Complex Numbers JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Complex Numbers

1. If $z = x + iy, \quad xy \neq 0$ satisfies the equation $z^2 + iz = 0$, then $|z^2|$; equal to **(+4, -1)**

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2. If z = x + iy satisfies | z | − 2 = 0 and |z − i| − | z + 5i| = 0, then **(+4, -1)**

   a. x + 2y − 4 = 0

   b. x^2 + y − 4 = 0

   c. x + 2y + 4 = 0

   d. x^2 − y + 3 = 0

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3. Let S be the set of (α,β),π<α,β<2π,for which the complex number **(+4, -1)**
   $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real,
   Let $Z\alpha\beta = \sin 2\alpha + i\cos 2\beta, (\alpha,\beta) \in S$. Then
   $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}}\right)$
   is equal to

   a. 3

   b. 3i

   c. 1

   d. 2-i

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4. Sum of squares of modulus of all the complex numbers z satisfying **(+4, -1)**
   $\overline{z} = iz^2 + z^2 - z$
   is equal to ________.

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5. The number of points of intersection of $|z-(4+3i)| = 2$ and $|z| + |z-4| = 6$, **(+4, -1)**
   $z \in \mathbb{C}$, is

   a. 0

b. 1

c. 2

d. 3

---

6. Let **(+4, -1)**

$A = \{z \in \mathbb{C} : |\frac{z+1}{z-1}| < 1\}$

and

$B = \{z \in \mathbb{C} : \arg(\frac{z-1}{z+1}) = \frac{2\pi}{3}\}$

Then $A \cap B$ is :

a. A portion of a circle centred at $(0, \ -\frac{1}{\sqrt{3}})$ that lies in the second and third quadrants only

b. A portion of a circle centred at $(0, \ -\frac{1}{\sqrt{3}})$ that lies in the second only

c. An empty set

d. A portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

---

7. Let arg(z) represent the principal argument of the complex number *z*. Then, |*z*| = 3 and arg(*z* – 1) – arg(*z* + 1) = π/4 intersect **(+4, -1)**

a. exactly at one point

b. exactly at two points

c. nowhere

d. at infinitely many points

---

8. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to **(+4, -1)**

a. 1

b. α

c. 1+α

d. 1+2α

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9. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point $z_2, Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true? **(+4, -1)**

a. $arg\ z_2 = \pi – tan^{-1} 3$

b. $(arg)(z_1 - 2z_2) = -tan^{-1}\frac{4}{3}$

c. $|z_2| = \sqrt{10}$

d. $|2z_1 - z_2| = 5$

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10. The number of elements in the set $\{\ z = a + ib\ \in C : a, b\ \in Z$ and $1 < |z - 3 + 2i| < 4\ \}$ is _______ . **(+4, -1)**

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11. Let $S = z \in C : |z - 3| <= 1$ and $z(4 + 3i) + z(4 - 3) \leq 24.$ **(+4, -1)**
If α + iβ is the point in S which is closest to 4i, then 25(α + β) is equal to ______.

---

12. If $z^2 + z + 1 = 0$, **(+4, -1)**
$z\ \in C$, then $\left|\sum_{n=1}^{15}\left(z_n + (-1)^n \frac{1}{z_n}\right)^2\right|$
is equal to ________.

---

13. Let z_1 and z_2 be two complex numbers such that **(+4, -1)**

$z_1 = iz_2\ and\ arg(\frac{z_1}{z_2}) = \pi.$

a. $arg\ z_2 = (\frac{\pi}{4})$

b. $arg\ z_2 = -(\frac{3\pi}{4})$

c. $arg\, z_1 = (\frac{\pi}{4})$

d. $arg\, z_1 = -(\frac{3\pi}{4})$

---

14. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point $z_2, \mathrm{Re}\,(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ? **(+4, -1)**

a. $\arg z_2 = \pi - \tan^{-1} 3$

b. $\arg\,(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

c. $|z_2| = \sqrt{10}$

d. $|2z_1 - z_2| = 5$

---

15. If and are different complex number with | | = 1, then find | $\frac{-}{1 - \ -}$ | **(+4, -1)**

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16. For all real values of , the minimum value of $\frac{1 - \ + \ ^2}{1 + \ + \ ^2}$ is: **(+4, -1)**

a. (A) 0

b. (B) 1

c. (C) $\frac{2}{3}$

d. (D) $\frac{1}{3}$

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17. The Integrating Factor of the differential equation — – = 2 ^{2} is: **(+4, -1)**

a. (A) $^-$

b. (B) $^-$

c. (C) ^{1}

d. (D)

---

18. If $= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $(\ ) = {}^{2} - 2 \ - 3$ then find the value of $(\ )$. **(+4, -1)**

a. (A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b. (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c. (C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

d. (D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

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19. If , , , then the value of determinant $\begin{vmatrix} (5 \ + 5^{-} \ )^2 & (5 \ - 5^{-} \ )^2 & 1 \\ (6 \ + 6^{-} \ )^2 & (6 \ - 6^{-} \ )^2 & 1 \\ (7 \ + 7^{-} \ )^2 & (7 \ - 7^{-} \ )^2 & 1 \end{vmatrix}$ is: **(+4, -1)**

a. (A) 10

b. (B) 12

c. (C) 1

d. (D) 0

---

20. The sine of the angle between vectors $= 3\hat{\ } - 2\hat{\ } + \hat{\ }$ and $= 4\hat{\ } + 3\hat{\ } - 6\hat{\ }$ is: **(+4, -1)**

a. (A) 0

b. (B) 1

c. (C) −1

d. (D) 2

---

21. Find the direction cosines of the sides of the triangle whose vertices are (3,5,-4),(-1,1,2) and (-5,-5,-2). **(+4, -1)**

a. (A) $\frac{6}{2\sqrt{42}}, \frac{-4}{2\sqrt{42}}, \frac{5}{2\sqrt{42}}$

b. (B) $\frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}$

c. (C) $\frac{-6}{2\sqrt{42}}, \frac{-3}{2\sqrt{42}}, \frac{5}{2\sqrt{42}}$

d. (D) $\frac{8}{2\sqrt{42}}, \frac{5}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}$

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22. The adjacent side of parallelogram is represented by vectors $2\hat{} + 3\hat{}$ and $\hat{} + 4\hat{}$. The area of the parallelogram is **(+4, -1)**

a. (A) 5 units

b. (B) 3 units

c. (C) 8 units

d. (D) 11 units

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23. A force $= 3\hat{} + 2\hat{} - 4\hat{}$ is applied at the point (1,−1,2) . What is the moment of the force about the point (2,−1,3) ? **(+4, -1)**

a. (A) $\hat{} + 4\hat{} + 4\hat{}$

b. (B) $2\hat{} + \hat{} + 2\hat{}$

c. (C) $2\hat{} - 7\hat{} - 2\hat{}$

d. (D) $2\hat{} + 4\hat{} - \hat{}$

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24. The number of terms of an is even. The sum of the odd terms is 24 and of the even terms is 30. The last term exceeds the first by $10\frac{1}{2}$. Then the number of terms in the series is ______. **(+4, -1)**

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25. If (1 −) is a root of quadratic equation ^{2} + + (1 −) = 0, then its roots are **(+4, -1)**

a. (A) 0, 1

b. (B) −1, 1
Wrong AnswerYour Answer

c. (C) 0, − 1
Correct Answer

d. (D) −1, 2

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26. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of the other two observations is: (+4, -1)

a. (A) 6: 7

b. (B) 10: 3

c. (C) 4: 9
Correct Answer

d. (D) 5: 8

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27. If $a \neq \pm b$ and are purely real, z ϵ complex number, $Re(az^2 + bz) = a$ and $Re(bz^2 + az) = b$ then number of value of z possible is (+4, -1)

a. 0

b. 1

c. 2

d. 3

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28. **The complex number $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to :** (+4, -1)

a. $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

b. $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

c. $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

d. $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

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29. Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x+y+2z-7=0=2x+3y+z-2$ be $ax+by+cz=65$ Then the distance of the point (a,b,c) from the plane $2x+2y-z+16=0$ is _______ **(+4, -1)**

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30. Let $z=1+i$ and $z_1=\frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$ Then $\frac{12}{\pi}\arg(z_1)$ is equal to________ **(+4, -1)**

Answers

1. **Answer: 1 - 1**

Explanation:

The Correct Answer is :1

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- “a” is a real number
- “b” is an imaginary number

The Complex Number consists of a symbol “i” which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

---

2. **Answer: c**

Explanation:

The correct answer is (C) : $x + 2y + 4 = 0$
$|z-i| = |z+5i|$
Thereafter, z lies on ⊥r bisector of (0, 1) and (0, −5)
i.e., line $y = -2$
as $|z| = 2$

$\Rightarrow$ z = −2i
x = 0 and y = −2Hence, x + 2y + 4 = 0

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---

3. Answer: c

Explanation:

The correct answer is (C) : 1
$\because \frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary
$\therefore \frac{1-i\sin\alpha}{1+2i\sin\alpha}+\frac{1+\sin\alpha}{1-2i\sin\alpha}=0$
$\Rightarrow 1-2\sin^2\alpha=0$
$\therefore\ \alpha=\frac{5\pi}{4},\frac{7\pi}{4}$
and
$\frac{1+i\cos\beta}{1-2i\cos\beta}$
is purely real.
$\frac{1-i\cos\beta}{1-2i\cos\beta}-\frac{1-i\cos\beta}{1+2i\cos\beta}=0$
$\Rightarrow cos\beta=0$
$\therefore\ \beta=\frac{3\pi}{2}$

$\therefore S = \{(\frac{5\pi}{4}, \frac{3\pi}{2}), (\frac{7\pi}{4}, \frac{3\pi}{2})\}$
$Z_{\alpha\beta}$ = 1-i and $Z_{\alpha\beta}$ = -1-i
$\therefore \sum_{(\alpha,\beta)\in S}(iZ_{\alpha\beta} + \frac{1}{i\overline{Z}_{\alpha\beta}})$
$= i(-2i) + \frac{1}{i}\left[\frac{1}{1+i} + \frac{1}{-1+i}\right]$
$= 2 + \frac{1}{i}\frac{2i}{-2}$
= 1

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4. Answer: 2 - 2

Explanation:

The correct answer is 2
Let z = x + iy
So 2x = (1 + i)($x^2 - y^2$ + 2xyi)
⇒ 2x = $x^2 - y^2$ – 2xy ...(i) and $x^2 - y^2$ + 2xy = 0 ...(ii)
From (i) and (ii) we get
x = 0 or y$= -\frac{1}{2}$
When x = 0 we get y = 0

When y$= -\frac{1}{2}$
we get $x^2 - x - \frac{1}{4} = 0$
$\Rightarrow x = \frac{-1\pm\sqrt{2}}{2}$
So there will be total 3 possible values of z, which are
$0, (\frac{-1+\sqrt{2}}{2}) - \frac{1}{2}i$ and $(\frac{-1-\sqrt{2}}{2}) - \frac{1}{2}i$
Sum of squares of modulus
$= 0 + (\frac{\sqrt{2}-1}{2})^2 + \frac{1}{4} + (\frac{\sqrt{2}+1}{2})^2 = +\frac{1}{4}$
= 2

Concepts:

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5. Answer: c

Explanation:

The correct answer is (C) : 2
$C_1 : |z - (4 + 3i)| = 2$ and $C_2 : |z| + |z - 4| = 6, z \in C$
C_1: represents a circle with centre (4, 3) and radius 2 and C_2 represents a ellipse with focii at (0, 0) and (4, 0) and length of major axis = 6, and length of semi-major axis $2\sqrt{5}$ and (4, 2) lies inside the both C_1 and C_2 and (4, 3) lies outside the C_2

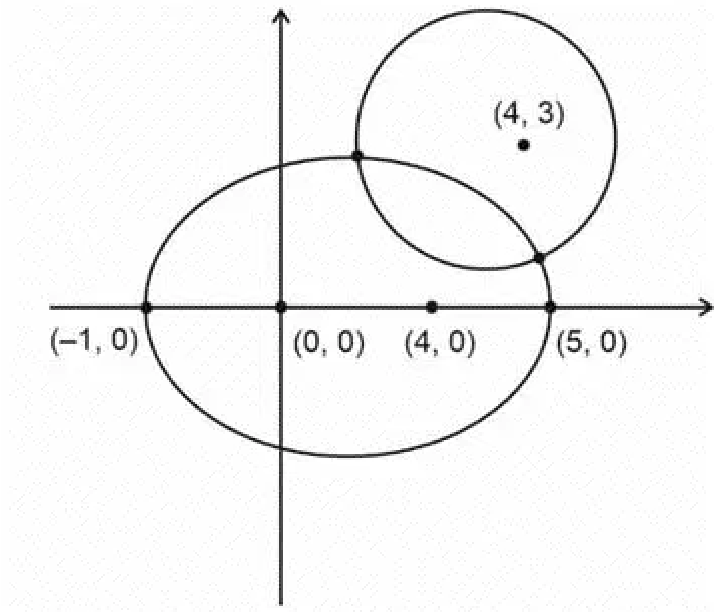

Fig.

Therefore,
no. of intersection points = 2

Concepts:

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6. Answer: b

Explanation:

The correct answer is (B) : A portion of a circle centred at $(0, \ -\frac{1}{\sqrt{3}})$ that lies in the second only

$|\frac{z+1}{z-1}| < 1 \Rightarrow |z+1| < |z-1| \Rightarrow Re(z) < 0$

and $arg(\frac{z-1}{z+1}) = \frac{2\pi}{3}$

is a part of circle as shown

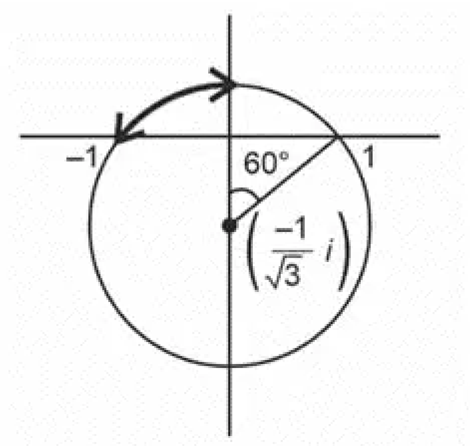

Fig.

Concepts:

1. Complex Number:

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7. **Answer: a**

Explanation:

The correct answer is (C) : nowhere

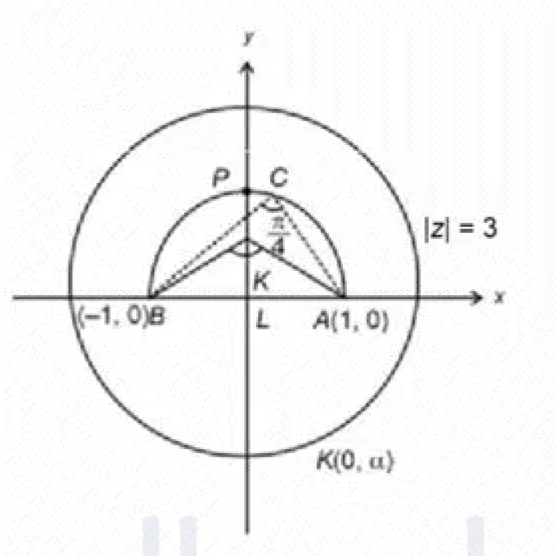

Fig.

|z| = 3
arg(z-1) - arg(z+1) = π/4
$\angle AKL = \angle ACB = \frac{\pi}{4}$
⇒ LK = AL = α = 1
K(0,1)
Radius = $\sqrt{2}$
PL = PK + KL = $\sqrt{2}+1$
P(0,1 + $\sqrt{2}$)
Therefore , Number of intersection = 0

Concepts:

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8. Answer: a

Explanation:

The correct answer is (A) : 1

$1 + x^2 + x^4 = 0$

$\omega^{1011} + \omega^{2022} - \omega^{3033}$

$= (\omega^3)^{337} + (\omega^3)^{674} - (\omega^3)^{1011}$

$= 1+1-1$

$= 1$

Concepts:

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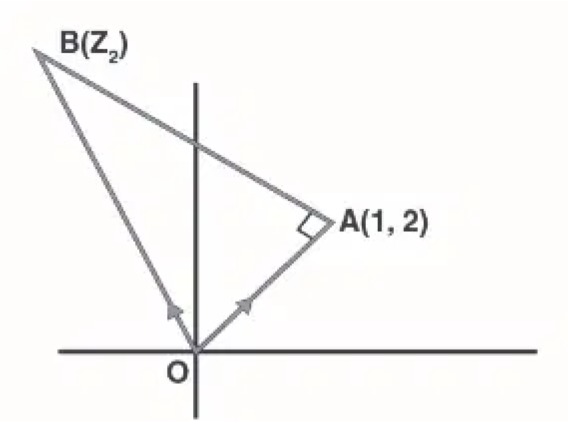
9. **Answer: d**

Explanation:



$\frac{z2-0}{(1+2i)-0} = \frac{|OB|}{|OA|} e^{\frac{i\pi}{4}}$

$\Rightarrow \frac{z2}{1+2i} = \sqrt{2} e^{i\pi} 4$

$z_2 = (1+2i)(1+i)$

$z_2 = -1+3i$

$arg\ z_2 = \pi – tan^{-1} 3$

$|z_2| = \sqrt{10}$

$z_1 – 2z_2 = (1+2i) + 2–6i$

$z_1 – 2z_2 = 3–4i$

$arg\ (z_1 - 2z_2) = -tan^{-1} \frac{4}{3}$

$|2z_1 - z_2| = |2+4i+1-3i|$

$|2z_1 - z_2| = |3+i|$

$= \sqrt{10}$

So, the correct option is (D): $|2z_1 - z_2| = 5$

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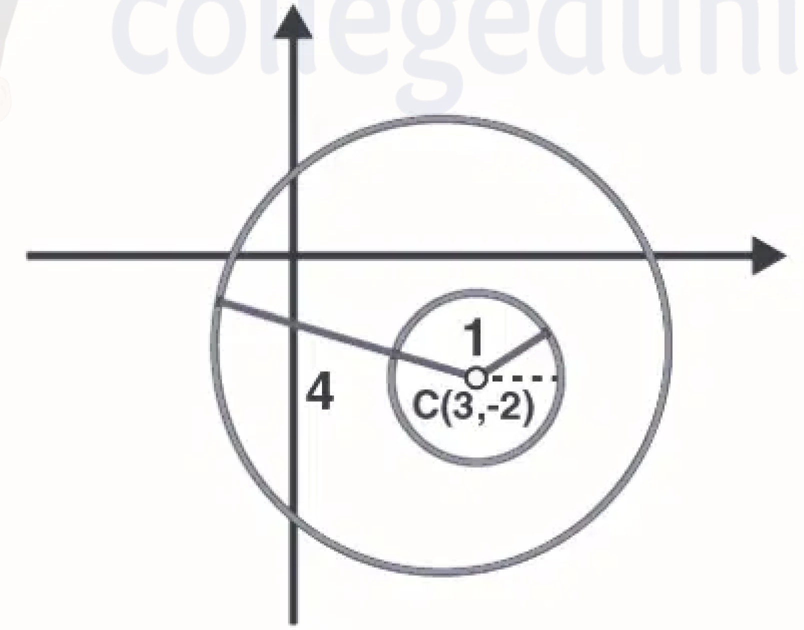
10. **Answer: 40 - 40**

Explanation:



At line $y = -2$, we have $(5, -2)$ $(6, -2)$ $(1, -2)$ $(0, -2)$
$\Rightarrow$ 4 points
At line $y = -1$, we have $(4, -1)$ $(5, -1)$ $(6, -1)$ $(2, -1)$ $(1, -1)$ $(0, -1)$
$\Rightarrow$ 6 points
At line $y = 0$, we have $(0, 0)$ $(1, 0)$ $(2, 0)$ $(3, 0)$ $(4, 0)$ $(5, 0)$ $(6, 0)$
$\Rightarrow$ 7 points
At line $y = 1$, we have $(1, 1)$, $(2, 1)$, $(3, 1)$, $(4, 1)$, $(5, 1)$ i.e. 5 points

Similarly,
At line $y = -5$, we have 5 points
At line $y = -4$, we have 7 points
At line $y = -3$, we have 6 points
Then, the total integral points = 2(5 + 7 + 6) + 4 = 40

So, the answer is 40.

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11. Answer: 80 - 80

Explanation:

The correct answer is 80
Here |z – 3| < 1
$\Rightarrow (x–3)^2 + y^2 < 1$
And
$z = (4+3i) + \overline{z}(4-3i) \le 24$
$\Rightarrow 4x - 3y \le 12$

$tan\theta = \frac{4}{3}$

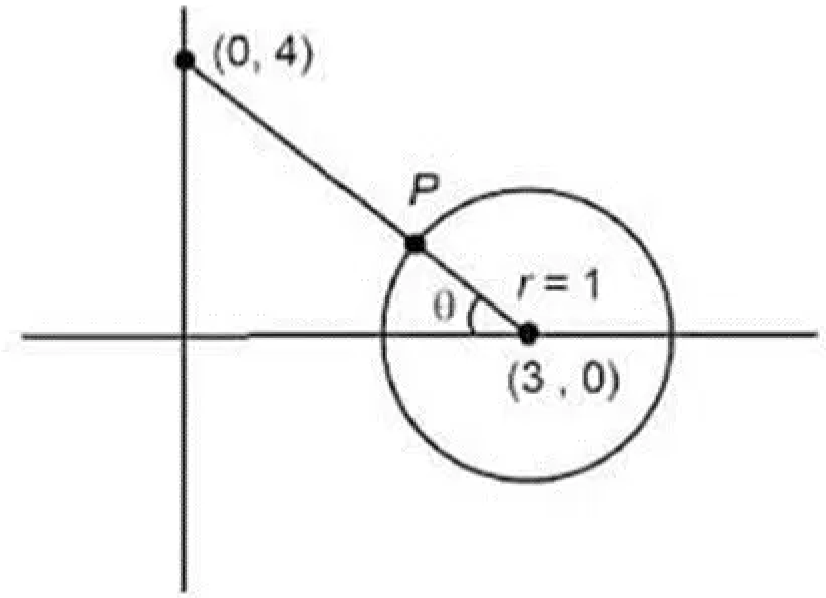

Figure.

∴ Coordinate of P = (3 – cosθ, sinθ)

$= (3 - \frac{3}{5}, \frac{4}{5})$

$\therefore\ \alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$

$\therefore 25(\alpha + \beta) = 80$

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12. **Answer: 2 - 2**

Explanation:

The correct answer is 2

$\because z^2 + z + 1 = 0$

$\Rightarrow \omega$ or ω^2

$\because \left|\sum_{n=1}^{15}\left(z_n + (-1)^n \frac{1}{z_n}\right)^2\right|$

$= \left|\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} z^{-2n} + 2.\sum_{n=1}^{15}(-1)^n\right|$

$= |0 + 0 - 2|$

$= 2$

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

---

13. **Answer: a**

Explanation:

The correct option is: (C).

$\because \frac{z_1}{z_2} = i \Rightarrow z_1 = -iz_2$

$\Rightarrow arg(z_1) = -\frac{\pi}{2} + arg(z_2).....(i)$

Also

arg(z1)-arg(z2)= π

$\Rightarrow arg(z_1) + arg(z_2) = \pi$ ...(ii)

From (i) and (ii), we get

$arg(z_1) = \frac{\pi}{4} and\, arg(z_2) = \frac{3\pi}{4}$

Concepts:

1. Complex Number:

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14. Answer: d

Explanation:

The correct option is (D): $|2z_1 - z_2| = 5$

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- “a” is a real number
- “b” is an imaginary number

The Complex Number consists of a symbol “i” which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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15. Answer: 1 - 1

Explanation:

Explanation:

We know that | $\frac{-}{1-}$ |^2 = ($\frac{-}{1-}$)($\frac{\overline{-}}{1-}$) = ($\frac{-}{1-}$)($\frac{- - -}{1-}$) = $\frac{- - - - - + -}{1- \quad - - \quad + \quad -}$

= $\frac{| - |^2 - - \quad - \quad - + | \quad |^2}{1- \quad - - \quad + | \quad |^2 \quad | \quad |^2}$ [$-$ = | $\quad$ |^2] = $\frac{1- - \quad - \quad - + | \quad |^2}{1- \quad - - \quad + | \quad |^2}$ [| $\quad$ | = 1] = 1 | $\frac{-}{1-}$ |^2 = 1

| $\frac{-}{1-}$ | = 1Hence, the correct answer is 1.

---

16. Answer: d

Explanation:

Explanation:

Let ( ) = $\frac{1- \quad + \quad ^2}{1+ \quad + \quad ^2}$ $\quad ^{\prime}$( ) = $\frac{(1+ \quad + \quad ^2)(-1+2 \quad)-(1- \quad + \quad ^2)(1+2 \quad)}{(1+ \quad + \quad ^2)^2}$

$= \frac{-1+2\ \ -\ \ +2\ ^2 -\ ^2 +2\ ^3 -1-2\ \ +\ \ +2\ ^2 -\ ^2 -2\ ^3}{(1+\ \ +\ ^2)^2} = \frac{2\ ^2 -2}{(1+\ \ +\ ^2)^2} = \frac{2(\ ^2 -1)}{(1+\ \ +\ ^2)^2}$ $'(\ ) = 0$ $^2 = 1$ ± 1 Now, $''(\ ) = \frac{2[(1+\ \ +\ ^2)^2(2\ ) - (\ ^2 - 1)(2)(1+\ \ +\ ^2)(1+2\ )]}{(1+\ \ +\ ^2)^4} = \frac{4[\ \ +\ ^2 +\ ^3 -\ ^2 -2\ ^3 +1+2\ ]}{(1+\ \ +\ ^2)^3} = \frac{4(1+3\ \ -\ ^3)}{(1+\ \ +\ ^2)^3}$ And , $''(1) = \frac{4(1+3-1)}{(1+1+1)^3} = \frac{4(3)}{(3)^3} > 0$ Also $''(-1) = \frac{4(1-3+1)}{(1-1+1)^3} = 4(-1) = -4 < 0$ By second derivative test, is the minimum at $= 1$ and the minimum value is given by $(1) = \frac{1-1+1}{1+1+1} = \frac{1}{3}$ .Hence, the correct option is (D).

---

17. Answer: c

Explanation:

Explanation:

The given differential equation is: $—-\ \ = 2\ ^2$ $—--= 2$ This is a linear differential equation of the form: $—+\ \ =$ (where $= -\frac{1}{}$ and $= 2$)Now, $.\ \ . =\ ^{\int}$ $=\ ^{\int(-\frac{1}{})} =\ ^{-\log} =\ ^{\log(\ ^{-1})} =\ ^{-1} = \frac{1}{}$Hence, the correct option is (C).

---

18. Answer: a

Explanation:

Explanation:

Given, $= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, and $(\ ) =\ ^2 - 2\ \ - 3$ The matrix A is also satisfying the above polynomial. $(\ ) =\ ^2 - 2\ \ - 3$ $(\ ) =\ \ - 2\ \ - 3$

$(\ ) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\ ) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$(\ ) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ $(\ ) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$Hence, the correct option is (A).

---

19. Answer: d

Explanation:

Explanation:

Given determinant is $\begin{vmatrix} (5 + 5^{-})^2 & (5 - 5^{-})^2 & 1 \\ (6 + 6^{-})^2 & (6 - 6^{-})^2 & 1 \\ (7 + 7^{-})^2 & (7 - 7^{-})^2 & 1 \end{vmatrix}$

$= \begin{vmatrix} (5^2 + 2 + 5^{-2}) & (5^2 - 2 + 5^{-2}) & 1 \\ (6^2 + 2 + 6^{-2}) & (6^2 - 2 + 6^{-2}) & 1 \\ (7^2 + 2 + 7^{-2}) & (7^2 - 2 + 7^{-2}) & 1 \end{vmatrix}$ Applying $_2 \quad _2 - \ _1$, we get

$= \begin{vmatrix} (5^2 + 2 + 5^{-2}) & -4 & 1 \\ (6^2 + 2 + 6^{-2}) & -4 & 1 \\ (7^2 + 2 + 7^{-2}) & -4 & 1 \end{vmatrix} = (-4) \begin{vmatrix} (5^2 + 2 + 5^{-2}) & 1 & 1 \\ (6^2 + 2 + 6^{-2}) & 1 & 1 \\ (7^2 + 2 + 7^{-2}) & 1 & 1 \end{vmatrix}$ In above determinant $_2$ and $_3$ are identical.We know that, if two coloums are identical in determinant, the value of the determinant is zero.So, If , , , then the value of determinant

$\begin{vmatrix} (5 + 5^{-})^2 & (5 - 5^{-})^2 & 1 \\ (6 + 6^{-})^2 & (6 - 6^{-})^2 & 1 \\ (7 + 7^{-})^2 & (7 - 7^{-})^2 & 1 \end{vmatrix}$ is 0.Hence, the correct option is (D).

---

20. Answer: b

Explanation:

Explanation:

$= 3\hat{} - 2\hat{} + \hat{}$ and $= 4\hat{} + 3\hat{} - 6\hat{}$. $= (3\hat{} - 2\hat{} + \hat{})\ (4\hat{} + 3\hat{} - 6\hat{})\ \ 12 - 6 - 6\ \ 0$
$\cos\ = \frac{}{|\ |\times|\ |}\cos\ = \frac{0}{|\ |\times|\ |}\cos\ = 0\ \ = 90$ Nowsin $= \sin 90\ = 1$Hence, the correct option is (B).

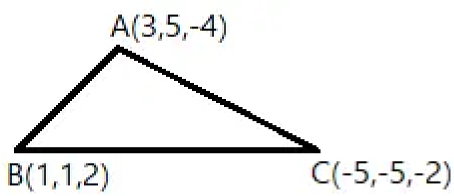
---

21. Answer: b

Explanation:

Explanation:

$(3, 5, -4),\ (-1, 1, 2),$ and $(-5, -5, -2)$



The direction ratios of the side are (-1-3),(1-5), and [2-(-4)] i.e., -4,-4 and 6.Then, $\sqrt{(-4)^2+(-4)^2+(6)^2}=\sqrt{16+16+36}=\sqrt{68}=2\sqrt{17}$Therefore, the direction cosines of are $=\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$, $m=\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$, $=\frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$i.e., $\frac{-4}{2\sqrt{17}},-\frac{4}{2\sqrt{17}},\frac{6}{2\sqrt{17}}$ $\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{3}{\sqrt{17}}$The direction ratios of are (-5-(-1)),(-5-1) and (-2-2) i.e., -4,-6 and -4 . Therefore, the direction cosines of are $\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$i.e., $\frac{-4}{2\sqrt{17}},\frac{-6}{2\sqrt{17}},\frac{-4}{2\sqrt{17}}$The direction ratios of are (-5-3),(-5-5), and [-2-(-4)] i.e., -8,-10 and 2 . Hence, the direction cosines of are$\frac{-8}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{-5}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{2}{\sqrt{(-8)^2+(10)^2+(2)^2}}$i.e., $\frac{-8}{2\sqrt{42}},\frac{-10}{2\sqrt{42}},\frac{2}{2\sqrt{42}}$Hence, the correct option is (B).

---

22. **Answer: a**

Explanation:

Explanation:

Given that,The one side of the vector equation is, $=2\hat{}+3\hat{}+0\hat{}$The other side of the vector equation is, $=\hat{}+4\hat{}+0\hat{}$Now,Area is given by $\times$ .Now performing the matrix multiplication, then $=\hat{}[(0\times3)-(4\times0)]-\hat{}[(0\times2)-(1\times0)]+\hat{}[(2\times4)-(1\times3)]$ $=\hat{}[0-0]-\hat{}[0-0]+\hat{}[8-3]$ $=0\hat{}-0\hat{}+5\hat{}$Taking modules, $=\sqrt{0^2+0^2+5^2}$ $=\sqrt{0+0+25}$ $=\sqrt{25}$ $=5$ units.Hence, the correct option is (A).

---

23. **Answer: c**

Explanation:

Explanation:

Let point is (1,–1,2) and point is (2,–1,3) Position vector of w.r.t is $=(1-2)\hat{}+(-1+1)\hat{}+(2-3)\hat{}$ $=-\hat{}+0\hat{}-\hat{}$ and $=3\hat{}+2\hat{}-4\hat{}$ Moment = $\times$

$= \begin{vmatrix} \hat{} & \hat{} & \hat{} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix} = \hat{}(0+2) - \hat{}(4+3) + \hat{}(-2+0) = 2\hat{} - 7\hat{} - 2\hat{}$ Hence, the correct option is (C).

---

24. **Answer: 8 - 8**

Explanation:

Explanation:
Let the total number of terms be 2 , and the first term is and common difference be . $+ (2 \quad - 1) \quad = 1$ ....(1)1 − = 10.5If we take odd terms only it will also be an A.P. of terms with a common difference of 2 .24 = $\frac{}{2}$\{2 + (− 1)2 \}24 = (+ (− 1))..... (2)30 = $\frac{}{2}$\{2(+) + (− 1)2 \}30 = (+).....(3)Put the value of equation (3) in equation (2)24 = 30 − = 6, [Putting in equation (1)]12 − = 10.5 = 1.5 So, = 4 = 1.5So no. of terms 8 .Series 1.5, 3, 4.5, 6, 7.5, 9, 10.5, 12Hence, the correct answer is 8.

---

25. **Answer: c**

Explanation:

Explanation:
Substituting = 1 − , we get$(1 - \;)^2 +$ (1 −) + (1 −) = 0(1 −)(1 − + + 1) = 0So, = 1 Thus, = 0 is the root of the equation.Now, the equation changes to $^2 +$ = 0 (+ 1) = 0 = 0 and = − 1Hence, the correct option is (C).

---

26. **Answer: c**

Explanation:

Explanation:
Let two observations are $_1$ and $_2$ mean = $\frac{\sum}{5}$ = 5 $1 + 3 + 8 + \;_1 + \;_2 = 25$ $_1 + \;_2 = 13$....(i)Variance (^{2}) = $\frac{\sum^2}{5}$ − 25 = 9.20 $\sum \;^2 = 171$ $^2_1 + \;^2_2 = 97$.....(ii)by (1) and (2)$(\;_1 + \;_2)^2 - 2\;_1\;_2 = 97$or $_1\;_2 = 36$ $_1 : \;_2 = 4 : 9$Hence, the correct option is (C).

27. **Answer: a**

Explanation:

$$a(x^2 - y^2) + bx = a \ldots..(i)$$
$$b(x^2 - y^2) + ax = b \ldots..(ii)$$
$$(i) - (ii)$$
$$(a - b)(x^2 - y^2) + (b - a)x = a - b \qquad (a \neq b)$$
$$\Rightarrow x^2 - y^2 - x = 1$$
$$(i) + (ii)$$
$$(a + b)(x^2 - y^2) + x(a + b) = a + b \qquad (a \neq -b)$$
$$\Rightarrow x^2 - y^2 + x = 1$$
$$\Rightarrow x = 0$$
$$\Rightarrow y^2 = -1$$

therefore, no complex number is possible.
The correct option is (A): 0

Concepts:

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a + ib

where,

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28. Answer: d

Explanation:

$Z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}} = \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$= \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i} \times \frac{\frac{1}{2}-\sqrt{\frac{3}{2}i}}{\frac{1}{2}-\sqrt{3/2i}} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$

Apply polar form,

$r\cos\theta = \frac{\sqrt{3}-1}{2}$

$r\sin\theta = \frac{\sqrt{3}+1}{2}$

Now, $\tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

So, $\theta = \frac{5\pi}{12}$

So, the correct option is (D) : $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

Concepts:

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29. Answer: 9 - 9

Explanation:

The correct answer is 9.

Equation of plane is

$(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$

$\therefore$ plane is $13x + 10y + 35z = 65$

Distance from given point to plane $= 9$

Concepts:

1. Complex Number:

A Complex Number is written in the form

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---

30. Answer: 9 - 9

Explanation:

The correct answer is 9.

$z - 1 + i$

$z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$

$z_1 = \frac{1+i(1 \quad i)}{(1-i)(1-1-i)+\frac{1}{1+i}}$

$= \frac{1+i-i^2}{(1-i)(-i)+\frac{1-i}{2}}$
$= \frac{2+i}{-3i-1} = \frac{4+2i}{-3i-1}$
$= \frac{-(4+2i)(3i-1)}{(3i)^2-(1)^2}$
$\text{Arg}(z_1) = \frac{3\pi}{4}$
$\therefore \frac{12}{\pi}\arg(z_1) = \frac{3\pi}{4}$

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

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