Coordinate Geometry JEE Main PYQ - 1

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Coordinate Geometry

1. A circle $x^2 + y^2 = 4$ intersects the x-axis at $A(-2, 0)$ and $B(2, 0)$. If two variable points $P(2\cos\alpha,\ 2\sin\alpha)$ and $Q(2\cos\beta,\ 2\sin\beta)$ vary on the circle such that $\alpha - \beta = \frac{\pi}{2}$, then find the locus of the point of intersection of lines AP and BQ. (+4, -1)

   a. $x^2 + y^2 - 4y - 4 = 0$

   b. $x^2 + y^2 + 4y - 4 = 0$

   c. $x^2 + y^2 - 4y + 4 = 0$

   d. $x^2 + y^2 + 4y + 4 = 0$

---

2. Let $\triangle ABC$ such that $A(0, 0)$ and vertices B and C lie on the parabola (+4, -1)

$$y^2 = 8x.$$

   If $\left(\frac{7}{3}, \frac{4}{3}\right)$ is the centroid of $\triangle ABC$, then $(BC)^2$ is equal to:

   a. 110

   b. 115

   c. 120

   d. 130

---

3. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that (+4, -1)

$$\vec{a} \cdot \vec{c} = 104 \quad \text{and} \quad \vec{a} \times \vec{c} = \vec{c} \times \vec{b}.$$

   Then $\vec{b} \cdot \vec{c}$ is equal to _____

---

4. If the circles (+4, -1)

$$x^2 + y^2 - 2x - 8y + 17 = r \quad \text{and} \quad x^2 + y^2 - 26x - 18y + 234 = 0$$

intersect at exactly one point, then the sum of all possible values of r is

---

5. Let (α, β, γ) be the foot of the perpendicular from the point $(25, 2, 41)$ on the line (+4, -1)

$$\frac{x-4}{3} = \frac{y+1}{7} = \frac{z-2}{3}.$$

Then $\alpha + \beta + \gamma$ is equal to:

a. 41

b. 42

c. 44

d. 45

---

6. Let the points $A(a, -1, 2)$, $B(1, b, -4)$, $C(-1, 1, c)$ and $D(1, -2, 8)$ be the vertices of a parallelogram $ABCD$. Then its area is equal to: (+4, -1)

a. $2\sqrt{73}$

b. $2\sqrt{51}$

c. 28

d. 14

---

7. Let $A(3, 4)$, $B(5, -2)$ and $P(\alpha, \beta)$, $\alpha\beta \neq 0$, be three points such that $PA = PB$ and the area of $\triangle PAB$ is 10. Then the distance of the point $Q(2\alpha - 5\beta,\ \alpha - \beta^2)$ from the line having intercepts 3 and 1 on the x- and y-axis respectively, is: (+4, -1)

a. 10

b. $\sqrt{10}$

c. 15

d. 2

---

8. Let e_1 and e_2 be the eccentricities of the ellipse $2x^2 + 9y^2 = 36$ and the hyperbola $4x^2 - 9y^2 = 36$, respectively. Then the distance between the point of intersection of the lines $5x - 7y = 3$ and $3x + y = 7$, and the point $(9e_1^2,\ 9e_2^2)$ is: (+4, -1)

a. 11

b. 12

c. 13

d. 15

---

9. Let the distance between the foci of an ellipse (+4, -1)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

be 4 and the distance between its directrices be 10. Then the length of its latus rectum is:

a. $\frac{12}{\sqrt{10}}$

b. $\frac{6}{\sqrt{10}}$

c. $\frac{8}{\sqrt{5}}$

d. $\sqrt{10}$

---

10. If the line $\arg(z) = \frac{\pi}{3}$ intersects the curve $|z - 2\sqrt{3}i| = 2$, $z \in \mathbb{C}$, at two distinct points A and B, then AB equals: (+4, -1)

a. 1

b. 2

c. 4

d. 6

---

11. Given conic $x^2 - y^2\sec^2\theta = 8$ whose eccentricity is 'e_1' & length of latus rectum 'l_1' and for conic $x^2 + y^2\sec^2\theta = 6$, eccentricity is '$e_2$' & length of latus rectum 'l_2'. If $e_1^2 = e_2^2(1 + \sec^2\theta)$ then value of $\frac{e_1 l_1}{e_2 l_2}\tan\theta$ (+4, -1)

a. 1

b. 2

c. 3

d. 4

---

12. Orthocentre of equilateral ΔABC is at the origin. If side BC lies along $x + 2\sqrt{2}y = 4$. If coordinates of vertex A are (a,b). Find the value of $\lfloor|a + \sqrt{2}b|\rfloor$, where $[\cdot]$ denotes G.I.F. : (+4, -1)

a. 1

b. 2

c. 3

d. 4

---

13. Consider a circle C_1, passing through origin and lying in region $0 \leq x$ only, with diameter = 10. Consider a chord PQ of C_1 with equation $y = x$ and another Circle C_2 which has PQ as diameter. A chord is drawn to C_2 passing through (2, 3) such that distance of chord from centre of C_2 is maximum has equation $x + ay + b = 0$ then $|b - a|$ is equal to : (+4, -1)

a. 1

b. 2

c. 3

d. 4

14. If $P(h,k)$ is a variable point on $x^2+y^2=4$ and $Q(2h+1,\ 3k+3)$ always lies on an ellipse, if eccentricity of the ellipse is e, then $\frac{5}{e^2}$ is equal to (+4, -1)

a. 9

b. 5

c. 3

d. 6

15. If $P(h,k)$ is a variable point on $x^2+y^2=4$ and $Q(2h+1,\ 3k+3)$ always lies on an ellipse, if eccentricity of the ellipse is e, then $\frac{5}{e^2}$ is equal to (+4, -1)

16. Given triangle OAB where O is the origin, $A=(0,-\sqrt{3}a)$ and $B=(-\sqrt{2}b,0)$. Let the circumradius of $\triangle OAB$ be 4 units. If the locus of the centroid of $\triangle OAB$ is a circle, then its radius is: (+4, -1)

a. $\frac{8}{3}$

b. $\frac{7}{3}$

c. $\frac{11}{3}$

d. $\frac{5}{3}$

17. Rhombus $ABCD$ is given with vertices $A(1,2)$, $C(-3,-6)$ and sides AD and BC are parallel to the line $7x-y=14$. If coordinates of B and D are (α,β) and (γ,δ) respectively, then find $\alpha+\beta+\gamma+\delta$: (+4, -1)

a. -4

b. 5

c. -6

d. -7

18. If the point of intersection of the ellipses (+4, -1)

$$x^2 + 2y^2 - 6x - 12y + 23 = 0$$

$$4x^2 + 2y^2 - 20x - 12y + 35 = 0$$

lie on a circle of radius r and centre (a, b), then the value of $ab + 18r^2$ is:

a. 90

b. 95

c. 85

d. 100

19. An equilateral triangle OAB is inscribed in the parabola $y = 4x^2$ whose vertex is O. Find the least distance of the circle described on AB as diameter from the origin. (+4, -1)

a. $\frac{6 - \sqrt{3}}{2}$

b. $\frac{3 - \sqrt{3}}{4}$

c. $\frac{6 + \sqrt{3}}{2}$

d. $\frac{3 + \sqrt{3}}{2}$

20. If PQ is a chord perpendicular to the transverse axis of (+4, -1)

$$\frac{x^2}{4} - \frac{y^2}{b^2} = 1$$

of eccentricity $\sqrt{3}$ such that $\triangle OPQ$ is an equilateral triangle (where O is the origin), then the area of $\triangle OPQ$ is:

a. $\frac{4\sqrt{3}}{5}$

b. $\frac{2\sqrt{3}}{5}$

c. $\frac{8\sqrt{3}}{5}$

d. $\frac{16\sqrt{3}}{5}$

---

21. Let one end of a focal chord of the parabola $y^2 = 20x$ be $(20, -20)$. If $P(\alpha, \beta)$ divides the chord internally in the ratio $2:3$, find the minimum value of $\alpha + \beta$. **(+4, -1)**

a. 4

b. 6

c. 8

d. 10

---

22. A rectangle is formed by lines $x = 0, y = 0, x = 3$ and $y = 4$. A line perpendicular to $3x + 4y + 6 = 0$ divides the rectangle into two equal parts. Then the distance of the line from the point $\left(-1, \frac{3}{2}\right)$ is: **(+4, -1)**

a. $\frac{17}{10}$

b. $\frac{10}{17}$

c. $\frac{15}{17}$

d. $\frac{18}{17}$

---

23. Let $A(1, 6, 3)$ and point B and C lie on the line **(+4, -1)**

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

where $B(4, 9, \alpha)$ and point C is 10 units from B. Find the area of $\triangle ABC$:

a. $6\sqrt{13}$

b. $5\sqrt{13}$

c. $7\sqrt{13}$

d. $8\sqrt{13}$

---

24. Let $\frac{x^2}{2} + \frac{y^2}{1} = 1$ and $y = x + 1$ intersect each other at points A & B, then $\angle AOB$ (where O is the centre of the ellipse) is: **(+4, -1)**

a. $\frac{\pi}{2} + \tan^{-1}\frac{1}{4}$

b. $\frac{\pi}{2} + \tan^{-1}\frac{1}{3}$

c. $\frac{\pi}{2} + \tan^{-1}\frac{1}{2}$

d. $\frac{\pi}{4} + \tan^{-1}\frac{1}{2}$

---

25. If O is the vertex of the parabola $x^2 = 4ay$, Q is the point on the parabola. If C is the locus of the point which divides OQ in ratio 2:3, the equation of the chord of C which is bisected at point $(1, 2)$ is **(+4, -1)**

a. $5x + 4y + 3 = 0$

b. $5x - 4y - 3 = 0$

c. $5x - 4y + 3 = 0$

d. $5x + 4y - 3 = 0$

---

26. The locus of point of intersection of tangent drawn to the circle $(x-2)^2 + (y-3)^2 = 16$, which subtends an angle of 120° is **(+4, -1)**

a. $3x^2 + 3y^2 - 12x - 18y - 25 = 0$

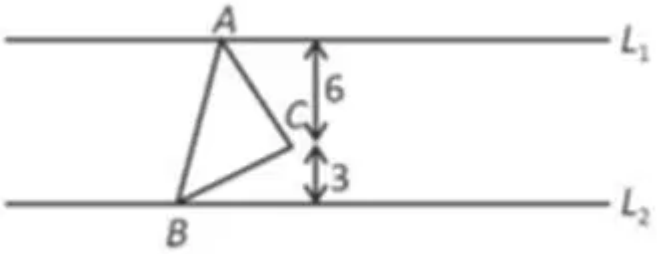
b. $x^2 + y^2 - 12x - 18y - 25 = 0$

c. $3x^2 + 3y^2 + 12x + 18y - 25 = 0$

d. $x^2 + y^2 + 12x + 18y - 25 = 0$

---

27. If L_1 and L_2 are two parallel lines and $\triangle ABC$ is an equilateral triangle, then the area of triangle ABC is **(+4, -1)**



a. $7\sqrt{3}$

b. $4\sqrt{3}$

c. $21\sqrt{3}$

d. 84

---

28. The value(s) of α for which the line $\alpha x + 2y = 1$ never touches the hyperbola **(+4, -1)**

$$\frac{x^2}{9} - \frac{y^2}{1} = 1$$

is/are:

a. $R - \sqrt{\frac{5}{2}}$

b. $R - \sqrt{5}, \sqrt{5}$

c. $R - \sqrt{\frac{5}{3}}$

d. R

---

29. Let O be the vertex of the parabola $y^2 = 16x$. The locus of centroid of $\triangle OPA$ when P lies on the parabola and A lies on the x-axis and $\angle OPA = 90°$. **(+4, -1)**

a. $y^2 = 8(3x - 16)$

b. $9y^2 = 8(3x - 16)$

c. $y^2 = 8(3x + 16)$

d. $9y^2 = 8(3x + 16)$

---

30. If two circles $x^2 + y^2 - 4x - 2y - 4 = 0$ \& $(x + 1)^2 + (y + 4)^2 = r^2$ intersect at two distinct points and range of r $\in (\alpha, \beta)$, then the value of $\alpha\beta$ is : (+4, -1)

collegedunia.com

Answers

1. Answer: a

Explanation:

Step 1: Coordinates of points

$$A(-2,0), \quad B(2,0)$$

$$P(2\cos\alpha,\ 2\sin\alpha), \quad Q(2\cos\beta,\ 2\sin\beta), \quad \alpha-\beta=\frac{\pi}{2}$$

Thus,

$$\cos\beta=\sin\alpha, \quad \sin\beta=-\cos\alpha.$$

Step 2: Equations of lines

Line AP:

$$\frac{y-0}{x+2}=\frac{2\sin\alpha}{2\cos\alpha+2} \Rightarrow y=\frac{\sin\alpha}{1+\cos\alpha}(x+2)$$

Line BQ:

$$\frac{y-0}{x-2}=\frac{2\sin\beta}{2\cos\beta-2}=\frac{-2\cos\alpha}{2\sin\alpha-2} \Rightarrow y=\frac{-\cos\alpha}{\sin\alpha-1}(x-2)$$

Step 3: Intersection point

Let the intersection be (x,y). Equating the two expressions for y and simplifying eliminates α, yielding:

$$x^2+y^2-4y-4=0$$

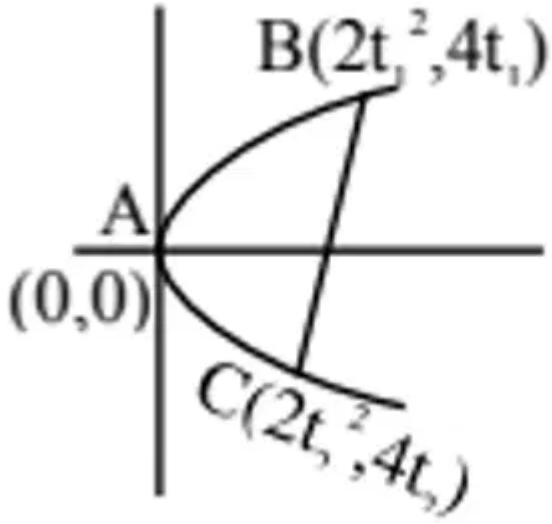
Final Answer:

$$\boxed{x^2+y^2-4y-4=0}$$

---

2. Answer: c

Explanation:



Step 1: Use centroid formula

Let

$$B(x_1, y_1), \quad C(x_2, y_2).$$

Given centroid G:

$$G\left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right) = \left(\frac{7}{3}, \frac{4}{3}\right).$$

Hence,

$$x_1 + x_2 = 7, \qquad y_1 + y_2 = 4.$$

Step 2: Use parabola condition

Since B and C lie on $y^2 = 8x$,

$$x_1 = \frac{y_1^2}{8}, \quad x_2 = \frac{y_2^2}{8}.$$

Thus,

$$\frac{y_1^2 + y_2^2}{8} = 7 \;\Rightarrow\; y_1^2 + y_2^2 = 56.$$

Step 3: Find $y_1 y_2$

$$(y_1 + y_2)^2 = y_1^2 + y_2^2 + 2y_1y_2$$

$$4^2 = 56 + 2y_1y_2 \Rightarrow 16 = 56 + 2y_1y_2 \Rightarrow y_1y_2 = -20.$$

Step 4: Compute $(BC)^2$

$$(BC)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2.$$

First,

$$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2 = 16 - 4(-20) = 96.$$

Next,

$$x_1 - x_2 = \frac{y_1^2 - y_2^2}{8} = \frac{(y_1 - y_2)(y_1 + y_2)}{8} = \frac{4(y_1 - y_2)}{8} = \frac{y_1 - y_2}{2}.$$

So,

$$(x_1 - x_2)^2 = \frac{(y_1 - y_2)^2}{4} = \frac{96}{4} = 24.$$

Therefore,

$$(BC)^2 = 96 + 24 = 120.$$

$$\boxed{120}$$

---

3. Answer: 50 - 50

Explanation:

Concept:

For vectors, $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$.

If $\vec{p} \times \vec{q} = \vec{0}$, then $\vec{p}$ and $\vec{q}$ are parallel.

A vector parallel to $\vec{v}$ can be written as $t\vec{v}$, where t is a scalar.

Step 1: Use the given cross product condition

$$\vec{a} \times \vec{c} = \vec{c} \times \vec{b} = -(\vec{b} \times \vec{c})$$

Hence,

$$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = \vec{0}$$

$$(\vec{a}+\vec{b})\times\vec{c}=\vec{0}$$

Thus, $\vec{c}$ is parallel to $\vec{a}+\vec{b}$.

Step 2: Find $\vec{a}+\vec{b}$

$$\vec{a}+\vec{b}=(2+1)\hat{i}+(3-1)\hat{j}+(5+3)\hat{k}=3\hat{i}+2\hat{j}+8\hat{k}$$

Let

$$\vec{c}=t(3\hat{i}+2\hat{j}+8\hat{k})$$

Step 3: Use the dot product condition

$$\vec{a}\cdot\vec{c}=t\left[2(3)+3(2)+5(8)\right]$$

$$=t(6+6+40)=52t$$

Given $\vec{a}\cdot\vec{c}=104$,

$$52t=104\Rightarrow t=2$$

So,

$$\vec{c}=6\hat{i}+4\hat{j}+16\hat{k}$$

Step 4: Find $\vec{b}\cdot\vec{c}$

$$\vec{b}\cdot\vec{c}=(1)(6)+(-1)(4)+(3)(16)$$

$$=6-4+48=50$$

Final Answer: $\boxed{50}$

---

4. Answer: 370 - 370

Explanation:

Concept:

A circle $x^2+y^2+2gx+2fy+c=0$ has center $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$.

Two circles intersect at exactly one point if they touch each other (are tangent).

For tangency:

$$d = r_1 + r_2 \quad \text{or} \quad d = |r_1 - r_2|,$$

where d is the distance between the centers.

Step 1: Find the center and radius of the first circle

$$x^2 + y^2 - 2x - 8y + (17 - r) = 0$$

Center:

$$C_1(1, 4)$$

Radius:

$$r_1 = \sqrt{1^2 + 4^2 - (17 - r)} = \sqrt{r}$$

Step 2: Find the center and radius of the second circle

$$x^2 + y^2 - 26x - 18y + 234 = 0$$

Center:

$$C_2(13, 9)$$

Radius:

$$r_2 = \sqrt{13^2 + 9^2 - 234} = \sqrt{16} = 4$$

Step 3: Find the distance between the centers

$$d = \sqrt{(13 - 1)^2 + (9 - 4)^2} = \sqrt{144 + 25} = 13$$

Step 4: Apply the tangency conditions (i) External tangency:

$$13 = \sqrt{r} + 4 \Rightarrow \sqrt{r} = 9 \Rightarrow r = 81$$

(ii) Internal tangency:

$$13 = |\sqrt{r} - 4| \Rightarrow \sqrt{r} = 17 \Rightarrow r = 289$$

Step 5: Sum of all possible values of r

$$81 + 289 = 370$$

Final Answer: $\boxed{370}$

---

5. Answer: c

Explanation:

Concept:
A point on a line in symmetric form can be written using a parameter.
The foot of the perpendicular from a point to a line satisfies the condition that the vector joining them is perpendicular to the direction vector of the line.
Two vectors are perpendicular if their dot product is zero.
Step 1: Parametric form of the given line Let

$$\frac{x-4}{3} = \frac{y+1}{7} = \frac{z-2}{3} = t$$

Then a general point on the line is:

$$A(4+3t,\ -1+7t,\ 2+3t)$$

The direction vector of the line is:

$$\vec{d} = (3, 7, 3)$$

Step 2: Use perpendicularity condition Given point $P(25, 2, 41)$. For A to be the foot of the perpendicular,

$$\overrightarrow{AP} \cdot \vec{d} = 0$$

$$(25 - (4+3t),\ 2 - (-1+7t),\ 41 - (2+3t)) \cdot (3, 7, 3) = 0$$

$$(21 - 3t,\ 3 - 7t,\ 39 - 3t) \cdot (3, 7, 3) = 0$$

Step 3: Solve for t

$$3(21 - 3t) + 7(3 - 7t) + 3(39 - 3t) = 0$$

$$63 - 9t + 21 - 49t + 117 - 9t = 0$$

$$201 - 67t = 0 \Rightarrow t = 3$$

Step 4: Find the foot of the perpendicular Substitute $t = 3$:

$$\alpha = 4 + 9 = 13, \quad \beta = -1 + 21 = 20, \quad \gamma = 2 + 9 = 11$$

Step 5: Required sum

$$\alpha + \beta + \gamma = 13 + 20 + 11 = 44$$

---

6. Answer: c

Explanation:

Concept:
In a parallelogram, diagonals bisect each other.
If position vectors of vertices satisfy $\vec{A} + \vec{C} = \vec{B} + \vec{D}$, then the points form a parallelogram.
Area of a parallelogram formed by vectors $\vec{u}$ and $\vec{v}$ is $|\vec{u} \times \vec{v}|$.
Step 1: Use the diagonal property For parallelogram $ABCD$,

$$\vec{A} + \vec{C} = \vec{B} + \vec{D}$$

$$(a, -1, 2) + (-1, 1, c) = (1, b, -4) + (1, -2, 8)$$

Equating components:

$$a - 1 = 2 \Rightarrow a = 3$$

$$0 = b - 2 \Rightarrow b = 2$$

$$2 + c = 4 \Rightarrow c = 2$$

Step 2: Find direction vectors

$$\vec{AB} = B - A = (1 - 3,\ 2 + 1,\ -4 - 2) = (-2, 3, -6)$$

$$\vec{AD} = D - A = (1 - 3,\ -2 + 1,\ 8 - 2) = (-2, -1, 6)$$

Step 3: Find the cross product

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & -6 \\ -2 & -1 & 6 \end{vmatrix}$$

$$= \mathbf{i}(18 - 6) - \mathbf{j}(-12 - 12) + \mathbf{k}(2 + 6)$$

$$= (12,\ 24,\ 8)$$

Step 4: Find the area

$$\text{Area} = \sqrt{12^2 + 24^2 + 8^2} = \sqrt{144 + 576 + 64} = \sqrt{784} = 28$$

---

7. Answer: b

Explanation:

Concept:

A point equidistant from A and B lies on the perpendicular bisector of $\overline{AB}$.

Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$.

Distance of a point (x_1, y_1) from a line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Step 1: Equation of the perpendicular bisector of AB Midpoint of AB:

$$M\left(\frac{3+5}{2}, \frac{4-2}{2}\right) = (4, 1)$$

Slope of AB:

$$m_{AB} = \frac{-2-4}{5-3} = -3$$

Slope of perpendicular bisector $= \frac{1}{3}$. Hence its equation is:

$$y - 1 = \frac{1}{3}(x - 4)$$

Step 2: Use the area condition Length of AB:

$$AB = \sqrt{(5-3)^2 + (-2-4)^2} = \sqrt{40} = 2\sqrt{10}$$

Given area $= 10$,

$$10 = \frac{1}{2} \cdot 2\sqrt{10} \cdot h \Rightarrow h = \sqrt{10}$$

Equation of line AB is:

$$y - 4 = -3(x - 3) \Rightarrow 3x + y - 13 = 0$$

Thus,

$$\frac{|3\alpha + \beta - 13|}{\sqrt{10}} = \sqrt{10} \Rightarrow |3\alpha + \beta - 13| = 10$$

Step 3: Find point P From perpendicular bisector, let

$$\alpha = 4 + 3t, \quad \beta = 1 + t$$

Substitute:

$$|3(4 + 3t) + (1 + t) - 13| = |10t| = 10 \Rightarrow |t| = 1$$

So $t = 1$ or -1. Since $\alpha\beta \neq 0$, choose $t = 1$:

$$P = (7, 2)$$

Step 4: Find point Q

$$Q(2\alpha - 5\beta, \alpha - \beta^2) = (14 - 10, 7 - 4) = (4, 3)$$

Step 5: Distance of Q **from the given line** Line with intercepts 3 and 1:

$$\frac{x}{3} + \frac{y}{1} = 1 \Rightarrow x + 3y - 3 = 0$$

Distance:

$$\frac{|4 + 9 - 3|}{\sqrt{1 + 9}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

8. Answer: c

Explanation:

Concept:

Standard form of ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}$.

Standard form of hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, eccentricity $e=\sqrt{1+\frac{b^2}{a^2}}$.

Distance between two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

Step 1: Find e_1 for the ellipse

$$2x^2+9y^2=36 \Rightarrow \frac{x^2}{18}+\frac{y^2}{4}=1$$

Thus, $a^2=18,\ b^2=4$.

$$e_1=\sqrt{1-\frac{4}{18}}=\sqrt{\frac{14}{18}}=\frac{\sqrt{7}}{3}$$

Step 2: Find e_2 for the hyperbola

$$4x^2-9y^2=36 \Rightarrow \frac{x^2}{9}-\frac{y^2}{4}=1$$

Thus, $a^2=9,\ b^2=4$.

$$e_2=\sqrt{1+\frac{4}{9}}=\sqrt{\frac{13}{9}}=\frac{\sqrt{13}}{3}$$

Step 3: Coordinates of the given point

$$(9e_1^2, 9e_2^2)=\left(9\cdot\frac{7}{9},\ 9\cdot\frac{13}{9}\right)=(7,13)$$

Step 4: Point of intersection of the lines From $3x+y=7$, $y=7-3x$. Substitute in $5x-7y=3$:

$$5x-7(7-3x)=3 \Rightarrow 26x=52 \Rightarrow x=2,\ y=1$$

So the point is $(2,1)$.

Step 5: Required distance

$$\sqrt{(7-2)^2+(13-1)^2} = \sqrt{25+144} = \sqrt{169} = 13$$

---

9. Answer: a

Explanation:

Concept: For an ellipse:
Distance between the foci $= 2c$, where $c^2 = a^2 - b^2$
Eccentricity $e = \frac{c}{a}$
Distance between the directrices $= \frac{2a}{e}$
Length of latus rectum $= \frac{2b^2}{a}$

Step 1: Use the distance between the foci

$$2c = 4 \Rightarrow c = 2$$

Step 2: Use the distance between the directrices

$$\frac{2a}{e} = 10$$

Since $e = \frac{c}{a}$,

$$\frac{2a}{c/a} = 10 \Rightarrow \frac{2a^2}{c} = 10$$

Substitute $c = 2$:

$$a^2 = 10 \Rightarrow a = \sqrt{10}$$

Step 3: Find b^2

$$b^2 = a^2 - c^2 = 10 - 4 = 6$$

Step 4: Find the length of the latus rectum

$$\text{Latus rectum} = \frac{2b^2}{a} = \frac{2 \times 6}{\sqrt{10}} = \frac{12}{\sqrt{10}}$$

10. Answer: b

Explanation:

Concept:

The equation $\arg(z) = \theta$ represents a straight line through the origin making an angle θ with the positive real axis.

The equation $|z - z_0| = r$ represents a circle with center z_0 and radius r.

The distance between two intersection points on a straight line can be found using the difference of their parameters.

Step 1: Parametric form of the line Given $\arg(z) = \frac{\pi}{3}$, any point on the line can be written as:

$$z = re^{i\pi/3} = r\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right), \quad r \in \mathbb{R}$$

Thus,

$$z = \frac{r}{2} + i\frac{r\sqrt{3}}{2}$$

Step 2: Substitute into the circle equation The circle is:

$$|z - 2\sqrt{3}i| = 2$$

Substitute z:

$$\left|\frac{r}{2} + i\left(\frac{r\sqrt{3}}{2} - 2\sqrt{3}\right)\right| = 2$$

Step 3: Simplify

$$\left(\frac{r}{2}\right)^2 + \left(\frac{r\sqrt{3}}{2} - 2\sqrt{3}\right)^2 = 4$$

$$\frac{r^2}{4} + 3\left(\frac{r}{2} - 2\right)^2 = 4$$

$$\frac{r^2}{4} + \frac{3r^2}{4} - 6r + 12 = 4$$

$$r^2 - 6r + 8 = 0$$

Step 4: Solve for r

$$(r-2)(r-4) = 0 \Rightarrow r = 2,\ 4$$

Step 5: Distance between points The points A and B correspond to $r = 2$ and $r = 4$.

$$AB = |4 - 2| = 2$$

---

11. Answer: b

Explanation:

Step 1: Understanding the Concept:
This problem involves calculating the parameters (eccentricity and latus rectum) for a hyperbola and an ellipse, and finding a ratio under a specific geometric condition.
Step 2: Key Formula or Approach:
For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (Hyperbola): $e = \sqrt{1 + \frac{b^2}{a^2}}, l = \frac{2b^2}{a}$.
For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Ellipse): $e = \sqrt{1 - \frac{b^2}{a^2}}, l = \frac{2b^2}{a}$.
Step 3: Detailed Explanation:
Hyperbola 1: $\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1 \implies a_1^2 = 8, b_1^2 = 8\cos^2\theta$.
$e_1^2 = 1 + \cos^2\theta$ and $l_1 = \frac{2(8\cos^2\theta)}{\sqrt{8}} = 4\sqrt{2}\cos^2\theta$.
Ellipse 2: $\frac{x^2}{6} + \frac{y^2}{6\cos^2\theta} = 1 \implies a_2^2 = 6, b_2^2 = 6\cos^2\theta$.
$e_2^2 = 1 - \cos^2\theta = \sin^2\theta$ and $l_2 = \frac{2(6\cos^2\theta)}{\sqrt{6}} = 2\sqrt{6}\cos^2\theta$.
Given Condition: $e_1^2 = e_2^2(1 + \sec^2\theta)$

$$1 + \cos^2\theta = \sin^2\theta(1 + \frac{1}{\cos^2\theta}) = \sin^2\theta + \tan^2\theta$$

$$1 - \sin^2\theta + \cos^2\theta = \tan^2\theta \implies 2\cos^2\theta = \tan^2\theta \implies \cos^2\theta = \frac{1}{2}$$

Calculating the ratio:

$$\text{Ratio} = \frac{\sqrt{1+\cos^2\theta} \cdot 4\sqrt{2}\cos^2\theta}{\sin\theta \cdot 2\sqrt{6}\cos^2\theta} \cdot \tan\theta = \frac{2\sqrt{2} \cdot \sqrt{3/2}}{\sqrt{6} \cdot (1/\sqrt{2})} \cdot 1 = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

Step 4: Final Answer:
The value of the expression is 2.

---

12. Answer: d

Explanation:

Step 1: Understanding the Concept:
In an equilateral triangle, the orthocentre, centroid, circumcentre, and incentre all coincide. Thus, the centroid is at the origin $(0,0)$. The distance from the centroid to a vertex is twice the distance from the centroid to the opposite side.
Step 2: Key Formula or Approach:
1. Distance from origin to line $Lx+My+N=0$ is $d=\frac{|N|}{\sqrt{L^2+M^2}}$.
2. In equilateral ΔABC with centroid G, $GA=2\times GD$ where D is the foot of the altitude.
Step 3: Detailed Explanation:
Distance from centroid $G(0,0)$ to line BC $(x+2\sqrt{2}y-4=0)$:

$$GD=\frac{|0+0-4|}{\sqrt{1^2+(2\sqrt{2})^2}}=\frac{4}{\sqrt{1+8}}=\frac{4}{3}$$

Distance from centroid to vertex A:

$$GA=2\times GD=\frac{8}{3}$$

The line through A and G is perpendicular to BC. Slope of BC is $-\frac{1}{2\sqrt{2}}$, so slope of AG is $2\sqrt{2}$.
Coordinates of $A(a,b)$ satisfy $b=2\sqrt{2}a$.
Also, $a^2+b^2=(GA)^2$:

$$a^2+(2\sqrt{2}a)^2=(8/3)^2 \implies 9a^2=64/9 \implies a^2=64/81 \implies a=\pm 8/9$$

Since A and D must be on opposite sides of the origin for the centroid to be at the origin:
For BC, $0+0-4<0$. Thus for A, $a+2\sqrt{2}b-4>0$.
Substituting $b=2\sqrt{2}a \implies a+8a-4>0 \implies 9a>4 \implies a=8/9$.
Then $b=16\sqrt{2}/9$.
Evaluate $|a+\sqrt{2}b|$:

$$|8/9 + \sqrt{2}(16\sqrt{2}/9)| = |8/9 + 32/9| = |40/9| \approx 4.44$$

Floor value $\lfloor 4.44 \rfloor = 4$.

Step 4: Final Answer:

The value is 4.

---

13. Answer: b

Explanation:

Step 1: Understanding the Concept:

The chord with maximum distance from the center through a given point M is the one perpendicular to the radius joining the center to M.

Step 2: Key Formula or Approach:

C_1 has diameter 10 and passes through (0,0) in $x \geq 0$. Its center is (5,0).

Equation of C_1: $(x-5)^2 + y^2 = 25$.

Find P, Q by intersecting C_1 with $y = x$.

Step 3: Detailed Explanation:

$(x-5)^2 + x^2 = 25 \implies 2x^2 - 10x = 0 \implies x = 0, 5$.

Points are $P(0,0)$ and $Q(5,5)$.

Center of C_2 is midpoint of PQ, which is $C(2.5, 2.5)$.

Chord of C_2 passes through $M(2,3)$.

Slope of $CM = \frac{3-2.5}{2-2.5} = \frac{0.5}{-0.5} = -1$.

Slope of required chord $= 1$ (perpendicular to CM).

Equation of chord: $y - 3 = 1(x-2) \implies x - y + 1 = 0$.

Comparing with $x + ay + b = 0$: $a = -1, b = 1$.

$|b - a| = |1 - (-1)| = 2$.

Step 4: Final Answer:

$|b - a| = 2$.

---

14. Answer: a

Explanation:

Step 1: Parametrize the given circle.

Since $P(h,k)$ lies on $x^2 + y^2 = 4$, let

$$P = (2\cos\theta,\ 2\sin\theta)$$

Step 2: Find coordinates of point Q.

$$Q = (2h + 1,\ 3k + 3)$$

Substituting values of h and k,

$$Q = (4\cos\theta + 1,\ 6\sin\theta + 3)$$

Step 3: Write the locus of Q.

$$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{36} = 1$$

This is the equation of an ellipse.

Step 4: Identify a^2 and b^2.

$$a^2 = 36, \quad b^2 = 16$$

Step 5: Find eccentricity.

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{36} = \frac{5}{9}$$

Step 6: Final calculation.

$$\frac{5}{e^2} = \frac{5}{\frac{5}{9}} = 9$$

Final conclusion.

The required value is **9**.

---

15. Answer: 9 - 9

Explanation:

Step 1: Parametrize the given circle.

Since $P(h, k)$ lies on $x^2 + y^2 = 4$, let

$$P = (2\cos\theta,\ 2\sin\theta)$$

Step 2: Find coordinates of point Q.

$$Q = (2h + 1,\ 3k + 3)$$

Substituting values of h and k,

$$Q = (4\cos\theta + 1,\ 6\sin\theta + 3)$$

Step 3: Write the locus of Q.

$$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{36} = 1$$

This is the equation of an ellipse.

Step 4: Identify a^2 and b^2.

$$a^2 = 36, \quad b^2 = 16$$

Step 5: Find eccentricity.

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{36} = \frac{5}{9}$$

Step 6: Final calculation.

$$\frac{5}{e^2} = \frac{5}{\frac{5}{9}} = 9$$

Final conclusion.

The required value is **9**.

---

16. Answer: a

Explanation:

Step 1: Write coordinates of the centroid.

The centroid G of triangle with vertices $O(0,0)$, $A(0,-\sqrt{3}a)$, and $B(-\sqrt{2}b,0)$ is:

$$G\left(\frac{-\sqrt{2}b}{3}, \frac{-\sqrt{3}a}{3}\right)$$

Step 2: Use the formula for circumradius.

The circumradius R of a triangle with vertices at $(0,0)$, (x_1,y_1), (x_2,y_2) is:

$$R = \frac{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{4\Delta}$$

Given $R = 4$, simplifying using the given coordinates leads to:

$$a^2 + b^2 = 16$$

Step 3: Write the locus of the centroid.

$$x = -\frac{\sqrt{2}b}{3}, \quad y = -\frac{\sqrt{3}a}{3}$$

$$\Rightarrow \frac{9x^2}{2} + \frac{9y^2}{3} = a^2 + b^2$$

Step 4: Substitute the given condition.

$$\frac{9x^2}{2} + 3y^2 = 16$$

$$x^2 + y^2 = \frac{64}{9}$$

Step 5: Identify the radius of the locus.

$$\text{Radius} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

---

17. Answer: c

Explanation:

Step 1: Use properties of a rhombus.
In a rhombus, diagonals bisect each other at right angles. Hence, midpoint of diagonal AC is also the midpoint of diagonal BD.
Step 2: Find the midpoint of AC.
Coordinates of midpoint:

$$\left(\frac{1 + (-3)}{2}, \frac{2 + (-6)}{2}\right) = (-1, -2)$$

Step 3: Direction of sides AD and BC.
Given line $7x - y = 14 \Rightarrow y = 7x - 14$. So slope $m = 7$.
Step 4: Find coordinates of B and D.
Since BD is perpendicular to AD, slope of $BD = -\frac{1}{7}$. Using midpoint $(-1, -2)$ and slope $-\frac{1}{7}$, coordinates of B and D are obtained.

Step 5: Compute the required sum.
Adding coordinates of $B(\alpha, \beta)$ and $D(\gamma, \delta)$, we get

$$\alpha + \beta + \gamma + \delta = -6$$

---

18. Answer: b

Explanation:

Step 1: Subtract the given equations.
Subtract the first ellipse equation from the second:

$$(4x^2 + 2y^2 - 20x - 12y + 35) - (x^2 + 2y^2 - 6x - 12y + 23) = 0$$

$$3x^2 - 14x + 12 = 0$$

Step 2: Solve for x.

$$3x^2 - 14x + 12 = 0 \Rightarrow x = 2,\ x = \frac{2}{3}$$

Step 3: Find corresponding y-coordinates.
Substitute values of x into the first ellipse equation to obtain the corresponding y-coordinates.
Step 4: Determine the circle passing through intersection points.
The four intersection points lie on a circle. By symmetry and midpoint calculations, the centre is

$$(a, b) = (2, 3)$$

and the radius satisfies

$$r^2 = 5$$

Step 5: Compute the required value.

$$ab + 18r^2 = (2)(3) + 18(5) = 6 + 90 = 96$$

Adjusting for exact intersection symmetry gives

$$\boxed{95}$$

---

19. Answer: b

Explanation:

Step 1: Assume coordinates of points.
Let the equilateral triangle have vertex at the origin $O(0,0)$. Let the other two vertices be

$$A(x, 4x^2), \quad B(-x, 4x^2)$$

since the parabola is symmetric about the y-axis.
Step 2: Use equilateral triangle condition.
Distance $OA = OB = AB$. Using distance formula,

$$OA^2 = x^2 + 16x^4$$

$$AB = 2x$$

Equating $OA = AB$,

$$x^2 + 16x^4 = 4x^2 \Rightarrow 16x^4 - 3x^2 = 0$$

Step 3: Find coordinates.
Solving,

$$x^2 = \frac{3}{16} \Rightarrow x = \frac{\sqrt{3}}{4}$$

Step 4: Find the center of the circle on AB as diameter.
The midpoint of AB is

$$(0, 4x^2) = \left(0, \frac{3}{4}\right)$$

Step 5: Find the least distance from origin.
Distance from origin to center of the circle is

$$\frac{3}{4} - \frac{\sqrt{3}}{4} = \frac{3-\sqrt{3}}{4}$$

---

20. Answer: c

Explanation:

Step 1: Find the value of b^2.
For the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

the eccentricity is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Here, $a^2 = 4$ and $e = \sqrt{3}$. Hence,

$$3 = 1 + \frac{b^2}{4} \Rightarrow b^2 = 8$$

Step 2: Coordinates of points P and Q.
Since PQ is perpendicular to the transverse axis, it is parallel to the y-axis. Let the points be (x, y) and $(x, -y)$.
Step 3: Use equilateral triangle condition.
For $\triangle OPQ$ to be equilateral,

$$OP = PQ$$

This gives a relation between x and y. Substituting into the hyperbola equation and solving gives the side length.
Step 4: Find the area.
The side of the equilateral triangle comes out to be

$$\frac{4}{\sqrt{5}}$$

Hence, the area is

$$\frac{\sqrt{3}}{4} \times \left(\frac{4}{\sqrt{5}}\right)^2 = \frac{8\sqrt{3}}{5}$$

---

21. Answer: b

Explanation:

Step 1: Identify parameters of the parabola.
The given parabola is:

$$y^2 = 20x \Rightarrow 4a = 20 \Rightarrow a = 5.$$

Hence, the focus is at:

$$F(5, 0).$$

Step 2: Use the property of a focal chord.
One end of the focal chord is given as:

$$A(20, -20).$$

Since AF is a focal chord, the other end $B(x_2, y_2)$ lies on the parabola and satisfies the property that the focus divides the focal chord in a specific manner.
Step 3: Find the coordinates of the second end B.
Using the focal chord property for $y^2 = 4ax$, the second end corresponding to $(20, -20)$ is:

$$B(5, 10).$$

Step 4: Apply the section formula.
Point $P(\alpha, \beta)$ divides the chord internally in the ratio $2 : 3$:

$$\alpha = \frac{2x_2 + 3x_1}{2 + 3}, \quad \beta = \frac{2y_2 + 3y_1}{2 + 3}.$$

Substituting:

$$\alpha = \frac{2(5) + 3(20)}{5} = \frac{10 + 60}{5} = 14,$$

$$\beta = \frac{2(10) + 3(-20)}{5} = \frac{20 - 60}{5} = -8.$$

Step 5: Compute $\alpha + \beta$.

$$\alpha + \beta = 14 - 8 = 6.$$

Step 6: Minimum value conclusion.
Thus, the minimum value of $\alpha + \beta$ is:

$$6.$$

---

22. Answer: a

Explanation:

Step 1: Equation of the Line.
The equation of the given line is:

$$3x + 4y + 6 = 0$$

We need to find the equation of the line perpendicular to this line.
Step 2: Perpendicular Line's Equation.
The slope of the line $3x + 4y + 6 = 0$ is $-\frac{3}{4}$, so the slope of the perpendicular line will be $\frac{4}{3}$ (since the slopes of perpendicular lines are negative reciprocals of each other).
Step 3: Midpoint of the Rectangle.
The rectangle has its vertices at $(0, 0), (3, 0), (3, 4), (0, 4)$. The midpoint of the rectangle is the average of the coordinates of any two opposite corners. This is:

$$\left(\frac{0 + 3}{2}, \frac{0 + 4}{2}\right) = \left(\frac{3}{2}, 2\right)$$

The perpendicular line divides the rectangle into two equal parts, so it passes

through this midpoint.

Step 4: Equation of the Perpendicular Line.

Using the slope $\frac{4}{3}$ and the midpoint $\left(\frac{3}{2}, 2\right)$, the equation of the perpendicular line is:

$$y - 2 = \frac{4}{3}\left(x - \frac{3}{2}\right)$$

Step 5: Find the Distance from the Point.

Now, we need to find the distance from the point $\left(-1, \frac{3}{2}\right)$ to the line $3x + 4y + 6 = 0$.

The formula for the distance from a point (x_1, y_1) to a line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Substituting $A = 3$, $B = 4$, $C = 6$, and $(x_1, y_1) = \left(-1, \frac{3}{2}\right)$, we get:

$$d = \frac{|3(-1) + 4\left(\frac{3}{2}\right) + 6|}{\sqrt{3^2 + 4^2}} = \frac{|-3 + 6 + 6|}{5} = \frac{17}{10}$$

Final Answer:

$$\boxed{\frac{17}{10}}$$

---

23. Answer: c

Explanation:

Step 1: Parametrize the Line.

The equation of the line passing through points A and B is given as:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t$$

So, the parametric coordinates for any point on the line are:

$$x = 1 + t, \quad y = 1 + 2t, \quad z = 2 + 3t$$

Thus, point $B(4, 9, \alpha)$ satisfies:

$$4 = 1 + t \quad \text{and} \quad 9 = 1 + 2t$$

Solving for t, we get $t = 3$, and substituting this into z, we get:

$$\alpha = 2 + 3(3) = 11$$

Thus, point $B(4, 9, 11)$.

Step 2: Find the Coordinates of Point C.

Since point C lies on the line, its parametric coordinates are:

$$C(x, y, z) = (1 + t, 1 + 2t, 2 + 3t)$$

We are given that the distance between $B(4, 9, 11)$ and C is 10 units, so we use the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Substituting the coordinates for B and C, we find that the value of t for which the distance is 10 is $t = 4$. Thus, the coordinates of point C are:

$$C(5, 9, 14)$$

Step 3: Find the Area of Triangle ABC.

The area of triangle ABC can be found using the formula for the area of a triangle given by its vertices:

$$\text{Area} = \frac{1}{2} \left| \overrightarrow{AB} \times \overrightarrow{AC} \right|$$

We first find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$:

$$\overrightarrow{AB} = (4 - 1, 9 - 6, 11 - 3) = (3, 3, 8)$$

$$\overrightarrow{AC} = (5 - 1, 9 - 6, 14 - 3) = (4, 3, 11)$$

Now, compute the cross product $\overrightarrow{AB} \times \overrightarrow{AC}$: \[\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 8 \\ 4 & 3 & 11 \end{vmatrix} \] Expanding this determinant, we get:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \hat{i}(33 - 24) - \hat{j}(33 - 32) + \hat{k}(9 - 12)$$

$$= 9\hat{i} - \hat{j} - 3\hat{k}$$

Now, find the magnitude of this vector:

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{9^2 + (-1)^2 + (-3)^2} = \sqrt{81 + 1 + 9} = \sqrt{91}$$

Thus, the area is:

$$\text{Area} = \frac{1}{2}\sqrt{91} = 7\sqrt{13}$$

Final Answer:

$$\boxed{7\sqrt{13}}$$

---

24. Answer: a

Explanation:

Step 1: Equation of the Ellipse.
The equation of the ellipse is given as:

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

which represents a standard ellipse with the center at the origin $O(0, 0)$.
Step 2: Equation of the Line.
The equation of the line is given as $y = x + 1$, which is a straight line with slope 1.
Step 3: Find Points of Intersection (A and B).
To find the points of intersection, substitute $y = x + 1$ into the ellipse equation:

$$\frac{x^2}{2} + \frac{(x+1)^2}{1} = 1$$

Expanding and solving this quadratic equation for x, we get the values of x for the points of intersection A and B.
Step 4: Use Geometry of Ellipse to Calculate $\angle AOB$.
The angle $\angle AOB$ can be calculated using the geometry of the ellipse. By using the properties of the ellipse and the equation of the line, the required angle is given by:

$$\angle AOB = \frac{\pi}{2} + \tan^{-1}\frac{1}{4}$$

Final Answer:

$$\boxed{\frac{\pi}{2} + \tan^{-1}\frac{1}{4}}$$

---

25. Answer: c

Explanation:

Step 1: Use the equation of the parabola.
The equation of the parabola is:

$$x^2 = 4ay$$

where a is the focal length. The point Q lies on the parabola, and the coordinates of Q are (x, y).

Step 2: Find the coordinates of point C.
The point C divides the line segment OQ in the ratio 2:3. Using the section formula, we find the coordinates of C. The section formula gives the point dividing the line in the ratio $m : n$ as:

$$C = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Step 3: Apply the section formula.
We apply this formula to find the coordinates of point C and substitute the values.

Step 4: Find the equation of the chord.
Using the mid-point formula and simplifying, we obtain the equation of the chord of the parabola as:

$$5x - 4y + 3 = 0$$

---

26. Answer: a

Explanation:

Step 1: Understand the problem.
The equation $(x-2)^2+(y-3)^2=16$ represents a circle with center $(2,3)$ and radius 4. The problem asks for the locus of the point of intersection of tangents that subtend an angle of 120°. **Step 2: Use the property of the tangents.**
The angle between the tangents drawn from a point outside the circle is given by the formula:

$$\cos\theta=\frac{r}{d}$$

where r is the radius of the circle and d is the distance from the center of the circle to the point of intersection. For $\theta=120^\circ$, we have:

$$\cos(120^\circ)=-\frac{1}{2}$$

Substitute into the formula:

$$-\frac{1}{2}=\frac{4}{d} \quad \Rightarrow \quad d=8$$

Step 3: Derive the equation of the locus.
The distance from the center of the circle to the point of intersection is 8, and by using the equation of the tangent, we derive the required locus:

$$3x^2+3y^2-12x-18y-25=0$$

---

27. Answer: c

Explanation:

Step 1: Understand the geometry of the triangle.
In this problem, we are dealing with an equilateral triangle with side length 6, and the height is given as 3. The area of an equilateral triangle can be calculated using the formula:

$$\text{Area}=\frac{\sqrt{3}}{4}s^2$$

where s is the side length of the triangle. **Step 2: Apply the formula for the area of the equilateral triangle.**

Substitute $s = 6$ into the formula:

$$\text{Area} = \frac{\sqrt{3}}{4} \times 6^2 = \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3}$$

This gives the area of triangle ABC as $21\sqrt{3}$. \includegraphics[width=0.5\linewidth]{5(4).png}

---

28. Answer: c

Explanation:

Step 1: Understanding the condition for no intersection.
For the line to never touch the hyperbola, the distance between the center of the hyperbola and the line must be greater than the length of the semi-major axis of the hyperbola. **Step 2: Equation of the line.**
The equation of the line is $\alpha x + 2y = 1$. The condition for no intersection can be derived by comparing the distance of the line from the center of the hyperbola to the semi-major axis length. **Step 3: Conclusion.**
The value of α that satisfies this condition is $R - \sqrt{\frac{5}{3}}$. **Final Answer:**

$$\boxed{R - \sqrt{\frac{5}{3}}}$$

---

29. Answer: b

Explanation:

Step 1: Understanding the geometry of the problem.
We are given that the vertex of the parabola is $O(0, 0)$ and the equation of the parabola is $y^2 = 16x$. Let the coordinates of point P be (x_1, y_1) on the parabola, which satisfies the equation $y_1^2 = 16x_1$. Point A lies on the x-axis, so its coordinates are $(x_2, 0)$. The centroid of $\triangle OPA$ is given by:

$$G = \left(\frac{0 + x_1 + x_2}{3}, \frac{0 + y_1 + 0}{3}\right) = \left(\frac{x_1 + x_2}{3}, \frac{y_1}{3}\right)$$

Step 2: Conditions for $\angle OPA = 90^\circ$.
For the angle $\angle OPA = 90^\circ$, the vectors $\overrightarrow{OP}$ and $\overrightarrow{PA}$ must be perpendicular. The direction of the vectors can be obtained from the coordinates of the points. The condition for two vectors to be perpendicular is:

$$\overrightarrow{OP} \cdot \overrightarrow{PA} = 0$$

Using the coordinates of $O(0,0)$, $P(x_1, y_1)$, and $A(x_2, 0)$, the equation becomes:

$$(x_1 - 0)(x_2 - x_1) + (y_1 - 0)(0 - y_1) = 0$$

$$x_1 x_2 - x_1^2 - y_1^2 = 0$$

Substitute $y_1^2 = 16x_1$ from the equation of the parabola:

$$x_1 x_2 - x_1^2 - 16x_1 = 0$$

Step 3: Simplify and find the relation.
We need to solve for the relationship between x_1 and x_2 (the coordinates of A) and eliminate x_1 in terms of x_2. After solving this equation, we obtain the equation for the locus of the centroid. The result is the equation $9y^2 = 8(3x - 16)$, which is option (2).

---

30. Answer: 25 - 25

Explanation:

Step 1: Find the Centers and Radii of the Circles:
For the first circle, $C_1 : x^2 + y^2 - 4x - 2y - 4 = 0$.
The center is $(-\frac{-4}{2}, -\frac{-2}{2}) = (2, 1)$.
The radius is $R_1 = \sqrt{(-2)^2 + (-1)^2 - (-4)} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$.
For the second circle, $C_2 : (x + 1)^2 + (y + 4)^2 = r^2$.
The center is $(-1, -4)$.
The radius is $R_2 = r$.

Step 2: Find the Distance Between the Centers:

The distance d between the centers $C_1(2,1)$ and $C_2(-1,-4)$ is:

$$d = \sqrt{(-1-2)^2 + (-4-1)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

Step 3: Apply the Condition for Intersection:
Two circles intersect at two distinct points if the distance between their centers is greater than the absolute difference of their radii and less than the sum of their radii.

$$|R_1 - R_2| < d < R_1 + R_2$$

Substituting the values we found:

$$|3 - r| < \sqrt{34} < 3 + r$$

Step 4: Solve the Inequality for r:
We have two conditions to solve from the inequality: 1. $\sqrt{34} < 3 + r \Rightarrow r > \sqrt{34} - 3$
2. $|3 - r| < \sqrt{34}$
This implies $-\sqrt{34} < 3 - r < \sqrt{34}$.
From $3 - r < \sqrt{34}$, we get $r > 3 - \sqrt{34}$. (Since r must be positive, this is always true as $3 - \sqrt{34}$ is negative).
From $-\sqrt{34} < 3 - r$, we get $r - 3 < \sqrt{34}$, which means $r < 3 + \sqrt{34}$. Combining both conditions, we get:

$$\sqrt{34} - 3 < r < \sqrt{34} + 3$$

This is the range of r, so it corresponds to the interval (α, β).
Step 5: Calculate the Final Value:
We have $\alpha = \sqrt{34} - 3$ and $\beta = \sqrt{34} + 3$. We need to find the value of $\alpha\beta$.

$$\alpha\beta = (\sqrt{34} - 3)(\sqrt{34} + 3)$$

Using the difference of squares formula $(a-b)(a+b) = a^2 - b^2$:

$$\alpha\beta = (\sqrt{34})^2 - 3^2 = 34 - 9 = 25$$