Functions JEE Main PYQ - 1

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Functions

1. **Statement 1 : The function $f : \mathbb{R} \to \mathbb{R}$ defined by** **(+4, -1)**

$$f(x) = \frac{x}{1+|x|}$$

is one–one.
Statement 2 : The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{x^2+4x-30}{x^2-8x+18}$$

is many–one.
Which of the following is correct?

a. Both Statements are correct

b. Both Statements are false

c. Statement 1 is false and Statement 2 is correct

d. Statement 1 is correct and Statement 2 is false

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2. **If $y = \text{sgn}(\sin x) + \text{sgn}(\cos x) + \text{sgn}(\tan x) + \text{sgn}(\cot x)$, where $\text{sgn}(p)$ denotes the signum function of p, then the sum of elements in the range of y is:** **(+4, -1)**

a. 4

b. -2

c. 0

d. 2

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3. $y = \log_5 \log_3 \log_7(9x - x^2 - 13)$, If its domain is (m, n) and **(+4, -1)**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

is a hyperbola having eccentricity $\frac{n}{3}$ and length of the latus rectum is $\frac{8m}{3}$, find $b^2 - a^2$:

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4. If $(f(x))^2 = 25 + \int_0^x \left[(f(x))^2 + (f'(x))^2\right] \, dx$, find the mean of $f(\ln 1) + f(\ln 2) + \cdots + f(\ln 625)$: **(+4, -1)**

a. 1561

b. 1675

c. 1465

d. 1565

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5. Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$ and R be a relation defined on set A such that $R = \{(x, y) : 2x + y \leq -2, x, y \in A\}$. Let l = number of elements in R, m = minimum number of elements to be added in R to make it reflexive relation, n = minimum number of elements to be added in R to make it symmetric relation, then $(l + m + n)$ is: **(+4, -1)**

a. 17

b. 18

c. 19

d. 20

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6. The sum of roots of the equation **(+4, -1)**

$$|x - 1|^2 - 5|x - 1| + 6 = 0$$

is

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7. If the domain of the function **(+4, -1)**

$$\cos^{-1}\left(\frac{2x - 5}{11x - 7}\right) + \sin^{-1}\left(2x^2 - 3x + 1\right)$$

is

$$[0, a] \cup [12/13, b]$$

then $\frac{1}{ab}$ is equal to}

a. -3

b. 3

c. 2

d. 4

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8. **Consider a set $S = \{a, b, c, d\}$. Then the number of reflexive as well as symmetric relations from $S \to S$ are** **(+4, -1)**

a. 1024

b. 256

c. 16

d. 64

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9. The number of solution(s) of the equation **(+4, -1)**

$$x|x+4| + 3|x+2| = 0$$

is/are equal to:

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10. Let $M = \{1, 2, 3, \ldots, 16\}$ and R be a relation on M defined by xRy if and only if $4y = 5x - 3$. Then, the number of elements required to be added in R to make it symmetric is: **(+4, -1)**

a. 2

b. 3

c. 4

d. 5

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11. If the domain of the function **(+4, -1)**

$$f(x) = \frac{1}{\ln(10-x)} + \sin^{-1}\left(\frac{x+2}{2x+3}\right)$$

is $(-\infty, -1) \cup (-1, b) \cup (b, c) \cup (c, \infty)$, then $(b + c + 3a)$ is equal to:

a. 22

b. 24

c. 23

d. 21

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12. Let $A = \{2, 3, 5, 7, 11\}$ and a relation R is defined as **(+4, -1)**

$$R = \{(x, y) : x, y \in A, 2x \leq 3y\}.$$

Then the minimum number of elements to be added to relation R such that R becomes symmetric is:

a. 4

b. 8

c. 7

d. 6

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13. If domain of $f(x) = \sin^{-1}\left(\frac{5-x}{2x+3}\right) + \frac{1}{\log_e(10-x)}$ is $(-\infty, \alpha] \cup (\beta, \gamma) - \{\delta\}$ then value of $6(\alpha + \beta + \gamma + \delta)$ is equal to : **(+4, -1)**

a. 60

b. 70

c. 80

d. 90

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14. If a + α = 1, b + β = 2 and a f(x) + α f(1/x) = b x + β / x, then [f(x) + f(1/x)] / [x + 1/x] is ________ **(+4, -1)**

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15. The real valued function $f(x) = \csc^{-1} x / \sqrt{(x - [x])}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to : (+4, -1)

a. all reals except integers

b. all reals except the interval [-1, 1]

c. all non-integers except the interval (-1, 1)

d. all integers except 0, -1, 1

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16. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{(1-x)}$, then what is the common domain of the following functions : f+g, f−g, f/g, g/f, g−f where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = f(x)/g(x)$ (+4, -1)

a. $0 \le x < 1$

b. $0 < x < 1$

c. $0 \le x \le 1$

d. $0 < x \le 1$

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17. Let $g : N \to N$ be defined as $g(3n+1) = 3n+2, g(3n+2) = 3n+3, g(3n+3) = 3n+1$, for all $n \ge 0$. Then which of the following statements is true ? (+4, -1)

a. $g \circ g \circ g = g$

b. There exists a one-one function $f : N \to N$ such that $f \circ g = f$.

c. There exists an onto function $f : N \to N$ such that $f \circ g = f$.

d. There exists a function $f : N \to N$ such that $g \circ f = f$.

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18. Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = (x - 2)/(x - 3)$. Let $g : R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = 13/2$ is equal to. (+4, -1)

a. 2

b. 5

c. 3

d. 7

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19. **Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 2x - 1$ and $g : \mathbb{R} \setminus \{1\} \to \mathbb{R}$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is :** **(+4, -1)**

a. one-one but not onto

b. onto but not one-one

c. neither one-one nor onto

d. both one-one and onto

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20. **Let f,g : ℕ → ℕ such that f(n+1) = f(n) + f(1) ∀ n ∈ ℕ and g be any arbitrary function. Which of the following statements is NOT true?** **(+4, -1)**

a. If f is onto, then f(n) = n ∀ n ∈ ℕ

b. f is one-one

c. If g is onto, then fog is one-one

d. If fog is one-one, then g is one-one

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21. **The minimum value of f(x) = $a^{a^x} + a^{1-a^x}$, where a, x $\in$ R and a>0, is equal to :** **(+4, -1)**

a. a + 1

b. a + $\frac{1}{a}$

c. 2a

d. $2\sqrt{a}$

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22. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1+a)^2 + b^2$ is equal to: **(+4, -1)**

a. 26

b. 29

c. 25

d. 30

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23. Let $A = \{1, 2, 3, ..., 10\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_3, a_4), (a_5, a_6), ..., (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to: **(+4, -1)**

a. 6

b. 7

c. 5

d. 8

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24. Let the domain of the function $f(x) = \log_2 \log_4 \log_6(3 + 4x - x^2)$ be (a, b). If **(+4, -1)**

$$\int_0^{b-a} [x^2]\, dx = p - \sqrt{q} - \sqrt{r}, \quad p, q, r \in \mathbb{N},\ \gcd(p, q, r) = 1,$$

where $[\,]$ is the greatest integer function, then $p + q + r$ is equal to

a. 10

b. 8

c. 11

d. 9

25. If the domain of the function $f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha, \beta]$, then $\alpha^2 + 4\beta$ is equal to **(+4, -1)**

a. 5

b. 4

c. 3

d. 7

26. **The number of points of discontinuity of the function $f(x) = \left\lfloor \frac{x^2}{2} \right\rfloor - \lfloor \sqrt{x} \rfloor, \quad x \in [0, 4]$, where $\lfloor \cdot \rfloor$ denotes the greatest integer function, is:** **(+4, -1)**

27. **Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_2(2x+5))$ be (γ, δ). Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to:** **(+4, -1)**

28. **Let $f(x) = \frac{x-5}{x^2-3x+2}$. If the range of $f(x)$ is $(-\infty, \alpha) \cup (\beta, \infty)$, then $\alpha^2 + \beta^2$ equals to.** **(+4, -1)**

a. 1

b. 2

c. 3

d. 4

29. **The domain of the function** **(+4, -1)**

$$f(x) = \frac{1}{\sqrt{10 + 3x - x^2}} + \frac{1}{\sqrt{x + |x|}}$$

is (a, b). Then $(1 + a^2) + b^2$ is:

a. 26

b. 30

c. 25

d. 29

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30. If the domain of the function $f(x) = \frac{1}{\sqrt{3x+10-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1+a)^2 + b^2$ **(+4, -1)** is equal to:

a. 25

b. 16

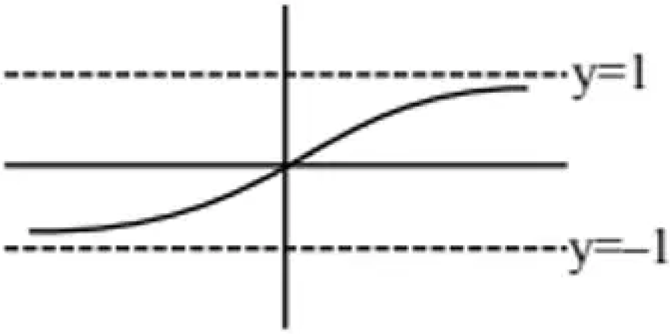
c. 24

d. 26

Answers

1. **Answer: a**

   Explanation:



Statement 1 Analysis:

$$f(x) = \frac{x}{1+|x|}$$

Write it piecewise:

$$f(x) = \begin{cases} \frac{x}{1+x}, & x \geq 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$$

For $x \geq 0$: $f'(x) = \frac{1}{(1+x)^2} > 0 \Rightarrow$ strictly increasing. \
For $x < 0$: $f'(x) = \frac{1}{(1-x)^2} > 0 \Rightarrow$ strictly increasing.

Also,

$$\lim_{x \to 0^-} f(x) = 0 = \lim_{x \to 0^+} f(x)$$

Thus, the function is strictly increasing on $\mathbb{R}$, hence **one–one**
. \[\Rightarrow **Statement 1 is correct.**
\] **Statement 2 Analysis:**

$$f(x) = \frac{x^2+4x-30}{x^2-8x+18}$$

Rewrite numerator using denominator:

$$x^2 + 4x - 30 = (x^2 - 8x + 18) + 12x - 48$$

$$f(x) = 1 + \frac{12x - 48}{x^2 - 8x + 18}$$

Since the function is a rational function of degree $2/2$, it is **not strictly monotonic** on its domain. Indeed,

$$f(2) = \frac{4 + 8 - 30}{4 - 16 + 18} = \frac{-18}{6} = -3$$

$$f(6) = \frac{36 + 24 - 30}{36 - 48 + 18} = \frac{30}{6} = 5$$

Also, multiple distinct x-values can give the same $f(x)$, hence the function is **many–one**

. \[\Rightarrow **Statement 2 is correct.**

\]

Final Conclusion:

Both statements are correct.

$$\boxed{\text{Option (1)}}$$

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2. Answer: d

Explanation:

Concept:

The signum function is defined as:

$$\text{sgn}(p) = \begin{cases} +1, & p > 0 \\ 0, & p = 0 \\ -1, & p < 0 \end{cases}$$

For this problem, we only consider intervals where the trigonometric functions are defined and non-zero.

Step 1: Analyze by Quadrants

Consider one full cycle $0 < x < 2\pi$. **First Quadrant** $(0, \frac{\pi}{2})$**:**

$$\sin x > 0,\ \cos x > 0,\ \tan x > 0,\ \cot x > 0$$

$$y = 1 + 1 + 1 + 1 = 4$$

Second Quadrant $(\frac{\pi}{2}, \pi)$**:**

$$\sin x > 0,\ \cos x < 0,\ \tan x < 0,\ \cot x < 0$$

$$y = 1 - 1 - 1 - 1 = -2$$

Third Quadrant $(\pi, \frac{3\pi}{2})$**:**

$$\sin x < 0,\ \cos x < 0,\ \tan x > 0,\ \cot x > 0$$

$$y = -1 - 1 + 1 + 1 = 0$$

Fourth Quadrant $(\frac{3\pi}{2}, 2\pi)$**:**

$$\sin x < 0,\ \cos x > 0,\ \tan x < 0,\ \cot x < 0$$

$$y = -1 + 1 - 1 - 1 = -2$$

Step 2: Determine the Range of y

Possible values of y:

$$\{4,\ -2,\ 0\}$$

Step 3: Sum of Elements in the Range

$$4 + (-2) + 0 = 2$$

$$\boxed{2}$$

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3. **Answer: 7 - 7**

Explanation:

Step 1: Find the domain of the function.

The given function is $y = \log_5 \log_3 \log_7(9x - x^2 - 13)$. The domain of the function is determined by the condition that the argument inside each logarithmic function must be positive. Start by analyzing the innermost logarithm $\log_7(9x - x^2 - 13)$. The argument $9x - x^2 - 13$ must be greater than 0:

$$9x - x^2 - 13 > 0$$

Solving this quadratic inequality, we find the domain (m, n) of x.
Step 2: Use the equation of the hyperbola.
We are given the equation of a hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The eccentricity e of a hyperbola is related to a and b by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Given that the eccentricity is $\frac{n}{3}$, we can express the relation between a and b in terms of n.
Step 3: Use the length of the latus rectum.
The length of the latus rectum of a hyperbola is given by:

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

We are given that the length of the latus rectum is $\frac{8m}{3}$. Using this information, we can set up an equation and solve for $b^2 - a^2$.
Step 4: Final calculation.
After solving for $b^2 - a^2$, we find that the value is 7. **Final Answer:**

$$\boxed{7}$$

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4. Answer: d

Explanation:

Step 1: Use the given equation.
We are given that:

$$(f(x))^2 = 25 + \int_0^x \left[(f(x))^2 + (f'(x))^2\right] \, dx$$

Differentiating both sides with respect to x, we get:

$$2f(x)f'(x) = (f(x))^2 + (f'(x))^2$$

Rearrange this to express in terms of $f(x)$ and its derivative $f'(x)$.
Step 2: Solve for $f(x)$.

Solving the differential equation, we find that:

$$f(x) = 5e^x$$

Step 3: Calculate the sum.
Now, calculate the sum of $f(\ln 1), f(\ln 2), \ldots, f(\ln 625)$:

$$f(\ln n) = 5e^{\ln n} = 5n$$

Thus, we need to find:

$$f(\ln 1) + f(\ln 2) + \cdots + f(\ln 625) = 5(1 + 2 + 3 + \cdots + 625)$$

The sum of the first 625 natural numbers is:

$$S = \frac{625(625+1)}{2} = \frac{625 \times 626}{2} = 195312.5$$

So, the total sum is:

$$5 \times 195312.5 = 976562.5$$

Step 4: Find the Mean.
The mean is the sum divided by the number of terms (625):

$$\text{Mean} = \frac{976562.5}{625} = 1565$$

Final Answer:

$$\boxed{1565}$$

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5. Answer: a

Explanation:

Step 1: Understand the Relation.
The relation R is defined by the condition $2x + y \leq -2$ for all pairs $(x, y) \in A \times A$, where $A = \{-2, -1, 0, 1, 2, 3, 4\}$. First, we calculate how many pairs satisfy this condition.
Step 2: Find the Number of Elements in R (l).
We check each pair $(x, y) \in A \times A$ that satisfies $2x + y \leq -2$. After listing all valid pairs, we find that there are $l = 10$ elements in R.

Step 3: Make the Relation Reflexive (m).
For the relation to be reflexive, we need $(x, x) \in R$ for all $x \in A$. Checking the pairs, we see that not all diagonal pairs are included, so we need to add 3 elements to make it reflexive. Therefore, $m = 3$.
Step 4: Make the Relation Symmetric (n).
For the relation to be symmetric, if $(x, y) \in R$, then (y, x) must also be in R. Checking the symmetry condition, we find that 4 additional pairs need to be added to make the relation symmetric. Therefore, $n = 4$.
Step 5: Calculate the Total.
Now, the total number of elements to be added is $l + m + n = 10 + 3 + 4 = 17$. **Final Answer:**

$$\boxed{17}$$

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6. **Answer: 4 - 4**

Explanation:

Step 1: Let $|x - 1| = t$.
Substitute $|x - 1| = t$ into the equation:

$$t^2 - 5t + 6 = 0$$

Solve this quadratic equation:

$$(t - 3)(t - 2) = 0$$

So, $t = 2$ or $t = 3$. **Step 2: Solve for x.**
For $t = 2$, $|x - 1| = 2$ gives $x = 3$ or $x = -1$. For $t = 3$, $|x - 1| = 3$ gives $x = 4$ or $x = -2$.
Step 3: Calculate the sum of the roots.
The roots are $3, -1, 4, -2$, so the sum is:

$$3 + (-1) + 4 + (-2) = 4$$

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7. **Answer: b**

Explanation:

Step 1: Solve for the domain of the first term.
The domain of the inverse cosine function is given by the condition:

$$\frac{2x-5}{11x-7} \in [-1,1]$$

Solving this inequality gives the domain for the first term. **Step 2: Solve for the domain of the second term.**
For the inverse sine function, the domain is given by:

$$-1 \le 2x^2 - 3x + 1 \le 1$$

Solving this inequality gives the domain for the second term. **Step 3: Determine the values of a and b.**
By solving the inequalities, we find that the domain is $[0,a] \cup [12/13, b]$, and $ab = 3$.
Therefore, $\frac{1}{ab} = 3$.

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8. **Answer: d**

Explanation:

Step 1: Understand the concept of reflexive and symmetric relations.
A reflexive relation is a relation where every element is related to itself, i.e., $(a,a),(b,b),(c,c),(d,d)$ must be included in the relation. A symmetric relation means that if (a,b) is in the relation, then (b,a) must also be in the relation. **Step 2: Calculate the number of such relations.**
The total number of possible relations from a set A to itself is given by $n(A) \times n(A) = N^2$, where N is the number of elements in the set. For $N = 4$, the total number of relations is $4^2 = 16$. The reflexive relations require that each element is related to itself, leaving $N^2 - N$ choices for the other elements. Each of these remaining relations has two choices: it can either be included or excluded from the relation. Hence, the number of reflexive and symmetric relations is:

$$2^{N^2-N} = 2^{4^2-4} = 2^{16-4} = 64$$

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9. **Answer: 3 - 3**

Explanation:

Step 1: Understanding the absolute values.
We need to consider different cases based on the values inside the absolute value functions. **Case 1: $x \geq -2$.**
For $x \geq -2$, the absolute value functions become:

$$|x+4| = x+4, \quad |x+2| = x+2$$

Substituting into the equation:

$$x(x+4) + 3(x+2) = 0$$

Simplify and solve the resulting quadratic equation. **Case 2: $-4 \leq x < -2$.**
For $-4 \leq x < -2$, we have:

$$|x+4| = x+4, \quad |x+2| = -(x+2)$$

Substitute and solve the equation. **Case 3: $x < -4$.**
For $x < -4$, we have:

$$|x+4| = -(x+4), \quad |x+2| = -(x+2)$$

Substitute and solve the equation. **Step 2: Conclusion.**
After solving all the cases, we find three solutions for x. **Final Answer:**

$$\boxed{3}$$

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10. Answer: a

Explanation:

Step 1: Understanding the relation.
The relation R is defined as xRy if and only if $4y = 5x - 3$. For the relation to be symmetric, if xRy is true, then yRx must also be true. **Step 2: Finding pairs for symmetry.**
For symmetry, we need to ensure that the relation $4y = 5x - 3$ also holds for $4x = 5y - 3$. Solving the system of equations for pairs of x and y, we find the pairs that violate symmetry. **Step 3: Conclusion.**
After analyzing, we find that 2 additional elements need to be added to make the relation symmetric. **Final Answer:**

$$\boxed{2}$$

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11. **Answer: b**

Explanation:

Step 1: Analyzing the domain of $\ln(10-x)$.
For the natural logarithm function $\ln(10-x)$, the argument must be positive:

$$10 - x > 0 \quad \Rightarrow x < 10$$

Thus, x must be less than 10. **Step 2: Analyzing the domain of $\sin^{-1}\left(\frac{x+2}{2x+3}\right)$.**
The inverse sine function requires that the argument must lie between -1 and 1:

$$-1 \leq \frac{x+2}{2x+3} \leq 1$$

This leads to two inequalities. Solving them gives the valid range for x. **Step 3: Conclusion.**
From the given domain conditions, we conclude that $b = 5$, $c = 7$, and $a = 6$. Hence, $b + c + 3a = 24$. **Final Answer:**

$$\boxed{24}$$

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12. **Answer: d**

Explanation:

Step 1: Understand the given relation.
We are given a set $A = \{2, 3, 5, 7, 11\}$ and a relation R defined by:

$$R = \{(x, y) : x, y \in A, 2x \leq 3y\}.$$

This means that a pair (x, y) belongs to R if and only if $2x \leq 3y$.
Step 2: Check if R is symmetric.
For a relation to be symmetric, for any pair (x, y) in the relation, the pair (y, x) must also be in the relation. Let's check the relation R for each element in A and find out which pairs are present: - For $x = 2$, we check $2x \leq 3y$ for all values of $y \in A$. - For $y = 3$, $2(2) = 4 \leq 3(3) = 9$, so $(2, 3)$ is in the relation. - For $y = 5$, $2(2) = 4 \leq 3(5) = 15$, so $(2, 5)$ is in the relation. - For $y = 7$, $2(2) = 4 \leq 3(7) = 21$, so $(2, 7)$ is in the relation. - For $y = 11$, $2(2) = 4 \leq 3(11) = 33$, so $(2, 11)$ is in the relation. Continue this process for each

element of A.
Step 3: Identify missing pairs for symmetry.
Once the pairs are identified, we need to add those missing pairs to ensure the relation is symmetric. For example, if $(2,3)$ is in the relation but $(3,2)$ is not, we add $(3,2)$ to make the relation symmetric.
Step 4: Conclusion.
By adding 6 pairs to the relation R, we can make it symmetric. Therefore, the minimum number of elements to be added is 6. Thus, the correct answer is (4).

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13. **Answer: b**

Explanation:

Step 1: Understanding the Question:
We need to find the domain of the function $f(x)$, which is the intersection of the domains of its two component parts. Then, we compare the resulting domain with the given format to find the values of $\alpha, \beta, \gamma, \delta$. There seems to be a typo in the provided format, we assume it should be $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$ based on standard results.

Step 2: Domain of the Inverse Sine Function:
For $g(x) = \sin^{-1}\left(\frac{5-x}{2x+3}\right)$, the argument must be in the interval $[-1,1]$. Also, the denominator cannot be zero, so $2x+3 \neq 0 \Rightarrow x \neq -3/2$.

$$-1 \leq \frac{5-x}{2x+3} \leq 1$$

This gives two inequalities:
Inequality 1: $\frac{5-x}{2x+3} \leq 1 \Rightarrow \frac{5-x}{2x+3} - 1 \leq 0 \Rightarrow \frac{5-x-(2x+3)}{2x+3} \leq 0 \Rightarrow \frac{2-3x}{2x+3} \leq 0 \Rightarrow \frac{3x-2}{2x+3} \geq 0.$
This holds for $x \in (-\infty, -3/2) \cup [2/3, \infty)$.
Inequality 2: $\frac{5-x}{2x+3} \geq -1 \Rightarrow \frac{5-x}{2x+3} + 1 \geq 0 \Rightarrow \frac{5-x+2x+3}{2x+3} \geq 0 \Rightarrow \frac{x+8}{2x+3} \geq 0.$
This holds for $x \in (-\infty, -8] \cup (-3/2, \infty)$.
The domain of the sine inverse part is the intersection of these two sets: $x \in (-\infty, -8] \cup [2/3, \infty)$.

Step 3: Domain of the Logarithmic Function:
For $h(x) = \frac{1}{\log_e(10-x)}$, we have two conditions:
The argument of the logarithm must be positive: $10 - x > 0 \Rightarrow x < 10$.

The denominator cannot be zero: $\log_e(10-x) \neq 0 \Rightarrow 10-x \neq 1 \Rightarrow x \neq 9$. So, the domain of the logarithmic part is $x \in (-\infty, 10)$ and $x \neq 9$.

Step 4: Finding the Overall Domain and the Final Answer:
The domain of $f(x)$ is the intersection of the domains from Step 2 and Step 3. We need to find the intersection of $(-\infty, -8] \cup [2/3, \infty)$ and $(-\infty, 10) - \{9\}$.

$$((-\infty, -8] \cup [2/3, \infty)) \cap (-\infty, 10) - \{9\}$$

$$= ((-\infty, -8] \cap (-\infty, 10)) \cup ([2/3, \infty) \cap (-\infty, 10)) - \{9\}$$

$$= (-\infty, -8] \cup [2/3, 10) - \{9\}$$

Comparing this with the format $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$ (assuming the typo correction), we get: $\alpha = -8$, $\beta = 2/3$, $\gamma = 10$, $\delta = 9$.
The required value is $6(\alpha + \beta + \gamma + \delta)$:

$$6\left(-8 + \frac{2}{3} + 10 + 9\right) = 6\left(11 + \frac{2}{3}\right) = 6\left(\frac{33+2}{3}\right) = 6\left(\frac{35}{3}\right) = 2 \times 35 = 70$$

---

14. Answer: 2 - 2

Explanation:

Step 1: Given $af(x) + \alpha f(1/x) = bx + \beta/x$ (i).
Step 2: Replace x by $1/x$: $af(1/x) + \alpha f(x) = b/x + \beta x$ (ii).
Step 3: Add (i) and (ii): $(a+\alpha)f(x) + (a+\alpha)f(1/x) = (b+\beta)x + (b+\beta)/x$.
Step 4: Substitute $a + \alpha = 1$ and $b + \beta = 2$: $1[f(x) + f(1/x)] = 2[x + 1/x]$.
Step 5: Result is 2.

---

15. Answer: c

Explanation:

Step 1: For $\csc^{-1} x$, the domain is $|x| \geq 1$. This means $x \in (-\infty, -1] \cup [1, \infty)$.
Step 2: For the denominator $\sqrt{x - [x]}$, the term inside the square root must be positive. Let $\{x\} = x - [x]$ be the fractional part. We need $\{x\} > 0$.
Step 3: The fractional part $\{x\}$ is zero if and only if x is an integer ($x \in \mathbb{Z}$). Thus, $x \notin \mathbb{Z}$.

Step 4: Combining these: $x \in ((-\infty, -1] \cup [1, \infty))$ AND $x \notin \mathbb{Z}$. This simplifies to all non-integers except the interval $(-1, 1)$.

---

16. **Answer: b**

Explanation:

Step 1: Find the domain of $f(x) = \sqrt{x}$. For $\sqrt{x}$ to be real, $x \geq 0$.
Step 2: Find the domain of $g(x) = \sqrt{1-x}$. For $\sqrt{1-x}$ to be real, $1 - x \geq 0 \implies x \leq 1$.
Step 3: The intersection for $f \pm g$ is $0 \leq x \leq 1$.
Step 4: For f/g, we must also ensure $g(x) \neq 0$. $\sqrt{1-x} \neq 0 \implies x \neq 1$.
Step 5: For g/f, we must also ensure $f(x) \neq 0$. $\sqrt{x} \neq 0 \implies x \neq 0$.
Step 6: The common intersection of all conditions is $0 < x < 1$.

---

17. **Answer: c**

Explanation:

Step 1: Understanding the Concept:
The function g partitions the set of natural numbers $\mathbb{N}$ into cycles of length 3:
$\{3n+1, 3n+2, 3n+3\} \to \{3n+2, 3n+3, 3n+1\}$.
This means $g(g(g(x))) = x$ for all x.
Step 3: Detailed Explanation:
1. **Analysis of g:**
Since $g(g(g(x))) = x$, then $g \circ g \circ g$ is the identity function I. Statement (A) is false.
2. **Analysis of $f \circ g = f$:**
This equation implies $f(g(x)) = f(x)$.
This means f must take the same value for all elements in a cycle: $f(3n+1) = f(3n+2) = f(3n+3) = c_n$.
If f takes the same value for different inputs, it cannot be one-one. Statement (B) is false.
3. **Analysis of onto function:**
Let $f(x) = \lceil x/3 \rceil$.
Then $f(3n+1) = f(3n+2) = f(3n+3) = n+1$.
Since n can be any non-negative integer, the range of f is $\{1, 2, 3, ...\}$, which is $\mathbb{N}$.
Thus f is onto. Statement (C) is true.

4. **Analysis of $g \circ f = f$:**
This implies $f(x)$ is a fixed point of g. But according to the definition of g, $g(k) \neq k$ for any k. So no such f exists. Statement (D) is false.
Step 4: Final Answer:
Statement (C) is true.

---

18. Answer: b

Explanation:

Step 1: Find $f^{-1}(x)$. Let $y = \frac{x-2}{x-3} \implies xy - 3y = x - 2 \implies x(y-1) = 3y - 2$. $x = \frac{3y-2}{y-1} \implies f^{-1}(x) = \frac{3x-2}{x-1}$.
Step 2: Find $g^{-1}(x)$. Let $y = 2x - 3 \implies x = \frac{y+3}{2} \implies g^{-1}(x) = \frac{x+3}{2}$.
Step 3: Solve $\frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$. $\frac{3x-2}{x-1} = \frac{13-x-3}{2} = \frac{10-x}{2}$. $2(3x-2) = (x-1)(10-x) \implies 6x - 4 = 10x - x^2 - 10 + x$. $x^2 - 5x + 6 = 0 \implies (x-2)(x-3) = 0$.
Step 4: Values of x are 2 and 3. Sum $= 2 + 3 = 5$.

---

19. Answer: a

Explanation:

Step 1: Calculate the composition $f(g(x))$. $f(g(x)) = 2\left(\frac{x-1/2}{x-1}\right) - 1 = \frac{2x-1}{x-1} - 1 = \frac{2x-1-(x-1)}{x-1} = \frac{x}{x-1}$.
Step 2: Check for one-one. Let $f(g(x_1)) = f(g(x_2)) \Rightarrow \frac{x_1}{x_1-1} = \frac{x_2}{x_2-1}$. $x_1x_2 - x_1 = x_1x_2 - x_2 \Rightarrow x_1 = x_2$. So, it is one-one.
Step 3: Check for onto. Let $y = \frac{x}{x-1} \Rightarrow yx - y = x \Rightarrow x(y-1) = y \Rightarrow x = \frac{y}{y-1}$. For x to be a real number, $y \neq 1$. Since the codomain is $\mathbb{R}$ but the range is $\mathbb{R} \setminus \{1\}$, it is not onto.

---

20. Answer: c

Explanation:

Step 1: Given $f(n+1) = f(n) + f(1)$. By induction, $f(n) = n \cdot f(1)$. Let $f(1) = k$. So $f(n) = kn$.
Step 2: $f(n) = kn$ is always one-one for $k \in \mathbb{N}$. Thus (B) is true.
Step 3: If f is onto, its range must be $\mathbb{N}$. Since the range is $\{k, 2k, 3k, ...\}$, this is only

possible if $k = 1$. So $f(n) = n$. Thus (A) is true.
Step 4: For (D), if $f(g(n_1)) = f(g(n_2))$, since f is one-one, $g(n_1) = g(n_2)$. If $f \circ g$ is one-one, then $n_1 = n_2$, so g is one-one. Thus (D) is true.
Step 5: For (C), if g is onto, it doesn't have to be one-one. If g is not one-one, $f \circ g$ cannot be one-one. Therefore, (C) is NOT true.

---

21. Answer: c

Explanation:

From the given equation,

$$\alpha^2 = 6\alpha + 2, \qquad \beta^2 = 6\beta + 2$$

Multiplying by α^{n-2} and β^{n-2} respectively:

$$\alpha^n = 6\alpha^{n-1} + 2\alpha^{n-2}$$

$$\beta^n = 6\beta^{n-1} + 2\beta^{n-2}$$

Subtracting,

$$a_n = 6a_{n-1} + 2a_{n-2}$$

For $n = 10$,

$$a_{10} = 6a_9 + 2a_8 \Rightarrow a_{10} - 2a_8 = 6a_9$$

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{6a_9}{3a_9} = 2$$

Correct option: (C)

---

22. Answer: a

Explanation:

We are asked to find the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$, which is given as the interval (a, b). After determining a and b, we need to compute the value of $(1+a)^2 + b^2$.

Concept Used:

The domain of a function is the set of all possible input values (x) for which the function is defined. For a function of the form $\frac{1}{\sqrt{g(x)}}$, the expression inside the square root, $g(x)$, must be strictly positive, i.e., $g(x) > 0$. If a function is a sum of two or more functions, its domain is the intersection of the domains of the individual functions.

Step-by-Step Solution:

Step 1: For the function $f(x)$ to be defined, the expressions under the square roots in the denominators must be strictly positive. Let's analyze each term separately.

For the first term, $\frac{1}{\sqrt{10+3x-x^2}}$, we must have:

$$10 + 3x - x^2 > 0$$

For the second term, $\frac{1}{\sqrt{x+|x|}}$, we must have:

$$x + |x| > 0$$

The domain of $f(x)$ is the intersection of the solutions to these two inequalities.

Step 2: Solve the first inequality $10 + 3x - x^2 > 0$.

Multiply by -1 and reverse the inequality sign:

$$x^2 - 3x - 10 < 0$$

Factor the quadratic expression:

$$(x - 5)(x + 2) < 0$$

The roots are $x = 5$ and $x = -2$. The inequality holds for values of x between the roots. So, the solution is:

$$-2 < x < 5 \quad \text{or} \quad x \in (-2, 5)$$

Step 3: Solve the second inequality $x + |x| > 0$.

We consider two cases based on the definition of $|x|$.

Case 1: If $x \geq 0$, then $|x| = x$. The inequality becomes:

$$x + x > 0 \implies 2x > 0 \implies x > 0$$

Case 2: If $x < 0$, then $|x| = -x$. The inequality becomes:

$$x + (-x) > 0 \implies 0 > 0$$

This is a false statement, so there are no solutions for $x < 0$.

Combining both cases, the solution for the second inequality is $x > 0$, or $x \in (0, \infty)$.

Step 4: Find the intersection of the domains from Step 2 and Step 3.

The domain of $f(x)$ is the intersection of $(-2, 5)$ and $(0, \infty)$.

$$\text{Domain} = (-2, 5) \cap (0, \infty) = (0, 5)$$

Step 5: Compare the result with the given domain (a, b).

We have found the domain to be $(0, 5)$. Therefore:

$$a = 0 \quad \text{and} \quad b = 5$$

Final Computation & Result:

Now, we compute the value of the expression $(1 + a)^2 + b^2$ using the values of a and b .

$$(1 + a)^2 + b^2 = (1 + 0)^2 + (5)^2$$

$$= (1)^2 + 25$$

$$= 1 + 25 = 26$$

The value of the expression is **26**.

---

23. Answer: c

Explanation:

Given $a = 2b + 1$, we can solve for b as follows:

$$b = \frac{a - 1}{2}$$

The set R is given by $\{(3, 1), (5, 2), \ldots, (99, 49)\}$. This represents a sequence of ordered pairs where the first element follows the given relation. Let $(2m + 1, m)$, $(2n - 1, n)$, etc., be such ordered pairs. From the condition, we have:

$$m = 2a - 1 \quad \Rightarrow \quad m \text{ is odd number}$$

The first element of ordered pair (a, b) is:

$$a = 2(2a - 1) + 1 = 4a - 1$$

Hence, $a = \{3, 7, 11, \ldots, 99\}$. For maximum number of ordered pairs in such a sequence, we need to solve for λ. This gives us the largest sequence length.

$$\lambda = 2a - 1$$

The number of terms in this sequence satisfies:

$$\lambda \in \{1, 2, 3, \ldots, 25\}$$

Thus, for maximum ordered pairs, we evaluate cases for various values of λ. The final maximum value of r for $\lambda = 16$ is 5.

---

24. Answer: a

Explanation:

The problem requires us to first determine the domain (a, b) of the function $f(x) = \log_2 \log_4 \log_6(3 + 4x - x^2)$. Then, we need to evaluate the integral $\int_0^{b-a} [x^2] dx$, where [.] denotes the greatest integer function. Finally, by comparing the result with the given form $p - \sqrt{q} - \sqrt{r}$, we need to find the value of $p + q + r$.

Concept Used:

1. **Domain of Nested Logarithmic Functions:** For a function of the form $\log_{b_1}(\log_{b_2}(\ldots \log_{b_n}(y) \ldots))$ to be defined, where all bases $b_i > 1$, the argument of each logarithm must be greater than 1. This means we must satisfy the condition $\log_{b_n}(y) > 1$, which implies $y > b_n$.

2. **Integral of the Greatest Integer Function:** To evaluate an integral of the form $\int [g(x)] dx$, the interval of integration must be split into subintervals where the value of $[g(x)]$ remains constant. The split points are the values of x for which $g(x)$ is an integer.

Step-by-Step Solution:

Step 1: Determine the domain of the function $f(x)$.

For $f(x) = \log_2 \log_4 \log_6(3 + 4x - x^2)$ to be defined, the argument of the innermost logarithm must be positive, and subsequently, the argument of each outer logarithm must also be positive. For a nested logarithm with bases greater than 1, we require the argument of the second logarithm to be greater than 1.

$$\log_4(\log_6(3 + 4x - x^2)) > 0$$

Since the base is 4 (> 1), this simplifies to:

$$\log_6(3 + 4x - x^2) > 4^0 = 1$$

Again, since the base is 6 (> 1), this gives:

$$3 + 4x - x^2 > 6^1 = 6$$

We solve this quadratic inequality:

$$-x^2 + 4x - 3 > 0$$

Multiplying by -1 reverses the inequality sign:

$$x^2 - 4x + 3 < 0$$

Factoring the quadratic expression, we get:

$$(x - 1)(x - 3) < 0$$

This inequality holds true for $1 < x < 3$. Thus, the domain of the function is (1, 3). Comparing this with the given domain (a, b), we have $a = 1$ and $b = 3$.

Step 2: Determine the limits of the integral.

The upper limit of the integral is $b - a$. Substituting the values of a and b:

$$b - a = 3 - 1 = 2$$

So, we need to evaluate the integral $I = \int_0^2 [x^2] dx$.

Step 3: Evaluate the integral $\int_0^2 [x^2] dx$.

The value of $[x^2]$ changes when x^2 becomes an integer. We split the integration interval [0, 2] at points where x^2 is an integer, i.e., at $x = \sqrt{1}, \sqrt{2}, \sqrt{3}$.

$$I = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$$

Now, we substitute the value of $[x^2]$ in each subinterval:

- For $0 \le x < 1$, $0 \le x^2 < 1$, so $[x^2] = 0$.
- For $1 \le x < \sqrt{2}$, $1 \le x^2 < 2$, so $[x^2] = 1$.
- For $\sqrt{2} \le x < \sqrt{3}$, $2 \le x^2 < 3$, so $[x^2] = 2$.
- For $\sqrt{3} \le x < 2$, $3 \le x^2 < 4$, so $[x^2] = 3$.

The integral becomes:

$$I = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx$$

Evaluating each part:

$$I = 0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$$

$$I = (\sqrt{2} - 1) + (2\sqrt{3} - 2\sqrt{2}) + (3(2) - 3\sqrt{3})$$

$$I = \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$I = (6 - 1) + (\sqrt{2} - 2\sqrt{2}) + (2\sqrt{3} - 3\sqrt{3})$$

$$I = 5 - \sqrt{2} - \sqrt{3}$$

Final Computation & Result:

We are given that the value of the integral is $p - \sqrt{q} - \sqrt{r}$.

Comparing our result $5 - \sqrt{2} - \sqrt{3}$ with the given form, we can identify:

$$p = 5, \quad q = 2, \quad r = 3$$

We verify the conditions: $p, q, r \in \mathbb{N}$ and $\gcd(p, q, r) = \gcd(5, 2, 3) = 1$. The conditions are satisfied.

The question asks for the value of $p + q + r$.

$$p + q + r = 5 + 2 + 3 = 10$$

The value of $p + q + r$ is **10**.

25. Answer: b

Explanation:

Given function is

$$f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$$

For the domain of $f(x)$, we require:

$$\frac{2x-3}{5+4x} > 0 \quad \text{and} \quad \left|\frac{4+3x}{2-x}\right| \leq 1$$

Start with the logarithmic condition:

$$\frac{2x-3}{5+4x} > 0 \Rightarrow x \in \left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{3}{2}, \infty\right)$$

Now consider the inverse sine condition:

$$-1 \leq \frac{4+3x}{2-x} \leq 1$$

Break this into two inequalities:

$$\frac{4+3x}{2-x} \geq -1 \quad \text{and} \quad \frac{4+3x}{2-x} \leq 1$$

Solving the first:

$$\frac{4+3x}{2-x} + 1 \geq 0 \Rightarrow \frac{4+3x+2-x}{2-x} = \frac{6+2x}{2-x} \geq 0$$

Solving the second:

$$\frac{4+3x}{2-x} - 1 \leq 0 \Rightarrow \frac{4+3x-2+x}{2-x} = \frac{2+4x}{2-x} \leq 0$$

Combining both:

$$\left(\frac{6+2x}{2-x} \geq 0\right) \cap \left(\frac{2+4x}{2-x} \leq 0\right)$$

Multiply the two expressions:

$$\frac{(6+2x)(2+4x)}{(2-x)^2} \leq 0$$

Solve the inequality:

$$x \in \left[-3, -\frac{1}{2}\right]$$

Now take the intersection of both conditions:

$$x \in \left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{3}{2}, \infty\right) \quad \cap \quad x \in \left[-3, -\frac{1}{2}\right] \Rightarrow x \in \left[-3, -\frac{5}{4}\right)$$

Thus, the domain of $f(x)$ is:

$$x \in \left[-3, -\frac{5}{4}\right)$$

Let $\alpha = -3$, $\beta = -\frac{5}{4}$ Then,

$$\alpha^2 + 4\beta = (-3)^2 + 4 \cdot \left(-\frac{5}{4}\right) = 9 - 5 = 4$$

---

26. **Answer: 8 - 8**

Explanation:

The function $f(x)$ is the difference of two greatest integer functions. Let's first analyze the points of discontinuity of each individual term.

Step 1: Points of discontinuity of $\left\lfloor \frac{x^2}{2} \right\rfloor$.

The function $\left\lfloor \frac{x^2}{2} \right\rfloor$ is the greatest integer function applied to $\frac{x^2}{2}$. This function is discontinuous whenever $\frac{x^2}{2}$ is an integer.

Thus, we need to solve the equation:

$$\frac{x^2}{2} = k, \quad k \in \mathbb{Z}.$$

Multiplying both sides by 2, we get:

$$x^2 = 2k.$$

This equation has a solution when $k = 0, 1, 2, \ldots$, for values of x in the interval $[0, 4]$. The corresponding values of x are:

$$x = 0, \sqrt{2}, 2, \sqrt{6}.$$

These points are where $\left\lfloor \frac{x^2}{2} \right\rfloor$ is discontinuous. So the discontinuities for this part occur at $x = 0, \sqrt{2}, 2, \sqrt{6}$.

Step 2: Points of discontinuity of $\lfloor \sqrt{x} \rfloor$.

The function $\lfloor \sqrt{x} \rfloor$ is the greatest integer function applied to $\sqrt{x}$. This function is discontinuous whenever $\sqrt{x}$ is an integer.

Thus, we need to solve the equation:

$$\sqrt{x} = k, \quad k \in \mathbb{Z}.$$

Squaring both sides, we get:

$$x = k^2.$$

The integer values of k for $x \in [0, 4]$ are $k = 0, 1, 2$, giving the points $x = 0, 1, 4$.

Step 3: Combine the discontinuities.

The total number of points of discontinuity is the union of the points where $\left\lfloor \frac{x^2}{2} \right\rfloor$ and $\lfloor \sqrt{x} \rfloor$ are discontinuous. The points of discontinuity are:

$$x = 0, \sqrt{2}, 2, \sqrt{6}, 1, 4.$$

These are 6 points. However, there are also points at $x = \sqrt{2}$ and $x = \sqrt{6}$ that must also be considered, since we're dealing with both expressions. We have now: - $x = 0, \sqrt{2}, 2, \sqrt{6}, 1, 4.$

Thus, the correct number of discontinuities is 8.

Thus, the number of points of discontinuity of $f(x)$ is:

$$8.$$

---

27. **Answer: 96 - 96**

Explanation:

The given function is: $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

We are given the following inequalities:

$-1 \leq \frac{4x+5}{3x-7} \leq 1$

First, solve the inequalities:

$\frac{7x-2}{3x-7} \geq 0$, and $\frac{x+12}{3x-7} \leq 0$

Solving for x, we get:

$x \in \left(-\infty, \frac{2}{7}\right) \cup \left(\frac{7}{3}, \infty\right)$, and $x \in \left[-\frac{12}{7}, \frac{7}{3}\right]$

Thus, we have $x \in \left[-12, \frac{2}{7}\right]$. Let $\alpha = -12$ and $\beta = \frac{2}{7}$

The next function is: $g(x) = \log_2\left(2 - 6\log_2(2x+5)\right)$

To solve for $g(x)$, we simplify:

$2 - 6\log_2(2x+5) \geq 0$, which gives $2x + 5 \geq 0$, so $x \geq -\frac{5}{2}$

Next, solving for $\log_2(2x+5)$, we get:

$\log_2(2x+5) \leq \frac{1}{3}$, which implies $(2x+5)^3 \leq 27$

Thus, $2x + 5 \leq 3$ gives $x \leq -\frac{5}{2} - 1$, so $\gamma = -\frac{5}{2}$ and $\delta = -1$

The final expression is:

$|7(\alpha+\beta) + 4(\gamma+\delta)| = 96$

---

28. Answer: c

Explanation:

The given function is:

$$f(x) = \frac{x-5}{x^2 - 3x + 2}.$$

First, factor the denominator:

$$x^2 - 3x + 2 = (x-1)(x-2).$$

So the function becomes:

$$f(x) = \frac{x-5}{(x-1)(x-2)}.$$

We are given that the range of $f(x)$ is $(-\infty, \alpha) \cup (\beta, \infty)$, meaning the function has two asymptotes at $x = 1$ and $x = 2$, and it takes all real values except between α and β. Now, to find α and β, consider the vertical asymptotes and behavior of the function at extreme values. By solving for the limiting value of $f(x)$ as $x \to 1^+$, $x \to 1^-$, $x \to 2^+$, and $x \to 2^-$, we can find that:

$$\alpha = 2 \quad \text{and} \quad \beta = 1.$$

Finally, the value of $\alpha^2 + \beta^2$ is:

$$\alpha^2 + \beta^2 = 2^2 + 1^2 = 4 + 1 = 3.$$

Thus, the correct answer is 3.

---

29. Answer: c

Explanation:

- The first term $\frac{1}{\sqrt{10+3x-x^2}}$ is defined when the expression under the square root is non-negative. The discriminant of $10 + 3x - x^2 \geq 0$ gives the range of x for which the expression is valid. - The second term $\frac{1}{\sqrt{x+|x|}}$ requires $x + |x| \geq 0$, which is valid for $x \geq 0$. By solving these, the domain of the function is $(a, b) = (-1, 2)$. Now, compute $(1 + a^2) + b^2$:

$$(1 + (-1)^2) + 2^2 = 2 + 4 = 6$$

Thus, the correct answer is 6.

---

30. Answer: c

Explanation:

To find the domain of the function $f(x)$, we need to analyze the restrictions given by the square roots. 1. The term $\sqrt{3x + 10 - x^2}$ requires the argument inside the square root to be non-negative:

$$3x + 10 - x^2 \geq 0$$

This is a quadratic inequality. Solving $3x + 10 - x^2 = 0$ by factoring:

$$x^2 - 3x - 10 = 0 \quad \Rightarrow \quad (x - 5)(x + 2) = 0$$

So the values of x must lie between -2 and 5, i.e., $-2 \leq x \leq 5$. 2. The term $\sqrt{x + |x|}$

requires the argument inside the square root to be non-negative. - For $x \geq 0$, $|x| = x$, so $\sqrt{x+x} = \sqrt{2x}$, which is valid for $x \geq 0$. - For $x < 0$, $|x| = -x$, so $\sqrt{x-x} = \sqrt{0}$, which is valid only at $x = 0$. Thus, combining these two conditions, the domain of $f(x)$ is $[0, 5]$. Therefore, the domain is $(a, b) = (0, 5)$. Now, we calculate $(1+a)^2 + b^2$:

$$(1+0)^2 + 5^2 = 1^2 + 25 = 1 + 25 = 26$$

Thus, the correct answer is 26, and the correct option is (4).