Conic Sections JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Conic Sections

1. If $y=m_1x+c_1$ and $y=m_2x+c_2$, $m_1 \neq m_2$ are two common tangents of circle $x_2+y_2=2$ and parabola $y^2=x$, then the value of $8|m_1m_2|$ is equal to : **(+4, -1)**

   a. $3+4\sqrt{2}$

   b. $-5+6\sqrt{2}$

   c. $-4+3\sqrt{2}$

   d. $7+6\sqrt{2}$

---

2. Let P be the plane containing the straight line $\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$ If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to : **(+4, -1)**

   a. $\frac{147}{2}$

   b. 96

   c. $\frac{32}{3}$

   d. 54

---

3. A line, with the slope greater than one, passes through the point A (4,3) and intersects the line x-y-2=0 at the point B If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line : **(+4, -1)**

   a. 2 x+y=9

   b. 3 x-2 y=7

   c. x+2 y=6

   d. 2 x -3 y =3

---

4. The set of all values of a^2 for which the line $x+y=0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2},\frac{1-a}{2}\right)$ on the circle $2x^2+2y^2-(1+a)x-(1-$ **(+4, -1)**

$a)y = 0$, is equal to :

a. $(8, \infty)$

b. $(0, 4]$

c. $(4, \infty)$

d. $(2, 12]$

---

5. **A parabola with focus (3, 0) and directrix x = −3. Points P and Q lie on the parabola and their ordinates are in the ratio 3 : 1. The point of intersection of tangents drawn at points P and Q lies on the parabola** **(+4, -1)**

a. $y^2 = 16x$

b. $y^2 = 4x$

c. $y^2 = 8x$

d. $x^2 = 4y$

---

6. Let the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{\alpha} = \frac{1}{25}$ coincide Then the length of the latus rectum of the hyperbola is:- **(+4, -1)**

a. $\frac{32}{9}$

b. $\frac{18}{5}$

c. $\frac{27}{4}$

d. $\frac{27}{10}$

---

7. The equations of two sides of a variable triangle are $x - 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is: **(+4, -1)**

a. $4y^2 - 18y + 3x + 18 = 0$

b. $4y^2 - 18y - 3x + 18 = 0$

c. $4y^2 + 18y + 3x + 18 = 0$

d. $4y^2 - 18y - 3x - 18 = 0$

---

8. Two circles with equal radii are intersecting at the points $(0, 1)$ and $(0, -1)$. The tangent at the point $(0, 1)$ to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is : **(+4, -1)**

a. 1

b. $\sqrt{2}$

c. $2\sqrt{2}$

d. 2

---

9. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is : **(+4, -1)**

a. $\frac{1}{8}$

b. $\frac{2}{3}$

c. $\frac{1}{2}$

d. $\frac{3}{2}$

---

10. Statement 1 : The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5\,is\,x^2 + y^2 - 6x + 2y = 0$. Statement 2 : $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$. **(+4, -1)**

a. Statement 1 is false; Statement 2 is true

b. Statement 1 is true; Statement 2 is true, Statement 2 is a correct explanation for Statement 1

c. Statement 1 is true; Statement 2 is false

d. Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1

---

11. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is : **(+4, -1)**

a. 2

b. 4

c. 6

d. 8

---

12. Two tangents are drawn from a point $(-2, -1)$ to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan\alpha|$ is equal to : **(+4, -1)**

a. $\frac{1}{3}$

b. $\frac{1}{\sqrt{3}}$

c. $\sqrt{3}$

d. 3

---

13. Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in R$, is a **(+4, -1)**

a. Straight line parallel to x-axis

b. Straight line parallel to y-axis

c. Circle of radius $\sqrt{2}$

d. Circle of radius $\sqrt{3}$

---

14. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is: **(+4, -1)**

a. 8

b. $4\sqrt{5}$

c. 12

d. $8\sqrt{5}$

---

15. Let the equations of two ellipses be $E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1 and E_2 : \frac{x^2}{16} + \frac{y^2}{b^2} = 1$, If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is: **(+4, -1)**

a. 8

b. 9

c. 4

d. 2

---

16. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If 'S'BS is a right angled triangle with right angle at B and area (Δ S'BS) = 8 s units, then the length of a latus rectum of the ellipse is : **(+4, -1)**

a. $2\sqrt{2}$

b. 2

c. 4

d. $4\sqrt{2}$

---

17. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divides PQ in the ratio $2 : 1$, then the locus of R is : **(+4, -1)**

a. $9y^2 = 4x$

b. $9y^2 = -4x$

c. $3y^2 = 2x$

d. $3y^2 = -2x$

---

18. Let P $(3\,sec\,\theta,\, 2\,tan\,\theta)$ and Q $(3\,sec\,\phi,\, 2\,tan\,\phi)$ where $\theta + \phi = \frac{\pi}{2}$, be two distinct points on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. Then the ordinate of the point of intersection of the normals at P and Q is : **(+4, -1)**

a. $\frac{11}{3}$

b. $\frac{-11}{3}$

c. $\frac{13}{2}$

d. $\frac{-13}{2}$

---

19. Let O be the vertex and Q be any point on the parabola, x^2 = 8y. If the point P divides the line segment OQ internally in the ratio $1:3$, then the locus of P is **(+4, -1)**

a. $x^2 = y$

b. $y^2 = x$

c. $y^2 = 2x$

d. $x^2 = 2y$

---

20. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$, then : **(+4, -1)**

a. $L_1 > L_2$

b. $L_1 = L_2$

c. $L_1 < L_2$

d. $\frac{L_1}{L_2} = \sqrt{2}$

---

21. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y) passing through origin and touching the circle C externally, then the radius of T is equal to **(+4, -1)**

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{\sqrt{3}}{2}$

d. $\frac{\sqrt{3}}{\sqrt{2}}$

---

22. If $y + 3x = 0$ is the equation of a chord of the circle, $x^2 + y^2 - 30x = 0$, then the equation of the circle with this chord as diameter is : **(+4, -1)**

a. $x^2 + y^2 + 3x + 9y = 0$

b. $x^2 + y^2 - 3x + 9y = 0$

c. $x^2 + y^2 - 3x - 9y = 0$

d. $x^2 + y^2 + 3x - 9y = 0$

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23. If two vertices of an equilateral triangle are $A(-a, 0)$ and $B(a, 0), a > 0$, and the third vertex C lies above x-axis then the equation of the circumcircle of ΔABC is : **(+4, -1)**

a. $3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$

b. $3x^2 + 3y^2 - 2ay = 3a^2$

c. $x^2 + y^2 - 2ay = a^2$

d. $x^2 + y^2 - \sqrt{3}ay = a^2$

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24. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, them the minimum area (in s units) of the triangle OAB is: **(+4, -1)**

a. $\frac{9}{2}$

b. $3\sqrt{3}$

c. $9\sqrt{3}$

d. 9

---

25. If the tangent at $(1,7)$ to the curve $x^2=y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is: **(+4, -1)**

a. 195

b. 185

c. 85

d. 95

---

26. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is : **(+4, -1)**

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d. $\frac{1}{9}$

---

27. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is : **(+4, -1)**

a. $\frac{60}{13}$

b. $\frac{120}{13}$

c. $\frac{13}{2}$

d. $\frac{13}{5}$

---

28. If one end of a focal chord of the parabola, $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is **(+4, -1)**

a. 25

b. 24

c. 20

d. 22

---

29. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is : **(+4, -1)**

a. $(x^2 + y^2)^2 = 4Rx^2y^2$

b. $(x^2 + y^2)(x + y) = R^2xy$

c. $(x^2 + y^2)^3 = 4R^2x^2y^2$

d. $(x^2 + y^2)^2 = 4R^2x^2y^2$

---

30. If a circle C passing through the point $(4, 0)$ touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point $(1, -1)$, then the radius of C is : **(+4, -1)**

a. $\sqrt{57}$

b. 4

c. $2\sqrt{5}$

d. 5

Answers

1. **Answer: c**

Explanation:

Suppose, tangent to $y^2 = x$ be $y = mx + \frac{1}{4m}$
For tangent to circle,
$|\frac{\frac{1}{4}m}{\sqrt{1+m^2}}| = \sqrt{2}$
$32m^4 + 32m^2 - 1 = 0$
According to the Sridharacharya formula,
$m_2 = \frac{-32 \pm \sqrt{(32)^2 + 4(32)}}{64}$
$8m_1 m_2 = -4 + 3\sqrt{2}$

So, the correct option is (C): $-4 + 3\sqrt{2}$

Concepts:

1. **Conic Sections:**

When a plane intersects a cone in multiple sections, several types of curves are obtained. These curves can be a circle, an ellipse, a parabola, and a hyperbola. When a plane cuts the cone other than the vertex then the following situations may occur:

Let 'β' is the angle made by the plane with the vertical axis of the cone

1. When β = 90º, we say the section is a **circle**
2. When α < β < 90º, then the section is an **ellipse**
3. When α = β; then the section is said to as a **parabola**
4. When 0 ≤ β < α; then the section is said to as a **hyperbola**

Read More: [Conic Sections]

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2. **Answer: c**

Explanation:

The correct answer is option (C): $\frac{32}{3}$
a(x - 3)+b(y + 4)+c(z - 7) = 0
P : 9a - b -5c = 0
-11a - b + 5c = 0
After solving DR's ∝ (1, -1 , 2)
Equation of plane
x - y + 2z = 21
$d = \frac{8}{\sqrt{6}}$
$d^2 = \frac{32}{2}$

Concepts:

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Read More: [Conic Sections]()

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3. Answer: c

Explanation:

The correct option is(C): x+2 y=6.

Concepts:

1. Conic Sections:

When a plane intersects a cone in multiple sections, several types of curves are obtained. These curves can be a circle, an ellipse, a parabola, and a hyperbola. When a plane cuts the cone other than the vertex then the following situations may occur:

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Read More: Conic Sections

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4. Answer: a

Explanation:

The correct option is (A) : $(8, \infty)$

Concepts:

1. Conic Sections:

When a plane intersects a cone in multiple sections, several types of curves are obtained. These curves can be a circle, an ellipse, a parabola, and a hyperbola. When a plane cuts the cone other than the vertex then the following situations may occur:

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Read More: Conic Sections

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5. Answer: a

Explanation:

Given parabola y2 = 12x

$\frac{t_1}{t_2} = 3 = t_1 = 3t_2....(i)$

Let point of intersection be (h, k)

$h = 3t_1t_2 \ ....(ii)$

$and \ k = 3(t_1 + t_2)........(iii)$

$\frac{k}{12}$....(i)

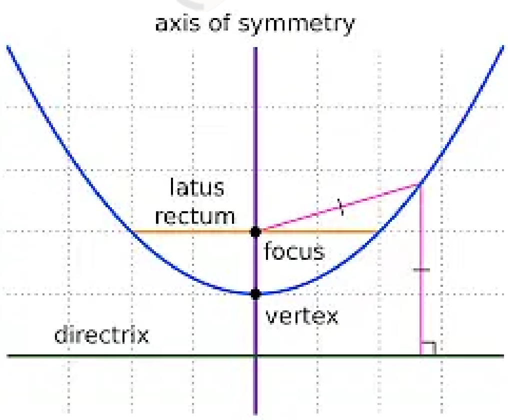
$9 \times \frac{k^2}{144}$

The correct option is (A): $y^2 = 16x$

Concepts:

1. Parabola:

Parabola is defined as the locus of points equidistant from a fixed point (called focus) and a fixed-line (called directrix).



Parabola

Standard Equation of a Parabola

For horizontal parabola

- Let us consider
- Origin (0,0) as the parabola's vertex A,

1. Two equidistant points S(a,0) as focus, and Z(- a,0) as a directrix point,
2. P(x,y) as the moving point.

- Let us now draw SZ perpendicular from S to the directrix. Then, SZ will be the axis of the parabola.
- The centre point of SZ i.e. A will now lie on the locus of P, i.e. AS = AZ.
- The x-axis will be along the line AS, and the y-axis will be along the perpendicular to AS at A, as in the figure.
- By definition PM = PS

=> $MP^2 = PS^2$

- So, $(a + x)^2 = (x - a)^2 + y^2$.
- Hence, we can get the equation of horizontal parabola as $y^2 = 4ax$.

For vertical parabola

- Let us consider
- Origin (0,0) as the parabola's vertex A

1. Two equidistant points, S(0,b) as focus and Z(0, -b) as a directrix point
2. P(x,y) as any moving point

- Let us now draw a perpendicular SZ from S to the directrix.
- Then SZ will be the axis of the parabola. Now, the midpoint of SZ i.e. A, will lie on P’s locus i.e. AS=AZ.
- The y-axis will be along the line AS, and the x-axis will be perpendicular to AS at A, as shown in the figure.
- By definition PM = PS

=> $MP^2 = PS^2$

So, $(b + y)^2 = (y - b)^2 + x^2$

- As a result, the vertical parabola equation is **x^2= 4by**.

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6. **Answer: d**

Explanation:

$\frac{x^2}{16}+\frac{y^2}{7}=1$
$\Rightarrow 7=16\left(1-e^2\right) \Rightarrow e=\frac{3}{4}$
Foci of ellipse is $(\pm ae, 0) \Rightarrow(\pm 3,0)$
Now hyperbola be $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$
$\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{\alpha}{25}}=1$
Now $a=\frac{12}{5}, b^2=\frac{\alpha}{25}$
Let eccentricity of hyperbola be e ae $=3$ (Given)
$\Rightarrow \frac{12}{5} e=3 \Rightarrow e=\frac{5}{4}$
$b^2=a^2\left(e^2-1\right)$
$\frac{\alpha}{25}=\frac{144}{25}\left(\frac{25}{16}-1\right) \Rightarrow \alpha=81$
Hyperbola is $\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{81}{25}}=1$
Now length of $LR=\frac{2b^2}{a}=\frac{27}{10}$

---

7. **Answer: c**

Explanation:

To determine the locus of the circumcentre, we consider the properties of the triangle and its relation to the parabola. The triangle's sides along the y-axis and the line y = 3 form a right angle at the origin. The third side, tangent to the parabola, gives a specific geometric condition that must be analyzed to find the tangent line's slope and intersection points. Using the derivative of the parabola y^2 = 6x:

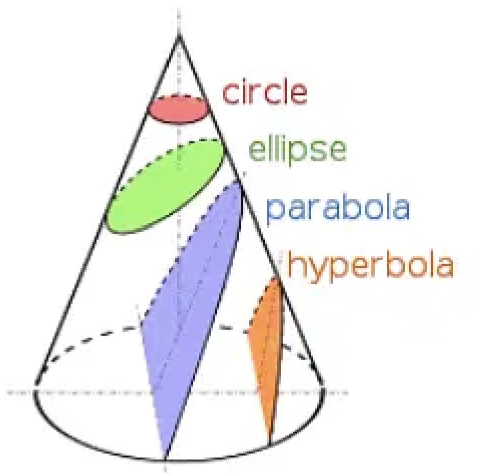
$\frac{dy}{dx}=\frac{6}{2y}=\frac{3}{y}$

Setting this equal to the slope of the tangent line and solving for y and x coordinates of the point of tangency, we can derive the general equation of the tangent line. Subsequent use of circumcentre formulae in a coordinate geometry setting yields the locus as a line equation.

Concepts:

1. Sections of a Cone:

There are four sections of a cone:



1. **Circle**- It is the locus of a point that moves in a certain plane around a fixed distance. The equation of a circle with center (h,k) and the radius r is: $(x–h)^2 + (y–k)^2 = r^2$
2. **Ellipse**- It is the set of all points in a plane, the sum of the whose distance from two fixed points in a plane is constant. With the foci on the x-axis, the equation of an ellipse is shown as: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
3. **Parabola**- It is a locus of a point that moves so that its distance from a fixed point is equivalent to the distance from the moving point to fixed straight lines. When the parabola has a focus at (a,0), where, {a > 0} and directrix {x = -a}, its equation is shown as $y^2 = 4ax$.
4. **Hyperbola**- It is the set of all points in a plane, the difference of whose distance from any two fixed points in the plane is constant. The equation of a hyperbola having its foci on the x-axis is: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Read More: Conic Section

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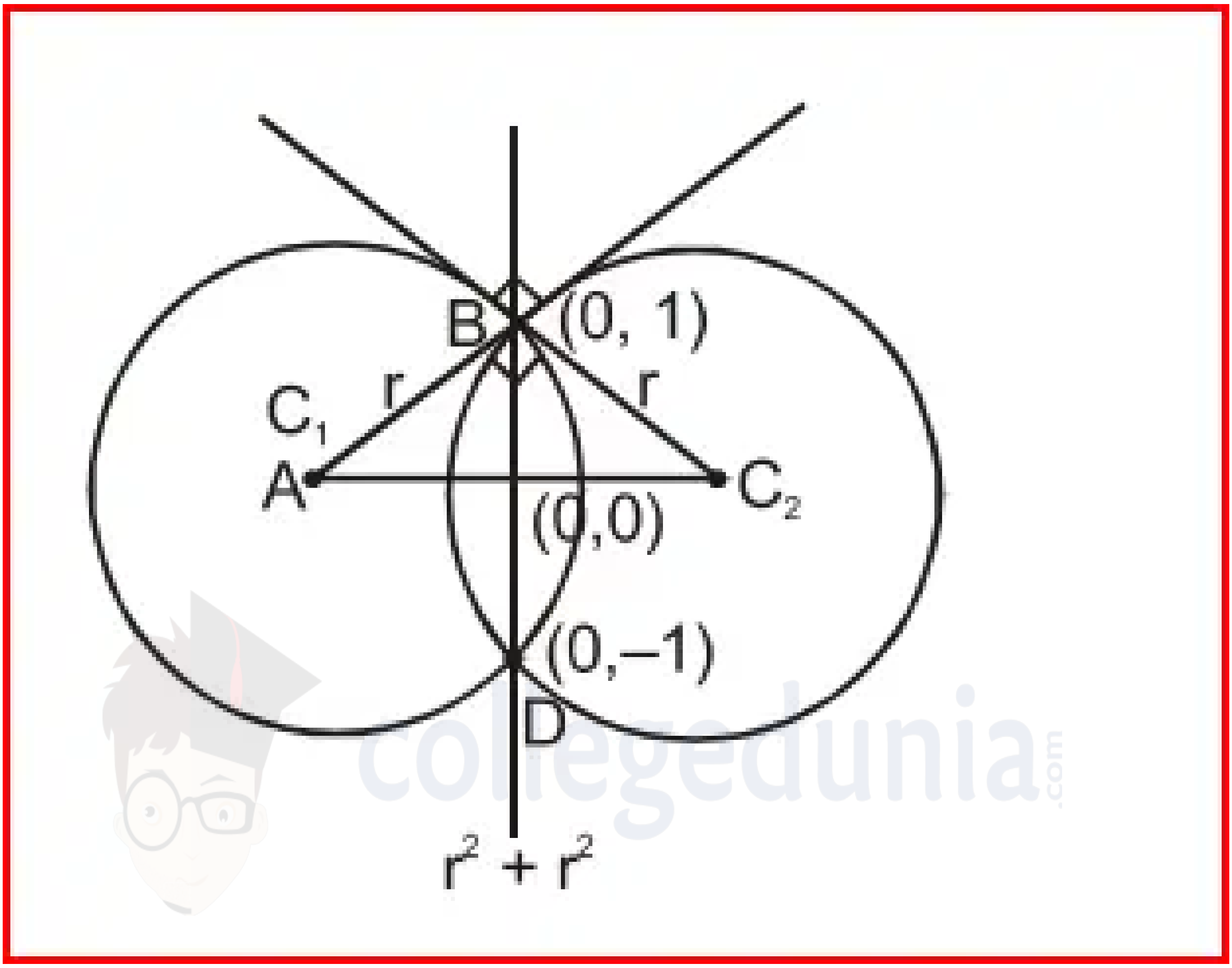
8. Answer: d

Explanation:

The correct option is(D): 4.



$DC_1 = \frac{\sqrt{2}r}{2}$

$= 1 + \frac{r^2}{2} = r^2$

$r = \sqrt{2}$

$C_1C_2 = 2$

Concepts:

1. Conic Sections:

When a plane intersects a cone in multiple sections, several types of curves are obtained. These curves can be a circle, an ellipse, a parabola, and a hyperbola. When a plane cuts the cone other than the vertex then the following situations may occur:

Let 'β' is the angle made by the plane with the vertical axis of the cone

1. When β = 90°, we say the section is a **circle**
2. When α < β < 90°, then the section is an **ellipse**
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Read More: Conic Sections

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9. **Answer: c**

Explanation:

$y^2 = 4x \quad \ldots(i)$ $x^2 = -32y \quad \ldots(2)$ m be slope of common tangent Equation of tangent (1) $y = mx + \frac{1}{m} \quad \ldots(i)$ Equation of tangent (2) $y = mx + 8m^2 \quad \ldots(iii)$ (i) and (ii) are identical $\frac{1}{m} = 8m^2 \Rightarrow m^3 = \frac{1}{8}$ $m = \frac{1}{2}$: Let tangent to $y^2 = 4x$ be $y = mx + \frac{1}{m}$ as this is also tangent to $x^2 = -32y$ Solving $x^2 + 32mx + \frac{32}{m} = 0$ Since roots are equal $\therefore$ $D = 0 \Rightarrow$ $(32)^2 - 4 \times \frac{32}{m} = 0 \Rightarrow m^3 = \frac{4}{32} \Rightarrow m = \frac{1}{2}$

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10. **Answer: a**

Explanation:

Circle: $x^2 + y^2 - 6x + 2y = 0$ ...(i) Line : $2 \times +3 - 1 = 5$ ...(ii) Centre=(3,-1) Now, $2 \times 3 - 1 = 5$, hence centre lies on the given line. Therefore line passes through the centre. The given line is normal to the circle. Thus statement-2 is true, hut statement-1 is not true as there are infinite circle according to the given conditions.

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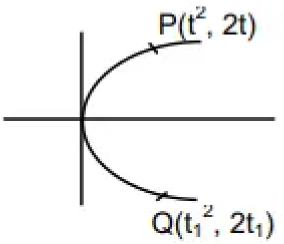
11. **Answer: d**

Explanation:



$t_1 = -t - \frac{2}{t}$ $t_1^2 = t^2 + \frac{4}{t^2} + 4$ min of $t_1^2 = 8$

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12. Answer: d

Explanation:

Let equation of tangent from $(-2, -1)$ be $y + 1 = m(x + 2)$

$\Rightarrow y = mx + (2m - 1)$

Condition of tangency, $C = \frac{a}{m}$

i.e., $2m - 1 = \frac{1}{m}$

$\Rightarrow 2m^2 - m - 1 = 0$

$(2m + 1)(m - 1) = 0$

$m = -\frac{1}{2}, 1$

Now, $|\tan\alpha| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right| = 3$

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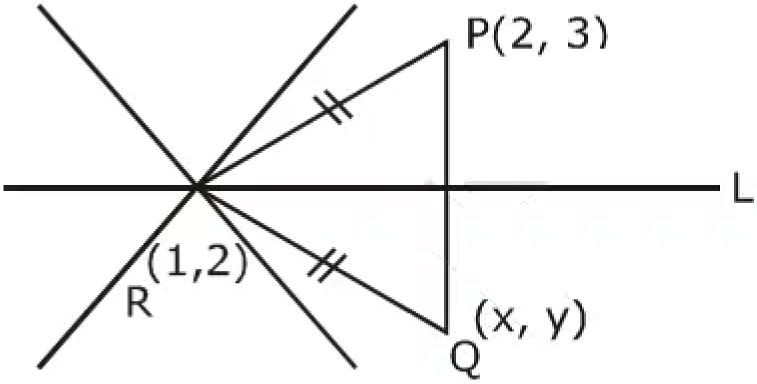
13. Answer: c

Explanation:



$PR = RQ$
$(x-1)^2 + (y-2)^2 = (2-1)^2 + (3-2)^2$
$(x-1)^2 + (y-2)^2 = 2$

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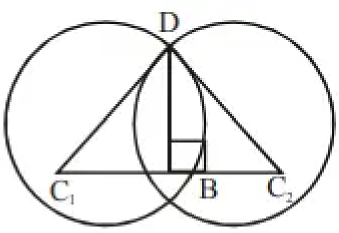
14. Answer: a

Explanation:

Length of latus rectum = 4



$DB = 2$
$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$
$C_1C_2 = 8$

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15. Answer: c

Explanation:

Given equations of ellipses

$E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1$

$\Rightarrow e_1 = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$

$E_2 : \frac{x^2}{61} + \frac{y^2}{b^2} = 1$

$\Rightarrow e_2 = \sqrt{\frac{1-b^2}{16}} = \sqrt{\frac{16-b^2}{4}}$

Also, given $e_1 \times e_2 = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{3}} \times \sqrt{\frac{16-b^2}{4}} = \frac{1}{2} \Rightarrow 16 - b^2 = 12$

$\Rightarrow b^2 = 4$

$\therefore$ Length of minor axis of

$E_2 = 2b = 2 \times 2 = 4$

---

16. **Answer: c**

Explanation:

$m_{SB}.m_{SB} = -1$

$b^2 = a^2e^2 \ ...(i)$

$\frac{1}{2}S'B.SB = 8$

$S'B.SB = 16$

$a^2c^2 + b^2 = 16 ....(ii)$

$b^2 = a^2 \ (1 - e^2) ....(iii)$

using (i),(ii), (iii) a = 4

$b = 2\sqrt{2}$

$e = \frac{1}{\sqrt{2}}$

$\therefore \ \ell(L.R) = \frac{2b^2}{a} = 4$

---

17. **Answer: b**

Explanation:

$Let P \left(-at_1^2, 2at_1\right)$,

$Q \left(-at_1^2, -2at_1\right)$ and $R\,(h,k)$

By using section formula, we have

$h = -at_1^2, k = \frac{-2at_1}{3}$

$k = -\frac{2at_1}{3}$
$\Rightarrow 3k = -2at_1$
$\Rightarrow 9k^2 = 4a^2\, t_1^2 = 4a\,(-h)$
$\Rightarrow 9k^2 = -4ah$
$\Rightarrow 9k^2 = -4h$
$\Rightarrow 9y^2 = -4x$

---

18. Answer: d

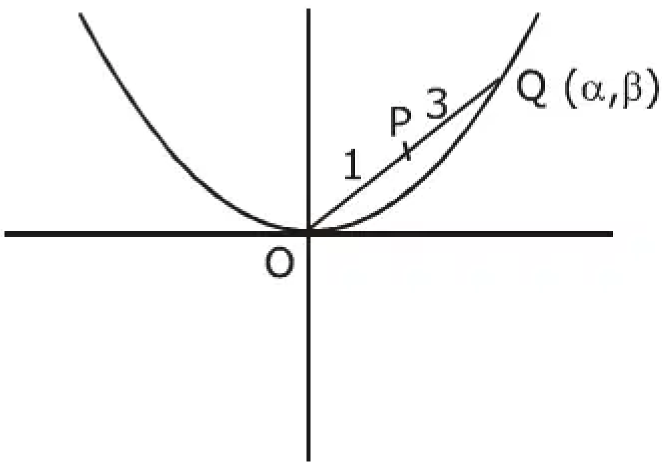
Explanation:

$p\,(3\,sec\,\theta,\ 2\,tan\,\theta) \quad Q = (3\,sec\,\phi\,,\ 2\,tan\phi)$
$\theta + \phi = \frac{\pi}{2} \quad Q = (3\,cosec\,\theta,\ 2\,cot\theta)$
Equation of normal at $p =$
$= 3x\,cos\,\theta + 2y\,cot\,\theta = 13$
$= 3x\,sin\,\theta\,cos\,\theta + 2y\,cos\,\theta = 13\,sin\theta \quad ...(1)$
equation of normal at Q $\Rightarrow$
$= 3x\,sin\,\theta + 2y\,tan\,\theta = 13$
$= 3x\,sin\,\theta\,cos\,\theta + 2y\,sin\,\theta = 13\,cos\theta \quad ...(2)$
$(1) - (2) \ \Rightarrow$
$2y\ (cos\,\theta - sin\,\theta) = 13\,(sin\,\theta - cos\,\theta)$
$2y = -13 \Rightarrow \frac{-13}{2}$

---

19. Answer: d

Explanation:



Let $P:(h,k)$

$h=\frac{1.\alpha+\beta.0}{4}$

$\Rightarrow \alpha=4h$

$k=\frac{1.\beta+3.0}{4}$

$\Rightarrow \beta=4k$

$\because (\alpha,\beta)$ on Parabola

$\Rightarrow \alpha^2=8\beta$

$\Rightarrow \left(4h^2\right)=8.4k$

$16h^2=32k$

$x^2=2y$

---

20. Answer: c

Explanation:

$x^2+y^2=9\,\&\,y^2=8x$

$L_2=$ L.R. of $y^2=8x\Rightarrow L_2=8$

Solve $x^2+8x=9\Rightarrow x=1,-9$

$x=-9 reject$

$\therefore y^2=8\,x$ so $y^2=8$

$y=\pm\sqrt{8}$

Point of intersection are $(1,\sqrt{8})\ (1,-\sqrt{8})$

So $L_1=2\sqrt{8}$

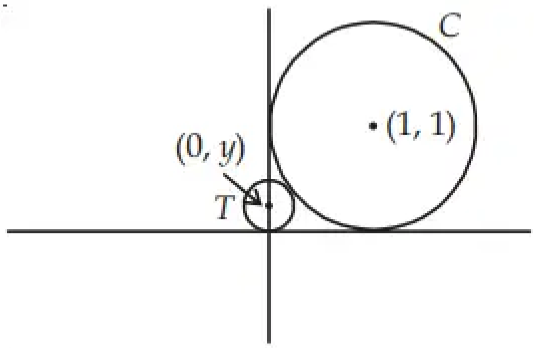
$\frac{L_1}{L_2}=\frac{2\sqrt{8}}{8}=\frac{2}{\sqrt{8}}=\frac{1}{\sqrt{2}}<1$

$L_1<L_2$

---

21. Answer: b

Explanation:

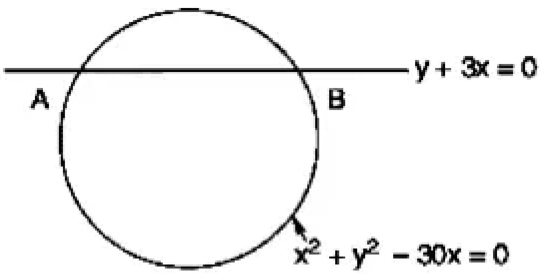


$C \equiv (x-1)^2 + (y-1)^2 = 1$
Radius of $T = |y|$
T touches C externally
$(0-1)^2 + (y-1)^2 = (1+|y|)^2$
$\Rightarrow 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$
If $y > 0$, $y^2 + 2 - 2y = y^2 + 1 + 2y$
$\Rightarrow 4y = 1$
$\Rightarrow y = \frac{1}{4}$
If $y

---

22. **Answer: d**

Explanation:



Given that $y + 3x = 0$ is the equation of a chord of the circle then
$y = -3x$....(i)
$\left(x^2\right) + (-3x)^2 - 30x = 0$
$10x^2 - 30x = 0$
$10x\,(x-3) = 0$

$x = 0,\ y = 0$
so the equation of the circle is
$(x?3)(x-0)+(y+9)(y-0)=0$
$x^2-3x+y^2+9y=0$
$x^2+y^2-3x+9y=0$

---

23. Answer: a

Explanation:

Let $C=(x,y)$
Now, $CA^2=CB^2=AB^2$
$\Rightarrow (x+a)^2+y^2=(x-a)^2+y^2=(2a)^2$
$\Rightarrow x^2+2ax+a^2+y^2=4a^2\ldots(i)$
and $x^2-2ax+a^2+y^2=4a^2\ldots(ii)$
From (i) and (ii), $x=0$ and $y=\pm\sqrt{3}a$
Since point $C(x,y)$ lies above the x-axis and $a>0$, hence $y=\sqrt{3}a$
$\therefore C=\left(0,\sqrt{3}a\right)$
Let the equation of circumcircle be
$x^2+y^2+2gx+2fy+C=0$
Since points $A(-a,0), B(a,0)$ and $C\left(0,\sqrt{3}a\right)$ lie on the circle, therefore
$a^2-2ga+C=0\ldots(iii)$
$a^2+2ga+C=0\ldots(iv)$
and $3a^2+2\sqrt{3}af+C=0\ldots(v)$
From $(iii),(iv)$, and (v)
$g=0, c=-a^2, f=-\frac{a}{\sqrt{3}}$
Hence equation of the circumcircle is
$x^2+y^2-\frac{2a}{\sqrt{3}}y-a^2=0$
$\Rightarrow x^2+y^2-\frac{2\sqrt{3}ay}{3}-a^2=0$
$\Rightarrow 3^22+3y^2-2\sqrt{3}ay=3a^2$

---

24. Answer: d

Explanation:

Equation of tangent to ellipse

$\frac{x}{\sqrt{27}} + \frac{y}{\sqrt{3}} = 1$

Area bounded by line and co-ordinate axis

$\frac{1}{2}\times$intercept on x-axis $\times$ intercept on y -axis

$\Delta = \frac{1}{2}.\frac{\sqrt{27m^2+3}}{m}.\sqrt{27m^2+3}sin$

$\frac{1}{2} \times \frac{(27m^2+3)}{m}$

now apply

AM≥GM

$\frac{27m+\frac{3}{m}}{2} \geq \sqrt{27m \times \frac{3}{m}} \geq 9$

Δ

$\Delta_{min} = 9$

---

25. Answer: d

Explanation:

Equation of tangent at $(1,7)$ to curve $x^2 = y - 6$ is

$x - 1 = \frac{1}{2}(y + 7) - 6$

$2x - y + 5 = 0 \quad ...(i)$

Centre of circle $= (-8, -6)$

Radius of circle $= \sqrt{64 + 36 - c} = \sqrt{100 - c}$

$\because$ Line (i) touches the circle

$\therefore \left|\frac{2(-8)-(-6)+5}{\sqrt{4+1}}\right| = \sqrt{100 - c}$

$\sqrt{5} = \sqrt{100 - c}$

$\Rightarrow c = 95$

---

26. Answer: b

Explanation:

Given: $\frac{2b^2}{a} = 4$

$\Rightarrow b^2 = 2a$

$b^2 = a^2\left(1 - e^2\right)$

$a(1-e)=\frac{3}{2}$
So $2=a(1-e)(1+e) \Rightarrow 2=\frac{3}{2}(1+e)$
$\Rightarrow 4=3+3e$
$\Rightarrow e=\frac{1}{3}$

---

27. Answer: b

Explanation:

Let length of common chord = $2x$
$\sqrt{25-x^2}+\sqrt{144-x^2}=13$
after solving
$x=\frac{12\times5}{13}$
$2x=\frac{120}{13}$

---

28. Answer: a

Explanation:

$y^2=4ax=16x \Rightarrow a=4$
A(1,4) $\Rightarrow$ 2.4.t_1 = 4 $\Rightarrow$ $t_1 = \frac{1}{2}$
$\therefore$ length of focal chord $= s\left(t+\frac{1}{t}\right)^2$
$=4\left(\frac{1}{2}+2\right)^2 = 4, \frac{25}{4} = 25$

---

29. Answer: c

Explanation:

Slope of $AB=\frac{-h}{k}$
Equation of AB is $hx+ky=h^2+k^2$
$A\left(\frac{h^2+k^2}{h},0\right), B\left(0,\frac{h^2+k^2}{k}\right)$
$AB=2R$
$\Rightarrow (h^2+k^2)^3=4R^2h^2k^2$
$\Rightarrow (x^2+y^2)^3 = 4R^2x^2y^2$

30. **Answer: d**

Explanation:

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at $(1, -1)$

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through $(4, 0)$:

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow \; 20 + \lambda(5) = 0$

$\Rightarrow \; \lambda = -4$

$\therefore \; (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16 + 25 - 16} = 5$