integers, it is not differentiable at any integer.

4. For non-integers, the function is locally constant, making it differentiable with a derivative of 0.

Thus, the function is not differentiable at all integers.

Final Answer : All integers

Answer: (C)

Q20.

Solution

Concept: The expression $a \sin x + b \cos x$ can be rewritten as $\sqrt{a^2 + b^2} \sin(x + \alpha)$, where the maximum value is $\sqrt{a^2 + b^2}$.

Solution: Let $f(x) = \sin x + \cos x$. Here, the coefficient of $\sin x$ is $a = 1$ and the coefficient of $\cos x$ is $b = 1$.

The maximum value of the function is:

$$\text{Maximum Value} = \sqrt{a^2 + b^2}$$

$$= \sqrt{1^2 + 1^2}$$

$$= \sqrt{1 + 1}$$

$$= \sqrt{2}.$$

Alternatively, using calculus: $f'(x) = \cos x - \sin x$. Setting $f'(x) = 0$ gives $\tan x = 1$, so $x = \pi/4$.

$f(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

Final Answer : $\sqrt{2}$

Answer: (C)

Q21.

Solution

Concept: The slope of the tangent to a curve $y = f(x)$ at a point (x_0, y_0) is given by the derivative $\frac{dy}{dx}$ at that point.

Solution: Given the curve $y = x^{1/5}$.

Step 1: Differentiate y with respect to x using the power rule:

$\frac{dy}{dx} = \frac{1}{5}x^{\frac{1}{5}-1} = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$.

Step 2: Evaluate the slope at the origin $(0, 0)$:

As $x \to 0$, $\frac{dy}{dx} = \frac{1}{5(0)^{4/5}} = \frac{1}{0} \to \infty$.

Step 3: Interpret the result. A slope that is undefined or tends to infinity indicates that the tangent line is a vertical line.

Therefore, at the origin, the curve $y = x^{1/5}$ has a vertical tangent.

Final Answer : A vertical tangent

Answer: (B)

Q22.

Solution

Concept: A function $f(x)$ is strictly increasing in an interval if its first derivative $f'(x) > 0$ for all x in that interval.

Solution: Given $f(x) = \frac{x}{\log x}$. For the function to be defined, $x > 0$ and $x \neq 1$.

Step 1: Find the derivative $f'(x)$ using the quotient rule $\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$:

$f'(x) = \frac{(\log x)\cdot\frac{d}{dx}(x)-x\cdot\frac{d}{dx}(\log x)}{(\log x)^2}$ $f'(x) = \frac{(\log x)(1)-x(\frac{1}{x})}{(\log x)^2} = \frac{\log x-1}{(\log x)^2}$.

Step 2: For strictly increasing, set $f'(x) > 0$:

$\frac{\log x-1}{(\log x)^2} > 0.$

Since the denominator $(\log x)^2$ is always positive for $x \in \text{Domain} \setminus \{1\}$, we solve:

$\log x - 1 > 0 \implies \log x > 1.$

Step 3: Convert the logarithmic inequality to exponential form (base e):

$x > e^1 \implies x > e.$

Thus, the function is strictly increasing in the interval (e, ∞).

Final Answer : (e, ∞)

Answer: (C)

Q23.

Solution

Concept: Related rates of change: Calculating the rate of change of area with respect to time by differentiating the geometric formula and applying the chain rule.

Solution: Let s be the side of the equilateral triangle and A be its area at any time t.

1. The formula for the area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}s^2$.

2. We are given that the side is increasing at a rate of $\frac{ds}{dt} = 2$ cm/s.

3. We need to find the rate of change of area, $\frac{dA}{dt}$, when the side $s = 10$ cm.

4. Differentiating the area formula with respect to time t using the chain rule:

$\frac{dA}{dt} = \frac{d}{ds}\left(\frac{\sqrt{3}}{4}s^2\right) \cdot \frac{ds}{dt}$ $\frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot (2s) \cdot \frac{ds}{dt} = \frac{\sqrt{3}}{2}s\frac{ds}{dt}$

5. Substituting the given values $s = 10$ and $\frac{ds}{dt} = 2$:

$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (10) \cdot (2) = 10\sqrt{3}\ \text{cm}^2/\text{s}$.

Final Answer : $10\sqrt{3}$ **cm^2/s**

Answer: (A)

Q24.

Solution

Concept: Applications of Derivatives: Finding a point on a curve where the tangent slope equals the slope of a given line (chord).

Solution: 1. Find the slope of the chord joining the points $(0, 0)$ and $(1, 1)$:

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{1-0}{1-0} = 1.$

2. The curve is given by $y = x^2$. The slope of the tangent at any point (x, y) is given by the derivative $\frac{dy}{dx}$:

$\frac{dy}{dx} = 2x.$

3. Since the tangent is parallel to the chord, their slopes must be equal:

$2x = 1 \implies x = \frac{1}{2}.$

4. Find the corresponding y-coordinate by substituting $x = \frac{1}{2}$ back into the curve equation $y = x^2$:

$y = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$

The required point is $(1/2, 1/4)$.

Final Answer : $(1/2, 1/4)$

Answer: (B)

Q25.

Solution

Concept: Optimization of a multivariable function using substitution to reduce it to a single-variable function.

Solution: We want to minimize $Z = 2x + 3y$ given the constraint $xy = 6$ (where $x, y > 0$).

1. From the constraint, express y in terms of x: $y = \frac{6}{x}$.

2. Substitute this into the expression for Z:

$Z(x) = 2x + 3\left(\frac{6}{x}\right) = 2x + \frac{18}{x}$.

3. To find the minimum, differentiate Z with respect to x and set it to zero:

$Z'(x) = 2 - \frac{18}{x^2}$.

$2 - \frac{18}{x^2} = 0 \implies \frac{18}{x^2} = 2 \implies x^2 = 9$.

Since $x > 0$, we take $x = 3$.

4. Calculate the corresponding y: $y = \frac{6}{3} = 2$.

5. The minimum value is $Z = 2(3) + 3(2) = 6 + 6 = 12$.

Final Answer : 12

Answer: (A)

Q26.

Solution

Concept: Monotonicity of functions: A function $f(x)$ is strictly decreasing in an interval if its derivative $f'(x) < 0$.

Solution: Given $f(x) = \sin x - \cos x$ for $0 \le x \le 2\pi$.

1. Differentiate the function:

$f'(x) = \cos x - (-\sin x) = \cos x + \sin x.$

2. For the function to be strictly decreasing, $f'(x) < 0$:

$\cos x + \sin x < 0.$

3. Multiply and divide by $\sqrt{2}$ to simplify:

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) < 0 \implies \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) < 0.$

4. The sine function is negative in the 3rd and 4th quadrants ($\pi < \theta < 2\pi$):

$\pi < x + \frac{\pi}{4} < 2\pi.$

5. Solving for x:

$\pi - \frac{\pi}{4} < x < 2\pi - \frac{\pi}{4} \implies \frac{3\pi}{4} < x < \frac{7\pi}{4}.$

Final Answer : $(3\pi/4, 7\pi/4)$

Answer: (B)

Q27.

Solution

Concept: Integration by substitution: Identifying a part of the integrand as the derivative of another part.

Solution: Let $I = \int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$.

1. Let $u = xe^x$.

2. Differentiate u with respect to x using the product rule:

$du = (x \cdot e^x + 1 \cdot e^x)dx = e^x(x+1)dx$.

3. Notice that the numerator of our integral is exactly du.

4. Substitute u and du into the integral:

$I = \int \frac{1}{\cos^2 u}du = \int \sec^2 u du$.

5. The standard integral of $\sec^2 u$ is $\tan u + C$.

6. Substitute back the value of u:

$I = \tan(xe^x) + C$.

Final Answer : $\tan(xe^x) + C$

Answer: (A)

Q28.

Solution

Concept: Definite integration using substitution and changing the limits of integration accordingly.

Solution: Let $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$.

1. Let $u = \tan^{-1} x$.

2. Differentiate u: $du = \frac{1}{1+x^2} dx$.

3. Change the limits of integration:

- Lower limit: When $x = 0$, $u = \tan^{-1}(0) = 0$.

- Upper limit: When $x = 1$, $u = \tan^{-1}(1) = \frac{\pi}{4}$.

4. Rewrite the integral in terms of u:

$I = \int_0^{\pi/4} u du$.

5. Integrate:

$I = \left[\frac{u^2}{2}\right]_0^{\pi/4} = \frac{(\pi/4)^2}{2} - \frac{0^2}{2} = \frac{\pi^2/16}{2} = \frac{\pi^2}{32}$.

Final Answer : $\pi^2/32$

Answer: (B)

Q29.

Solution

Concept: Evaluating integrals involving quadratic denominators by completing the square and using the standard integral $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$.

Solution: Let $I = \int \frac{dx}{x^2+2x+2}$.

1. Complete the square for the quadratic expression in the denominator:

$x^2 + 2x + 2 = (x^2 + 2x + 1) + 1 = (x + 1)^2 + 1^2$.

2. The integral becomes:

$I = \int \frac{dx}{(x+1)^2+1^2}$.

3. This is in the standard form $\int \frac{1}{u^2+a^2}du$ with $u = x + 1$ and $a = 1$.

4. Applying the formula:

$I = \tan^{-1}(x + 1) + C$.

Final Answer : $\tan^{-1}(x + 1) + C$

Answer: (A)

Q30.

Solution

Concept: Symmetry properties of definite integrals: The integral of an odd function over a symmetric interval $[-a, a]$ is always zero.

Solution: Let $I = \int_{-\pi/2}^{\pi/2} \sin^7 x dx$.

1. Identify the function: $f(x) = \sin^7 x$.

2. Check if the function is even or odd:

$f(-x) = \sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$.

3. Since $f(-x) = -f(x)$, the function is an odd function.

4. The limits of integration are symmetric about the origin ($-\pi/2$ to $\pi/2$).

5. Using the property $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is odd:
$I = 0$.

Final Answer : 0

Answer: (B)

Q31.

Solution

Concept: Logarithmic integration: Evaluating an integral where the numerator is the negative of the derivative of the denominator.

Solution: Let $I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$.

1. Let $u = \sin x + \cos x$.

2. Differentiate u with respect to x:

$\frac{du}{dx} = \cos x - \sin x$.

3. Rearranging gives: $du = (\cos x - \sin x)dx$.

4. Observe the numerator of the original integral: $(\sin x - \cos x)dx = -(\cos x - \sin x)dx = -du$.

5. Substitute u and $-du$ into the integral:

$I = \int \frac{-du}{u} = -\int \frac{1}{u} du$.

6. Integrate:

$I = -\log|u| + C$. 7. Replace u with its original expression:

$I = -\log|\sin x + \cos x| + C$.

Final Answer : $-\log|\sin x + \cos x| + C$

Answer: (B)

Q32.

Solution

Concept: Property of definite integrals: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. This is often used to eliminate an x term in the numerator.

Solution: Let $I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$ (i)

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$I = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$

Since $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$ (so $\cos^2(\pi - x) = \cos^2 x$):

$I = \int_0^\pi \frac{(\pi-x)\sin x}{1+\cos^2 x}dx$ (ii)

Adding (i) and (ii):

$2I = \int_0^\pi \frac{x\sin x+(\pi-x)\sin x}{1+\cos^2 x}dx = \int_0^\pi \frac{\pi\sin x}{1+\cos^2 x}dx$

$2I = \pi \int_0^\pi \frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x = t$, then $-\sin x dx = dt$.

When $x = 0, t = 1$; when $x = \pi, t = -1$.

$2I = \pi \int_1^{-1} \frac{-dt}{1+t^2} = \pi \int_{-1}^1 \frac{dt}{1+t^2}$

$2I = \pi[\tan^{-1} t]_{-1}^1 = \pi[\tan^{-1}(1) - \tan^{-1}(-1)]$

$2I = \pi[\pi/4 - (-\pi/4)] = \pi[\pi/2] = \pi^2/2$

$I = \pi^2/4$.

Final Answer : $\pi^2/4$

Answer: (A)

Q33.

Solution

Concept: Integration by parts: $\int u \cdot v \, dx = u \int v \, dx - \int (u' \int v \, dx) dx$.

Solution: Let $I = \int_1^e \log x \, dx$. To integrate $\log x$, we treat it as $\log x \cdot 1$.

Let $u = \log x$ (first function) and $v = 1$ (second function).

$\int \log x \cdot 1 \, dx = (\log x)(x) - \int (\frac{1}{x} \cdot x) dx$

$= x \log x - \int 1 dx = x \log x - x.$

Now applying the limits from 1 to e:

$I = [x \log x - x]_1^e$

$I = (e \log e - e) - (1 \log 1 - 1)$

Since $\log e = 1$ and $\log 1 = 0$:

$I = (e \cdot 1 - e) - (0 - 1)$

$I = (e - e) - (-1) = 0 + 1 = 1.$

Final Answer : 1

Answer: (B)

Q34.

Solution

Concept: Substitution method and standard logarithmic integral form $\int \frac{du}{\sqrt{u^2-a^2}} = \log|u + \sqrt{u^2 - a^2}|$.

Solution: Given $I = \int \frac{dx}{\sqrt{1-e^{2x}}}$.

Divide numerator and denominator by e^x:

$$I = \int \frac{e^{-x}dx}{\sqrt{e^{-2x}(1-e^{2x})}} = \int \frac{e^{-x}dx}{\sqrt{e^{-2x}-1}}.$$

Let $u = e^{-x}$. Differentiating both sides: $du = -e^{-x}dx \implies -du = e^{-x}dx$.

Substitute u and du into the integral:

$$I = \int \frac{-du}{\sqrt{u^2-1}}.$$

Using the standard formula $\int \frac{du}{\sqrt{u^2-1}} = \log|u + \sqrt{u^2 - 1}| + C$:

$$I = -\log|u + \sqrt{u^2 - 1}| + C.$$

Substituting back $u = e^{-x}$:

$$I = -\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C.$$

Final Answer : $-\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$

Answer: (C)

Q35.

Solution

Concept: The total area bounded by a curve $y = f(x)$ is the sum of the absolute values of the integrals over sub-intervals where the function changes sign.

Solution: We need to find the area under $y = \cos x$ for $x \in [0, 2\pi]$.

1. $\cos x \geq 0$ for $x \in [0, \pi/2]$ and $x \in [3\pi/2, 2\pi]$.

2. $\cos x \leq 0$ for $x \in [\pi/2, 3\pi/2]$.

Total Area $A = \int_0^{\pi/2} \cos x dx + |\int_{\pi/2}^{3\pi/2} \cos x dx| + \int_{3\pi/2}^{2\pi} \cos x dx$

$A = [\sin x]_0^{\pi/2} + |[\sin x]_{\pi/2}^{3\pi/2}| + [\sin x]_{3\pi/2}^{2\pi}$

$A = (\sin \frac{\pi}{2} - \sin 0) + |(\sin \frac{3\pi}{2} - \sin \frac{\pi}{2})| + (\sin 2\pi - \sin \frac{3\pi}{2})$

$A = (1 - 0) + |(-1 - 1)| + (0 - (-1))$

$A = 1 + |-2| + 1 = 1 + 2 + 1 = 4.$

Final Answer : 4

Answer: (D)

Q36.

Solution

Concept: Calculating the area of a region bounded by a curve and vertical lines using definite integration.

Solution: Given $y^2 = 9x$. In the first quadrant, $y = \sqrt{9x} = 3\sqrt{x}$.

The region is bounded by $x = 2$ and $x = 4$.

Area $A = \int_2^4 y dx = \int_2^4 3x^{1/2} dx$.

$A = 3\left[\frac{x^{3/2}}{3/2}\right]_2^4 = 3 \cdot \frac{2}{3}[x^{3/2}]_2^4$

$A = 2[4^{3/2} - 2^{3/2}]$.

Since $4^{3/2} = (2^2)^{3/2} = 2^3 = 8$ and $2^{3/2} = 2\sqrt{2}$:

$A = 2[8 - 2\sqrt{2}] = 16 - 4\sqrt{2}$.

Final Answer : $16 - 4\sqrt{2}$

Answer: (A)

Q37.

Solution

Concept: The degree of a differential equation is the power of the highest order derivative when the equation is expressed in polynomial form with respect to derivatives.

Solution: The given differential equation is: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$.

To determine the degree, we must eliminate the fractional exponent (3/2).

Square both sides of the equation:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2=\left(\frac{d^2y}{}dx^2\right)^2$$

$[1+(\frac{dy}{dx})^2]^3=(\frac{d^2y}{dx^2})^2.$

The highest order derivative present is $\frac{d^2y}{dx^2}$ (which is order 2).

The power of this highest order derivative is 2.

Therefore, the degree is 2.

Final Answer : 2

Answer: (B)

Q38.

Solution

Concept: Solving differential equations using the Variable Separable method and tangent subtraction formula.

Solution: Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

Separating variables x and y:

$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$.

Integrating both sides:

$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + \tan^{-1} C$.

Rearranging:

$\tan^{-1} y - \tan^{-1} x = \tan^{-1} C$.

Using the identity $\tan^{-1} A - \tan^{-1} B = \tan^{-1}(\frac{A-B}{1+AB})$:

$\tan^{-1}(\frac{y-x}{1+xy}) = \tan^{-1} C$

$\frac{y-x}{1+xy} = C \implies y - x = C(1 + xy)$.

Final Answer : $y - x = C(1 + xy)$

Answer: (D)

Q39.

Solution

Concept: Integrating Factor (IF) of a linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is $e^{\int P(x)dx}$.

Solution: The given equation is $\cos x \frac{dy}{dx} + y \sin x = 1$.

Divide the entire equation by $\cos x$ to put it in the standard linear form:

$\frac{dy}{dx} + y \frac{\sin x}{\cos x} = \frac{1}{\cos x}$

$\frac{dy}{dx} + (\tan x)y = \sec x.$

Comparing with $\frac{dy}{dx} + Py = Q$, we find $P = \tan x$.

$IF = e^{\int P dx} = e^{\int \tan x dx}$

$IF = e^{\log|\sec x|}$

Since $e^{\log f(x)} = f(x)$, we get:

$IF = \sec x.$

Final Answer : $\sec x$

Answer: (B)

Q40.

Solution

Concept: General solution of a linear differential equation is given by $y \cdot (IF) = \int Q \cdot (IF) dx + C$.

Solution: Given: $\frac{dy}{dx} + y \tan x = \sec x$. This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$.

Here $P = \tan x$ and $Q = \sec x$.

1. Find the Integrating Factor (IF):

$IF = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$.

2. Write the general solution:

$y \cdot (\sec x) = \int (\sec x) \cdot (\sec x) dx + C$

$y \sec x = \int \sec^2 x dx + C$

$y \sec x = \tan x + C$.

Final Answer : $y \sec x = \tan x + C$

Answer: (A)

Q41.

Solution

Concept: To form a differential equation from a family of curves, differentiate the given equation and eliminate the arbitrary constant.

Solution: Given $y = e^{mx}$ (where m is an arbitrary constant).

1. Differentiate both sides with respect to x:

$$\frac{dy}{dx} = m \cdot e^{mx} \tag{i}$$

2. From the original equation $y = e^{mx}$, take the natural log:

$$\log y = mx \implies m = \frac{\log y}{x}.$$

3. Substitute the value of m into equation (i):

$$\frac{dy}{dx} = \left(\frac{\log y}{x}\right) \cdot e^{mx}$$

4. Since $e^{mx} = y$:

$$\frac{dy}{dx} = \frac{\log y}{x} \cdot y$$

5. Rearrange:

$$x\frac{dy}{dx} = y \log y.$$

Final Answer : $x\frac{dy}{dx} = y \log y$

Answer: (A)

Q42.

Solution

Concept: Vector magnitude properties: $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$.

Solution: Given $\vec{a}$ and $\vec{b}$ are unit vectors, so $|\vec{a}| = 1$ and $|\vec{b}| = 1$.

The dot product $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = 1 \cdot 1 \cdot \cos\theta = \cos\theta$.

Consider the squared magnitude of the difference:

$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$

$= 1 + 1 - 2\cos\theta = 2(1 - \cos\theta)$.

Using the trigonometric identity $1 - \cos\theta = 2\sin^2(\theta/2)$:

$|\vec{a} - \vec{b}|^2 = 2 \cdot (2\sin^2(\theta/2)) = 4\sin^2(\theta/2)$.

Taking the square root of both sides:

$|\vec{a} - \vec{b}| = 2\sin(\theta/2)$.

Final Answer : $2\sin(\theta/2)$

Answer: (B)

Q43.

Solution

Concept: Condition for Coplanarity: Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if their scalar triple product is zero, i.e., $[\vec{a}\ \vec{b}\ \vec{c}] = 0$. This is equivalent to the determinant of their components being zero.

Solution: The given vectors can be written in matrix form based on their coefficients:

$\vec{v_1} = (\lambda, 1, 2)$, $\vec{v_2} = (1, \lambda, -1)$, and $\vec{v_3} = (2, -1, \lambda)$.

Setting the determinant to zero:

$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$ Expanding along the first row:

$\lambda(\lambda^2 - (-1)(-1)) - 1(\lambda - (-2)) + 2(-1 - 2\lambda) = 0$

$\lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$

$\lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$

$\lambda^3 - 6\lambda - 4 = 0.$

We test the provided options:

- For $\lambda = -2$: $(-2)^3 - 6(-2) - 4 = -8 + 12 - 4 = 0.$

Since $\lambda = -2$ satisfies the characteristic equation, the vectors are coplanar for this value.

Final Answer : -2

Answer: (A)

Q44.

Solution

Concept: The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle between the vectors.

Solution: We are given the condition: $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$.

Comparing this given equation with the standard definition of the dot product:

$|\vec{a}||\vec{b}| \cos\theta = -|\vec{a}||\vec{b}|$.

Assuming $\vec{a}$ and $\vec{b}$ are non-zero vectors, we can divide both sides by $|\vec{a}||\vec{b}|$:

$\cos\theta = -1$. The cosine function takes the value of -1 when the angle θ is π radians (or 180°).

This indicates that the two vectors are anti-parallel (pointing in exactly opposite directions).

Final Answer : π

Answer: (C)

Q45.

Solution

Concept: The foot of the perpendicular (x, y, z) from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by: $\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C} = -\frac{Ax_1+By_1+Cz_1+D}{A^2+B^2+C^2}$.

Solution: The point is $(0, 0, 0)$ and the plane is $2x - 3y + 4z - 6 = 0$.

Here, $A = 2, B = -3, C = 4, D = -6$.

Let the foot be (x, y, z). Using the formula:

$\frac{x-0}{2} = \frac{y-0}{-3} = \frac{z-0}{4} = -\frac{2(0)-3(0)+4(0)-6}{2^2+(-3)^2+4^2}$

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{4} = -\frac{-6}{4+9+16} = \frac{6}{29}$.

Solving for x, y, z:

$x = 2 \times \frac{6}{29} = \frac{12}{29}$

$y = -3 \times \frac{6}{29} = -\frac{18}{29}$

$z = 4 \times \frac{6}{29} = \frac{24}{29}$.

The coordinates are $(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29})$.

Final Answer : $(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29})$

Answer: (A)

Q46.

Solution

Concept: The angle θ between two lines with direction vectors $\vec{d}_1$ and $\vec{d}_2$ is found using $\cos\theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{|\vec{d}_1||\vec{d}_2|}$.

Solution: 1. For Line 1: $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$.

The $z = 2$ part means the change in z is zero. So, the direction vector is $\vec{d}_1 = (3, -2, 0)$.

2. For Line 2: $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$.

Rewrite in standard form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$:

$\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$.

The direction ratios are $(1, 3/2, 2)$, which can be simplified by multiplying by 2 to get $\vec{d}_2 = (2, 3, 4)$.

3. Calculate the dot product $\vec{d}_1 \cdot \vec{d}_2$:

$\vec{d}_1 \cdot \vec{d}_2 = (3)(2) + (-2)(3) + (0)(4) = 6 - 6 + 0 = 0$.

Since the dot product is zero, the lines are perpendicular, and the angle is $\pi/2$.

Final Answer : $\pi/2$

Answer: (A)

Q47.

Solution

Concept: The distance d between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is $d = \frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Plane 1: $2x - y + 2z + 3 = 0$

Plane 2: $4x - 2y + 4z + 5 = 0$

To use the formula, the coefficients of x, y, z must be identical. Divide Plane 2 by 2:

$2x - y + 2z + \frac{5}{2} = 0.$

Now, $A = 2, B = -1, C = 2, D_1 = 3, D_2 = 2.5.$

$d = \frac{|3-2.5|}{\sqrt{2^2+(-1)^2+2^2}} = \frac{0.5}{\sqrt{4+1+4}} = \frac{0.5}{}\sqrt{9} = \frac{0.5}{3} = \frac{1}{6}.$

Final Answer : 1/6

Answer: (A)

Q48.

Solution

Concept: A feasible region exists if there is at least one point (x, y) that satisfies all the given linear inequalities simultaneously.

Solution: The given inequalities are:

1. $x \geq 6$

2. $y \geq 2$

3. $2x + y \leq 10$

From (1) and (2), the minimum values are $x = 6$ and $y = 2$.

Let's substitute these minimum values into the third inequality:

$2(6) + 2 = 12 + 2 = 14.$

For any values of $x \geq 6$ and $y \geq 2$, the sum $2x + y$ will always be greater than or equal to 14.

However, the constraint (3) requires $2x + y \leq 10$.

Since 14 is never less than or equal to 10, there is no point that satisfies all three conditions.

Therefore, the feasible region is an empty set.

Final Answer : Empty set

Answer: (C)

Q49.

Solution

Concept: Corner Point Theorem: The maximum or minimum value of a linear objective function Z occurs at one of the vertices (corner points) of the feasible region defined by linear constraints.

Solution: The constraints are $x + y \leq 4, x \geq 0, y \geq 0$.

The feasible region is a triangle with vertices at the intersections of the boundary lines:

1. Intersection of $x = 0$ and $y = 0$: $(0, 0)$

2. Intersection of $x + y = 4$ and $y = 0$: $(4, 0)$

3. Intersection of $x + y = 4$ and $x = 0$: $(0, 4)$

Evaluate the objective function $Z = 3x + 4y$ at these corner points:

- At $(0, 0) : Z = 3(0) + 4(0) = 0$

- At $(4, 0) : Z = 3(4) + 4(0) = 12$

- At $(0, 4) : Z = 3(0) + 4(4) = 16$

The highest value obtained is 16.

Final Answer : 16

Answer: (B)

Q50.

Solution

Concept: General Probability Rules: 1. $P(B|A) = \frac{P(A \cap B)}{P(A)}$ (Conditional Probability) 2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Addition Rule)

Solution: Given: $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$.

Step 1: Calculate $P(A \cap B)$ using the conditional probability formula:

$P(A \cap B) = P(B|A) \times P(A) = 0.6 \times 0.4 = 0.24$.

Step 2: Calculate $P(A \cup B)$ using the addition rule:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.4 + 0.8 - 0.24$

$P(A \cup B) = 1.2 - 0.24 = 0.96$.

Final Answer : 0.96

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	A	4	A	5	A
6	A	7	B	8	C	9	C	10	C
11	A	12	A	13	A	14	B	15	C
16	B	17	A	18	B	19	C	20	C
21	B	22	C	23	A	24	B	25	A
26	B	27	A	28	B	29	A	30	B
31	B	32	A	33	B	34	C	35	D
36	A	37	B	38	D	39	B	40	A
41	A	42	B	43	A	44	C	45	A
46	A	47	A	48	C	49	B	50	A