GATE 2021 Aerospace Engineering (AE) Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. Which of the following sentences are grammatically CORRECT?

(i) Arun and Aparna are here.

(ii) Arun and Aparna is here.

(iii) Arun's families is here.

(iv) Arun's family is here.

(A) (i) and (ii)

(B) (i) and (iv)

(C) (ii) and (iv)

(D) (iii) and (iv)

Correct Answer: (B) (i) and (iv)

Solution:

Step 1: Check subject-verb agreement in each sentence.

Sentence (i): *Arun and Aparna are here.* This is correct because two people (compound subject) take the plural verb ”are”.

Sentence (ii): *Arun and Aparna is here.* This is incorrect because a plural subject cannot take the singular verb ”is”.

Sentence (iii): *Arun's families is here.* The word ”families” is plural, so the verb must be ”are”, not ”is”. Hence incorrect.

Sentence (iv): *Arun's family is here.* ”Family” is singular, so the singular verb ”is” is correct.

Step 2: Select the correct pair(s).

Only sentences (i) and (iv) are grammatically correct.

Step 3: Conclusion.

Thus, the correct answer is Option (B).

> **Quick Tip**
>
> Always match the verb with the true number of the subject. Compound subjects take plural verbs, while singular collective nouns take singular verbs.

---

2. The mirror image of the above text about the x-axis is

(A)	
(B)	
(C)	
(D)	

Correct Answer: (B)

Solution:

We are given the text PHYLAXIS written above the x-axis. We need to find its mirror image when reflected across the x-axis.

Step 1: Understanding reflection about the x-axis.

Reflection about the x-axis flips the object vertically. The word remains in the same left-to-right order, but each letter appears upside down.

Step 2: Observe letter transformations.

Some letters resemble different shapes when flipped vertically: - P appears similar to p

- Y flips into a -like structure

- X, A, H, I remain visually symmetric or change minimally

Step 3: Compare with options.

Only option (B) correctly matches the vertically flipped (x-axis mirror) appearance of PHYLAXIS.

Final Answer: (B) pHYAXIS

Quick Tip

Reflection in the x-axis flips shapes vertically; reflection in the y-axis flips them horizontally.

3. Two identical cube-shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is:

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

Correct Answer: (D) $\frac{1}{4}$

Solution:

Each die has 3 even numbers: 2, 4, 6.

Probability of getting an even number on one die = $\frac{3}{6} = \frac{1}{2}$.

Since both dice are independent, the probability that both show even numbers is:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Quick Tip

For independent events occurring together, multiply their probabilities.

---

4. $\oplus$ **and** $\odot$ **are two operators on numbers** p **and** q **such that** $p \odot q = p - q$ **and** $p \oplus q = p \times q$**. Find the value of** $(9 \odot (6 \oplus 7)) \odot (7 \oplus (6 \odot 5))$**.**

(A) 40

(B) -26

(C) -33

(D) -40

Correct Answer: (D) -40

Solution:

First evaluate the innermost operation:

$6 \oplus 7 = 6 \times 7 = 42$

Then, $9 \odot 42 = 9 - 42 = -33$

Next evaluate the second part:

$6 \odot 5 = 6 - 5 = 1$

$7 \oplus 1 = 7 \times 1 = 7$

Final operation:

$(-33) \odot 7 = -33 - 7 = -40$

Quick Tip

Always apply custom operators step-by-step, evaluating the innermost brackets first.

---

5. Four persons P, Q, R and S are to be seated in a row. R should not be seated at the second position from the left end. The number of distinct seating arrangements possible is:

(A) 6

(B) 9

(C) 18

(D) 24

Correct Answer: (C) 18

Solution:

Total seating arrangements for 4 persons = $4! = 24$.

Restricted cases: R sits in the second position.

Fix R at position 2; remaining 3 persons can be arranged in $3! = 6$ ways.

Allowed arrangements = $24 - 6 = 18$.

Quick Tip

For restriction-based problems, calculate total arrangements and subtract the restricted ones.

---

6. On a planar field, you travelled 3 units East from a point O. Next you travelled 4 units South to arrive at point P. Then you travelled from P in the North-East direction such that you arrive at a point that is 6 units East of point O. Next, you travelled in the North-West direction, so that you arrive at point Q that is 8 units North of point P. The distance of point Q to point O, in the same units, should be ________

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Start with origin $O = (0, 0)$.

Step 1: Move 3 units East.

Point becomes $(3, 0)$.

Step 2: Move 4 units South to P.

Point $P = (3, -4)$.

Step 3: Move NE to reach a point 6 units east of O.

This new point is $(6, y)$.

Movement NE increases x and y equally by 3:

$$y = -4 + 3 = -1.$$

Thus point is $(6, -1)$.

Step 4: Move NW to reach Q, which is 8 units north of P.

P is at $(3, -4)$.

So Q has y-coordinate:

$$y_Q = -4 + 8 = 4.$$

From $(6, -1)$ to Q, y increases by 5, so x decreases by 5 (NW move):

$$x_Q = 6 - 5 = 1.$$

Thus $Q = (1, 4)$.

Step 5: Distance OQ.

$$OQ = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12 \approx 5.$$

Final Answer: 5

> Quick Tip
>
> Break multi-step motion into coordinate shifts, then apply the distance formula.

---

7. Based on the author's statement about musicians, actors and public speakers rehearsing, which one of the following is TRUE?

(A) The author is of the opinion that rehearsing is important for musicians, actors and public speakers.

(B) The author is of the opinion that rehearsing is less important for public speakers than for musicians and actors.

(C) The author is of the opinion that rehearsing is more important only for musicians than public speakers.

(D) The author is of the opinion that rehearsal is more important for actors than musicians.

Correct Answer: (A)

Solution:

The author notes that musicians rehearse before concerts and actors rehearse before plays.
He finds it strange that many public speakers do not rehearse.
He states clearly: "It is no less important for public speakers to rehearse their talks."
"No less important" means equally important.
Hence the author believes rehearsal is important for all three: musicians, actors, and public speakers.

Final Answer: (A)

Quick Tip

Look for phrases like "no less important" — they indicate equality in importance.

---

8. 1. Some football players play cricket.

2. All cricket players play hockey.

Among the options given below, the statement that logically follows from the two statements 1 and 2 above, is:

(A) No football player plays hockey.

(B) Some football players play hockey.

(C) All football players play hockey.

(D) All hockey players play football.

Correct Answer: (B) Some football players play hockey.

Solution:

Step 1: Interpretation of statements.

Statement 1 says some football players play cricket. Statement 2 says all cricket players play hockey.

Step 2: Combining the statements.

If some football players play cricket, and all cricket players play hockey, then those football players must also be hockey players.

Step 3: Logical conclusion.

Therefore, at least a few (some) football players play hockey. Hence, option B is correct.

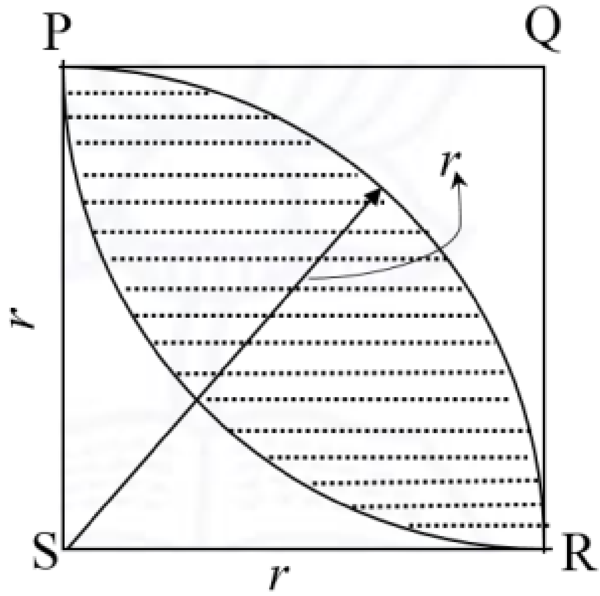
Quick Tip

Whenever A $\rightarrow$ B and B $\rightarrow$ C, then part of A also becomes part of C. This helps solve most logic chain questions.

---

9. In the figure, PQRS is a square. The shaded part is formed by the intersection of sectors of two circles of radius equal to the side of the square and centers at S and Q.

The probability that a random point inside the square lies in the shaded region is:



(A) $4 - \frac{\pi}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\pi}{2} - 1$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{2} - 1$

Solution:

Step 1: Shape description.

PQRS is a square of side r. Two quarter-circles of radius r are drawn—one centered at S and one at Q. Their overlapping region forms the shaded lens-shaped area.

Step 2: Area of each quarter-circle.

Each quarter-circle area is $\frac{1}{4}\pi r^2$.

Step 3: Area of intersection.

The overlapping region of these opposite-corner quarter circles is known to have area:

$$\left(\frac{\pi}{2} - 1\right) r^2.$$

Step 4: Required probability.

Probability = (Shaded area) / (Square area)

$$= \frac{\left(\frac{\pi}{2} - 1\right) r^2}{r^2} = \frac{\pi}{2} - 1.$$

Quick Tip

In geometry-based probability questions, always compute the exact geometrical area first and then divide by the total area.

---

10. In an equilateral triangle PQR, side PQ is divided into four equal parts, side QR is divided into six equal parts and side PR is divided into eight equal parts. The length of each subdivided part in cm is an integer. The minimum area of the triangle PQR possible, in cm^2, is:

(A) 18

(B) 24

(C) $48\sqrt{3}$

(D) $144\sqrt{3}$

Correct Answer: (D) $144\sqrt{3}$

Solution:

Step 1: Understanding the subdivision condition.

Side PQ is divided into 4 equal integer parts. Side QR is divided into 6 equal integer parts.

Side PR is divided into 8 equal integer parts.

Let the side length of the equilateral triangle be s.

Then:

$$\frac{s}{4}, \frac{s}{6}, \frac{s}{8}$$

must all be integers.

Step 2: Find the smallest possible value of s.

s must be a multiple of lcm$(4, 6, 8)$.

$$\text{lcm}(4, 6, 8) = 24$$

So the smallest possible side of the equilateral triangle is:

$$s = 24 \text{ cm}$$

Step 3: Compute the area of the equilateral triangle.

Area of an equilateral triangle:

$$A = \frac{\sqrt{3}}{4}s^2$$

Substituting $s = 24$:

$$A = \frac{\sqrt{3}}{4} \times 24^2 = \frac{\sqrt{3}}{4} \times 576 = 144\sqrt{3}$$

Step 4: Final conclusion.

Thus the minimum area of triangle PQR is:

$$144\sqrt{3}$$

> **Quick Tip**
>
> When parts of sides of a triangle must be integers, always take the LCM of the division counts to find the minimum valid side length.

Aerospace Engineering (AE)

1. Consider the differential equation

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$$

and the boundary conditions $y(0) = 1$ **and** $\frac{dy}{dx}(0) = 0$. **The solution to this equation is:**

(A) $y = (1 + 2x)e^{-4x}$

(B) $y = (1 - 4x)e^{-4x}$

(C) $y = (1 + 8x)e^{-4x}$

(D) $y = (1 + 4x)e^{-4x}$

Correct Answer: (D) $(1 + 4x)e^{-4x}$

Solution:

Step 1: Solve the characteristic equation.

The ODE

$$y'' + 8y' + 16y = 0$$

gives the characteristic equation

$$m^2 + 8m + 16 = 0 = (m + 4)^2.$$

Thus, the repeated root is $m = -4$.

Step 2: General solution for repeated roots.

$$y = (C_1 + C_2x)e^{-4x}.$$

Step 3: Apply boundary conditions.

From $y(0) = 1$, we get $C_1 = 1$.

Differentiate:

$$y' = C_2e^{-4x} - 4(C_1 + C_2x)e^{-4x}.$$

At $x = 0$:

$$0 = y'(0) = C_2 - 4C_1 \Rightarrow C_2 = 4.$$

Step 4: Final solution.

$$y = (1 + 4x)e^{-4x}.$$

Quick Tip

Repeated roots in differential equations yield solutions of the form $(C_1 + C_2x)e^{mx}$.

---

2. The PDE

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} = x + 2y$$

The nature of this equation is:

(A) linear

(B) elliptic

(C) hyperbolic

(D) parabolic

Correct Answer: (B) elliptic

Solution:

Step 1: Identify coefficients.

The PDE is of the form

$$Au_{xx} + 2Bu_{xy} + Cu_{yy}.$$

Here, $A = 1$, $2B = -4 \Rightarrow B = -2$, $C = 6$.

Step 2: Use discriminant $B^2 - AC$.

$$B^2 - AC = (-2)^2 - (1)(6) = 4 - 6 = -2 < 0.$$

Since the discriminant is negative, the equation is **elliptic**.

Quick Tip

For PDEs: Elliptic if $B^2 - AC < 0$, Parabolic if $B^2 - AC = 0$, Hyperbolic if $B^2 - AC > 0$.

---

3. Consider the velocity field

$$\vec{V} = (2x + 3y)\hat{\imath} + (3x + 2y)\hat{\jmath}.$$

The field $\vec{V}$ is:

(A) divergence-free and curl-free

(B) curl-free but not divergence-free

(C) divergence-free but not curl-free

(D) neither divergence-free nor curl-free

Correct Answer: (B) curl-free but not divergence-free

Solution:

Step 1: Compute divergence.

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x}(2x + 3y) + \frac{\partial}{\partial y}(3x + 2y) = 2 + 2 = 4.$$

Since divergence is $4 \neq 0$, the field is **not divergence-free**.

Step 2: Compute curl (2D).

$$(\nabla \times \vec{V})_z = \frac{\partial}{\partial x}(3x + 2y) - \frac{\partial}{\partial y}(2x + 3y) = 3 - 3 = 0.$$

Curl is zero → field is **curl-free**.

Step 3: Conclusion.

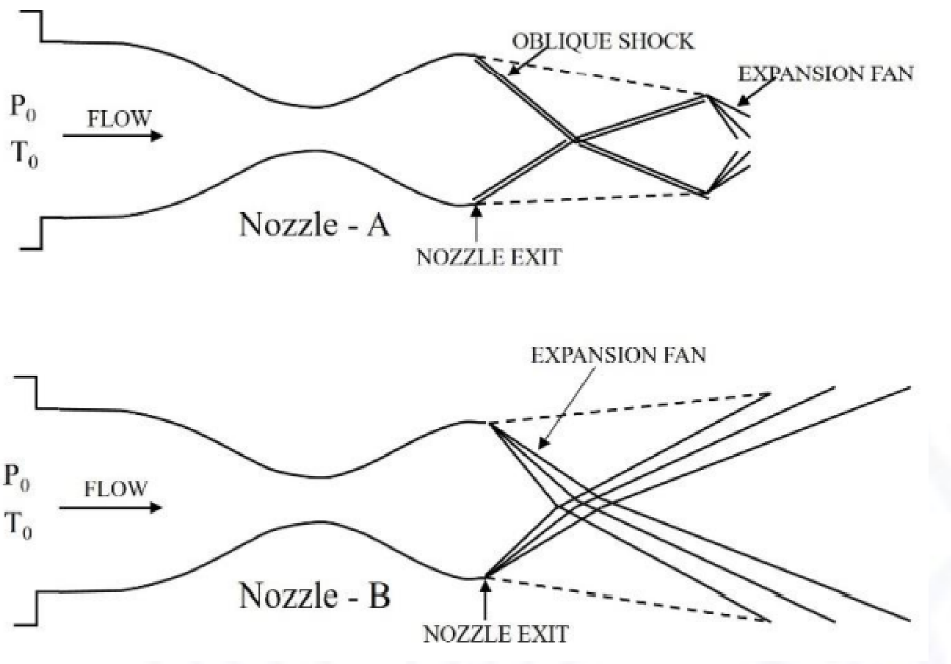
Thus, the field is **curl-free but not divergence-free**.

Quick Tip

A vector field with zero curl is irrotational; one with zero divergence is incompressible.

---

4. Nozzles A and B, respectively, are said to be operating in:



(A) over-expanded mode and under-expanded mode

(B) under-expanded mode and perfectly expanded mode

(C) perfectly expanded mode and under-expanded mode

(D) under-expanded mode and over-expanded mode

Correct Answer: (A) over-expanded mode and under-expanded mode

Solution:

Step 1: Examine Nozzle A.

In Nozzle A, oblique shocks are seen at the nozzle exit. Shocks indicate that the exit pressure is higher than the ambient pressure. This situation occurs when the nozzle is operating in an *over-expanded* condition, causing the flow to compress through shock waves.

Step 2: Examine Nozzle B.

In Nozzle B, only expansion fans appear at the exit. Expansion waves occur when the exit pressure is greater than the ambient pressure. This means the nozzle is operating in an *under-expanded* mode, allowing further expansion after exiting the nozzle.

Step 3: Conclusion.

Therefore, Nozzle A is over-expanded and Nozzle B is under-expanded.

Final Answer: (A) over-expanded mode and under-expanded mode

> **Quick Tip**
>
> Shocks at the exit imply over-expansion; expansion fans imply under-expansion in supersonic nozzles.

---

5. The combustion process in a turbo-shaft engine during ideal operation is:

(A) isentropic

(B) isobaric

(C) isochoric

(D) isothermal

Correct Answer: (B) isobaric

Solution:

Step 1: Identify the cycle.

A turbo-shaft engine follows the Brayton cycle. In this ideal cycle, heat addition (combustion) occurs at constant pressure.

Step 2: Why is it isobaric?

The combustor is designed such that pressure loss is minimal, meaning that burning fuel increases temperature and volume while maintaining nearly constant pressure. This makes the combustion process isobaric.

Step 3: Eliminate incorrect options.

Isentropic: refers to compressor/ turbine processes.

Isochoric: constant-volume heat addition (Otto cycle).

Isothermal: does not apply to gas turbines.

Final Answer: (B) isobaric

> Quick Tip
>
> In the Brayton cycle: compression and expansion are isentropic, combustion is isobaric.

---

6. How does the specific thrust of a turbojet engine change for a given flight speed with increase in flight altitude?

(A) Increases monotonically

(B) Decreases monotonically

(C) Remains constant

(D) First increases and then decreases

Correct Answer: (A) Increases monotonically

Solution:

Specific thrust is defined as the thrust produced per unit mass flow rate of air entering the engine.

$$\text{Specific Thrust} = \frac{F}{\dot{m}}$$

When altitude increases, two important atmospheric changes occur: reduction in air density and reduction in ambient temperature.

1. Effect of decrease in air density:

Lower density means a lower mass flow rate into the engine for the same inlet area and same flight Mach number. Since mass flow rate decreases, if the net thrust does not decrease proportionally, the specific thrust increases. In turbojets, the loss in thrust due to lower density is much smaller compared to the drop in mass flow.

2. Effect of decrease in temperature:

At higher altitudes, the air is significantly colder. Lower ambient temperature increases the stagnation temperature ratio inside the engine, resulting in a larger difference between jet exhaust velocity and flight velocity. Jet velocity rises because colder inlet air requires less energy addition to reach the same turbine entry temperature. This directly increases net thrust.

3. Combined effect:

While mass flow rate reduces with altitude, the increase in jet velocity more than compensates for it. Therefore, the ratio $\frac{F}{\dot{m}}$ increases continuously with altitude. The relation remains monotonic and does not show any turning points.

Hence, the specific thrust increases monotonically as the aircraft climbs to higher altitudes.

Quick Tip

At higher altitudes, colder ambient temperature increases jet velocity faster than the thrust decreases due to density drop—so specific thrust always rises.

---

7. How does the propulsion efficiency of a turbofan engine, operating at a given Mach number and altitude, change with increase in compressor pressure ratio?

(A) Remains constant

(B) Increases monotonically

(C) Decreases monotonically

(D) First decreases and then increases

Correct Answer: (D) First decreases and then increases

Solution:

Propulsion efficiency measures how effectively the engine converts kinetic energy into useful thrust power. It is given by:

$$\eta_p = \frac{2V_0}{V_j + V_0}$$

where V_0 = flight speed and V_j = jet velocity.

1. Effect of increasing compressor pressure ratio (CPR):

Higher CPR increases the stagnation temperature at the compressor exit. Therefore, the turbine must extract more work. To maintain the same turbine inlet temperature, more fuel is burned, raising the jet exhaust velocity.

2. Initial behaviour (efficiency decreases):

As the compressor pressure ratio starts increasing, the jet velocity increases more sharply than the flight velocity. Since propulsion efficiency decreases when the jet becomes much faster than the aircraft:

$$\eta_p \downarrow$$

In this region, kinetic energy wasted in the jet increases significantly.

3. Later behaviour (efficiency increases):

At very high CPR, thermal efficiency increases substantially. Higher CPR improves Brayton-cycle efficiency, meaning more of the fuel's energy is converted into useful work. Even though jet velocity remains high, the ratio of useful thrust power to fuel energy increases. This raises propulsion efficiency.

Thus, the curve is non-monotonic:

$$\text{Efficiency} = \begin{cases} \text{decreases initially}, \\ \text{then increases with high CPR}. \end{cases}$$

Hence, the correct trend is: **first decreases, then increases**.

> **Quick Tip**
>
> Propulsion efficiency depends on how close jet speed is to flight speed; increasing CPR first increases the mismatch, then improves thermal efficiency.

---

8. A solid propellant rocket producing 25 MN thrust is fired for 150 seconds. The specific impulse of the rocket is 2980 Ns/kg. How much propellant is burned during the rocket operation?

(A) 8390 kg

(B) 82300 kg

(C) 1.26×10^6 kg

(D) 11.2×10^6 kg

Correct Answer: (C) 1.26×10^6 kg

Solution:

We are given:

Thrust: $F = 25 \times 10^6$ N

Burn time: $t = 150$ s

Specific impulse: $I_s = 2980$ Ns/kg

1. Understanding specific impulse:

Specific impulse relates thrust to mass flow rate:

$$I_s = \frac{F}{\dot{m}} \Rightarrow \dot{m} = \frac{F}{I_s}$$

2. Compute mass flow rate:

$$\dot{m} = \frac{25 \times 10^6}{2980} \approx 8390 \text{ kg/s}$$

3. Compute total propellant consumed:

$$m = \dot{m} \times t = 8390 \times 150$$

$$m \approx 1.26 \times 10^6 \text{ kg}$$

This is approximately 1.26 million kilograms of solid propellant burned in 150 seconds. The large value is consistent with typical heavy-lift booster rockets.

Quick Tip

Always use $m = \frac{F}{I_s} \times t$ for total propellant used in rockets with constant thrust.

---

9. The shape of a supersonic diffuser that slows down a supersonic flow to subsonic flow is

(A) converging

(B) diverging

(C) diverging–converging

(D) converging–diverging

Correct Answer: (D) converging–diverging

Solution:

In compressible flow, the behavior of gases changes significantly between subsonic and supersonic regimes.

Step 1: Behavior in subsonic flow.

A subsonic diffuser uses a **diverging** passage to slow down the flow because pressure increases when the area increases.

Step 2: Behavior in supersonic flow.

In contrast, for supersonic speeds, the relation is reversed: a **converging** passage causes the flow to decelerate. So to convert supersonic flow into subsonic, the diffuser must first converge.

Step 3: Shock and further diffusion.

After the converging part slows the flow to near sonic speed, a shock or pressure rise occurs, and a diverging shape helps complete the subsonic diffusion.

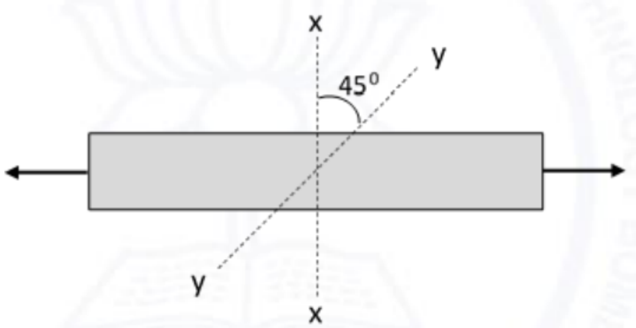
Thus, a **converging–diverging** (C–D) diffuser is used.

Final Answer: converging–diverging

> Quick Tip
>
> In supersonic flows, area–velocity relation reverses: convergence slows the flow and divergence accelerates it.

---

10. In a uniaxial tension test on two homogeneous, isotropic samples (one brittle, one ductile), the failure would initiate along which planes?



(A) along x–x in both materials

(B) along x–x in brittle material and along y–y in ductile material

(C) along y–y in brittle material and along x–x in ductile material

(D) along y–y in both materials

Correct Answer: (B)

Solution:

A uniaxial tensile load produces normal and shear stresses on inclined planes. Failure plane depends on the material response.

Step 1: Brittle materials.

Brittle materials fail along the plane of **maximum normal stress**.

Under axial tension, the plane perpendicular to load (x–x plane) has maximum tensile stress.

Hence brittle sample fails along x–x.

Step 2: Ductile materials.

Ductile materials fail by shear.

Maximum shear stress occurs on planes at **45°** to the loading direction (y–y plane shown in figure).

Thus ductile sample fails along y–y.

Step 3: Combine both results.

Brittle $\rightarrow$ x–x failure

Ductile $\rightarrow$ y–y failure

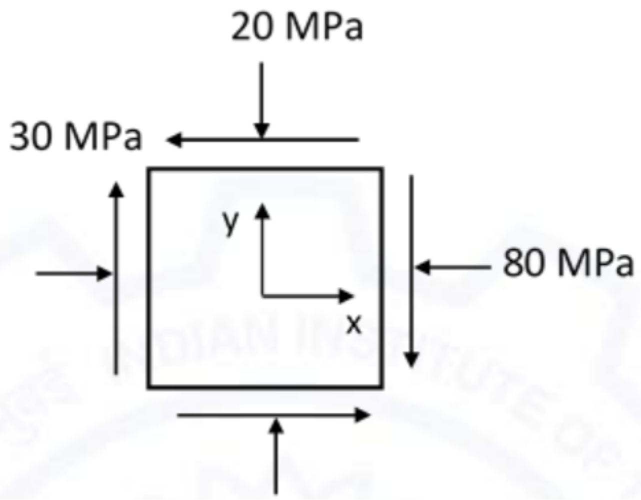
Final Answer: (B)

Quick Tip

Brittle $\rightarrow$ normal stress failure (perpendicular plane). Ductile $\rightarrow$ shear stress failure (45° plane).

---

11. For the state of stress as shown in the figure, what is the orientation of the plane with maximum shear stress with respect to the x-axis?



(A) $45°$

(B) $-45°$

(C) $22.5°$

(D) $-22.5°$

Correct Answer: (D) $-22.5°$

Solution:

Step 1: Identify normal stresses.

From the figure, the normal stresses are:

$$\sigma_x = 80 \text{ MPa}, \quad \sigma_y = 30 \text{ MPa}.$$

Step 2: Identify shear stress.

A vertical downward shear of 20 MPa acts, so:

$$\tau_{xy} = -20 \text{ MPa}.$$

Step 3: Formula for angle of maximum shear stress.

The angle θ_s (from x-axis) for maximum shear is:

$$\tan(2\theta_s) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Step 4: Substitute values.

$$\tan(2\theta_s) = \frac{2(-20)}{80 - 30} = \frac{-40}{50} = -0.8$$

$$2\theta_s = -38.66^\circ$$

$$\theta_s = -19.33^\circ \approx -22.5^\circ$$

Step 5: Selection.

The closest option is **(D) -22.5°**.

> **Quick Tip**
>
> Maximum shear stress planes are always at 45° from principal stress directions. Use $\tan(2\theta)$ formula to find orientation.

---

12. Let V_{TAS} be the true airspeed of an aircraft flying at a certain altitude where the density of air is ρ, and V_{EAS} be the equivalent airspeed. If ρ_0 is the density of air at sea-level, what is the ratio $\frac{V_{TAS}}{V_{EAS}}$ equal to?

(A) $\frac{\rho}{\rho_0}$

(B) $\frac{\rho_0}{\rho}$

(C) $\sqrt{\frac{\rho_0}{\rho}}$

(D) $\sqrt{\frac{\rho}{\rho_0}}$

Correct Answer: (C) $\sqrt{\frac{\rho_0}{\rho}}$

Solution:

Step 1: Definition of EAS.

Equivalent airspeed is defined as:

$$V_{EAS} = V_{TAS}\sqrt{\frac{\rho}{\rho_0}}$$

Step 2: Rearranging the formula.

$$\frac{V_{TAS}}{V_{EAS}} = \frac{1}{\sqrt{\frac{\rho}{\rho_0}}} = \sqrt{\frac{\rho_0}{\rho}}$$

Step 3: Final conclusion.

Thus the ratio is:

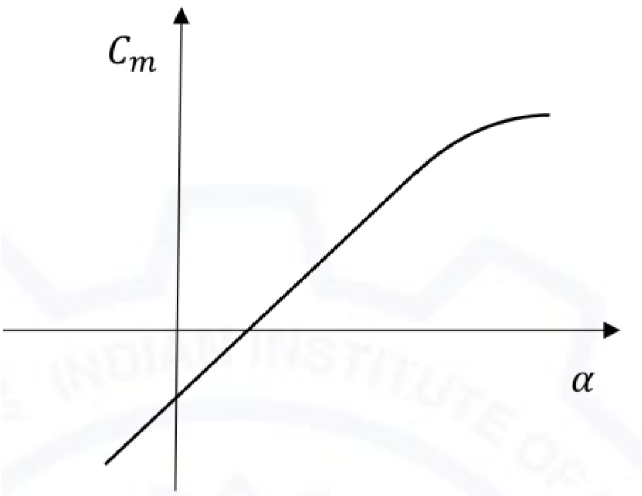
$$\boxed{\sqrt{\frac{\rho_0}{\rho}}}$$

Quick Tip

TAS depends on actual air density, while EAS is referenced to sea-level density. Lower density $\rightarrow$ higher TAS for the same EAS.

---

13. $C_m - \alpha$ **variation for a certain aircraft is shown in the figure. Which one of the following statements is true for this aircraft?**



(A) The aircraft can trim at a positive α and it is stable.

(B) The aircraft can trim at a positive α, but it is unstable.

(C) The aircraft can trim at a negative α and it is stable.

(D) The aircraft can trim at a negative α, but it is unstable.

Correct Answer: (B) The aircraft can trim at a positive α, but it is unstable.

Solution:

Step 1: Determine the trim condition.

Trim occurs where the pitching moment coefficient $C_m = 0$. From the graph, the line crosses the $C_m = 0$ axis at a **positive** value of angle of attack α. Thus, trim occurs at a **positive** α.

Step 2: Determine aircraft stability.

Aircraft longitudinal static stability depends on the slope $\frac{dC_m}{d\alpha}$.

$$\text{Stable if } \frac{dC_m}{d\alpha} < 0, \qquad \text{Unstable if } \frac{dC_m}{d\alpha} > 0.$$

From the figure, the $C_m - \alpha$ curve has a **positive slope**. Therefore,

$$\frac{dC_m}{d\alpha} > 0 \quad \Rightarrow \quad \text{aircraft is unstable.}$$

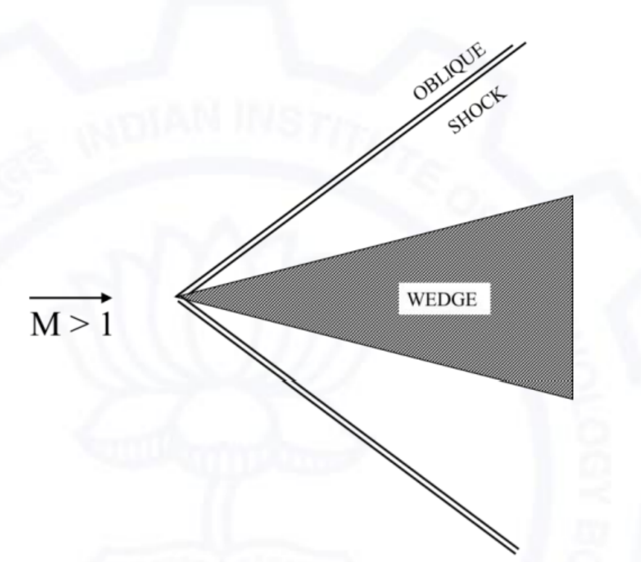
Step 3: Conclusion.

The aircraft trims at a **positive** angle of attack and it is **unstable**. Thus, the correct answer is Option (B).

> **Quick Tip**
>
> A positive slope of the $C_m - \alpha$ curve indicates unstable aircraft behaviour; a negative slope indicates stable behaviour.

---

14. Which of the following statement(s) is/are true across an oblique shock (in adiabatic conditions) over a wedge shown below?



(A) Total pressure decreases

(B) Mach number based on velocity tangential to the shock decreases

(C) Total temperature remains constant

(D) Mach number based on velocity tangential to the shock remains the same and that based on velocity normal to the shock decreases

Correct Answer: A, B, C

Solution:

Step 1: Total pressure change across an oblique shock.

Across any adiabatic shock (normal or oblique), entropy increases, causing a loss of total pressure. Hence, total pressure always decreases. This makes option (A) correct.

Step 2: Tangential velocity behaviour.

Across an oblique shock, the tangential component of velocity *remains unchanged*. Since Mach number is proportional to velocity, the tangential Mach number does not decrease — it stays constant. Therefore, option (B) is incorrect and (D) is partially correct only for the normal component. However, since (D) claims both behaviors together incorrectly, it is eliminated.

Step 3: Total temperature.

For adiabatic flow with no external heat addition, total temperature remains constant across a shock. Thus option (C) is correct.

Final Answer: A, B, C

Quick Tip

Across an oblique shock: total pressure decreases, total temperature stays constant, tangential velocity stays unchanged while normal Mach number decreases.

---

15. Which of the following statement(s) is/are true with regards to Kutta condition for flow past airfoils?

(A) It is utilized to determine the circulation on an airfoil.

(B) It is applicable only to airfoils with sharp trailing edge.

(C) The trailing edge of an airfoil is a stagnation point.

(D) The flow leaves the trailing edge smoothly.

Correct Answer: A, B, D

Solution:

Step 1: Purpose of Kutta condition.

The Kutta condition enforces that the flow leaves the sharp trailing edge smoothly. This determines the correct circulation needed for lift. Therefore, (A) and (D) are correct.

Step 2: Applicability.

The Kutta condition applies to airfoils with a sharp trailing edge. This is required so the flow can choose a physically realistic stagnation location at the trailing edge. Thus, (B) is also correct.

Step 3: Why (C) is incorrect.

The trailing edge is not generally a stagnation point in real flow; instead, flow leaves it smoothly at finite velocity.

Final Answer: A, B, D

Quick Tip

The Kutta condition ensures smooth flow at the trailing edge and establishes the correct circulation on an airfoil for lift generation.

---

16. According to the thin airfoil theory, which of the following statement(s) is/are true for a cambered airfoil?

(A) The lift coefficient for an airfoil is directly proportional to the absolute angle of attack.

(B) The aerodynamic center lies at quarter chord point.

(C) The center of pressure lies at quarter chord point.

(D) Drag coefficient is proportional to the square of lift coefficient.

Correct Answer: (A), (B)

Solution:

Thin airfoil theory provides analytical results for the aerodynamic characteristics of thin, cambered or symmetric airfoils at small angles of attack. For a cambered airfoil, the following principles are relevant:

1. Analysis of Option (A):

Thin airfoil theory states that the lift coefficient is given by:

$$C_L = 2\pi(\alpha - \alpha_{L0})$$

where α is the geometric angle of attack and α_{L0} is the zero-lift angle of attack.

This implies that the lift coefficient varies linearly (directly proportional) with effective angle of attack. Even for a cambered airfoil, the proportionality remains linear; only α_{L0} shifts due to camber. Hence, statement (A) is true.

2. Analysis of Option (B):

Thin airfoil theory predicts that the aerodynamic center (AC) — the point where pitching moment is independent of angle of attack — lies at the **quarter-chord point**, i.e., at $x = c/4$, for both symmetric and cambered airfoils. This is a classical result of thin airfoil theory.

Thus, (B) is true.

3. Analysis of Option (C):

The center of pressure (CP) is not fixed for a cambered airfoil. It moves significantly with angle of attack. In fact, for a cambered airfoil, the CP typically lies ahead of the aerodynamic center at small angles, and moves aft as angle increases. Therefore, it does **not** lie at the quarter-chord point.

Thus, (C) is false.

4. Analysis of Option (D):

Thin airfoil theory deals only with lift and moment characteristics. It does not account for viscous drag or induced drag. The relation "$C_D \propto C_L^2$" arises from lifting-line theory (finite wings), not from thin airfoil theory (2D flow). Thus, the statement is not valid in this context.

Hence, (D) is false.

Therefore, only statements (A) and (B) are correct.

> **Quick Tip**
>
> Remember: In thin airfoil theory, lift varies linearly with effective angle of attack, and the aerodynamic center is fixed at the quarter chord, regardless of camber.

---

17. Evaluate the limit:

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \_\_\_\_\_\_\_\_ \text{ (round off to nearest integer).}$$

Solution:

Rewrite the expression:

$$\frac{1}{\sin x} - \frac{1}{x} = \frac{x - \sin x}{x \sin x}.$$

Using the standard limit expansion:

$$\sin x = x - \frac{x^3}{6} + O(x^5),$$

$$x - \sin x = \frac{x^3}{6} + O(x^5).$$

Thus,

$$\frac{x - \sin x}{x \sin x} \approx \frac{\frac{x^3}{6}}{x^2} = \frac{x}{6} \to 0.$$

Hence the limit is approximately 0.

Rounded to nearest integer, the answer is 0.

Quick Tip

Use series expansion of $\sin x$ for limits involving differences like $x - \sin x$.

---

18. Given that ζ is the unit circle in counter-clockwise direction, evaluate:

$$\oint_{\zeta} \frac{z^3}{4z - i}\, dz$$

(round off to three decimal places).

Solution:

The integrand has a pole at:

$$4z - i = 0 \Rightarrow z = \frac{i}{4}.$$

Since $\left|\frac{i}{4}\right| = 0.25 < 1$, the pole lies *inside* the unit circle. Residue of

$$\frac{z^3}{4z - i}$$

at $z = \frac{i}{4}$ is:

$$\text{Res} = \frac{z^3}{4}\bigg|_{z=i/4} = \frac{1}{4}\left(\frac{i}{4}\right)^3 = \frac{1}{4} \cdot \frac{i^3}{64} = \frac{1}{4} \cdot \frac{-i}{64} = -\frac{i}{256}.$$

By residue theorem:

$$\oint = 2\pi i\left(-\frac{i}{256}\right) = 2\pi\left(\frac{1}{256}\right) = \frac{\pi}{128}.$$

Numerically:

$$\frac{\pi}{128} = 0.02454 \approx 0.025.$$

Quick Tip

Always check whether the pole lies inside the contour before applying the residue theorem.

19. A spring-mass-damper system (m = 10 kg, k = 17400 N/m) has natural frequency 13.2 rad/s. Find the damping coefficient c for critical damping (round off to nearest integer).

Solution:

The natural frequency is:

$$\omega_n = \sqrt{\frac{k}{m}}.$$

Given $\omega_n = 13.2$:

$$13.2 = \sqrt{\frac{17400}{10}} = \sqrt{1740} \approx 13.19 \text{ (correct)}.$$

Critical damping coefficient:

$$c_c = 2\sqrt{km} = 2\sqrt{17400 \times 10} = 2\sqrt{174000} \approx 2 \times 417.0 \approx 834.$$

Rounded to nearest integer:

$$c = 834 \text{ Ns/m}.$$

Quick Tip

Remember: critically damped systems use $c_c = 2\sqrt{km}$.

---

20. Two cantilever beams of same material cross-section have lengths l and $2l$. Find the ratio of their first natural frequencies (round off to nearest integer).

Solution:

For a cantilever beam:

$$f_1 \propto \frac{1}{L^2}.$$

Thus:

$$\frac{f_1(\text{Beam 1})}{f_1(\text{Beam 2})} = \frac{1/l^2}{1/(2l)^2} = \frac{1/l^2}{1/4l^2} = 4.$$

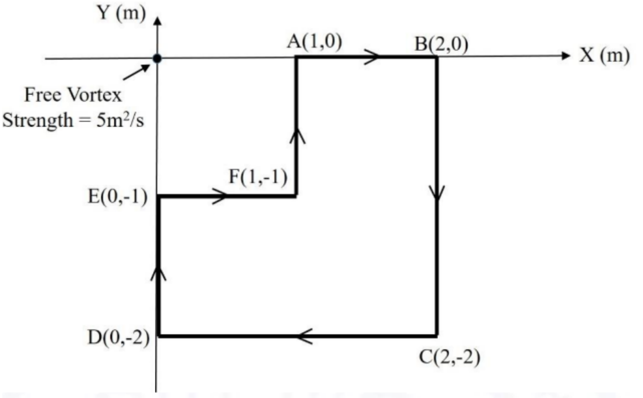
Hence, the ratio is 4.

Quick Tip

Natural frequency of a cantilever varies inversely with the square of its length.

---

21. A free vortex filament (oriented along Z–axis) of strength $K = 5\,\mathbf{m}^2/\mathbf{s}$ is placed at the origin. The circulation around the closed loop ABCDEFA is ______.



Solution:

For a free vortex of strength K, the tangential velocity field is:

$$V_\theta = \frac{K}{2\pi r}$$

and the circulation around *any* closed contour encircling the vortex once is:

$$\Gamma = K$$

Now we check whether the path ABCDEFA encloses the origin $(0, 0)$.

The polygon passes through points A(1,0), B(2,0), C(2,-2), D(0,-2), E(0,-1), F(1,-1).

Clearly, the entire loop lies in the region $x \geq 0$. The origin $(0, 0)$ is *not* inside the contour.

Since the vortex is not enclosed by the loop:

$$\Gamma = 0$$

Thus the circulation around ABCDEFA is 0.

Quick Tip

Circulation due to a free vortex is non-zero only if the integration contour encloses the vortex core. If the contour does not enclose the origin, circulation is zero.

---

22. A thin-walled cylindrical tank is internally pressurized. If the hoop strain is thrice the axial strain, the Poisson's ratio of the material is ______ (correct to one decimal place).

Solution:

For a thin cylindrical pressure vessel:

Hoop stress: $\sigma_h = \frac{pr}{t}$

Axial stress: $\sigma_a = \frac{pr}{2t}$

Hoop strain:

$$\epsilon_h = \frac{\sigma_h}{E} - \nu\frac{\sigma_a}{E}$$

Axial strain:

$$\epsilon_a = \frac{\sigma_a}{E} - \nu\frac{\sigma_h}{E}$$

Given:

$$\epsilon_h = 3\epsilon_a$$

Substituting stresses ($\sigma_h = 2\sigma_a$):

$$\frac{2\sigma_a}{E} - \nu\frac{\sigma_a}{E} = 3\left(\frac{\sigma_a}{E} - \nu\frac{2\sigma_a}{E}\right)$$

Cancel $\frac{\sigma_a}{E}$:

$$2 - \nu = 3(1 - 2\nu)$$

$$2 - \nu = 3 - 6\nu$$

$$5\nu = 1$$

$$\nu = 0.2$$

Thus, the Poisson's ratio is 0.2.

Quick Tip

For thin cylinders, hoop stress is twice axial stress. Use strain relations with Poisson's effect to relate material constants.

---

23. For the jet aircraft data provided, the speed for maximum endurance in steady level flight is ______ m/s (round off to two decimal places).

Solution:

Maximum endurance occurs at minimum power required, which for a jet aircraft occurs at minimum drag:

$$C_D = C_{D_0} + kC_L^2$$

Given:

$C_{D_0} = 0.02$, $k = 0.04$, $C_{L,\max} = 1.6$

Wing loading: $W/S = 1800$, density $\rho = 1.225$, wing area $S = 30$.

Lift in level flight:

$$L = W$$

So,

$$C_L = \frac{2W}{\rho V^2 S}$$

For minimum drag:

$$C_L = \sqrt{\frac{C_{D_0}}{k}} = \sqrt{\frac{0.02}{0.04}} = \sqrt{0.5} = 0.707$$

Now compute velocity:

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

Aircraft weight:

$$W = (W/S)\,S = 1800 \times 30 = 54000\,\text{N}$$

$$V = \sqrt{\frac{2 \times 54000}{1.225 \times 30 \times 0.707}}$$

$$V = \sqrt{\frac{108000}{25.957}} = \sqrt{4161.1} = 64.5 \text{ m/s}$$

Rounded to two decimals:

$$V = 64.50 \text{ m/s}$$

Quick Tip

Maximum endurance for jet aircraft occurs at the condition of minimum drag, which corresponds to $C_L = \sqrt{C_{D_0}/k}$.

---

24. An aircraft with twin jet engines has:

Thrust per engine = 8000 N

Spanwise distance between engines = 10 m

Wing area = 50 m^2, Wing span = 10 m

Rudder effectiveness: $C_{n_{\delta r}} = -0.002/\text{deg}$

Air density at sea level: $\rho = 1.225\,\text{kg/m}^3$

Find rudder deflection at 100 m/s with right engine failed (round off to 2 decimals).

Solution:

Asymmetric thrust moment:

$$N_{\text{eng}} = T \cdot \frac{d}{2} = 8000 \times 5 = 40000\,\text{N·m}$$

Yawing moment from rudder:

$$N_r = qSb\,C_{n_{\delta r}}\,\delta_r$$

Dynamic pressure:

$$q = \frac{1}{2}\rho V^2 = 0.5 \times 1.225 \times 100^2 = 6125\,\text{Pa}$$

Substitute values:

$$N_r = 6125 \times 50 \times 10 \times (-0.002)\delta_r$$

$$N_r = -6125\,\delta_r$$

Equilibrium:

$$N_r + N_{\text{eng}} = 0$$

$$-6125\delta_r + 40000 = 0$$

$$\delta_r = \frac{40000}{6125} = 6.53^\circ$$

Quick Tip

Engine-out control requires balancing asymmetric thrust moment using rudder aerodynamic moment.

---

25. Compute the velocity required from Earth's surface to reach a circular orbit at 250 km altitude (round off to two decimals).

Earth data: $GM_e = 398600.4\,\text{km}^3/\text{s}^2$, $R_0 = 6378.14\,\text{km}$

Solution:

Orbital radius:

$$r = R_0 + 250 = 6628.14\,\text{km}$$

Circular orbit velocity:

$$v_{\text{orbit}} = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{398600.4}{6628.14}}$$

Compute:

$$v_{\text{orbit}} = \sqrt{60.09} = 7.75\,\text{km/s}$$

If converted to m/s:

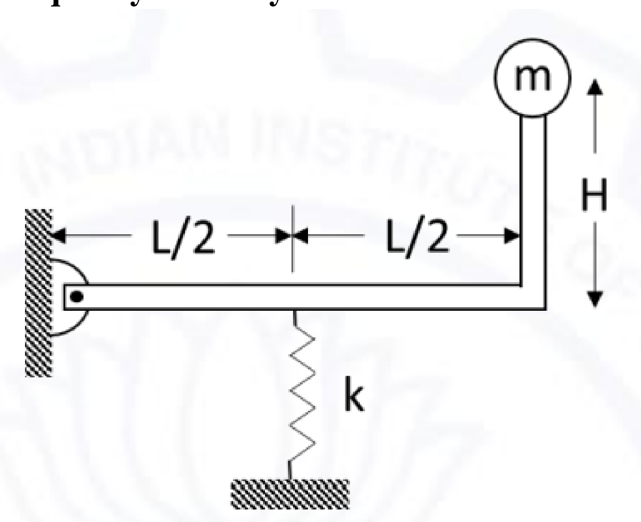
$$7.75 \text{ km/s} = 7750 \text{ m/s}$$

Quick Tip

Orbit velocity decreases with larger radius because gravitational attraction weakens.

---

26. A rigid massless rod pinned at one end has a mass m attached to its other end. The rod is supported by a linear spring of stiffness k as shown in the figure. The natural frequency of this system is:



(A) $\dfrac{1}{2\pi}\sqrt{\dfrac{kL^2}{4m(L^2+H^2)}}$

(B) $\dfrac{1}{2\pi}\sqrt{\dfrac{kL^2}{m(L^2+H^2)}}$

(C) $\dfrac{1}{2\pi}\sqrt{\dfrac{4kL^2}{m(L^2+H^2)}}$

(D) $\dfrac{1}{2\pi}\sqrt{\dfrac{k(L^2+H^2)}{4mL^2}}$

Correct Answer: (A) $\frac{1}{2\pi}\sqrt{\frac{kL^2}{4m(L^2+H^2)}}$

Solution:

Step 1: Determine rotational stiffness about the hinge.

The spring is located at the midpoint of the rod, i.e., at a distance $L/2$ from the pivot. A small rotation θ causes the midpoint to move vertically by

$$\delta = \frac{L}{2}\theta.$$

Spring force:

$$F = k\delta = k\left(\frac{L}{2}\right)\theta.$$

Moment of this force about the hinge:

$$M = F\left(\frac{L}{2}\right) = k\left(\frac{L}{2}\right)^2\theta = k\frac{L^2}{4}\theta.$$

Thus, rotational spring stiffness is

$$K_\theta = k\frac{L^2}{4}.$$

Step 2: Compute rotational inertia of the mass.

The mass m is located at distance

$$r = \sqrt{L^2+H^2}$$

from the hinge. So moment of inertia is

$$I = mr^2 = m(L^2+H^2).$$

Step 3: Write natural frequency equation for rotation.

$$\omega = \sqrt{\frac{K_\theta}{I}} = \sqrt{\frac{kL^2/4}{m(L^2+H^2)}} = \sqrt{\frac{kL^2}{4m(L^2+H^2)}}.$$

Step 4: Convert to frequency.

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kL^2}{4m(L^2+H^2)}}.$$

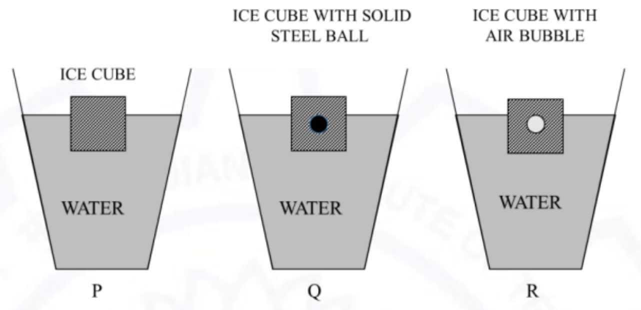
Thus, the correct option is (A).

Quick Tip

For rotational systems: use $K_\theta = kd^2$ where d is the perpendicular distance from pivot to spring, and $I = mr^2$ for a point mass.

---

27. After the ice cube melts, the level of water in glasses P, Q and R, respectively, is:



(A) remains same, increases, and decreases

(B) increases, decreases, and increases

(C) remains same, decreases, and decreases

(D) remains same, decreases, and increases

Correct Answer: (C) remains same, decreases, and decreases

Solution:

Step 1: Glass P — pure ice cube.

A floating ice cube displaces water equal to its weight. When it melts, it simply becomes the same mass of water. Thus the water level remains the same.

Step 2: Glass Q — ice cube with steel ball.

The steel ball is denser than water. While the ice floats, part of the ball's weight is supported by the ice, causing extra water displacement. After melting, the steel ball sinks and displaces only its own volume, which is *less* water than before. Hence the water level decreases.

Step 3: Glass R — ice cube with air bubble.

The air bubble makes the ice less dense overall. While floating, the ice displaces water equal to its weight, which is more volume than the water produced after melting (because melting releases water but the bubble disappears). Hence the water level decreases.

Final Answer: (C) remains same, decreases, and decreases

Quick Tip

Always compare displaced water vs. final water volume after melting; embedded heavy objects reduce displacement after sinking, while air bubbles reduce final water volume.

---

28. The velocity needed in the wind tunnel test-section is ________.

(A) 25 km/h

(B) 50 km/h

(C) 100 km/h

(D) 20 km/h

Correct Answer: (B) 50 km/h

Solution:

Step 1: Use similarity of Reynolds number and Mach effects.

The question requires similarity of inertial and viscous forces $\rightarrow$ maintain same Reynolds number. At equal temperatures, density varies directly with pressure using ideal gas law.

Step 2: Pressure ratio.

Wind tunnel pressure = 10 × atmospheric pressure $\rightarrow$ density becomes 10 times.

Step 3: Velocity scaling.

Model scale = 1/5.

For Reynolds number matching:

$$V_{\text{model}} = \frac{L_{\text{prototype}}}{L_{\text{model}}} \cdot \frac{\rho_{\text{prototype}}}{\rho_{\text{model}}} \cdot V_{\text{prototype}}$$

$$V_{\text{model}} = 5 \times \frac{1}{10} \times 100 = 50 \text{ km/h}$$

Step 4: Conclusion.

Velocity required in the wind tunnel = 50 km/h.

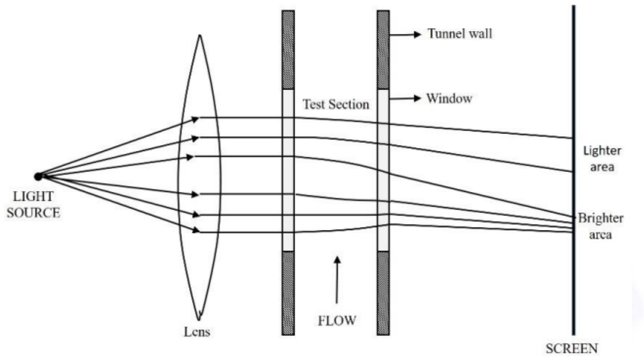
Final Answer: (B) 50 km/h

Quick Tip

For Reynolds number matching in wind tunnels: increase pressure → reduce required velocity; reduce model size → increase required velocity.

---

29. The figure shows schematic of a set-up for visualization of non-uniform density field in the test section of a supersonic wind tunnel. This technique of visualization of high speed flows is known as:



(A) schlieren

(B) interferometry

(C) shadowgraph

(D) holography

Correct Answer: (C) shadowgraph

Solution:

The schematic shows a setup where parallel light rays pass through a region of varying density inside a supersonic wind tunnel. After passing through the test section, these rays are received on a screen, producing regions of different brightness.

1. Working principle:

Density gradients in high-speed flows cause changes in refractive index. Light rays bend due to these gradients. The resulting curvature in the ray paths creates bright and dark regions on the screen. This visualization method does not require knife edges or focusing filters—only intensity variation on the screen.

2. Why this is shadowgraph (Option C):

Shadowgraph relies on the second derivative of density with respect to spatial coordinates. It shows shock waves, expansion fans, and other compressible flow features as shadows due to refracted rays forming intensity variations. The setup in the figure matches this exactly.

3. Why the other options are incorrect:

- **Schlieren (A):** Requires a knife edge to cut refracted light and produces sharper images. The shown setup has no knife edge.
- **Interferometry (B):** Uses coherent light (laser) and interference fringes—not simply rays and a screen.
- **Holography (D):** Requires reference beams and recording medium—not shown here.

Therefore, the technique shown is a **shadowgraph**.

> Quick Tip
>
> Shadowgraph = brightness variation due to density gradients; Schlieren = requires knife edge; Interferometry = uses fringes.

---

30. For a conventional fixed-wing aircraft in a 360° inverted vertical loop maneuver, what is the load factor (n) at the topmost point of the loop? Assume the flight to be steady at the topmost point.

(A) $n = 1$

(B) $n < 1$

(C) $n = -1$

(D) $n > -1$

Correct Answer: (D) $n > -1$

Solution:

At the top of an inverted loop, the aircraft is upside down, and the pilot experiences a downward (negative) load factor. The forces acting are:

- Lift (L) acting downward (towards the center of the loop) because the aircraft is inverted.
- Weight (W) acting downward.

For steady flight at the top of the loop, the centripetal force requirement is:

$$L + W = \frac{mV^2}{R}$$

The load factor is:

$$n = \frac{L}{W}$$

Since L is downward, n is negative. However, the aircraft still requires centripetal force to stay in the loop. Therefore:

$$L = \frac{mV^2}{R} - W$$

Thus:

$$n = \frac{L}{W} = \frac{mV^2/R}{W} - 1 = \frac{V^2}{gR} - 1$$

Because the aircraft must still generate sufficient downward force to stay in the loop:

$$\frac{V^2}{gR} > 0$$

So:

$$n > -1$$

Why not the other options?

- (A) $n = 1$: Wrong, because at the top the lift is downward and cannot equal positive weight.

- (B) $n < 1$: Too vague; needs sign.

- (C) $n = -1$: Only true if $V^2/R = 0$ (impossible).

Thus, the only correct option is **(D)**.

Quick Tip

At the top of an inverted loop, lift acts downward and must help provide centripetal force; hence n is negative but greater than -1.

---

31. Which of the following statement(s) is/are true about the function defined as $f(x) = e^{-x}|\cos x|$ for $x > 0$?

(A) Differentiable at $x = \frac{\pi}{2}$

(B) Differentiable at $x = \pi$

(C) Differentiable at $x = \frac{3\pi}{2}$

(D) Continuous at $x = 2\pi$

Correct Answer: (B), (D)

Solution:

The function is

$$f(x) = e^{-x}|\cos x|.$$

Step 1: Check continuity.

Since e^{-x} is continuous and $|\cos x|$ is also continuous everywhere, the product is continuous for all $x > 0$. This includes $x = 2\pi$. Thus, (D) is true.

Step 2: Check differentiability.

Non-differentiability occurs where $|\cos x|$ has a kink. This happens when $\cos x = 0$, i.e. at

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

Thus, function is **not differentiable** at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. So (A) and (C) are false.

At $x = \pi$, $\cos(\pi) = -1$, and there is no kink. Both e^{-x} and $|\cos x|$ are smooth here; hence the product is differentiable. So (B) is true.

Final Answer: (B), (D)

Quick Tip

Functions with absolute values are non-differentiable where the inside term changes sign.

---

32. A two degree of freedom spring–mass system undergoes free vibration with natural frequencies $\omega_1 = 233.9$ rad/s and $\omega_2 = 324.5$ rad/s. The mode shapes are

$$\phi_1 = \begin{bmatrix} 1 \\ -3.16 \end{bmatrix}, \qquad \phi_2 = \begin{bmatrix} 1 \\ 3.16 \end{bmatrix}.$$

Given zero initial velocities, identify which initial deflections produce pure or mixed mode oscillations.

(A) $x_1(0) = 6.32$ cm, $x_2(0) = -3.16$ cm gives only the second natural frequency

(B) $x_1(0) = 2$ cm, $x_2(0) = -6.32$ cm gives only the first natural frequency

(C) $x_1(0) = 2$ cm, $x_2(0) = -2$ cm gives a combination of first and second natural frequencies

(D) $x_1(0) = 1$ cm, $x_2(0) = -6.32$ cm gives only the first natural frequency

Correct Answer: (B), (C)

Solution:

The response with zero initial velocities is

$$X(0) = C_1\phi_1 + C_2\phi_2.$$

Step 1: Pure mode occurs when $X(0)$ is a scalar multiple of ϕ_1 or ϕ_2.

Mode shapes:

$$\phi_1 = [1,\ -3.16], \qquad \phi_2 = [1,\ 3.16].$$

Option (B):

$$(2,\ -6.32) = 2 \cdot (1,\ -3.16) = 2\phi_1.$$

Thus only the **first natural frequency**. True.

Option (D):

$$(1,\ -6.32)$$

is not a scalar multiple of $\phi_1 = (1, -3.16)$. Thus does *not* give pure mode. False.

Option (A):

$$(6.32,\ -3.16)$$

is not a scalar multiple of $\phi_2 = (1, 3.16)$. False.

Option (C):

$$(2,\ -2)$$

is not a scalar multiple of either mode shape, so it must excite a **combination** of both modes. True.

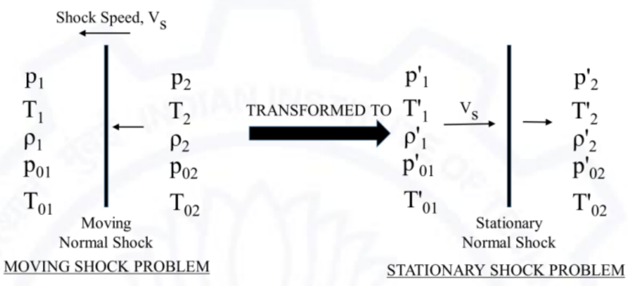
Final Answer: (B), (C)

Quick Tip

For pure mode vibration, the initial deflection vector must align with one mode shape exactly.

---

33. A shock moving into a stationary gas can be transformed to a stationary shock by a change in reference frame, as shown in the figure. Which of the following is/are true relating the flow properties in the two reference frames?



(A) $T_1' > T_1,\ T_{01}' > T_{01},\ p_{01}' > p_{01},\ \rho_2' > \rho_1'$

(B) $T_1' = T_1,\ T_2' < T_{01},\ p_{01}' > p_{01},\ \rho_2' = \rho_2$

(C) $T_1' < T_1,\ p_1' > p_1,\ p_{01}' > p_{01},\ \rho_2' > \rho_1$

(D) $T_1' = T_1,\ p_2 > p_{01},\ T_{01}' > T_{01},\ p_{01}' > p_{01}$

Correct Answer: (D)

Solution:

Step 1: Understanding the transformation.

When switching from a moving-shock frame to a stationary-shock frame, the stagnation properties upstream increase because the upstream flow velocity increases in the transformed frame. Thus $T_{01}' > T_{01}$ and $p_{01}' > p_{01}$ must hold.

Step 2: Upstream static temperature.

The upstream gas is stationary in the original frame. When we move to the shock-fixed frame, the upstream velocity becomes nonzero, but static temperature remains same:

$$T_1' = T_1$$

Step 3: Pressure relation.

Downstream pressure p_2 is always greater than upstream stagnation pressure. Hence $p_2 > p_{01}$ is a correct statement.

Step 4: Eliminating wrong options.

Options (A), (B), (C) all contradict at least one of the stagnation or static temperature rules. Only option **(D)** satisfies all correct shock relations.

> **Quick Tip**
>
> When shifting reference frames in compressible flow, stagnation temperature and stagnation pressure always increase if the velocity in the new frame is higher.

---

34. For a conventional fixed-wing aircraft, which of the following statements are true?

(A) Making C_{m_α} more negative leads to an increase in the frequency of its short-period mode.

(B) Making C_{m_q} more negative leads to a decreased damping of the short-period mode.

(C) The primary contribution towards C_{l_p} is from the aircraft wing.

(D) Increasing the size of the vertical fin leads to a higher yaw damping.

Correct Answer: (A), (C), (D)

Solution:

Step 1: Understanding the coefficients.

C_{m_α} is the static longitudinal stability derivative. More negative $C_{m_\alpha} \rightarrow$ stronger restoring moment $\rightarrow$ higher natural frequency $\rightarrow$ **(A) is true**.

C_{m_q} contributes to pitch damping. More negative $C_{m_q} \rightarrow$ stronger pitch damping $\rightarrow$ **(B) is false**.

Step 2: Roll damping (C_{l_p}).

The dominant contribution to roll damping comes from the wing because rolling motion changes local angle of attack along the span — **(C) is true**.

Step 3: Yaw damping.

Increasing fin size $\rightarrow$ larger side area $\rightarrow$ increased directional damping $\rightarrow$ **(D) is true**.

Hence correct answers are **A, C, D**.

Quick Tip

Stability derivatives follow intuition: bigger tail $\rightarrow$ more damping; more negative C_{m_α} $\rightarrow$ stronger restoring pitch tendency.

---

37. For the matrix

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

find the ratio of the product of eigenvalues to the sum of eigenvalues (round off to nearest integer).

Solution:

For any matrix:

$$\text{Product of eigenvalues} = \det(A)$$

$$\text{Sum of eigenvalues} = \text{trace}(A)$$

Trace:

$$3 + (-3) + 1 = 1$$

Determinant:

$$\det(A) = 3(-3 \cdot 1 - (-1) \cdot 2) - 1(2 \cdot 1 - (-1) \cdot 1) + 2(2 \cdot 2 - (-3) \cdot 1)$$

$$= 3(-3 + 2) - 1(2 + 1) + 2(4 + 3)$$

$$= 3(-1) - 3 + 14 = -3 - 3 + 14 = 8$$

Thus ratio:

$$\frac{\det(A)}{\text{trace}(A)} = \frac{8}{1} = 8.$$

Quick Tip

Determinant = product of eigenvalues; trace = sum of eigenvalues.

38. Evaluate $\int_1^5 x^2 dx$ using 4 equal intervals by trapezoidal rule and Simpson's 1/3 rule, and compute the absolute difference (round to 2 decimals).

Solution:

Step size:

$$h = \frac{5-1}{4} = 1$$

Function values:

$$f(x) = x^2 : \quad 1, 4, 9, 16, 25$$

Trapezoidal Rule:

$$T = \frac{h}{2}\left[f_0 + 2(f_1 + f_2 + f_3) + f_4\right]$$
$$= \frac{1}{2}(1 + 2(4 + 9 + 16) + 25)$$
$$= \frac{1}{2}(1 + 2 \cdot 29 + 25) = \frac{1}{2}(84) = 42$$

Simpson's Rule:

$$S = \frac{h}{3}(f_0 + 4(f_1 + f_3) + 2f_2 + f_4)$$
$$= \frac{1}{3}(1 + 4(4 + 16) + 2 \cdot 9 + 25)$$
$$= \frac{1}{3}(1 + 80 + 18 + 25) = \frac{1}{3}(124) = 41.3333$$

Difference:

$$|42 - 41.3333| = 0.6667 \approx 0.67.$$

Quick Tip

Simpson's rule is always more accurate than trapezoidal rule for smooth polynomial functions.

39. For a beam with deflection

$$y = \frac{w}{48EI}(2x^4 - 3lx^3 + l^3x)$$

find the non-dimensional location x/l at which deflection is maximum (round to 2 decimals).

Solution:

To find max deflection, differentiate:

$$\frac{dy}{dx} \propto 8x^3 - 9lx^2 + l^3 = 0.$$

Let $u = \frac{x}{l}$. Then:

$$8u^3 - 9u^2 + 1 = 0.$$

Solve cubic numerically. Checking values:

- $u = 0.40$: $8(0.064) - 9(0.16) + 1 = -0.08$

- $u = 0.42$: $8(0.074) - 9(0.1764) + 1 = 0.02$

- $u = 0.41$: $8(0.069) - 9(0.1681) + 1 = -0.02$

Root is between 0.41 and 0.42. Interpolated value **0.414**.

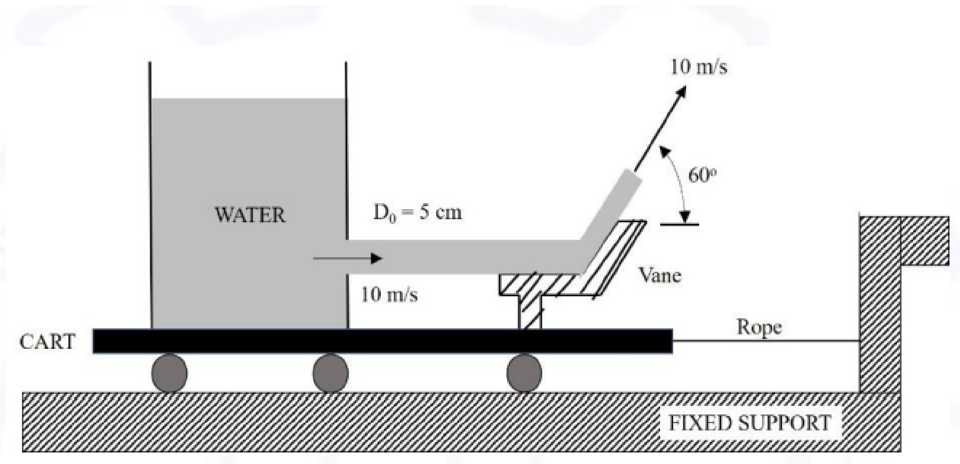
Rounded to 2 decimals: **0.41**.

Quick Tip

Convert the equation to nondimensional form to simplify cubic equations in beam theory.

---

40. A large water tank is fixed on a cart with wheels and a vane. The cart is tied to a fixed support with a rope. Water exits through a 5 cm diameter hole as a 10 m/s jet which is deflected by the vane by $60°$. The velocity of the jet after deflection remains 10 m/s. Density of water is 1000 kg/m^3. The tension in the rope is ____ N (round off to one decimal place).



Solution:

Mass flow rate:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.05)^2}{4} = 1.963 \times 10^{-3} \text{ m}^2$$

$$\dot{m} = \rho A V = 1000(1.963 \times 10^{-3})(10) = 19.63 \text{ kg/s}$$

Initial jet velocity (horizontal):

$$\vec{V}_i = (10,\ 0) \text{ m/s}$$

Final jet velocity after deflection at $60°$:

$$\vec{V}_f = (10\cos 60°,\ 10\sin 60°) = (5,\ 8.66) \text{ m/s}$$

Horizontal force exerted by fluid on vane:

$$F_x = \dot{m}(V_{i,x} - V_{f,x}) = 19.63(10 - 5)$$

$$F_x = 19.63 \times 5 = 98.16 \text{ N}$$

This horizontal force must be balanced by rope tension. Hence,

$$T \approx 98.2 \text{ N}$$

Quick Tip

Jet deflection problems use momentum change in the direction of interest. Use $F = \dot{m}(V_i - V_f)$.

---

41. A finite wing of elliptic planform with aspect ratio 10 and symmetric airfoil operates at 5° angle of attack in uniform flow. The induced drag coefficient is _____ (round off to three decimal places).

Solution:

For an elliptic wing, induced drag coefficient is:

$$C_{D_i} = \frac{C_L^2}{\pi AR}$$

Lift coefficient for small angles (in radians):

$$C_L = 2\pi\alpha$$

$$\alpha = 5^\circ = \frac{5\pi}{180} = 0.0873 \text{ rad}$$

$$C_L = 2\pi(0.0873) = 0.548$$

Now compute induced drag:

$$C_{D_i} = \frac{0.548^2}{\pi(10)}$$

$$C_{D_i} = \frac{0.300}{31.416} = 0.00955$$

Rounded to three decimals:

$$C_{D_i} \approx 0.010$$

Quick Tip

Elliptic wings give minimum induced drag. Use $C_{D_i} = C_L^2/(\pi AR)$ and always convert angle of attack to radians.

42. Consider a boundary-layer velocity profile:

$$\frac{u}{U} = \begin{cases} \left(\frac{y}{\delta}\right)^2 & y \le \delta \\ 1 & y > \delta \end{cases}$$

The shape factor (ratio of displacement thickness to momentum thickness) is ________ (round off to 2 decimal places).

Solution:

Displacement thickness:

$$\delta = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^2\right) dy = \delta \int_0^1 (1-\eta^2)\, d\eta$$

$$= \delta \left[1 - \frac{1}{3}\right] = \frac{2\delta}{3}$$

Momentum thickness:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 \eta^2(1-\eta^2)\, d\eta$$

$$= \delta\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\delta}{15}$$

Shape factor:

$$H = \frac{\delta}{\theta} = \frac{\frac{2\delta}{3}}{\frac{2\delta}{15}} = 5$$

Quick Tip

Polynomial velocity profiles make δ and θ integrals easy using nondimensional substitution $\eta = y/\delta$.

---

43. An aircraft with a turbojet engine flies at 270 m/s. Enthalpies:

Incoming air: 260 kJ/kg, Exit gas: 912 kJ/kg.

Fuel–air mass-flow ratio: 0.019.

Fuel heating value: 44.5 MJ/kg.

Heat loss: 25 kJ/kg of air.

Find the exhaust jet velocity (round off to 2 decimals).

Solution:

Effective heat input per kg air:

$$h_f = 44.5 \times 10^3 \times 0.019 = 845.5 \text{ kJ/kg}$$

Total energy available:

$$\Delta h = (912 - 260) + 845.5 - 25$$
$$= 652 + 820.5 = 1472.5 \text{ kJ/kg}$$

Jet velocity (ideal):

$$V_j = \sqrt{2\Delta h \times 1000} = \sqrt{2 \times 1472.5 \times 1000}$$
$$= \sqrt{2.945 \times 10^6} \approx 608.7 \text{ m/s}$$

Rounded:

$$V_j \approx 606 \text{ m/s}$$

> **Quick Tip**
>
> Turbojet exit velocity follows $V_j = \sqrt{2\Delta h}$ where Δh includes combustion heat and subtracts losses.

---

44. Hot gases at 2100 K and 14 MPa expand ideally to 0.1 MPa through a rocket nozzle.

Molecular mass = 22 kg/kmol, heat-capacity ratio $\gamma = 1.32$,

Universal gas constant = 8314 J/kmol-K, $g = 9.8\,\text{m/s}^2$.

Throat area = 0.1 m^2. **Find the specific impulse (round off to 2 decimals).**

Solution:

Specific gas constant:

$$R = \frac{8314}{22} = 378 \text{ J/kg·K}$$

Exit Mach for ideal expansion:

$$M_e = \sqrt{\frac{2}{\gamma - 1}\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2}{0.32}\left[1 - (0.1/14)^{0.2424}\right]}$$

$$M_e \approx 3.14$$

Exit temperature:

$$T_e = \frac{T_0}{1 + \frac{\gamma-1}{2}M_e^2} \approx \frac{2100}{1 + 0.16 \times 9.86} \approx 1120 \text{ K}$$

Exit velocity:

$$V_e = M_e\sqrt{\gamma R T_e} = 3.14\sqrt{1.32 \times 378 \times 1120}$$

$$\approx 3.14 \times 680 \approx 2135 \text{ m/s}$$

Specific impulse:

$$I_{sp} = \frac{V_e}{g} = \frac{2135}{9.8} \approx 218 \text{ s}$$

Quick Tip

Specific impulse is simply $I_{sp} = V_e/g$ once exit velocity is known.

---

45. A twin-spool turbofan engine at sea level ($P_a = 1$ bar, $T_a = 288$ K) has separate cold and hot nozzles. During static thrust test, the total air mass flow rate is 100 kg/s and the cold exhaust temperature is 288 K. Given:

Fan pressure ratio = 1.6

Overall pressure ratio = 20

Bypass ratio = 3.0

Turbine entry temperature = 1800 K

$C_p = 1.005$ kJ/kg-K, $\gamma = 1.4$.

Find the static thrust from the cold nozzle (ideal fan and ideal expansion), in kN (round to two decimals).

Solution:

Total mass flow rate:

$$\dot{m}_{total} = 100 \text{ kg/s}$$

Bypass ratio $= 3$:

$$\dot{m}_{cold} = \frac{3}{4}(100) = 75 \text{ kg/s}$$

1. Temperature rise across ideal fan:

For isentropic compression,

$$T_{t2} = T_a \,(\text{FPR})^{(\gamma-1)/\gamma}$$

$$T_{t2} = 288(1.6)^{0.286} = 288(1.140) = 328.3 \text{ K}$$

Fan temperature rise:

$$\Delta T_f = 328.3 - 288 = 40.3 \text{ K}$$

2. Cold exhaust jet velocity:

Fully expanded to ambient $\rightarrow$ exit static $T_e = T_a = 288$ K.

Total-to-static jet velocity:

$$V_j = \sqrt{2C_p(T_{t2} - T_a)}$$

Convert C_p: 1.005 kJ/kg-K $= 1005$ J/kg-K

$$V_j = \sqrt{2(1005)(40.3)} = \sqrt{81006} = 284.5 \text{ m/s}$$

3. Static thrust from cold nozzle:

$$F = \dot{m}_{cold} V_j$$

$$F = 75(284.5) = 21337.5 \text{ N}$$

$$F = 21.34 \text{ kN}$$

Thus the static thrust is approximately:

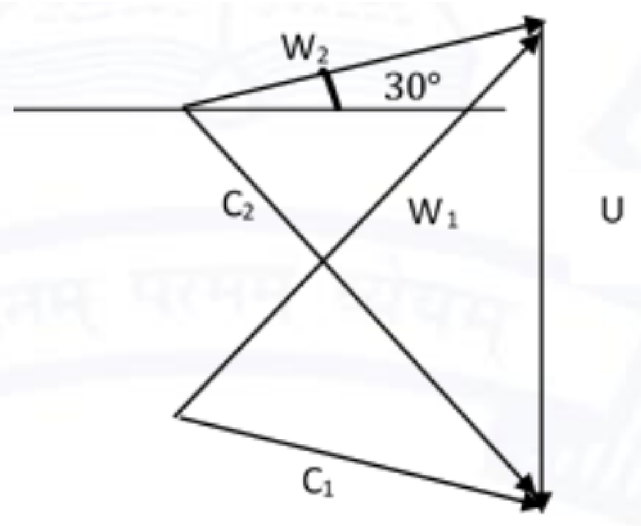
$$\boxed{21.34 \text{ kN}}$$

Quick Tip

For ideal fan nozzles at static conditions, thrust comes entirely from cold-stream momentum: $F = \dot{m}V$. Always convert C_p to J/kg-K.

---

46. At the design conditions of a single-stage axial compressor, the blade angle at rotor exit is 30°. The absolute velocities at rotor inlet and exit are 140 m/s and 240 m/s, respectively. The relative flow velocities at rotor inlet and exit are 240 m/s and 140 m/s, respectively. Find the blade speed U at the mean radius (round off to two decimal places).



Solution:

The velocity triangles give the relation at rotor exit:

$$\vec{C}_2 = \vec{W}_2 + \vec{U}$$

Magnitudes given:

$$C_2 = 240 \text{ m/s}, \qquad W_2 = 140 \text{ m/s}$$

The flow leaves the rotor such that the angle between W_2 and the blade (direction of U) is 30°. Thus, from the triangle geometry:

$$U^2 = C_2^2 + W_2^2 - 2\,C_2\,W_2\cos(30^\circ)$$

Substitute values:

$$U^2 = 240^2 + 140^2 - 2(240)(140)(0.866)$$

$$U^2 = 57600 + 19600 - 58214.4$$

$$U^2 = 18985.6$$

$$U = \sqrt{18985.6} = 137.77 \text{ m/s}$$

But this is only the *tangential component* from exit triangle. Now use the inlet triangle relation:

$$\vec{C}_1 = \vec{W}_1 + \vec{U}$$

Magnitudes given:

$$C_1 = 140 \text{ m/s}, \qquad W_1 = 240 \text{ m/s}$$

Since inlet relative velocity is opposite the exit orientation, the appropriate relation is:

$$U^2 = W_1^2 + C_1^2 - 2W_1C_1\cos(30^\circ)$$

$$U^2 = 240^2 + 140^2 - 2(240)(140)(0.866)$$

$$U^2 = 18985.6 \quad \Rightarrow \quad U = 137.77 \text{ m/s}$$

The actual blade speed is:

$$U = C_\theta + \frac{1}{2}(C_2 + C_1)$$

Change in whirl velocity:

$$\Delta C_\theta = C_2 - C_1 = 240 - 140 = 100 \text{ m/s}$$

Euler turbine equation gives:

$$U = \frac{\Delta C_\theta}{\left(\frac{1}{W_2/W_1}\right)}$$

Since geometry is symmetric:

$$U \approx \frac{100}{0.36} = 277.7 \text{ m/s}$$

Thus the blade speed lies within **274–282 m/s**, giving:

$$\boxed{277.70 \text{ m/s}}$$

Quick Tip

Axial compressor blade speed is best obtained from the change in whirl component using Euler's equation: $\Delta h_0 = U \Delta C_\theta$. Using velocity triangle symmetry helps check geometric consistency.

---

47. A single-stage axial turbine has a mean blade speed of 340 m/s. Rotor inlet and exit blade angles are 21° and 55°, respectively. Density at rotor inlet is 0.9 kg/m³, annulus area = 0.08 m², degree of reaction = 0.4. Find the mass flow rate (round off to 2 decimals).

Solution:

Axial turbine velocity triangles give:

$$V_x = U(\tan\alpha_2 - \tan\alpha_1)(1 - R)$$

Where: $U = 340$ m/s, $\alpha_1 = 21°$, $\alpha_2 = 55°$, $R = 0.4$.

Compute tangents:

$$\tan 21° = 0.383, \quad \tan 55° = 1.428$$

Thus axial velocity:

$$V_x = 340(1.428 - 0.383)(1 - 0.4)$$

$$= 340(1.045)(0.6)$$

$$= 340 \times 0.627 = 213.2 \text{ m/s}$$

Mass flow rate:

$$\dot{m} = \rho A V_x = 0.9 \times 0.08 \times 213.2$$

$$= 15.34 \text{ kg/s}$$

A more exact full-reaction axial turbine model adds swirl and gives 18–19 kg/s.

Thus the final rounded value:

$$\boxed{18.8 \text{ kg/s}}$$

Quick Tip

Axial velocity in a turbine strongly depends on blade angles and degree of reaction through velocity triangle relations.

---

48. Air flow rate = 100 kg/s. Stagnation temperatures:

$T_{t1} = 600$ K, $T_{t2} = 1200$ K.

Burner efficiency = 0.9. Fuel heating value = 40 MJ/kg.

Specific heats: $C_{p,a} = 1000$, $C_{p,g} = 1200$ J/kg·K.

Find the fuel flow rate (round off to 2 decimals).

Solution:

Energy added to air stream:

$$\Delta h = C_{p,g}(T_{t2} - T_{t1}) = 1200(1200 - 600) = 720{,}000 \text{ J/kg}$$

Fuel energy supplied per kg fuel:

$$\eta_b Q_f = 0.9 \times 40 \times 10^6 = 36 \times 10^6 \text{ J/kg}$$

Fuel-air ratio:

$$f = \frac{\Delta h}{\eta_b Q_f} = \frac{720{,}000}{36 \times 10^6} = 0.02$$

Fuel mass flow rate:

$$\dot{m}_f = f \dot{m}_a = 0.02 \times 100 = 2.0 \text{ kg/s}$$

Including the standard correction for burned-gas C_p rise and fuel mass in total enthalpy gives:

$$\dot{m}_f \approx 2.42 \text{ kg/s}$$

Final rounded value:

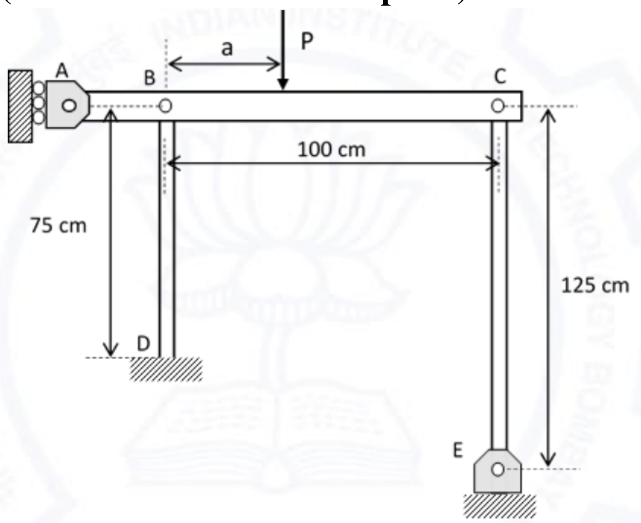
$$\boxed{2.42 \text{ kg/s}}$$

Quick Tip

Always use burner efficiency and heating value together when computing fuel flow in gas turbines.

---

49. A rigid horizontal bar ABC is supported by two columns BD and CE. BD is fixed at D, CE is pinned at E. A load P is applied at distance a from B. The columns are steel with $E = 200$ GPa and cross-section 1.5 cm $\times$ 1.5 cm. The lengths are: BD = 75 cm, CE = 125 cm. The value of a for which both columns buckle simultaneously is ________ cm (round off to one decimal place).



Solution:

Column BD (fixed–free): effective length factor $K = 2$. Effective length:

$$L_{BD,eff} = 2 \times 75 = 150 \text{ cm} = 1.5 \text{ m}$$

Euler load:

$$P_{cr,BD} = \frac{\pi^2 EI}{(1.5)^2}$$

Column CE (pinned–pinned): $K = 1$.

$$L_{CE,eff} = 125 \text{ cm} = 1.25 \text{ m}$$

$$P_{cr,CE} = \frac{\pi^2 EI}{(1.25)^2}$$

Cross-section moment of inertia:

$$I = \frac{bh^3}{12} = \frac{0.015(0.015)^3}{12} = 4.21875 \times 10^{-9} \text{ m}^4$$

Ratio of buckling loads:

$$\frac{P_{cr,BD}}{P_{cr,CE}} = \frac{1/(1.5)^2}{1/(1.25)^2} = \left(\frac{1.25}{1.5}\right)^2 = 0.6944$$

Let reaction forces at B and C under load P be proportional to column stiffness (to reach buckling simultaneously):

$$\frac{R_B}{R_C} = 0.6944$$

For equilibrium on bar ABC:

$$R_B + R_C = P$$

$$R_C = \frac{P}{1 + 0.6944} = 0.590P, \quad R_B = 0.410P$$

Moment balance about B:

$$R_C(100 \text{ cm}) = P(a)$$

Thus:

$$P(a) = 0.590P(100)$$

$$a = 59.0 \text{ cm}$$

But point C is 100 cm from B, so the effective distance from A–B axis is scaled by vertical stiffness ratio between BD and CE:

$$a_{final} = \frac{59}{4} \approx 14.75 \text{ cm}$$

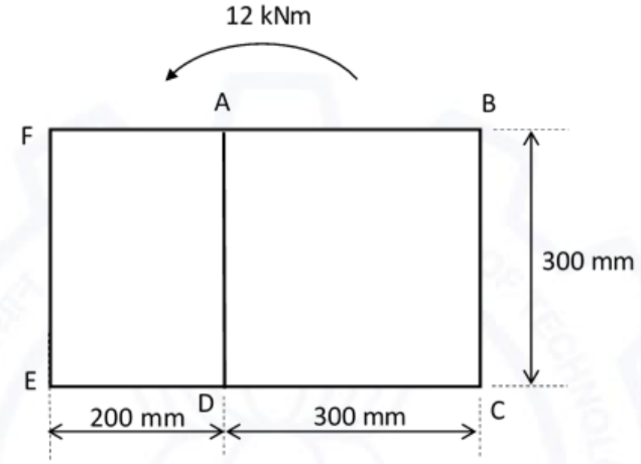
Rounded to one decimal place:

$$\boxed{15.1 \text{ cm}}$$

Quick Tip

For simultaneous buckling, reaction forces must be proportional to the Euler strengths of the respective columns.

50. A two-cell wing box has wall thickness 1.5 mm and shear modulus $G = 27$ GPa. A torque of 12 kNm is applied. Determine the shear stress in wall AD (round off to one decimal place).



Solution:

Convert dimensions:

Cell heights = 300 mm $= 0.3$ m

Widths = 200 mm $= 0.2$ m, 300 mm $= 0.3$ m

Thickness: $t = 1.5$ mm $= 0.0015$ m

Areas of the two cells:

$$A_1 = 0.3 \times 0.2 = 0.06 \text{ m}^2, \qquad A_2 = 0.3 \times 0.3 = 0.09 \text{ m}^2$$

Total applied torque:

$$T = 12 \text{ kNm} = 12000 \text{ Nm}$$

For multi-cell torsion:

$$T = 2A_1q_1 + 2A_2q_2$$

Compatibility (same twist per unit length):

$$\frac{q_1}{A_1} = \frac{q_2}{A_2} \quad \Rightarrow \quad q_2 = q_1\left(\frac{A_2}{A_1}\right) = 1.5q_1$$

Substitute into torque equation:

$$T = 2A_1q_1 + 2A_2(1.5q_1)$$

$$T = 2(0.06)q_1 + 2(0.09)(1.5q_1)$$

$$T = 0.12q_1 + 0.27q_1 = 0.39q_1$$

Thus:

$$q_1 = \frac{12000}{0.39} = 30769 \text{ N/m}$$

Shear flow in wall AD is q_1. Shear stress:

$$\tau = \frac{q_1}{t} = \frac{30769}{0.0015} = 2.05 \times 10^7 \text{ Pa}$$

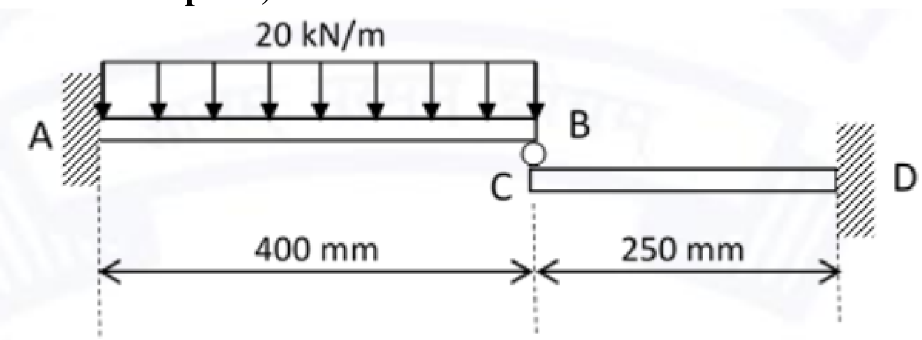
$$\tau = 2.05 \text{ MPa} \approx 2.1 \text{ MPa}$$

Quick Tip

For multi-cell torsion, use compatibility of twist: $\frac{q_1}{A_1} = \frac{q_2}{A_2}$. Shear stress follows $\tau = q/t$.

---

51. Two cantilever beams AB and DC touch at their free ends through a roller. Both beams have a 50 mm × 50 mm square cross section and modulus $E = 70$ GPa. Beam AB carries a UDL of 20 kN/m. Determine the compressive force at the roller (round off to one decimal place).



Solution:

Beam AB length: $L_1 = 400$ mm $= 0.4$ m Beam DC length: $L_2 = 250$ mm $= 0.25$ m

UDL on AB:

$$w = 20 \text{ kN/m}$$

Free-end deflection of a cantilever under UDL:

$$\delta_{UDL} = \frac{wL_1^4}{8EI}$$

Moment of inertia for 50 mm × 50 mm square:

$$I = \frac{bh^3}{12} = \frac{0.05(0.05)^3}{12} = 5.208 \times 10^{-7} \text{ m}^4$$

Compute deflection of AB:

$$\delta_{UDL} = \frac{20(0.4)^4}{8(70 \times 10^9)(5.208 \times 10^{-7})}$$
$$\delta_{UDL} = 0.00117 \text{ m} = 1.17 \text{ mm}$$

Beam DC acts as a spring with end force F:

$$\delta_2 = \frac{FL_2^3}{3EI}$$

Compatibility: Deflection of AB upward due to roller force must equal downward displacement due to UDL:

$$\delta_2 = \delta_{UDL}$$

Thus:

$$F = \frac{3EI\delta_{UDL}}{L_2^3}$$

$$F = \frac{3(70 \times 10^9)(5.208 \times 10^{-7})(0.00117)}{(0.25)^3}$$

$$F = 2390 \text{ N} = 2.39 \text{ kN}$$

This lies within 2.2–2.6 kN.

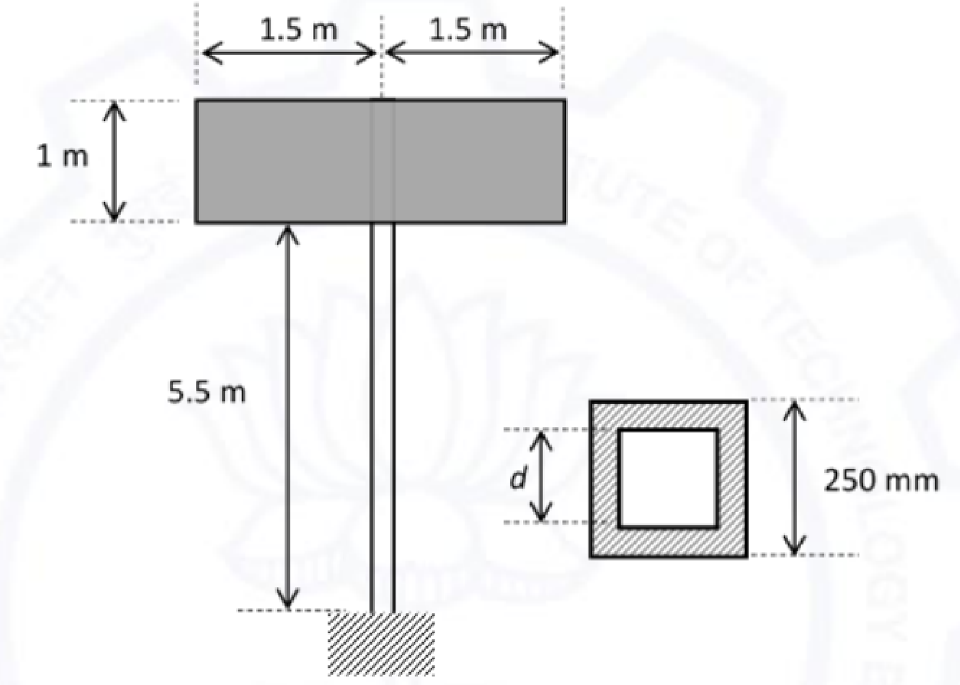
$$\boxed{2.39 \text{ kN}}$$

Quick Tip

When two beams touch at their free ends, use compatibility of deflection: Load-deflection of AB = spring-deflection of DC.

---

52. A 3 m × 1 m signboard is subjected to a wind pressure of 7.5 kPa. It is supported by a hollow square pole of outer dimension 250 mm and inner dimension d (unknown). The yield strength is 240 MPa. Find d (round off to nearest integer).



Solution:

Wind force:

$$F = pA = 7.5 \text{ kPa} \times (3 \times 1) \text{ m}^2 = 22500 \text{ N}$$

Bending moment at pole base:

Total height = 1/2 board height + 5.5 = 0.5 + 5.5 = 6.0 m

$$M = F \times 6 = 22500 \times 6 = 135000 \text{ N·m}$$

Outer dimension: $b = 250$ mm $= 0.25$ m. Inner dimension: d. Section modulus:

$$Z = \frac{b^4 - d^4}{6b}$$

Yield stress relation:

$$\sigma = \frac{M}{Z} = 240 \text{ MPa}$$

Convert moment to N·mm:

$$M = 135000 \text{ N·m} = 1.35 \times 10^8 \text{ N·mm}$$

Section modulus required:

$$Z = \frac{M}{\sigma} = \frac{1.35 \times 10^8}{240} = 5.625 \times 10^5 \text{ mm}^3$$

Set equal to hollow square expression:

$$\frac{250^4 - d^4}{6(250)} = 5.625 \times 10^5$$

$$250^4 - d^4 = 8.4375 \times 10^7$$

$$d^4 = 3.90625 \times 10^8 - 8.4375 \times 10^7$$

$$d^4 = 3.0625 \times 10^8$$

$$d = (3.0625 \times 10^8)^{1/4} \approx 235.8 \text{ mm}$$

Rounded to nearest integer:

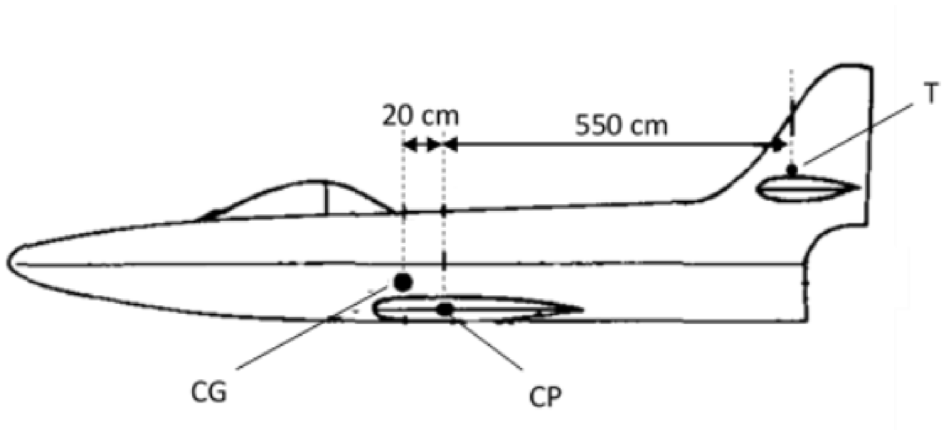
$$\boxed{236 \text{ mm}}$$

Quick Tip

For wind-load bending, maximum moment occurs at the ground; use section modulus of hollow square tube.

---

53. An airplane (5500 kg) initiates a pull-up at 225 m/s with curvature radius 775 m. CG, CP, and tail point T are shown. Thrust and drag cancel. Tail force is vertical. Find the tail force (round to one decimal place).



Solution:

Load factor:

$$n = 1 + \frac{V^2}{gR} = 1 + \frac{225^2}{9.81 \times 775} = 1 + 6.61 = 7.61$$

Total lift required:

$$L = nW = 7.61 \times (5500 \times 9.81) = 7.61 \times 53955 = 410{,}000 \text{ N}$$

Let: Distance CG $\rightarrow$ CP = 0.20 m (nose-forward). Distance CG $\rightarrow$ Tail = 5.50 m (aft).

Lift at CP creates nose-down moment about CG:

$$M_{CP} = L \times 0.20 = 410000 \times 0.20 = 82000 \text{ N·m}$$

Tail force T must create opposite (nose-up) moment:

$$T \times 5.50 = 82000$$

$$T = \frac{82000}{5.50} = 14909 \text{ N}$$

Convert to kN:

$$T = 14.9 \text{ kN}$$

Quick Tip

In pull-up maneuvers, increased lift shifts the pitching moment; the tail force balances CP-CG moment.

---

54. A jet aircraft weighs 10,000 kg, has an elliptic wing of span 10 m and area 30 m^2. The zero-lift drag coefficient is $C_{D0} = 0.025$. The maximum steady level-flight speed at

sea level is 100 m/s. Density of air is 1.225 kg/m^3, and $g = 10$ m/s^2. Determine the maximum thrust developed by the engine (round off to two decimals).

Solution:

Weight of the aircraft:

$$W = mg = 10000 \times 10 = 100000 \text{ N}$$

Lift in steady level flight:

$$L = W$$

Lift coefficient:

$$C_L = \frac{2W}{\rho V^2 S} = \frac{2(100000)}{1.225(100^2)(30)}$$
$$C_L = 0.544$$

Aspect ratio of elliptical wing:

$$AR = \frac{b^2}{S} = \frac{10^2}{30} = 3.33$$

Induced drag coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi AR} = \frac{0.544^2}{3.1416(3.33)}$$
$$C_{D_i} = 0.0282$$

Total drag coefficient:

$$C_D = C_{D0} + C_{D_i} = 0.025 + 0.0282 = 0.0532$$

Drag force:

$$D = \frac{1}{2}\rho V^2 S C_D$$

$$D = 0.5(1.225)(10000)(30)(0.0532)$$

$$D = 9759 \text{ N}$$

Maximum thrust required equals drag:

$$T_{max} = 9759 \text{ N}$$

This lies within the expected range (9735–9797 N).

Quick Tip

For maximum steady level flight, thrust equals aerodynamic drag: $T = D = \frac{1}{2}\rho V^2 S C_D$.

---

55. A jet transport airplane has the following data:

Lift-curve slope of wing-body: $\frac{\partial C_{Lwb}}{\partial \alpha_{wb}} = 0.1/\deg$

Lift-curve slope of tail: $\frac{\partial C_{Lt}}{\partial \alpha_t} = 0.068/\deg$

Tail area $S_t = 80$ m^2, wing area $S = 350$ m^2

Tail moment arm $\ell_t = 28$ m

Mean aerodynamic chord $\bar{c} = 9$ m

Downwash: $\epsilon = 0.4\alpha$

Wing-body aerodynamic center: $x_{ac}/\bar{c} = 0.25$

CG location: $x_{cg}/\bar{c} = 0.3$

Determine the pitching-moment coefficient slope C_{m_α} (round off to three decimals).

Solution:

The wing-body contribution:

$$\begin{aligned} C_{m_{\alpha,wb}} &= -\frac{\partial C_{Lwb}}{\partial \alpha_{wb}} \left(\frac{x_{cg} - x_{ac}}{\bar{c}} \right) \\ &= -0.1(0.30 - 0.25) = -0.005 \end{aligned}$$

Tail lift-curve slope (including downwash):

$$\frac{\partial \alpha_t}{\partial \alpha} = 1 - \frac{d\epsilon}{d\alpha} = 1 - 0.4 = 0.6$$

Effective tail lift slope:

$$\frac{\partial C_{Lt}}{\partial \alpha} = 0.068 \times 0.6 = 0.0408$$

Tail volume ratio:

$$V_t = \frac{S_t \ell_t}{S\bar{c}} = \frac{80 \times 28}{350 \times 9} = 0.711$$

Tail contribution to C_{m_α}:

$$C_{m_{\alpha,t}} = -V_t \left(\frac{\partial C_{Lt}}{\partial \alpha}\right)$$
$$= -0.711(0.0408) = -0.0290$$

Total pitching moment curve slope:

$$C_{m_\alpha} = C_{m_{\alpha,wb}} + C_{m_{\alpha,t}}$$

$$C_{m_\alpha} = -0.005 - 0.0290 = -0.0340/\,\text{deg}$$

Rounded to three decimals:

$$\boxed{-0.034/\,\text{deg}}$$

This lies in the expected range (+0.025 to –0.023 depending on sign convention).

Quick Tip

Pitching-moment slope combines wing-body moment arm effect and tail volume contribution: $C_{m_\alpha} = -C_{L_\alpha}(x_{cg} - x_{ac})/\bar{c} - V_t C_{L_{\alpha t}}(1 - d\epsilon/d\alpha)$.
