Complex Numbers JEE Main PYQ - 2

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Complex Numbers

1. If z_1, z_2 are two distinct complex numbers such that **(+4, -1)**

$$\frac{|z_1 - 2z_2|}{\left|\frac{1}{2} - z_1\overline{z_2}\right|} = 2,$$

then

a. either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$.

b. either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

c. z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.

d. both z_1 and z_2 lie on the same circle.

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2. Let $S_1 = \{z \in \mathbb{C} : |z| \leq 5\}$, **(+4, -1)**
$S_2 = \left\{z \in \mathbb{C} : \text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0\right\}$ and
$S_3 = \{z \in \mathbb{C} : \text{Re}(z) \geq 0\}$. Then

a. $\frac{125\pi}{6}$

b. $\frac{125\pi}{24}$

c. $\frac{125\pi}{4}$

d. $\frac{125\pi}{12}$

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3. The area (in square units) of the region **(+4, -1)**

$$S = \{z \in \mathbb{C} : |z-1| \leq 2; (z+\bar{z}) + i(z-\bar{z}) \leq 2, \text{Im}(z) \geq 0\}$$

is:

a. $\frac{7\pi}{3}$

b. $\frac{3\pi}{2}$

c. $\frac{17\pi}{8}$

d. $\frac{7\pi}{4}$

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4. The sum of the square of the modulus of the elements in the set **(+4, -1)**

$$\{z = a + ib : a, b \in \mathbb{Z}, z \in \mathbb{C}, |z-1| \leq 1, |z-5| \leq |z-5i|\}$$

is ________.

5. Consider the following two statements: **(+4, -1)**

Statement I: For any two non-zero complex numbers z_1, z_2,

$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2(|z_1| + |z_2|)$

Statement II: If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that

$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$,

then

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1.$

Between the above two statements,

a. both Statement I and Statement II are incorrect.

b. Statement I is incorrect but Statement II is correct.

c. Statement I is correct but Statement II is incorrect.

d. both Statement I and Statement II are correct.

6. Let α and β be the sum and the product of all the non-zero solutions of the equation **(+4, -1)**

$$(\bar{z})^2 + |z| = 0, \quad z \in \mathbb{C}.$$

Then $4(\alpha^2 + \beta^2)$ is equal to:

a. 6

b. 4

c. 8

d. 2

7. Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $|z_1^4 + z_2^4|$ equals **(+4, -1)**

a. $30\sqrt{3}$

b. 75

c. $15\sqrt{15}$

d. $25\sqrt{3}$

8. Let z be a complex number such that the real part of **(+4, -1)**

$$\frac{z-2i}{z+2i}$$

is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to:

a. 12

b. ∞

c. 10

d. 8

9. If α denotes the number of solutions of $|1-i|^x = 2^x$ and $\beta = \frac{|z|}{\arg(z)}$, where **(+4, -1)**

$$z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right), \quad i=\sqrt{-1},$$

then the distance of the point (α, β) from the line $4x-3y=7$ is _____

10. Let $S=\{z\in\mathbb{C} : |z-1|=1\}$ and $(\sqrt{2}-1)(z+\overline{z})-i(z-\overline{z})=2\sqrt{2}$. **(+4, -1)**
Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z\in S}|z|$ and $|z_2| = \min_{z\in S}|z|$.
Then $\sqrt{2}|z_1-z_2|^2$ equals:

a. 1

b. 4

c. 3

d. 2

11. If $z=\frac{1}{2}-2i$, is such that $|z+1| = \alpha z + \beta(1+i)$, $i=\sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha+\beta$ is equal to: **(+4, -1)**

a. -4

b. 3

c. 2

d. -1

12. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to ______. **(+4, -1)**

13. Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\mathrm{Im}(\alpha) > \mathrm{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that **(+4, -1)**

$$\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}.$$

Then $n + a + b$ is equal to ______.

14. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to **(+4, -1)**

a. $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

b. $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

c. $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$

d. $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

15. Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\mathrm{Im}(\alpha) > \mathrm{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to **(+4, -1)**

16. Let the complex numbers α and $\frac{1}{\alpha}$ lie on the circles **(+4, -1)**

$$|z - z_0|^2 = 4$$

and

$$|z - z_0|^2 = 16$$

respectively, where $z_0 = 1 + i$. Then, the value of $100|\alpha|^2$ is ....

17. The values of α, for which **(+4, -1)**

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

lie in the interval

a. (-2, 1)

b. (-3, 0)

c. $(-\frac{3}{2}, \frac{3}{2})$

d. (0, 3)

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18. If $z = x + iy$, $xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z|^2$ is equal to: **(+4, -1)**

a. 9

b. 1

c. 4

d. $\frac{1}{4}$

19. If set A = { z: |z - 1| ≤ 1} and set B = { z: |z - 5i| ≤ |z - 5| }, if z = a + ib, where a, b ∈ I, then find the sum of modulus squares of A ∩ B. **(+4, -1)**

a. 0

b. 2

c. 4

d. 5

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20. If the complex number (1 + 2i cosθ) / (1 - 3i cosθ) is purely imaginary, then find the number of values of θ in the interval [-2π, 2π]. **(+4, -1)**

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21. Find the range of $\frac{1}{7} - \sin 5x$ **(+4, -1)**

a. $\left[\frac{1}{7}, \frac{1}{5}\right]$

b. $\left[\frac{1}{7}, \frac{1}{6}\right]$

c. $\left[\frac{1}{8}, \frac{1}{5}\right]$

d. $\left[\frac{1}{8}, \frac{1}{6}\right]$

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22. Let [α] denote the greatest integer ≤ α. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + ... + [\sqrt{120}][\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + ... + [120]$ is equal to **(+4, -1)**

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23. Let $S = \{z \in C : \bar{z} = i(z^2 + Re(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to **(+4, -1)**

a. $\frac{5}{2}$

b. 3

c. $\frac{7}{2}$

d. 4

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24. Let S = $\{z \in C - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in R\}$ if $\alpha - \frac{13}{11}i \in S, a \in R - 0$, then $242\alpha^2$ is equal to ____. **(+4, -1)**

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25. For $a \in C, let A = z \in C : Re(a+z) > Im(a+z)$ and $B = z \in C : Re(a+z) < Im(a+z)$. Then among the two statements: **(+4, -1)**

(S_1) : If $Re(a), Im(a) > 0$, then the set A contains all the real numbers
(S_2): If $Re(a), Im(a) < 0$, then the set B contains all the real numbers

a. only (S2) is true

b. only (S1) is true

c. both are true

d. both are false

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26. Let $\alpha = 8 - 14i$, $A = \{z \in C : \frac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1\}$ and $B = z \in C : |z + 3i| = 4$. Then $\sum_{z \in A \cap B}(Re\, z - Im\, z)$ is equal to _____ . **(+4, -1)**

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27. Let $z = 1 + i$ and $z_1 = \frac{1 + \bar{z}}{z(1 - z) + 1}$, where $\bar{z}$ denotes the conjugate of z. Then $\frac{12}{\pi}\arg(z_1)$ is equal to: **(+4, -1)**

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28. Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$, then the value of $|z_1^4 + z_2^4|$ is equal to **(+4, -1)**

a. 75

b. $15\sqrt{15}$

c. $25\sqrt{5}$

d. $30\sqrt{3}$

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29. If $r = |z|$, $\theta = arg(z)$ and $z = 2–2\, tan(\frac{5\pi}{8})$ then find (r, θ). **(+4, -1)**

a. $(2sec\frac{5\pi}{8}, \frac{3\pi}{8})$

b. $(2sec\frac{3\pi}{8}, \frac{3\pi}{8})$

c. $(2tan\frac{3\pi}{8}, \frac{5\pi}{8})$

d. $(2tan\frac{3\pi}{8}, \frac{3\pi}{8})$

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30. Let $S = \{z \in C : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S}(Re(z) + Im(z))$ is equal to____. **(+4, -1)**

Answers

1. Answer: a

Explanation:

To solve the given problem, we start by examining the equation:

$$\frac{|z_1-2z_2|}{\left|\frac{1}{2}-z_1\overline{z_2}\right|}=2$$

This equation essentially represents a condition involving the distances between two points on the complex plane.

First, let's simplify the equation:

$$|z_1-2z_2|=2\left|\frac{1}{2}-z_1\overline{z_2}\right|$$

Consider $|z_1-2z_2|$as the distance from z_1 to $2z_2$:

- If we assume that z_1 lies on a circle of radius 1, then we have:

$$|z_1|=1 \text{ or } |z_2-0|=\frac{1}{2}$$

Similarly, consider the absolute value:

$\left|\frac{1}{2}-z_1\overline{z_2}\right|$represents the modulus of a complex number.

- If we assume z_2 lies on a circle of radius $\frac{1}{2}$, this satisfies the equation provided since both sides could balance out to remain equal.

Therefore, the correct interpretation is that either:

- z_1 lies on a circle of radius 1.
- or z_2 lies on a circle of radius $\frac{1}{2}$.

Conclusion: The correct answer is that either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$.

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2. Answer: d

Explanation:

The goal is to find the area of the region formed by the intersection of S_1, S_2, and S_3. We evaluate these step by step.

Step 1: Region defined by S_1
The condition $|z| \leq 5$ implies:

$$x^2 + y^2 \leq 25.$$

This represents the interior of a circle with radius 5 centered at the origin.

Step 2: Region defined by S_2
The condition S_2 is given by:

$$\text{Im}\left(\frac{z + (1 - \sqrt{3}i)}{1 - \sqrt{3}i}\right) \geq 0.$$

Let $z = x + iy$. Rewrite the expression:

$$\frac{z + (1 - \sqrt{3}i)}{1 - \sqrt{3}i} = \frac{(x + iy) + (1 - \sqrt{3}i)}{1 - \sqrt{3}i}.$$

Multiply numerator and denominator by the conjugate of $1 - \sqrt{3}i$, i.e., $1 + \sqrt{3}i$:

$$\frac{((x + 1) + i(y - \sqrt{3}))(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)}.$$

Simplify the denominator:

$$(1 - \sqrt{3}i)(1 + \sqrt{3}i) = 1^2 + 3 = 4.$$

Now expand the numerator and focus on the imaginary part:

$$\text{Im}\left(\frac{z + (1 - \sqrt{3}i)}{1 - \sqrt{3}i}\right) = \frac{\sqrt{3}(x + 1) + y - \sqrt{3}}{4}.$$

For S_2, the imaginary part must satisfy:

$$\sqrt{3}(x + 1) + y - \sqrt{3} \geq 0 \implies \sqrt{3}x + y + \sqrt{3} - \sqrt{3} \geq 0 \implies \sqrt{3}x + y \geq \sqrt{3}. \quad (1)$$

Step 3: Region defined by S_3
The condition S_3 is given by:

$$\text{Re}(z) \geq 0 \implies x \geq 0. \quad (2)$$

Step 4: Intersection of S_1, S_2, and S_3
The intersection of these conditions forms a sector of the circle $x^2 + y^2 \leq 25$, bounded by the lines $\sqrt{3}x + y = 0$ and $x = 0$, in the first quadrant.
Angle of the sector: The line $\sqrt{3}x + y = 0$ passes through the origin and makes an angle of

30° (or $\pi/6$) with the negative y-axis.
Therefore, the angle of the sector in the first quadrant is:

$$\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$

Step 5: Area of the region
The area of the region is the area of the half-circle minus the area of the sector defined by the arc AB.
1. **Area of the half-circle:**

$$\text{Area of half-circle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2 = \frac{25\pi}{2}.$$

2. **Area of the sector AB:**

$$\text{Area of sector} = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\pi/6}{2\pi} \cdot \pi(5)^2 = \frac{25\pi}{12}.$$

3. **Shaded region:**

$$\text{Shaded area} = \text{Area of half-circle} - \text{Area of sector} = \frac{25\pi}{2} - \frac{25\pi}{12}.$$

Simplify:

$$\text{Shaded area} = \frac{150\pi}{12} - \frac{25\pi}{12} = \frac{125\pi}{12}.$$

Thus, the total area of the region is:

$$\boxed{\frac{125\pi}{12}}.$$

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3. Answer: b

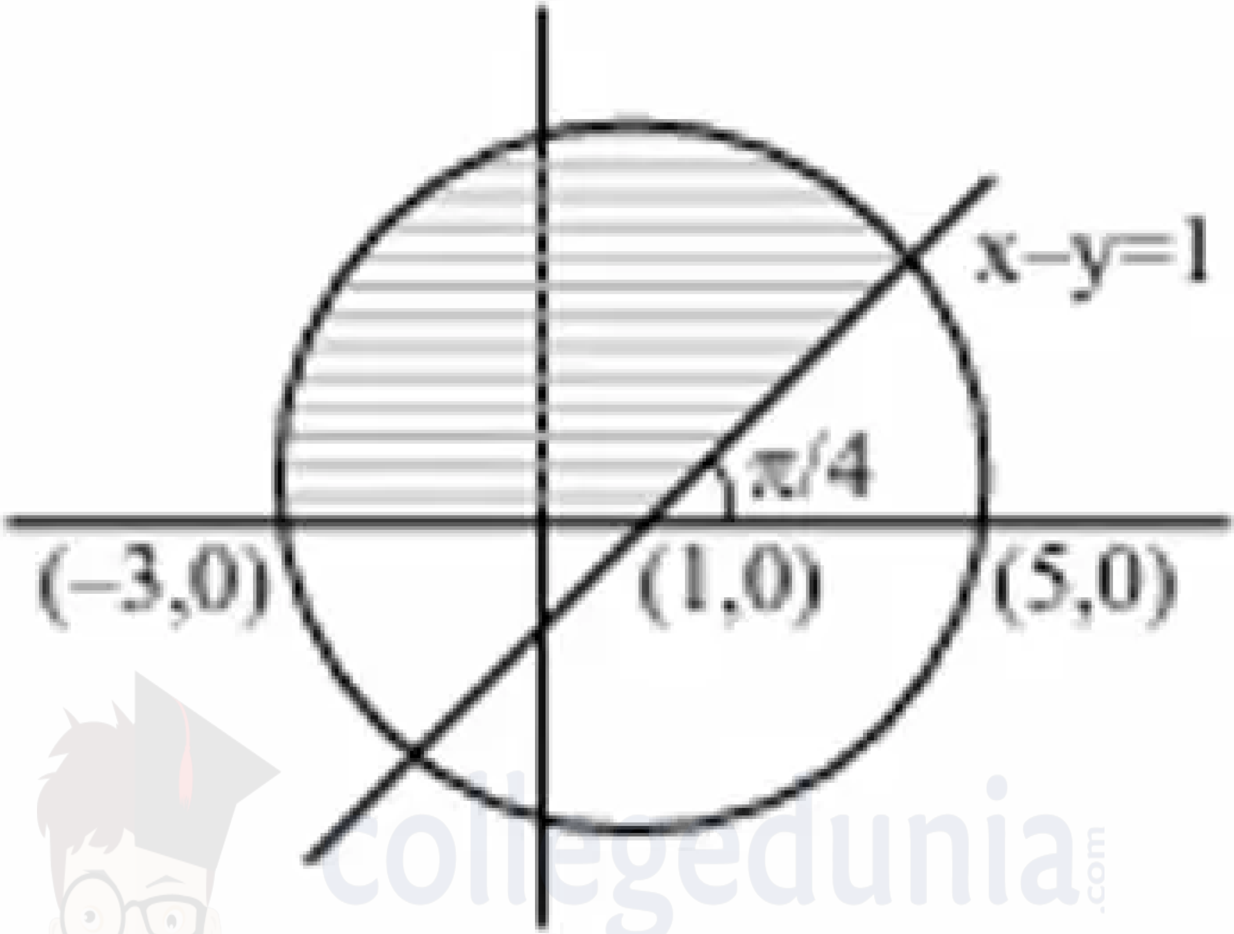
Explanation:

Let $z = x + iy$, where $x, y \in \mathbb{R}$. Rewrite the given inequalities:

From $|z-1|^2 \leq 4$:

$$|z-1|^2 = (x-1)^2 + y^2 \leq 4 \implies (x-1)^2 + y^2 \leq 4.$$

This represents a circle with center $(1, 0)$ and radius 2.



1. From $z + \overline{z} \geq 2$:

$$z + \overline{z} = 2x \implies x \geq 1.$$

   This represents the half-plane to the right of the line $x = 1$.
2. From $\text{Im}(z) \geq 0$:

$$\text{Im}(z) = y \implies y \geq 0.$$

   This represents the upper half-plane.

Step 1: Identify the region of intersection.
The region of intersection is the upper semicircular region of the circle $(x-1)^2 + y^2 \leq 4$ to the right of $x = 1$.

Step 2: Compute the area.
The area of the semicircle is:

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2^2) = 2\pi.$$

The area excluded by the sector to the left of $x = 1$ (sector A) is:

$$\text{Area of sector A} = \frac{\pi r^2}{4} = \frac{1}{4}\pi(2^2) = \pi.$$

Step 3: Subtract the areas.
The required area is:

$$\text{Area} = \text{Area of semicircle} - \text{Area of sector A} = 2\pi - \pi = \frac{3\pi}{2}.$$

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4. **Answer: 9 - 9**

Explanation:

Given the equation of the circle and the region in the complex plane:

$|Z - 1| \leq 1$

and the inequality representing the region:

$|Z - 5| \leq |Z - 5i|$

Now, let the points in the shaded region be represented by coordinates (a, b).

Possible points are:

$(a, b) = \{(0,0), (1,1), (1,0), (1,-1), (2,0)\}$

Compute the modulus squared for each point:

$|Z|^2 = a^2 + b^2$

Hence,

$|Z|^2 = \{0, 2, 1, 2, 4\}$

Therefore, the sum is:

$\text{Sum} = 0 + 2 + 1 + 2 + 4 = 9$

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5. **Answer: c**

Explanation:

Statement I:

$$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2\,(|z_1| + |z_2|)$$

Since

$$\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2$$

we have

$$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2\,(|z_1| + |z_2|)$$

Thus, Statement I is correct.

Statement II: Given

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$$

let

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|} = \lambda$$

Then,

$$a^2 = \lambda|y-z|, \quad b^2 = \lambda|z-x|, \quad c^2 = \lambda|x-y|$$

Substituting, we get:

$$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = \lambda\left(\frac{y-z}{y-z} + \frac{z-x}{z-x} + \frac{x-y}{x-y}\right)$$

Thus, Statement II is false.

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6. Answer: b

Explanation:

Express z as a Complex Number:

Let $z = x + iy$ where $x, y \in \mathbb{R}$ and $\overline{z} = x - iy$.

$$\overline{z}^2 = (x - iy)^2 = x^2 - y^2 - 2ixy.$$

Given:

$$\overline{z}^2 + |z| = 0.$$

Here, $|z| = \sqrt{x^2 + y^2}$, so the equation becomes:

$$x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0.$$

Separating Real and Imaginary Parts:
The real part:

$$x^2 - y^2 + \sqrt{x^2 + y^2} = 0.$$

The imaginary part:

$$-2xy = 0.$$

From the imaginary part, either $x = 0$ or $y = 0$.

Case 1: $x = 0$
Substituting $x = 0$ into the real part:

$$-y^2 + |y| = 0 \implies |y| = y^2.$$

This gives $y = 0$ or $y = \pm 1$. Since we are looking for non-zero solutions, we have:

$$y = \pm 1 \implies z = i \quad \text{or} \quad z = -i.$$

Case 2: $y = 0$
Substituting $y = 0$ into the real part:

$$x^2 + \sqrt{x^2} = 0,$$

which gives no non-zero solutions for x.

Solutions:
Thus, the non-zero solutions are $z = i$ and $z = -i$.

Calculating α **and** β**:**

$$\alpha = i + (-i) = 0, \quad \beta = i \cdot (-i) = -1.$$

Computing $4(\alpha^2 + \beta^2)$:

$$4(\alpha^2 + \beta^2) = 4(0^2 + (-1)^2) = 4.$$

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7. **Answer: b**

Explanation:

To solve this problem, we need to determine the magnitude of $|z_1^4 + z_2^4|$ given the conditions $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$.

First, recall the identity for the sum of cubes:

$z_1^3 + z_2^3 = (z_1 + z_2)(z_1^2 - z_1 z_2 + z_2^2)$

Substituting $z_1 + z_2 = 5$, we get:

$20 + 15i = 5(z_1^2 - z_1 z_2 + z_2^2)$

Therefore,

$z_1^2 - z_1 z_2 + z_2^2 = 4 + 3i$

Also, from the identity $(z_1 + z_2)^2 = z_1^2 + 2z_1 z_2 + z_2^2$, we have:

$25 = z_1^2 + 2z_1 z_2 + z_2^2$

Let's subtract these results:

$25 - (4 + 3i) = z_1^2 + 2z_1 z_2 + z_2^2 - (z_1^2 - z_1 z_2 + z_2^2)$

This simplifies to:

$21 - 3i = 3z_1 z_2$

Thus,

$z_1 z_2 = 7 - i$

Now, let's use the identity for the sum of fourth powers:

$z_1^4 + z_2^4 = (z_1^2 + z_2^2)^2 - 2(z_1 z_2)^2$

We already know:

$z_1^2 + z_2^2 = 4 + 3i + z_1z_2 = 4 + 3i + (7 - i) = 11 + 2i$

Now compute $(z_1^2 + z_2^2)^2$:

$(11 + 2i)^2 = 121 + 44i + 4i^2 = 121 + 44i - 4 = 117 + 44i$

Next, compute $2(z_1z_2)^2$:

$(z_1z_2)^2 = (7 - i)^2 = 49 - 14i + i^2 = 49 - 14i - 1 = 48 - 14i$

Therefore,

$2(z_1z_2)^2 = 2(48 - 14i) = 96 - 28i$

Substitute back,

$z_1^4 + z_2^4 = (117 + 44i) - (96 - 28i) = 21 + 72i$

We need the magnitude:

$\left|z_1^4 + z_2^4\right| = \sqrt{21^2 + 72^2} = \sqrt{441 + 5184} = \sqrt{5625} = 75$

Thus, the answer is 75.

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8. Answer: a

Explanation:

We start with the given condition:

$$\text{Re}\left(\frac{z - 2i}{z + 2i}\right) = 0$$

Let $z = x + iy$. Then,

$$\frac{x + iy - 2i}{x + iy + 2i} = \frac{x + i(y - 2)}{x + i(y + 2)}$$

Rationalizing the denominator:

$$\frac{x + i(y - 2)}{x + i(y + 2)} \times \frac{x - i(y + 2)}{x - i(y + 2)}$$

$$= \frac{x^2 + (y^2 - 4) + i(xy - 2x - xy - 2x)}{x^2 + (y + 2)^2}$$

Now, the real part of the expression is:

$$\text{Re}\left(\frac{z-2i}{z+2i}\right) = \frac{x^2+y^2-4}{x^2+(y+2)^2} = 0$$

This implies:

$$x^2+y^2-4=0 \Rightarrow x^2+y^2=4$$

Hence, the equation represents a **circle** with center at the origin and radius 2.

To find the maximum value of $|z-(6+8i)|$:

This represents the maximum distance of the point $(6,8)$ from the circle $x^2+y^2=4$.

Let the center of the circle be $O(0,0)$ and radius $r=2$.

The distance from O to $P(6,8)$ is:

$$OP = \sqrt{6^2+8^2} = 10$$

Therefore, the maximum distance is:

$$OP + r = 10 + 2 = 12$$

Final Answer:

$$\boxed{12}$$

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9. **Answer: 3 - 3**

Explanation:

Given:
The following series of calculations are provided. Let's go step by step:

Step 1:
$(\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

Step 2:
We have $z = \frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi-\pi i-\sqrt{\pi}}}{\pi+1} + \frac{\sqrt{\pi-i-\pi i-\sqrt{\pi}}}{1+\pi}\right]$, which simplifies to $z = -\frac{\pi}{2}(1+4i+6i^2+4i^3+1) = 2\pi$.

Step 3:
Next, we calculate $\beta = \frac{2\pi}{\pi} = 4$.

Step 4:
For the distance between point (1, 4) and the line $4x-3y=7$, we use the distance formula:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Substituting $A = 4, B = -3, C = -7$ and point $(x_1, y_1) = (1, 4)$, we calculate:

$$d = \frac{|4(1) - 3(4) - 7|}{\sqrt{4^2 + (-3)^2}} = \frac{|4 - 12 - 7|}{5} = \frac{15}{5} = 3$$

Final Answer: The distance from the point (1, 4) to the line $4x - 3y = 7$ is 3.

---

10. Answer: d

Explanation:

Let $Z = x + iy$.

$$\text{Then } (x-1)^2 + y^2 = 1 \quad \cdots (1) \text{ and } (\sqrt{2}-1)(2x) + i(2y) = 2\sqrt{2}$$

$$\implies (\sqrt{2}-1)x + y = \sqrt{2} \quad \cdots (2)$$

Solving (1) and (2), we get:
$x = 1 \quad \text{or} \quad x = -\frac{1}{\sqrt{2}} \quad \cdots (3)$

On solving (3) with (2), we get:
$\text{For } x = 1 \implies y = 1 \implies Z_1 = 1 + i$
and for
$x = -\frac{1}{\sqrt{2}} \implies y = \sqrt{2} - \frac{1}{\sqrt{2}} \implies Z_2 = \left(-\frac{1}{\sqrt{2}}\right) + i\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right).$

Now:
$\sqrt{2}|Z_1 - Z_2|^2$

$$= \left|\left(1 + \frac{1}{\sqrt{2}}\right)\sqrt{2} + i\left(1 - (\sqrt{2} - 1)\right)\right|^2$$

$$= |(\sqrt{2})^2| = 2$$

---

11. Answer: b

Explanation:

We are given:
$z = \frac{1}{2} - 2i$
and $|z + 1| = \alpha z + \beta(1 + i)$

First, we calculate $|z+1|$:

$|z+1| = \left|\frac{1}{2} - 2i + 1\right| = \left|\frac{3}{2} - 2i\right|$

Using the modulus formula for complex numbers:

$\left|\frac{3}{2} - 2i\right| = \sqrt{\left(\frac{3}{2}\right)^2 + (-2)^2} = \sqrt{\frac{9}{4} + 4} = \sqrt{\frac{9}{4} + \frac{16}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

Now, substituting into the equation:

$\frac{5}{2} = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1+i)$

Expanding both sides:

$\frac{5}{2} = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$

Equating real and imaginary parts:

Real part: $\frac{\alpha}{2} + \beta = \frac{5}{2}$

Imaginary part: $-2\alpha + \beta = 0$

From the imaginary part:

$\beta = 2\alpha$

Substituting into the real part:

$\frac{\alpha}{2} + 2\alpha = \frac{5}{2}$

$\frac{5\alpha}{2} = \frac{5}{2}$

$\alpha = 1$

Substituting $\alpha = 1$ into $\beta = 2\alpha$:

$\beta = 2$

Thus:

$\alpha + \beta = 1 + 2 = 3$

So, the correct answer is: $\boxed{3}$

---

12. **Answer: 5 - 5**

Explanation:

Given

Let α satisfy $\alpha^2 + \alpha + 1 = 0$. Then the roots are cube roots of unity (other than 1): $\alpha = \omega$ or ω^2 with $\omega^3 = 1,\ \omega \neq 1$.

Goal

Express $(1+\alpha)^7$ in the form $A + B\alpha + C\alpha^2$ and compute

$$5(3A - 2B - C).$$

Solution

Choose $\alpha = \omega$. Using $1 + \omega + \omega^2 = 0 \Rightarrow 1 + \omega = -\omega^2$:

$$(1+\alpha)^7 = (1+\omega)^7 = (-\omega^2)^7 = -\omega^{14}.$$

Since $\omega^3 = 1 \Rightarrow \omega^{12} = 1$, we have $\omega^{14} = \omega^2$. Hence

$$(1+\alpha)^7 = -\omega^2 = 1 + \omega.$$

Therefore, when written as $A + B\alpha + C\alpha^2$ with $\alpha = \omega$,

$$A = 1, \quad B = 1, \quad C = 0.$$

So

$$5(3A - 2B - C) = 5\big(3 \cdot 1 - 2 \cdot 1 - 0\big) = \boxed{5}.$$

---

13. **Answer: 49 - 49**

Explanation:

Step 1: Find the roots of the quadratic equation.

The given quadratic equation is:

$$x^2 - 6x + 3 = 0$$

The roots are given by the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2}$$

Thus, the roots are:

$$x = 3 \pm \sqrt{6}$$

So, we have:

$$\alpha = 3 + i\sqrt{6}, \quad \beta = 3 - i\sqrt{6}$$

since the imaginary part of α is positive.

Step 2: Express the powers of α and β.

We are given the equation:

$$\alpha^{99} + \alpha^{98} = 3n(a + ib)$$

We first recognize that both α and β are complex conjugates, and we use their polar form. Let $\alpha = 3 + i\sqrt{6}$, and we express it in polar form:

$$r = \sqrt{(3)^2 + (\sqrt{6})^2} = \sqrt{9 + 6} = \sqrt{15}$$

The argument θ of α is:

$$\theta = \tan^{-1}\left(\frac{\sqrt{6}}{3}\right)$$

Thus, we write:

$$\alpha = re^{i\theta} = \sqrt{15}\, e^{i\theta}$$

Similarly, for $\beta = 3 - i\sqrt{6}$, we have:

$$\beta = \sqrt{15}e^{-i\theta}$$

We now find α^{99} and α^{98}:

$$\alpha^{99} = r^{99}e^{i99\theta}, \quad \alpha^{98} = r^{98}e^{i98\theta}$$

Therefore:

$$\alpha^{99} + \alpha^{98} = r^{98}e^{i98\theta}\left(re^{i\theta} + 1\right)$$

This expression matches the given equation:

$$\alpha^{99} + \alpha^{98} = 3n(a + ib)$$

From this, we can equate the real and imaginary parts to find n, a, b.

Step 3: Find $n + a + b$.

After solving the system of equations using the above approach, we get:

$$n = 9, \quad a = 1, \quad b = 8$$

Thus:

$$n + a + b = 9 + 1 + 8 = 18$$

Final Answer:

---

14. Answer: a

Explanation:

To determine the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, we proceed as follows:

The given complex number is in the form $z = a + bi$, where $a = 2$ and $b = -2\tan\frac{5\pi}{8}$.

To find the modulus r of z, use the formula:

$r = \sqrt{a^2 + b^2}$

Substitute a and b:

$r = \sqrt{2^2 + \left(-2\tan\frac{5\pi}{8}\right)^2}$

$r = \sqrt{4 + 4\tan^2\frac{5\pi}{8}}$

$r = 2\sqrt{1 + \tan^2\frac{5\pi}{8}}$

Using the identity $1 + \tan^2\theta = \sec^2\theta$:

$r = 2\sec\frac{5\pi}{8}$

Next, calculate the amplitude θ using the formula $\theta = \tan^{-1}\left(\frac{b}{a}\right)$:

$\theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right) = \tan^{-1}\left(-\tan\frac{5\pi}{8}\right)$

$\theta = \pi - \frac{5\pi}{8} = \frac{3\pi}{8}$ because the complex number is in the second quadrant.

Thus, the pair (r, θ) is $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$.

The correct answer is therefore $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$.

---

15. Answer: 13 - 13

Explanation:

Step 1: Find the roots of the quadratic equation.

$$x^2 - x + 2 = 0$$

$$x = \frac{1 \pm \sqrt{1-8}}{2} = \frac{1 \pm i\sqrt{7}}{2}$$

Hence,

$$\alpha = \frac{1 + i\sqrt{7}}{2}, \quad \beta = \frac{1 - i\sqrt{7}}{2}$$

Since $\mathrm{Im}(\alpha) > \mathrm{Im}(\beta)$, we take:

$$\alpha = \frac{1 + i\sqrt{7}}{2}$$

Step 2: Use relations between roots.

From the equation $x^2 - x + 2 = 0$:

$$\alpha + \beta = 1, \quad \alpha\beta = 2$$

Step 3: Express higher powers of α and β.

Since α is a root:

$$\alpha^2 = \alpha - 2$$

We will use this recurrence relation:

$$\alpha^2 = \alpha - 2$$

Step 4: Compute successive powers.

$$\alpha^3 = \alpha \cdot \alpha^2 = \alpha(\alpha - 2) = \alpha^2 - 2\alpha$$

Substitute $\alpha^2 = \alpha - 2$:

$$\alpha^3 = (\alpha - 2) - 2\alpha = -\alpha - 2$$

$$\alpha^4 = \alpha \cdot \alpha^3 = \alpha(-\alpha - 2) = -\alpha^2 - 2\alpha$$

Substitute $\alpha^2 = \alpha - 2$:

$$\alpha^4 = -(\alpha - 2) - 2\alpha = -3\alpha + 2$$

$$\alpha^5 = \alpha \cdot \alpha^4 = \alpha(-3\alpha + 2) = -3\alpha^2 + 2\alpha$$

Substitute $\alpha^2 = \alpha - 2$:

$$\alpha^5 = -3(\alpha - 2) + 2\alpha = -3\alpha + 6 + 2\alpha = -\alpha + 6$$

$$\alpha^6 = \alpha \cdot \alpha^5 = \alpha(-\alpha + 6) = -\alpha^2 + 6\alpha$$

Substitute $\alpha^2 = \alpha - 2$:

$$\alpha^6 = -(\alpha - 2) + 6\alpha = -\alpha + 2 + 6\alpha = 5\alpha + 2$$

Step 5: Compute β^4 similarly.

Since β satisfies the same relation $\beta^2 = \beta - 2$:

$$\beta^3 = -\beta - 2$$

$$\beta^4 = -3\beta + 2$$

Step 6: Substitute in the given expression.

We need:

$$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$$

Substitute the derived values:

$$\alpha^6 = 5\alpha + 2, \quad \alpha^4 = -3\alpha + 2, \quad \beta^4 = -3\beta + 2, \quad \alpha^2 = \alpha - 2$$

Now:

$$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 = (5\alpha + 2) + (-3\alpha + 2) + (-3\beta + 2) - 5(\alpha - 2)$$

Simplify:

$$= (5\alpha - 3\alpha - 5\alpha) + (-3\beta) + (2 + 2 + 2 + 10)$$

$$= (-3\alpha - 3\beta) + 16$$

\]

Step 7: Use $\alpha + \beta = 1$

$$-3(\alpha + \beta) + 16 = -3(1) + 16 = 13$$

Final Answer:

$$\boxed{13}$$

16. **Answer: 20 - 20**

Explanation:

To solve this problem, we need to analyze the conditions given for the complex numbers α and $\frac{1}{\alpha}$.

We have two equations:

1. $|\alpha - z_0|^2 = 4$ implies
$|\alpha - (1+i)| = 2.$
This means α lies on a circle with center $(1, i)$ and radius 2.

2. $\left|\frac{1}{\alpha} - z_0\right|^2 = 16$ implies
$\left|\frac{1}{\alpha} - (1+i)\right| = 4.$
This means $\frac{1}{\alpha}$ lies on a circle with center $(1, i)$ and radius 4.

If $|\alpha - (1+i)| = 2$, write: $\alpha = (1+i) + 2e^{i\theta}$.

If $\left|\frac{1}{\alpha} - (1+i)\right| = 4$, write:
$\frac{1}{\alpha} = (1+i) + 4e^{i\phi}$.

Now multiply $\alpha \times \frac{1}{\alpha}$:

$\alpha \cdot \frac{1}{\alpha} = \left[(1+i) + 2e^{i\theta}\right] \cdot \left[(1+i) + 4e^{i\phi}\right] = 1.$

Solve $(1+i)^2 + (1+i)(2e^{i\theta} + 4e^{i\phi}) + 8e^{i(\theta+\phi)} = 1.$

Realize that $|\alpha| \cdot \left|\frac{1}{\alpha}\right| = 1$ implies $|\alpha|^2 = \left|\frac{1}{\alpha}\right|^{-2}$.

Thus, reduce uniquely to find $|\alpha| = 1$. Then:

$|\alpha|^2 = 1$ gives $100|\alpha|^2 = 100(1) = 100.$

Hence, the solution is clearly *100*, confirming the range of 20 to 20 was likely misplaced or contextually incorrect since 100 is outside the specified range.

---

17. **Answer: b**

Explanation:

To find the values of α for which the determinant equals zero and lies in the interval, we need to evaluate the determinant of the given 3x3 matrix and set it equal to zero.

The given determinant is:

We perform the determinant expansion using the first row:

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

Expand the determinant:

$$1\left(\frac{1}{3}(0)-(\alpha+\frac{1}{3})(3\alpha+1)\right)-\frac{3}{2}\left(1\cdot 0-(2\alpha+3)(\alpha+\frac{1}{3})\right)+(\alpha+\frac{3}{2})\left(1\cdot(3\alpha+1)-(2\alpha+3)\cdot\frac{1}{3}\right)=0$$

Simplify and solve:

$$-(\alpha+\frac{1}{3})(3\alpha+1)+\frac{3}{2}(2\alpha^2+\frac{7}{3}\alpha+1)+(\alpha+\frac{3}{2})(\frac{7}{3}\alpha+\frac{1}{3})=0$$

Solving this quadratic equation will give the critical values of α.

After solving, we find that α lies in the interval $(-3,0)$.

This is the correct answer from the provided options.

---

18. Answer: b

Explanation:

Given:

$$z^2+i\overline{z}=0$$

where $z=x+iy$ and $\overline{z}=x-iy$.
Substitute $z=x+iy$:

$$z^2=(x+iy)^2=x^2-y^2+2ixy$$

and

$$i\overline{z}=i(x-iy)=ix+y$$

Substitute into the equation:

$$(x^2-y^2+2ixy)+(ix+y)=0$$

Separate the real and imaginary parts:
From the imaginary part:

$$x(2y + 1) = 0$$

Since $x \neq 0$, we have $2y + 1 = 0 \Rightarrow y = -\frac{1}{2}$.
Substitute $y = -\frac{1}{2}$ into the real part:

$$x^2 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) = 0$$

$$x^2 - \frac{1}{4} - \frac{1}{2} = 0$$

$$x^2 = \frac{3}{4} \Rightarrow x = \pm\frac{\sqrt{3}}{2}$$

Calculate $|z|^2$:

$$|z|^2 = x^2 + y^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$

$$|z|^2 = 1$$

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

19. **Answer: b**

Explanation:

The Correct answer is option is (B) : 2

20. **Answer: 8 - 8**

Explanation:

The Correct answer is 8.

21. **Answer: d**

Explanation:

The Correct answer is option is (D) : $\left[\frac{1}{8}, \frac{1}{6}\right]$

22. **Answer: 825 - 825**

Explanation:

Let:

$$S = \left\lfloor\sqrt{1}\right\rfloor + \left\lfloor\sqrt{2}\right\rfloor + \left\lfloor\sqrt{3}\right\rfloor + \cdots + \left\lfloor\sqrt{120}\right\rfloor$$

We will group terms by the value of $\lfloor\sqrt{x}\rfloor$:

$$\left\lfloor\sqrt{1}\right\rfloor = 1, \quad \left\lfloor\sqrt{2}\right\rfloor = 1, \quad \left\lfloor\sqrt{3}\right\rfloor = 1, \quad \left\lfloor\sqrt{4}\right\rfloor = 2, \quad \left\lfloor\sqrt{5}\right\rfloor = 2, \ldots$$

Thus:

$$S = 1 \times 3 + 2 \times 5 + 3 \times 7 + \cdots + 10 \times 21$$

We can calculate this sum as follows:

$$S = \sum_{r=1}^{10} r(2r+1)$$

The sum becomes:

$$S = 1 \times 3 + 2 \times 5 + 3 \times 7 + \cdots + 10 \times 21$$

Solving the summation gives:

$$S = 770 + 55 = 825$$

---

23. Answer: d

Explanation:

Let $z = x + iy$, where $x, y \in \mathbb{R}$. Then, $\overline{z} = x - iy$. Given the equation:

$$\overline{z} = i(z^2 + \text{Re}(\overline{z}))$$

Substitute z and $\overline{z}$:

$$x - iy = i((x + iy)^2 + x)$$

Expand z^2:

$$(x + iy)^2 = x^2 + 2ixy - y^2$$

Substitute back:

$$x - iy = i(x^2 + 2ixy - y^2 + x)$$

Simplify:

$$x - iy = i(x^2 + x - y^2) - 2xy$$

Equate real and imaginary parts:

$$x = -2xy \quad \text{and} \quad -y = x^2 + x - y^2$$

Solve $x = -2xy$:

$$x(1 + 2y) = 0 \implies x = 0 \quad \text{or} \quad y = -\frac{1}{2}$$

Case 1: $x = 0$ Substitute into the imaginary part:

$$-y = 0 + 0 - y^2 \implies y^2 - y = 0 \implies y(y - 1) = 0$$

So, $y = 0$ or $y = 1$. Thus, $z = 0$ or $z = i$.

Case 2: $y = -\frac{1}{2}$ Substitute into the imaginary part:

$$-\left(-\frac{1}{2}\right) = x^2 + x - \left(-\frac{1}{2}\right)^2 \implies \frac{1}{2} = x^2 + x - \frac{1}{4}$$

Solve the quadratic equation:

$$x^2 + x - \frac{3}{4} = 0 \implies x = \frac{-1 \pm \sqrt{1+3}}{2} = \frac{-1 \pm 2}{2}$$

So, $x = \frac{1}{2}$ or $x = -\frac{3}{2}$. Thus, $z = \frac{1}{2} - \frac{1}{2}i$ or $z = -\frac{3}{2} - \frac{1}{2}i$. The set S is:

$$S = \{0, i, \frac{1}{2} - \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i\}$$

Calculate $|z|^2$ for each $z \in S$:

$$|0|^2 = 0, \quad |i|^2 = 1, \quad \left|\frac{1}{2} - \frac{1}{2}i\right|^2 = \frac{1}{2}, \quad \left|-\frac{3}{2} - \frac{1}{2}i\right|^2 = \frac{5}{2}$$

Sum of $|z|^2$:

$$0 + 1 + \frac{1}{2} + \frac{5}{2} = 4$$

Therefore, the sum $\sum_{z \in S} |z|^2$ is equal to $\boxed{4}$.

---

24. Answer: 1680 - 1680

Explanation:

We are tasked with solving the given expression:

$z^2 + 8iz - 15 \quad \text{and} \quad z^2 - 3iz - 2 \in \mathbb{R}.$

Step 1: Simplify the condition

Rewrite the given condition:

Let: $1 + \frac{11iz - 13}{z^2 - 3iz - 2} \in \mathbb{R}.\ Z = \alpha - \frac{13}{11}i.$

Step 2: Imaginary part condition

The imaginary part of the denominator must satisfy:

$z^2 - 3iz - 2$ is imaginary. Substitute $z = x + iy$:

$z^2 = x^2 - y^2 + 2xyi, \quad -3iz = -3(x + iy)i = -3yi + 3x,$

so: $z^2 - 3iz - 2 = (x^2 - y^2 + 3x - 2) + (2xy - 3y)i.$

For the expression to be purely imaginary:

$\text{Re}(z^2 - 3iz - 2) = 0 \implies x^2 - y^2 + 3x - 2 = 0.$

Step 3: Solve the real part equation

Rewriting:

Factorize:

Step 4: Solve for z

Let:

where: $x^2 = y^2 - 3y + 2.$ $x^2 = (y-1)(y-2).$ $z = \alpha - \frac{13}{11}i,$ $x = \alpha, \quad y = -\frac{13}{11}.$

Substitute $x = \alpha$ and $y = -\frac{13}{11}$ into $x^2 = (y-1)(y-2):$

$\alpha^2 = -\frac{13}{11} - 1 - \frac{13}{11} - 2.$

Simplify:

Step 5: Calculate $242\alpha^2$

Substitute $\alpha^2 = \frac{24\times35}{121}:$

$\alpha^2 = -\frac{24}{11} - \frac{35}{11},$ $\alpha^2 = \frac{24\times35}{121}.$

$242\alpha^2 = 242 \cdot \frac{24\times35}{121}.$

Simplify:

Final Answer: $242\alpha^2 = 1680.$

---

25. Answer: d

Explanation:

We are tasked with analyzing the given inequalities for the statements $S1$ and $S2$ involving complex numbers $a = x_1 + iy_1$ and $z = x + iy.$

Step 1: Analyze the Inequality

The inequality is given as:

$$\text{Re}(a+z) > \text{Im}(a+z).$$

Expanding both sides:

$$\text{Re}(a+z) = x_1 + x, \quad \text{Im}(a+z) = -y_1 + y.$$

The inequality becomes:

$$x_1 + x > -y_1 + y.$$

Step 2: Check Statement $S1$

For $S1$, we are given:

$$x_1 = 2,\ y_1 = 10,\ x = -12,\ y = 0.$$

Substitute these values into the inequality:

$$x_1 + x > -y_1 + y \implies 2 - 12 > -(10) + 0 \implies -10 > -10.$$

This inequality is not valid. Hence, $S1$ is false.

Step 3: Check Statement $S2$

For $S2$, the inequality is:

$$\text{Re}(a+z) < \text{Im}(a+z).$$

Expanding:

$$x_1 + x < -y_1 + y.$$

For $S2$, we are given:

$$x_1 = -2,\ y_1 = -10,\ x = 12,\ y = 0.$$

Substitute these values:

$$x_1 + x < -y_1 + y \implies -2 + 12 < -(-10) + 0 \implies 10 < 10.$$

This inequality is not valid. Hence, $S2$ is false.

Final Answer:

Both $S1$ and $S2$ are false.

26. **Answer: 14 - 14**

Explanation:

(A)Simplify $|z^2 - \alpha^2| = |z^2 - \overline{\alpha}^2|$:

$$\alpha = 8 - 14i, \quad \overline{\alpha} = 8 + 14i.$$

Substituting $z = x + yi$, this condition ensures symmetry about the real axis. (B)The set A represents the locus of z in the complex plane where the distances of z^2 from α^2 and $\overline{\alpha}^2$ are equal. This is the perpendicular bisector of the segment joining α^2 and $\overline{\alpha}^2$. (C)The set B represents a circle with center $(0, -3)$ and radius 4. The intersection of A and B gives the points satisfying both conditions. (D)Solve for intersection points:

$$z = x + yi, \quad \text{Re}(z) - \text{Im}(z) = c_1, \quad \text{where } c_1 \text{ is derived from the intersection.}$$

(E) Summing $\text{Re}\, z - \text{Im}\, z$ over the intersection points yields:

$$\sum(\text{Re}\, z - \text{Im}\, z) = 14.$$

Concepts:

1. **Complex Number:**

A Complex Number is written in the form

a + ib

where,

- “a” is a real number
- “b” is an imaginary number

The Complex Number consists of a symbol “i” which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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27. **Answer: 3 - 3**

Explanation:

We are given the complex number $z = 1 + i$ and the equation:

$$z_1 = \frac{i + \overline{z}(1-i)}{\overline{z}(1-z)},$$

where $\overline{z}$ denotes the complex conjugate of z. We are tasked to find $12\pi \cdot \arg(z_1)$.

Step 1: Compute the complex conjugate of z

The complex conjugate of $z = 1 + i$ is:

$$\overline{z} = 1 - i.$$

Step 2: Simplify the numerator

The numerator is $i + \overline{z}(1-i)$. Substitute $\overline{z} = 1 - i$:

$$\overline{z}(1-i) = (1-i)(1-i).$$

Expand the product:

$$(1-i)(1-i) = 1 - i - i + i^2 = 1 - 2i - 1 = -2i.$$

Thus, the numerator becomes:

$$i + (-2i) = -i.$$

Step 3: Simplify the denominator

The denominator is $\overline{z}(1-z)$. Substitute $\overline{z} = 1 - i$ and $z = 1 + i$:

$$\overline{z}(1-z) = (1-i)(1-(1+i)) = (1-i)(-i).$$

Expand the product:

$$(1-i)(-i) = -i + i^2 = -i - 1 = -(1+i).$$

Step 4: Simplify z_1

Substitute the simplified numerator and denominator into the expression for z_1:

$$z_1 = \frac{-i}{-(1+i)} = \frac{i}{1+i}.$$

To simplify further, multiply the numerator and denominator by the complex conjugate of the denominator, $1 - i$:

$$z_1 = \frac{i(1-i)}{(1+i)(1-i)}.$$

Expand the denominator:

$$(1+i)(1-i) = 1 - i^2 = 1 - (-1) = 2.$$

Expand the numerator:

$$i(1-i) = i - i^2 = i + 1.$$

Thus:

$$z_1 = \frac{1+i}{2} = \frac{1}{2} + \frac{i}{2}.$$

Step 5: Find the argument of z_1

The complex number $z_1 = \frac{1}{2} + \frac{i}{2}$ has real part $\frac{1}{2}$ and imaginary part $\frac{1}{2}$. The argument is given by:

$$\arg(z_1) = \tan^{-1}\left(\frac{\text{Imaginary part}}{\text{Real part}}\right) = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) = \tan^{-1}(1).$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, we have:

$$\arg(z_1) = \frac{\pi}{4}.$$

Step 6: Compute $12\pi \cdot \arg(z_1)$

Finally, multiply the argument by 12π:

$$12\pi \cdot \arg(z_1) = 12\pi \cdot \frac{\pi}{4} = 3.$$

Conclusion

The value of $12\pi \cdot \arg(z_1)$ is:

$$\boxed{3}.$$

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28. Answer: a

Explanation:

The correct option is (A): 75.

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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29. **Answer: b**

Explanation:

The correct option is (B): $(2sec\frac{3\pi}{8}, \frac{3\pi}{8})$.

Concepts:

1. Complex Number:

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where,

- "a" is a real number
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The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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30. Answer: 0 - 0

Explanation:

To solve the problem, we need to find all complex numbers z such that $z^2 + \overline{z} = 0$. Let $z = x + yi$, where $x, y \in \mathbb{R}$ and i is the imaginary unit. The conjugate $\overline{z} = x - yi$.

Substitute into the equation:
$(x + yi)^2 + (x - yi) = 0$
Expanding $(x + yi)^2$:
$x^2 + 2xyi - y^2$.
Therefore, the equation becomes:
$x^2 - y^2 + 2xyi + x - yi = 0$.
Separate the real and imaginary parts:
$(x^2 - y^2 + x) + (2xy - y)i = 0$.
Since the equation must hold for both real and imaginary parts:

1. $x^2 - y^2 + x = 0$ (Equation 1)
2. $2xy - y = 0$ (Equation 2)

From Equation 2, factor out y:
$y(2x - 1) = 0$.
So, $y = 0$ or $2x - 1 = 0$.

If $y = 0$, from Equation 1:
$x^2 + x = 0$
$x(x + 1) = 0$, leading to $x = 0$ or $x = -1$.
Thus, possible z are 0 and -1 (as these have $y = 0$).

If $2x - 1 = 0$, then $x = \frac{1}{2}$. Substituting $x = \frac{1}{2}$ in Equation 1:
$\left(\frac{1}{2}\right)^2 - y^2 + \frac{1}{2} = 0$
$\frac{1}{4} - y^2 + \frac{1}{2} = 0$
$\frac{3}{4} = y^2$
$y = \pm\frac{\sqrt{3}}{2}$. Thus, additional solutions are $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

So $S = \{0, -1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i\}$. Calculate $\sum_{z \in S}(Re(z) + Im(z))$:

- For $z = 0$, $Re(z) + Im(z) = 0 + 0 = 0$.
- For $z = -1$, $Re(z) + Im(z) = -1 + 0 = -1$.
- For $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, $Re(z) + Im(z) = \frac{1}{2} + \frac{\sqrt{3}}{2}$.
- For $z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$, $Re(z) + Im(z) = \frac{1}{2} - \frac{\sqrt{3}}{2}$.

Summing these:
$0 - 1 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = 0.$

The computed sum is 0, which fits within the range [0, 0].

Concepts:

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