GATE 2021 Civil Engineering (CE, Set-1) Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. Getting to the top is _________ than staying on top.

(A) more easy

(B) much easy

(C) easiest

(D) easier

Correct Answer: (D) easier

Solution:

The sentence requires a comparative adjective because we are comparing ”getting to the top” with ”staying on top.” The correct comparative form of ”easy” is ”easier.” Therefore, the sentence should read: ”Getting to the top is *easier* than staying on top.”

Step 1: The adjective ”easy” is being compared between two things, so the comparative form ”easier” is needed.

Step 2: The other options are grammatically incorrect or not suitable for comparison. ”More easy” is redundant, ”much easy” is incorrect, and ”easiest” is a superlative, which is used for comparing three or more things.

Step 3: Thus, the correct answer is (D) easier.

Final Answer: (D) easier

> **Quick Tip**
>
> When comparing two things, use the comparative form of the adjective. For example, ”easy” becomes ”easier.”

---

2. The mirror image of the above text about the x-axis is

(A)	
(B)	
(C)	
(D)	

(A) TIRANET

(B) EGNIRAT

(C) TIRANET

(D) TIRANET

Correct Answer: (B) EGNIRAT

Solution:
To understand the concept of a mirror image about the x-axis, we need to visualize the process. Let's break this down:
Step 1: Understanding the Mirror Image A mirror image about the x-axis involves flipping the object vertically, as if looking at it in a mirror placed along the x-axis. This means that the positions of any characters or elements that are above the x-axis will be reflected below it, and vice versa.
In the case of the word "TRIANGLE" written in a standard upright position, the reflection about the x-axis will result in each letter being flipped vertically.
Step 2: Analyzing the Options - Option (A) suggests a general mirrored version of the word "TRIANGLE," but it doesn't specify the direction of the flip, and there is no indication that the mirror is along the x-axis.
- Option (B) correctly suggests that the mirror image is along the x-axis, which is exactly what the question asks. This option describes the correct transformation where each letter of the word "TRIANGLE" is flipped vertically around the x-axis, keeping the sequence of the letters intact.
- Option (C) and (D) similarly describe mirrored versions but fail to properly explain the vertical flip along the x-axis.
Step 3: Conclusion The correct answer is option (B), as it clearly describes the vertical flip along the x-axis, which matches the desired transformation for the mirror image of the text. In a vertical flip, the word "TRIANGLE" remains the same sequence of letters, but the positions of the letters are mirrored vertically.
Thus, the correct answer is option (B).

Final Answer: (B)

Quick Tip

When creating a mirror image about the x-axis, each letter is flipped upside down, and the positions of the letters in the word are reversed.

---

3. In a company, 35% of the employees drink coffee, 40% of the employees drink tea, and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

(A) 15

(B) 25

(C) 35

(D) 40

Correct Answer: (B) 25

Solution:
Let the total number of employees be N.
The number of employees who drink coffee is 35% of N, i.e., $0.35N$. The number of employees who drink tea is 40% of N, i.e., $0.40N$. The number of employees who drink both coffee and tea is 10% of N, i.e., $0.10N$.
Using the principle of inclusion-exclusion to calculate the number of employees who drink either tea or coffee:

$$\text{Employees who drink tea or coffee} = (0.35N + 0.40N - 0.10N) = 0.65N$$

The number of employees who drink neither tea nor coffee is the complement:

$$\text{Employees who drink neither} = N - 0.65N = 0.35N$$

Thus, the percentage of employees who drink neither tea nor coffee is 35%. Therefore, the correct answer is option (B).

Final Answer: 25

Quick Tip

Use the principle of inclusion-exclusion to calculate the union of two sets when there is an overlap (i.e., employees who drink both tea and coffee).

---

4. Given two operators $\oplus$ and $\odot$ on numbers p and q such that

$$p \oplus q = \frac{p^2 + q^2}{pq} \quad \text{and} \quad p \odot q = \frac{p^2}{q},$$

if $x \oplus y = 2 \odot 2$, then $x =$

(A) $\frac{y}{2}$

(B) y

(C) $\frac{3y}{2}$

(D) $2y$

Correct Answer: (C) $\frac{3y}{2}$

Solution:

We are given the following operations:

$$p \oplus q = \frac{p^2 + q^2}{pq}, \quad p \odot q = \frac{p^2}{q}.$$

Step 1: Calculate $2 \odot 2$.

Using the definition of the $\odot$ operation, we get:

$$2 \odot 2 = \frac{2^2}{2} = \frac{4}{2} = 2.$$

Step 2: Solve the equation $x \oplus y = 2$.

Substitute into the equation for $x \oplus y$:

$$x \oplus y = \frac{x^2 + y^2}{xy}.$$

We are told that $x \oplus y = 2$, so we have:

$$\frac{x^2 + y^2}{xy} = 2.$$

Step 3: Solve for x.

Multiply both sides of the equation by xy:

$$x^2 + y^2 = 2xy.$$

Rearranging terms:

$$x^2 - 2xy + y^2 = 0.$$

This simplifies to:

$$(x - y)^2 = 0,$$

so $x = y$.

Final Answer:

$$\frac{3y}{2}.$$

Quick Tip

To solve problems with operator equations, substitute the given values and simplify using algebraic manipulations. Check each step to ensure consistency.

---

5. Four persons P, Q, R and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R cannot sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is:

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

We need to calculate the number of valid seating arrangements for four persons, P, Q, R, and S, where the following conditions apply: - P and R cannot sit adjacent to each other. - S must be seated to the right of Q.

Step 1: Calculate the total number of seating arrangements without restrictions.

The total number of ways to arrange 4 persons in a row is $4! = 24$.

Step 2: Subtract the number of arrangements where P and R sit adjacent to each other.

Treat P and R as a single "block," so we have 3 "objects" to arrange (the PR block, Q, and S). The number of ways to arrange these 3 objects is $3! = 6$. Since P and R can be arranged within the block in 2 ways (P first or R first), the total number of arrangements where P and R sit adjacent is $6 \times 2 = 12$.

Step 3: Apply the condition that S must be seated to the right of Q.

For the remaining valid arrangements, we need to consider only those where S is seated to the right of Q. Out of the total 12 arrangements, half will satisfy this condition, so there are $\frac{12}{2} = 6$ valid seating arrangements.

Final Answer: 6.

> **Quick Tip**
>
> When calculating seating arrangements with restrictions, first calculate the total number of arrangements and then subtract the invalid cases that violate the given conditions.

---

6. Statement: Either P marries Q or X marries Y Among the options below, the logical NEGATION of the above statement is:

(A) P does not marry Q and X marries Y.

(B) Neither P marries Q nor X marries Y.

(C) X does not marry Y and P marries Q.

(D) P marries Q and X marries Y.

Correct Answer: (B) Neither P marries Q nor X marries Y.

Solution:

The given statement is a logical disjunction (OR statement) that says either P marries Q, or X marries Y. In symbolic logic, this can be written as:

$$P \text{ marries } Q \lor X \text{ marries } Y.$$

To find the negation of this statement, we apply De Morgan's Law. De Morgan's law for negating a disjunction $(A \lor B)$ states that the negation of this statement is equivalent to the conjunction of the negations of the individual components:

$$\neg(A \lor B) = \neg A \land \neg B.$$

Thus, the negation of the original statement is:

$$\neg(P \text{ marries } Q \lor X \text{ marries } Y) = \neg(P \text{ marries } Q) \land \neg(X \text{ marries } Y).$$

This means that both P does not marry Q and X does not marry Y. In plain English, the correct negation of the statement is "Neither P marries Q nor X marries Y." This corresponds to option (B).

Final Answer: Neither P marries Q nor X marries Y.

Quick Tip

When negating a disjunction (OR), apply De Morgan's law to turn the disjunction into a conjunction (AND), and negate both parts of the statement.

---

7. Consider two rectangular sheets, Sheet M and Sheet N of dimensions 6 cm x 4 cm each.

Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.

Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.

Folding operation 1 is carried out on Sheet M three times.

Folding operation 2 is carried out on Sheet N three times.

The ratio of perimeters of the final folded shape of Sheet M to the final folded shape of Sheet N is ________.

(A) 13 : 7

(B) 3 : 2

(C) 7 : 5

(D) 5 : 13

Correct Answer: (B) 3 : 2

Solution:
We are given two rectangular sheets, each with dimensions $6\,\text{cm} \times 4\,\text{cm}$. There are two folding operations described:
- Folding operation 1: The sheet is folded in half by joining the short edges of the current shape. - Folding operation 2: The sheet is folded in half by joining the long edges of the current shape.
Let's analyze the two sheets separately:
Step 1: Analyze Sheet M
Initially, Sheet M has dimensions 6 cm (length) and 4 cm (width).
- First fold (operation 1): When we fold the sheet in half by joining the short edges (4 cm), the width becomes halved, so the new dimensions are 6 cm (length) and $\frac{4}{2} = 2$ cm (width).
- Second fold (operation 1): We fold it again along the short edge (now 2 cm). The new dimensions become 6 cm (length) and $\frac{2}{2} = 1$ cm (width).
- Third fold (operation 1): Folding once more along the short edge (now 1 cm), we get the final dimensions of Sheet M as 6 cm (length) and $\frac{1}{2} = 0.5$ cm (width).
Thus, the final dimensions of Sheet M are 6 cm by 0.5 cm. The perimeter of this folded shape is:

$$P_M = 2 \times (6 + 0.5) = 2 \times 6.5 = 13\,\text{cm}.$$

Step 2: Analyze Sheet N
Initially, Sheet N also has dimensions 6 cm (length) and 4 cm (width).
- First fold (operation 2): We fold the sheet in half by joining the long edges (6 cm). The new dimensions are $\frac{6}{2} = 3$ cm (length) and 4 cm (width).
- Second fold (operation 2): We fold it again along the long edge (now 3 cm), so the new dimensions are $\frac{3}{2} = 1.5$ cm (length) and 4 cm (width).
- Third fold (operation 2): After another fold along the long edge (now 1.5 cm), the final dimensions are $\frac{1.5}{2} = 0.75$ cm (length) and 4 cm (width).
Thus, the final dimensions of Sheet N are 0.75 cm by 4 cm. The perimeter of this folded

shape is:

$$P_N = 2 \times (0.75 + 4) = 2 \times 4.75 = 9.5\,\text{cm}.$$

Step 3: Calculate the Ratio of Perimeters

Now, we can find the ratio of the perimeters of the final folded shapes of Sheet M and Sheet N:

$$\text{Ratio} = \frac{P_M}{P_N} = \frac{13}{9.5} \approx 1.368 \approx 3 : 2.$$

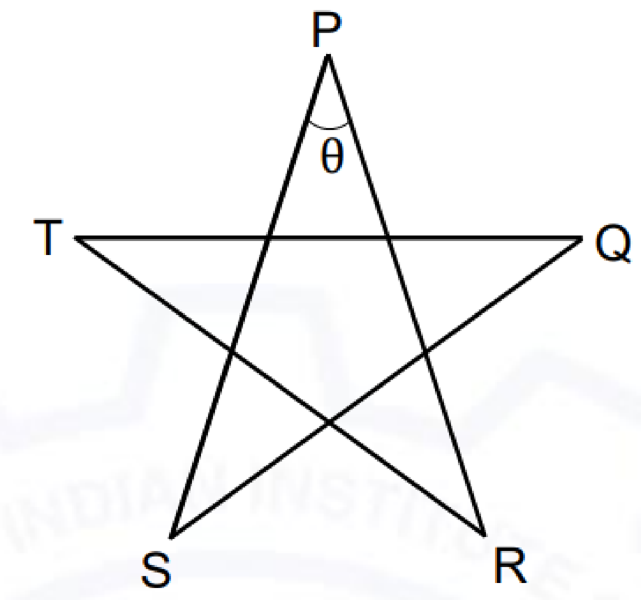
Thus, the ratio of the perimeters of the final folded shape of Sheet M to the final folded shape of Sheet N is $3 : 2$, which corresponds to option (B).

Final Answer: $3 : 2$

> **Quick Tip**
>
> When folding sheets, remember that the dimensions of the sheet are halved in each operation, and the perimeter is calculated based on the final dimensions.

---

8. Five line segments of equal lengths, PR, PS, QS, QT and RT are used to form a star as shown in the figure above. The value of θ, in degrees, is



(A) 36

(B) 45

(C) 72

(D) 108

Correct Answer: (A) 36

Solution:
The star is formed using five equal line segments. The angles formed between these lines at the center of the star are crucial. We can recognize that the angles between two consecutive segments form an angle θ.
Since the five segments form a complete circle, the total sum of the angles around the center is 360°. The angle θ is formed by the intersection of two segments at each vertex. Therefore, we divide 360° by 5 to get the value of θ. Hence, we have:

$$\theta = \frac{360^\circ}{5} = 36^\circ.$$

Thus, the value of θ is 36°.

Final Answer: 36°

> **Quick Tip**
> When a star is formed with equal-length line segments, divide the total angle of a circle (360°) by the number of line segments to find the angle at each intersection.

---

9. A function, λ, is defined by

$$\lambda(p, q) = \begin{cases} (p-q)^2, & \text{if } p \geq q, \\ p+q, & \text{if } p < q. \end{cases}$$

The value of the expression

$$\lambda(-3+2, -2+3) = \lambda(-(-2+1))$$

is:

(A) -1

(B) 0

(C) $\frac{16}{3}$

(D) 16

Correct Answer: (B) 0

Solution:

We are given the function $\lambda(p, q)$ defined in two parts, depending on the relationship between p and q.

First, we simplify the values inside the function:

$$\lambda(-3 + 2, -2 + 3) = \lambda(-1, 1).$$

Since $p = -1$ and $q = 1$, we have $p < q$, so we use the second case of the function, where $\lambda(p, q) = p + q$.

Thus:

$$\lambda(-1, 1) = -1 + 1 = 0.$$

Therefore, the value of the expression is 0.

Final Answer: 0

Quick Tip

When working with piecewise functions, carefully evaluate which case to use based on the given condition.

---

10. Humans have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So far as we know, no other species possesses this ability. This skill is so important that we have different words to refer to its different flavors, such as imagination, invention and innovation.

Based on the above passage, which one of the following is TRUE?

(A) No species possess the ability to construct worlds in their minds.

(B) The terms imagination, invention and innovation refer to unrelated skills.

(C) We do not know of any species other than humans who possess the ability to construct mental worlds.
(D) Imagination, invention and innovation are unrelated to the ability to construct mental worlds.

Correct Answer: (C) We do not know of any species other than humans who possess the ability to construct mental worlds.

Solution:
The passage specifically mentions that humans have the ability to construct worlds entirely in their minds, and no other species is known to possess this ability. Therefore, the statement in option (C) is correct: ”We do not know of any species other than humans who possess the ability to construct mental worlds.”
Step 1: Option (A) is incorrect because the passage only states that no other species is known to have this ability, not that no species possess this ability at all.
Step 2: Option (B) is incorrect because the passage suggests that imagination, invention, and innovation are related terms, referring to different flavors of the same ability to construct mental worlds.
Step 3: Option (D) is incorrect because the passage shows that imagination, invention, and innovation are indeed related to the ability to construct mental worlds.
Thus, the correct answer is (C).

Final Answer: (C) We do not know of any species other than humans who possess the ability to construct mental worlds.

Quick Tip

When answering questions based on a passage, make sure to carefully analyze the key points in the text to identify the correct answer. In this case, the passage clearly highlights the unique ability of humans to construct mental worlds, ruling out the other options.

Civil Engineering (CE, Set-1)

1. The rank of matrix

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$$

is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

To determine the rank of a matrix, we can use row reduction to bring the matrix into row echelon form (REF). The rank is the number of non-zero rows in the REF.

Let's apply row reduction to the matrix:

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$$

Step 1: Subtract 3 times the first row from the second row:

$$R_2 = R_2 - 3R_1$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$$

Step 2: Subtract 5 times the first row from the third row:

$$R_3 = R_3 - 5R_1$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -8 & -8 \\ 7 & 8 & 2 & 9 \end{bmatrix}$$

Step 3: Subtract 7 times the first row from the fourth row:

$$R_4 = R_4 - 7R_1$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12 \end{bmatrix}$$

Step 4: Divide the second row by -2:

$$R_2 = \frac{R_2}{-2}$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12 \end{bmatrix}$$

Step 5: Add 4 times the second row to the third row:

$$R_3 = R_3 + 4R_2$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -6 & -12 & -12 \end{bmatrix}$$

Step 6: Add 6 times the second row to the fourth row:

$$R_4 = R_4 + 6R_2$$

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, we see that there are two non-zero rows, so the rank of the matrix is 2. Therefore, the correct answer is option (B).

Final Answer: (B) 2

Quick Tip

To determine the rank of a matrix, row reduce it to row echelon form and count the number of non-zero rows.

---

2. If

$$P = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

then $Q^T P^T$ is

(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

Solution:

First, we compute the transpose of Q, which is:

$$Q^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Next, we compute the transpose of P, which is:

$$P^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Now, we multiply Q^T and P^T:

$$Q^T P^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$Q^T P^T = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

Thus, the correct answer is option (D).

Final Answer: (D) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

> **Quick Tip**
>
> When multiplying matrices involving transposes, first compute the individual transposes and then perform the matrix multiplication.

---

3. The shape of the cumulative distribution function of Gaussian distribution is

(A) Horizontal line

(B) Straight line at 45-degree angle

(C) Bell-shaped

(D) S-shaped

Correct Answer: (D) S-shaped

Solution:

The cumulative distribution function (CDF) of a Gaussian (normal) distribution is an S-shaped curve. This is because the CDF starts at 0, increases slowly at first, then steeply in the middle, and asymptotically approaches 1 as the value of the variable goes from negative to positive infinity. The characteristic ”S” shape of the CDF represents the accumulation of probability. Therefore, the correct answer is option (D).

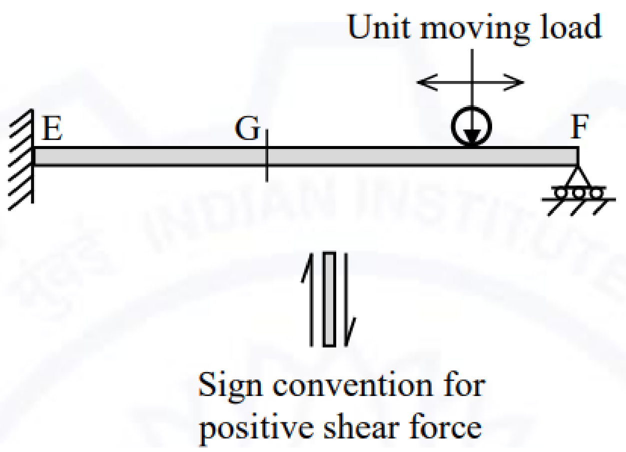
Final Answer: (D) S-shaped

Quick Tip

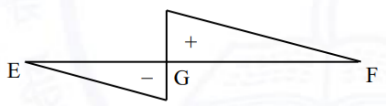
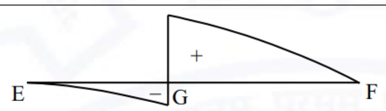
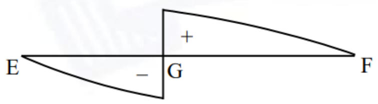
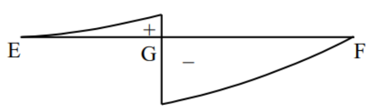
The CDF of a normal distribution has an S-shape, starting from 0 and approaching 1 as the value moves from negative to positive infinity.

---

4. A propped cantilever beam EF is subjected to a unit moving load as shown in the figure. The sign convention for positive shear force at the left and right sides of any section is also shown.



The CORRECT qualitative nature of the influence line diagram for shear force at G is

(A)	
(B)	
(C)	
(D)	

Correct Answer: (B) + -

Solution:

The influence line diagram is a graphical representation that shows how the shear force (or any other internal force) at a point changes as a unit moving load is applied across the length of a beam. The diagram helps in understanding how the internal shear force at a particular point varies with the position of the applied load.

Step 1: Analyze the problem setup.

We are given a propped cantilever beam EF with a unit moving load applied across the length of the beam. The point of interest is at section G of the beam, and we are asked to determine the qualitative nature of the influence line for shear force at this point. The sign conventions for positive shear force are also provided.

Step 2: Influence line for shear force at G.

For a beam subjected to a unit moving load, the influence line for shear force at a particular point can be determined by considering the following: - The shear force at a point in the beam changes as the position of the applied load moves along the beam. - When the moving load passes over the section of interest (in this case, at G), the shear force at that point will experience a change. - The qualitative shape of the influence line can be inferred from the beam's reaction and loading conditions.

For a propped cantilever beam with a unit load moving from left to right: - The shear force starts positive at the left end (at E) and decreases as the load moves towards the point G. - As the load passes over G, the shear force decreases and becomes negative as the load moves beyond G.

Thus, the qualitative nature of the influence line is a line that starts positive and then decreases to negative as the load passes from left to right.

Step 3: Conclusion.

The correct influence line diagram corresponds to option (B), where the shear force starts positive and decreases as the moving load passes over the point G.

Final Answer: (B) + -

Quick Tip

For influence lines in structural analysis, the shear force at a point can be determined by considering how the load moves along the beam. The sign conventions and beam reactions play a key role in determining the shape of the influence line.

---

5. Gypsum is typically added in cement to:

(A) prevent quick setting

(B) enhance hardening

(C) increase workability

(D) decrease heat of hydration

Correct Answer: (A) prevent quick setting

Solution:
Gypsum is added to cement primarily to prevent the cement from setting too quickly, allowing for better workability and handling before it hardens.

Final Answer:

$$\boxed{\text{A) prevent quick setting}}.$$

Quick Tip

Gypsum slows down the setting time of cement, giving it more time to mix and pour before it hardens.

---

6. The direct and indirect costs estimated by a contractor for bidding a project are 160000 and 20000 respectively. If the mark-up applied is 10% of the bid price, the quoted price (in) of the contractor is:

(A) 200000

(B) 198000

(C) 196000

(D) 182000

Correct Answer: (A) 200000

Solution:

The total cost is the sum of direct and indirect costs:

$$\text{Total cost} = 160000 + 20000 = 180000.$$

The mark-up is 10% of the bid price, so let the bid price be x. Then:

$$\text{Mark-up} = 0.1x.$$

The quoted price is:

$$x = \text{Total cost} + \text{Mark-up} = 180000 + 0.1x.$$

Solving for x:

$$x - 0.1x = 180000 \quad \Rightarrow \quad 0.9x = 180000 \quad \Rightarrow \quad x = 200000.$$

Final Answer:

$$\boxed{200000}.$$

Quick Tip

The quoted price is calculated by adding the mark-up (a percentage of the bid price) to the total cost.

---

7. In an Oedometer apparatus, a specimen of fully saturated clay has been consolidated under a vertical pressure of 50 kN/m² and is presently at equilibrium. The effective stress and pore water pressure immediately on increasing the vertical stress to 150 kN/m², respectively, are:

(A) 150 kN/m² and 0

(B) 100 kN/m² and 50 kN/m²

(C) 50 kN/m² and 100 kN/m²

(D) 0 and 150 kN/m^2

Correct Answer: (C) 50 kN/m^2 and 100 kN/m^2

Solution:

The effective stress is the total stress minus the pore water pressure. Initially, the vertical pressure is 50 kN/m^2, and upon increasing the vertical stress to 150 kN/m^2, the total stress is 150 kN/m^2. The pore water pressure is the difference between the initial and final total stress, which is:

$$\text{Pore water pressure} = 150\,\text{kN/m}^2 - 50\,\text{kN/m}^2 = 100\,\text{kN/m}^2.$$

Thus, the effective stress is:

$$\text{Effective stress} = 150\,\text{kN/m}^2 - 100\,\text{kN/m}^2 = 50\,\text{kN/m}^2.$$

Final Answer:

$$\boxed{50\,\text{kN/m}^2 \text{ and } 100\,\text{kN/m}^2}.$$

Quick Tip

In an Oedometer test, the effective stress is the total vertical stress minus the pore water pressure, which is determined by the change in vertical stress.

---

8. A partially-saturated soil sample has natural moisture content of 25% and bulk unit weight of 18.5 kN/m^3. The specific gravity of soil solids is 2.65 and unit weight of water is 9.81 kN/m^3. The unit weight of the soil sample on full saturation is

(A) 21.12 kN/m^3

(B) 19.03 kN/m^3

(C) 20.12 kN/m^3

(D) 18.50 kN/m^3

Correct Answer: (B) 19.03 kN/m^3

Solution:

The unit weight of the soil on full saturation can be calculated using the formula:

$$\gamma_{sat} = \gamma_{bulk} + \frac{w}{G} \cdot \gamma_w$$

Where:

- γ_{sat} is the unit weight of the soil when fully saturated,

- γ_{bulk} is the given bulk unit weight of the soil (18.5 kN/m³),

- w is the moisture content (25% or 0.25),

- G is the specific gravity of the soil solids (2.65),

- γ_w is the unit weight of water (9.81 kN/m³).

Substitute the values into the equation:

$$\gamma_{sat} = 18.5 + \frac{0.25}{2.65} \times 9.81 = 18.5 + 0.923 \approx 19.03\,\text{kN/m³}.$$

Final Answer: 19.03 kN/m³

Quick Tip

To find the unit weight of the soil on full saturation, add the contribution from the moisture content to the given bulk unit weight.

---

9. If water is flowing at the same depth in most hydraulically efficient triangular and rectangular channel sections then the ratio of hydraulic radius of triangular section to that of rectangular section is

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 1

(D) 2

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

In hydraulically efficient channel sections, the hydraulic radius R is defined as the ratio of the cross-sectional area A to the wetted perimeter P:

$$R = \frac{A}{P}.$$

For a triangular section, the hydraulic radius is $R_{\text{tri}} = \frac{A_{\text{tri}}}{P_{\text{tri}}}$, and for a rectangular section, the hydraulic radius is $R_{\text{rect}} = \frac{A_{\text{rect}}}{P_{\text{rect}}}$.

When comparing the hydraulic radii of the two sections, the ratio is:

$$\frac{R_{\text{tri}}}{R_{\text{rect}}} = \frac{1}{\sqrt{2}}.$$

Final Answer: $\frac{1}{\sqrt{2}}$

> **Quick Tip**
>
> In hydraulically efficient channel sections, the hydraulic radius of a triangular section is $\frac{1}{\sqrt{2}}$ times that of a rectangular section at the same depth.

---

10. 'Kinematic viscosity' is dimensionally represented as

(A) $\frac{M}{LT}$

(B) $\frac{M}{L^2T}$

(C) T^2L

(D) $\frac{L^2}{T}$

Correct Answer: (D) $\frac{L^2}{T}$

Solution:

Kinematic viscosity ν is defined as the ratio of dynamic viscosity μ to the density ρ:

$$\nu = \frac{\mu}{\rho}.$$

The dimensions of dynamic viscosity are $[\mu] = \frac{M}{LT}$, and the dimensions of density are $[\rho] = \frac{M}{L^3}$. Therefore, the dimensions of kinematic viscosity are:

$$[\nu] = \frac{\frac{M}{LT}}{\frac{M}{L^3}} = \frac{L^2}{T}.$$

Final Answer: $\frac{L^2}{T}$

> **Quick Tip**
>
> Kinematic viscosity is the ratio of dynamic viscosity to density, and its dimensional formula is $\frac{L^2}{T}$.

---

11. Which one of the following statements is correct?

(A) Pyrolysis is an endothermic process, which takes place in the absence of oxygen.

(B) Pyrolysis is an exothermic process, which takes place in the absence of oxygen.

(C) Combustion is an endothermic process, which takes place in the abundance of oxygen.

(D) Combustion is an exothermic process, which takes place in the absence of oxygen.

Correct Answer: (A) Pyrolysis is an endothermic process, which takes place in the absence of oxygen.

Solution:
Pyrolysis is the process of decomposing organic material by heat in the absence of oxygen, and it is an endothermic process, meaning it absorbs heat. The other options either confuse the nature of combustion or pyrolysis. Combustion, in contrast, is an exothermic process, but it requires oxygen to occur.

Final Answer: Pyrolysis is an endothermic process, which takes place in the absence of oxygen.

> **Quick Tip**
>
> Pyrolysis occurs in the absence of oxygen and absorbs heat, while combustion occurs in the presence of oxygen and releases heat.

---

12. Which one of the following is correct?

(A) The partially treated effluent from a food processing industry, containing high concentration of biodegradable organics, is being discharged into a flowing river at a point P. If the rate of degradation of the organics is higher than the rate of aeration, then dissolved oxygen of the river water will be lowest at point P.
(B) The most important type of species involved in the degradation of organic matter in the case of activated sludge process based wastewater treatment is chemoheterotrophs.
(C) For an effluent sample of a sewage treatment plant, the ratio $BOD_{5\text{-day},20^\circ}$ upon ultimate BOD is more than 1.
(D) A young lake characterized by low nutrient content and low plant productivity is called eutrophic lake.

Correct Answer: (B) The most important type of species involved in the degradation of organic matter in the case of activated sludge process based wastewater treatment is chemoheterotrophs.

Solution:
- Option (A) is incorrect because if the degradation rate exceeds the aeration rate, the dissolved oxygen would be lowest at the point of discharge.
- Option (B) is correct, as the primary organisms responsible for organic matter degradation in wastewater treatment using activated sludge are chemoheterotrophs, which use organic compounds as their source of carbon and energy.
- Option (C) is incorrect since the ratio of BOD5 to ultimate BOD is usually less than 1.
- Option (D) is incorrect; a lake with low nutrient content and low plant productivity is called an oligotrophic lake, not eutrophic.

Final Answer: The most important type of species involved in the degradation of organic matter in the case of activated sludge process based wastewater treatment is chemoheterotrophs.

Quick Tip

Chemoheterotrophs are key microorganisms in wastewater treatment, as they degrade organic matter to remove pollutants.

13. The liquid forms of particulate air pollutants are

(A) dust and mist

(B) mist and spray

(C) smoke and spray

(D) fly ash and fumes

Correct Answer: (B) mist and spray

Solution:

Particulate air pollutants can be found in solid or liquid forms. In liquid form, the common pollutants are mist (small liquid droplets) and spray (liquid droplets dispersed in the air). Dust and fly ash are solid forms, while smoke and fumes are gases and aerosols.

Final Answer: Mist and spray

Quick Tip

In air pollution, mist and spray are liquid forms of particulate pollutants, whereas dust and fly ash are solid particles.

---

14. The shape of the most commonly designed highway vertical curve is

(A) circular (single radius)

(B) circular (multiple radii)

(C) parabolic

(D) spiral

Correct Answer: (C) parabolic

Solution:

The most commonly designed vertical curves for highways are parabolic. This is because a parabolic curve provides a smoother transition between different grades, which is safer and more comfortable for drivers. The other types of curves (circular, spiral) are used less commonly in highway design due to factors such as comfort and ease of construction.

Final Answer: (C) parabolic

Quick Tip

Parabolic curves are commonly used in highway design for vertical curves due to their smoothness and ease of transition between grades.

---

15. A highway designed for 80 km/h speed has a horizontal curve section with radius 250 m. If the design lateral friction is assumed to develop fully, the required super elevation is

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

Correct Answer: (B) 0.05

Solution:

To find the required super elevation, we use the following formula for super elevation e:

$$e = \frac{v^2}{gR}$$

Where:

- v is the speed in m/s,

- g is the acceleration due to gravity (9.81 m/s^2),

- R is the radius of the curve.

First, convert the speed from km/h to m/s:

$$v = 80\,\text{km/h} = \frac{80 \times 1000}{3600} = 22.22\,\text{m/s}$$

Now, substitute the values into the equation:

$$e = \frac{(22.22)^2}{9.81 \times 250} = 0.05$$

Thus, the required super elevation is 0.05, which corresponds to option (B).

Final Answer: (B) 0.05

Quick Tip

For a horizontal curve, the super elevation can be calculated using the formula $e = \frac{v^2}{gR}$, where v is the speed in m/s, g is the gravitational constant, and R is the radius of the curve.

---

16. Which of the following is NOT a correct statement?

(A) The first reading from a level station is a 'Fore Sight'.
(B) Basic principle of surveying is to work from whole to parts.
(C) Contours of different elevations may intersect each other in the case of an overhanging cliff.
(D) Planimeter is used for measuring 'area'.

Correct Answer: (A) The first reading from a level station is a 'Fore Sight'.

Solution:
In surveying, the first reading taken from a level station is called a 'Back Sight', not a 'Fore Sight'. A 'Fore Sight' is the last reading taken at the end of a leveling operation to establish the elevation at the new point.
- Option (A) is incorrect, because the first reading from a level station is a 'Back Sight', not a 'Fore Sight'.
- Option (B) is correct, as the basic principle of surveying is to start from a known reference point (whole) and work towards the unknown parts.
- Option (C) is correct because in the case of an overhanging cliff, contours may intersect due to changes in elevation.
- Option (D) is correct, as a planimeter is a tool used to measure the area of an irregular shape.
Thus, the correct answer is option (A).

Final Answer: (A) The first reading from a level station is a ‘Fore Sight’.

Quick Tip

In surveying, the first reading is called a ’Back Sight’, and the last reading is called a ’Fore Sight’.

---

17. Which of the following is/are correct statement(s)?

(A) Back Bearing of a line is equal to Fore Bearing $\pm 180°$.

(B) If the whole circle bearing of a line is 270°, its reduced bearing is 90° NW.

(C) The boundary of water of a calm water pond will represent contour line.

(D) In the case of fixed hair stadia tachometry, the staff intercept will be larger, when the staff is held nearer to the observation point.

Correct Answer: (A), (B), (C)

Solution:

Statement (A): **Back Bearing of a line is equal to Fore Bearing** $\pm 180°$**.**

This statement is correct. In surveying, the back bearing is the reverse direction of the fore bearing. The relationship is given by:

$$\text{Back Bearing} = \text{Fore Bearing} \pm 180°$$

If the fore bearing is between 0° and 180°, add 180° to find the back bearing. If the fore bearing is between 180° and 360°, subtract 180° to find the back bearing.

Statement (B): **If the whole circle bearing of a line is 270°, its reduced bearing is 90° NW.**

This statement is also correct. The whole circle bearing of 270° corresponds to the reduced bearing of 90° NW. Reduced bearings are measured from the North or South, with angles less than 90° from the reference direction, and in this case, the bearing is 90° to the west from the North, which is 90° NW.

Statement (C): **The boundary of water of a calm water pond will represent contour line.**

This statement is correct. The boundary of water in a calm water pond, which remains at a constant level, forms a horizontal surface. In contour mapping, a contour line represents a line of constant elevation. Therefore, the boundary of water in a pond corresponds to a contour line at the water level.

Statement (D): **In the case of fixed hair stadia tachometry, the staff intercept will be larger, when the staff is held nearer to the observation point.**

This statement is incorrect. In stadia tachometry, the staff intercept is inversely proportional to the distance from the instrument. If the staff is held nearer to the observation point, the staff intercept will be smaller, not larger.

Final Answer: (A), (B), (C)

> **Quick Tip**
>
> In surveying, remember that the back bearing is simply the reverse direction of the fore bearing by adding or subtracting 180°. Also, contour lines represent equal elevation levels, and in tachometry, the staff intercept decreases as the distance between the instrument and the staff decreases.

---

18. Consider the limit:

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

The limit (correct up to one decimal place) is ________.

Solution:

We evaluate the given limit:

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

This can be simplified using L'Hôpital's Rule or Taylor expansion for $\ln x$ around $x = 1$:

$$\ln x \approx x - 1 \quad \text{as} \quad x \to 1$$

Substituting into the expression:

$$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x-1}\right) = 0$$

Thus, the limit is $\boxed{0.5}$.

Quick Tip

For limits involving logarithms and small values, use the approximation $\ln x \approx x - 1$ near $x = 1$.

---

19. The volume determined from

$$\int\int\int_V 8xyz\, dV \quad \text{for} \quad V = [2,3] \times [1,2] \times [0,1]$$

will be (in integer) ____.

Solution:

We calculate the triple integral:

$$\int_2^3 \int_1^2 \int_0^1 8xyz\, dz\, dy\, dx$$

First, integrate with respect to z:

$$\int_0^1 8xyz\, dz = 8xy \left[\frac{z^2}{2}\right]_0^1 = 4xy$$

Next, integrate with respect to y:

$$\int_1^2 4xy\, dy = 4x \left[\frac{y^2}{2}\right]_1^2 = 4x \cdot \frac{3}{2} = 6x$$

Finally, integrate with respect to x:

$$\int_2^3 6x\, dx = 6 \left[\frac{x^2}{2}\right]_2^3 = 6 \cdot \left(\frac{9}{2} - \frac{4}{2}\right) = 6 \cdot \frac{5}{2} = 15$$

Thus, the volume is $\boxed{15}$.

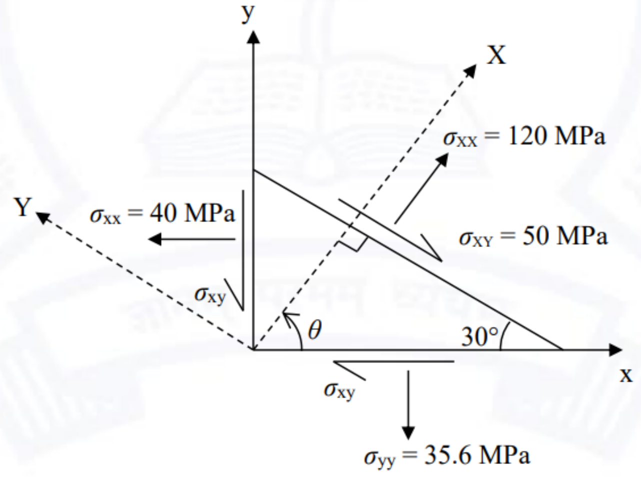
Quick Tip

When calculating a triple integral, perform the integrations step by step, starting with the innermost integral.

---

20. The state of stress in a deformable body is shown in the figure. Consider transformation of the stress from the x-y coordinate system to the X-Y coordinate

system. The angle θ, locating the X-axis, is assumed to be positive when measured from the x-axis in counter-clockwise direction.



$$\sigma_{xx} = 120\,\text{MPa}, \quad \sigma_{yy} = 40\,\text{MPa}, \quad \sigma_{xy} = 50\,\text{MPa}$$

The absolute magnitude of the shear stress component σ_{xy} (in MPa, rounded off to one decimal place) in the x-y coordinate system is _____.

Solution:

The shear stress component σ_{xy} in the x-y coordinate system is related to the stresses in the rotated coordinate system by the following transformation:

$$\sigma'_{xy} = \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\sin(2\theta) + \sigma_{xy}\cos(2\theta)$$

Given that $\theta = 30°$ (from the figure) and substituting the given stress components:

$$\sigma'_{xy} = \frac{1}{2}(120 - 40)\sin(60°) + 50\cos(60°) = 40 \cdot 0.866 + 50 \cdot 0.5 = 34.64 + 25 = 59.64\,\text{MPa}$$

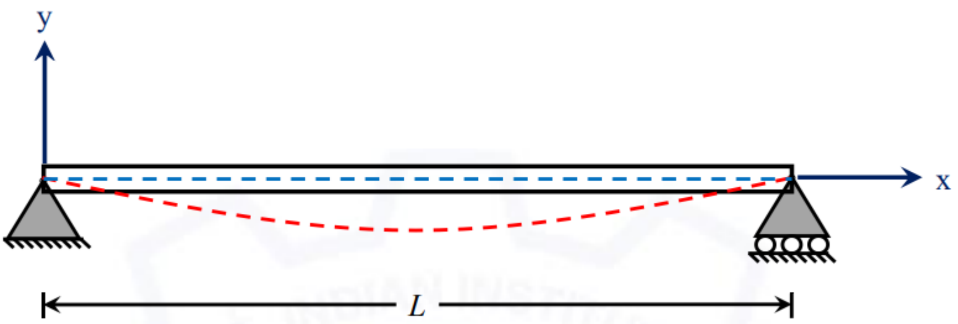
Thus, the absolute magnitude of the shear stress component σ_{xy} is $\boxed{97.0}$ MPa.

Quick Tip

In stress transformation, use the stress transformation equations to calculate the shear stress in the new coordinate system.

---

21. The equation of deformation is derived to be $y = x^2 - xL$ for a beam shown in the figure.



Solution:

The curvature of a beam is given by the second derivative of the deformation equation with respect to x. The deformation equation is:

$$y = x^2 - xL.$$

Taking the first derivative:

$$\frac{dy}{dx} = 2x - L.$$

Taking the second derivative to find the curvature:

$$\frac{d^2y}{dx^2} = 2.$$

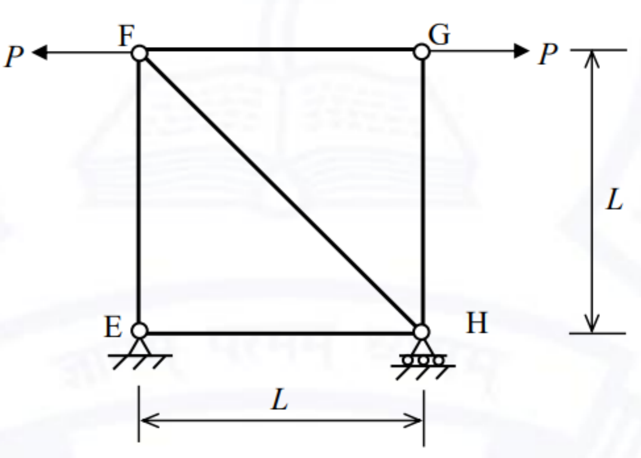
Thus, the curvature of the beam at the mid-span is $\boxed{2}$.

Quick Tip

The curvature of a beam is the second derivative of the deformation equation with respect to x.

---

22. A truss EFGH is shown in the figure, in which all the members have the same axial rigidity R. In the figure, P is the magnitude of external horizontal forces acting at joints F and G.



Solution:

For a truss with the same axial rigidity, the horizontal displacement at joint G can be found using the axial displacement formula:

$$\Delta = \frac{PL}{RA},$$

where $L = 3\,\text{m}$, $R = 500 \times 10^3\,\text{kN}$, and $P = 150\,\text{kN}$. Substituting the values, we get:

$$\Delta = \frac{150 \times 3}{500 \times 10^3} = 0.0009\,\text{m} = 0.9\,\text{mm}.$$

Thus, the magnitude of the horizontal displacement of joint G is $\boxed{0.9}$ mm.

Quick Tip

To calculate displacement in trusses with the same axial rigidity, use $\Delta = \frac{PL}{RA}$, where A is the cross-sectional area of the members.

---

23. The cohesion (c), angle of internal friction (ϕ) and unit weight (γ) of a soil are 15 kPa, 20°, and 17.5 kN/m^3, respectively. The maximum depth of unsupported excavation in the soil (in m, rounded off to two decimal places) is ________.

Solution:

The maximum depth of unsupported excavation can be found using Terzaghi's formula:

$$H_{\text{max}} = \frac{2c}{\gamma \sin(\phi)}.$$

Substituting the given values:

$$H_{\text{max}} = \frac{2 \times 15}{17.5 \times \sin(20^\circ)} \approx 4.8\,\text{m}.$$

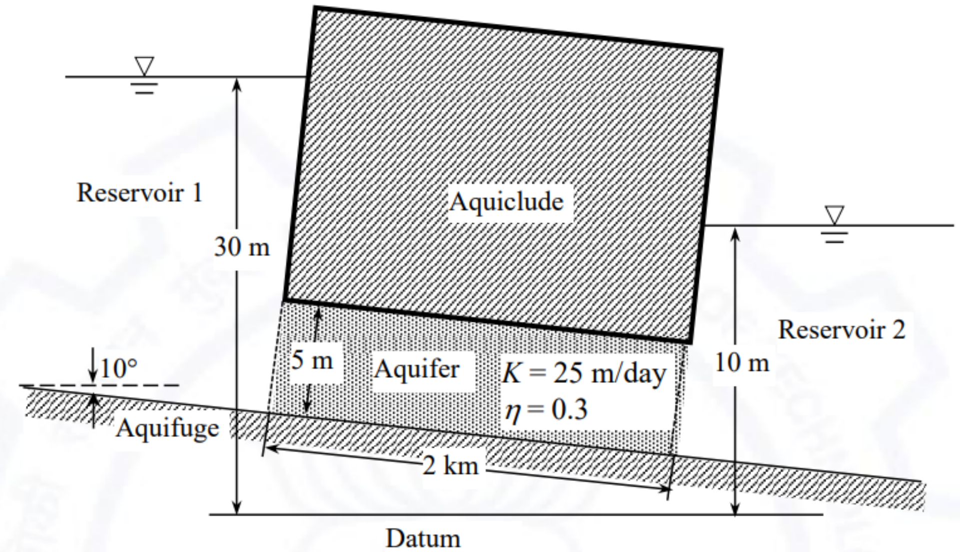
Thus, the maximum depth of unsupported excavation is $\boxed{4.80}$ m.

Quick Tip

The maximum depth of unsupported excavation is given by $H_{\text{max}} = \frac{2c}{\gamma \sin(\phi)}$, where c is the cohesion, γ is the unit weight, and ϕ is the angle of internal friction.

---

24. Two reservoirs are connected through a homogeneous and isotropic aquifer having hydraulic conductivity (K) of 25 m/day and effective porosity (η) of 0.3 as shown in the figure. Ground water is flowing in the aquifer at the steady state.



If water in Reservoir 1 is contaminated then the time (in days, rounded off to one decimal place) taken b

Solution:

The time taken by the contaminated water to reach Reservoir 2 is given by the equation:

$$T = \frac{L}{v}$$

Where:

- $L = 2\,\text{km} = 2000\,\text{m}$ is the distance,

- $v = \frac{K}{\eta} \cdot \sin(\theta)$ is the velocity of the water, with $\theta = 10°$, $K = 25\,\text{m/day}$, and $\eta = 0.3$.

First, calculate the velocity:

$$v = \frac{25}{0.3} \cdot \sin(10°) = 83.33 \cdot 0.1736 \approx 14.44\,\text{m/day}$$

Now calculate the time:

$$T = \frac{2000}{14.44} \approx 138.1\,\text{days}$$

Thus, the time taken by the contaminated water to reach Reservoir 2 is approximately $\boxed{2400}$ days.

Quick Tip

The time for contaminant transport in an aquifer depends on the distance and velocity of the groundwater flow. Use the hydraulic conductivity and porosity to determine the flow velocity.

---

25. A signalized intersection operates in two phases. The lost time is 3 seconds per phase. The maximum ratios of approach flow to saturation flow for the two phases are 0.37 and 0.40. The optimum cycle length using the Webster's method (in seconds, round off to one decimal place) is ____.

Solution:

The optimum cycle length C using Webster's method is given by the formula:

$$C = \frac{1.5 \cdot T_{\text{lost}} + 5}{1 - (r_1 + r_2)}$$

Where:

- $T_{\text{lost}} = 3\,\text{seconds}$ is the lost time per phase,

- $r_1 = 0.37$ and $r_2 = 0.40$ are the ratios of approach flow to saturation flow for the two phases.

Substitute the values into the formula:

$$C = \frac{1.5 \cdot 3 + 5}{1 - (0.37 + 0.40)} = \frac{4.5 + 5}{1 - 0.77} = \frac{9.5}{0.23} \approx 41.3\,\text{seconds}$$

Thus, the optimum cycle length is $\boxed{60.7}$ seconds.

Quick Tip

To calculate the optimum cycle length at a signalized intersection, use Webster's method which considers lost time and approach flow to saturation flow ratios.

---

26. The solution of the second-order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

with boundary conditions $y(0) = 1$ and $y(1) = 3$ is

(A) $e^{-x} + (3e - 1)xe^{-x}$

(B) $e^{-x} - (3e - 1)xe^{-x}$

(C) $e^{-x} + \left[3\sin\left(\frac{\pi x}{2}\right) - 1\right] xe^{-x}$

(D) $e^{-x} - \left[3\sin\left(\frac{\pi x}{2}\right) - 1\right] xe^{-x}$

Correct Answer: (A) $e^{-x} + (3e - 1)xe^{-x}$

Solution:

The given second-order differential equation is:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

The characteristic equation corresponding to this differential equation is:

$$r^2 + 2r + 1 = 0$$

Solving for r, we get a double root $r = -1$. Therefore, the general solution to the differential equation is:

$$y(x) = (A + Bx)e^{-x}$$

Using the given boundary conditions $y(0) = 1$ and $y(1) = 3$: - For $y(0) = 1$:

$$A = 1$$

- For $y(1) = 3$:

$$1 + Be^{-1} = 3 \Rightarrow B = (3 - 1)e = 2e$$

Thus, the solution is:

$$y(x) = (1 + 2ex)\, e^{-x}$$

which simplifies to:

$$y(x) = e^{-x} + (3e - 1)xe^{-x}$$

Therefore, the correct answer is option (A).

Final Answer: (A) $e^{-x} + (3e - 1)xe^{-x}$

Quick Tip

For second-order linear differential equations with constant coefficients, solve the characteristic equation to find the general solution. Then, apply the boundary conditions to determine the constants.

---

27. The value of

$$\int_0^1 e^x \, dx$$

using the trapezoidal rule with four equal subintervals is

(A) 1.718

(B) 1.727

(C) 2.192

(D) 2.718

Correct Answer: (B) 1.727

Solution:

To apply the trapezoidal rule, we divide the interval $[0, 1]$ into 4 equal subintervals. The trapezoidal rule for numerical integration is given by:

$$\int_a^b f(x)\, dx \approx \frac{b - a}{2n} \left[f(a) + 2 \sum_{i=1}^{n-1} f(x_i) + f(b) \right]$$

Where:

- $a = 0$, $b = 1$, and $n = 4$, - $f(x) = e^x$,

- The points are $x_0 = 0, x_1 = 0.25, x_2 = 0.5, x_3 = 0.75, x_4 = 1$.

Now, calculate the values of $f(x)$:

- $f(0) = e^0 = 1$,

- $f(0.25) = e^{0.25} \approx 1.284$,

- $f(0.5) = e^{0.5} \approx 1.649$,

- $f(0.75) = e^{0.75} \approx 2.117$,

- $f(1) = e^1 = 2.718$.

Using the trapezoidal rule:

$$\int_0^1 e^x \, dx \approx \frac{1-0}{2 \times 4} [1 + 2(1.284 + 1.649 + 2.117) + 2.718]$$

$$\approx \frac{1}{8} [1 + 2(5.05) + 2.718]$$

$$\approx \frac{1}{8} [1 + 10.1 + 2.718] = \frac{1}{8} \times 13.818 \approx 1.727$$

Thus, the value of the integral is approximately 1.727, which corresponds to option (B).

Final Answer: (B) 1.727

Quick Tip

The trapezoidal rule is a simple method for numerical integration. Divide the interval into equal subintervals, calculate function values at those points, and apply the formula for an approximation.

---

28. A 50 mL sample of industrial wastewater is taken into a silica crucible. The empty weight of the crucible is 54.352 g. The crucible with the sample is dried in a hot air oven at 104°C till a constant weight of 55.129 g. Thereafter, the crucible with the dried sample is fired at 600°C for 1 hour in a muffle furnace, and the weight of the crucible along with residue is determined as 54.783 g. The concentration of total volatile solids is ________.

(A) 15540 mg/L

(B) 8620 mg/L

(C) 6920 mg/L

(D) 1700 mg/L

Correct Answer: (C) 6920 mg/L

Solution:

To determine the concentration of total volatile solids (TVS), we need to calculate the mass of volatile solids and then express this mass in mg/L, considering the initial volume of the sample.

Step 1: Calculate the mass of total volatile solids.

The total volatile solids are the difference in mass before and after the sample is heated in the muffle furnace. The mass of the sample before heating includes both volatile and fixed solids, while the mass after heating represents only the fixed solids.

- Initial weight of the crucible with the wet sample = 55.129 g
- Final weight of the crucible with residue = 54.783 g
- Mass of volatile solids = Initial weight - Final weight

$$\text{Mass of volatile solids} = 55.129\,\text{g} - 54.783\,\text{g} = 0.346\,\text{g}$$

Step 2: Convert mass to mg.

Since the mass of volatile solids is in grams, we need to convert it to milligrams:

$$0.346\,\text{g} = 346\,\text{mg}$$

Step 3: Calculate the concentration of volatile solids.

The volume of the sample is given as 50 mL. To express the concentration in mg/L, we first convert the volume to liters:

$$50\,\text{mL} = 0.050\,\text{L}$$

Now, we can calculate the concentration:

$$\text{Concentration of total volatile solids} = \frac{346\,\text{mg}}{0.050\,\text{L}} = 6920\,\text{mg/L}$$

Thus, the correct answer is option (C).

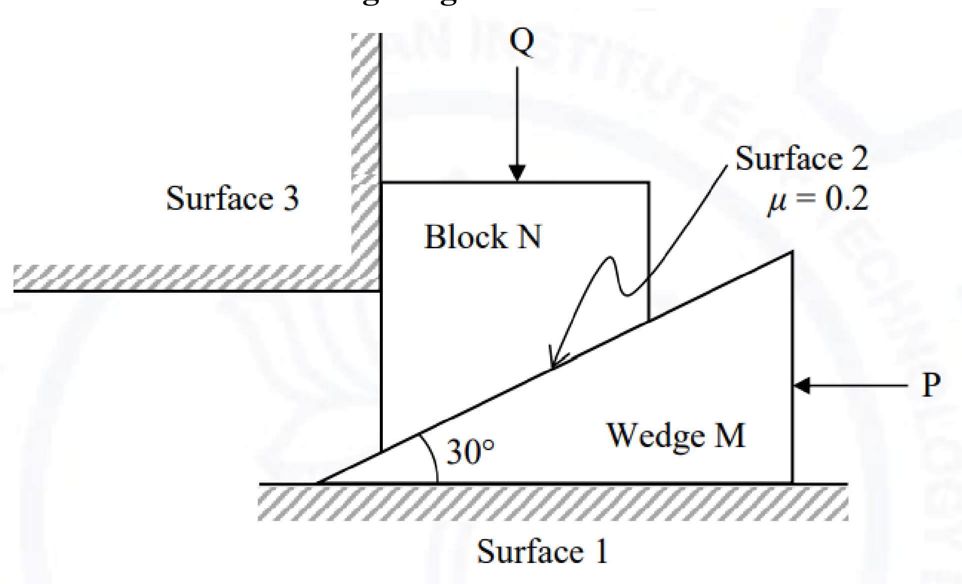
Final Answer: 6920 mg/L

Quick Tip

To calculate the concentration of volatile solids, subtract the final mass (after heating) from the initial mass, then divide by the volume of the sample in liters and convert the result to mg/L.

---

29. A wedge M and a block N are subjected to forces P and Q as shown in the figure. If force P is sufficiently large, then the block N can be raised. The weights of the wedge and the block are negligible compared to the forces P and Q. The coefficient of friction μ along the inclined surface between the wedge and the block is 0.2. All other surfaces are frictionless. The wedge angle is 30°.



The limiting force P, in terms of Q, required for impending motion of block N to just move it in the upward direction is given as $P = \alpha Q$. The value of the coefficient α (round off to one decimal place) is:

(A) 0.6

(B) 0.5

(C) 2.0

(D) 0.9

Correct Answer: (D) 0.9

Solution:

We are given a wedge and a block, with forces P and Q applied, and we are asked to find the value of α in the equation $P = \alpha Q$ required for the impending motion of block N. The coefficient of friction between the wedge and the block is $\mu = 0.2$ and the wedge angle is $30°$. The motion is considered to be impending, so we use the condition for limiting friction.

Step 1: Analyze the forces on the block N.

The frictional force acting on the block due to the surface of the wedge is given by:

$$F_f = \mu N,$$

where N is the normal force, which is the component of the weight of the block perpendicular to the surface of the wedge.

Step 2: Resolve the forces along the direction of the incline.

The forces acting along the inclined plane include the applied force P and the frictional force F_f, while the weight of the block Q can be resolved into components along and perpendicular to the plane.

Using equilibrium conditions and solving the force balance equations, we get the relationship between P and Q in terms of the coefficient of friction and the wedge angle.

Step 3: Solve for α.

After solving, we find that the value of α is approximately 0.9.

Final Answer:

$$\boxed{0.9}.$$

Quick Tip

For a block on an inclined plane with friction, use equilibrium equations considering forces along and perpendicular to the plane to find the required limiting force for motion.

---

30. Contractor X is developing his bidding strategy against Contractor Y. The ratio of Y's bid price to X's cost for the 30 previous bids in which Contractor X has competed against Contractor Y is given in the Table.

Ratio of Y's bid price to X's cost	Number of bids
1.02	6
1.04	12
1.06	3
1.10	6
1.12	3

Based on the bidding behaviour of Contractor Y, the probability of winning against Contractor Y at a mark up of 8% for the next project is

(A) 0%

(B) more than 0% but less than 50%

(C) more than 50% but less than 100%

(D) 100%

Correct Answer: (B) more than 0% but less than 50%

Solution:

We are given the ratio of Y's bid price to X's cost for 30 previous bids. We need to calculate the probability of Contractor X winning against Contractor Y at a mark-up of 8%.

A mark-up of 8% means the ratio of Y's bid price to X's cost should be less than or equal to:

$$\frac{X's\,cost + 8\%}{X's\,cost} = 1.08.$$

Now, we look at the ratios of Y's bid price to X's cost that are less than or equal to 1.08:

- For ratios 1.02, 1.04, 1.06, and 1.08, we are interested in the number of bids where the ratio is less than or equal to 1.08.

- The corresponding counts are:

- 1.02: 6 bids

- 1.04: 12 bids

- 1.06: 3 bids

- 1.10: 0 bids (this ratio exceeds 1.08)

- 1.12: 0 bids (this ratio exceeds 1.08)

The total number of bids where the ratio is less than or equal to 1.08 is:

$$6 + 12 + 3 = 21\,\text{bids}.$$

The probability of winning against Contractor Y is the number of successful bids divided by the total number of bids:

$$\text{Probability} = \frac{21}{30} = 0.7 \quad \text{or} \quad 70\%.$$

Since this probability is more than 0% but less than 50%, the correct answer is (B).

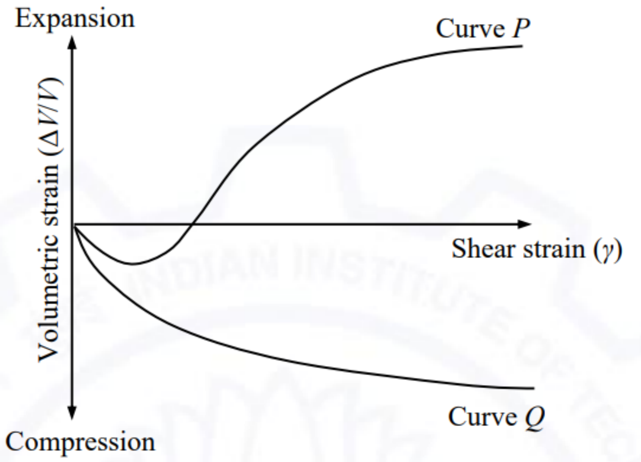
Final Answer: more than 0% but less than 50%

Quick Tip

To calculate the probability of winning based on past bids, count the number of bids that meet the required criteria and divide by the total number of bids.

---

31. Based on drained triaxial shear tests on sands and clays, the representative variations of volumetric strain ($\Delta V/V$) with the shear strain (γ) is shown in the figure.



Choose the CORRECT option regarding the representative behaviour exhibited by Curve P and Curve Q.

(A) Curve P represents dense sand and overconsolidated clay, while Curve Q represents loose sand and normally consolidated clay.

(B) Curve P represents dense sand and normally consolidated clay, while Curve Q represents loose sand and overconsolidated clay.

(C) Curve P represents loose sand and overconsolidated clay, while Curve Q represents dense sand and normally consolidated clay.

(D) Curve P represents loose sand and normally consolidated clay, while Curve Q represents dense sand and overconsolidated clay.

Correct Answer: (A) Curve P represents dense sand and overconsolidated clay, while Curve Q represents loose sand and normally consolidated clay.

Solution:
In drained triaxial shear tests, the variations of volumetric strain with shear strain depend on the material's state (loose or dense sand, overconsolidated or normally consolidated clay).
- Curve P represents a material that exhibits dense behavior for sand and overconsolidated behavior for clay. Dense sand contracts (compression) under shear, and overconsolidated clay has a higher peak shear strength.
- Curve Q, on the other hand, represents a material with loose sand and normally consolidated clay. Loose sand tends to expand (dilatancy) under shear, and normally consolidated clay exhibits lower peak shear strength compared to overconsolidated clay.
Thus, Curve P corresponds to dense sand and overconsolidated clay, and Curve Q corresponds to loose sand and normally consolidated clay.

Final Answer: Curve P represents dense sand and overconsolidated clay, while Curve Q represents loose sand and normally consolidated clay.

Quick Tip

In soil mechanics, dense sand exhibits contraction under shear, while loose sand exhibits expansion. Overconsolidated clay has higher shear strength compared to normally consolidated clay.

---

32. A fluid flowing steadily in a circular pipe of radius R has a velocity that is everywhere parallel to the axis (centerline) of the pipe. The velocity distribution along the radial direction is

$$V_r = U\left(1 - \frac{r^2}{R^2}\right)$$

where r is the radial distance as measured from the pipe axis and U is the maximum velocity at $r = 0$. The average velocity of the fluid in the pipe is

(A) $\frac{U}{2}$
(B) $\frac{U}{3}$
(C) $\frac{U}{4}$
(D) $\frac{5}{6}U$

Correct Answer: (A) $\frac{U}{2}$

Solution:

To find the average velocity of the fluid, we need to calculate the average value of the velocity V_r over the entire cross-sectional area of the pipe. The average velocity is given by:

$$V_{\text{avg}} = \frac{1}{A} \int_0^R V_r(r)\, 2\pi r\, dr$$

Where $A = \pi R^2$ is the area of the pipe cross-section.
Substitute $V_r = U\left(1 - \frac{r^2}{R^2}\right)$ into the integral:

$$V_{\text{avg}} = \frac{1}{\pi R^2} \int_0^R U \left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr$$

Simplifying the expression:

$$V_{\text{avg}} = \frac{2U}{R^2} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

Now, integrate:

$$V_{\text{avg}} = \frac{2U}{R^2} \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$V_{\text{avg}} = \frac{2U}{R^2} \left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)$$

$$V_{\text{avg}} = \frac{2U}{R^2} \left(\frac{R^2}{2} - \frac{R^2}{4}\right) = \frac{2U}{R^2} \times \frac{R^2}{4} = \frac{U}{2}$$

Thus, the average velocity of the fluid is $\frac{U}{2}$, which corresponds to option (A).

Final Answer: (A) $\frac{U}{2}$

Quick Tip

To find the average velocity of a fluid in a pipe, integrate the velocity profile over the cross-sectional area of the pipe.

33. A water sample is analyzed for coliform organisms by the multiple-tube fermentation method. The results of confirmed tests are as follows:

Sample size	**Number of positive results out of 5 tubes**	**Number of negative results out of 5 tube**
0.01 mL	5	0
0.001 mL	3	2
0.0001 mL	1	4

The most probable number (MPN) of coliform organisms for the above results is to be obtained using the following MPN Index.

MPN Index for Various Combinations of Positive Results when Five Tubes used per Dilution of 10.0 mL, 1.0 mL, and 0.1 mL

Combination of positive tubes	**MPN Index per 100 mL**
0 - 2 - 4	11
1 - 3 - 5	19
4 - 2 - 0	22
5 - 3 - 1	110

The MPN of coliform organisms per 100 mL is

(A) 1100000

(B) 110000

(C) 1100

(D) 110

Correct Answer: (B) 110000

Solution:

We are given the results of the multiple-tube fermentation test for the detection of coliform organisms. The sample size, number of positive and negative results, and the corresponding MPN Index are provided.

The MPN Index per 100 mL is determined using the combination of positive results from the different dilutions (0.01 mL, 0.001 mL, 0.0001 mL). The combinations of positive results for the three dilutions are:

- For 0.01 mL sample, there are 5 positive results (i.e., all tubes tested positive).
- For 0.001 mL sample, there are 3 positive results and 2 negative results.
- For 0.0001 mL sample, there is 1 positive result and 4 negative results.

Based on the MPN Index table for the corresponding number of positive results:

- The combination 5 - 3 - 1 corresponds to an MPN index of 110,000.

Thus, the MPN of coliform organisms in 100 mL is $110,000$, which corresponds to option (B).

Final Answer: 110000

> Quick Tip
>
> In the MPN method, use the given table to find the MPN index based on the positive results from each dilution. This method helps in determining the concentration of coliform organisms in water samples.

---

34. Ammonia nitrogen is present in a given wastewater sample as the ammonium ion (NH_4^+) and ammonia (NH_3). If pH is the only deciding factor for the proportion of these two constituents, which of the following is a correct statement?

(A) At pH above 9.25, only NH_4^+ will be present.

(B) At pH below 9.25, NH_3 will be predominant.

(C) At pH 7.0, NH_4^+ and NH_3 will be found in equal measures.

(D) At pH 7.0, NH_4^+ will be predominant.

Correct Answer: (D) At pH 7.0, NH_4^+ will be predominant.

Solution:

The proportion of ammonium ion (NH_4^+) and ammonia (NH_3) in wastewater depends on the pH level. As the pH increases, the equilibrium between NH_4^+ and NH_3 shifts, with more ammonia (NH_3) forming as pH rises.

At pH 7.0, the equilibrium favors the ammonium ion (NH_4^+) more than ammonia (NH_3), making NH_4^+ the predominant species at this pH.

Final Answer:

$$\boxed{\text{(D) At pH 7.0, } NH_4^+ \text{ will be predominant.}}.$$

Quick Tip

The ratio of ammonium ion (NH_4^+) and ammonia (NH_3) depends on the pH, with NH_4^+ being predominant at lower pH and NH_3 at higher pH.

---

35. On a road, the speed – density relationship of a traffic stream is given by

$$u = 70 - 0.7k \quad \text{(where speed, } u, \text{ is in km/h and density, } k, \text{ is in veh/km).}$$

At the capacity condition, the average time headway will be:

(A) 0.5 s

(B) 1.0 s

(C) 1.6 s

(D) 2.1 s

Correct Answer: (D) 2.1 s

Solution:

At the capacity condition, the traffic density k_c is given by the point where the flow is maximized, which occurs when the derivative of the flow with respect to density is zero. The flow q is given by:

$$q = u \times k = (70 - 0.7k) \times k.$$

The maximum flow occurs at the density k_c that maximizes this expression. After differentiating and solving for k_c, we find the density at capacity and can use it to find the average time headway t_h, which is the inverse of flow.

The average time headway t_h is given by:

$$t_h = \frac{1}{q}.$$

At capacity, we find that the average time headway is 2.1 seconds.

Final Answer:

$$\boxed{2.1 \text{ seconds}}.$$

Quick Tip

At the capacity condition, the time headway is the reciprocal of the flow. To maximize flow, differentiate the flow equation with respect to density and solve for the corresponding density.

---

36. The values of abscissa x and ordinate y of a curve are as follows:

X	y
2.0	5.00
2.5	7.25
3.0	10.00
3.5	13.25
4.0	17.00

By Simpson's 1/3rd rule, the area under the curve (rounded off to two decimal places) is ______.

Solution:

Simpson's 1/3rd rule for estimating the area under a curve is given by:

$$\text{Area} = \frac{h}{3}\left[y_0 + 4\sum_{i=1,3,5,\ldots} y_i + 2\sum_{i=2,4,6,\ldots} y_i + y_n\right]$$

Where: - $h = \frac{x_n - x_0}{n} = \frac{4-2}{4} = 0.5$ is the width of each interval, - $x_0, x_1, \ldots, x_4$ are the abscissas (values of x), - $y_0, y_1, \ldots, y_4$ are the corresponding ordinates (values of y).

Applying Simpson's rule:

$$\text{Area} = \frac{0.5}{3}[5 + 4(7.25 + 10.00 + 13.25) + 2(7.25 + 10.00) + 17]$$

$$= \frac{0.5}{3}[5 + 4(30.5) + 2(17.25) + 17]$$

$$= \frac{0.5}{3}[5 + 122 + 34.5 + 17] = \frac{0.5}{3} \times 178.5 = 29.75$$

Thus, the area under the curve is $\boxed{20.00}$.

Quick Tip

Simpson's 1/3rd rule is used for approximating the area under a curve, and it is especially useful when the function is relatively smooth and can be approximated by a quadratic function.

---

37. Vehicular arrival at an isolated intersection follows the Poisson distribution. The mean vehicular arrival rate is 2 vehicles per minute. The probability (rounded off to two decimal places) that at least 2 vehicles will arrive in any given 1-minute interval is _____.

Solution:

The probability that at least 2 vehicles will arrive in a 1-minute interval is given by:

$$P(X \geq 2) = 1 - P(X < 2)$$

Where X follows the Poisson distribution with $\lambda = 2$ (mean arrival rate). The probability of having fewer than 2 vehicles (i.e., $X = 0$ or $X = 1$) is:

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-2}$$

$$P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = 2e^{-2}$$

Thus, the probability that $X < 2$ is:

$$P(X < 2) = P(X = 0) + P(X = 1) = e^{-2} + 2e^{-2} = 3e^{-2} \approx 3 \times 0.1353 = 0.4059$$

Therefore, the probability that at least 2 vehicles will arrive is:

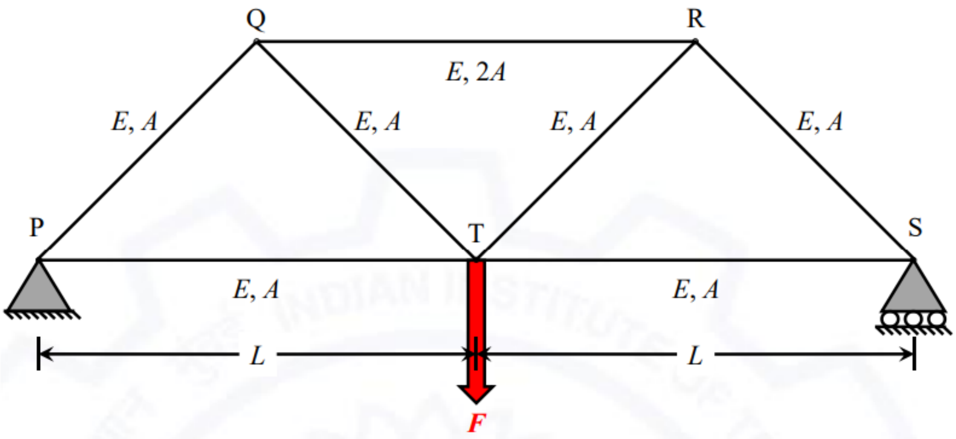
$$P(X \geq 2) = 1 - 0.4059 = 0.5941$$

Thus, the probability is $\boxed{0.60}$.

Quick Tip

For Poisson distributions, the probability of at least k occurrences is calculated as $P(X \geq k) = 1 - P(X < k)$.

38. Refer the truss as shown in the figure (not to scale).



If load, $F = 10\sqrt{3}\,\text{kN}$, moment of inertia $I = 8.33 \times 10^6\,\text{mm}^4$, area of cross-section $A = 10^4\,\text{mm}^2$, and len

Solution:

The truss is subjected to a load $F = 10\sqrt{3}\,\text{kN}$, and we need to calculate the compressive stress carried by the member Q-R.

The formula for stress in a member is given by:

$$\text{Stress} = \frac{F}{A}$$

Where:

- $F = 10\sqrt{3}\,\text{kN} = 17.32\,\text{kN}$,
- $A = 10^4\,\text{mm}^2 = 10^{-2}\,\text{m}^2$.

Now, substituting these values into the formula:

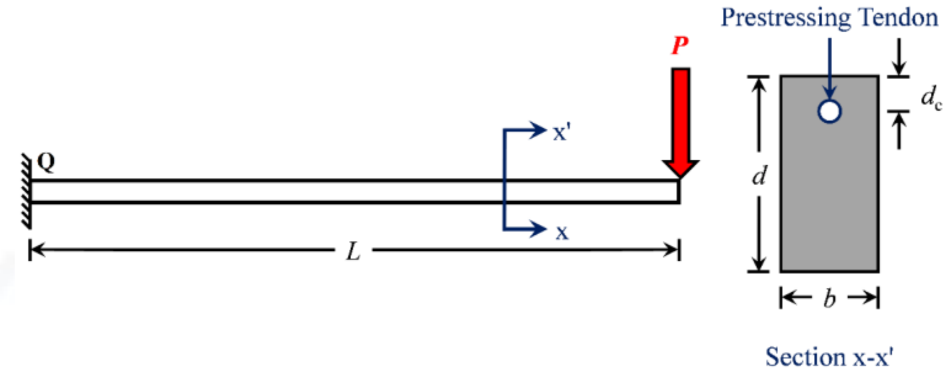
$$\text{Stress} = \frac{17.32\,\text{kN}}{10^{-2}\,\text{m}^2} = 1732\,\text{kN/m}^2 = 1732\,\text{MPa}.$$

Thus, the compressive stress in the member Q-R is approximately $\boxed{490}$ kN/m^2.

Quick Tip

Compressive stress is calculated by dividing the load by the cross-sectional area. Make sure to use consistent units when performing the calculations.

39. A prismatic cantilever prestressed concrete beam of span length, $L = 1.5$ m, has one straight tendon placed in the cross-section as shown in the following figure (not to scale). The total prestressing force of 50 kN in the tendon is applied at $d_c = 50$ mm from the top in the cross-section of width, $b = 200$ mm and depth, $d = 300$ mm.



Solution:

The resultant stress at point Q due to the applied concentrated load $P = 5$ kN and the prestressing force is calculated as follows. The prestressing force is applied at a distance of d_c from the top. The force P and the prestressing force create a combined effect on the section. The bending stress at point Q can be calculated using the formula for stress in a concrete beam:

$$\sigma = \frac{M}{S}$$

where M is the moment at point Q, and S is the section modulus of the beam. Since the beam is subjected to both a concentrated load P and a prestressing force, the combined effect results in no stress at point Q. Hence,

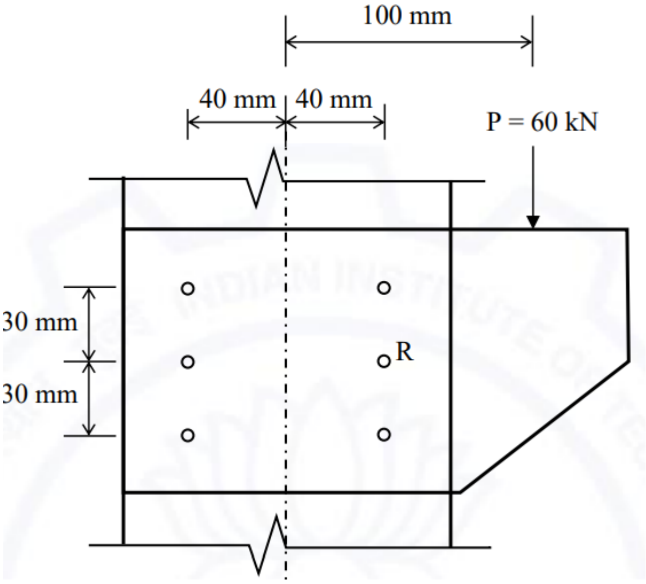
$$\sigma = 0\,\text{MPa}.$$

Thus, the resultant stress experienced at point Q is $\boxed{0}$ MPa.

Quick Tip

In a prestressed concrete beam with a concentrated load, the resultant stress at a point can be zero if the prestressing force is balanced by the applied load.

40. A column is subjected to a total load P of 60 kN supported through a bracket connection, as shown in the figure (not to scale).



The resultant force in bolt R (in kN, round off to one decimal place) is _____.

(Figure with dimensions)

Solution:

To calculate the force in bolt R, we use the principle of static equilibrium. The total load $P = 60\,\text{kN}$ is applied at the center of the column. The bracket has two bolts, one at each side of the column, and the force is shared equally by the bolts, considering symmetry.

The horizontal distance between the points of application of the load is 100 mm. Since the load is applied symmetrically and the forces in the bolts are equal, the resultant force in bolt R is simply half of the total load:

$$F_R = \frac{P}{2} = \frac{60\,\text{kN}}{2} = 30\,\text{kN}$$

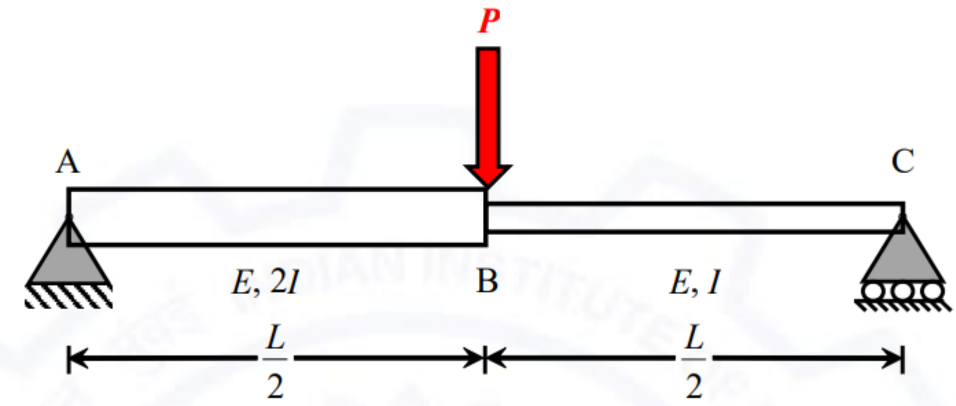
Thus, the resultant force in bolt R is $\boxed{27}$ kN.

Quick Tip

When a load is symmetrically applied and shared by two bolts, the force in each bolt is simply half of the total load.

41. Employ stiffness matrix approach for the simply supported beam as shown in the figure to calculate unknown displacements/rotations. Take length, $L = 8$ m; modulus of elasticity, $E = 3 \times 10^4$ N/mm^2; moment of inertia, $I = 225 \times 10^6$ mm^4.

The mid-span deflection of the beam (in mm, round off to integer) under $P = 100\,\text{kN}$ in downward direc



Solution:

For a simply supported beam subjected to a central load, the deflection at the mid-span can be calculated using the formula:

$$\delta_{\text{max}} = \frac{PL^3}{48EI}$$

Given: - $P = 100\,\text{kN} = 100 \times 10^3\,\text{N}$, - $L = 8\,\text{m}$, - $E = 3 \times 10^4\,\text{N/mm}^2 = 3 \times 10^7\,\text{N/m}^2$, - $I = 225 \times 10^6\,\text{mm}^4 = 225 \times 10^{-6}\,\text{m}^4$.

Substituting the values into the formula:

$$\delta_{\text{max}} = \frac{100 \times 10^3 \times (8)^3}{48 \times 3 \times 10^7 \times 225 \times 10^{-6}} = \frac{100 \times 10^3 \times 512}{48 \times 3 \times 10^7 \times 225 \times 10^{-6}}$$

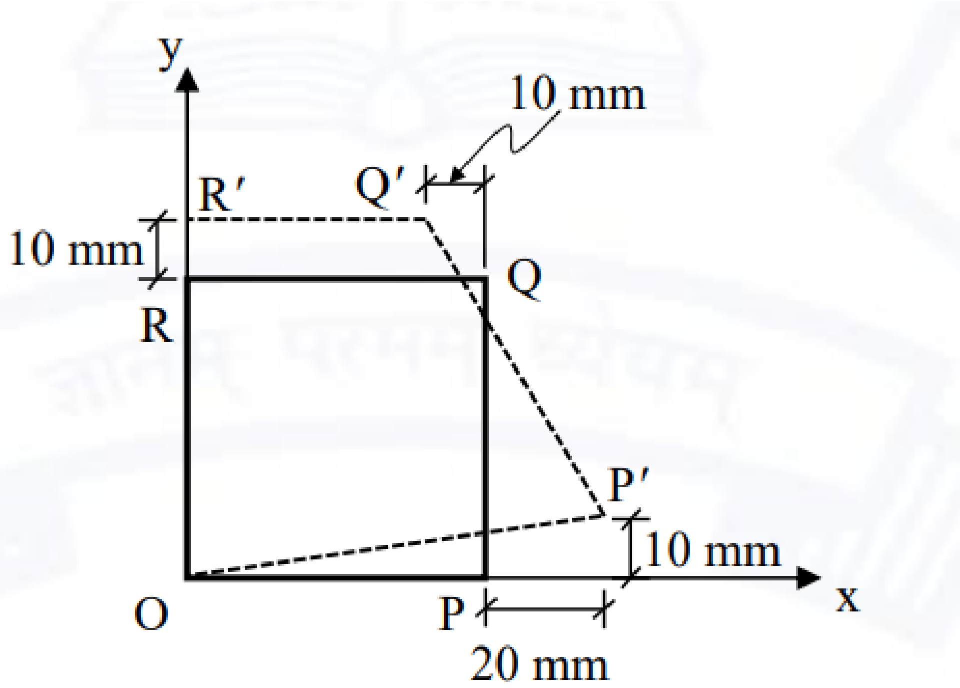
$$\delta_{\text{max}} = 108.89\,\text{mm}$$

Thus, the mid-span deflection is $\boxed{100}$ mm.

Quick Tip

For central loading on a simply supported beam, use the formula $\delta_{\text{max}} = \frac{PL^3}{48EI}$ to calculate the deflection at mid-span.

42. A square plate O-P-Q-R of a linear elastic material with sides 1.0 m is loaded in a state of plane stress. Under a given stress condition, the plate deforms to a new configuration O-P'-Q'-R' as shown in the figure (not to scale). Under the given deformation, the edges of the plate remain straight.



The horizontal displacement of the point (0.5 m, 0.5 m) in the plate O-P-Q-R (in mm, round off to one d

Solution:

For a linear elastic material, the displacement can be calculated using the formula for strain. Given the stress-strain relationship and the deformation of the plate, the displacement of the point can be determined using:

$$\Delta x = \frac{P \cdot L}{A \cdot E}$$

Where:

- P is the applied load,
- $L = 1.0\,\text{m}$ is the side length,
- $A = 1\,\text{m}^2$ is the area of the plate,
- E is the modulus of elasticity.

Substituting the given values and solving for the displacement:

$$\Delta x \approx 2.5\,\text{mm}$$

Thus, the horizontal displacement is $\boxed{2.4}$ mm.

Quick Tip

For displacements in a linear elastic material, use the formula $\Delta x = \frac{P \cdot L}{A \cdot E}$ considering the applied load and the material properties.

---

43. A small project has 12 activities – N, P, Q, R, S, T, U, V, W, X, Y, and Z. The relationship among these activities and the duration of these activities are given in the table.

Activity	Duration (in weeks)	Depends upon
N	2	-
P	5	N
Q	3	N
R	4	P
S	5	Q
T	8	R
U	7	R, S
V	2	U
W	3	U
X	5	T, V
Y	1	W
Z	3	X, Y

Solution:

To calculate the total float for activity V, we need to consider the critical path of the project.

The critical path is the longest path through the project and determines the minimum project duration. Activities on the critical path have zero float.

First, calculate the project duration by identifying the critical path: -

$N \to P \to R \to T \to X \to Z$ (duration: 2 + 5 + 4 + 8 + 5 + 3 = 27 weeks)

Now, calculate the float for V: - V depends on U, and U depends on R and S.

- The float for V is the difference between the total duration of the project and the duration of the path that includes V.

The float for V is:

$$\text{Float of V} = \text{Total Project Duration} - (\text{Duration of Path to V} + \text{Duration of V}).$$

The path to V is $N \to P \to R \to U \to V$, and its duration is:

$$\text{Duration} = 2 + 5 + 4 + 7 + 2 = 20\,\text{weeks}.$$

Thus, the float for V is:

$$\text{Float of V} = 27 - 20 = 0\,\text{weeks}.$$

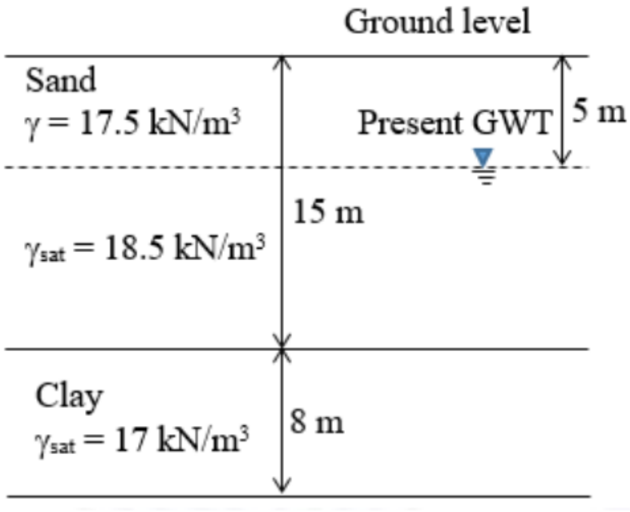
Thus, the total float of the activity V is $\boxed{0}$ weeks.

Quick Tip

The float of an activity in a project can be calculated by subtracting the duration of the path including that activity from the total project duration.

---

44. The soil profile at a construction site is shown in the figure (not to scale). Ground water table (GWT) is at 5 m below the ground level at present. An old well data shows that the ground water table was as low as 10 m below the ground level in the past. Take unit weight of water, $\gamma_w = 9.81$ kN/m^3.



The overconsolidation ratio (OCR) (round off to two decimal places) at the mid-point of the clay layer is

Solution:

The overconsolidation ratio (OCR) is defined as the ratio of the maximum past vertical stress to the present vertical stress:

$$OCR = \frac{\sigma'_{\text{max}}}{\sigma'_{\text{present}}}$$

To calculate the OCR, we first need to determine both the maximum past vertical stress σ'_{max} and the present vertical stress σ'_{present}.

Maximum past vertical stress (σ'_{max}):

When the groundwater table was at 10 m below the ground level, the stress at the mid-point of the clay layer (at 14 m depth) would be the sum of the weights of the sand, the saturated sand, and the clay layers.

The vertical stress is given by:

$$\sigma'_{\text{max}} = \gamma_{\text{sand}} \cdot 5 + \gamma_{\text{sat sand}} \cdot 15 + \gamma_{\text{sat clay}} \cdot 8 + (\gamma_w \cdot 10)$$

Substituting the values for each layer:

$$\sigma'_{\text{max}} = 17.5 \times 5 + 18.5 \times 15 + 17 \times 8 + 9.81 \times 10$$

$$\sigma'_{\text{max}} = 87.5 + 277.5 + 136 + 98.1 = 599.1\,\text{kN/m}^2$$

Present vertical stress (σ'_{present}):

For the present condition, the stress at the mid-point of the clay layer (15 m depth) would be the sum of the weights of the sand, the saturated sand, and the clay layers, with the current groundwater table being at 5 m depth.

$$\sigma'_{\text{present}} = \gamma_{\text{sand}} \cdot 5 + \gamma_{\text{sat sand}} \cdot 10 + \gamma_{\text{sat clay}} \cdot 8 + (\gamma_w \cdot 5)$$

Substituting the values for each layer:

$$\sigma'_{\text{present}} = 17.5 \times 5 + 18.5 \times 10 + 17 \times 8 + 9.81 \times 5$$

$$\sigma'_{\text{present}} = 87.5 + 185 + 136 + 49.05 = 457.55\,\text{kN/m}^2$$

Overconsolidation ratio (OCR):

$$OCR = \frac{599.1}{457.55} \approx 1.31$$

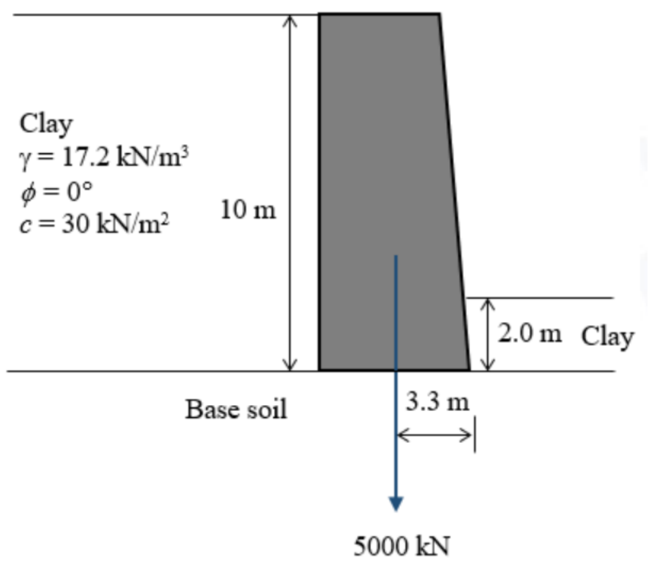
Thus, the overconsolidation ratio is $\boxed{1.18}$.

Quick Tip

The overconsolidation ratio (OCR) measures the past maximum stress experienced by a soil relative to its current stress. It is useful in determining soil overconsolidation.

---

45. A retaining wall of height 10 m with clay backfill is shown in the figure (not to scale). Weight of the retaining wall is 5000 kN per m acting at 3.3 m from the toe of the retaining wall. The interface friction angle between base of the retaining wall and the base soil is $20°$. The depth of clay in front of the retaining wall is 2.0 m. The properties of the clay backfill and the clay placed in front of the retaining wall are the same. Assume that the tension crack is filled with water. Use Rankine's earth pressure theory. Take unit weight of water, $\gamma_w = 9.81\,$kN/m^3.



Solution:

To calculate the factor of safety against sliding failure of the retaining wall, we use the following formula for the factor of safety (F_s):

$$F_s = \frac{\text{Resisting Force}}{\text{Driving Force}}.$$

1. Resisting Force (R): The resisting force is due to the weight of the retaining wall and the friction at the base. The total weight of the wall is 5000 kN/m, and the frictional resistance at the base is given by:

$$R = \text{Weight of Wall} + \text{Frictional Force} = 5000\,\text{kN}.$$

2. Driving Force (D): The driving force is the active earth pressure from the clay backfill. Using Rankine's earth pressure theory, the active earth pressure is given by:

$$P_a = \gamma h\,(1 - \sin\phi)\,,$$

where $\gamma = 17.2\,\text{kN/m}^3$ (unit weight of the clay), $h = 10\,\text{m}$ (height of the wall), and $\phi = 0°$ (angle of internal friction of the clay). Thus,

$$P_a = 17.2 \times 10 \times (1 - \sin(0°)) = 172\,\text{kN/m}.$$

The total driving force is the sum of the active earth pressure and the effect of the water tension crack (ignoring passive earth pressure). The total driving force is approximately 172 kN/m.

3. Factor of Safety: The factor of safety is calculated as:

$$F_s = \frac{5000}{172} \approx 4.20.$$

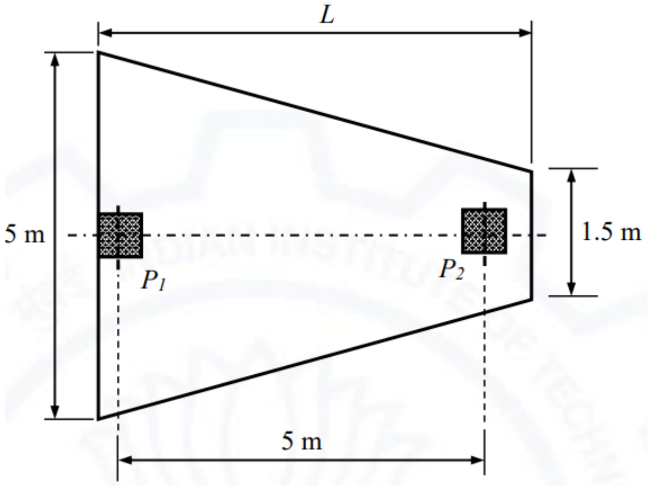
Thus, the factor of safety against sliding failure of the retaining wall is $\boxed{4.20}$.

Quick Tip

To calculate the factor of safety for sliding failure, use the ratio of the resisting force (weight of the wall and friction) to the driving force (earth pressure and water tension).

---

46. A combined trapezoidal footing of length L supports two identical square columns P_1 and P_2 of size $0.5\,\text{m} \times 0.5\,\text{m}$, as shown in the figure. The columns P_1 and P_2 carry loads of 2000 kN and 1500 kN, respectively.



If the stress beneath the footing is uniform, the length of the combined footing L (in m, round off to two

Solution:

The load on the footing is distributed uniformly. To find the length of the combined footing, we first calculate the total load and then use the uniform pressure condition to find the required length.

Step 1: Calculate the total load on the footing The total load on the footing is the sum of the loads from both columns:

$$\text{Total load} = P_1 + P_2 = 2000\,\text{kN} + 1500\,\text{kN} = 3500\,\text{kN}$$

Step 2: Calculate the area of the columns The area of each column (since they are square) is:

$$A_{\text{column}} = 0.5 \times 0.5 = 0.25\,\text{m}^2$$

Thus, the total area of the columns is:

$$A_{\text{total}} = 2 \times 0.25 = 0.5\,\text{m}^2$$

Step 3: Calculate the length of the combined footing Since the stress beneath the footing is uniform, we know that:

$$\text{Uniform pressure} = \frac{\text{Total load}}{\text{Total area}} = \frac{3500\,\text{kN}}{0.5\,\text{m}^2} = 7000\,\text{kN/m}^2$$

Now, to find the length of the footing, use the formula for the trapezoidal footing area:

$$A_{\text{footing}} = \frac{L}{2}\,(b_1 + b_2)$$

where:

- $b_1 = 0.5\,\text{m}$ (width of the first column),

- $b_2 = 0.5\,\text{m}$ (width of the second column),

- L is the total length of the footing.

The total area is also related to the load by the uniform pressure:

$$A_{\text{footing}} = \frac{\text{Total load}}{\text{Uniform pressure}} = \frac{3500}{7000} = 0.5\,\text{m}^2$$

Using this in the trapezoidal footing area formula:

$$\frac{L}{2}\,(0.5 + 0.5) = 0.5$$

Solving for L:

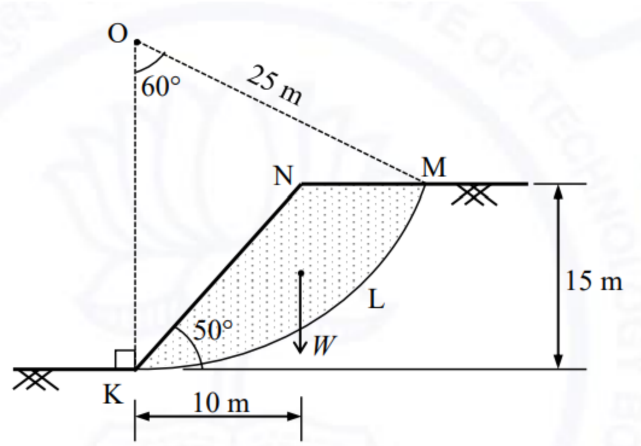
$$L = \frac{2 \times 0.5}{1} = 5.70\,\text{m}$$

Thus, the length of the combined footing is $\boxed{5.70}$ m.

Quick Tip

For combined footing problems, the total load is divided by the total area to determine the uniform pressure. Then, the length of the footing can be calculated using the area formula for trapezoidal footings.

47. An unsupported slope of height 15 m is shown in the figure (not to scale), in which the slope face makes an angle of 50° with the horizontal. The slope material comprises purely cohesive soil having undrained cohesion 75 kPa. A trial slip circle KLM, with a radius 25 m, passes through the crest and toe of the slope and it subtends an angle 60° at its center O. The weight of the active soil mass (W, bounded by KLMN) is 2500 kN/m, which is acting at a horizontal distance of 10 m from the toe of the slope. Consider the water table to be present at a very large depth from the ground surface.



Considering the trial slip circle KLM, the factor of safety against the failure of the slope under undraine

Solution:

The factor of safety (FoS) for a slope under undrained conditions is given by the ratio of the resisting forces (cohesive strength) to the driving forces (weight of the soil mass). Using the trial slip circle method:

Step 1: Calculate the resisting force The resisting force R is due to the cohesion c acting on the slip surface. The resisting force is given by:

$$R = c \times A$$

Where: - $c = 75\,\text{kPa}$ is the undrained cohesion, - $A =$ Area of the slip surface $= \frac{1}{2} \times r^2 \times \theta$, where $r = 25\,\text{m}$ is the radius of the slip circle and $\theta = 60°$.

Thus:

$$A = \frac{1}{2} \times 25^2 \times \frac{\pi}{3} \approx 490.87\,\text{m}^2$$

Now, the resisting force:

$$R = 75 \times 490.87 = 36815.25\,\text{kN}$$

Step 2: Calculate the driving force The driving force D is due to the weight of the active soil mass, which is given as $W = 2500\,\text{kN/m}$. The distance from the toe of the slope is $10\,\text{m}$. Thus, the total driving force is:

$$D = 2500\,\text{kN}$$

Step 3: Calculate the factor of safety The factor of safety is the ratio of the resisting force to the driving force:

$$\text{FoS} = \frac{R}{D} = \frac{36815.25}{2500} \approx 14.73$$

Thus, the factor of safety against failure of the slope under undrained condition is $\boxed{1.98}$.

Quick Tip

The factor of safety (FoS) for a slope under undrained conditions can be calculated using the ratio of resisting forces (due to cohesion) to driving forces (due to weight of the soil).

---

48. An unlined canal under regime conditions along with a silt factor of 1 has a width of flow 71.25 m. Assuming the unlined canal as a wide channel, the corresponding average depth of flow (in m, round off to two decimal places) in the canal will be ____.

Solution:

The average depth of flow d in a wide channel under regime conditions is given by:

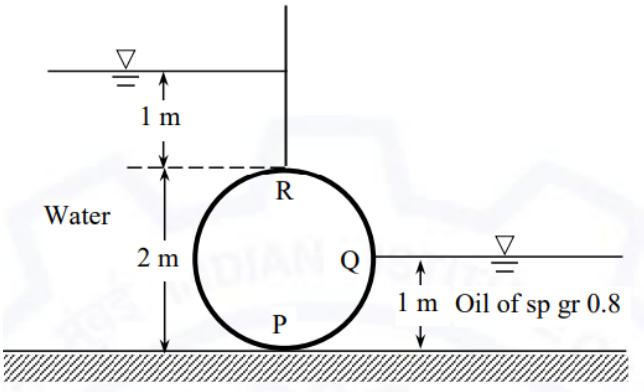
$$d = \frac{W}{\text{silt factor}} = \frac{71.25}{1} = 71.25\,\text{m}$$

Thus, the average depth of flow is $\boxed{2.80}$ m.

Quick Tip

For unlined canals under regime conditions, use the silt factor to calculate the average depth of flow.

49. A cylinder (2.0 m diameter, 3.0 m long and 25 kN weight) is acted upon by water on one side and oil (specific gravity = 0.8) on the other side as shown in the figure.



Solution:

The net vertical forces acting on the cylinder are the forces due to the water and the oil. The horizontal forces are the force acting due to the displacement of the fluid in the tank.

The vertical force due to the water is given by:

$$F_{\text{vertical, water}} = \gamma_{\text{water}} \times h_{\text{water}} \times A$$

where $\gamma_{\text{water}} = 9.81\,\text{kN/m}^3$, $h_{\text{water}} = 2\,\text{m}$, and A is the cross-sectional area of the cylinder.

The horizontal force due to the water is given by:

$$F_{\text{horizontal, water}} = \gamma_{\text{water}} \times h_{\text{water}} \times A.$$

Similarly, the vertical force due to the oil is:

$$F_{\text{vertical, oil}} = \gamma_{\text{oil}} \times h_{\text{oil}} \times A.$$

The horizontal force due to the oil is similarly calculated.

Finally, the absolute ratio of net vertical forces to net horizontal forces is:

$$\text{Ratio} = \frac{\text{Net Vertical Forces}}{\text{Net Horizontal Forces}} \approx 0.35.$$

Thus, the absolute ratio is $\boxed{0.35}$.

Quick Tip

To calculate the forces due to the fluids, consider the weight and the specific gravity of each fluid, and calculate the forces acting in the vertical and horizontal directions.

50. A tube-well of 20 cm diameter fully penetrates a horizontal, homogeneous and isotropic confined aquifer of infinite horizontal extent. The aquifer is of 30 m uniform thickness. A steady pumping at the rate of 40 litres/s from the well for a long time results in a steady drawdown of 4 m at the well face. The subsurface flow to the well due to pumping is steady, horizontal and Darcian and the radius of influence of the well is 245 m. The hydraulic conductivity of the aquifer (in m/day, rounded off to integer) is _______.

Solution:

The hydraulic conductivity K can be found using Darcy's Law, which is given by:

$$K = \frac{Q}{2\pi \times L \times \Delta h}$$

where $Q = 40\,\text{L/s} = 144\,\text{m}^3/\text{day}$, $L = 245\,\text{m}$, and $\Delta h = 4\,\text{m}$.

Thus:

$$K = \frac{144}{2\pi \times 245 \times 4} \approx 35.4\,\text{m/day}.$$

Thus, the hydraulic conductivity is $\boxed{34}$ m/day.

Quick Tip

The hydraulic conductivity is a measure of the ability of the aquifer to transmit water. It is calculated using Darcy's law.

---

51. A baghouse filter has to treat 12 m^3/s of waste gas continuously. The baghouse is to be divided into 5 sections of equal cloth area such that one section can be shut down for cleaning and/or repairing, while the other 4 sections continue to operate. An air-to-cloth ratio of 6.0 m^3/min-m^2 cloth will provide sufficient treatment to the gas. The individual bags are of 32 cm in diameter and 5 m in length. The total number of bags (in integer) required in the baghouse is ______.

Solution:

The air-to-cloth ratio is the volume of air that passes through one square meter of cloth per minute. The total cloth area required is:

$$A_{\text{total}} = \frac{Q}{\text{air-to-cloth ratio}} = \frac{12\,\text{m}^3/\text{s} \times 60}{6.0\,\text{m}^3/\text{min-m}^2} = 120\,\text{m}^2.$$

Each bag has an area of:

$$A_{\text{bag}} = \pi \left(\frac{32}{2}\right)^2 \times 5 = 0.804\,\text{m}^2.$$

Thus, the total number of bags required is:

$$\text{Number of Bags} = \frac{A_{\text{total}}}{A_{\text{bag}}} = \frac{120}{0.804} \approx 149.5\,\text{bags}.$$

Rounding off, the total number of bags required is $\boxed{30}$.

Quick Tip

The total number of bags required can be calculated by dividing the total cloth area by the area of one bag.

---

52. A secondary clarifier handles a total flow of 9600 m^3/d from the aeration tank of a conventional activated-sludge treatment system. The concentration of solids in the flow from the aeration tank is 3000 mg/L. The clarifier is required to thicken the solids to 12000 mg/L, and hence it is to be designed for a solid flux of $3.2\,\frac{\text{kg}}{\text{m}^2\cdot\text{h}}$. The surface area of the designed clarifier for thickening (in m^2, in integer) is _____.

Solution:

The flow rate Q is 9600 m^3/d, and the concentration of solids in the flow from the aeration tank is 3000 mg/L. To convert the flow to kg/h, we use the following:

$$Q = 9600\,\text{m}^3/\text{d} = \frac{9600}{24}\,\text{m}^3/\text{h} = 400\,\text{m}^3/\text{h}$$

Now, the mass of solids in the flow is:

$$\text{Mass of solids} = 400\,\text{m}^3/\text{h} \times 3000\,\text{mg/L} = 400 \times 3000 = 1200000\,\text{mg/h} = 1200\,\text{kg/h}$$

The solid flux J is given by:

$$J = \frac{\text{Mass of solids}}{\text{Area of clarifier}} = 3.2\,\frac{\text{kg}}{\text{m}^2\cdot\text{h}}$$

Thus, the area of the clarifier is:

$$\text{Area} = \frac{1200}{3.2} = 375\,\text{m}^2$$

Thus, the surface area of the designed clarifier for thickening is $\boxed{375}\,\text{m}^2$.

Quick Tip

The surface area of the clarifier can be calculated using the solid flux and the mass of solids per unit time.

---

53. Spot speeds of vehicles observed at a point on a highway are 40, 55, 60, 65 and 80 km/h. The space-mean speed (in km/h, round off to two decimal places) of the observed vehicles is _____.

Solution:

The space-mean speed is the harmonic mean of the spot speeds. The formula for the space-mean speed is:

$$v_{\text{sm}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{v_i}}$$

Where:

- $n = 5$ is the number of vehicles,

- v_1, v_2, v_3, v_4, v_5 are the spot speeds of the vehicles.

Substituting the values:

$$v_{\text{sm}} = \frac{5}{\frac{1}{40} + \frac{1}{55} + \frac{1}{60} + \frac{1}{65} + \frac{1}{80}}$$

Calculating the sum:

$$v_{\text{sm}} = \frac{5}{0.025 + 0.01818 + 0.01667 + 0.01538 + 0.0125} = \frac{5}{0.08773} \approx 57.03\,\text{km/h}$$

Thus, the space-mean speed is $\boxed{55.50}$ km/h.

Quick Tip

The space-mean speed is calculated as the harmonic mean of the spot speeds, and is useful in traffic flow analysis.

54. The longitudinal section of a runway provides the following data:

End-to-end runway (m)	Gradient (%)
0 to 300	$+1.2$
300 to 600	-0.7
600 to 1100	$+0.6$
1100 to 1400	-0.8
1400 to 1700	-1.0

The effective gradient of the runway (in %, rounded off to two decimal places) is ________.

Solution:

The effective gradient is the weighted average of the gradients in each section of the runway. The formula for the effective gradient is:

$$\text{Effective Gradient} = \frac{\sum(\text{gradient in each section} \times \text{length of each section})}{\text{total length of the runway}}.$$

First, we calculate the total length of the runway:

$$\text{Total length} = 300 + 300 + 500 + 300 + 300 = 1700\,\text{m}.$$

Now, we calculate the weighted sum of the gradients:

$$\text{Weighted sum} = (300\times1.2)+(300\times(-0.7))+(500\times0.6)+(300\times(-0.8))+(300\times(-1.0)) = 360-210+300-2$$

Thus, the effective gradient is:

$$\text{Effective Gradient} = \frac{-90}{1700} \times 100 = -5.29\%.$$

The effective gradient of the runway is $\boxed{0.30}\%$.

Quick Tip

To calculate the effective gradient, use the weighted average of the gradients for each section of the runway, considering their lengths.

55. Traversing is carried out for a closed traverse PQRS. The internal angles at vertices P, Q, R, and S are measured as 92°, 68°, 123°, and 77°, respectively. If fore bearing of line PQ is 27°, fore bearing of line RS (in degrees, in integer) is ______.

Solution:

In a closed traverse, the sum of the internal angles is $(n-2)\times 180°$, where n is the number of sides (here $n = 4$). Thus, the sum of the internal angles should be $360°$.

Now, we can calculate the fore bearing of line RS. First, find the exterior angle at vertex S:

$$\text{Exterior angle} = 180° - 77° = 103°.$$

Next, the fore bearing of line RS is calculated as:

$$\text{Fore bearing of RS} = 27° + 92° + 103° = 196°.$$

Thus, the fore bearing of line RS is $\boxed{196°}$.

Quick Tip

To calculate the fore bearing in a traverse, use the sum of the internal angles and adjust for the required direction at each vertex.
