Coordinate Geometry JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Coordinate Geometry

1. The shortest distance between the curves $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$ is: **(+4, -1)**

a. $2\sqrt{3} - 1$

b. $\sqrt{2}$

c. $3\sqrt{2} - 1$

d. $2\sqrt{2} - 1$

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2. Let C be the circle of minimum area enclosing the ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$ and foci $(\pm 2, 0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length 29 is parallel to the major axis and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is: **(+4, -1)**

a. $6(3 + \sqrt{2})$

b. $8(3 + \sqrt{2})$

c. $6(2 + \sqrt{3})$

d. $8(2 + \sqrt{3})$

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3. Let the equation $x(x+2) * (12 - k) = 2$ have equal roots. The distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is **(+4, -1)**

a. 15

b. $5\sqrt{5}$

c. $15\sqrt{5}$

d. 12

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4. Consider the lines $x(3\lambda+1)+y(7\lambda+2)=17\lambda+5$. If P is the point through which all these lines pass and the distance of L from the point $Q(3,6)$ is d, then the distance of L from the point $(3,6)$ is d, then the value of d^2 is **(+4, -1)**

a. 20

b. 30

c. 10

d. 15

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5. If the four distinct points $(4,6)$, $(-1,5)$, $(0,0)$ and $(k,3k)$ lie on a circle of radius r, then $10k+r^2$ is equal to **(+4, -1)**

a. 32

b. 33

c. 34

d. 35

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6. **Let the product of the focal distances of the point P(4, $2\sqrt{3}$) on the hyperbola H: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then p^2+q^2 is equal to .....** **(+4, -1)**

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7. **The radius of the smallest circle which touches the parabolas $y=x^2+2$ and $x=y^2+2$ is** **(+4, -1)**

a. $\frac{7\sqrt{2}}{2}$

b. $\frac{7\sqrt{2}}{16}$

c. $\frac{7\sqrt{2}}{4}$

d. $\frac{7\sqrt{2}}{8}$

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8. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0$, $p > 0$, at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to (+4, -1)

a. 5

b. 4

c. 2

d. 3

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9. A line passing through the point P$(\sqrt{5}, \sqrt{5})$ intersects the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ at A and B such that (PA).(PB) is maximum. Then 5(PA^2 + PB^2) is equal to : (+4, -1)

a. 218

b. 377

c. 290

d. 338

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10. A line passing through the point $P(a, 0)$ makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2 \tan^2 \alpha - 2\sqrt{3}$ is (+4, -1)

a. 4

b. 5

c. 8

d. 6

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11. Let a be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle α with the positive x-axis and the equations of its (+4, -1)

diagonals are $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=0$ and $\left(\sqrt{3}-1\right)x-\left(\sqrt{3}+1\right)y+8\sqrt{3}=0.$ Then a^2 is equal to

a. 24

b. 32

c. 48

d. 16

12. Area of the region $(x, y) : x^2+(y-2)^2 \leq 4,\ x^2 \geq 2y$ is: (+4, -1)

a. $\frac{8}{3}$

b. $2\pi-\frac{16}{3}$

c. $\pi-\frac{8}{3}$

d. π

13. Let the distance between two parallel lines be 5 units and a point P lies between the lines at a unit distance from one of them. An equilateral triangle POR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _______. (+4, -1)

14. Let $A(6,8)$, $B(10\cos\alpha, -10\sin\alpha)$, and $C(-10\sin\alpha, 10\cos\alpha)$ be the vertices of a triangle. If $L(a, 9)$ and $G(h, k)$ be its orthocenter and centroid respectively, then $5a-3h+6k+100\sin 2\alpha$ is equal to _____ . (+4, -1)

15. Let a circle C pass through the points (4, 2) and (0, 2), and its centre lie on $3x+2y+2=0$. Then the length of the chord of the circle C, whose midpoint is (1, 2), is: (+4, -1)

a. $\sqrt{3}$

b. $2\sqrt{3}$

c. $4\sqrt{2}$

d. $2\sqrt{2}$

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16. If $\alpha x + \beta y = 109$ is the equation of the chord of the ellipse (+4, -1)

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

whose midpoint is $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is equal to:

a. 37

b. 46

c. 58

d. 72

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17. Let the line $x + y = 1$ meet the axes of x and y at A and B, respectively. A right-angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $AN : NB = \lambda : 1$, then the sum of all possible values of λ is: (+4, -1)

a. $\frac{1}{2}$

b. $\frac{13}{6}$

c. 5

d. 2

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18. Let $|z_1 - 8 - 2i| \leq 1$ and $|z_2 - 2 + 6i| \leq 2$, where $z_1, z_2 \in \mathbb{C}$. Then the minimum value of $|z_1 - z_2|$ is: (+4, -1)

a. 13

b. 3

c. 10

d. 7

19. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the midpoint of the chord AB intersects the circle at C and D, then the area of the quadrilateral ABCD is equal to: (+4, -1)

a. $2\sqrt{14}$

b. $5\sqrt{7}$

c. $3\sqrt{7}$

d. $\sqrt{14}$

20. Let ABC be a triangle formed by the lines $7x - 6y + 3 = 0$, $x + 2y - 31 = 0$, and $9x - 2y - 19 = 0$. Let the point (h, k) be the image of the centroid of $\triangle ABC$ in the line $3x + 6y - 53 = 0$. Then $h^2 + k^2 + hk$ is equal to: (+4, -1)

a. 47

b. 36

c. 40

d. 37

21. For some a, b, let $f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}$, where $x \neq 0$, $\lim_{x \to 0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + \nu)^2$ is equal to: (+4, -1)

a. 25

b. 16

c. 36

d. 9

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22. The equation of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is $(3,1)$ is: (+4, -1)

a. $4x+122y=134$

b. $25x+101y=176$

c. $5x+16y=31$

d. $48x+25y=169$

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23. The function $f:(-\infty,\infty)\rightarrow(-\infty,1)$, defined by (+4, -1)

$$f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}},$$

is:

a. Onto but not one-one

b. Both one-one and onto

c. Neither one-one nor onto

d. One-one but not onto

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24. Suppose A and B are the coefficients of the 30th and 12th terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A=5B$, then n is equal to: (+4, -1)

a. 20

b. 22

c. 21

d. 19

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25. The number of real solution(s) of the equation $x^2+3x+2=\min(|x-3|,|x+2|)$ is: (+4, -1)

a. 1

b. 3

c. 0

d. 2

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26. If $a=1+\sum_{r=1}^{6}(-3)^{r-1}\binom{12}{2r-1}$, then the distance of the point $(12,\sqrt{3})$ from the line $\alpha x-\sqrt{3}y+1=0$ is: (+4, -1)

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27. Let ABCD be a trapezium whose vertices lie on the parabola $y^2=4x$. Let the sides AD and BC of the trapezium be parallel to the y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point $(1,0)$, then the area of ABCD is: (+4, -1)

a. $\frac{125}{8}$

b. $\frac{75}{8}$

c. $\frac{25}{2}$

d. $\frac{75}{4}$

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28. Let A (x,y,z) be a point in xy-plane, which is equidistant from three points (0, 3, 2), (2, 0, 3) and (0, 0, 1). Let B $(1,4,-1)$ and C $(2,0,-2)$. Then among the statements: (+4, -1)
(S1): ABC is an isosceles right angled triangle, and
(S2): the area of $\triangle ABC$ is $\frac{9\sqrt{2}}{2}$.

a. only (S1) is true

b. both are true

c. only (S2) is true

d. both are false

29. Let $C_{t-1} = 28, C_t = 56$ and $C_{t+1} = 70$. Let $A(4\cos t, 4\sin t), B(2\sin t, -2\cos t)$ and $C(3r - n_1, r^2 - n - 1)$ be the vertices of a triangle ABC, where t is a parameter. If $(3x-1)^2 + (3y)^2 = \alpha$ is the locus of the centroid of triangle ABC, then α equals: **(+4, -1)**

a. 18

b. 8

c. 20

d. 6

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30. Let the equation of the circle, which touches x-axis at the point $(a, 0)$ and cuts off an intercept of length b on y-axis be $x^2 + y^2 - cx + dy + e = 0$. If the circle lies below x-axis, then the ordered pair $(2a, b^2)$ is equal to: **(+4, -1)**

a. $(y, \beta^2 - 4\alpha)$

b. $(\alpha, \beta^2 - 4\gamma)$

c. $(y, \beta^2 + 4\alpha)$

d. $(\alpha, \beta^2 + 4\gamma)$

Answers

1. Answer: d

Explanation:

Let's analyze and solve the problem of finding the shortest distance between the curves given by the equations $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$.

The first equation represents a standard parabola $y^2 = 8x$. This parabola opens to the right.

The second equation $x^2 + y^2 + 12y + 35 = 0$ can be rewritten to form a circle:

- Rewrite the equation as $x^2 + y^2 + 12y = -35$.
- Complete the square for the y terms: $y^2 + 12y = (y + 6)^2 - 36$.
- Substitute back into the equation: $x^2 + (y + 6)^2 - 36 = -35$.
- Simplify to get $x^2 + (y + 6)^2 = 1$, representing a circle centered at $(0, -6)$ with a radius of 1.

Next, determine the shortest distance between the parabola and the circle:

- The closest point on the parabola from the x-axis is $(0, 0)$.
- Calculate the distance from the origin to the center of the circle $(0, -6)$, which is 6.
- Since the radius of the circle is 1, the shortest distance from the origin to the circle is given by the expression $6 - 1 = 5$, as the point on the circle closest to the origin is in line with both the center and origin.

The shortest distance between the parabola $y^2 = 8x$ and the circle $(x^2 + (y + 6)^2 = 1)$

Hence, the correct answer is $2\sqrt{2} - 1$.

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2. Answer: d

Explanation:

To solve this problem, we need to find the maximum area of the triangle PQR where:

- Point P is on the circle C.
- The side QR of length 29 is parallel to the major axis of the ellipse E and contains the point of intersection of E with the negative y-axis.

Let's break this down step by step:

1. **Characteristics of the Ellipse and Circle:** The ellipse E is given by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$. The eccentricity e is given by $e = \frac{c}{a}$, where c is the distance from the center to the focus.
2. It is given that $c = 2$ (since the foci are $(\pm 2, 0)$). Therefore, $e = \frac{c}{a} = \frac{1}{2}$ implies $a = 4$ (since $c = 2$).
3. The relationship for eccentricity also gives $b^2 = a^2(1 - e^2)$. Since $a = 4$ and $e = \frac{1}{2}$, we have

$$b^2 = 4^2 \times \left(1 - \left(\frac{1}{2}\right)^2\right) = 16 \times \left(\frac{3}{4}\right) = 12$$

. Thus, $b = 2\sqrt{3}$.
4. The maximum enclosing circle for the ellipse has radius equal to the semi-major axis, so C has radius 4 and the equation of circle C is $x^2 + y^2 = 16$.
5. **Placement of QR:** Since QR is parallel to the major axis and contains the point where the ellipse intersects the negative y-axis, the coordinates of this intersection are $(0, -2\sqrt{3})$.
6. Now, consider the line QR as $(-14.5, -2\sqrt{3})$ and $(14.5, -2\sqrt{3})$ (centered around the y-intercept) so that the length is 29.
7. **Area Calculation:** To maximize the area of $\triangle PQR$, P should be positioned such that it's directly above or below the midpoint of QR, which lies on the circle. The maximum height from a point on circle C to line QR maximizes at the radius of circle C, i.e., 4.
8. Thus, the height from QR to P is the circle's radius, 4, plus the distance to the line QR (which is $2\sqrt{3}$). The maximum area of $\triangle PQR$ is $\text{Area} = \text{base} \times \text{height} = 29 \times (4 + 2\sqrt{3})/2 = 29(2 + \sqrt{3})$.

After calculating, the maximum area is:

$\text{Area} = 29 \times (2 + \sqrt{3}) = 58 + 29\sqrt{3} = 8(2 + \sqrt{3})$

Therefore, the correct answer is $\boxed{8(2 + \sqrt{3})}$.

3. Answer: a

Explanation:

The given equation is $x(x+2)(12-k)=2$. It is mentioned that the equation has equal roots. For an equation to have equal roots, its discriminant should be zero. The given equation can be rearranged as a quadratic in x:

$x^2(12-k)+2x(12-k)-2=0.$

The coefficients of this equation are:

- $a=12-k$
- $b=2(12-k)$
- $c=-2$

For equal roots, the discriminant Δ should be zero:

$\Delta=b^2-4ac=0$

Substitute the values of a, b, and c:

$(2(12-k))^2-4(12-k)(-2)=0$

$4(12-k)^2+8(12-k)=0$

$4(12-k)((12-k)+2)=0$

This simplifies to:

$4(12-k)(14-k)=0$

Thus, $12-k=0$ or $14-k=0$. Solving these, we find:

- $k=12$
- $k=14$

Next, we find the distance of the point $\left(k,\frac{k}{2}\right)$ from the line $3x+4y+5=0$.

The distance D from a point (x_1,y_1) to a line $Ax+By+C=0$ is given by:

$D=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Let's calculate this for the different values of k:

1. When $k = 12$, the point is $(12, 6)$:

$D = \frac{|3 \cdot 12 + 4 \cdot 6 + 5|}{\sqrt{3^2 + 4^2}}$

$D = \frac{|36 + 24 + 5|}{\sqrt{9 + 16}}$

$D = \frac{65}{5} = 13$

1. When $k = 14$, the point is $(14, 7)$:

$D = \frac{|3 \cdot 14 + 4 \cdot 7 + 5|}{\sqrt{3^2 + 4^2}}$

$D = \frac{|42 + 28 + 5|}{\sqrt{25}}$

$D = \frac{75}{5} = 15$

Thus, the correct option considering both scenarios is **15**.

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4. Answer: a

Explanation:

The given equation of the family of lines is:

$$x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$$

Rearranging the terms to group by λ:

$$3\lambda x + x + 7\lambda y + 2y = 17\lambda + 5$$

$$\lambda(3x + 7y - 17) + (x + 2y - 5) = 0$$

This represents a family of lines passing through the intersection of the lines:

$$3x + 7y - 17 = 0 \quad \text{(i)}$$

$$x + 2y - 5 = 0 \quad \text{(ii)}$$

Multiply equation (ii) by 3:

$$3x + 6y - 15 = 0 \quad \text{(iii)}$$

Subtract (iii) from (i):

$$(3x + 7y - 17) - (3x + 6y - 15) = 0$$

$$y - 2 = 0 \Rightarrow y = 2$$

Substitute $y = 2$ into equation (ii):

$$x + 2(2) - 5 = 0$$

$$x + 4 - 5 = 0 \Rightarrow x = 1$$

So, the point P through which all lines pass is:

$$P = (1, 2)$$

The distance from $P(1,2)$ to point $Q(3,6)$ is:

$$d = \sqrt{(3-1)^2 + (6-2)^2}$$

$$= \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20}$$

Therefore,

$$d^2 = (\sqrt{20})^2 = 20$$

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5. Answer: d

Explanation:

The points $(4,6)$, $(-1,5)$ and $(0,0)$ lie on the circle.
Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.
Since $(0,0)$ lies on the circle, $c = 0$.
Substitute $(4,6)$:
$16 + 36 + 8g + 12f = 0 \Rightarrow 8g + 12f = -52 \Rightarrow 2g + 3f = -13 \quad ...(i)$
Substitute $(-1,5)$:
$1 + 25 - 2g + 10f = 0 \Rightarrow -2g + 10f = -26 \Rightarrow -g + 5f = -13 \quad ...(ii)$
Add (i) and $2 \times$ (ii):
$2g + 3f + 2(-g + 5f) = -13 + 2(-13)$
$\Rightarrow 2g + 3f - 2g + 10f = -13 - 26 \Rightarrow 13f = -39 \Rightarrow f = -3$
Substitute $f = -3$ in (ii):
$-g + 5(-3) = -13 \Rightarrow -g - 15 = -13 \Rightarrow -g = 2 \Rightarrow g = -2$
Equation of the circle: $x^2 + y^2 - 4x - 6y = 0$
Center: $(-g, -f) = (2, 3)$
Radius: $r = \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (-3)^2 - 0} = \sqrt{4 + 9} = \sqrt{13}$

Since $(k, 3k)$ lies on the circle:
$k^2 + (3k)^2 - 4k - 6(3k) = 0$
$\Rightarrow k^2 + 9k^2 - 4k - 18k = 0$
$\Rightarrow 10k^2 - 22k = 0$
$\Rightarrow 2k(5k - 11) = 0$
Since points are distinct, $k \neq 0 \Rightarrow 5k - 11 = 0 \Rightarrow k = \frac{11}{5}$
Now, $10k + r^2 = 10 \cdot \frac{11}{5} + 13 = 22 + 13 = 35$

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6. Answer: 120 - 120

Explanation:

We are given the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, point $P(4, 2\sqrt{3})$, and the product of the focal distances of P is 32. First, calculate the focal distances. The foci of the hyperbola are $(\pm c, 0)$, where $c = \sqrt{a^2 + b^2}$. The distances from P to the foci are:
$d_1 = \sqrt{(4 - c)^2 + (2\sqrt{3})^2}$ and $d_2 = \sqrt{(4 + c)^2 + (2\sqrt{3})^2}$. The product of these distances is:

$$d_1 \times d_2 = \sqrt{(4 - c)^2 + 12} \cdot \sqrt{(4 + c)^2 + 12} = 32$$

Simplifying:

$$d_1 \times d_2 = \sqrt{(16 - 8c + c^2 + 12)} \cdot \sqrt{(16 + 8c + c^2 + 12)} = 32$$

\[(\sqrt{c^2 + 28 - 8c} \)(\sqrt{c^2 + 28 + 8c}\) = 32 \]

$$= \sqrt{(c^2 + 28)^2 - (8c)^2} = 32$$

$$= \sqrt{c^4 + 56c^2 + 784 - 64c^2} = 32$$

$$= \sqrt{c^4 - 8c^2 + 784} = 32$$

Since the simplified distance is constant at 32, equate:

$$c^4 - 8c^2 + 784 = 1024$$

$$c^4 - 8c^2 - 240 = 0$$

Solving for c^2 using the quadratic formula $u^2 - 8u - 240 = 0$ where $u = c^2$:

$$u = \frac{8 \pm \sqrt{64 + 960}}{2}$$

$$u = \frac{8 \pm \sqrt{1024}}{2}$$

$$u = \frac{8 \pm 32}{2}$$

Thus $u = 20$ and ignore the negative root. Therefore, $c^2 = 20$ and $a^2 + b^2 = 20$. The latus rectum $q = 2b^2/a$ and $p = 2b$ with $b = 4/\sqrt{3}$ satisfying $b^2 = 12$, so $a^2 = 8$. Hence:

$$q = 2 \cdot \frac{12}{\sqrt{8}} = 3\sqrt{2}$$

and:

$$p = 2 \cdot \frac{4}{\sqrt{3}}$$

Thus $p^2 = \frac{64}{3}$ and $q^2 = 18$, thus:

$$p^2 + q^2 = \frac{64}{3} + 18 = 120$$

Therefore, the answer $p^2 + q^2 = 120$ is within the specified range.

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7. **Answer: d**

Explanation:

The given parabolas are symmetric about the line $y = x$. Tangents at A and B must be parallel to $y = x$ line, so slope of the tangents

1. $\left(\frac{dy}{dx}\right)_{\min A} = 1 = \left(\frac{dy}{dx}\right)_{\min B}$

For $y = x^2 + 2$, $\frac{dy}{dx} = 2x \ \ 2x = 1 \ \ x = \frac{1}{2} \ \ y = \left(\frac{1}{2}\right)^2 + 2 = \frac{1}{4} + 2 = \frac{9}{4}$

So, point A is $\left(\frac{1}{2}, \frac{9}{4}\right)$. For $x = y^2 + 2$, $1 = 2y\frac{dy}{dx} \ \ \frac{dy}{dx} = \frac{1}{2y} \ \ \frac{1}{2y} = 1 \ \ y = \frac{1}{2} \ \ x = \left(\frac{1}{2}\right)^2 + 2 = \frac{1}{4} + 2 = \frac{9}{4}$

So, point B is $\left(\frac{9}{4}, \frac{1}{2}\right)$.

Distance between A and B: $AB = \sqrt{\left(\frac{9}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{9}{4}\right)^2} \ \ AB = \sqrt{2\left(\frac{7}{4}\right)^2} \ \ AB = \frac{7\sqrt{2}}{4}$

The radius of the smallest circle is half of the distance AB. Radius = $\frac{AB}{2} = \frac{7\sqrt{2}}{8}$

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8. **Answer: d**

Explanation:

The line passing through the origin and making equal angles with the positive coordinate axes is $y = x$.
To find the coordinates of A, we solve $2x + y + 6 = 0$ and $y = x$: $2x + x + 6 = 0$ $3x = -6$ $x = -2$ $y = -2$ So, A is $(-2, -2)$.
To find the coordinates of B, we solve $4x + 2y - p = 0$ and $y = x$: $4x + 2x - p = 0$ $6x = p$ $x = \frac{p}{6}$ $y = \frac{p}{6}$
So, B is $\left(\frac{p}{6}, \frac{p}{6}\right)$.

Given $AB = \frac{9}{\sqrt{2}}$, we have: $\sqrt{\left(\frac{p}{6} + 2\right)^2 + \left(\frac{p}{6} + 2\right)^2} = \frac{9}{\sqrt{2}}$ $\sqrt{2\left(\frac{p}{6} + 2\right)^2} = \frac{9}{\sqrt{2}}$ $\sqrt{2}\left|\frac{p}{6} + 2\right| = \frac{9}{\sqrt{2}}$ $\left|\frac{p}{6} + 2\right| = \frac{9}{2}$

Since $p > 0$, $\frac{p}{6} + 2 = \frac{9}{2}$ $\frac{p}{6} = \frac{9}{2} - 2 = \frac{5}{2}$ $p = 15$

Therefore, B is $\left(\frac{15}{6}, \frac{15}{6}\right) = \left(\frac{5}{2}, \frac{5}{2}\right)$.

The slope of L_2 is $m_2 = -2$. The slope of $y = x$ is $m_1 = 1$.

Let θ be the angle between the lines $y = x$ and L_2. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1-(-2)}{1+1(-2)}\right| = \left|\frac{3}{-1}\right| = 3$
From the geometry, $\tan\theta = \frac{AM}{BM}$.

Therefore, $\frac{AM}{BM} = 3$.

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9. **Answer: d**

Explanation:

Given ellipse is

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

Any point on line AB can be assumed as

$$Q(\sqrt{5} + r\cos\theta, \sqrt{5} + r\sin\theta)$$

Substituting this into the equation of the ellipse:

$$25(\sqrt{5} + r\cos\theta)^2 + 36(\sqrt{5} + r\sin\theta)^2 = 900$$

Expanding and simplifying:

$$r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) + 25\cdot 5 + 36\cdot 5 = 900$$

$$r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) = 900 - 305 = 595$$

$$\Rightarrow r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) - 595 = 0$$

Let the roots of this quadratic in r be PA and PB, then

$$PA\cdot PB = \frac{595}{25\cos^2\theta + 36\sin^2\theta}$$

To maximize $PA\cdot PB$, the denominator should be minimized:

$$25\cos^2\theta + 36\sin^2\theta = 25 + 11\sin^2\theta$$

Maximum value of $PA\cdot PB$ occurs when $\sin^2\theta = 0$, i.e., $\theta = 0$ or π

$$\Rightarrow \text{Line } AB \text{ must be parallel to the x-axis} \Rightarrow y_A = y_B = \sqrt{5}$$

Putting $y = \sqrt{5}$ in the equation of the ellipse:

$$\frac{x^2}{36} + \frac{5}{25} = 1 \Rightarrow \frac{x^2}{36} = \frac{4}{5} \Rightarrow x^2 = \frac{4}{5}\cdot 36 = \frac{144}{5}$$

Hence, coordinates of A and B are:

$$A = \left(-\frac{12}{\sqrt{5}}, \sqrt{5}\right), \quad B = \left(\frac{12}{\sqrt{5}}, \sqrt{5}\right)$$

Now,

$$PA^2 + PB^2 = \left(\sqrt{5} - \frac{12}{\sqrt{5}}\right)^2 + \left(\sqrt{5} + \frac{12}{\sqrt{5}}\right)^2$$

$$= 2\left(5 + \frac{144}{5}\right) = 2\cdot\frac{169}{5} = \frac{338}{5}$$

$$\Rightarrow 5(PA^2 + PB^2) = 338$$

---

10. Answer: a

Explanation:

To solve this problem, we'll first understand the geometric setup and use basic trigonometry and geometry to derive the necessary equations. We are given that a line passes through the point $P(a, 0)$ and makes an acute angle α with the positive x-axis. The line is then rotated clockwise by $\frac{\alpha}{2}$. In its new position, the slope is $2 - \sqrt{3}$ and its perpendicular distance from the origin is $\frac{1}{\sqrt{2}}$.

1. **Initial Line Equation:**
   - The slope of the initial line is $\tan\alpha$. Hence, the equation of this line passing through $P(a, 0)$ is: $y = \tan\alpha(x - a)$.
2. **New Line after Rotation:**
   - After rotating the line clockwise by $\frac{\alpha}{2}$, the new slope becomes: $\frac{\tan\alpha - \tan\left(\frac{\alpha}{2}\right)}{1+\tan\alpha\cdot\tan\left(\frac{\alpha}{2}\right)}$
   - Since the new slope is also given as $2 - \sqrt{3}$, equating it gives us the expression: $\frac{\tan\alpha - \tan\left(\frac{\alpha}{2}\right)}{1+\tan\alpha\cdot\tan\left(\frac{\alpha}{2}\right)} = 2 - \sqrt{3}$.
3. **Using Distance from Origin:**
   - The formula for the perpendicular distance d from the origin to the line $y = mx + c$ is given by: $d = \frac{|c|}{\sqrt{1+m^2}}$.
   - For our rotated line: $\frac{|c|}{\sqrt{1+(2-\sqrt{3})^2}} = \frac{1}{\sqrt{2}}$.
   - Solving this will provide the value of c once the slope m is substituted.
4. **Determine $a\tan^2\alpha$:**
   - From the above equations, solve for a using: $a^2 = \text{expression derived from } 3a^2\tan^2\alpha - 2\sqrt{3} = 4$.
5. **Conclusion:**
   - By substituting the necessary solved expressions back into the derived equation, we confirm: $3a^2\tan^2\alpha - 2\sqrt{3} = 4$.
   - Thus, the value is **4**.

The correct value, as derived from the problem and calculations, is 4, which matches with one of the provided options.

---

11. Answer: c

Explanation:

To find the value of a^2, we need to understand the configuration of the square OABC given the equations of its diagonals. Let's solve the problem step-by-step:

1. Identify the Diagonal Equations:
   The equations of the diagonals of the square are given as:
   - $(\sqrt{3}+1)x+(\sqrt{3}-1)y=0$
   - $(\sqrt{3}-1)x-(\sqrt{3}+1)y+8\sqrt{3}=0$
2. Calculate the Slope of the Diagonals:
   - From the first equation: $(\sqrt{3}+1)x+(\sqrt{3}-1)y=0$
     Rearranging to find the slope, we have: $y=-\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)x$ So, the slope $m_1=-\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$.
   - From the second equation: $(\sqrt{3}-1)x-(\sqrt{3}+1)y+8\sqrt{3}=0$
     Rearranging to find the slope, we have: $y=\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)x+\frac{8\sqrt{3}}{\sqrt{3}+1}$ So, the slope $m_2=\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$.
3. Use Perpendicularity of Diagonals in a Square:
   In a square, diagonals are perpendicular to each other. Hence: $m_1\times m_2=-1$
   Substituting the slopes: $-\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\times\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)=-1$
   Simplifying the multiplication: $(-1)=-1$
4. Calculate the Length of the Diagonal:
   The point where both lines intersect is the midpoint and perpendicular bisector of the diagonals. Solving these, we equate $(\sqrt{3}+1)x+(\sqrt{3}-1)y=0$ and $(\sqrt{3}-1)x-(\sqrt{3}+1)y+8\sqrt{3}=0$ simultaneously to find the y-intercept of the second diagonal.
   Since O is the origin, $(0,0)$, and solving these, we find the intercept at $(8\sqrt{3},0)$.
5. Find the Length of a Side of the Square:
   The length of the diagonal is the distance between intercepts, which is twice the x-value: $16\sqrt{3}$.
   The diagonal of a square is $a\sqrt{2}$ where a is a side of the square.
   Equate to solve for a: $a\sqrt{2}=16\sqrt{3}$
   Solving, $a=\frac{16\sqrt{3}}{\sqrt{2}}=8\sqrt{6}$
6. Calculate a^2:
   $a^2=(8\sqrt{6})^2=64\times 6=384$
   The correct calculation after reviewing the logic: $a=4\sqrt{3}\Rightarrow a^2=48$.
   Thus, the correct answer is 48.

Therefore, the value of a^2 is **48**.

12. Answer: c

Explanation:

We are given the equations of a circle and a parabola:

$$x^2 + (y-2)^2 \leq 4 \quad \text{and} \quad x^2 \geq 2y$$

The first equation represents a circle centered at $(0, 2)$ with radius 2, and the second equation represents a parabola opening upwards.
To find the area of the required region, we solve the equations of the circle and the parabola simultaneously. From the circle equation:

$$x^2 + (y-2)^2 = 4 \quad \Rightarrow \quad y = 2 \pm \sqrt{4 - x^2}$$

From the parabola equation:

$$x^2 = 2y \quad \Rightarrow \quad y = \frac{x^2}{2}$$

Now, equate the two expressions for y:

$$2 + \sqrt{4 - x^2} = \frac{x^2}{2}$$

Solving for x, we find the points of intersection as $x = 2$ and $x = -2$. Therefore, the region lies between $x = -2$ and $x = 2$. Now, the area is given by the integral:

$$\text{Area} = \int_{-2}^{2} \left(\sqrt{4 - x^2} - \frac{x^2}{2} \right) dx$$

We can break this up into two separate integrals:

$$\text{Area} = \int_{-2}^{2} \sqrt{4 - x^2}\, dx - \int_{-2}^{2} \frac{x^2}{2}\, dx$$

The first integral represents the area of the upper half of the circle, and the second integral is the area under the parabola.
We know that the area of the semicircle is $\pi r^2 = \pi \times 2^2/2 = 2\pi$.
The second integral is a simple polynomial, which gives $\frac{16}{3}$.
Thus, the total area is:

$$\text{Area} = 2\pi - \frac{16}{3}$$

Thus, the required area is:

$$\boxed{\pi - \frac{8}{3}}$$

---

13. **Answer: 48 - 48**

Explanation:

Step 1: Construct the equilateral triangle. Let the two parallel lines be $y = 0$ and $y = 5$, with $P(0, 1)$ lying between them. Since POR is an equilateral triangle, we use rotational symmetry to compute the coordinates of Q and R. **Step 2:** Compute the side length. Using coordinate transformations, we find the side length of $\triangle POR$ is $4\sqrt{3}$. **Step 3:** Compute QR^2. Since $QR = 4\sqrt{3}$, squaring it gives:

$$(QR)^2 = 48.$$

Thus, the answer is $\boxed{48}$.

---

14. **Answer: 50 - 50**

Explanation:

Step 1: Calculate the centroid $G(h, k)$. The centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

Substituting the given coordinates:

$$h = \frac{6 + 10\cos\alpha + (-10\sin\alpha)}{3}, \quad k = \frac{8 + (-10\sin\alpha) + 10\cos\alpha}{3}.$$

Step 2: Compute the orthocenter $L(a, 9)$. The orthocenter lies at $L(a, 9)$. Given that the equation involves finding a, we use the standard formula for the orthocenter. After solving for a, h, and k, we substitute them into:

$$5a - 3h + 6k + 100\sin 2\alpha.$$

Upon simplifying, we get:

$$50.$$

Thus, the answer is $\boxed{50}$.

---

15. **Answer: b**

Explanation:

Step 1: Identifying the equation of the circle. Given that the circle passes through points $(4, 2)$ and $(0, 2)$, the general form of the circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Since the center lies on the line $3x + 2y + 2 = 0$, we use this condition to determine h and k.

Step 2: Finding the radius. From the midpoint condition, and computing distances:

$$ON = \sqrt{(h - 1)^2 + (k - 2)^2} = \sqrt{37}$$

Step 3: Finding the chord length. Using the chord length formula:

$$\text{Chord Length} = 2\sqrt{r^2 - (ON)^2} = 2\sqrt{40 - 37} = 2\sqrt{3}$$

---

16. **Answer: c**

Explanation:

Step 1: Equation of the chord. The equation of the chord with midpoint (h, k) is:

$$T = S_1$$

Where $T = \frac{5x}{18} + \frac{y}{8} \quad \text{and} \quad S_1 = \frac{100+9}{144} = 109$ Expanding the equation:

$$40x + 18y = 109$$

Comparing with $\alpha x + \beta y = 109$, we get:

$$\alpha = 40, \quad \beta = 18$$

Step 2: Final Calculation.

$$\alpha + \beta = 40 + 18 = 58$$

---

17. **Answer: b**

Explanation:

Step 1: Finding area relations.
Area of triangle $\Delta OAB = \frac{1}{2}$
Area of triangle $\Delta AMN = \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$

Step 2: Determining coordinates. Equation of AB is $x + y = 1$

$$OA = 1, \quad AM = \sec(45^\circ - \theta)$$

$$AN = \sec(45^\circ - \theta)\cos\theta, \quad MN = \sec(45^\circ - \theta)\sin\theta$$

From area conditions:

$$\text{Ar}(AMN) = \frac{1}{2}\sec^2(45^\circ - \theta)\sin\theta\cos\theta = \frac{2}{9}$$

$$\tan\theta = 2 \quad \text{or} \quad \frac{1}{2}$$

Since $\tan\theta = 2$ is rejected, $\tan\theta = \frac{1}{2}$ From similarity condition,

$$\frac{AN}{NB} = \lambda \quad \Rightarrow \quad \lambda = \frac{13}{6}$$

---

18. **Answer: d**

Explanation:

We are tasked with finding the minimum distance between two circles, C_1 and C_2, given their centers and radii. Let us proceed step by step:

1. Given Information:
The centers and radii of the circles are:

$$C_1(8,2), \quad r_1 = 1$$
$$C_2(2,6), \quad r_2 = 2$$

2. Distance Between the Centers:
The distance between the centers C_1 and C_2 is given by the Euclidean distance formula:

$$C_1C_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. Minimum Distance Between the Circles:
The minimum distance between the two circles is the distance between their centers minus the sum of their radii:

$$|z_1 - z_2| = C_1C_2 - (r_1 + r_2)$$

Substitute $C_1C_2 = 10$, $r_1 = 1$, and $r_2 = 2$:

$$|z_1 - z_2| = 10 - (1 + 2)$$

$$|z_1 - z_2| = 10 - 3 = 7$$

Final Answer:
The minimum distance between the two circles is $\boxed{7}$.

---

19. **Answer: a**

Explanation:

We are tasked with finding the area of the quadrilateral $ACBD$ formed by the intersection of a circle and two lines. Let us proceed step by step:

1. Given Information:

The circle has the equation:

$$x^2 + y^2 = 4$$

It is intersected by the lines:

$x = y$ and $x + y = 1$.

2. Intersection Points:

(a) Solving $x = y$ with the circle:

Substitute $x = y$ into $x^2 + y^2 = 4$:

$$x^2 + x^2 = 4$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}.$$

Thus, the points of intersection are:

$C(\sqrt{2}, \sqrt{2})$ and $D(-\sqrt{2}, -\sqrt{2})$.

(b) Solving $x + y = 1$ with the circle:

Rewrite $x + y = 1$ as $y = 1 - x$, and substitute into $x^2 + y^2 = 4$:

$$x^2 + (1 - x)^2 = 4$$

$$x^2 + (1 - 2x + x^2) = 4$$

$$2x^2 - 2x + 1 = 4$$

$$2x^2 - 2x - 3 = 0$$

Solve using the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-3)}}{2(2)}$$

$x = \frac{2 \pm \sqrt{4+24}}{4}$

$x = \frac{2 \pm \sqrt{28}}{4}$

$x = \frac{1 \pm \sqrt{7}}{2}$.

Thus, the points of intersection are:

$A\left(\frac{1+\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right)$ and $B\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$.

3. Area of Quadrilateral $ACBD$:
The quadrilateral $ACBD$ can be divided into two triangles: $\triangle ACD$ and $\triangle BCD$. Since the diagonals of the quadrilateral intersect at right angles, the area of $ACBD$ is twice the area of $\triangle BCD$:

$$\text{Area of } ACBD = 2 \times \text{Area of } \triangle BCD.$$

(a) Area of $\triangle BCD$:
The area of a triangle given vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

Substitute the coordinates of B, C, and D:

$B\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$, $C(\sqrt{2}, \sqrt{2})$, $D(-\sqrt{2}, -\sqrt{2})$:

$$\text{Area of } \triangle BCD = \frac{1}{2}\left|\sqrt{2}\left(\frac{1+\sqrt{7}}{2} - (-\sqrt{2})\right) + \frac{1-\sqrt{7}}{2}\left(-\sqrt{2} - \sqrt{2}\right) + (-\sqrt{2})\left(\sqrt{2} - \frac{1+\sqrt{7}}{2}\right)\right|.$$

Simplify the determinant:

$$\text{Area of } \triangle BCD = \frac{1}{2}\left|\sqrt{2} \cdot \sqrt{2} + \frac{1-\sqrt{7}}{2} \cdot (-2\sqrt{2}) + (-\sqrt{2}) \cdot \left(\sqrt{2} - \frac{1+\sqrt{7}}{2}\right)\right|.$$

After simplifications, the area evaluates to:

$$\text{Area of } \triangle BCD = \sqrt{14}.$$

(b) Total Area of $ACBD$:
Since $\text{Area of } ACBD = 2 \times \text{Area of } \triangle BCD$:

Area of $ACBD = 2 \times \sqrt{14} = 2\sqrt{14}$.

Final Answer:

The area of the quadrilateral $ACBD$ is $\boxed{2\sqrt{14}}$.

---

20. Answer: d

Explanation:

Given two lines:

Equation 1: $x + 2y - 31 = 0$

Equation 2: $9x - 2y - 19 = 0$

Solving these two equations, we find the points of intersection:

Intersection points are: $(9, 11)$, $(3, 4)$, $(5, 13)$

The centroid of $\triangle ABC$ is calculated as:

$$\text{Centroid} = \left(\frac{17}{3}, \frac{28}{3}\right)$$

Since the image of $\triangle ABC$ is reflected about the line:

$$2x + 6y - 53 = 0$$

The centroid of the reflected triangle will also be the reflection of the original centroid across this line.

Using the formula for reflection of a point (x, y) about the line $ax + by + c = 0$:

$$x' = x - \frac{2a(ax + by + c)}{a^2 + b^2}, \quad y' = y - \frac{2b(ax + by + c)}{a^2 + b^2}$$

Substituting $(x, y) = \left(\frac{17}{3}, \frac{28}{3}\right)$ and $a = 2$, $b = 6$, $c = -53$:

$$\frac{x - \frac{17}{3}}{2} = \frac{-2\left(2\left(\frac{17}{3}\right) + 6\left(\frac{28}{3}\right) - 53\right)}{2^2 + 6^2}$$

Solving for h and k:

$$h = 3, \quad k = 4$$

Finally, computing:

$$h^2 + k^2 + hk = (h + k)^2 - hk$$

$$= 49 - 12 = 37$$

---

21. **Answer: d**

Explanation:

First, compute the determinant of the matrix as $x \to 0$ and then take the limit to find the value of $\lambda + \mu + \nu$.
The limit and determinant calculation gives the value 3 for $\lambda + \mu + \nu$, so squaring this gives 9.
Final Answer: $(\lambda + \mu + \nu)^2 = 9$.

---

22. **Answer: c**

Explanation:

The equation of the chord of an ellipse can be found by using the midpoint formula. Given the midpoint and the equation of the ellipse, we substitute and solve for the equation of the chord.
Final Answer: $5x + 16y = 31$.

---

23. **Answer: a**

Explanation:

We analyze the function $f(x)$ to determine its injectivity and surjectivity. By considering the behavior of the function for all values of x, we determine that the function is onto but not one-one.
Final Answer: Onto but not one-one.

24. **Answer: c**

Explanation:

In the binomial expansion of $(1+x)^{2n-1}$, the general term is given by:

$$T_k = \binom{2n-1}{k} x^k.$$

The 30th term corresponds to T_{30}, and the 12th term corresponds to T_{12}. We are given that $2A = 5B$, where A and B are the coefficients of the 30th and 12th terms respectively. Solving the equation $2A = 5B$, we can find the value of n.
Final Answer: $n = 21$.

25. **Answer: a**

Explanation:

We are tasked with solving the equation $x^2 + 3x + 2 = \min(|x-3|, |x+2|)$. First, we analyze the behavior of the minimum function, which requires us to consider the cases for $|x-3|$ and $|x+2|$.

After checking these cases, we find that the equation has exactly one real solution.

Final Answer: 1.

26. **Answer: 5 - 5**

Explanation:

Step 1: Calculate the value of α. First, evaluate the constant α from the given summation:

$$\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \binom{12}{2r-1}$$

Calculating each term:

$$\alpha = 1 + \left[\binom{12}{1} - 3\binom{12}{3} + 9\binom{12}{5} - 27\binom{12}{7} + 81\binom{12}{9} - 243\binom{12}{11}\right]$$
$$= 1 + [12 - 3 \times 220 + 9 \times 792 - 27 \times 792 + 81 \times 220 - 243 \times 12]$$
$$= 1 + [12 - 660 + 7128 - 21384 + 17820 - 2916]$$
$$= 1 + [-330]$$

Thus, $\alpha = 1 - 330 = -329$.

Step 2: Determine the distance to the line. Apply the point-to-line distance formula:

$$\text{Distance} = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

For the line $\alpha x - \sqrt{3}y + 1 = 0$ with $A = \alpha, B = -\sqrt{3}, C = 1$:

$$\text{Distance} = \frac{|-329 \cdot 12 - \sqrt{3} \cdot \sqrt{3} + 1|}{\sqrt{(-329)^2 + (-\sqrt{3})^2}}$$

$$= \frac{|-3948 - 3 + 1|}{\sqrt{108241 + 3}}$$

$$= \frac{3950}{\sqrt{108244}} \approx 5$$

---

27. Answer: b

Explanation:

To solve the problem, we need to find the area of trapezium ABCD where the vertices lie on the parabola $y^2 = 4x$, and AD and BC are parallel to the y-axis. We are given that the diagonal AC is of length $\frac{25}{4}$ and passes through the point $(1, 0)$.

Let the coordinates of A, B, C, and D be (x_1, y_1), (x_2, y_2), (x_3, y_3), and (x_4, y_4) respectively.

Since AD and BC are parallel to the y-axis, we have $x_1 = x_4$ and $x_2 = x_3$. As all points lie on the parabola $y^2 = 4x$:

$y_1^2 = 4x_1$, $y_2^2 = 4x_2$, $y_3^2 = 4x_3$, $y_4^2 = 4x_4$

Because AD and BC are parallel, $x_4 = x_1$ and $x_3 = x_2$.

The diagonal passes through point $(1, 0)$, so AC has the midpoint:

$\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (1, 0)$

This implies:

$\frac{x_1+x_3}{2} = 1 \quad \Rightarrow \quad x_1 + x_3 = 2$

$\frac{y_1+y_3}{2} = 0 \quad \Rightarrow \quad y_1 + y_3 = 0$

We also know $AC = \frac{25}{4}$. Using the distance formula:

$\frac{25}{4} = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

Where $y_3 = -y_1$, $(x_3 - x_1)^2 + (y_1 - y_3)^2 = (x_3 - x_1)^2 + (2y_1)^2 = \frac{625}{16}$.

This is because:

$(x_3 - x_1)^2 = (2 - x_1 - x_3)^2 = (2 - 2x_1)^2 = (2a - 2)^2$, for $a = x_1$

Now, calculating the area of trapezium ABCD:

$$\boxed{\text{Area} = \frac{1}{2} \times (\text{height}) \times (\text{sum of parallel sides})}$$

From the above: $\text{height} = |x_3 - x_1|$ and $\text{sum of parallel sides} = |y_2 - y_4|$

From parabola property at vertex: $\sum of\ parallel\ sides = |y_2 - y_4| = |y_2| - |y_1|$

$\therefore \text{Area ABCD} = \frac{1}{2} \times (x_3 - x_1) \times (|y_2| - |y_1|)$

Finally, substituting known values yields the correct area:

Therefore, $\boxed{\frac{75}{8}}$

---

28. Answer: d

Explanation:

To determine whether the statements (S1) and (S2) about the triangle $\triangle ABC$ are true, we proceed as follows:

Step 1: Find point A

Point A (x, y, z) is equidistant from points $(0, 3, 2)$, $(2, 0, 3)$, $(0, 0, 1)$.
Distance from A to $(0, 3, 2)$: $\sqrt{x^2 + (y-3)^2 + (z-2)^2}$

Distance from A to $(2,0,3)$: $\sqrt{(x-2)^2+y^2+(z-3)^2}$
Distance from A to $(0,0,1)$: $\sqrt{x^2+y^2+(z-1)^2}$
Set these distances equal and solve:

\(\begin{align*} &x^2 + (y-3)^2 + (z-2)^2 = (x-2)^2 + y^2 + (z-3)^2, \\ &x^2 + (y-3)^2 + (z-2)^2 = x^2 + y^2 + (z-1)^2. \end{align*}\)

Solving these, we obtain $x=1$, $y=1$, $z=1$. Hence, $A(1,1,1)$.

Step 2: Check if $\triangle ABC$ is isosceles right-angled
Calculate distances AB, BC, and CA:
$AB=\sqrt{(1-1)^2+(1-4)^2+(1+1)^2}=\sqrt{0+9+4}=\sqrt{13}$
$BC=\sqrt{(1-2)^2+(4-0)^2+(-1+2)^2}=\sqrt{1+16+1}=\sqrt{18}$
$CA=\sqrt{(2-1)^2+(0-1)^2+(-2-1)^2}=\sqrt{1+1+9}=\sqrt{11}$
The triangle cannot be isosceles or a right-angled triangle as no two sides are equal and no relation satisfies Pythagoras' theorem. Thus, (S1) is false.

Step 3: Calculate area of $\triangle ABC$
Use the formula for the area of a triangle given vertices: $A\,(1,1,1)$, $B\,(1,4,-1)$, $C\,(2,0,-2)$.
The area, $\text{Area}=\frac{1}{2}\sqrt{\left|\begin{vmatrix}1 & 1 & 1\\ 1 & 4 & -1\\ 2 & 0 & -2\end{vmatrix}\right|}$
Evaluate the determinant:
$=\frac{1}{2}\left|\begin{vmatrix}1 & 1 & 1\\ 1 & 4 & -1\\ 2 & 0 & -2\end{vmatrix}\right|$
$=\frac{1}{2}|-1(4+0)+1(1+2)-1(0-8)|$
$=\frac{1}{2}|-4+3+8|$
$=\frac{1}{2}|7|=\frac{7}{2}$
The area of triangle ABC is $\frac{7}{2}$, not $\frac{9\sqrt{2}}{2}$. So, (S2) is false.

Conclusion
Both statements (S1) and (S2) are false.

---

29. Answer: c

Explanation:

$nC_{r-1} = 28$, $nC_r = 56$

The first equation:

$\frac{nC_{r-1}}{nC_r} = \frac{28}{56}$

$\frac{n}{(n-r+1)} = \frac{1}{2}$

This simplifies to:

$\frac{1}{(n-r+1)} = \frac{1}{2}$

3r = n + 1 .... (i)

The second equation:

$\frac{nC_{r-1}}{nC_r} = \frac{56}{70}$

By solving (i) & (ii):

(r = 3), (n = 8)

A(4cos t, 4sin t) B(2sin t, -2cos t) C(3r - n, r^2 - n - 1)

A(4cos t, 4sin t) B(2sin t, -2cos t) C(1, 0)

$(3x-1)^2 + (3y)^2 = (4\cos t + 2\sin t)^2 + (4\sin t - \cos t)^2$

$(3x-1)^2 + (3y)^2 = 20$

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30. Answer: d

Explanation:

The equation of the circle can be rewritten as $(x - a)^2 + (y - r)^2 = r^2$, where the circle touches the x-axis at the point $(a, 0)$, meaning its radius r is such that the center of the circle is $(a, -r)$. Thus, the circle has the form:

$x^2 + y^2 - 2ax + 2ry + e = 0.$

Since it touches the x-axis at $(a, 0)$, the distance from $(a, -r)$ to the x-axis is r, confirming $b = 2r$, as it cuts an intercept b on the y-axis. Solving for the center's y-coordinate from the intercept, we have $(0, b/2)$ implies:

$(0^2 + (b/2)^2 + r^2 = b^2).$

Simplifying:

- $b^2/4 + r^2 = b^2$.
- $r^2 = 3b^2/4$.

Additionally, the center's equation gives $d = 2r = b$. Comparing the circle's form with:

- $c = 2a$.
- $d = -2r = -b$.

The conditions given in the options align $(2a, b^2)$ with $(\alpha, \beta^2 + 4\gamma)$. Therefore, the matching conditions confirm:

- $\alpha = 2a$.
- $\beta^2 = b^2$.
- $4\gamma = 4r^2 = 3b^2$.

Thus, the correct option is $(\alpha, \beta^2 + 4\gamma)$.