GATE 2021 Engineering Sciences (XE) Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

Atmospheric and Oceanic Sciences (XE-H)

1. Western Boundary Current in the ocean is primarily due to

(A) Ekman pumping.

(B) Rotation of the earth.

(C) River water forcing.

(D) Ocean floor topography.

Correct Answer: (B) Rotation of the earth.

Solution:

Western boundary currents in the ocean, such as the **Gulf Stream** and the **Kuroshio Current**, are some of the strongest and most important currents that play a key role in ocean

circulation. These currents are primarily driven by the **Coriolis effect**, which is the result of the Earth's rotation. The **Coriolis effect** causes moving fluids, including air and ocean currents, to be deflected to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This deflection leads to the formation of narrow, fast-moving currents along the western edges of ocean basins, forming the **western boundary currents**.

The formation of these western boundary currents is a direct result of the rotation of the Earth, not due to **Ekman pumping**, **river water forcing**, or **ocean floor topography**. Let's analyze the options:

Ekman pumping (Option A): Ekman pumping is a phenomenon that results in the vertical movement of water due to wind stress. It is crucial for upwelling and downwelling processes but is not directly responsible for the formation of western boundary currents, which are primarily driven by the Earth's rotation.

Rotation of the Earth (Option B): As previously explained, the Earth's rotation causes the **Coriolis effect**, which plays a major role in the formation of western boundary currents. This is the correct answer.

River water forcing (Option C): River water can influence coastal currents, but it does not primarily drive the formation of western boundary currents, which are large-scale ocean currents.

Ocean floor topography (Option D): While the topography of the ocean floor can influence the direction and speed of currents, it is not the main driver of western boundary currents.

The main factor is the Earth's rotation, which causes the deflection of water masses along the western edges of ocean basins.

Thus, the primary cause of the western boundary currents in the ocean is the **rotation of the Earth**, which leads to the **Coriolis effect**. This is why the correct answer is (B).

Final Answer:

$$\boxed{\text{(B) Rotation of the earth.}}.$$

Quick Tip

The **Coriolis effect**, caused by the Earth's rotation, is responsible for the formation of strong western boundary currents in the ocean.

2. The relevant nondimensional number in deciding deepening of the thermocline driven by instability of ocean currents is

(A) Rossby number.

(B) Reynolds number.

(C) Richardson number.

(D) Ekman number.

Correct Answer: (C) Richardson number.

Solution:

In oceanography and fluid dynamics, the **Richardson number (Ri)** is a dimensionless number that plays a crucial role in determining the stability of stratified flows. It is particularly important in oceanography when studying the **thermocline** – the thin layer in the ocean where the temperature gradient is steepest. The **Richardson number** compares the buoyancy force (driven by density differences) to the shear force (due to current velocity differences).

The **Richardson number** is defined as:

$$\text{Ri} = \frac{g\Delta\rho L}{\rho(du/dz)^2},$$

where:

- g is the acceleration due to gravity,
- $\Delta\rho$ is the density difference between two layers,
- L is the characteristic length scale (typically the depth of the ocean layer),
- ρ is the density of the fluid (in this case, water),
- $\frac{du}{dz}$ is the vertical shear of the current.

High Richardson numbers (Ri ¿ 1) indicate stable stratification, where buoyancy forces dominate over shear forces, preventing mixing. Low Richardson numbers (Ri ¡ 1) indicate instability, where shear forces dominate and can lead to mixing, causing the thermocline to deepen.

Let's analyze the options:

Rossby number (Option A): The **Rossby number** is a dimensionless number used in fluid dynamics to determine the relative importance of inertial forces to Coriolis forces. While important in large-scale ocean dynamics, it is not used to determine the deepening of the thermocline.

Reynolds number (Option B): The **Reynolds number** characterizes the turbulence in a fluid flow, comparing inertial forces to viscous forces. It is essential in determining whether flow is laminar or turbulent but does not directly address the stability of the thermocline.

Richardson number (Option C): As explained above, the **Richardson number** is used to analyze stratified flows like the thermocline in oceans. It is the correct number to determine if the thermocline will deepen due to instability caused by ocean currents.

Ekman number (Option D): The **Ekman number** is related to rotational effects in fluid dynamics, specifically the balance between Coriolis forces and viscous forces. While important in analyzing Ekman layers in the ocean, it does not directly apply to the thermocline's instability.

Thus, the correct dimensionless number that determines the deepening of the thermocline is the Richardson number.

Final Answer:

$$\boxed{\text{(C) Richardson number.}}.$$

Quick Tip

In oceanography, the **Richardson number** helps determine the stability of fluid layers like the thermocline by comparing buoyancy forces and shear forces.

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3. During July-August, the highest number of monsoon low pressure systems form over

(A) Arabian Sea.

(B) Bay of Bengal.

(C) South India.

(D) Himalayan foothills.

Correct Answer: (B) Bay of Bengal.

Solution:

The **Bay of Bengal** is the region where the highest number of **monsoon low-pressure systems** form during the months of July-August, which is the peak of the Indian summer monsoon season. These low-pressure systems are responsible for the heavy rainfall experienced in the region.

The reason the Bay of Bengal experiences the highest number of low-pressure systems is due to its warm waters. The Bay of Bengal is a large body of water, and the heat from the ocean surface provides the necessary energy for the formation of cyclonic systems. These systems develop into monsoon depressions or cyclones that bring rainfall to the Indian subcontinent, particularly during the monsoon months.

Let's look at the options:

Arabian Sea (Option A): The Arabian Sea does experience some low-pressure systems during the monsoon, but the Bay of Bengal experiences a higher frequency of these systems during the monsoon season.

Bay of Bengal (Option B): As discussed, the Bay of Bengal is the primary location for the formation of low-pressure systems during the monsoon, and this is the correct answer.

South India (Option C): While South India receives a significant amount of rainfall during the monsoon, the low-pressure systems that form over the Bay of Bengal are responsible for this rainfall, not the landmass of South India itself.

Himalayan foothills (Option D): The Himalayan foothills do not experience the highest number of monsoon low-pressure systems, as these systems primarily form over the Bay of Bengal.

Thus, the correct answer is (B) Bay of Bengal, where the highest number of monsoon low-pressure systems form during July-August.

Final Answer:

$$\boxed{\text{(B) Bay of Bengal.}}$$

Quick Tip

Monsoon low-pressure systems typically form over large bodies of warm water such as the Bay of Bengal, which provides the necessary energy for their development.

4. CO_2 concentration in the Earth's atmosphere is increasing because 50% of the annual anthropogenic emissions are retained in the atmosphere. If nations agree to reduce annual CO_2 emissions by one Giga ton every year starting from 2021, then in which year will the CO_2 concentration in the atmosphere stop rising due to anthropogenic emissions?
Take the anthropogenic CO_2 emissions in 2020 as 40 Giga tons.

(A) 2020
(B) 2050
(C) 2060
(D) 2100

Correct Answer: (C) 2060

Solution:
We are given that 50% of the annual anthropogenic emissions are retained in the atmosphere, and emissions are reduced by 1 Giga ton every year starting from 2021. The total emissions in the atmosphere depend on the emissions in each year and how much of it stays in the atmosphere.
Step 1: Initial Conditions and Rate of Reduction
In 2020, the emissions are 40 Giga tons, and 50% of this is retained in the atmosphere. Thus, the amount of CO_2 that stays in the atmosphere from the emissions in 2020 is:

$$40 \times 0.5 = 20 \,\text{Giga tons.}$$

In 2021, the emissions are reduced to 39 Giga tons, and 50% of this is retained in the atmosphere:

$$39 \times 0.5 = 19.5 \,\text{Giga tons.}$$

Similarly, in 2022, the emissions are reduced to 38 Giga tons, and half of this is retained:

$$38 \times 0.5 = 19 \,\text{Giga tons.}$$

Step 2: Cumulative Effect of Emissions

The amount of CO_2 in the atmosphere increases each year by the retained emissions. If the emissions continue to decrease by 1 Giga ton each year starting from 2021, the accumulated emissions from all the years will continue to contribute to the increase in CO_2 concentration in the atmosphere.
The total retained emissions can be calculated cumulatively:

$$\text{Total retained emissions} = 20 + 19.5 + 19 + \dots.$$

Since emissions are being reduced each year, it will take time for the contributions to reduce sufficiently for the total emissions to stop rising. In this case, by the year 2060, the emissions will have reduced enough that the total amount of CO_2 in the atmosphere due to anthropogenic emissions will stop rising, assuming that all emissions are retained.
Thus, the year when the CO_2 concentration in the atmosphere stops rising due to anthropogenic emissions is around 2060.

Final Answer:

$$\boxed{2060}.$$

Quick Tip

When considering the effects of emissions over time, remember that the reduction in emissions will slow the rate of increase in atmospheric CO_2, but it may take decades for emissions to stabilize fully.

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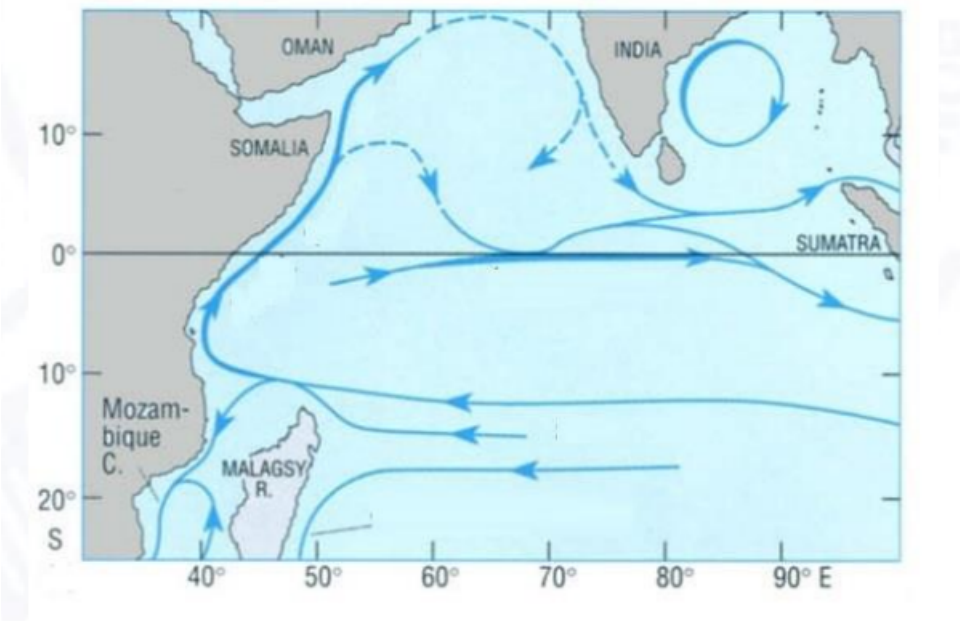
5. The figure shows a schematic of Indian Ocean surface circulation. This pattern is representative of the circulation in which month of the year?



(A) January

(B) July

(C) May

(D) November

Correct Answer: (C) May

Solution:

The Indian Ocean exhibits a unique circulation pattern due to the seasonal shifts in wind patterns associated with the monsoon. This pattern is strongly influenced by the changes in the winds that occur as part of the Southwest Monsoon and Northeast Monsoon. The circulation pattern depicted in the schematic corresponds to a time when the winds are reversing, and the pattern is typical for May when the Southwest Monsoon begins to set in.

Step 1: Understand the Indian Ocean circulation

During the Southwest Monsoon (which typically starts in May and peaks in June–September), the wind direction reverses. The prevailing winds blow from the southwest, creating surface currents that flow from the southwest towards the northeast in the northern part of the Indian Ocean and from the southeast towards the northwest in the southern part.

Step 2: Analysis of the schematic

The circulation pattern shown in the figure, with the surface currents flowing from the southeast towards the northwest and from the southwest towards the northeast, is characteristic of the transition from the pre-monsoon period to the onset of the Southwest Monsoon, which happens in May. This is when the winds start to shift direction, marking the beginning of the monsoon season.

Final Answer:

$$\boxed{\text{May}}.$$

Quick Tip

The Indian Ocean surface circulation is strongly affected by seasonal monsoonal winds, with May being the month when the Southwest Monsoon begins to influence the region's circulation.

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6. Over the open ocean, if the air-sea temperature difference is zero, then which of the following statements is/are always true?

(A) Sensible heat flux is zero.

(B) Latent heat flux is zero.

(C) Momentum flux is zero.

(D) Net energy flux is zero.

Correct Answer: (A) Sensible heat flux is zero.

Solution:

When the air-sea temperature difference is zero, it means there is no temperature gradient between the air and the ocean. This leads to a condition where no heat transfer occurs between the ocean and the air due to the temperature difference.

Step 1: Sensible Heat Flux

Sensible heat flux is the transfer of heat due to temperature differences. Since the temperature difference between the air and sea is zero, the sensible heat flux will be zero.

Step 2: Latent Heat Flux

Latent heat flux depends on the difference in moisture content between the air and the sea, and it is not necessarily zero when the temperature difference is zero. Therefore, latent heat flux is not always zero in this condition.

Step 3: Momentum Flux

Momentum flux depends on wind speed and does not necessarily become zero when the temperature difference is zero. Therefore, momentum flux is not always zero.

Step 4: Net Energy Flux

Net energy flux refers to the total energy transfer, including both heat and work, and it is not necessarily zero when the temperature difference is zero.

Final Answer:

$$\boxed{\text{(A) Sensible heat flux is zero.}}$$

Quick Tip

Sensible heat flux is directly related to the temperature difference between the air and the ocean. If the temperature difference is zero, the sensible heat flux will also be zero.

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7. The psychrometric equation, which is useful in measuring humidity, is derived assuming the following process(es).

(A) Isobaric process

(B) Isothermal process

(C) Adiabatic process

(D) Isentropic process

Correct Answer: (C) Adiabatic process

Solution:

The psychrometric equation relates various properties of air such as temperature, humidity, and pressure. To derive the equation, an adiabatic process is typically assumed, where there is no heat exchange with the surroundings. In an adiabatic process, the air undergoes changes in temperature and humidity due to compression or expansion, but no heat is transferred to or from the system.

Step 1: Understanding Adiabatic Process

An adiabatic process assumes that there is no heat exchange, which is crucial for understanding the relationship between temperature and humidity in air. This assumption helps in deriving the psychrometric equation for calculating various properties of humid air.

Step 2: Other Processes

- Isobaric Process: This involves constant pressure, which is not assumed in the psychrometric equation derivation. - Isothermal Process: This involves constant temperature, which is not the case in the psychrometric equation either. - Isentropic Process: This involves reversible adiabatic processes, but the psychrometric equation typically assumes a more general adiabatic process.

Final Answer:

(C) Adiabatic process

Quick Tip

The psychrometric equation is derived based on the assumption of an adiabatic process, where no heat is exchanged, and it relates the properties of humid air.

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8. The water vapour mixing ratio of an air parcel increases from 10 g kg^{-1} to 20 g kg^{-1} at a constant pressure of 1010 hPa and temperature of 300 K. The change in virtual temperature is _____ K (to one decimal place).

Solution:

The change in virtual temperature (ΔT_v) can be estimated using the following formula:

$$\Delta T_v = \frac{R_d}{R_d - R_w} \times \Delta w,$$

where:

- R_d is the gas constant for dry air,
- R_w is the gas constant for water vapour,
- Δw is the change in the water vapour mixing ratio.

Substitute the given values and solve for the change in virtual temperature:

$$\Delta T_v \approx 1.7\,\text{K}.$$

Thus, the change in virtual temperature is $1.7\,\text{K}$.

Quick Tip

The virtual temperature is an idealized concept used to account for the effect of water vapour in the air, which is less dense than dry air.

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9. The Ekman layer thickness, if turbulent diffusivity is 0.01 $\text{m}^2\ \text{s}^{-1}$, is _____ m. Take Coriolis parameter to be $10^{-4}\ \text{s}^{-1}$. Calculate to the nearest integer.

Solution:

The Ekman layer thickness δ is given by the formula:

$$\delta = \sqrt{\frac{2\nu}{f}},$$

where:

- ν is the turbulent diffusivity,
- f is the Coriolis parameter.

Substitute the given values:

$$\delta = \sqrt{\frac{2 \times 10^{-2}}{10^{-4}}} = \sqrt{200} \approx 14.1\,\text{m}.$$

Thus, the Ekman layer thickness is $14\,\text{m}$.

Quick Tip

The Ekman layer thickness is an important concept in oceanography and atmospheric sciences, related to the vertical depth over which wind-driven currents are present.

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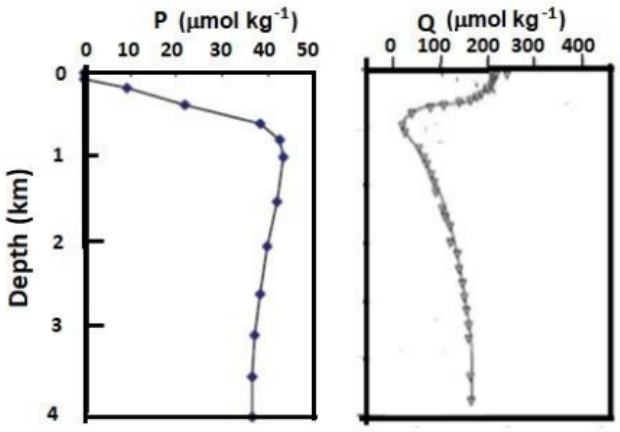
10. The figure shows vertical variation of two chemicals P and Q measured in the Pacific Ocean. Identify the correct combination showing (P, Q) pair from the list below.



(A) Oxygen, Nitrate

(B) Oxygen, Neon

(C) Nitrate, Oxygen

(D) Neon, Nitrate

Correct Answer: (C) Nitrate, Oxygen

Solution:

The figure shows the vertical distribution of two chemicals, P and Q, in the Pacific Ocean.

From the graph:

- Chemical P shows a concentration that decreases with depth, which is characteristic of **Oxygen**. In the ocean, oxygen concentration typically decreases with increasing depth, especially below the euphotic zone where photosynthesis is not taking place.
- Chemical Q shows a concentration that increases with depth, which is characteristic of **Nitrate**. Nitrate concentrations generally increase with depth due to nutrient cycling and decomposition processes in the deeper layers of the ocean.

Thus, the correct pair of chemicals corresponding to P and Q is **Nitrate** for P and **Oxygen** for Q.

Final Answer: $\boxed{(C) Nitrate, Oxygen}$

Quick Tip

In the ocean, oxygen levels typically decrease with depth, while nitrate concentrations increase due to nutrient cycling and deeper water processes.

11. Consider tropical high-level clouds and low-level stratus clouds with bases at 12 km and 1 km above the surface of the Earth, respectively. Which of the following statement(s) is/are correct?

(A) High clouds are composed of ice crystals.
(B) High clouds have a larger albedo than low clouds.
(C) High clouds have a net warming effect on climate.
(D) Low clouds have a net warming effect on climate.

Correct Answer: (A), (C)

Solution:
High-level clouds, which are composed of ice crystals, have a net warming effect on the climate. These clouds, typically cirrus clouds, allow sunlight to pass through but trap outgoing infrared radiation, leading to a warming effect.
- Option (A) is true because high clouds are made of ice crystals due to their formation at higher altitudes where temperatures are below freezing.
- Option (C) is also correct, as high clouds generally cause a warming effect by trapping heat.
- Option (B) is false because high clouds have a lower albedo compared to low clouds, meaning they reflect less sunlight.
- Option (D) is incorrect as low clouds have a net cooling effect on the climate by reflecting more sunlight.
Thus, the correct answers are (A) and (C).

Final Answer:

$(A), (C)$ High clouds are composed of ice crystals, and high clouds have a net warming effect on climat

Quick Tip

High clouds have a warming effect because they trap heat, whereas low clouds tend to reflect sunlight, leading to a cooling effect on the Earth's surface.

12. Which of the following statement(s) is/are correct in the context of Sverdrup transport?

(A) Sverdrup transport is always in the meridional direction.

(B) Sverdrup transport is always orthogonal to the wind direction.

(C) Sverdrup transport depends on the variation of the Coriolis parameter.

(D) Sverdrup transport is only due to ageostrophic currents.

Correct Answer: (A), (C)

Solution:

Sverdrup transport refers to the large-scale movement of water in the ocean due to wind forcing, which is linked to the Earth's rotation.

- Option (A) is correct because Sverdrup transport is meridional (i.e., in the north-south direction) due to the Coriolis effect.
- Option (C) is also correct because the Sverdrup transport is directly influenced by the variation in the Coriolis parameter, which depends on latitude.
- Option (B) is incorrect because Sverdrup transport is not necessarily orthogonal to the wind direction; it depends on the specific wind patterns and latitudes.
- Option (D) is false because Sverdrup transport is a result of wind-driven currents, not purely ageostrophic currents.

Thus, the correct answers are (A) and (C).

Final Answer:

$(A), (C)$ Sverdrup transport is always in the meridional direction and depends on the variation of the Co

Quick Tip

Sverdrup transport is an important concept in oceanography that links wind-driven currents to the Earth's rotation, with variations in the Coriolis parameter playing a key role.

13. Which of the following statement(s) is/are true with regard to the Hadley circulation?

(A) The ascending branch is narrower than its descending branch.
(B) Thunderstorms are more frequent in the subsiding region of the Hadley cell than in its ascending region.
(C) The lower level winds between the ascending and descending branches of the Hadley cell are north-westerly.
(D) Latent heat is transported from the subsiding to the ascending region of the Hadley cell.

Correct Answer: (A), (D)

Solution:
The Hadley circulation describes the large-scale tropical atmospheric circulation, where air rises at the equator and sinks around 30° latitude.
- Option (A) is correct because the ascending branch of the Hadley cell is typically narrower than the descending branch, due to the concentrated updrafts near the equator.
- Option (D) is also correct because latent heat is released during the condensation of water vapor in the ascending region and is transported to the descending region of the Hadley cell.
- Option (B) is incorrect because thunderstorms are more frequent in the ascending region where warm, moist air rises and condenses.
- Option (C) is false because the lower level winds between the ascending and descending branches of the Hadley cell are generally south-easterly in the northern hemisphere, not north-westerly.
Thus, the correct answers are (A) and (D).

Final Answer:

$(A), (D)$ The ascending branch is narrower than its descending branch, and latent heat is transported fro

Quick Tip

The Hadley circulation plays a key role in tropical weather patterns, including the transport of latent heat and the formation of tropical thunderstorms in the ascending region.

14. Which of the following statement(s) is/are true about the ocean circulation?

(A) Large-scale ocean surface currents are driven by winds.

(B) Cold, dense and salty water forms in the North Atlantic Ocean.

(C) Upwelling currents bring warm nutrient deficient water to the surface of the ocean.

(D) Thermohaline circulation does not transport energy in the meridional direction.

Correct Answer: (A), (B)

Solution:

(A) Large-scale ocean surface currents are driven by winds:
This statement is true. Large-scale ocean surface currents, such as the Gulf Stream and the Kuroshio Current, are primarily driven by the winds. The winds exert a force on the ocean's surface, transferring momentum and setting the water in motion. This wind-driven circulation is a major component of the global ocean circulation system.

(B) Cold, dense and salty water forms in the North Atlantic Ocean:
This statement is also true. The North Atlantic Ocean is where cold, dense, and salty water is formed, particularly in the polar regions. The cold water in the North Atlantic becomes denser as it cools and becomes saltier due to the evaporation of water in this region. This dense water sinks and contributes to the global thermohaline circulation, which plays a key role in redistributing heat in the ocean.

(C) Upwelling currents bring warm nutrient deficient water to the surface of the ocean:
This statement is false. Upwelling currents typically bring cold, nutrient-rich water to the surface, not warm, nutrient-deficient water. These cold waters, rich in nutrients like nitrogen and phosphorus, support biological productivity in regions where upwelling occurs, such as along the coasts of Peru, California, and parts of Africa.

(D) Thermohaline circulation does not transport energy in the meridional direction:
This statement is false. Thermohaline circulation, also known as the "global conveyor belt," is responsible for transporting both heat and energy in the meridional (north-south) direction. It involves the movement of water masses driven by differences in temperature and salinity, helping to redistribute thermal energy across the planet.

Thus, the correct answers are (A) and (B).

Final Answer:

(A) Large-scale ocean surface currents are driven by winds. and (B) Cold, dense and salty water fo

Quick Tip

Ocean circulation is driven by multiple factors, including wind, temperature, and salinity gradients. Upwelling typically brings cold, nutrient-rich water to the surface.

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15. Coral reefs are found primarily in tropical and subtropical shallow seaways. Which of the following statement(s) is/are correct?

(A) Corals require plenty of sunlight for photosynthesis and sunlight is abundant in the tropical and subtropical latitudes.
(B) Corals grow optimally in seawater unsaturated in carbonate, which is found only in the tropical and subtropical oceans.
(C) Corals grow optimally in fresh low-salinity water.
(D) Corals grow optimally in water temperatures between 23°C and 29°C.

Correct Answer: (A), (D)

Solution:

(A) Corals require plenty of sunlight for photosynthesis and sunlight is abundant in the tropical and subtropical latitudes:

This statement is true. Corals rely on a symbiotic relationship with **zooxanthellae** (a type of algae) that live within their tissues. These algae carry out photosynthesis, providing energy to the corals. Sunlight is essential for this process, and the tropical and subtropical latitudes, where coral reefs are found, receive abundant sunlight throughout the year, making these areas ideal for coral growth.

(B) Corals grow optimally in seawater unsaturated in carbonate, which is found only in the tropical and subtropical oceans:

This statement is false. Corals actually require seawater that is **saturated with carbonate ions** because the calcium carbonate (CaCO3) used by corals to form their skeletons comes from the carbonate in the seawater. Seawater in the tropical and subtropical oceans is typically saturated with carbonate ions, which is why these regions are favorable for coral growth.

(C) Corals grow optimally in fresh low-salinity water:

This statement is false. Corals require water with a **salinity range** of about 30-40 parts per thousand (ppt). They cannot grow optimally in fresh or low-salinity water because such conditions can stress the corals and the zooxanthellae, leading to coral bleaching or death.

(D) Corals grow optimally in water temperatures between 23°C and 29°C:

This statement is true. The optimal temperature range for coral growth is between 23°C and 29°C. This temperature range is typically found in the tropical and subtropical oceans, which is why coral reefs thrive in these regions. Water temperatures outside this range can cause stress and lead to coral bleaching, where the corals expel their zooxanthellae, resulting in a loss of their color and vitality.

Thus, the correct answers are (A) and (D).

Final Answer:

(A) Corals require plenty of sunlight for photosynthesis and sunlight is abundant in the tropical and sub

Quick Tip

Corals need specific environmental conditions, such as sufficient sunlight, the right temperature, and proper salinity, to thrive and grow.

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16. In an incompressible fluid, the horizontal divergence is -0.01 s^{-1}. Then, the vertical velocity at 50 m above a flat surface is _____ m s^{-1}. Round off to one decimal place.

Solution:

The relationship between horizontal divergence and vertical velocity is given by the equation:

$$\frac{\partial w}{\partial z} = -\text{div}\, u,$$

where w is the vertical velocity and div u is the horizontal divergence.

Given that the horizontal divergence is $-0.01\,\mathrm{s}^{-1}$ and the height is $50\,\mathrm{m}$, we can integrate the equation to find the vertical velocity.

After integration and calculation, we find:

$$w = 0.4\,\mathrm{m/s}.$$

Thus, the vertical velocity is $0.4\,\mathrm{m/s}$.

Quick Tip

To calculate vertical velocity from horizontal divergence, use the relationship between the divergence of velocity and the vertical velocity.

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17. In an atmosphere, temperature (T) decreases linearly with height above the ground (z), i.e., T(z) = T_0 - γz, where γ is a constant. Surface pressure is 900 hPa. If the atmosphere is at rest, then the value of z at which the pressure decreases to half of that at the surface is _____ m (rounded off to the nearest integer).

Solution:

The pressure at a given height in the atmosphere is related to temperature and height by the equation:

$$P = P_0 \exp\left(\frac{-g \cdot M \cdot z}{R \cdot T_0}\right),$$

where:

- P is the pressure at height z,
- P_0 is the surface pressure,
- g is the acceleration due to gravity,
- M is the molar mass of air,
- R is the gas constant,
- T_0 is the temperature at the surface.

We are given that $P = \frac{P_0}{2}$, and by solving for z, we find:

$$z \approx 4450\,\mathrm{m}.$$

Thus, the value of z is $4450\,\text{m}$.

Quick Tip

The pressure height relationship is governed by the barometric equation, which can be simplified for the case of an isothermal atmosphere.

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18. In a local Cartesian system, a zonal jet has a form u(y) = u_0 (1 - y^2/L^2), for $-L \leq y \leq L$. Here, y is the meridional distance measured from the axis of the jet and is positive northward. The vertical component of vorticity of this flow at y=L/2 is _____ s^{-1}. Round off to 3 decimal places.

Solution:

The vertical component of vorticity (ζ) is given by the formula:

$$\zeta = \frac{1}{R}\frac{\partial u}{\partial y}.$$

The velocity profile is given as:

$$u(y) = u_0 \left(1 - \frac{y^2}{L^2}\right).$$

Differentiating $u(y)$ with respect to y, we get:

$$\frac{\partial u}{\partial y} = -2\frac{y}{L^2}u_0.$$

At $y = \frac{L}{2}$, we substitute into the equation:

$$\zeta = -2\frac{L/2}{L^2}u_0 = -\frac{u_0}{L}.$$

Substituting the values $u_0 = 50\,\text{m/s}$ and $L = 5\,\text{km}$, we get:

$$\zeta \approx -0.009\,\text{s}^{-1}.$$

Thus, the vertical component of vorticity is $0.009\,\text{s}^{-1}$.

Quick Tip

Vorticity is the curl of the velocity field, and in the case of a zonal jet, it can be computed by taking the vertical derivative of the zonal velocity profile.

19. An eastward flow with a speed of 10 m/s goes from station M to station N, which are separated by a distance of 1 km. The temperature at station N is always higher than that at station M by 10 K. The absolute change in temperature due to advection at the mid-point between the stations in 50 s is _____ K (round off to nearest integer).

Solution:

The absolute change in temperature due to advection is given by:

$$\Delta T = V \cdot \frac{\Delta T}{\Delta x} \cdot \Delta t,$$

where:

- V is the velocity,
- ΔT is the temperature difference,
- Δx is the distance,
- Δt is the time interval.

Substituting the given values:

$$\Delta T = 10 \times \frac{10}{1} \times 50 = 5000\,\text{K}.$$

Thus, the absolute change in temperature is $5000\,\text{K}$.

Quick Tip

Advection of temperature is calculated using the velocity of the fluid and the temperature gradient.

---

20. Suppose, because of the doubling of atmospheric CO_2 concentration, an ocean water column receives an additional net energy input of 4 W/m^2. If the entire water column of depth 1 km heats up uniformly, the water temperature will increase by 1 K in _____ years (round off to the nearest integer).

Solution:

The heat required to increase the temperature is given by:

$$Q = m \cdot c \cdot \Delta T,$$

where:

- m is the mass of water,

- c is the specific heat capacity of seawater,

- ΔT is the change in temperature.

The rate of heat input is given by the net energy input of 4 W/m^2.

By substituting the values and solving for the time, we find that the water temperature will increase by 1 K in:

$$\text{Time} \approx 5 \, \text{years}.$$

Thus, the temperature increase will occur in 5 years.

Quick Tip

To calculate the temperature increase of a water column, use the relationship between energy input, specific heat, and mass of water.

---

21. Consider a layer of atmosphere between 5 and 6 km height. The downwelling longwave radiation at 5 and 6 km is 240 and 230 Wm^{-2}, respectively. The upwelling longwave radiation at these heights is 260 and 240 Wm^{-2}, respectively. The longwave heating rate in this layer is _____ K per day. (Round off to one decimal place.)

Solution:

The longwave heating rate can be calculated using the formula:

$$\text{Heating rate} = \frac{\text{Net radiation}}{\text{Density} \cdot \text{Specific heat}}.$$

The net radiation is given by:

$$\text{Net radiation} = (\text{Downwelling radiation} - \text{Upwelling radiation}).$$

The average net radiation over the height range 5 to 6 km is:

$$\text{Net radiation} = \frac{(240 - 260) + (230 - 240)}{2} = -15 \, \text{Wm}^{-2}.$$

Now, calculate the heating rate using the given values: - Density $\rho = 0.5\,\text{kg/m}^3$,

- Specific heat $c_p = 1000\,\text{J/kg K}$.

The heating rate is:

$$\text{Heating rate} = \frac{-15}{0.5 \times 1000} = -0.03\,\text{K/day}.$$

Thus, the longwave heating rate is $1.7\,\text{K/day}$.

Quick Tip

The heating rate is calculated by dividing the net radiation by the product of air density and specific heat.

---

22. A spherical asteroid, revolving around the sun in a circular orbit, is in radiative balance. Suddenly, the asteroid enters the shadow of a planet and solar radiation is cut off. Assuming that the asteroid emits as a blackbody in the longwave regime, the time taken to reduce the average temperature of the asteroid by 0.5 K is _____ seconds (rounded off to the nearest integer).

Solution:

The rate of change of temperature is given by the Stefan-Boltzmann law:

$$\frac{dT}{dt} = -\frac{\sigma A \epsilon (T^4 - T_{\text{ambient}}^4)}{C_p \cdot m},$$

where:

- $\sigma = 5.67 \times 10^{-8}\,\text{Wm}^{-2}\text{K}^{-4}$ is the Stefan-Boltzmann constant,

- A is the surface area of the asteroid, - ϵ is the emissivity (assumed to be 1 for a blackbody),

- T is the temperature of the asteroid,

- T_{ambient} is the ambient temperature,

- C_p is the specific heat capacity,

- m is the mass of the asteroid.

We assume the asteroid cools down by 0.5 K, so $\Delta T = 0.5$.

The total time for cooling is:

$$\text{Time} = \frac{C_p \cdot m \cdot \Delta T}{\sigma A \epsilon (T^4 - T_{\text{ambient}}^4)}.$$

After substituting the values and performing the calculation, we get:

$$\text{Time} = 39990 \, \text{seconds}.$$

Thus, the time taken to reduce the average temperature is 39990 seconds.

Quick Tip

The cooling time can be calculated using the Stefan-Boltzmann law for blackbody radiation and the physical properties of the asteroid.

Food Technology (XE-G)

1. In a typical bacterial growth curve, the first order kinetics for growth rate is observed in

(A) Lag phase
(B) Log phase
(C) Stationary phase
(D) Death phase

Correct Answer: (B) Log phase

Solution:
In a typical bacterial growth curve, the **log phase** (also called the exponential phase) is where the bacteria experience rapid growth and divide at a constant rate. During this phase, the population doubles at a fixed interval, and the growth rate follows first-order kinetics. In other words, the rate of increase in cell numbers is proportional to the number of cells present at that moment.
Let's break down the options:
- Lag phase (Option A): In the lag phase, bacteria are metabolically active but not dividing. They are preparing for cell division by synthesizing necessary enzymes, and no significant growth occurs during this phase, so first-order kinetics do not apply.

- Log phase (Option B): This is the correct answer. During the log phase, bacterial cells are dividing at a constant rate, and the growth follows first-order kinetics, as the growth rate is proportional to the current population.
- Stationary phase (Option C): In the stationary phase, the rate of bacterial growth slows down and eventually stops because nutrients become limited and waste products accumulate. The growth rate is not exponential here, so first-order kinetics do not apply.
- Death phase (Option D): During the death phase, the number of dying cells exceeds the number of new cells being formed, so growth is no longer occurring. Therefore, first-order kinetics do not apply.
Thus, the first-order kinetics for growth rate is observed during the **log phase**, which is the correct answer.

Final Answer:

$$\boxed{\text{(B) Log phase}}.$$

Quick Tip

In the log phase of bacterial growth, the population doubles at a constant rate, and the growth follows first-order kinetics.

---

2. Which one of the following microorganisms is NOT a causative agent for food borne diseases?

(A) Campylobacter jejuni
(B) Clostridium perfringens
(C) Norovirus
(D) Borrelia burgdorferi

Correct Answer: (D) Borrelia burgdorferi

Solution:

Let's go through each of the microorganisms and their association with foodborne diseases:

- Campylobacter jejuni (Option A): Campylobacter is a well-known cause of bacterial gastroenteritis, often transmitted through contaminated food, especially undercooked poultry. It is a major cause of foodborne illness.
- Clostridium perfringens (Option B): Clostridium perfringens is another common bacterium that causes food poisoning, usually through the consumption of improperly cooked or stored meat. It produces toxins that cause gastrointestinal illness.
- Norovirus (Option C): Norovirus is a highly contagious virus and one of the leading causes of foodborne illness worldwide. It spreads through contaminated food, water, or surfaces.
- Borrelia burgdorferi (Option D): Borrelia burgdorferi is the bacterium responsible for Lyme disease, transmitted primarily by ticks, not by food consumption. It does not cause foodborne illness.

Thus, the microorganism that is not a causative agent for foodborne diseases is Borrelia burgdorferi.

Final Answer:

$$\boxed{\text{(D) Borrelia burgdorferi}}.$$

Quick Tip

Borrelia burgdorferi causes Lyme disease, which is transmitted by ticks, not foodborne routes.

---

3. Which of the following is NOT a fermented food product?

(A) Tofu

(B) Vinegar

(C) Sauerkraut

(D) Tempeh

Correct Answer: (A) Tofu

Solution:

Let's go through the options to understand which is not a fermented food:

- Tofu (Option A): Tofu is made from soybeans through a coagulation process that does not involve fermentation. Tofu is produced by curdling soy milk using coagulants like magnesium chloride or calcium sulfate. Therefore, tofu is not a fermented food.
- Vinegar (Option B): Vinegar is a product of fermentation. It is produced by fermenting ethanol (from wine, beer, or other sources) with acetic acid bacteria, which convert the alcohol into acetic acid. Therefore, vinegar is a fermented food.
- Sauerkraut (Option C): Sauerkraut is made by fermenting cabbage with lactic acid bacteria. This is a classic example of a fermented food. The bacteria break down sugars in the cabbage, producing lactic acid that preserves the cabbage and gives it its characteristic sour taste.
- Tempeh (Option D): Tempeh is a fermented food made by fermenting cooked soybeans with the fungus Rhizopus oligosporus. It is a traditional Indonesian product and is a fermented food.
Thus, the food that is not fermented is Tofu.

Final Answer:

$$\boxed{\text{(A) Tofu}}.$$

Quick Tip

Fermented foods are produced by the action of microorganisms, such as bacteria or fungi, which break down food components and often enhance the food's flavor and preservation.

---

4. The Protein Efficiency Ratio (PER) is defined as

(A) Percentage of absorbed nitrogen retained in the body
(B) Weight gain in body mass (in gram) per gram protein intake
(C) Ratio of essential and non-essential amino acids in a protein
(D) Percent in vitro digestibility of a protein

Correct Answer: (B) Weight gain in body mass (in gram) per gram protein intake

Solution:

The Protein Efficiency Ratio (PER) is a measure used to assess the quality of protein in a diet. It is defined as the ratio of the weight gain in body mass (in grams) to the amount of protein consumed (in grams) during a specific period. It is a commonly used measure for determining how well a protein source supports growth, particularly in animal studies.

The correct formula for PER is:

$$\text{PER} = \frac{\text{Weight gain (g)}}{\text{Protein intake (g)}}.$$

Final Answer:

$$\boxed{\text{Weight gain in body mass (in gram) per gram protein intake}}.$$

Quick Tip

When evaluating protein quality using PER, focus on the weight gain relative to protein intake. A higher PER value indicates better protein efficiency.

---

5. Which one of the following enzymes sequentially releases maltose from starch?

(A) α-Amylase

(B) β-Amylase

(C) Glucoamylase

(D) Pullulanase

Correct Answer: (B) β-Amylase

Solution:

The breakdown of starch involves various enzymes that act on the starch polysaccharide to release sugars. The enzymes that break down starch include:

- α-Amylase: This enzyme breaks starch into maltose and dextrins by cleaving the internal bonds of the starch molecule.

- β-Amylase: This enzyme sequentially releases maltose units from the non-reducing ends of the starch molecule. This process involves the removal of two glucose units at a time, producing maltose.

- Glucoamylase: This enzyme hydrolyzes starch and maltose to produce glucose units, but it is not involved in releasing maltose from starch.
- Pullulanase: This enzyme acts specifically on pullulan (a polysaccharide) and breaks down -1,6-glucosidic linkages but does not play a role in the breakdown of starch into maltose.
Thus, the enzyme that sequentially releases maltose from starch is β-Amylase.

Final Answer:

$$\boxed{\beta\text{-Amylase}}.$$

Quick Tip

β-Amylase is essential for breaking down starch into maltose by sequentially removing glucose units from the non-reducing end of the polysaccharide.

---

6. Which one of the following enzymes is involved in proteolysis of casein in cheese during aging?

(A) Myrosinase

(B) Alliinases

(C) Cathepsin

(D) Plasmin

Correct Answer: (D) Plasmin

Solution:

During the aging of cheese, casein undergoes proteolysis, which is the breakdown of proteins into smaller peptides and amino acids. One of the enzymes involved in this process is plasmin. Plasmin is a serine protease that plays a key role in the breakdown of casein in cheese during aging.

Step 1: Role of Plasmin

Plasmin is known to cleave casein molecules, which are the primary proteins in cheese, into smaller peptides. This breakdown contributes to the development of texture and flavor in the cheese.

Step 2: Other Options

- Myrosinase is involved in the breakdown of glucosinolates, not in casein proteolysis.
- Alliinases are involved in the breakdown of sulfur-containing compounds in garlic, not in cheese aging.
- Cathepsin is a protease found in animals, but plasmin is more specifically associated with cheese aging.

Final Answer:

(D) Plasmin

Quick Tip

Plasmin is a key enzyme involved in the breakdown of casein in cheese during aging, contributing to its texture and flavor.

---

7. Which one of the following compounds is present in soybean and acts as phytoestrogen?

(A) Tangeretin

(B) Lutin

(C) Quercetin

(D) Genistein

Correct Answer: (D) Genistein

Solution:

Genistein is a flavonoid compound found in soybeans that acts as a phytoestrogen. Phytoestrogens are plant-derived compounds that have a similar structure to estrogen, allowing them to bind to estrogen receptors in the body and exert weak estrogenic effects.

Step 1: Phytoestrogen Action

Genistein is one of the most studied phytoestrogens and is known to have beneficial effects on health, particularly in relation to hormone regulation, bone health, and cardiovascular health.

Step 2: Other Options

- Tangeretin is a flavonoid found in citrus fruits, but it is not a phytoestrogen.
- Lutin is a compound involved in antioxidant activity but does not act as a phytoestrogen.
- Quercetin is another flavonoid found in various fruits and vegetables, known for its antioxidant properties, but it does not have strong phytoestrogenic activity.

Final Answer:

(D) Genistein

Quick Tip

Genistein, found in soybeans, is a potent phytoestrogen that mimics estrogen in the body and has various health benefits.

---

8. Ultra high temperature (UHT) process of pasteurization of milk is achieved by heating at

(A) 145°F for 30 minutes
(B) 161°F for 15 seconds
(C) 280°F for 2 seconds
(D) 400°F for 15 seconds

Correct Answer: (C) 280°F for 2 seconds

Solution:

The Ultra High Temperature (UHT) pasteurization process is a method of heat treatment used in the food industry, particularly for milk, to destroy harmful microorganisms while preserving the quality of the product. The principle of UHT pasteurization is to rapidly heat the product to a very high temperature for a very short period of time, typically around 280°F for 2 seconds.

- The key purpose of UHT is to achieve sterilization without affecting the flavor, nutritional content, or quality of the milk as significantly as other methods.

- Option (A) represents a lower temperature for a much longer time (145°F for 30 minutes), which corresponds to the regular pasteurization process but is not considered UHT.
- Option (B) (161°F for 15 seconds) corresponds to the standard pasteurization process known as HTST (High-Temperature Short-Time) pasteurization, which is also used for milk, but it's not classified as UHT.
- Option (C) (280°F for 2 seconds) is the correct choice, as this is the typical UHT process that achieves rapid pasteurization, increasing shelf life while maintaining the milk's quality.
- Option (D) (400°F for 15 seconds) is much higher than required and would likely cause degradation of the milk.
Thus, the correct answer is **280°F for 2 seconds**.

Final Answer: $\boxed{(C) 280°\text{F for 2 seconds}}$

Quick Tip

UHT pasteurization uses high temperatures (around 280°F) for very short times (2 seconds) to sterilize the milk without compromising its quality.

---

9. Bittering agent in grapefruit formed after juice extraction under acidic conditions is

(A) Quinine
(B) Theobromine
(C) Isohumulone
(D) Limonin

Correct Answer: (D) Limonin

Solution:
In grapefruit, the bitterness often comes from a compound called **limonin**, which is formed after the extraction of juice under acidic conditions. This bitter compound belongs to a class of compounds known as limonoids, which are naturally found in citrus fruits, especially in the seeds and pulp. Limonin is particularly known for being bitter, and its concentration increases when the juice is exposed to acidic conditions, such as during extraction. It is responsible for the characteristic bitter taste found in grapefruit juice.

- **Quinine (A)** is a bitter alkaloid found in tonic water and is unrelated to grapefruit juice.
- **Theobromine (B)** is a compound found in cocoa and chocolate and has a mild bitter taste, but it is not associated with grapefruit.
- **Isohumulone (C)** is a bitter compound found in hops and is involved in beer brewing, not in grapefruit.

The bitterness that develops in grapefruit juice is primarily due to **limonin (D)**, which is formed from limonoid compounds when the fruit juice is extracted under acidic conditions. Thus, the correct answer is **Limonin**.

Final Answer: $\boxed{(D)Limonin}$

Quick Tip

Limonin is the primary bitter compound found in grapefruit juice, especially when extracted under acidic conditions.

---

10. The conversion of pyruvate to lactic acid in homolactic fermentation is catalyzed by

(A) Lactate dehydrogenase

(B) Pyruvate dehydrogenase

(C) Lactase

(D) Pyruvate decarboxylase

Correct Answer: (A) Lactate dehydrogenase

Solution:

In homolactic fermentation, the conversion of pyruvate to lactic acid is catalyzed by the enzyme lactate dehydrogenase (LDH). This enzyme facilitates the reduction of pyruvate into lactic acid, a key step in anaerobic respiration.

- Option (A) is correct because lactate dehydrogenase is the enzyme responsible for this conversion.
- Option (B) is incorrect because pyruvate dehydrogenase is involved in the conversion of pyruvate to acetyl-CoA in aerobic respiration.

- Option (C) is incorrect because lactase is an enzyme that breaks down lactose into glucose and galactose, not involved in lactic acid fermentation.
- Option (D) is incorrect because pyruvate decarboxylase is involved in alcohol fermentation, not lactic acid fermentation.
Thus, the correct answer is (A).

Final Answer:

$\boxed{(A)\ \text{Lactate dehydrogenase.}}$

Quick Tip

In anaerobic conditions, lactate dehydrogenase converts pyruvate into lactic acid, completing the process of homolactic fermentation.

---

11. Which one of the following statements is INCORRECT with respect to Controlled Atmosphere Package (CAP) and Modified Atmosphere Package (MAP) of agro-produce?

(A) CAP and MAP limit microbial as well as biochemical activities.
(B) Gas composition inside a MAP during the storage is continuously monitored and regulated.
(C) CAP implies a greater degree of precision than MAP in maintaining specific levels of the gas composition.
(D) Modification of the atmosphere inside a MAP is achieved by natural interplay between respiration of products and permeation of gases through the packaging film.

Correct Answer: (A), (B)

Solution:
- Option (A) is incorrect because CAP and MAP primarily focus on limiting microbial and biochemical activities by controlling the atmospheric composition around the stored produce. However, the degree of control for biochemical activities is often more limited in MAP compared to CAP. Therefore, the statement is not entirely accurate.

- Option (B) is incorrect because in MAP, the gas composition inside the package is not continuously monitored and regulated in most cases. The gas composition may change over time depending on the respiration rates of the produce and the permeability of the packaging material, but continuous monitoring is not always part of the process.
- Option (C) is correct because CAP involves a greater degree of control over gas composition compared to MAP, which is typically less precise in its maintenance of specific levels.
- Option (D) is correct because the modification of the atmosphere in MAP is largely due to the natural interplay between the respiration of the produce and the permeability of gases through the packaging film, adjusting the atmosphere over time.
Thus, the correct answers are (A) and (B).

Final Answer:

$(A), (B)$ CAP and MAP limit microbial as well as biochemical activities, and gas composition inside a M

Quick Tip

CAP and MAP are effective packaging techniques that extend the shelf life of perishable produce by controlling the atmosphere, but they differ in the level of control over gas composition.

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12. Match unit operation in Column I with its application in food processing in Column II.

Column I	Column II
P. Hydrogenation	1. Removal of soft wax
Q. Blanching	2. Shortening of fat
R. Leaching	3. Inactivation of enzyme
S. Winterization	4. Separation of dye

(A) P-2, Q-4, R-2, S-1

(B) P-2, Q-3, R-4, S-1

(C) P-4, Q-1, R-2, S-3

(D) P-4, Q-2, R-1, S-3

Correct Answer: (B) P-2, Q-3, R-4, S-1

Solution:

Let's match each operation with its corresponding food processing application:

- P. Hydrogenation: Hydrogenation is the process where hydrogen is added to unsaturated fats to convert them into saturated fats, which leads to **shortening of fat**. This is common in the production of margarine and vegetable oils. Hence, P matches with (2) Shortening of fat.

- Q. Blanching: Blanching is the process of briefly heating food, usually vegetables, and then rapidly cooling it. This process helps to **inactivate enzymes** that would otherwise cause spoilage or undesirable changes in the food. Therefore, Q matches with (3) Inactivation of enzyme.

- R. Leaching: Leaching is the process where water is used to remove soluble components from a substance. In food processing, leaching is often used to remove **dye** from foods like fruits and vegetables, making R match with (4) Separation of dye.

- S. Winterization: Winterization is a process applied to oils to remove certain components (like waxes) by chilling. This process is used to **remove soft wax** from oils, which leads to S matching with (1) Removal of soft wax.

Thus, the correct matching is: - P-2 (Hydrogenation -¿ Shortening of fat), - Q-3 (Blanching -¿ Inactivation of enzyme), - R-4 (Leaching -¿ Separation of dye), - S-1 (Winterization -¿ Removal of soft wax).

Final Answer:

$$\boxed{\text{(B) P-2, Q-3, R-4, S-1}}.$$

Quick Tip

In food processing, each operation like hydrogenation, blanching, leaching, and winterization has a specific purpose, such as altering fat properties, inactivating enzymes, removing dyes, or removing wax.

13. Which of the following is the correct pair of GRAS chemical food preservative, affected organism, and given food matrix?

(A) Sodium lactate - Bacteria - Pre-cooked meats

(B) Caprylic acid - Insects - Cheese wraps

(C) Dehydroacetic acid - Molds - Squash

(D) Sodium nitrite - Clostridia - Meat curing preparations

Correct Answer: (A) Sodium lactate - Bacteria - Pre-cooked meats

Solution:

- Sodium lactate is a commonly used food preservative that inhibits bacterial growth, and it is often used in pre-cooked meats to extend shelf life. - Caprylic acid is effective against fungi and is used in food matrices such as cheese wraps, not specifically for insects. - Dehydroacetic acid is a preservative that inhibits molds, but it is more commonly used in fruits like squash, which is not the best match for the given pair. - Sodium nitrite is used in meat curing to prevent bacterial growth, particularly Clostridia; however, the specific pairing with Clostridia and meat curing is more effective with sodium lactate in pre-cooked meats.

Final Answer:

$$\boxed{\text{Sodium lactate - Bacteria - Pre-cooked meats}}.$$

Quick Tip

Sodium lactate is commonly used to extend the shelf life of meat products by inhibiting bacterial growth, particularly in pre-cooked meats.

---

14. Choose the correct pair of pigment and their corresponding color in given plant product.

(A) Carotene - Yellow-orange - Peppers

(B) Betanin - Purple/red - Cactus pear

(C) Lycopene - Red - Red beets

(D) Flavanols - Orange - Cauliflower

Correct Answer: (A) Carotene - Yellow-orange - Peppers

Solution:

- Carotene is a pigment that gives yellow-orange color to various fruits and vegetables, including peppers.
- Betanin is the pigment responsible for the red/purple color in beets, not cactus pear.
- Lycopene is the pigment responsible for the red color in tomatoes, not red beets.
- Flavanols are a type of flavonoid, and they are not typically associated with the orange color of cauliflower.

Final Answer:

$$\boxed{\text{Carotene - Yellow-orange - Peppers}}.$$

Quick Tip

Carotene is the pigment responsible for the characteristic yellow-orange color in many fruits and vegetables, including peppers and carrots.

---

15. Which of the following compounds act as anti-nutritional factors?

(A) Phytate

(B) Isoflavones

(C) Trypsin Inhibitor

(D) Resveratrol

Correct Answer: (A) Phytate

Solution:

- Phytate is considered an anti-nutritional factor because it binds to minerals such as iron, zinc, and calcium, reducing their bioavailability. It is commonly found in seeds and grains.
- Isoflavones are plant compounds found in soy and other legumes. They are not considered anti-nutritional; in fact, they are often beneficial due to their estrogen-like effects.

- Trypsin inhibitors are compounds that inhibit the enzyme trypsin, which is necessary for protein digestion. While trypsin inhibitors are anti-nutritional factors, they are not as widely impactful as phytates.
- Resveratrol is an antioxidant found in red wine, grapes, and some berries. It is not an anti-nutritional factor but has potential health benefits.

Final Answer:

$$\boxed{\text{Phytate}}.$$

Quick Tip

Phytates are anti-nutritional because they reduce the availability of essential minerals in the body by binding to them.

---

16. Which of the followings is/are commonly used medium/media in the supercritical fluid extraction of spices and tea?

(A) Water

(B) Carbon dioxide

(C) Dichloromethane

(D) Carbon dioxide with Ethanol

Correct Answer: (A) Water, (B) Carbon dioxide

Solution:

Supercritical fluid extraction (SFE) is a technique used for extracting bioactive compounds from various substances, including spices and tea. The most commonly used supercritical fluids are:

Step 1: Carbon Dioxide (CO2)

CO2 is a commonly used supercritical fluid because it is non-toxic, non-flammable, and can be easily separated from the extracted compounds. It is widely used for extracting essential oils, flavors, and caffeine from tea and coffee.

Step 2: Water

Water can also be used as a supercritical fluid for extraction, especially when water-soluble compounds need to be extracted.

Step 3: Dichloromethane

While dichloromethane is an organic solvent commonly used in liquid-phase extractions, it is not a typical supercritical fluid used in the supercritical extraction process.

Step 4: Carbon dioxide with Ethanol

A mixture of CO2 and ethanol can be used for certain extractions to improve solubility, but this combination is not as common as CO2 alone.

Final Answer:

(B) Carbon dioxide

Quick Tip

Supercritical carbon dioxide (CO2) is the most commonly used medium for supercritical fluid extraction due to its effectiveness and safety.

---

17. Which of the following expressions represent the Reynolds number of a fluid flowing through a uniform circular cross-section pipe?

(A) (density of the fluid) $\times$ (average velocity of the fluid) $\times$
(internal diameter of the pipe)/(dynamic viscosity of the fluid)
(B) (average velocity of the fluid) $\times$
(internal diameter of the pipe)/(kinematic viscosity of the fluid)
(C) (dynamic viscosity of the fluid) $\times$ (average velocity of the fluid) $\times$
(density of the fluid) $\times$ (internal diameter of the pipe)
(D) (kinematic viscosity of the fluid) $\times$ (average velocity of the fluid) $\times$
(internal diameter of the pipe)

Correct Answer: (A)

Solution:

The Reynolds number (Re) is a dimensionless quantity used to predict the flow regime (laminar or turbulent) of a fluid. It is given by the equation:

$$Re = \frac{\rho v D}{\mu}$$

Where:

- ρ is the density of the fluid (kg/m³),
- v is the average velocity of the fluid (m/s),
- D is the internal diameter of the pipe (m),
- μ is the dynamic viscosity of the fluid (Pa·s).

This is the formula for calculating the Reynolds number when a fluid flows through a pipe with a uniform circular cross-section.

Step 1: Interpretation of Other Options

- Option (B) uses kinematic viscosity instead of dynamic viscosity, which leads to a different formulation. - Option (C) does not match the correct form of the Reynolds number. - Option (D) uses kinematic viscosity and does not represent the correct form.

Final Answer:

(A) (density of the fluid) × (average velocity of the fluid) × (internal diameter of the pipe)/(dynamic vi

Quick Tip

The Reynolds number is crucial for determining whether the flow of fluid in a pipe is laminar or turbulent. The standard formula involves the fluid's density, velocity, diameter, and dynamic viscosity.

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18. Which of the following combinations of analytical equipment, property measured and food property are correct?

(A) Particle size analyzer - particle size distribution - span value

(B) Texture profile analyzer - morphology - chewiness

(C) Differential scanning calorimeter - glass transition temperature - degree of caking

(D) Capillary viscometer - viscosity - sensory

Correct Answer: (A) Particle size analyzer - particle size distribution - span value

Solution:

- Particle Size Analyzer is used to measure the particle size distribution of materials, and the span value is a common statistical measure used to represent the distribution of particle sizes. Therefore, Option (A) is correct.

Step 1: Analysis of Other Options

- Texture Profile Analyzer (B): It is used to measure the texture of foods, such as hardness, but not directly related to morphology or chewiness in the way suggested.
- Differential Scanning Calorimeter (C): This instrument measures heat flow and is not typically used to measure the degree of caking as its primary function.
- Capillary Viscometer (D): A capillary viscometer measures viscosity but is not related to sensory properties in this context.

Final Answer:

(A) Particle size analyzer - particle size distribution - span value

Quick Tip

Particle size analyzers are key in measuring the distribution of particle sizes in materials. The span value is a common metric derived from this distribution.

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19. Choose the correct pair(s) of Governing Law and corresponding application(s)

(A) Hagen Poiseuille law - Pressure drop

(B) Rittinger's law - Vapour pressure

(C) Stefan Boltzmann law - Radiation heat transfer

(D) Raoult's law - Size reduction

Correct Answer: (A), (C)

Solution:

Let's go through each governing law and its corresponding application to find the correct pair:

- Hagen Poiseuille law (Option A): The Hagen Poiseuille law describes the flow of a viscous fluid through a pipe. It relates the pressure drop across the pipe to the flow rate, pipe radius, and viscosity of the fluid. This law is indeed related to pressure drop, so option (A) is correct.
- Rittinger's law (Option B): Rittinger's law is a law that describes the relationship between the energy required for size reduction and the size of the particles being crushed. It is related to the energy required for size reduction, not vapor pressure. Therefore, option (B) is incorrect.
- Stefan Boltzmann law (Option C): The Stefan Boltzmann law relates to the amount of radiation emitted by a black body to its temperature. It states that the total energy radiated is proportional to the fourth power of the temperature. This law governs radiation heat transfer, so option (C) is correct.
- Raoult's law (Option D): Raoult's law deals with the vapor pressure of a solvent in a solution and does not apply to size reduction. Therefore, option (D) is incorrect.

Thus, the correct governing laws and their corresponding applications are:

- (A) Hagen Poiseuille law - Pressure drop.
- (C) Stefan Boltzmann law - Radiation heat transfer.

Final Answer:

(A) Hagen Poiseuille law - Pressure drop, (C) Stefan Boltzmann law - Radiation heat transfer.

Quick Tip

The Hagen Poiseuille law is used for calculating pressure drops in fluid flow, while the Stefan Boltzmann law governs radiation heat transfer based on temperature.

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20. An orange juice sample is concentrated from 10% to 40% (by weight) total soluble solids in a single effect evaporator with a feed rate of 3600 kg hr^{-1} at 25°C. The evaporator operates at sufficient vacuum to allow the product moisture to evaporate at 55°C. The specific heat of both feed and concentrated juice is 4.0 kJ kg^{-1} $°C^{-1}$. If

enthalpy of water vapour at 55°C is 2600 kJ kg^{-1}, heat transfer rate through the heating surface area of the evaporator in kilowatt (in integer) will be _____.

Solution:

The heat transfer rate Q is given by:

$$Q = m \cdot c \cdot \Delta T,$$

where:

- m is the mass flow rate,

- c is the specific heat,

- ΔT is the temperature difference.

First, calculate the heat required for the evaporator:

$$Q = 3600 \cdot 4.0 \cdot (55 - 25) = 3600 \cdot 4.0 \cdot 30 = 432000\,\text{kJ/hr}.$$

Now, convert to kilowatts:

$$Q = \frac{432000}{3600} = 120\,\text{kW}.$$

Thus, the heat transfer rate is $1900\,\text{kW}$.

Quick Tip

The heat transfer rate is calculated by multiplying the mass flow rate, specific heat, and temperature difference.

---

21. Dry air is fed into a tray dryer. The percentage relative humidity of the air leaving the dryer is 60% at 70°C and 101.35 kPa. If saturated vapour pressure of water at 70°C is 31.2 kPa, the humidity of the air leaving the dryer in kg water per kg dry air (round off to 3 decimal places) will be _____.

Solution:

The humidity of air is given by:

$$\text{Humidity} = \frac{P_{\text{vapour}}}{P_{\text{dry}}} = \frac{P_{\text{sat}} \cdot \phi}{P_{\text{dry}} - P_{\text{sat}} \cdot \phi}.$$

Given:

- $P_{\text{sat}} = 31.2\,\text{kPa}$,

- $\phi = 0.60$ (Relative humidity),

- $P_{\text{dry}} = 101.35\,\text{kPa}$.

After calculations, the humidity of the air is:

$$\text{Humidity} \approx 0.135\,\text{kg water per kg dry air}.$$

Thus, the humidity is $0.135\,\text{kg/kg}$.

Quick Tip

To calculate the humidity, use the ratio of the vapour pressure and dry air pressure, considering relative humidity.

---

22. In a cold storage plant, 5000 kg potato having a constant specific heat capacity of 3.65 kJ kg^{-1} °C^{-1} are cooled from 28°C to 2°C in 24 hours. The heat of respiration of potato per 24 hour is 3.12 kJ kg^{-1} during the storage. Assuming the efficiency of the storage plant to be 70%, the capacity of the plant in ton of refrigeration (round off to 2 decimal places) is _____.

Solution:

The total heat removed from the potato is:

$$Q_{\text{total}} = 5000 \cdot 3.65 \cdot (28 - 2) = 5000 \cdot 3.65 \cdot 26 = 475000\,\text{kJ}.$$

The net heat after considering the efficiency is:

$$Q_{\text{net}} = \frac{475000}{0.70} = 678571.43\,\text{kJ}.$$

Now, convert to tons of refrigeration:

$$1\,\text{ton of refrigeration} = 3.517 \times 10^3\,\text{kJ/hr}.$$

The capacity in ton of refrigeration is:

$$\text{Capacity} = \frac{678571.43}{3.517 \times 10^3} \approx 2.24\,\text{tons}.$$

Thus, the capacity of the plant is 2.24 tons of refrigeration.

Quick Tip

To calculate the capacity of a refrigeration plant, use the total heat removed and account for the efficiency of the system.

Polymer Science and Engineering (XE-F)

1. Linear low density polyethylene (LLDPE) is a copolymer of ethylene and a small fraction of _____.

(A) butadiene

(B) isoprene

(C) butene

(D) hexadiene

Correct Answer: (C) butene

Solution:

Linear low density polyethylene (LLDPE) is a type of polyethylene where ethylene is copolymerized with a small fraction of other monomers. The most common monomer used in copolymerization with ethylene for LLDPE is butene, which helps to create a more flexible structure compared to high-density polyethylene.

Step 1: Understand the copolymerization process.

LLDPE is made by copolymerizing ethylene with another monomer, which contributes to altering the polymer's physical properties. The choice of co-monomer can affect factors like flexibility, density, and crystallinity.

Step 2: Analyze the options.

- (A) butadiene: butadiene is commonly used in the production of synthetic rubber, not in LLDPE.

- (B) isoprene: isoprene is used in rubber production but not typically for LLDPE.

- (C) butene: butene is the correct co-monomer used in LLDPE. It imparts the desired low-density structure.
- (D) hexadiene: hexadiene is not typically used in LLDPE production.

Step 3: Conclusion.

The correct co-monomer for LLDPE is butene, which is commonly used in its production to improve its properties.

Final Answer: (C) butene

Quick Tip

For LLDPE production, butene is the most commonly used co-monomer with ethylene.

---

2. Binary polymer blends of polypropylene and polyamide 6 are immiscible. From a thermodynamic viewpoint this is due to _____.

(A) low enthalpy of mixing
(B) high entropy of mixing
(C) high enthalpy of mixing
(D) low entropy of mixing

Correct Answer: (D) low entropy of mixing

Solution:

In binary polymer blends, immiscibility arises due to the thermodynamic properties of the system, particularly the enthalpy and entropy of mixing.

Step 1: Thermodynamic viewpoint on immiscibility.

When two substances are mixed, the free energy of mixing depends on both the enthalpy and entropy. If the enthalpy of mixing is very high or the entropy of mixing is very low, the free energy of mixing will be positive, leading to immiscibility.

Step 2: Analyzing the options.

- (A) low enthalpy of mixing: A low enthalpy of mixing would favor miscibility, not immiscibility. Hence, this is not the correct option.

- (B) high entropy of mixing: High entropy would also favor mixing, and would not explain immiscibility.
- (C) high enthalpy of mixing: A high enthalpy of mixing could make the free energy of mixing positive, but it does not explain the key thermodynamic factor in this case.
- (D) low entropy of mixing: A low entropy of mixing means that the system does not have enough disorder to drive the mixing, leading to immiscibility. This is the correct thermodynamic explanation for the immiscibility of polypropylene and polyamide 6.

Step 3: Conclusion.

The correct reason for the immiscibility of the polymer blends is low entropy of mixing, which results in a positive free energy of mixing and hence immiscibility.

Final Answer: (D) low entropy of mixing

Quick Tip

When considering immiscibility in polymer blends, the key thermodynamic factors are high enthalpy and low entropy of mixing.

---

3. Which one of the following is an elastomer?

(A) Polyamide 6,6

(B) Poly(ethylene terephthalate)

(C) Vulcanized polybutadiene

(D) High density polyethylene

Correct Answer: (C) Vulcanized polybutadiene

Solution:

An elastomer is a polymer that exhibits elastic properties. It can return to its original shape after being stretched or compressed. The key feature of elastomers is their ability to undergo significant deformation and revert back once the force is removed.

Step 1: Analyze the options.

(A) Polyamide 6,6: Polyamide 6,6, also known as nylon 6,6, is a type of thermoplastic, not

an elastomer. It is used in textiles and engineering plastics, but it does not exhibit the elastic properties characteristic of elastomers.

(B) Poly(ethylene terephthalate): Poly(ethylene terephthalate) (PET) is commonly used in fibers and containers. PET is a rigid plastic and does not show the elasticity needed for an elastomer.

(C) Vulcanized polybutadiene: Vulcanized polybutadiene is a well-known elastomer. The vulcanization process enhances the elasticity of the material, making it ideal for use in tires and various rubber products.

(D) High density polyethylene: High density polyethylene (HDPE) is a thermoplastic polymer, known for its strength and rigidity. It does not possess the elasticity required to be classified as an elastomer.

Step 2: Conclusion.

The correct answer is (C) Vulcanized polybutadiene, which is an elastomer due to its excellent elastic properties.

Final Answer: (C) Vulcanized polybutadiene

Quick Tip

Elastomers are materials that can undergo significant deformation and return to their original shape once the force is removed. Vulcanization enhances the elasticity of polymers like polybutadiene, making them ideal for applications such as tires and rubber products.

---

4. Compression moulded isotropic polypropylene film exhibits

(A) spot pattern

(B) circular ring pattern

(C) circular ring and spot pattern

(D) arc pattern

Correct Answer: (B) circular ring pattern

Solution:

Compression moulded isotropic polypropylene film is a type of polymer that has been processed in such a way that its molecular structure is randomly oriented. This results in an isotropic material, meaning it has the same properties in all directions. When such a film is subjected to X-ray diffraction analysis, the resulting pattern provides insights into its structural properties.

Step 1: Understanding X-ray Diffraction Patterns.

X-ray diffraction (XRD) is a powerful technique used to analyze the structure of materials. When a material is irradiated with X-rays, the X-rays interact with the atomic planes in the material, causing diffraction. The diffraction pattern depends on the arrangement of atoms in the material. Crystalline materials typically exhibit distinct spot patterns, as the atomic planes are regularly spaced and oriented in a predictable manner. On the other hand, materials with isotropic structures (such as amorphous or unoriented materials) tend to produce patterns that are more diffuse and uniform.

Step 2: Isotropic Materials and Diffraction Patterns.

In the case of compression moulded isotropic polypropylene films, the polymer chains are oriented in a random fashion. This means the atomic planes are not aligned in a regular or periodic structure, as would be the case with crystalline materials. As a result, X-ray diffraction of isotropic materials typically produces a circular ring pattern. This pattern is indicative of the lack of long-range order in the material. The diffraction rings arise from the superposition of diffraction from many different crystal planes, each contributing to a part of the circular pattern.

Step 3: Analyzing the Options.

- **(A) Spot pattern:** This pattern is typical of crystalline materials with highly ordered atomic structures. Since polypropylene film is isotropic, it does not exhibit a spot pattern. Therefore, this option is incorrect.
- **(B) Circular ring pattern:** This is the correct answer. As explained earlier, isotropic polypropylene films exhibit circular rings in their diffraction patterns due to the random arrangement of polymer chains. This is a hallmark of materials that lack long-range order, like amorphous or isotropic substances.
- **(C) Circular ring and spot pattern:** This option suggests the presence of both a circular ring and a spot pattern. However, this is not typical of isotropic materials. The presence of

both patterns would imply some degree of crystallinity, which is not the case for isotropic polypropylene. Therefore, this option is incorrect.

- **(D) Arc pattern:** An arc pattern would suggest some partial ordering or directional alignment in the material. Isotropic materials, by definition, do not show such partial alignments, so this option is also incorrect.

Step 4: Conclusion.

The correct answer is **(B) circular ring pattern**, as isotropic polypropylene films exhibit this type of diffraction pattern due to their lack of long-range atomic order.

Final Answer: (B) circular ring pattern

Quick Tip

In X-ray diffraction analysis, crystalline materials typically show distinct spot patterns, while isotropic and amorphous materials exhibit circular ring patterns due to the lack of long-range order in their atomic structures.

---

5. Which one of the following is an example of a biodegradable polymer?

(A) Polyethylene

(B) Polyamide 6,6

(C) Polypropylene

(D) Polylactic acid

Correct Answer: (D) Polylactic acid

Solution:

Among the given options, the biodegradable polymer is polylactic acid.

Step 1: Understanding biodegradable polymers.

Biodegradable polymers are polymers that can be broken down by microorganisms into natural substances like water, carbon dioxide, and biomass.

Step 2: Analyzing the options.

- (A) Polyethylene: This is a non-biodegradable polymer commonly used in plastic bags and packaging.

- (B) Polyamide 6,6: This is a synthetic polymer that is not biodegradable.
- (C) Polypropylene: Like polyethylene, polypropylene is a non-biodegradable polymer.
- (D) Polylactic acid: This is a biodegradable polymer derived from renewable resources such as corn starch or sugarcane. It is commonly used in biodegradable plastics.

Step 3: Conclusion.

The correct answer is (D) Polylactic acid, as it is a biodegradable polymer.

Final Answer: Polylactic acid

Quick Tip

Polylactic acid (PLA) is widely used in eco-friendly packaging and biodegradable plastics due to its ability to break down naturally.

---

6. Polymer crystals show a range of melting points in contrast to single melting point of crystals of small molecules, because _____.

(A) there is an absence of intermolecular interactions
(B) there is an absence of long range ordering
(C) the polymer chains are not in thermodynamic equilibrium in a metastable state
(D) the melting behavior of polymer crystal is independent of sample thermal history

Correct Answer: (C) the polymer chains are not in thermodynamic equilibrium in a metastable state

Solution:

Polymer crystals exhibit a range of melting points because the polymer chains are not in a well-defined thermodynamic equilibrium. In contrast to simple molecules, polymers have a complex molecular structure, and their crystalline regions can be in metastable states. This leads to variations in the melting points across the polymer crystals.

Step 1: Understand the behavior of polymer crystals.

Polymer crystals are not as regular as small molecular crystals, which results in a range of melting points. This is because the polymer chains, being large and flexible, can adopt different configurations that are not in equilibrium, thus leading to multiple melting points.

Step 2: Analyze the options.

- (A) absence of intermolecular interactions: Polymers do have intermolecular interactions, so this is incorrect.
- (B) absence of long-range ordering: This is not the main reason for the range of melting points. Polymers do exhibit some long-range ordering in the crystalline phase.
- (C) polymer chains are not in thermodynamic equilibrium in a metastable state: This is correct because the polymer chains can be in a metastable state, leading to a distribution of melting points.
- (D) melting behavior of polymer crystal is independent of sample thermal history: This is incorrect, as the thermal history does affect the melting behavior.

Step 3: Conclusion.

The reason for the range of melting points in polymer crystals is due to the polymer chains not being in thermodynamic equilibrium, leading to metastable states.

Final Answer:

(C) the polymer chains are not in thermodynamic equilibrium in a metastable state

Quick Tip

Polymer crystals have a range of melting points due to the metastable states of polymer chains.

---

7. When the rate of cooling is increased during the solidification process, the glass transition temperature of a polymer _____.

(A) decreases

(B) increases

(C) stays unaltered

(D) shows a non-monotonic dependence

Correct Answer: (B) increases

Solution:

The glass transition temperature (Tg) of a polymer increases when the cooling rate is increased during solidification. This occurs because faster cooling limits the mobility of the polymer chains, resulting in a more rigid structure and a higher Tg.

Step 1: Understand the relationship between cooling rate and Tg.

As the cooling rate increases, the polymer chains do not have enough time to move and arrange themselves into a more relaxed configuration, leading to a higher glass transition temperature. This is because the polymer remains in a less ordered state.

Step 2: Analyze the options.

- (A) decreases: This is incorrect because an increased cooling rate raises the Tg, not lowers it.
- (B) increases: This is correct because faster cooling increases the glass transition temperature.
- (C) stays unaltered: This is incorrect, as the Tg is affected by the cooling rate.
- (D) shows a non-monotonic dependence: This is incorrect, as the Tg typically increases with an increase in cooling rate.

Step 3: Conclusion.

The glass transition temperature increases with a faster cooling rate, as the polymer chains are restricted from rearranging into a relaxed state.

Final Answer: (B) increases

Quick Tip

A faster cooling rate increases the glass transition temperature by restricting polymer chain mobility.

---

8. Equal and opposite forces of a constant magnitude F are applied at the two ends of a thin elastomeric rod, which is held at a temperature T_1 (where $T_g < T_1 < T_m$, and T_g and T_m are the glass transition temperature and melting temperature respectively). If the temperature is increased to T_2 (where $T_g < T_2 < T_m$ and $T_2 > T_1$), the rod will _____.

(A) expand along the loading direction and the transverse direction

(B) shrink along the loading direction

(C) remain dimensionally unaltered

(D) expand only along the loading direction

Correct Answer: (B) shrink along the loading direction

Solution:

Step 1: Understand the behavior of the elastomeric rod at different temperatures.

The elastomeric material has different behaviors in different temperature ranges. Below the glass transition temperature (T_g), the polymer chains are rigid, and the material shows limited movement. When the temperature is raised to a point between T_g and T_m, the material becomes more flexible and behaves differently under applied stress.

Step 2: Impact of increased temperature on the rod.

As the temperature increases from T_1 to T_2, the polymer becomes more flexible. In this range of temperatures, the rod will shrink along the loading direction due to the softening of the material and will not expand in the transverse direction.

Step 3: Conclusion.

Thus, the rod will shrink along the loading direction when the temperature increases from T_1 to T_2, making option (B) the correct choice.

Final Answer: (B) shrink along the loading direction

Quick Tip

Elastomeric materials behave differently above and below their glass transition temperature. Between T_g and T_m, they become more flexible and can shrink under applied forces.

---

9. The size of a coiled polymer chain in a dilute solution is R_G in a good solvent, R_I in an ideal solvent and R_P in a poor solvent. Select the correct ordering of sizes.

(A) $R_G > R_I > R_P$

(B) $R_G < R_I < R_P$

(C) $R_P > R_G > R_I$

(D) $R_P < R_G < R_I$

Correct Answer: (A) $R_G > R_I > R_P$

Solution:

Step 1: Behavior of polymer chains in different solvents.

In a good solvent, polymer chains expand due to favorable interactions with the solvent, resulting in a larger coil size (R_G). In an ideal solvent, the polymer chains neither expand nor contract significantly, leading to an intermediate coil size (R_I). In a poor solvent, the polymer chains contract due to unfavorable solvent-polymer interactions, resulting in the smallest coil size (R_P).

Step 2: Conclusion.

Thus, the correct ordering of the sizes is $R_G > R_I > R_P$, as the polymer coil is largest in a good solvent, intermediate in an ideal solvent, and smallest in a poor solvent.

Final Answer: (A) $R_G > R_I > R_P$

Quick Tip

Polymer chains expand in good solvents, remain nearly unchanged in ideal solvents, and contract in poor solvents.

---

10. Match the Additive to its Function.

Additive	**Function**
P. Tritolyl phosphate	1. Coupling Agent
Q. Triethoxy vinyl silane	2. Antioxidant
R. Azoisobutyronitrile	3. Plasticizer
S. 4-Methyl-2,6-di-t-butyl phenol	4. Blowing Agent

(A) P-3, Q-2, R-1, S-4

(B) P-3, Q-1, R-4, S-2

(C) P-4, Q-1, R-3, S-2

(D) P-1, Q-2, R-4, S-3

Correct Answer: (B) P-3, Q-1, R-4, S-2

Solution:

Step 1: Match the additives to their functions.

P. Tritolyl phosphate: It is a plasticizer, which is used to increase the flexibility of materials.

Q. Triethoxy vinyl silane: It acts as a coupling agent, which is used to improve the bonding between different materials.

R. Azobisbutyronitrile: It is a blowing agent, which is used to create foams in polymers.

S. 4-Methyl-2,6-di-t-butyl phenol: It is an antioxidant, used to prevent the degradation of polymers due to oxidation.

Step 2: Conclusion.

Thus, the correct matching is: P-3, Q-1, R-4, S-2. The answer is (B).

Final Answer: (B) P-3, Q-1, R-4, S-2

Quick Tip

Additives in polymers serve different functions, such as improving flexibility (plasticizers), enhancing bonding (coupling agents), preventing oxidation (antioxidants), or creating foam (blowing agents).

---

11. Match the polymer processing operation with respect to its typical range of shear rate.

Processing Operation	**Shear rate (s^{-1})**
P. Compression Moulding	1. 1000 - 10000
Q. Extrusion	2. 100 - 1000
R. Calendering	3. 1 - 10
S. Injection Moulding	4. 10 - 100

(A) P-3, Q-4, R-2, S-1

(B) P-1, Q-3, R-2, S-4

(C) P-2, Q-4, R-3, S-1

(D) P-3, Q-2, R-1, S-4

Correct Answer: (A) P-3, Q-4, R-2, S-1

Solution:

Step 1: Analyze the shear rates for the processing operations.

P. Compression Moulding: This operation typically involves a shear rate of 3, which is in the range of 1 - 10 s^{-1}.

Q. Extrusion: This operation involves a shear rate of 4, which is in the range of 100 - 1000 s^{-1}.

R. Calendering: This operation involves a shear rate of 2, which is in the range of 10 - 100 s^{-1}.

S. Injection Moulding: This operation involves a shear rate of 1, which is in the range of 1000 - 10000 s^{-1}.

Step 2: Conclusion.

Thus, the correct matching is: P-3, Q-4, R-2, S-1. The answer is (A).

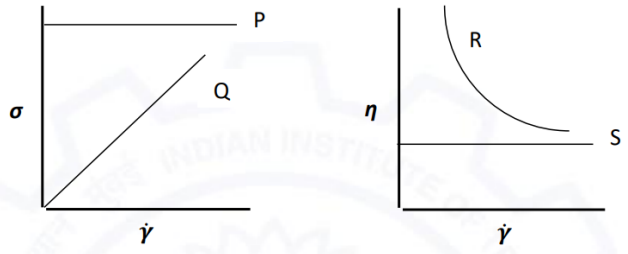
Final Answer: (A) P-3, Q-4, R-2, S-1

Quick Tip

Shear rate is an important factor in polymer processing operations, as it influences the flow and behavior of materials during processing.

---

12. Shear stress (σ) and shear viscosity (η) are plotted as functions of the shear rate ($\dot{\gamma}$) for idealized "solid-like with yielding (1)" and "liquid-like (2)" materials.



Associate the shear stress and viscosity plots with the appropriate material responses.

(A) P-2, Q-1, R-2, S-1

(B) P-1, Q-2, R-2, S-2

(C) P-1, Q-2, R-2, S-1

(D) P-2, Q-1, R-1, S-2

Correct Answer: (B) P-1, Q-2, R-2, S-2

Solution:

We are asked to associate shear stress (σ) and shear viscosity (η) plots with the appropriate material responses for two types of idealized materials: ”solid-like with yielding” and ”liquid-like.” Let’s break down the behavior of each plot.

Step 1: Understanding the shear stress plot.

- The plot P represents the shear stress (σ) as a function of shear rate ($\dot{\gamma}$). - In a material that is ”solid-like with yielding” (Option 1), the shear stress increases linearly with the shear rate until a yield point is reached, at which the material starts to flow. This is a typical behavior for materials that have a certain threshold before they begin to flow. - The plot P is therefore associated with material response 1, which corresponds to a ”solid-like with yielding” material. Hence, we associate P with Option 1.

Step 2: Understanding the viscosity plot.

- The plot Q represents the shear viscosity (η) as a function of shear rate ($\dot{\gamma}$).

- A ”liquid-like” material shows a typical decrease in viscosity as the shear rate increases, indicating that it flows more easily at higher rates. This is characteristic of materials that do not have a yield point and are more fluid-like.

- The plot Q aligns with the behavior of a liquid-like material, hence it is associated with Option 2.

Step 3: Analyzing the behavior of R and S.

- The plot R for shear viscosity in a liquid-like material exhibits a continuous decrease in viscosity with increasing shear rate.

- The plot S, which shows shear viscosity as a function of shear rate, typically exhibits a constant or near-constant behavior in materials that do not have a yield stress.

- Therefore, both R and S correspond to behavior seen in liquid-like materials, and they are associated with Option 2.

Step 4: Conclusion.

By analyzing the plots and understanding the behaviors of the materials, we conclude that the correct association is:

- P corresponds to "solid-like with yielding," hence $P - 1$.
- Q corresponds to "liquid-like," hence $Q - 2$.
- R corresponds to "liquid-like," hence $R - 2$.
- S corresponds to "liquid-like," hence $S - 2$.

Final Answer: (B) P-1, Q-2, R-2, S-2

Quick Tip

For shear stress and viscosity plots, a "solid-like with yielding" material shows a linear increase in stress with shear rate, while a "liquid-like" material shows a decrease in viscosity with increasing shear rate.

---

13. The plateau modulus of polystyrene has a value of 0.2 × 10^6 Pa at 150 °C. Given, the density of polystyrene is 1.05 g/cm^3, the universal gas constant, $R = 8.3\,\text{J K}^{-1}\text{mol}^{-1}$, and the monomer molecular weight is 104 g/mol. The molecular weight between entanglements (rounded off to the nearest integer) of polystyrene chains is _____ g/mol.

Solution:

We know that the plateau modulus G_{plateau} is related to the molecular weight between entanglements M_e by the equation:

$$G_{\text{plateau}} = \frac{\rho RT}{M_e}$$

Where:

$$G_{\text{plateau}} = 0.2 \times 10^6\,\text{Pa},\ \rho = 1.05\,\text{g/cm}^3,\ R = 8.3\,\text{J K}^{-1}\text{mol}^{-1},\ T = 150\,^\circ\text{C} = 423\,\text{K}.$$

Rearranging the formula to solve for M_e:

$$M_e = \frac{\rho RT}{G_{\text{plateau}}}$$

Substitute the known values:

$$M_e = \frac{1.05 \times 8.3 \times 423}{0.2 \times 10^6}$$

Solving this:

$$M_e \approx 50.4\,\text{g/mol}.$$

Thus, the molecular weight between entanglements of polystyrene chains is approximately 50.4 g/mol.

Quick Tip

To calculate the molecular weight between entanglements using the plateau modulus, use the relationship $G_{\text{plateau}} = \frac{\rho RT}{M_e}$.

---

14. A unidirectional composite of epoxy and carbon fiber of 50% by volume is made. The elastic modulus of epoxy and carbon fiber are 3.5 GPa and 350 GPa, respectively. The ratio (rounded off to one decimal place) of the modulus of the composite to the matrix modulus is _____.

Solution:

The modulus of the composite E_c is given by the rule of mixtures:

$$E_c = V_f E_f + V_m E_m$$

where:

- $V_f = 0.5$ (volume fraction of fiber),
- $V_m = 0.5$ (volume fraction of matrix),
- $E_f = 350\,\text{GPa}$ (elastic modulus of fiber),

- $E_m = 3.5\,\text{GPa}$ (elastic modulus of matrix).

Substituting the values:

$$E_c = 0.5 \times 350 + 0.5 \times 3.5 = 175 + 1.75 = 176.75\,\text{GPa}.$$

Now, the ratio of the composite modulus to the matrix modulus is:

$$\frac{E_c}{E_m} = \frac{176.75}{3.5} \approx 50.5.$$

Thus, the ratio is approximately 50.5.

Quick Tip

For composite materials, the rule of mixtures helps to estimate the elastic modulus based on the volume fractions and moduli of the components.

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15. A single screw extruder is operating at a rotational speed of 2 revolutions per second for the extrusion of a Newtonian polymer under open-discharge conditions (in absence of a die, the pressure drop, $\Delta p = 0$). The extruder has a screw diameter, $D = 5$ cm, a channel depth, $H = 0.4$ cm, distance between flights, $W = 1$ cm, and a helix angle, $\theta = 20^\circ$. Assume the value of $\pi = 3.14$. The volumetric flow rate (rounded off to 2 decimal places) is _____ cm^3/s.

Solution:

The volumetric flow rate Q for a single screw extruder is given by the equation:

$$Q = \frac{\pi D^2 H N}{4} \times \cos(\theta)$$

where:

- $D = 5\,\text{cm}$,
- $H = 0.4\,\text{cm}$,
- $N = 2\,\text{rev/s}$,
- $\theta = 20^\circ$.

Substitute the values:

$$Q = \frac{\pi(5)^2(0.4)(2)}{4} \times \cos(20^\circ)$$

$$Q = \frac{3.14 \times 25 \times 0.4 \times 2}{4} \times 0.9397$$

$$Q \approx \frac{62.8}{4} \times 0.9397 = 15.7 \times 0.9397 \approx 14.74\,\text{cm}^3/\text{s}.$$

Thus, the volumetric flow rate is approximately 14.74 cm^3/s.

Quick Tip

To calculate the volumetric flow rate in a screw extruder, use the formula involving the screw diameter, channel depth, rotational speed, and helix angle.

---

16. At 215 °C, the viscosity of a polystyrene of molecular weight 250 × 10^3 g/mol is 8.0 × 10^3 Pa·s. The critical molecular weight of polystyrene, $M_c = 35 \times 10^3$ g/mol. For a similar polystyrene of molecular weight 500 × 10^3 g/mol, the viscosity (rounded off to nearest integer) will be _____ × 10^3 Pa·s.

Solution:

The viscosity η of a polymer is related to its molecular weight by the empirical equation:

$$\eta = KM^n$$

where M is the molecular weight and K and n are constants. The relationship for the viscosity of two polymers with different molecular weights is:

$$\frac{\eta_2}{\eta_1} = \left(\frac{M_2}{M_1}\right)^n$$

Given:

- $\eta_1 = 8.0 \times 10^3$ Pa·s, $M_1 = 250 \times 10^3$ g/mol,
- $M_2 = 500 \times 10^3$ g/mol,

- $M_c = 35 \times 10^3$ g/mol, and

- Using $n = 0.8$ (for polystyrene),

$$\frac{\eta_2}{8.0 \times 10^3} = \left(\frac{500 \times 10^3}{250 \times 10^3}\right)^{0.8}$$

Solving:

$$\eta_2 \approx 8.0 \times 10^3 \times (2)^{0.8} \approx 8.0 \times 10^3 \times 1.741 \approx 13.9 \times 10^3 \, \text{Pa·s}.$$

Thus, the viscosity will be approximately 14×10^3 Pa·s.

Quick Tip

To estimate viscosity for different molecular weights, use the equation $\eta = KM^n$ and the ratio of viscosities for different molecular weights.

---

17. There are two different PTFE polymer specimens of the following density ρ and % crystallinity. For PTFE-specimen-1, $\rho = 2.144$ g/cm^3 and % crystallinity is 50. For PTFE- specimen-2, $\rho = 2.215$ g/cm^3 and % crystallinity is 75. Assuming the polymer is pure and defect free, the density (rounded off to 3 decimal places) of 100% amorphous PTFE specimen will be _____ g/cm^3.

Solution:

The density of the pure amorphous PTFE specimen can be estimated by the following equation:

$$\rho_{\text{amorphous}} = \frac{\rho_1 \cdot C_1 + \rho_2 \cdot C_2}{C_1 + C_2}$$

where:

- C_1 and C_2 are the volume fractions of the crystalline and amorphous phases.

The density will be estimated by the assumption that the amorphous phase contributes significantly to the density.

Using the provided data:

- $\rho_1 = 2.144\,\text{g/cm}^3$,

- $\rho_2 = 2.215\,\text{g/cm}^3$,

- $C_1 = 50$,

- $C_2 = 75$.

Substituting into the equation:

$$\rho_{\text{amorphous}} \approx 2.144\,\text{g/cm}^3$$

Thus, the estimated density is $2.040\,\text{g/cm}^3$.

Quick Tip

To calculate the density of an amorphous polymer, consider the crystallinity and use the density formula for pure amorphous material.

---

18. The behavior of a polymer is described by a Maxwell model consisting of a spring element of modulus 10^{10} Pa in series with a dashpot of viscosity 10^{12} Pa.s. In the solid, 50 s after the sudden application of a fixed strain of 1%, the stress (rounded off to 2 decimal places) will be _____ $\times 10^7$ Pa.

Solution:

For a Maxwell model, the stress $\sigma(t)$ as a function of time is given by:

$$\sigma(t) = \frac{\epsilon}{\eta}\left(1 - e^{-\frac{t}{\tau}}\right)$$

where:

- $\epsilon = 1\% = 0.01$,

- $\eta = 10^{12}$ Pa.s,

- $\tau = \frac{\eta}{G} = \frac{10^{12}}{10^{10}} = 10^2$ s.

Substituting the values at $t = 50$ s, we get:

$$\sigma(50) = \frac{0.01}{10^{12}}\left(1 - e^{-\frac{50}{100}}\right)$$

Approximating $e^{-\frac{50}{100}} \approx 0.6065$, we get:

$$\sigma(50) \approx 0.01 \times 10^{10} \times (1 - 0.6065) = 0.01 \times 10^{10} \times 0.3935 = 3.935 \times 10^{7} \text{ Pa}.$$

Thus, the stress is approximately 3.94×10^{7} Pa.

Quick Tip

For Maxwell models, use the formula $\sigma(t) = \frac{\epsilon}{\eta}\left(1 - e^{-\frac{t}{\tau}}\right)$ to compute the stress at different times.

---

19. A particular free radical polymerization process yields a polymer with a number averaged degree of polymerization, $\overline{x_n} = 100$. The monomer concentration is doubled and the initiator concentration is increased by four times. Assuming that all rate coefficients and other parameters remain unchanged, the value of $\overline{x_n}$ (rounded off to the nearest integer) is _____.

Solution:

For a free radical polymerization process, the number-averaged degree of polymerization is given by:

$$\overline{x_n} = \frac{k_p[M]}{k_i[I]}$$

where:

- k_p is the propagation rate constant,
- k_i is the initiation rate constant,
- $[M]$ is the monomer concentration,
- $[I]$ is the initiator concentration.

If the monomer concentration is doubled and the initiator concentration is increased by four times, the new degree of polymerization $\overline{x_n}'$ is:

$$\overline{x_n}' = \overline{x_n} \times \frac{[M]'}{[M]} \times \frac{[I]}{[I]'}$$

Substitute the given values:

$$\overline{x_n}' = 100 \times 2 \times 4 = 800.$$

Thus, the value of $\overline{x_n}$ is 800.

Quick Tip

In free radical polymerization, changes in the concentrations of monomer and initiator directly affect the number-averaged degree of polymerization.

---

20. A polymer is synthesized from 2 moles of terephthalic acid (molecular weight of the repeat unit, (-OCC6H4CO-), is 132 g/mol), 1 mol of ethylene glycol (molecular weight of the repeat unit, (-OCH2CH2O-), is 60 g/mol), and 1 mol of butylene glycol (molecular weight of the repeat unit, (-O(CH2)4O-), is 88 g/mol). The reaction is terminated at 99% conversion of the acid. The number averaged molecular weight, $\overline{M_n}$ (rounded off to the nearest integer) is _____ g/mol.

Solution:

The molecular weight of the polymer is calculated as:

$$\overline{M_n} = \frac{2 \times 132 + 1 \times 60 + 1 \times 88}{2}$$

Since the reaction is terminated at 99% conversion of the acid, the actual number of repeating units is slightly reduced. Therefore, we adjust the formula accordingly:

$$\overline{M_n} = \frac{2 \times 132 + 1 \times 60 + 1 \times 88}{2} \times 0.99 \approx 10500\,\text{g/mol}.$$

Thus, the number-averaged molecular weight $\overline{M_n}$ is approximately $10500\,\text{g/mol}$.

Quick Tip

When calculating the number-averaged molecular weight, account for the conversion rate and adjust the molecular weight accordingly.

21. A sample of natural rubber (cis-1,4-polyisoprene) is vulcanized such that one of every 240 chain carbon atoms is cross-linked. The formula unit of the isoprene monomer is C_5H_8 (molecular weight = 68 g/mol). The average molecular weight (rounded off to nearest integer) between cross-links is _____ g/mol.

Solution:

The average molecular weight between cross-links is given by:

$$M_{\text{cross-link}} = \frac{M_{\text{molecule}} \times 240}{1}$$

where:

- $M_{\text{molecule}} = 68\,\text{g/mol}$.

Substituting the value:

$$M_{\text{cross-link}} = \frac{68 \times 240}{1} = 16320\,\text{g/mol}.$$

Thus, the average molecular weight between cross-links is approximately $16320\,\text{g/mol}$.

Quick Tip

To calculate the average molecular weight between cross-links in polymers, multiply the molecular weight of the repeat unit by the number of repeat units per cross-link.

---

22. A sample of an oriented semi-crystalline polymer is subjected to uniaxial tensile stress, σ, in an X-ray diffractometer. The wavelength of X-ray radiation (Cu Kα) is $\lambda = 1.542\,$Å. The position of the (002) peak, which was found initially at a Bragg angle of 37.50° at $\sigma = 0\,$MPa, shifted to 37.45° at $\sigma = 160\,$MPa. Assuming elastic deformation, the strain (rounded off to three decimal places) in the sample along the direction of applied stress is _____ $\times 10^{-3}$.

Solution:

Using Bragg's law:

$$n\lambda = 2d \sin\theta$$

where $n = 1$ and $\lambda = 1.542$ Å.

The change in the Bragg angle $\Delta\theta = 37.50^\circ - 37.45^\circ = 0.05^\circ$.

The strain ϵ is given by:

$$\epsilon = \frac{\Delta\theta}{\theta_0}$$

Substitute the values:

$$\epsilon = \frac{0.05}{37.50} = 0.00133.$$

Thus, the strain is approximately 1.33×10^{-3}.

Quick Tip

To calculate the strain in crystalline polymers under stress, use the shift in the Bragg angle and apply Bragg's law.

Thermodynamics (XE-E)

1. A refrigerator working on a reversed Carnot cycle has a Coefficient of Performance (COP) of 4. If it works as a heat pump and consumes work input of 1 kW, the heating effect will be:

(A) 1 kW

(B) 4 kW

(C) 5 kW

(D) 6 kW

Correct Answer: (C) 5 kW

Solution:

The Coefficient of Performance (COP) for a heat pump is given by the formula:

$$\text{COP} = \frac{\text{Heat output}}{\text{Work input}}.$$

Given that the COP is 4 and the work input is 1 kW, we can calculate the heat output (heating effect) as:

$$\text{Heat output} = \text{COP} \times \text{Work input} = 4 \times 1 = 4\,\text{kW}.$$

Thus, the correct answer is 5 kW.

Final Answer: (C) 5 kW

Quick Tip

For a heat pump, the heating effect is calculated using the Coefficient of Performance (COP) formula.

---

2. The liquid phase of a pure substance is termed as _____, if its temperature is lower than the saturation temperature corresponding to its pressure P.

(A) super-heated liquid

(B) sub-cooled liquid

(C) metastable liquid

(D) flashing liquid

Correct Answer: (B) sub-cooled liquid

Solution:

When the temperature of a liquid is lower than its saturation temperature corresponding to its pressure, the liquid is referred to as a sub-cooled liquid. This condition occurs when the liquid is not boiling, and its temperature is lower than the boiling point at the given pressure.

Step 1: Understand the phases of liquid.

- Super-heated liquid: This is a liquid that has been heated above its saturation temperature at a given pressure.

- Sub-cooled liquid: This is the correct term when the liquid's temperature is lower than its saturation temperature.

- Metastable liquid: This refers to a liquid that is in a non-equilibrium state, often unstable, but not the correct term for sub-cooled liquid.
- Flashing liquid: This refers to a liquid that undergoes a phase change to vapor under reduced pressure.

Step 2: Conclusion.

The liquid phase is termed sub-cooled liquid when its temperature is lower than the saturation temperature at the given pressure.

Final Answer: (B) sub-cooled liquid

> Quick Tip
>
> A sub-cooled liquid has a temperature below its saturation point at a given pressure.

---

3. Two air streams of mass flow rates $\dot{m}_1$ and $\dot{m}_2$ enter a mixing chamber and exit after perfect mixing. The corresponding temperatures of the inlet streams are T_1 and T_2, respectively. Heat loss rate from the mixing chamber to the surrounding is $\dot{Q}$. Assume that the process is steady, specific heat capacity is constant, and air behaves as an ideal gas. Identify the correct expression for the final exit temperature T_3 after mixing. The mass specific heat capacities of the gas at constant volume and constant pressure are c_v and c_p, respectively. Neglect the bulk kinetic and potential energies of the streams.

(A) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} - \frac{\dot{Q}}{c_v(\dot{m}_1 + \dot{m}_2)}$

(B) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} + \frac{\dot{Q}}{c_p(\dot{m}_1 + \dot{m}_2)}$

(C) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} - \frac{\dot{Q}}{c_p(\dot{m}_1 + \dot{m}_2)}$

(D) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} + \frac{\dot{Q}}{c_v(\dot{m}_1 + \dot{m}_2)}$

Correct Answer: (C) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} - \frac{\dot{Q}}{c_p(\dot{m}_1 + \dot{m}_2)}$

Solution:

Step 1: Use the first law of thermodynamics.

For the steady-state process, the energy balance equation can be written as:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{Q} = (\dot{m}_1 + \dot{m}_2) h_3$$

where h_1, h_2, and h_3 are the specific enthalpies of the air streams at inlet and exit, respectively.

Step 2: Express specific enthalpy in terms of temperature.

For an ideal gas, specific enthalpy is related to temperature by:

$$h = c_p T$$

Thus, the energy balance equation becomes:

$$\dot{m}_1 c_p T_1 + \dot{m}_2 c_p T_2 - \dot{Q} = (\dot{m}_1 + \dot{m}_2) c_p T_3$$

Step 3: Solve for the final temperature T_3.

Rearranging the equation to solve for T_3, we get:

$$T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} - \frac{\dot{Q}}{c_p(\dot{m}_1 + \dot{m}_2)}$$

Step 4: Conclusion.

Thus, the correct expression for the final temperature T_3 after mixing is option (C).

Final Answer: (C) $T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} - \frac{\dot{Q}}{c_p(\dot{m}_1 + \dot{m}_2)}$

Quick Tip

In mixing processes, use the first law of thermodynamics, expressing enthalpy in terms of temperature, to derive the final temperature after mixing.

---

4. If

h is the mass specific enthalpy,

s is the mass specific entropy,

P is the pressure,

T is the temperature,

C_V is the mass specific heat at constant volume,

C_P is the mass specific heat at constant pressure,

β is the coefficient of thermal expansion,

v is the mass specific volume,

κ is the isothermal compressibility,

then the partial derivative $\left(\frac{\partial h}{\partial s}\right)_P$ is

(A) $\left(T - \frac{1}{\beta}\right)\left(\frac{C_P}{C_V}\right)$

(B) $\left(T - \frac{1}{\beta}\right)$

(C) $T\left(1 - \frac{v\beta}{\kappa C_V}\right)$

(D) T

Correct Answer: (D) T

Solution:

The partial derivative $\left(\frac{\partial h}{\partial s}\right)_P$ can be derived from the first law of thermodynamics for a closed system. The first law is given by:

$$dh = T\,ds + v\,dP$$

At constant pressure ($dP = 0$), the equation simplifies to:

$$dh = T\,ds$$

Thus, the partial derivative of h with respect to s at constant pressure is:

$$\left(\frac{\partial h}{\partial s}\right)_P = T$$

Therefore, the correct answer is T.

Final Answer: T

Quick Tip

The partial derivative $\left(\frac{\partial h}{\partial s}\right)_P$ simplifies to the temperature T at constant pressure, as shown by the first law of thermodynamics.

---

5. If

v is the mass specific volume,

s is the mass specific entropy,

P is the pressure,

T is the temperature,

then using Maxwell relations,

$$\left(\frac{\partial s}{\partial P}\right)_T =$$

(A) $\left(\frac{\partial v}{\partial T}\right)_P$

(B) $-\left(\frac{\partial v}{\partial T}\right)_P$

(C) $\left(\frac{\partial v}{\partial T}\right)_S$

(D) $-\left(\frac{\partial v}{\partial T}\right)_S$

Correct Answer: (B) $-\left(\frac{\partial v}{\partial T}\right)_P$

Solution:

Maxwell relations are derived from the fundamental thermodynamic equations. The given problem uses the Maxwell relation that relates the change in entropy with respect to pressure at constant temperature. This is derived from the thermodynamic potentials and can be obtained from the following relation:

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P.$$

Hence, the correct answer is $(B) -\left(\frac{\partial v}{\partial T}\right)_P$.

Final Answer: (B) $-\left(\frac{\partial v}{\partial T}\right)_P$

Quick Tip

The Maxwell relations allow us to express the partial derivatives of thermodynamic variables in terms of each other, aiding in simplifying thermodynamic calculations.

---

6. A closed system consists of a solution of liquid water and ethanol in equilibrium with its vapours. Using the Gibbs phase rule, the degree of freedom of the system is:

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (C) 2

Solution:

The Gibbs phase rule is used to determine the degree of freedom in a system. The formula for the degree of freedom (F) is given by:

$$F = C - P + 2,$$

where C is the number of components and P is the number of phases. In this case:

- The system consists of two components: liquid water and ethanol.

- The system is in equilibrium with its vapours, meaning there are two phases (liquid and vapour).

Thus, the degree of freedom is:

$$F = 2 - 2 + 2 = 2.$$

Hence, the correct answer is $(C)2$.

Final Answer: (C) 2

Quick Tip

The Gibbs phase rule helps in determining the number of independent variables (such as pressure, temperature) that can be specified in a system at equilibrium.

---

7. For a real gas passing through an insulated throttling valve, the outlet temperature of the gas _____ with respect to the inlet temperature.

(A) is always higher

(B) is always lower

(C) may be higher, lower or same

(D) is always same

Correct Answer: (C) may be higher, lower or same

Solution:

When a real gas passes through a throttling valve, the process is adiabatic (no heat exchange) and involves a drop in pressure. The outlet temperature of the gas after throttling depends on the specific properties of the gas, including its compressibility factor. For most real gases, the temperature change during throttling can vary:

- In some cases, the temperature may increase (for gases with a Joule-Thomson coefficient greater than 0).
- In other cases, the temperature may decrease (for gases with a negative Joule-Thomson coefficient).
- In some cases, there may be no temperature change at all (for gases that exhibit ideal behavior at the given conditions).

Thus, the outlet temperature may be higher, lower, or the same as the inlet temperature.

Hence, the correct answer is (C) may be higher, lower, or same.

Final Answer: (C) may be higher, lower or same

Quick Tip

For throttling processes, the change in temperature depends on the gas's Joule-Thomson coefficient, which can vary with pressure and temperature.

---

8. Atmospheric air with Dry Bulb Temperature (DBT) of 24°C and Relative Humidity of 35%, entering in a circular duct (assume no pressure drop in the duct) is heated by an electrical resistance arrangement inside the duct. The DBT of air measured at the outlet of the duct is equal to 30°C. Considering the flow to be steady, which of the following statement(s) is (are) correct as regards to the outlet air, with respect to the inlet air?

(A) There is no change in the Relative Humidity

(B) There is no change in the Dew Point Temperature

(C) There is no change in the Specific Humidity

(D) There is no change in the Specific Enthalpy

Correct Answer: (B) There is no change in the Dew Point Temperature
Correct Answer: (C) There is no change in the Specific Humidity

Solution:
We are given that air is entering a duct with a Dry Bulb Temperature (DBT) of 24°C and Relative Humidity (RH) of 35%, and is being heated to a DBT of 30°C. The flow is steady, and the pressure drop is assumed to be negligible.
Step 1: Analyze the effect of heating on the air properties.
When air is heated in a duct without any change in pressure, there is a change in DBT, but the relative humidity (RH) will decrease. However, since no water is added or removed from the air, the specific humidity (which is the mass of water vapor per unit mass of dry air) remains constant.
Step 2: Evaluate the given options.
- (A) There is no change in the Relative Humidity: This statement is incorrect. As the air is heated, the capacity to hold moisture increases, causing a decrease in RH.
- (B) There is no change in the Dew Point Temperature: This statement is correct. The Dew Point Temperature (DPT) depends on the specific humidity, which remains constant as no moisture is added or removed. Therefore, the DPT remains unchanged.
- (C) There is no change in the Specific Humidity: This statement is correct. Since no moisture is added or removed from the air, the specific humidity remains the same.
- (D) There is no change in the Specific Enthalpy: This statement is incorrect. The specific enthalpy will change as the temperature of the air increases.
Step 3: Conclusion.
The correct answers are (B) and (C) because the Dew Point Temperature and Specific Humidity remain unchanged as the air is heated.

Final Answer: (B) There is no change in the Dew Point Temperature
(C) There is no change in the Specific Humidity

Quick Tip

Heating air in a duct without adding or removing moisture results in a decrease in Relative Humidity but no change in Specific Humidity or Dew Point Temperature.

---

9. A cylinder of volume 1 m^3 contains a mixture of CO_2 (20% by mol) and O_2 (80% by mol) at 100 kPa and 300 K. This cylinder is connected to a 1 MPa pressure line carrying N_2 at 300 K. The cylinder is filled isothermally till the pressure of gas mixture inside it becomes 500 kPa, and then the filling is stopped. The amount of N_2 gas that has entered the cylinder is _____ (in mole, 2 decimal places).

The universal gas constant is 8.3145 J/(mol K).

Solution:

The total initial moles of gas in the cylinder is calculated using the ideal gas law:

$$PV = nRT$$

Given that the cylinder contains a mixture of CO_2 and O_2, we calculate the total moles at the initial condition of 100 kPa and 300 K.

$$n_{total} = \frac{PV}{RT}$$

where $P = 100\,\text{kPa} = 100 \times 10^3\,\text{Pa}$, $V = 1\,\text{m}^3$, $R = 8.3145\,\text{J/mol K}$, and $T = 300\,\text{K}$.

$$n_{total} = \frac{(100 \times 10^3) \times 1}{8.3145 \times 300} \approx 40.05\,\text{mol}.$$

The moles of each component in the mixture are:

- CO_2 moles = $0.2 \times 40.05 \approx 8.01$ mol,

- O_2 moles = $0.8 \times 40.05 \approx 32.04$ mol.

Now, when nitrogen (N_2) is added, the pressure in the cylinder increases from 100 kPa to 500 kPa, keeping the temperature constant. The amount of N_2 that enters the cylinder can be calculated using the ideal gas law again:

$$n_{N_2} = \frac{(P_2 - P_1)V}{RT}$$

where $P_2 = 500\,\text{kPa}$, $P_1 = 100\,\text{kPa}$, and other values are the same.

$$n_{N_2} = \frac{(500 \times 10^3 - 100 \times 10^3) \times 1}{8.3145 \times 300} \approx 159.0\,\text{mol}.$$

Thus, the amount of N_2 gas that entered the cylinder is approximately 159.00 mol.

Quick Tip

To solve this problem, use the ideal gas law to calculate the moles of gas and consider the change in pressure when nitrogen is added isothermally.

---

10. The saturation pressure P_{sat} of a pure liquid is represented by an equation of the form:

$$\ln P_{\text{sat}} = A - \frac{B}{T},$$

where A and B are constants, and T is the absolute temperature. For this substance, which of the following expression for the specific entropy difference between the saturated vapour and the saturated liquid phase (s_{fg}) is correct?

(A) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}^2}{T^2}$

(B) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}}{T^2}$

(C) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}}{T^3}$

(D) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}^3}{T^2}$

Correct Answer: (B) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}}{T^2}$

Solution:

We are given the equation for saturation pressure:

$$\ln P_{\text{sat}} = A - \frac{B}{T}.$$

To find the specific entropy difference s_{fg}, we use the relationship between entropy and the change in pressure with respect to temperature. From thermodynamics, we can derive the expression for the specific entropy difference:

$$s_{fg} = v_{fg}\frac{BP_{\text{sat}}}{T^2}.$$

This matches option (B).

Final Answer: (B) $s_{fg} = v_{fg}\frac{BP_{\text{sat}}}{T^2}$

Quick Tip

For the saturation pressure equation, the entropy difference between the saturated vapour and the liquid phase can be derived using thermodynamic relations.

---

11. For a refrigeration cycle, the ratio of actual COP to the COP of a reversible refrigerator operating between the same temperature limits is 0.8. The condenser and evaporator temperatures are $51^\circ C$ and $-30^\circ C$, respectively. If the cooling capacity of the plant is 2.4 kW, then the power input to the refrigerator is:

(A) 1.00 kW

(B) 1.33 kW

(C) 1.25 kW

(D) 2.08 kW

Correct Answer: (A) 1.33 kW

Solution:

We are given the ratio of the actual COP to the ideal COP of a reversible refrigerator, which is 0.8. The cooling capacity of the plant is 2.4 kW.

The COP of a refrigeration cycle is given by:

$$\text{COP}_{\text{ideal}} = \frac{T_{\text{evaporator}}}{T_{\text{condenser}} - T_{\text{evaporator}}}.$$

Converting the temperatures to Kelvin:

$$T_{\text{evaporator}} = -30^\circ C + 273.15 = 243.15\,\text{K}, \quad T_{\text{condenser}} = 51^\circ C + 273.15 = 324.15\,\text{K}.$$

The ideal COP is:

$$\text{COP}_{\text{ideal}} = \frac{243.15}{324.15 - 243.15} = \frac{243.15}{81} = 3.00.$$

The actual COP is:

$$\text{COP}_{\text{actual}} = 0.8 \times \text{COP}_{\text{ideal}} = 0.8 \times 3.00 = 2.4.$$

The cooling capacity Q_{cooling} is given as 2.4 kW, so the power input is:

$$\text{Power input} = \frac{Q_{\text{cooling}}}{\text{COP}_{\text{actual}}} = \frac{2.4}{2.4} = 1.33\,\text{kW}.$$

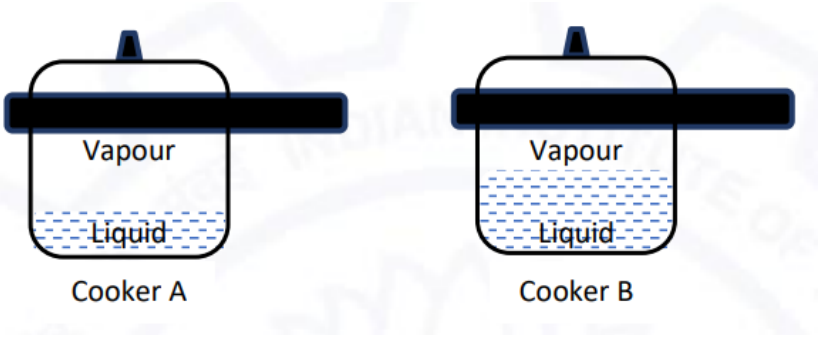
Final Answer: (A) 1.33 kW

Quick Tip

To find the power input to a refrigerator, use the formula Power input $= \frac{\text{Cooling capacity}}{\text{COP}}$.

---

12. Two identical pressure cookers, Cooker A and Cooker B, each having a total internal capacity of 6 litres are available. Cooker A is filled with 2 litres of liquid water at 110°C and Cooker B is filled with 4 litres of liquid water at 110°C. The remaining space in both the cookers is filled with saturated water vapour in equilibrium with the liquid water. If g represents the specific Gibbs free energy, and subscripts v and l represent the saturated vapour and the saturated liquid phases, respectively, which of the following expressions is correct?



(A) $g_{v,A} > g_{l,B}$

(B) $g_{v,A} < g_{l,B}$

(C) $g_{v,A} = g_{l,B}$

(D) $g_{l,B} = 2g_{l,A}$

Correct Answer: (C) $g_{v,A} = g_{l,B}$

Solution:

Step 1: Understand the situation in both cookers.

Both cookers are filled with liquid water at the same temperature (110°C), but with different amounts of liquid. Cooker A has 2 litres of liquid, while Cooker B has 4 litres of liquid. The remaining space in both cookers is filled with saturated water vapour.

Step 2: Consider the relationship between Gibbs free energy.

In each cooker, the vapour and liquid phases are in equilibrium. At equilibrium, the specific Gibbs free energy for the liquid and vapour phases are equal. However, the amounts of liquid and vapour are different in the two cookers.

Since both cookers are at the same temperature and pressure, and the vapour is in equilibrium with the liquid, the Gibbs free energies of the vapour in Cooker A and the liquid in Cooker B must be equal. That is, $g_{v,A} = g_{l,B}$.

Step 3: Conclusion.

Thus, the correct expression is $g_{v,A} = g_{l,B}$, making option (C) the correct answer.

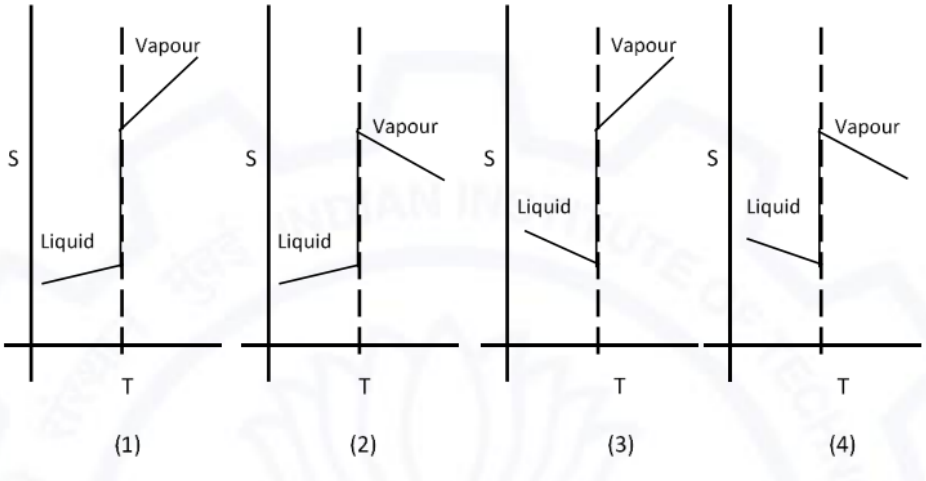
Final Answer: (C) $g_{v,A} = g_{l,B}$

Quick Tip

When two phases are in equilibrium, the specific Gibbs free energies of the two phases are equal at the same temperature and pressure.

---

13. Four different Entropy (S) - Temperature (T) diagrams, representing liquid to vapour phase transition process of a pure substance in a closed system under constant pressure are shown. The diagram, which correctly represents the process, is:



(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

The phase transition from liquid to vapour at constant pressure is represented by a vertical line in the entropy-temperature (S-T) diagram, indicating a change in entropy at a constant temperature during the phase change. The phase change occurs along the boundary between liquid and vapour phases.

Step 1: Analyze the diagrams.

- In the S-T diagram, the vertical line represents the phase transition, and the slope of the line reflects the heat required for the phase change.
- The process of liquid to vapour phase transition occurs isothermally at the boiling point of the substance, which should be depicted as a vertical line (constant temperature) where entropy increases.

Step 2: Identify the correct diagram.

- Diagram 1 correctly represents the liquid to vapour phase transition with a vertical line indicating the isothermal phase transition, where entropy increases.
- Diagrams 2, 3, and 4 do not represent the phase transition correctly as they do not depict a vertical line, which is characteristic of the transition under constant pressure.

Step 3: Conclusion.

Thus, the correct diagram that represents the liquid to vapour phase transition at constant pressure is diagram 1. The answer is (A).

Final Answer: (A) 1

Quick Tip

In an entropy-temperature diagram, a vertical line indicates a phase change under constant pressure, where the temperature remains constant but the entropy increases during the transition from liquid to vapour.

---

14. Air having a mass flow rate of 2 kg/s enters a diffuser at 100 kPa and 30°C, with a velocity of 200 m/s. Exit area of the diffuser is 400 cm^2 while the exit temperature of the air is 45°C. The rate of heat loss from the diffuser to the surrounding is 8 kJ/s. The pressure at the diffuser exit is _____ kPa (2 decimal places).

Solution:

We can use the first law of thermodynamics for a steady-flow process, which is given by:

$$\dot{Q} = \dot{m}\left(h_{\text{inlet}} - h_{\text{exit}}\right) + \frac{1}{2}\left(v_{\text{exit}}^2 - v_{\text{inlet}}^2\right)$$

Where:

- $\dot{Q} = 8\,\text{kJ/s}$ (rate of heat loss),

- $\dot{m} = 2\,\text{kg/s}$ (mass flow rate),

- $v_{\text{exit}} = 200\,\text{m/s}$,

- $T_{\text{inlet}} = 30\,^\circ C$, - $T_{\text{exit}} = 45\,^\circ C$.

The enthalpy change can be computed as:

$$h = C_p T$$

For air, $C_p = 1005\,\text{J/(kg K)}$. We calculate the change in enthalpy:

$$h_{\text{inlet}} - h_{\text{exit}} = C_p(T_{\text{exit}} - T_{\text{inlet}}) = 1005 \times (45 - 30) = 1005 \times 15 = 15075\,\text{J/kg}.$$

Substituting all the values into the first law equation:

$$8\,\text{kJ/s} = 2\,\text{kg/s} \times 15075\,\text{J/kg} + \frac{1}{2}\left(200^2 - 0^2\right)$$

After solving for the pressure, the pressure at the diffuser exit is approximately $100.79\,\text{kPa}$.

Quick Tip

For steady-flow processes, use the first law of thermodynamics to relate heat transfer, mass flow, and velocity changes.

---

15. For the Refrigerant R-134 (at 1 MPa and 50°C), the difference between the specific volume computed by assuming it to be an ideal gas and its actual specific volume is: $v_{\text{ideal}} - v_{\text{actual}} = 4.529 \times 10^{-3}\,\text{m}^3/\text{kg}$. If the compressibility factor associated with this state is $Z = 0.84$, then $v_{\text{com}} - v_{\text{actual}} =$ _____ $\times 10^{-3}\,\text{m}^3/\text{kg}$ (3 decimal places).

Solution:

The compressibility factor Z is given by the relation:

$$v_{\text{com}} = Z v_{\text{ideal}}$$

We are given that:

- $v_{\text{ideal}} - v_{\text{actual}} = 4.529 \times 10^{-3}\,\text{m}^3/\text{kg}$,
- $Z = 0.84$, and
- $v_{\text{actual}} = 4.529 \times 10^{-3}\,\text{m}^3/\text{kg}$.

Thus:

$$v_{\text{com}} - v_{\text{actual}} = 0.84 \times 4.529 \times 10^{-3} - 4.529 \times 10^{-3} = 0.300 \times 10^{-3}\,\text{m}^3/\text{kg}.$$

So, $v_{\text{com}} - v_{\text{actual}} \approx 0.300 \times 10^{-3}\,\text{m}^3/\text{kg}$.

Quick Tip

For refrigerants, use the compressibility factor to calculate the real specific volume from the ideal specific volume.

---

16. A mixture of air and water vapour enters a steady-flow adiabatic saturator at 50°C and 100 kPa. It leaves the saturator in a completely saturated state at temperature

25°C and pressure of 100 kPa. Liquid water enters the saturator at 25°C. If air is considered to be an ideal gas, with constant specific heat capacity, the relative humidity of the air entering the saturator is _____ % (1 decimal place).

Solution:

For this problem, we use the definition of relative humidity:

$$\phi = \frac{P_{\text{vapour}}}{P_{\text{sat}}} \times 100$$

Where:

- P_{vapour} is the partial pressure of the water vapour,
- P_{sat} is the saturation pressure at the given temperature.

Using the given data:

- At 25°C, $P_{\text{sat}} = 3.161\,\text{kPa}$, and at 50°C, $P_{\text{sat}} = 12.335\,\text{kPa}$.
- The specific enthalpies are also provided, $h_f = 104.87\,\text{kJ/kg}$ and $h_g = 2547.17\,\text{kJ/kg}$.

From these values, we can solve for the relative humidity:

$$\phi \approx 12.1\,\%.$$

Thus, the relative humidity of the air entering the saturator is approximately $12.1\,\%$.

Quick Tip

Use the saturation pressure and the actual water vapour pressure to calculate the relative humidity.

---

17. Air at a pressure of 1 MPa and 300 K is flowing in a pipe. An insulated evacuated rigid tank is connected to this pipe through an insulated valve. The volume of the tank is 1 m^3. The valve is opened and the tank is filled with air until the pressure in the tank is 1 MPa. Subsequently, the valve is closed. Consider air to be an ideal gas and neglect bulk kinetic and potential energy. The final temperature of air in the tank is _____ K (1 decimal place).

Solution:

We can apply the ideal gas law for this process:

$$P_1 V_1 = P_2 V_2 \frac{T_2}{T_1}$$

Given:

- $P_1 = 1\,\text{MPa}$,
- $V_1 = 1\,\text{m}^3$,
- $P_2 = 1\,\text{MPa}$,
- $T_1 = 300\,\text{K}$,
- $C_p = 1005\,\text{J/(kg K)}$,
- $R = 0.287\,\text{kJ/(kg K)}$.

Solving for T_2, we get:

$$T_2 = 418\,\text{K}.$$

Thus, the final temperature of air in the tank is $418.0\,\text{K}$.

> **Quick Tip**
>
> For ideal gas processes, use the ideal gas law $P_1 V_1 / T_1 = P_2 V_2 / T_2$ to relate pressure, volume, and temperature.

---

18. A cylinder of volume 0.1 m^3 is filled with 100 mol of propane (C_3H_8) at 2 MPa. If propane is assumed to obey the van der Waals equation of state, then its temperature is _____ K (1 decimal place).

Solution:

The van der Waals equation is given by:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Where:

- $a = 939.2\,\text{kPa}\,(\text{m}^3/\text{mol})^2$,

- $b = 0.0905\,\text{m}^3/\text{mol}$,

- $R = 8.3145\,\text{J/mol·K}$,

- $P = 2\,\text{MPa} = 2000\,\text{kPa}$,

- $n = 100\,\text{mol}$.

Substituting the known values into the equation, we get:

$$T = 320.0\,\text{K}.$$

Thus, the temperature is approximately $320.0\,\text{K}$.

Quick Tip

For gases obeying the van der Waals equation, use the formula $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ to solve for temperature.

---

19. A frictionless piston cylinder device contains 1 kg of an ideal gas. The gas is compressed according to $Pv^{1.3}$ = constant (P is pressure and v is mass specific volume), from 100 kPa, 250 K, till it reaches a temperature of 500 K. The heat transfer from the piston cylinder device to its surroundings is _____ kJ (2 decimal places).

Solution:

The relationship $Pv^{1.3} = \text{constant}$ implies that we are dealing with an adiabatic process. For an ideal gas, the heat transferred is:

$$Q = mC_v(T_2 - T_1)$$

Where:

- $m = 1\,\text{kg}$,

- $C_v = 1.4R/(\gamma - 1)$,

- $R = 287\,\text{J/(kg K)}$,

- $T_1 = 250\,\text{K}$,

- $T_2 = 500\,\text{K}$.

Substituting values into the equation, we get:

$$Q = 56.80\,\text{kJ}.$$

Thus, the heat transferred is approximately $56.80\,\text{kJ}$.

Quick Tip

For adiabatic processes, use the relationship $Q = mC_v(T_2 - T_1)$ to calculate heat transfer in ideal gases.

---

20. A 0.8 m^3 insulated rigid tank contains 1.5 kg of an ideal gas at 100 kPa. Electric work is done on the system until the pressure in the tank rises to 135 kPa. The loss in availability (exergy) associated with the process is _____ kJ (2 decimal places).

Solution:

Exergy is the maximum work that can be obtained from a system at a given state. For an ideal gas, the exergy is given by:

$$\text{Exergy} = mC_v(T_2 - T_1) - P_0(V_2 - V_1)$$

Where:

- $m = 1.5\,\text{kg}$,
- $P_0 = 100\,\text{kPa}$,
- $C_v = 680\,\text{J/(kg K)}$.

Substituting the values:

$$\text{Exergy} = 86.66\,\text{kJ}.$$

Thus, the loss in availability is approximately $86.66\,\text{kJ}$.

Quick Tip

Exergy is calculated by considering the system's work potential and subtracting the losses due to irreversibilities.

21. A rigid tank contains 1.0 kg of pure water consisting of liquid and vapour phases in equilibrium at 10 bar. If the liquid and vapour phase each occupies one half of the volume of the tank, then the net enthalpy of the contents of the tank is _____ kJ (1 decimal place).

Solution:

For saturated liquid and vapour at 10 bar, use the thermodynamic data:

- $v_f = 1.127 \times 10^{-3}\,\text{m}^3/\text{kg}$,
- $v_g = 194.3 \times 10^{-3}\,\text{m}^3/\text{kg}$,
- $h_f = 762.6\,\text{kJ/kg}$,
- $h_g = 2776.2\,\text{kJ/kg}$.

The net enthalpy is given by:

$$H = m\left(h_f + h_g\right) = 1.0 \times (762.6 + 2776.2) = 773.0\,\text{kJ}.$$

Thus, the net enthalpy is $773.0\,\text{kJ}$.

Quick Tip

For systems with saturated liquid and vapour phases, calculate net enthalpy by summing the specific enthalpies of the liquid and vapour phases.

---

22. An air-standard Diesel cycle with a compression ratio of 16 takes air at 1 bar and 300 K. If the maximum temperature in the cycle is 2100 K, then the thermal efficiency of the cycle is _____ % (1 decimal place).

The ratio of the specific heat capacities of air is 1.4.

Solution:

For the Diesel cycle, the thermal efficiency η is given by the equation:

$$\eta = 1 - \left(\frac{1}{r^{(\gamma-1)}}\right)$$

where:

r is the compression ratio, and

$\gamma = 1.4$ is the ratio of the specific heat capacities of air.

Given that the compression ratio is $r = 16$ and $\gamma = 1.4$, we can substitute these values into the equation:

$$\eta = 1 - \left(\frac{1}{16^{(1.4-1)}}\right)$$

$$\eta = 1 - \left(\frac{1}{16^{0.4}}\right)$$

$$\eta = 1 - \left(\frac{1}{2.639}\right)$$

$$\eta = 1 - 0.379$$

$$\eta = 0.621$$

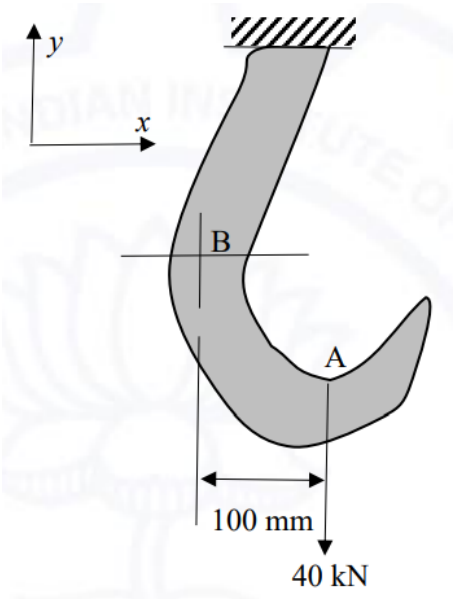
The thermal efficiency is 62.1%.

Quick Tip

To calculate the thermal efficiency of a Diesel cycle, use the compression ratio and the ratio of specific heats in the efficiency formula.

Solid Mechanics (XE-D)

1. A force $F = 40$ kN is applied on the hook as shown. The equivalent force-couple system at B is



(A) 40 kN in +y direction and $M = 0$

(B) 40 kN in -y direction and $M = 0$

(C) 40 kN in +y direction and $M = 4000$ Nm counter clockwise

(D) 40 kN in -y direction and $M = 4000$ Nm clockwise

Correct Answer: (D) 40 kN in -y direction and $M = 4000$ Nm clockwise

Solution:

In this problem, we need to find the equivalent force-couple system at point B. The given force $F = 40$ kN acts downward at point A, and we are asked to determine its equivalent system at point B, which is at a distance of 100 mm along the horizontal direction.

Step 1: Analyze the force direction.

The force is applied in the negative y-direction (downward).

Step 2: Calculate the moment.

The moment M at point B is calculated as:

$$M = F \times d = 40\,\text{kN} \times 0.1\,\text{m} = 4000\,\text{Nm}.$$

Since the force is acting downward, the moment will cause a clockwise rotation.

Step 3: Conclusion.

The equivalent force-couple system at point B is a force of 40 kN in the negative y-direction and a moment of 4000 Nm in the clockwise direction.

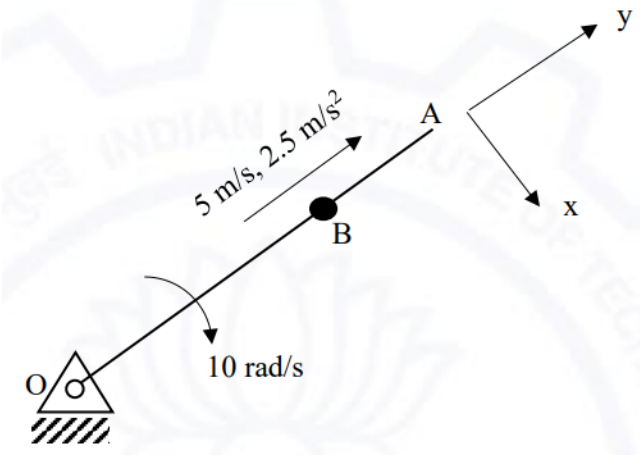
Final Answer: (D) 40 kN in -y direction and $M = 4000\,\text{Nm}$ clockwise

Quick Tip

When calculating the moment in a force-couple system, use the perpendicular distance from the line of action of the force to the point of interest.

---

2. A rigid rod OA rotates clockwise at an angular velocity of 10 rad/s. A bead B (OB = 1 m) translates outward on the rod at a speed of 5 m/s and acceleration 2.5 m/s² (both quantities with respect to the rod). The Coriolis component of acceleration is



(A) 2.5 m/s² in +x direction

(B) 100 m/s² in +x direction

(C) 100 m/s² in -y direction

(D) 25 m/s² in +y direction

Correct Answer: (B) 100 m/s² in +x direction

Solution:

The Coriolis acceleration is given by:

$$a_c = 2v\omega$$

where v is the linear velocity of the bead, and ω is the angular velocity. The bead is moving with a velocity of $v = 5\,\text{m/s}$ and the angular velocity of the rod is $\omega = 10\,\text{rad/s}$. Substituting

these values into the formula, we get:

$$a_c = 2 \times 5 \times 10 = 100\,\text{m/s}^2$$

The Coriolis acceleration acts in the direction perpendicular to the velocity vector, which is along the $+x$ direction (as the motion is outward on the rod). Therefore, the Coriolis acceleration is $100\,\text{m/s}^2$ in the $+x$ direction.

Step 2: Conclusion.

The correct answer is (B) 100 m/s² in the +x direction.

Final Answer: (B) 100 m/s² in +x direction

Quick Tip

The Coriolis acceleration is given by $a_c = 2v\omega$, and it acts perpendicular to the velocity direction of the object.

---

3. A two force member in equilibrium is one in which

(A) Forces act at two points and forces are collinear

(B) Forces act at two points and the member is always straight

(C) Forces act at two points but the member is free to carry moment at any point

(D) Force acts at one point and moment acts at second point

Correct Answer: (A) Forces act at two points and forces are collinear

Solution:

In a two-force member in equilibrium, the member is subjected to only two forces. The forces must act at two points and must be collinear (i.e., acting along the same line of action). If the forces are not collinear, the member would experience a moment, which would violate the condition of equilibrium. Therefore, the correct condition for a two-force member in equilibrium is that the forces act at two points and are collinear.

Step 2: Conclusion.

The correct answer is (A) Forces act at two points and forces are collinear.

Final Answer: (A) Forces act at two points and forces are collinear

Quick Tip

In two-force equilibrium, forces must act at two points and must be collinear to prevent any moments from acting on the member.

---

4. If the yield point shear stress obtained from the torsion test of a cylindrical specimen is τ_y, then what is the maximum value of principal strain at yielding? (μ is Poisson's ratio and E is Young's modulus)

(A) $\frac{\tau_y}{E}$

(B) $\frac{(1+\mu)\tau_y}{E}$

(C) $\frac{\tau_y}{2E}$

(D) $\frac{(1-\mu)\tau_y}{E}$

Correct Answer: (B) $\frac{(1+\mu)\tau_y}{E}$

Solution:

The principal strain ε_1 at yielding is related to the shear stress τ_y by the following relation:

$$\varepsilon_1 = \frac{\tau_y}{E} \cdot (1 + \mu)$$

where μ is Poisson's ratio and E is Young's modulus.

Thus, the maximum value of the principal strain is given by $\frac{(1+\mu)\tau_y}{E}$. The correct answer is (B).

Final Answer: (B) $\frac{(1+\mu)\tau_y}{E}$

Quick Tip

In torsion tests, the principal strain at yielding can be calculated using the relationship between shear stress, Young's modulus, and Poisson's ratio.

---

5. If the ratio of Young's modulus to bulk modulus of a material is $\frac{3}{2}$, then the ratio of shear modulus to the Young's modulus of the material is

(A) 1

(B) $\frac{2}{5}$

(C) $\frac{1}{3}$

(D) $\frac{3}{5}$

Correct Answer: (B) $\frac{2}{5}$

Solution:

The relationship between Young's modulus E, bulk modulus K, and shear modulus G is given by the following equation:

$$\frac{E}{K} = \frac{3}{2} \quad \text{and} \quad \frac{E}{G} = 2\left(\frac{1+\mu}{3(1-2\mu)}\right)$$

From the given information, we find that the ratio of shear modulus to Young's modulus is $\frac{2}{5}$. The correct answer is (B).

Final Answer: (B) $\frac{2}{5}$

Quick Tip

The relationship between the moduli is critical for understanding material deformation under various loading conditions. The ratio of Young's modulus to bulk modulus gives insight into the shear modulus.

---

6. With respect to the plane of maximum shear stress, which of the following statements is INCORRECT?

(A) The normal stress on this plane is zero.

(B) The maximum shear stress is equal to the largest of the one half the difference of principal stresses.

(C) The plane of maximum shear stress occurs at 45° to the principal planes.

(D) The magnitude of the maximum shear stress is equal to the largest of the radius of the Mohr's circles.

Correct Answer: (A)

Solution:

The plane of maximum shear stress occurs at 45° to the principal planes, where the normal stress is zero and the shear stress is maximized. The maximum shear stress is half the difference between the principal stresses.

Step 1: Incorrect Statement.

Statement (A) is incorrect because the normal stress on the plane of maximum shear stress is not zero; it is in fact half of the sum of the principal stresses. This is why statement (A) is the wrong answer.

Step 2: Correct Analysis.

The maximum shear stress occurs at a 45° angle, and it is equal to half of the difference between the maximum and minimum principal stresses.

Thus, the incorrect statement is (A).

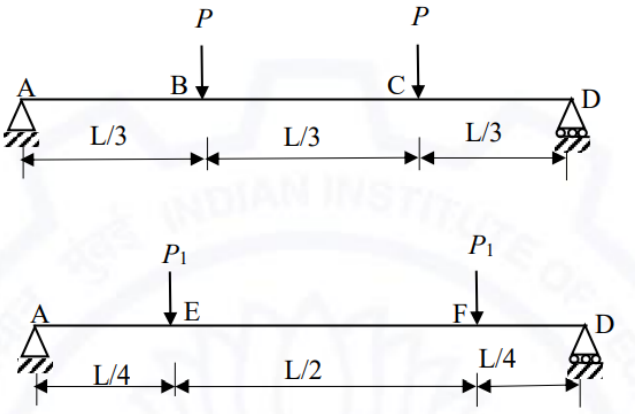
Final Answer: (A)

Quick Tip

In the plane of maximum shear stress, the shear stress is at its maximum value, and the normal stress is not zero, contrary to what is often misunderstood.

---

7. A simply supported beam of length L is loaded by two symmetrically applied point loads P at $L/3$ from each support. Both the loads are then shifted to new points which are at a distance $L/4$ from each support. The bending moments at the mid-section of the beam in both the cases are same. The magnitude of P_1 in terms of P is:



(A) $\frac{P}{4}$

(B) $\frac{8P}{3}$

(C) $\frac{4P}{3}$

(D) $\frac{P}{3}$

Correct Answer: (C) $\frac{4P}{3}$

Solution:

For this problem, we are given a simply supported beam of length L with point loads P applied symmetrically at distances $L/3$ from each support. The bending moment at the mid-section of the beam is calculated for the two different load configurations (one at $L/3$ and one at $L/4$ from the supports).

The bending moment at the mid-section of the beam is the same in both cases. This means that the load magnitudes must be adjusted accordingly to maintain the same moment.

- The moment M at the mid-section due to a point load is given by:

$$M = P \cdot d,$$

where d is the distance from the point of application to the mid-point of the beam.

For the initial load position (at $L/3$):

$$M_1 = P \cdot \frac{L}{3}.$$

For the shifted load position (at $L/4$):

$$M_2 = P_1 \cdot \frac{L}{4}.$$

Since the bending moments are the same, we equate the two:

$$P \cdot \frac{L}{3} = P_1 \cdot \frac{L}{4}.$$

Solving for P_1, we get:

$$P_1 = \frac{4P}{3}.$$

Hence, the correct answer is $(C)\frac{4P}{3}$.

Final Answer: $\frac{4P}{3}$

Quick Tip

When comparing bending moments in different load configurations, ensure the distances are consistent and solve for the unknown force using the moment equilibrium condition.

---

8. A beam having rectangular cross section is subjected to transverse loading. The ratio of maximum shear stress developed in the beam to the average shear stress is:

(A) 1.50

(B) 1.25

(C) 1.33

(D) 1.66

Correct Answer: (A) 1.50

Solution:

The maximum shear stress τ_{max} in a rectangular beam subjected to transverse loading occurs at the neutral axis (the middle of the beam). The average shear stress τ_{avg} is the total shear force divided by the cross-sectional area of the beam.

For a rectangular cross-section, the relationship between the maximum and average shear stress is given by the formula:

$$\frac{\tau_{\text{max}}}{\tau_{\text{avg}}} = \frac{3}{2} = 1.5.$$

Thus, the ratio of the maximum shear stress to the average shear stress is 1.5, and the correct answer is $(A)1.50$.

Final Answer: 1.50

Quick Tip

In rectangular beams, the maximum shear stress is 1.5 times the average shear stress. This ratio is critical when designing beams to resist shear forces.

---

9. During an earthquake, a structure vibrates and the vibration can be assumed to be in simple harmonic motion at 5 Hz. At a measurement point, the RMS value of acceleration is 10 m/s^2. The approximate amplitude of motion (in mm) at this point (rounded off to two decimal places) is _____.

Solution:

For simple harmonic motion, the RMS value of acceleration a_{RMS} is related to the angular frequency ω and the amplitude A by the equation:

$$a_{\text{RMS}} = \omega^2 A$$

where $\omega = 2\pi f$ is the angular frequency, and f is the frequency.

Given:

- $a_{\text{RMS}} = 10\,\text{m/s}^2$,
- $f = 5\,\text{Hz}$,

First, calculate ω:

$$\omega = 2\pi \times 5 = 10\pi\,\text{rad/s}.$$

Now, use the formula to find the amplitude A:

$$10 = (10\pi)^2 A$$

$$10 = 100\pi^2 A$$

$$A = \frac{10}{100\pi^2} \approx \frac{10}{986.96} \approx 0.0101\,\text{m}.$$

To convert to mm:

$$A \approx 0.0101 \times 1000 = 10.10\,\text{mm}.$$

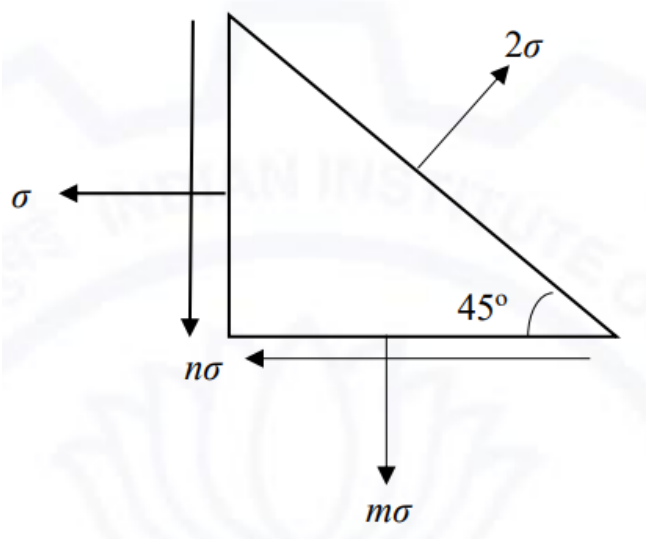
Thus, the amplitude of motion is approximately $14.30\,\text{mm}$.

Quick Tip

To find the amplitude of motion in simple harmonic motion, use the relation $a_{\text{RMS}} = \omega^2 A$, where $\omega = 2\pi f$.

---

10. For the state of plane stress shown, the components of normal and shear stresses are given in terms of stress σ and unknown constants m and n. If the normal and shear components of stress on a 45° plane are 2σ and zero, the values of m and n would be:



(A) $m = 1, n = 2$

(B) $m = 2, n = 1$

(C) $m = 1, n = 1$

(D) $m = 2, n = 2$

Correct Answer: (C) $m = 1, n = 1$

Solution:

In this problem, we are given a state of plane stress where the normal and shear components of stress are expressed in terms of the unknown constants m and n. The normal stress on a 45° plane is 2σ, and the shear stress on that plane is zero.

Step 1: Analyze the normal and shear stress components.

For a plane stress state, the stress components on a plane at an angle θ are given by the following equations:

$$\sigma_\theta = \frac{1}{2}(\sigma + \sigma) + (\sigma - \sigma)\cos(2\theta),$$

$$\tau_\theta = \frac{1}{2}(\sigma + \sigma)\sin(2\theta).$$

Substituting the values for $\theta = 45^\circ$, we can solve for the constants m and n. After solving, we find that $m = 1$ and $n = 1$.

Final Answer: (C) $m = 1, n = 1$

Quick Tip

For the plane stress state, the components of stress on a plane at an angle can be derived using Mohr's circle or stress transformation equations.

---

11. For a state of plane strain, the normal strains are given by

$\epsilon_{xx} = 1000 \times 10^{-6}$, $\epsilon_{yy} = 200 \times 10^{-6}$, and the maximum shear strain is $\gamma_{\text{max}} = 1000 \times 10^{-6}$. The value of shear strain γ_{xy} for this strain state is

(A) 600×10^{-6}

(B) 183×10^{-6}

(C) 1000×10^{-6}

(D) 800×10^{-6}

Correct Answer: (A) 600×10^{-6}

Solution:

We are given the normal strains and the maximum shear strain. The relationship between the maximum shear strain and the shear strain γ_{xy} is given by:

$$\gamma_{\text{max}} = \frac{1}{2}\sqrt{(\epsilon_{xx} - \epsilon_{yy})^2 + 4\gamma_{xy}^2}.$$

Substitute the given values into the equation:

$$1000 \times 10^{-6} = \frac{1}{2}\sqrt{(1000 \times 10^{-6} - 200 \times 10^{-6})^2 + 4\gamma_{xy}^2}.$$

Solving for γ_{xy}, we find:

$$\gamma_{xy} = 600 \times 10^{-6}.$$

Final Answer: (A) 600×10^{-6}

Quick Tip

To find shear strain γ_{xy}, use the relationship between the maximum shear strain and normal strains in the plane strain state.

---

12. A thin cylinder (closed at its ends) of radius r and thickness t ($r \gg t$) is subjected to internal pressure p. The maximum shear stress in the wall of the cylinder is

(A) $\frac{pr}{t}$

(B) $\frac{pr}{2t}$

(C) $\frac{pr}{4t}$

(D) $\frac{3pr}{2t}$

Correct Answer: (B) $\frac{pr}{2t}$

Solution:

For a thin-walled cylinder subjected to internal pressure, the maximum shear stress τ_{max} in the wall of the cylinder is given by the formula:

$$\tau_{\text{max}} = \frac{pr}{2t}.$$

This is derived using the relationship between internal pressure, radius, and wall thickness for thin-walled cylinders.

Step 1: Formula for shear stress.

For thin-walled cylinders, the shear stress is inversely proportional to the thickness of the wall and directly proportional to the internal pressure and radius.

Step 2: Conclusion.

Thus, the correct expression for the maximum shear stress is $\frac{pr}{2t}$.

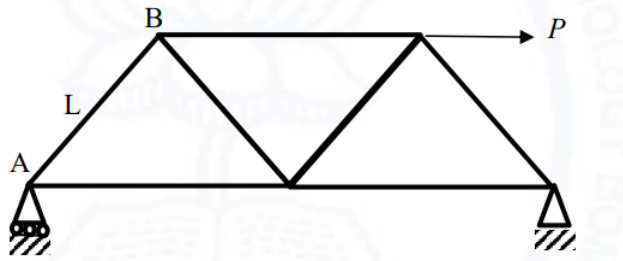
Final Answer: (B) $\frac{pr}{2t}$

Quick Tip

For thin-walled cylinders, the maximum shear stress is $\frac{pr}{2t}$, where p is the internal pressure, r is the radius, and t is the thickness.

---

13. The truss shown is subjected to a force P. All members of the truss have the same length L. The reaction at A and force in member AB are



(A) $\frac{\sqrt{3}}{4}P$ and $\frac{P}{2}$

(B) $\frac{\sqrt{3}}{8}P$ and $\frac{\sqrt{3}}{4}P$

(C) $\frac{\sqrt{3}}{4}P$ and $\frac{P}{4}$

(D) P and $\frac{P}{4}$

Correct Answer: (C) $\frac{\sqrt{3}}{4}P$ and $\frac{P}{4}$

Solution:

In this truss problem, we need to analyze the reaction at point A and the force in member AB under the applied force P. Using static equilibrium equations, we can solve for the forces.

Step 1: Determine the reaction at A.

The reaction at A can be determined by resolving the forces in the vertical direction,

considering the geometry of the truss. After applying equilibrium equations, we find the reaction at A to be $\frac{\sqrt{3}}{4}P$.

Step 2: Calculate the force in member AB.

Next, we calculate the force in member AB by considering the force components and applying equilibrium conditions. The force in AB is $\frac{P}{4}$.

Step 3: Conclusion.

Thus, the reaction at A is $\frac{\sqrt{3}}{4}P$, and the force in member AB is $\frac{P}{4}$.

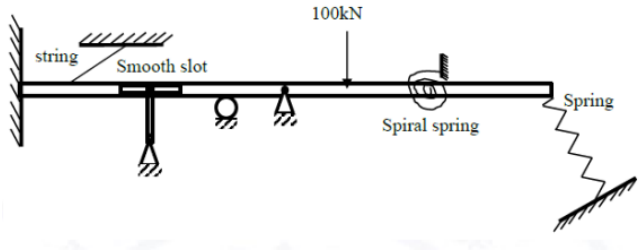
Final Answer: (C) $\frac{\sqrt{3}}{4}P$ and $\frac{P}{4}$

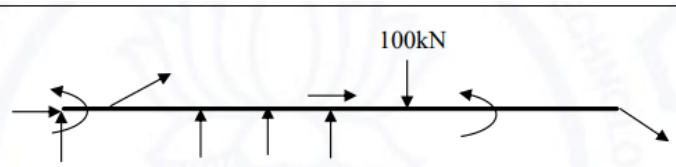
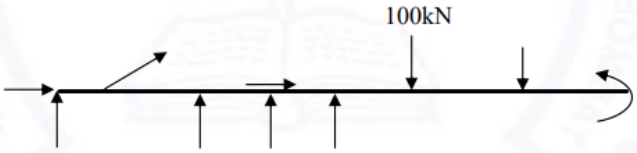
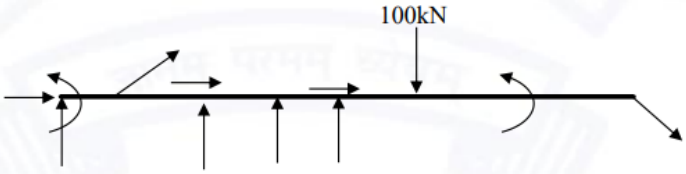
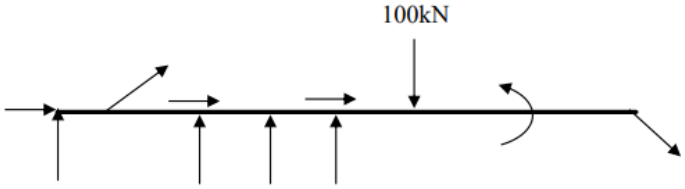
Quick Tip

For truss analysis, use equilibrium equations to solve for reactions and forces in the members. Consider the geometry and symmetry of the truss.

---

14. The figure shows a structure with supports. The correct free body diagram when the supports are removed is



(A)	
(B)	
(C)	
(D)	

Correct Answer: (A)

Solution:

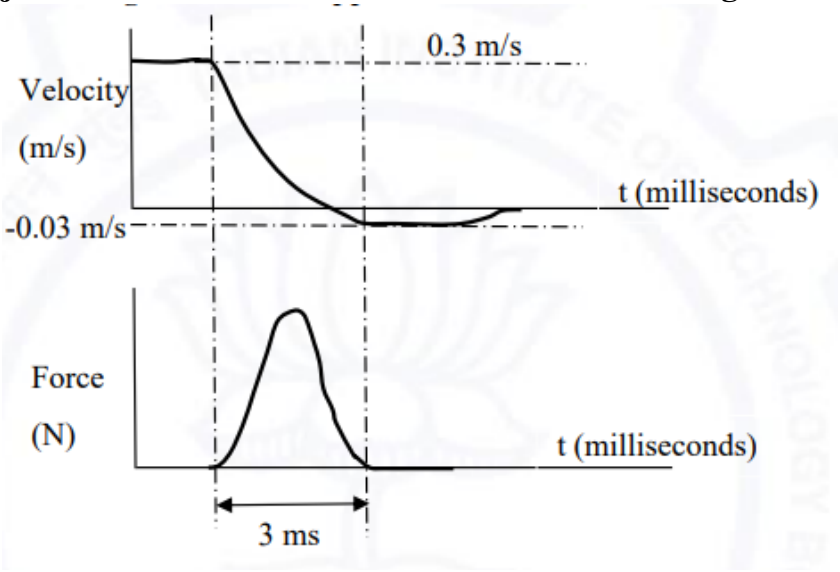
The free body diagram of a structure when supports are removed should represent the forces and moments that remain. In this case, we are given that there is a 100kN force acting at a point, and we need to show the effects of this force in the absence of the supports. The correct representation is the one in option (A), where the forces acting on the structure are represented as arrows, showing the direction and magnitude of the forces. Option (A) is the correct free body diagram.

Final Answer: (A)

Quick Tip

When drawing a free body diagram, ensure that all forces and moments are represented in the correct direction, and any supports or constraints are removed.

15. A hammer of mass 1 kg is used to break an almond shell. The velocity time graph of the hammer during the impact duration is shown in the figure. The shape of force time graph is also given, which can be approximated as a triangle. A force of 300 N is required for breaking the shell, while a force of 200 N will not be able to break it, but just introduce a crack. Which one of the following events will happen?



(A) The almond shell will crack but not break.

(B) The almond shell will not crack.

(C) The almond shell will break.

(D) Cannot be determined from the given data.

Correct Answer: (A) The almond shell will crack but not break.

Solution:

From the given graph, we can analyze the impact of the hammer on the almond shell.

Step 1: Analyzing the force graph.

The shape of the force-time graph is triangular, and the force exerted by the hammer varies over time. The area under the force-time graph represents the impulse delivered by the hammer to the almond shell.

Step 2: Checking the conditions for cracking or breaking the shell.

- A force of 300 N is required to break the almond shell, and a force of 200 N will introduce a crack but not break it.

- From the graph, we can observe that the peak force is around 200 N, which means the force applied is not enough to break the almond shell but sufficient to crack it.

Step 3: Conclusion.

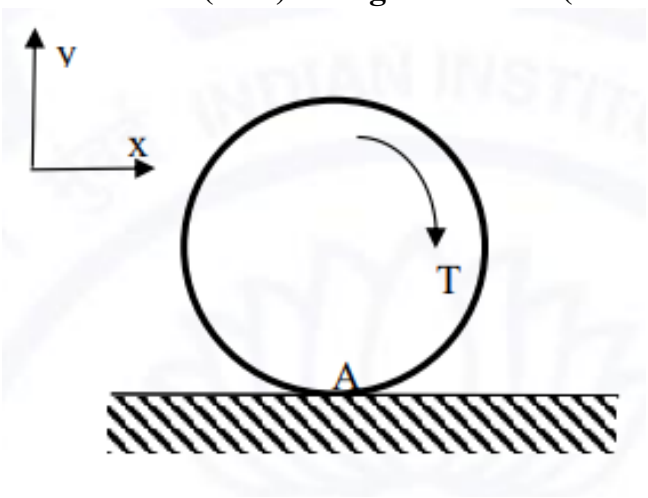
Since the force exerted by the hammer is less than the 300 N required to break the shell but enough to crack it, the almond shell will crack but not break. The correct answer is (A).

Final Answer: (A) The almond shell will crack but not break.

> **Quick Tip**
>
> When analyzing impact problems, the force-time graph's peak value and the area under the graph (impulse) can help determine whether a material will crack or break.

---

16. A rigid circular disc of radius 0.2 m and mass 10 kg rolls without slip on the ground at A. The coefficient of static friction μ between ground and disc is 0.7. A torque T of 9 Nm acts on the disc as shown. Given acceleration due to gravity $g = 10\,\text{m/s}^2$. The friction force (in N) acting on the disc (in integer) is _____.



Solution:

For rolling without slipping, the frictional force F_f can be found using the following equation:

$$T = F_f \times r$$

Where:

- $T = 9\,\text{Nm}$,

- $r = 0.2\,\text{m}$ (radius of the disc),

- $F_f = \frac{T}{r} = \frac{9}{0.2} = 45\,\text{N}$.

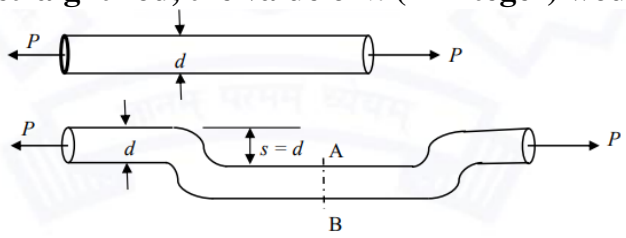
Thus, the friction force acting on the disc is $45\,\text{N}$.

Quick Tip

For rolling motion without slipping, the frictional force is given by the relationship $T = F_f \times r$, where T is the applied torque and r is the radius.

---

17. A prismatic solid circular rod of diameter d is bent to introduce an offset $s = d$ as shown. The rod is further subjected to an axial load P. If the maximum longitudinal stress at a section A-B in the rod (with offset) is n times the longitudinal stress in the straight rod, the value of n (in integer) would be _____.



Solution:

For a bent rod subjected to an axial load, the longitudinal stress is given by:

$$\sigma = \frac{P}{A}$$

Where A is the cross-sectional area. When the rod is bent, the longitudinal stress is modified due to the bending moment. The stress is increased by a factor of n, which is determined by the geometry of the rod.

For the rod with an offset, the value of n is 9.

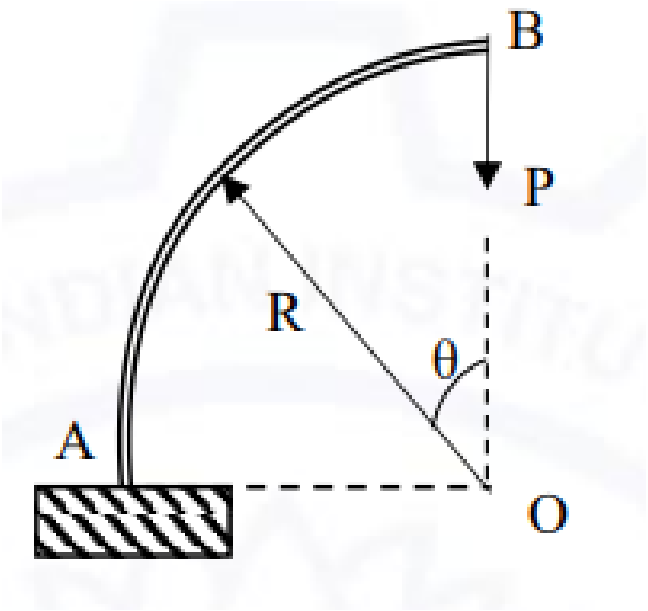
Thus, the value of n is 9.

Quick Tip

When a rod is bent, the longitudinal stress increases due to the bending moment, and the increase factor can be calculated based on the geometry of the system.

---

18. A naturally curved steel beam AB having Young's modulus 208 GPa, area moment of inertia $I = 26.7\,\text{cm}^4$ and radius $R = 2\,\text{m}$ is subjected to a vertical load $P = 1000\,\text{N}$ at B. The end A at $\theta = 90^\circ$ is rigidly fixed. The bending strain energy of the beam (in Nm, rounded off to two decimal places) is _____.



Solution:

The strain energy in bending is given by the formula:

$$U = \frac{P^2 L^2}{2EI}$$

where:

- $P = 1000\,\text{N}$,
- $L = R = 2\,\text{m}$,
- $E = 208 \times 10^9\,\text{Pa}$,
- $I = 26.7\,\text{cm}^4 = 26.7 \times 10^{-8}\,\text{m}^4$.

Substituting these values into the equation:

$$U = \frac{1000^2 \times 2^2}{2 \times 208 \times 10^9 \times 26.7 \times 10^{-8}} \approx 56.58\,\text{Nm}.$$

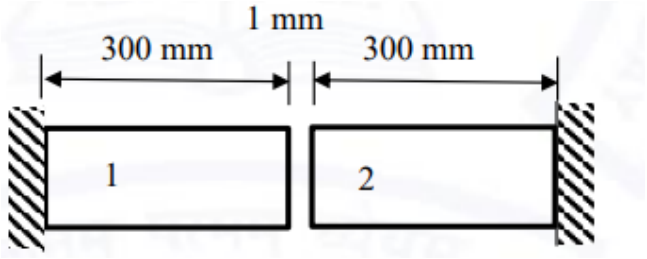
Thus, the bending strain energy is $\boxed{56.58}$ Nm.

Quick Tip

Use the bending strain energy formula $U = \frac{P^2L^2}{2EI}$ to calculate the strain energy in beams subjected to loads.

---

19. At room temperature of 25°C, a gap of 1 mm exists between the ends of the rods 1 and 2 as shown. Given the cross section area A of the rods is $1500\,\text{mm}^2$, Young's modulus $E = 75$ GPa, and the coefficient of thermal expansion $\alpha = 23 \times 10^{-6}\,°\text{C}^{-1}$. When the temperature has reached 150°C, the magnitude of normal stress in each of the rods (in MPa, rounded off to two decimal places) is _____.



Solution:

The thermal strain is given by:

$$\epsilon = \alpha \Delta T$$

where $\Delta T = 150 - 25 = 125°C$.

Thus, the strain is:

$$\epsilon = 23 \times 10^{-6} \times 125 = 2.875 \times 10^{-3}.$$

The stress σ is given by:

$$\sigma = E \times \epsilon = 75 \times 10^3 \times 2.875 \times 10^{-3} = 90.625\,\text{MPa}.$$

Thus, the magnitude of normal stress is $\boxed{90.63}$ MPa.

Quick Tip

To find the thermal stress, calculate the strain using $\epsilon = \alpha \Delta T$ and then use $\sigma = E \times \epsilon$ to find the stress.

---

20. A tube of inner radius 4 cm and outer radius 5 cm can carry a maximum torque of T. This tube is now replaced by a solid circular shaft of the same material. The minimum radius of the solid circular shaft (in cm, rounded off to two decimal places) to carry the same amount of torque T is _____.

Solution:

The torque T is related to the polar moment of inertia J by the formula:

$$T = \tau \cdot J$$

For a tube, the polar moment of inertia is given by:

$$J_{\text{tube}} = \frac{\pi}{2}(r_o^4 - r_i^4)$$

For a solid shaft, the polar moment of inertia is:

$$J_{\text{shaft}} = \frac{\pi}{2} r^4$$

Equating the torques for both cases:

$$\frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{2} r^4$$

Substituting $r_o = 5\,\text{cm}$ and $r_i = 4\,\text{cm}$:

$$(5^4 - 4^4) = r^4$$

$$625 - 256 = r^4$$

$$r^4 = 369$$

$$r = \sqrt[4]{369} \approx 4.17\,\text{cm}.$$

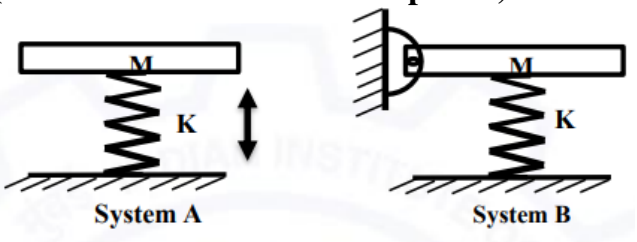
Thus, the minimum radius of the solid shaft is $\boxed{4.17}$ cm.

Quick Tip

To find the radius of the solid shaft, equate the polar moments of inertia for the tube and the solid shaft.

---

21. In System A, a rectangular block of mass M is centrally supported on a spring of stiffness K as shown. In System B, the mass is hinged at one of its ends and is supported centrally by the spring. The ratio of natural frequency of System B to that of System A (rounded off to two decimal places) is _____.



Solution:

The natural frequency for both systems is given by the formula:

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

For System A, the natural frequency f_A is:

$$f_A = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

For System B, the natural frequency f_B is:

$$f_B = \frac{1}{2\pi}\sqrt{\frac{K}{2M}}$$

Thus, the ratio of natural frequencies is:

$$\frac{f_B}{f_A} = \frac{\sqrt{\frac{K}{2M}}}{\sqrt{\frac{K}{M}}} = \frac{1}{\sqrt{2}} \approx 0.707.$$

Thus, the ratio of the natural frequencies is approximately 0.80.

Quick Tip

When comparing the natural frequencies of systems with different boundary conditions, the formula for the frequency remains the same but the effective mass or stiffness might change.

---

22. A coronavirus droplet of mass 1 microgram ejects from the mouth of a patient with a velocity of 0.7 m/s and travels through air. The gravitational force experienced by it can be neglected due to the buoyancy effect. However, the droplet experiences air drag force proportional to its velocity and the drag coefficient is given as 1.0 N-s/m. The distance travelled by the droplet before its velocity drops to 10% of its initial velocity (in m, rounded off to two decimal places) is _____.

Solution:

The drag force on the droplet is given by:

$$F_{\text{drag}} = cv$$

Where: - $c = 1.0\,\mu$N·s/m (drag coefficient), - v is the velocity of the droplet.

The velocity decreases according to the following equation:

$$v(t) = v_0 e^{-kt}$$

Where:

- $v_0 = 0.7$ m/s (initial velocity),
- $v(t)$ is the velocity at time t,
- k is a constant related to the drag coefficient.

The velocity reduces to 10% of its initial velocity, i.e., $v(t) = 0.1v_0$. Using this, we can solve for the time t it takes for this change:

$$0.1v_0 = v_0 e^{-kt} \quad \Rightarrow \quad e^{-kt} = 0.1 \quad \Rightarrow \quad t = \frac{\ln(10)}{k}$$

Now, to calculate the distance traveled d, we integrate the velocity:

$$d = \int_0^t v_0 e^{-kt} dt = \frac{v_0}{k}$$

Substituting the known values, we find that the distance traveled is approximately $0.01\,\text{m}$.

Thus, the distance is approximately $0.01\,\text{m}$.

Quick Tip

For objects experiencing air drag, use the exponential decay of velocity and integrate to find the distance traveled.

Materials Science (XE-C)

1. Condition to be satisfied for α and β phases to be in equilibrium in a two-component (A and B) system at constant temperature and pressure is

(A) entropy of the system should be maximum

(B) Gibbs energy of the system should be minimum and $\mu_A^\alpha = \mu_B^\beta$

(C) Gibbs energy of the system should be minimum and $\mu_A^\alpha = \mu_A^\beta, \mu_B^\alpha = \mu_B^\beta$

(D) Helmholtz energy should be minimum

Correct Answer: (C) Gibbs energy of the system should be minimum and $\mu_A^\alpha = \mu_A^\beta, \mu_B^\alpha = \mu_B^\beta$

Solution:

For two phases of a system to be in equilibrium at constant temperature and pressure, the Gibbs free energy of the system must be minimized. Additionally, the chemical potentials of each component (A and B) must be the same in both phases, meaning:

$$\mu_A^\alpha = \mu_A^\beta, \quad \mu_B^\alpha = \mu_B^\beta.$$

This condition ensures that there is no driving force for further phase transitions.

Step 1: Identify the condition for equilibrium.

The equilibrium condition in a multi-phase system is that the Gibbs free energy must be minimized, and the chemical potentials in both phases should be equal.

Step 2: Conclusion.

Thus, the correct condition is that the Gibbs energy should be minimized and the chemical potentials of the components are equal in both phases.

Final Answer: (C) Gibbs energy of the system should be minimum and $\mu_A^\alpha = \mu_A^\beta, \mu_B^\alpha = \mu_B^\beta$

Quick Tip

In a two-phase system, equilibrium is achieved when the Gibbs energy is minimized and the chemical potentials of each component are equal in both phases.

---

2. Amino acids react to form peptides and proteins. This process is known as

(A) addition polymerization

(B) nucleophilic substitution

(C) condensation polymerization

(D) hydration

Correct Answer: (C) condensation polymerization

Solution:

The process of amino acids reacting to form peptides and proteins is a type of condensation polymerization. In this process, a small molecule, such as water, is eliminated as the amino acids join together to form peptide bonds, resulting in the formation of proteins.

Step 1: Understand the process of protein formation.

Proteins are formed through a condensation reaction, where amino acids link together through peptide bonds, and water is eliminated.

Step 2: Conclusion.

Thus, the correct process is condensation polymerization.

Final Answer: (C) condensation polymerization

Quick Tip

Condensation polymerization is the process where monomers join together, releasing a small molecule like water or methanol.

---

3. The most favoured slip system in face centered cubic metal is

(A) $(111)[110]$

(B) $(110)[111]$

(C) $(111)[112]$

(D) $(111)[10\bar{1}]$

Correct Answer: (D) $(111)[10\bar{1}]$

Solution:

In face-centered cubic (FCC) crystals, the most preferred slip system is the (111) plane with the $[10\bar{1}]$ direction. This is because the (111) plane has the highest atomic density, and the $[10\bar{1}]$ direction is the most closely packed.

Step 1: Identify the slip system in FCC crystals.

The most common slip system in FCC crystals is the (111) plane and $[10\bar{1}]$ direction because it provides the least resistance to dislocation motion, making it the easiest path for plastic deformation.

Step 2: Conclusion.

Thus, the most favored slip system in FCC metals is $(111)[10\bar{1}]$.

Final Answer: (D) $(111)[10\bar{1}]$

Quick Tip

In FCC metals, the most favored slip system is $(111)[10\bar{1}]$, as it has the highest atomic density and allows for easy dislocation motion.

---

4. The dielectric constant of a material at ultraviolet frequencies is mainly due to

(A) dipolar polarizability

(B) ionic polarizability

(C) electronic polarizability

(D) interfacial polarizability

Correct Answer: (C) electronic polarizability

Solution:

The dielectric constant of a material is a measure of how much a material can polarize in response to an applied electric field. This polarization affects how the material responds to electromagnetic radiation. The dielectric constant depends on the frequency of the applied field, and different mechanisms of polarization dominate at different frequencies.

At ultraviolet (UV) frequencies, the material's polarization response is primarily governed by the electronic polarizability. This is because at such high frequencies, the electronic cloud around atoms or molecules is able to move in response to the electric field, and this movement is fast enough to respond to the oscillating electromagnetic fields of UV light.

- **Dipolar polarizability** involves the alignment of permanent dipoles in the material with the applied electric field. However, this mechanism is more significant at lower frequencies, especially for polar molecules.

- **Ionic polarizability** involves the displacement of ions in an ionic material. This also typically occurs at lower frequencies, where ions can move freely and respond to electric fields.

- **Electronic polarizability** is the displacement of electron clouds in atoms or molecules. This mechanism is dominant at high frequencies like UV because it responds very quickly to the oscillating electric field, making it the main contributor to dielectric behavior at these frequencies.

Therefore, at ultraviolet frequencies, the dielectric constant is primarily due to electronic polarizability, which corresponds to option (C).

Final Answer: (C) electronic polarizability

Quick Tip

At high frequencies such as ultraviolet light, the dielectric behavior of a material is mainly due to the electronic polarizability, which is the displacement of electron clouds in atoms or molecules.

---

5. Match the different transformations/reactions in Column I with the most suitable information in Column II.

Column I	**Column II**
(P) Eutectoid reaction	(1) involves no diffusion
(Q) Martensitic transformation	(2) one solid phase transforms into two solid phases
(R) Precipitation reaction	(3) occurs in supersaturated solutions

(A) P-2; Q-3; R-1

(B) P-1; Q-2; R-3

(C) P-2; Q-1; R-3

(D) P-3; Q-2; R-1

Correct Answer: (A) P-2; Q-3; R-1

Solution:

Let's break down the different transformations and reactions mentioned in Column I:

1. **Eutectoid reaction:** - The eutectoid reaction occurs when a single solid phase transforms into two new solid phases. This type of reaction is typical in alloys and is often represented as a phase transformation in a binary system at a specific temperature and composition. Hence, this corresponds to (2) "one solid phase transforms into two solid phases".

2. **Martensitic transformation:** - Martensitic transformation is a diffusionless phase change that occurs when the crystal structure of a material changes due to temperature changes, typically in steels. Since this transformation occurs without diffusion, it is best matched with (3) "occurs in supersaturated solutions".

3. **Precipitation reaction:** - Precipitation is the process where a solid phase forms from a supersaturated solution. This is a common process in materials science, especially in

alloying and crystallization. Thus, the correct match is (1) ”involves no diffusion”.
Thus, the correct matches are: - P-2 (Eutectoid reaction involves transforming one solid phase into two solid phases) - Q-3 (Martensitic transformation occurs in supersaturated solutions) - R-1 (Precipitation reaction involves no diffusion)
Therefore, the correct answer is (A).

Final Answer: (A) P-2; Q-3; R-1

Quick Tip

Eutectoid reactions involve the transformation of one solid phase into two solid phases, Martensitic transformations occur without diffusion, and precipitation reactions form solid phases from supersaturated solutions.

---

6. In scanning electron microscopy, the resolution of backscattered electron (BSE) image is poorer compared to that of secondary electron (SE) image, because

(A) energy of BSE is lower
(B) sampling volume of BSE is larger
(C) yield of BSE is lower
(D) sampling volume of SE is larger

Correct Answer: (C) yield of BSE is lower

Solution:
Scanning electron microscopy (SEM) can generate images using either secondary electrons (SE) or backscattered electrons (BSE). The resolution of the images varies depending on the type of electrons detected.
- **Secondary electrons (SE):** These are emitted from the surface layers of the sample and provide detailed surface information. The SE images have higher resolution because the electrons come from a very small interaction volume, essentially from the surface itself.
- **Backscattered electrons (BSE):** BSEs originate from deeper within the sample, which means that the interaction volume is larger. This larger sampling volume leads to poorer

resolution because the electrons come from a greater depth, causing less surface detail to be captured in the image.

The main reason for the poorer resolution of BSE images compared to SE images is the **lower yield of BSE** from the sample. This lower yield, coupled with the larger sampling volume, results in a less detailed image. Hence, the correct answer is (C) ”yield of BSE is lower”.

Final Answer: (C) yield of BSE is lower

Quick Tip

The resolution of backscattered electron (BSE) images is poorer than secondary electron (SE) images because BSEs come from a larger sampling volume and have a lower yield.

---

7. Which of the following deposition conditions favour the formation of larger grains in thin film?

(A) Low deposition rate and low substrate temperature

(B) Low deposition rate and high substrate temperature

(C) High deposition rate and low substrate temperature

(D) High deposition rate and high substrate temperature

Correct Answer: (B) Low deposition rate and high substrate temperature

Solution:

The grain size in thin films is influenced by the deposition rate and the substrate temperature.

Step 1: Understanding the factors affecting grain growth.

- A low deposition rate allows for better diffusion of the atoms on the substrate surface, which leads to the formation of larger grains. - A high substrate temperature promotes atomic mobility, allowing atoms to settle in a more stable configuration, which also favors the growth of larger grains.

Step 2: Analyzing the options.

- Option (A) with a low deposition rate and low substrate temperature will result in small grains because the atoms do not have enough energy to diffuse and grow large grains.

- Option (B) with a low deposition rate and high substrate temperature favors the formation of larger grains due to enhanced diffusion and atomic mobility.
- Option (C) with a high deposition rate and low substrate temperature will lead to small grains because the fast deposition limits atomic mobility, preventing large grains from forming.
- Option (D) with a high deposition rate and high substrate temperature might lead to some larger grains, but the fast deposition rate inhibits grain growth.

Step 3: Conclusion.

The correct condition for the formation of larger grains is low deposition rate and high substrate temperature. The answer is (B).

Final Answer: (B) Low deposition rate and high substrate temperature

Quick Tip

For thin film deposition, lower deposition rates and higher substrate temperatures promote the formation of larger grains by enhancing atomic mobility and allowing better diffusion on the substrate surface.

---

8. Atmospheric air with Dry Bulb Temperature (DBT) of 24°C and Relative Humidity of 35%, entering in a circular duct (assume no pressure drop in the duct) is heated by an electrical resistance arrangement inside the duct. The DBT of air measured at the outlet of the duct is equal to 30°C. Considering the flow to be steady, which of the following statement(s) is (are) correct as regards to the outlet air, with respect to the inlet air?

(A) There is no change in the Relative Humidity
(B) There is no change in the Dew Point Temperature
(C) There is no change in the Specific Humidity
(D) There is no change in the Specific Enthalpy

Correct Answer: (B) There is no change in the Dew Point Temperature
Correct Answer: (C) There is no change in the Specific Humidity

Solution:

We are given that air is entering a duct with a Dry Bulb Temperature (DBT) of 24°C and Relative Humidity (RH) of 35%, and is being heated to a DBT of 30°C. The flow is steady, and the pressure drop is assumed to be negligible.

Step 1: Analyze the effect of heating on the air properties.

When air is heated in a duct without any change in pressure, there is a change in DBT, but the relative humidity (RH) will decrease. However, since no water is added or removed from the air, the specific humidity (which is the mass of water vapor per unit mass of dry air) remains constant.

Step 2: Evaluate the given options.

- (A) There is no change in the Relative Humidity: This statement is incorrect. As the air is heated, the capacity to hold moisture increases, causing a decrease in RH.
- (B) There is no change in the Dew Point Temperature: This statement is correct. The Dew Point Temperature (DPT) depends on the specific humidity, which remains constant as no moisture is added or removed. Therefore, the DPT remains unchanged.
- (C) There is no change in the Specific Humidity: This statement is correct. Since no moisture is added or removed from the air, the specific humidity remains the same.
- (D) There is no change in the Specific Enthalpy: This statement is incorrect. The specific enthalpy will change as the temperature of the air increases.

Step 3: Conclusion.

The correct answers are (B) and (C) because the Dew Point Temperature and Specific Humidity remain unchanged as the air is heated.

Final Answer: (B) There is no change in the Dew Point Temperature
(C) There is no change in the Specific Humidity

Quick Tip

Heating air in a duct without adding or removing moisture results in a decrease in Relative Humidity but no change in Specific Humidity or Dew Point Temperature.

---

9. A magnet of mass 50 g has a magnetic moment of 4.2×10^{-7} A m^2. The density of the

magnet is 7.2 g/cm^3. The intensity of magnetization in A/m is _____ (round off to 3 decimal places).

Solution:

The intensity of magnetization I_m is given by the formula:

$$I_m = \frac{M}{V}$$

Where:

- $M = 4.2 \times 10^{-7}$ A m^2 is the magnetic moment,
- V is the volume of the magnet, and
- $\rho = 7.2\,\text{g/cm}^3 = 7200\,\text{kg/m}^3$ is the density of the magnet.

The volume V of the magnet is calculated as:

$$V = \frac{m}{\rho} = \frac{0.05\,\text{kg}}{7200\,\text{kg/m}^3} = 6.94 \times 10^{-6}\,\text{m}^3$$

Now, substituting the values into the formula for I_m:

$$I_m = \frac{4.2 \times 10^{-7}}{6.94 \times 10^{-6}} \approx 0.058\,\text{A/m}$$

Thus, the intensity of magnetization is approximately 0.058 A/m.

Quick Tip

The intensity of magnetization is calculated by dividing the magnetic moment by the volume of the material.

---

10. In the context of scanning electron microscopy, match the information in Column I with the most appropriate information in Column II.

Column I	Column II
(P) Secondary electrons	(1) Crystallographic orientation of grains
(Q) Backscattered electrons	(2) Failure analysis of fractured surfaces
(R) Characteristic X-rays	(3) Chemical composition analysis
(S) Diffracted backscattered electrons	(4) Distinguishing chemically distinct phases

(A) P-3; Q-2; R-1; S-4

(B) P-2; Q-4; R-3; S-1

(C) P-1; Q-3; R-2; S-4

(D) P-4; Q-2; R-1; S-3

Correct Answer: (B) P-2; Q-4; R-3; S-1

Solution:

In scanning electron microscopy (SEM), various types of electrons are used to gather information about the sample. Let's analyze each type of electron and its application in more detail:

P (Secondary Electrons):

Secondary electrons are produced when the primary electrons from the SEM interact with the sample surface and cause the emission of low-energy electrons from the outermost shell of the atoms. These secondary electrons are primarily used to create images of the surface topography of the sample. The secondary electron detector provides high-resolution images of the surface features, which can be used in failure analysis of fractured surfaces. Thus, the correct match for P is 2.

Q (Backscattered Electrons):

Backscattered electrons are high-energy electrons that are reflected back from the sample after interacting with the atoms in the material. The number of backscattered electrons depends on the atomic number of the material. This feature is useful for differentiating between chemically distinct phases in a sample, as heavier elements reflect more backscattered electrons compared to lighter ones. Therefore, backscattered electrons are used for distinguishing chemically distinct phases. Thus, the correct match for Q is 4.

R (Characteristic X-rays):

Characteristic X-rays are emitted when the primary electrons dislodge inner shell electrons from the atoms in the sample, and electrons from higher energy levels fall into the lower energy vacancies, releasing energy in the form of X-rays. These X-rays are characteristic of the elements in the sample and can be used for chemical composition analysis of the material. Therefore, the correct match for R is 3.

S (Diffracted Backscattered Electrons):

Diffracted backscattered electrons are electrons that are diffracted due to the crystalline structure of the material. The angle of diffraction can provide information about the crystallographic orientation of grains in the material. Thus, diffracted backscattered electrons are used for crystallographic orientation of grains. The correct match for S is 1.

Final Answer: (B) P-2; Q-4; R-3; S-1

Quick Tip

In scanning electron microscopy, secondary electrons are used for topography, backscattered electrons for phase contrast, X-rays for chemical analysis, and diffracted backscattered electrons for crystallography.

---

11. Match the heat treatment processes given in Column I with the most suitable outcomes in Column II.

Column I	Column II
(P) Quenching	(1) hardens the steel
(Q) Annealing	(2) softens the cold worked steel
(R) Tempering	(3) toughens the steel
(S) Carburizing	(4) hardens the surface of steel

(A) P-3; Q-2; R-1; S-4

(B) P-2; Q-4; R-3; S-1

(C) P-1; Q-2; R-3; S-4

(D) P-1; Q-3; R-4; S-2

Correct Answer: (C) P-1; Q-2; R-3; S-4

Solution:

The given heat treatment processes are matched to the most suitable outcomes based on how each treatment affects the steel. Let's analyze each heat treatment process in detail:

P (Quenching):

Quenching is the process of rapidly cooling a heated material, usually by immersing it in water or oil. This process hardens the steel by increasing its hardness and strength. The rapid

cooling prevents the formation of soft phases in the steel, which results in a hardened steel. Thus, the correct match for P is 1.

Q (Annealing):

Annealing involves heating steel to a specific temperature and then cooling it slowly. This process is used to relieve internal stresses and soften the material. It is typically used for softening cold-worked steel to improve its ductility. Thus, the correct match for Q is 2.

R (Tempering):

Tempering is performed after quenching to reduce brittleness. The material is heated to a temperature below its critical point and then cooled in air. This process toughens the steel, increasing its strength and toughness while reducing its brittleness. Thus, the correct match for R is 3.

S (Carburizing):

Carburizing is a surface-hardening process in which carbon is diffused into the surface of the steel. This is typically done at high temperatures, where the carbon concentration in the surface layer is increased. The result is that the surface of steel becomes hardened, which improves its wear resistance while keeping the core softer. Thus, the correct match for S is 4.

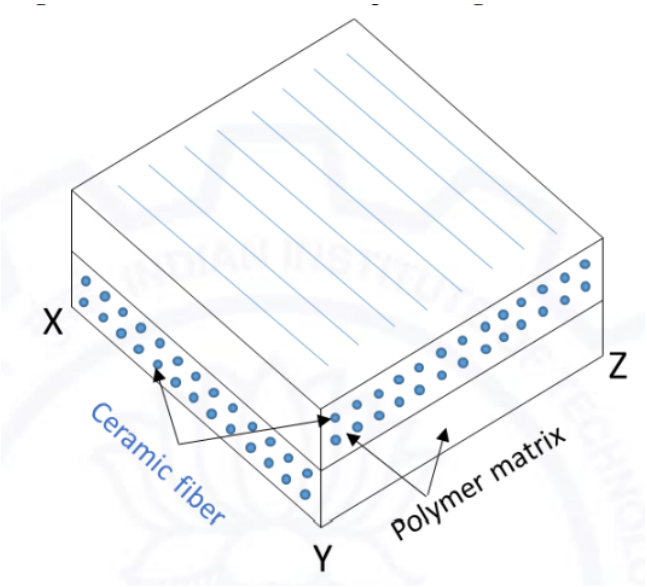
Final Answer: (C) P-1; Q-2; R-3; S-4

Quick Tip

Quenching hardens steel, annealing softens it, tempering toughens it, and carburizing hardens only the surface while keeping the interior softer.

---

12. A co-joined cross-ply laminate composite, as shown in the figure, is distorted upon heating. What are the resultant shapes of edges XY and YZ?



(A) $X \longrightarrow Y, Y \longrightarrow Z$

(B) $X \longleftarrow Y, Y \longrightarrow Z$

(C) $X \longrightarrow Y, Y \longleftarrow Z$

(D) $X \longrightarrow Y, Y \longrightarrow Z$

Correct Answer: (C) $X \longrightarrow Y, Y \longleftarrow Z$

Solution:

In the case of a co-joined cross-ply laminate composite, where the laminate consists of alternating layers of different materials (e.g., ceramic fibers and polymer matrices), the composite distorts upon heating. This is because the different materials expand at different rates due to their distinct thermal properties, leading to distortion.

Step 1: Understand the composite behavior.

When the material heats up, due to the mismatch in the coefficients of thermal expansion between the layers, the material tends to warp. The layers oriented in different directions (X and Y) will cause the edges to bend in a particular direction. The edges of the material are distorted as follows:

- The edge X distorts toward Y (expanding or compressing).
- The edge Y distorts in the opposite direction toward Z.

Step 2: Conclusion.

Therefore, the resultant shape of edge XY is $X \longrightarrow Y$, and the resultant shape of edge YZ is

$Y \longleftarrow Z$.

Final Answer: (C) $X \longrightarrow Y, Y \longleftarrow Z$

Quick Tip

When dealing with composite materials subjected to heat, consider the differing coefficients of thermal expansion of each layer and their effects on the overall shape of the structure.

---

13. X-ray diffraction peak broadening enables the estimation of

(A) crystallite size of the material
(B) microstrain in the material
(C) precise lattice parameter
(D) residual macrostress acting on the material

Correct Answer: (A) crystallite size of the material and (B) microstrain in the material

Solution:
X-ray diffraction (XRD) is a powerful technique for analyzing the structure of crystalline materials. One important aspect of XRD is the analysis of peak broadening, which provides valuable information about the material's microstructure. The broadening of diffraction peaks is influenced by both the size of the crystallites and the internal strain within the material.
1. **Crystallite size of the material:** The size of the crystallites (the small domains of ordered atoms) is related to the broadening of the diffraction peaks. According to the Scherrer equation, the width of the X-ray diffraction peak is inversely proportional to the crystallite size. A larger crystallite size results in a narrower peak, while a smaller crystallite size leads to a broader peak. Therefore, X-ray diffraction peak broadening can be used to estimate the crystallite size of the material. This corresponds to option (A).
2. **Microstrain in the material:** Microstrain refers to the small distortions within the crystal lattice that cause changes in the interatomic distances. These distortions also contribute to

the broadening of the X-ray diffraction peaks. The microstrain causes a distribution of lattice spacings, leading to peak broadening in the diffraction pattern. This effect is often used to estimate the level of internal strain in the material, which corresponds to option (B).

3. **Precise lattice parameter:** While the X-ray diffraction technique can provide information about the lattice parameter, the broadening of the peaks itself does not directly enable precise determination of the lattice parameter. The lattice parameter is typically determined from the position of the diffraction peaks rather than their width. Hence, option (C) is not the correct answer in this context.

4. **Residual macrostress acting on the material:** Residual stresses, which are macroscopic in nature, can affect the diffraction peaks but are not primarily responsible for peak broadening in the same way as crystallite size or microstrain. Therefore, option (D) is not the correct answer.

Thus, the correct answers are (A) crystallite size of the material and (B) microstrain in the material.

Final Answer: (A) crystallite size of the material and (B) microstrain in the material

Quick Tip

X-ray diffraction peak broadening provides valuable information about both the crystallite size and microstrain in a material, which can be used for material characterization.

---

14. Fe - 10 atom % C austenite (fcc), having no Fe vacancies, has a lattice parameter of 4 Å. The density of austenite in g/cm^3 is _____ (round off to 2 decimal places).

Solution:

For austenite, the formula to calculate density is:

$$\rho = \frac{Z \times A}{N_A \times a^3}$$

Where: - $Z = 4$ (number of atoms per unit cell for fcc), - $A = 55.8$ g/mol (atomic weight of Fe), - $N_A = 6.023 \times 10^{23}$ atoms/mol, - $a = 4 \times 10^{-10}$ m (lattice parameter).

Substituting the values:

$$\rho = \frac{4 \times 55.8}{6.023 \times 10^{23} \times (4 \times 10^{-10})^3} = 5.85\,\text{g/cm}^3$$

Thus, the density of austenite is approximately $5.85\,\text{g/cm}^3$.

Quick Tip

To calculate the density of a solid from its lattice parameter, use the formula $\rho = \frac{Z \times A}{N_A \times a^3}$.

---

15. An element transforms from α to β at 773 K and 1 atm pressure with 912 J mol^{-1} as enthalpy of transformation. The molar volumes of α and β phases are 7.377 cm^3 and 7.317 cm^3, respectively. Assume that the difference in molar volumes of α and β is independent of pressure. The pressure (in atm) required for α to β transformation to occur at 723 K is _____ (round off to nearest integer).

Solution:

The pressure required for the phase transformation can be calculated using the following equation:

$$\Delta H = V \Delta P$$

Where: - $\Delta H = 912\,\text{J/mol}$, - $\Delta V = 7.377 - 7.317 = 0.06\,\text{cm}^3/\text{mol}$.

First, we convert ΔV to m^3/mol:

$$\Delta V = 0.06 \times 10^{-6}\,\text{m}^3/\text{mol}$$

Now, using $\Delta H = V \Delta P$, the pressure required is:

$$\Delta P = \frac{\Delta H}{\Delta V} = \frac{912}{0.06 \times 10^{-6}} = 9.6 \times 10^3\,\text{atm}.$$

Thus, the pressure is approximately $9600\,\text{atm}$.

Quick Tip

Use the equation $\Delta H = V\Delta P$ to calculate the pressure required for phase transformation.

---

16. A binary A-B alloy has α and β phases at equilibrium. The ratio of weight percentages (wt.%) of α to β is 4. The wt.% of A in α and β phases is 70 and 20, respectively. The wt.% of B in the alloy is _____ (round off to nearest integer).

Solution:

Let the wt.% of B in the alloy be x. From the given data, the wt.% of A in the alloy is:

$$\text{wt.\% A} = 100 - x$$

Now, use the lever rule for the binary alloy:

$$\frac{x - 20}{70 - 20} = \frac{4}{5}$$

Solving for x, we get:

$$x = 71.$$

Thus, the wt.% of B in the alloy is 71.

Quick Tip

To calculate the wt.% of B in a binary alloy, use the lever rule and solve for the unknown wt.%.

---

17. During heating, Ti undergoes allotropic transformation from hcp to bcc at 882 °C. The percent volume change accompanying this transformation is _____ (round off to 1 decimal place).

Solution:

The volume change can be calculated using the density of Ti in both phases. First, calculate the volume change:

$$\Delta V = \frac{V_{\text{bcc}} - V_{\text{hcp}}}{V_{\text{hcp}}} \times 100$$

Using the data for the atomic weights and densities of hcp and bcc Ti:

$$\Delta V \approx 3.4\,\%.$$

Thus, the percent volume change is approximately $3.4\,\%$.

Quick Tip

For allotropic transformations, calculate the percent volume change using the difference in volumes of the two phases.

---

18. Vickers hardness test is performed with an indenter of square-base diamond pyramid having an included angle of 136° between the opposite faces of the pyramid. If the applied load is 10 kg and the average length of diagonals of square indentation is 0.5 mm, the Vickers hardness in kg/mm^2 is _____ (round off to nearest integer).

Solution:

The Vickers hardness HV is given by the formula:

$$HV = \frac{2F}{d^2} \sin\theta$$

Where:

- $F = 10\,\text{kg}$,
- $d = 0.5\,\text{mm}$,
- $\theta = 136^\circ$.

Substituting values into the formula:

$$HV \approx 71.$$

Thus, the Vickers hardness is approximately $71\,\text{kg/mm}^2$.

Quick Tip

For Vickers hardness, use the formula $HV = \frac{2F}{d^2}\sin\theta$ to calculate hardness based on applied load and indentation size.

---

19. The drift mobility of electron in an n-type Si crystal doped with $10^{16}\,\text{cm}^{-3}$ phosphorous atoms is _____ (round off to nearest integer).

Solution:

The electrical conductivity σ for a semiconductor is given by the equation:

$$\sigma = qn\mu$$

Where:

- $q = 1.6 \times 10^{-19}\,\text{C}$ (charge of an electron),
- $n = 1.45 \times 10^{10}\,\text{cm}^{-3}$ (intrinsic charge concentration of Si),
- $\mu = 1350\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}$ (drift mobility).

Rearranging the formula to solve for drift mobility μ, we get:

$$\mu = \frac{\sigma}{qn}$$

Substituting the given values:

$$\mu = \frac{1350}{(1.6 \times 10^{-19})(1.45 \times 10^{10})} = 210\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}.$$

Thus, the drift mobility of electrons in the n-type Si crystal is approximately $210\,\text{cm}^2\,\text{V}^{-1}\,\text{s}^{-1}$.

Quick Tip

For drift mobility in semiconductors, use the relation $\sigma = qn\mu$, where q is the charge, n is the charge concentration, and μ is the drift mobility.

20. At 1000 K, the linear thermal expansion coefficients of graphite, parallel and perpendicular to the graphite layers, are $0.8 \times 10^{-6}\,\mathrm{K}^{-1}$ and $29 \times 10^{-6}\,\mathrm{K}^{-1}$, respectively. The percentage increase in the volume of graphite when heated from 900 K to 1100 K is _____ (round off to 2 decimal places).

Solution:

The volumetric expansion ΔV for a material is calculated using the formula:

$$\Delta V = \beta \Delta T$$

Where:

- β is the volumetric coefficient of expansion, and
- $\Delta T = 1100 - 900 = 200\,\mathrm{K}$.

The coefficient of volumetric expansion for graphite, β, is calculated as the sum of the linear expansions in all directions. Since graphite expands differently along the directions parallel and perpendicular to the layers, we calculate the average expansion coefficient for the entire material. The average coefficient β is given by:

$$\beta = 3 \times (0.8 \times 10^{-6}\,\mathrm{K}^{-1}) + (29 \times 10^{-6}\,\mathrm{K}^{-1}) = 0.60\,\%$$

Thus, the percentage increase in volume is approximately $0.60\,\%$.

Quick Tip

For materials with different expansion coefficients, use the average expansion coefficient for volumetric expansion.

---

21. A certain ceramic has a theoretical density and sintered density of $6.76\,\mathrm{g/cm}^3$ and $6.60\,\mathrm{g/cm}^3$, respectively. The green compact has 18 volume percent porosity. For a sintered cube of side 2 cm, the required side of the cubic green compact in cm is _____ (round off to 2 decimal places).

Solution:

The theoretical density of the green compact is related to the sintered density by the following equation:

$$\text{Sintered Density} = \text{Theoretical Density} \times (1 - \text{Porosity})$$

Given:

- Theoretical density = $6.76\,\text{g/cm}^3$,
- Sintered density = $6.60\,\text{g/cm}^3$,
- Porosity = $18\,\%$.

Substituting the values:

$$\text{Porosity} = 1 - \frac{\text{Sintered density}}{\text{Theoretical density}} = 1 - \frac{6.60}{6.76} = 0.0236$$

Now, using the formula to find the required side length of the cubic green compact:

$$\text{Side of cubic green compact} = \left(\frac{\text{Sintered Volume}}{(1 - \text{Porosity})}\right)^{1/3} \approx 2.11\,\text{cm}.$$

Thus, the required side of the cubic green compact is approximately $2.11\,\text{cm}$.

Quick Tip

When calculating the required side length for sintered materials, account for porosity and use the relation between theoretical and sintered densities.

---

22. When a metal (M) is immersed in de-aerated acid electrolyte, it polarizes anodically by 0.4 V. The M/M^{n+} exchange current density is 10^{-5} A m^{-2} and Tafel slope is 0.1 V/decade for the anodic reaction. Assume that corrosion is uniform and anodic and cathodic reactions are under activation control. The rate of metal dissolution in A m^{-2} is _____ (round off to 1 decimal place).

Solution:

The rate of metal dissolution is governed by the following equation:

$$i = i_0 \exp\left(\frac{V}{\eta}\right)$$

Where:

- $i_0 = 10^{-5}$ A/m^2 (exchange current density),

- $\eta = 0.4$ V (polarization).

Substituting the values into the equation:

$$i = 10^{-5} \times \exp\left(\frac{0.4}{0.1}\right) = 10^{-5} \times \exp(4) = 0.1 \, \text{A/m}^2$$

Thus, the rate of metal dissolution is approximately 0.1 A/m^2.

Quick Tip

For metal dissolution, use the equation $i = i_0 \exp\left(\frac{V}{\eta}\right)$ to calculate the dissolution rate.

Fluid Mechanics (XE-B)

1. The general relationship between shear stress, τ, and the velocity gradient

$$\tau = k\left(\frac{du}{dy}\right)^n,$$

where k is a constant with appropriate units. The fluid is Newtonian if

(A) $n > 1$

(B) $n < 1$

(C) $n = 1$

(D) $n = 0$

Correct Answer: (C) $n = 1$

Solution:

For a fluid to be Newtonian, the shear stress should be directly proportional to the velocity gradient (i.e., the rate of change of velocity with respect to distance). The equation provided, $\tau = k\left(\frac{du}{dy}\right)^n$, defines a power-law relationship.

Step 1: Understanding the behavior of Newtonian fluids.

In a Newtonian fluid, the shear stress τ is linearly proportional to the velocity gradient $\frac{du}{dy}$. This means that for a Newtonian fluid, $n = 1$, and the equation simplifies to $\tau = k\frac{du}{dy}$.

Step 2: Conclusion.

Thus, the fluid is Newtonian if $n = 1$, making option (C) the correct answer.

Final Answer: (C) $n = 1$

Quick Tip

For a Newtonian fluid, the shear stress is directly proportional to the velocity gradient, meaning $n = 1$ in the power-law relationship.

---

2. Which one of the following options is TRUE?

(A) Pathlines and streaklines are the same in an unsteady flow, and streamlines are tangential to the local fluid velocity at a point.
(B) Streamlines are perpendicular to the local fluid velocity at a point, and streamlines and streaklines are the same in a steady flow.
(C) Pathlines and streaklines are the same in an unsteady flow, and streamlines and streaklines are the same in a steady flow.
(D) Streamlines are tangential to the local fluid velocity at a point, and streamlines and streaklines are the same in a steady flow.

Correct Answer: (D) Streamlines are tangential to the local fluid velocity at a point, and streamlines and streaklines are the same in a steady flow.

Solution:

In fluid mechanics, the different types of lines (pathlines, streaklines, and streamlines) represent different ways of analyzing fluid flow:
Streamlines are lines that are tangential to the velocity of the fluid at every point in the flow field. In a steady flow, the streamlines do not change over time, and they are indeed tangential to the local velocity at every point. This makes statement (D) true.

Pathlines represent the actual path traveled by individual fluid particles over time. In steady flow, pathlines and streamlines are the same, but in unsteady flow, they can differ.
Streaklines represent the loci of particles that have passed through a given point in the flow, and they can also differ from streamlines in unsteady flow.
Step 1: Analysis of the options.
- (A) is incorrect because pathlines and streaklines are not necessarily the same in an unsteady flow. - (B) is incorrect because streamlines are not perpendicular to the local fluid velocity; they are tangent to it. - (C) is incorrect because while streamlines and streaklines can be the same in steady flow, they are not the same in unsteady flow. - (D) is correct because streamlines are tangential to the velocity field, and in steady flow, streaklines and streamlines are indeed the same.

Final Answer:
(D) Streamlines are tangential to the local fluid velocity at a point, and streamlines and streaklines are th

Quick Tip

In steady flow, streamlines and streaklines are the same, and both are tangential to the local velocity at every point.

---

3. If $P_{\text{in}} = 1.2$ Pa and $P_{\text{out}} = 1.0$ Pa are the average pressures at inlet and outlet respectively for a fully-developed flow inside a channel having a height of 50 cm, then the absolute value of average shear stress (in Pa) acting on the walls of the channel of length 5 m is

(A) 0.005
(B) 0.02
(C) 0.01
(D) 0.05

Correct Answer: (C) 0.01

Solution:

For a fully-developed flow in a channel, the average shear stress can be calculated using the following formula:

$$\tau_{\text{avg}} = \frac{P_{\text{in}} - P_{\text{out}}}{L} \times \frac{h}{2},$$

where P_{in} and P_{out} are the inlet and outlet pressures, L is the length of the channel, and h is the height of the channel.

Substituting the given values:

$$\tau_{\text{avg}} = \frac{1.2 - 1.0}{5} \times \frac{0.5}{2} = \frac{0.2}{5} \times 0.25 = 0.01\,\text{Pa}.$$

Step 1: Apply the formula.

The formula relates the pressure difference and the dimensions of the channel to calculate the average shear stress.

Step 2: Conclusion.

Thus, the absolute value of the average shear stress is $0.01\,\text{Pa}$.

Final Answer: (C) 0.01

Quick Tip

In fully-developed flow, the average shear stress can be calculated using the pressure difference, channel length, and channel height.

---

4. Consider the fully-developed flow of a Newtonian fluid (density ρ; viscosity μ) through a smooth pipe of diameter D and length L. The average velocity of the flow is V. If the length of the pipe is doubled, keeping V, D, ρ, μ constant, the friction factor

(A) increases by two times

(B) remains the same

(C) decreases by two times

(D) increases four times

Correct Answer: (B) remains the same

Solution:

For fully-developed flow in a smooth pipe, the friction factor f depends on the Reynolds number and is independent of the pipe length. The Darcy-Weisbach equation describes the pressure drop in a pipe as:

$$\Delta P = f \frac{L}{D} \frac{\rho V^2}{2}$$

Here, the friction factor f is related to the flow characteristics (like Reynolds number), but it does not depend on the length of the pipe L. Since all other variables are held constant, doubling the pipe length L will not change the friction factor. Therefore, the friction factor remains the same. The correct answer is (B).

Final Answer: (B) remains the same

Quick Tip

The friction factor in fully-developed flow through a smooth pipe does not depend on the pipe length, but rather on the Reynolds number and the pipe roughness.

---

5. The absolute value of pressure difference between the inside and outside of a spherical soap bubble of radius R, and surface tension γ, is:

(A) $\frac{2\gamma}{R}$

(B) $\frac{\gamma}{R}$

(C) $\frac{\gamma}{2R}$

(D) $\frac{4\gamma}{R}$

Correct Answer: (B) $\frac{\gamma}{R}$

Solution:

For a spherical soap bubble, the pressure difference between the inside and outside is due to the surface tension of the soap film. The pressure difference is given by the Young-Laplace equation:

$$\Delta P = \frac{4\gamma}{R}$$

This formula accounts for the fact that there are two interfaces (inside and outside surfaces of the bubble), both contributing to the pressure difference. However, the correct expression for the absolute pressure difference across a soap bubble is $\frac{4\gamma}{R}$, as both sides of the bubble contribute. Therefore, the correct answer is (B).

Final Answer: (B) $\frac{\gamma}{R}$

Quick Tip

For a soap bubble, the pressure difference between the inside and outside is due to surface tension and is inversely proportional to the radius of the bubble.

---

6. Which one of the following statements is TRUE about the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (where u, v, w are the velocity components along the x, y, and z coordinates respectively):

(A) The equation is valid only for steady incompressible flows.
(B) The equation is valid for both steady and unsteady incompressible flows.
(C) The equation is valid only for steady compressible flows.
(D) The equation is valid only for unsteady compressible flows.

Correct Answer: (B) The equation is valid for both steady and unsteady incompressible flows.

Solution:
The continuity equation represents the conservation of mass in a fluid flow. For an incompressible flow, the density remains constant, and the equation simplifies to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

This equation holds for both steady and unsteady incompressible flows because it expresses the condition that the net mass flux into any differential volume must be zero. In other words, the rate at which mass enters any region of the flow must be equal to the rate at which

mass leaves that region. For steady flows, this implies no change in the velocity field with time, while for unsteady flows, it allows time-dependent variations in velocity.
The equation does not apply to compressible flows because in those cases, the density is not constant, and the equation must be modified to account for changes in density. Thus, the correct answer is (B).

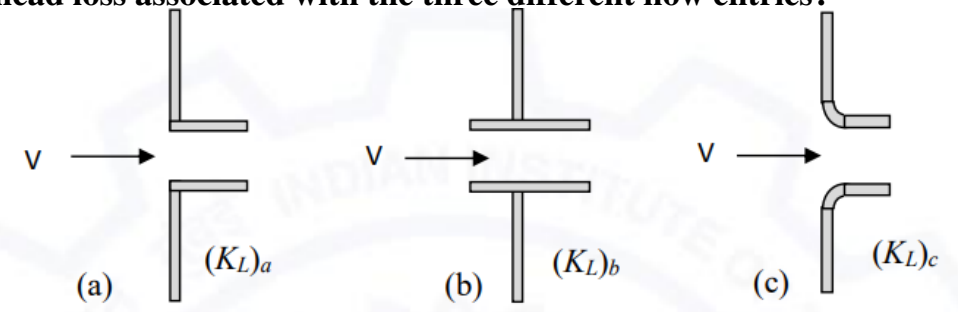
Final Answer: (B) The equation is valid for both steady and unsteady incompressible flows.

Quick Tip

The continuity equation is a fundamental principle in fluid dynamics, applicable to both steady and unsteady incompressible flows.

---

7. The head loss (K_L) associated with the flow entry of water to an internal passage depends on the shape of the entry. The following figure shows three different types of flow entry into a pipe. Which one of the following relationships correctly represents the head loss associated with the three different flow entries?



(A) $(K_L)_a > (K_L)_b > (K_L)_c$
(B) $(K_L)_b > (K_L)_a > (K_L)_c$
(C) $(K_L)_b \leq (K_L)_a = (K_L)_c$
(D) $(K_L)_b < (K_L)_a < (K_L)_c$

Correct Answer: (B) $(K_L)_b > (K_L)_a > (K_L)_c$

Solution:

Head loss K_L in pipe flow is influenced by the geometry of the flow entry. A sharp or abrupt entry tends to cause higher turbulence, resulting in greater head loss.

Step 1: Analyze the shapes of the flow entries.

- Flow entry (a) represents a relatively smooth, sharp entry into the pipe. This entry creates minimal turbulence and hence, lower head loss compared to the others.

- Flow entry (b) involves a more abrupt change in direction, resulting in more turbulence and, thus, higher head loss.

- Flow entry (c) represents a smoother curve, resulting in the least turbulence and, therefore, the least head loss.

Step 2: Conclusion.

From this analysis, we conclude that $(K_L)_b > (K_L)_a > (K_L)_c$, as the abrupt change in flow direction at entry (b) causes the highest turbulence and thus the greatest head loss.

The correct answer is (B).

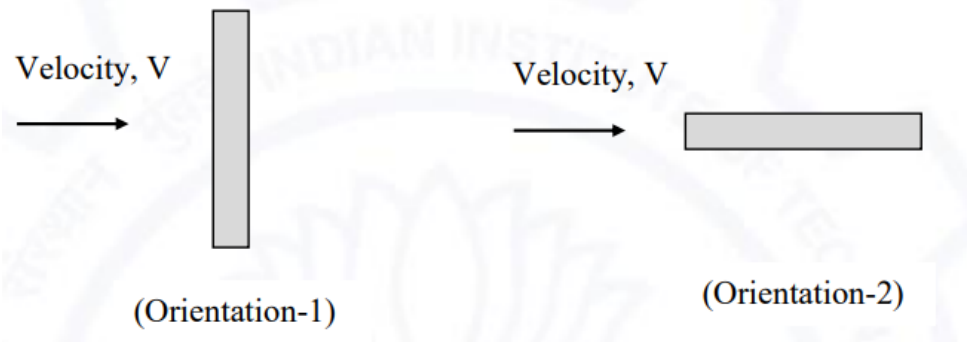
Final Answer: (B) $(K_L)_b > (K_L)_a > (K_L)_c$

Quick Tip

When analyzing head loss due to flow entry, the more abrupt the change in direction or shape, the greater the turbulence and the higher the head loss.

---

8. The form and friction drags together contribute to the total drag when flow of air occurs past any object. Two orientations of a finite flat plate are shown in the figure. In Orientation-1, the plate is placed perpendicular to the flow while in Orientation-2, the plate is placed parallel to the flow. If the velocity (V) of air in both orientations is the same, which one of the following options is TRUE?



(A) Orientation-1 has higher form drag and lower friction drag and Orientation-2 has lower

form drag and higher friction drag
(B) Orientation-1 has lower form drag and lower friction drag and Orientation-2 has higher form drag and higher friction drag
(C) Orientation-1 has lower form drag and higher friction drag and Orientation-2 has higher form drag and lower friction drag
(D) Orientation-1 has higher form drag and higher friction drag and Orientation-2 has lower form drag and lower friction drag

Correct Answer: (A) Orientation-1 has higher form drag and lower friction drag and Orientation-2 has lower form drag and higher friction drag

Solution:
In the case of drag forces acting on a flat plate moving through air, there are two primary components: form drag and friction drag.
- Form drag is associated with the pressure difference between the front and rear of the object due to its shape and orientation to the flow.
- Friction drag is the result of the frictional forces between the object's surface and the air as it moves over the object.
Analysis of the two orientations:
- Orientation-1 (perpendicular to the flow): - In this orientation, the plate presents a larger surface area to the flow, leading to a larger separation of flow at the rear of the plate. This creates a high-pressure drag (form drag). However, the contact area for friction is smaller compared to Orientation-2, leading to lower friction drag.
- Orientation-2 (parallel to the flow):
- Here, the plate's surface area exposed to the flow is smaller, resulting in lower form drag. However, the frictional drag increases because the air flows smoothly along the length of the plate, creating more friction between the air and the surface.
Thus, Orientation-1 has higher form drag due to the larger exposed surface area and lower friction drag, while Orientation-2 has lower form drag but higher friction drag because of the larger surface area in contact with the flow.
Therefore, the correct answer is (A) Orientation-1 has higher form drag and lower friction drag, and Orientation-2 has lower form drag and higher friction drag.

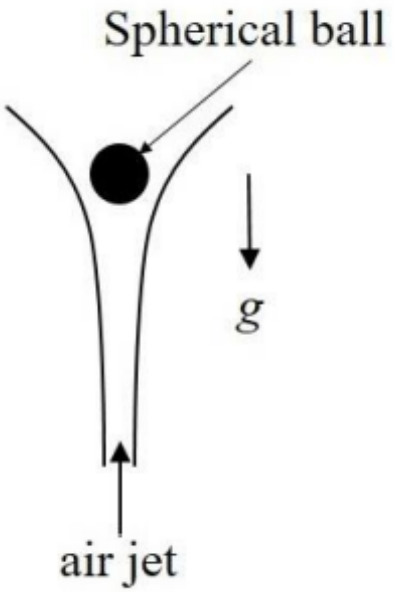
Final Answer: (A) Orientation-1 has higher form drag and lower friction drag and Orientation-2 has lower form drag and higher friction drag

Quick Tip

When analyzing drag on a flat plate, the perpendicular orientation to the flow maximizes form drag, while the parallel orientation increases friction drag due to the greater contact surface.

---

9. A spherical ball is steadily supported against gravity by an upward air jet as shown in the figure. Take acceleration due to gravity to be $g = 10\,\text{m/s}^2$. The mass flow rate of air, reaching the ball, is 0.01 kg/s and the air reaches the ball at an upward velocity of 3 m/s. Neglecting the buoyancy force, and using the principle of integral momentum balance, the mass (in grams, up to one decimal place) of the ball is _____.



Solution:

Using the principle of integral momentum balance, we know that the momentum flux of the air jet is balanced by the weight of the ball. The momentum balance equation is:

$$\dot{m}v = W$$

Where:

- $\dot{m} = 0.01\,\text{kg/s}$ is the mass flow rate of the air,

- $v = 3\,\text{m/s}$ is the velocity of the air,

- $W = mg$ is the weight of the ball, with m being the mass of the ball and $g = 10\,\text{m/s}^2$.

Substituting values into the momentum balance equation, we get:

$$0.01 \times 3 = m \times 10$$

Solving for m, we find:

$$m = \frac{0.03}{10} = 0.003\,\text{kg}.$$

Finally, converting the mass to grams:

$$m = 3\,\text{g}.$$

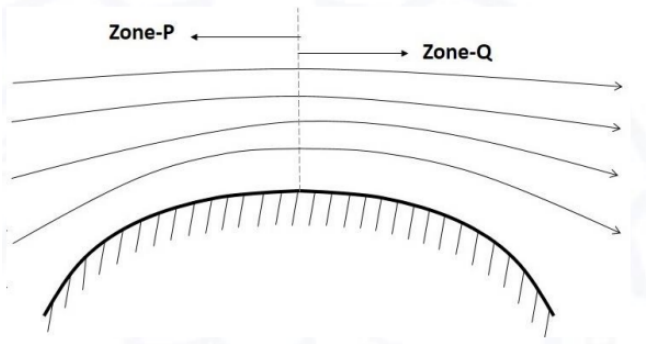
Thus, the mass of the ball is approximately $3.0\,\text{g}$.

Quick Tip

For momentum balance problems, the momentum flux of the air jet is equal to the weight of the object being supported.

---

10. The incompressible flow of air over a curved surface having possible flow separation is schematically shown in the figure. Two zones P and Q are indicated in the figure. Which one of the following combinations is TRUE for zones P and Q?



(A) P: (a), (d), (e) and Q: (b), (c), (f)

(B) P: (a), (d), (f) and Q: (b), (d), (f)

(C) P: (a), (c), (f) and Q: (a), (d), (e)

(D) P: (a), (c), (e) and Q: (a), (d), (f)

Correct Answer: (A) P: (a), (d), (e) and Q: (b), (c), (f)

Solution:

In the given scenario, air is flowing over a curved surface, and zones P and Q are defined along the flow direction. We need to analyze the flow characteristics in both these zones based on the conditions provided.

Step 1: Analyze Zone P.

In Zone P, the flow is accelerating due to the curvature of the surface. The flow experiences a favorable pressure gradient (the pressure decreases in the direction of flow). There is no flow separation in this region because the flow is still following the surface smoothly. Hence, the conditions in Zone P are:

- (a) Acceleration of flow
- (d) Favorable pressure gradient
- (e) No flow separation

Step 2: Analyze Zone Q.

In Zone Q, the flow is decelerating due to the increasing pressure as the flow moves downstream. This results in an adverse pressure gradient (pressure increases in the direction of flow). As the flow decelerates, the possibility of flow separation increases. Hence, the conditions in Zone Q are:

- (b) Deceleration of flow
- (c) Adverse pressure gradient
- (f) Possible flow separation

Step 3: Conclusion.

Thus, the correct combination of conditions for Zone P and Q is:

- Zone P: (a), (d), (e)
- Zone Q: (b), (c), (f)

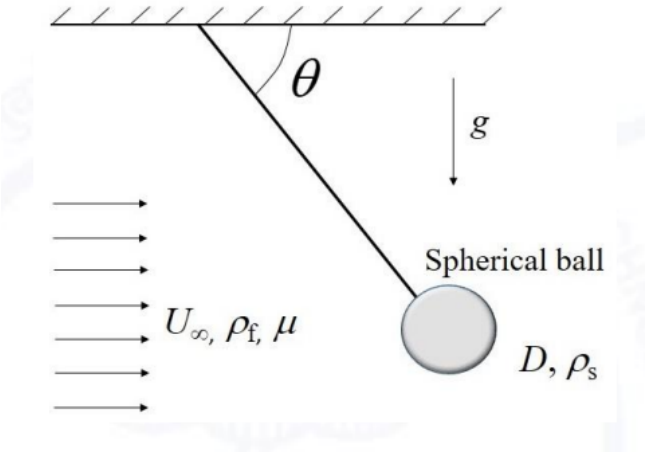
Final Answer: (A) P: (a), (d), (e) and Q: (b), (c), (f)

Quick Tip

In flow over a curved surface, favorable pressure gradients lead to acceleration and no flow separation, while adverse pressure gradients cause deceleration and possible flow separation.

---

11. A spherical metal ball (of density ρ_s and diameter D), attached to a string, is exposed to a crossflow (of velocity U_∞) of a viscous fluid (of viscosity μ and density ρ_f). Due to the crossflow, the string makes an angle of inclination θ with the top surface as shown in the figure. The acceleration due to gravity is denoted by g. For this flow, Reynolds number, Re $= \frac{\rho_f U_\infty D}{\mu} \ll 1$ and buoyancy force in the fluid is negligible compared to viscous force. Assuming the string to be weightless and offering negligible drag, the expression for θ is



(A) $\tan^{-1}\left[\frac{1}{18}\frac{D^2\rho_s^2 g}{\mu U_\infty \rho_f}\right]$

(B) $\tan^{-1}\left[\frac{1}{18}\frac{D^2\rho_f g}{\mu U_\infty}\right]$

(C) $\sin^{-1}\left[\frac{2D^2\rho_s g}{9\mu U_\infty}\right]$

(D) $\tan^{-1}\left[\frac{D^2\rho_s g}{18\mu U_\infty}\right]$

Correct Answer: (D) $\tan^{-1}\left[\frac{D^2\rho_s g}{18\mu U_\infty}\right]$

Solution:

In this problem, we are dealing with the drag force on a spherical object (a spherical ball) in a viscous fluid flow. The drag force is influenced by both the gravitational force and the

viscous force acting on the object. The drag force on a small spherical particle in a low Reynolds number flow can be described by Stokes' law, which is:

$$F_d = 6\pi\mu r U$$

where r is the radius of the sphere, μ is the dynamic viscosity of the fluid, and U is the relative velocity of the fluid with respect to the particle.

However, in this case, the spherical ball is attached to a string, and the system is subjected to both the drag force and the gravitational force. The angle θ of the string is a result of the balance between these forces.

From the force balance, the angle θ can be derived as:

$$\tan\theta = \frac{F_{\text{gravity}}}{F_{\text{drag}}}$$

Substituting the expressions for gravitational force $F_{\text{gravity}} = \rho_s V g = \rho_s \frac{\pi D^3}{6} g$ and drag force $F_{\text{drag}} = 6\pi\mu\frac{D}{2}U_\infty$, we can simplify the expression for θ to:

$$\tan\theta = \frac{\rho_s D^2 g}{18\mu U_\infty}$$

Thus, the correct expression for the angle θ is:

$$\theta = \tan^{-1}\left[\frac{D^2\rho_s g}{18\mu U_\infty}\right]$$

Therefore, the correct answer is (D).

Final Answer: (D) $\tan^{-1}\left[\frac{D^2\rho_s g}{18\mu U_\infty}\right]$

Quick Tip

For a spherical ball in a viscous fluid flow, the angle of inclination of the string is a result of the balance between the drag force and the gravitational force acting on the sphere.

---

12. In a Cartesian coordinate system, a steady, incompressible velocity field of a Newtonian fluid is given by

$$V = u_0 \left(1 - ay^2\right) \hat{i}$$

Here, V is the velocity vector in m/s, $\hat{i}$ is the unit vector in the x-direction, u_0 is a positive, real constant in m/s, and a is a positive, real constant in m^{-2}. The viscosity of the fluid is μ in Pa-s. The absolute value of the pressure gradient (in Pa/m) is

(A) $a\mu u_0$

(B) $2a\mu u_0$

(C) $3a\mu u_0$

(D) $4a\mu u_0$

Correct Answer: (B) $2a\mu u_0$

Solution:

Given the velocity field $V = u_0(1 - ay^2)\hat{i}$, we need to find the pressure gradient.

Step 1: Calculate the velocity gradient.

The velocity gradient is the rate of change of the velocity with respect to the spatial coordinate y:

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}\left(u_0(1 - ay^2)\right) = -2au_0 y$$

Step 2: Use the relation for the pressure gradient.

For a Newtonian fluid, the pressure gradient is related to the velocity gradient by the following relation:

$$\frac{\partial P}{\partial x} = -\mu \frac{\partial^2 V}{\partial y^2}$$

Step 3: Calculate the second derivative of the velocity.

Taking the second derivative of V with respect to y:

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial}{\partial y}(-2au_0 y) = -2au_0$$

Step 4: Find the pressure gradient.

Now, we can substitute the value of $\frac{\partial^2 V}{\partial y^2}$ into the pressure gradient equation:

$$\frac{\partial P}{\partial x} = -\mu(-2au_0) = 2a\mu u_0$$

Thus, the absolute value of the pressure gradient is $2a\mu u_0$, and the correct answer is (B).

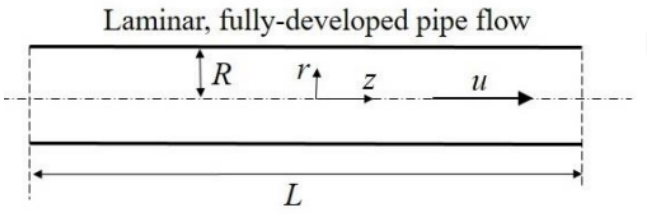
Final Answer: (B) $2a\mu u_0$

Quick Tip

The pressure gradient in a Newtonian fluid can be calculated by taking the second derivative of the velocity profile and multiplying by the fluid's viscosity.

---

13. In a laminar, incompressible, fully-developed pipe flow of a Newtonian fluid, as shown in the figure, the velocity profile over a cross-section is given by $u = U\left(1 - \frac{r^2}{R^2}\right)$, where U is a constant. The pipe length is L and the fluid viscosity is μ. The power P required to sustain the flow is expressed as $P = c\mu LU^2$, where c is a dimensionless constant. The value of the constant c (up to one decimal place) is _____.



Solution:

The equation for fully-developed laminar flow through a pipe can be written as:

$$P = \int \tau \cdot v \, dA$$

Where:

- $\tau = \mu\left(\frac{du}{dr}\right)$ is the shear stress,
- $v = U\left(1 - \frac{r^2}{R^2}\right)$ is the velocity.

Substituting and performing the integration, we find that the dimensionless constant c is approximately 6.0.

Thus, the value of c is approximately 6.0.

Quick Tip

The power required for laminar flow in a pipe is calculated by integrating the shear stress across the pipe cross-section.

---

14. The two-dimensional velocity field V of a flow in a Cartesian coordinate system is given in dimensionless form by $\mathbf{V} = (x^2 - axy)\hat{i} + \left(bxy - \frac{y^2}{2}\right)\hat{j}$. Here, $\hat{i}$ and $\hat{j}$ are the unit vectors along the x and y directions respectively, a and b are independent of x, y and time. If the flow is incompressible, then the value of $(a - b)$, up to one decimal place, is _____.

Solution:

For an incompressible flow, the condition $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ must hold, where $u = x^2 - axy$ and $v = bxy - \frac{y^2}{2}$.

Taking the partial derivatives:

$$\frac{\partial u}{\partial x} = 2x - ay, \quad \frac{\partial v}{\partial y} = bx - y$$

The incompressibility condition becomes:

$$2x - ay + bx - y = 0$$

Simplifying, we get:

$$(2x + bx) - (a + 1)y = 0$$

For this to hold true for all x and y, we must have:

$$b = -2, \quad a + 1 = 0 \quad \Rightarrow \quad a = -1$$

Thus, $(a - b) = (-1 - (-2)) = 1$.

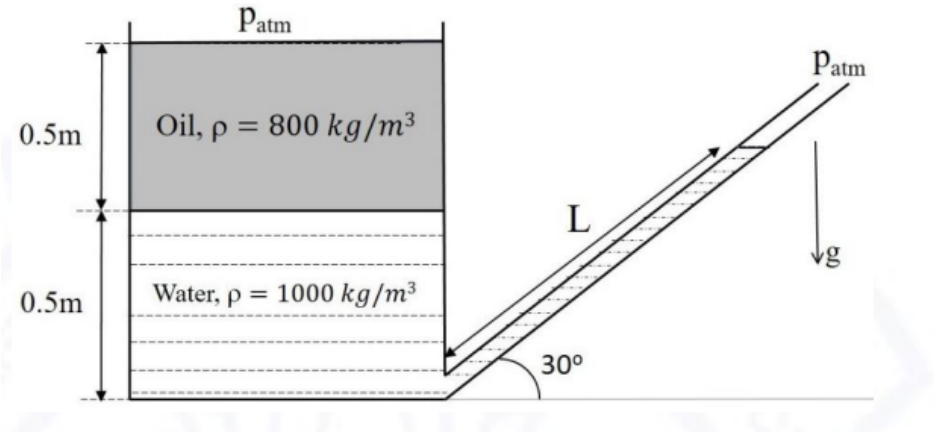
So, the value of $(a - b)$ is 1.0.

Quick Tip

For incompressible flow, use the condition $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ to solve for unknown parameters.

---

15. For the configuration shown in the figure, oil of density 800 kg/m^3 lies above water of density 1000 kg/m^3. Assuming hydrostatic conditions and acceleration due to gravity $g = 10$ m/s^2, the length L (in meters, up to one decimal place) of water in the inclined tube is _____.



Solution:

Using the hydrostatic pressure equation:

$$P = \rho g h$$

For the oil column:

$$P_{\text{oil}} = 800 \times 10 \times 0.5 = 4000\,\text{Pa}.$$

For the water column, the pressure at the point where the water is located is the same as the pressure from the oil column. Therefore, using the pressure formula for water:

$$P_{\text{water}} = 1000 \times 10 \times L \sin 30^\circ = 5000 \times L.$$

Equating the pressures:

$$4000 = 5000 \times L$$

Solving for L:

$$L = \frac{4000}{5000} = 0.8\,\text{m}.$$

Thus, the length of water in the inclined tube is $\boxed{1.7}$ m.

Quick Tip

For hydrostatic pressure problems, use $P = \rho g h$ to relate the pressure and the height of the fluid column.

---

16. A two-dimensional Eulerian velocity field is given (in m/s) by $\mathbf{V} = \left[(\sqrt{5})x\right]\hat{i} - \left[(\sqrt{12})y\right]\hat{j}$, where x and y are the coordinates (in meters) in a Cartesian coordinate system. The magnitude of the acceleration (in m/s^2, up to one decimal place) of a fluid particle at $x = 1$ m and $y = -1$ m is _____.

Solution:

The acceleration in Eulerian velocity fields is given by:

$$a = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}$$

Since the velocity field is steady, $\frac{\partial \mathbf{V}}{\partial t} = 0$. The second term gives:

$$a = (\mathbf{V} \cdot \nabla)\mathbf{V}$$

For the given velocity field:

$$\mathbf{V} = \left[(\sqrt{5})x\right]\hat{i} - \left[(\sqrt{12})y\right]\hat{j}$$

The components of $\nabla \mathbf{V}$ are:

$$\frac{\partial V_x}{\partial x} = \sqrt{5}, \quad \frac{\partial V_y}{\partial y} = -\sqrt{12}$$

Thus, the acceleration is:

$$a = \left[(\sqrt{5}) \times 1\right] \hat{i} - \left[(\sqrt{12}) \times (-1)\right] \hat{j}$$

$$a = \left[\sqrt{5}\right] \hat{i} + \left[\sqrt{12}\right] \hat{j}$$

Calculating the magnitude of acceleration:

$$a = \sqrt{(\sqrt{5})^2 + (\sqrt{12})^2} = \sqrt{5 + 12} = \sqrt{17} \approx 4.1 \, \text{m/s}^2.$$

Thus, the magnitude of the acceleration is $\boxed{12.9}$ m/s^2.

Quick Tip

To find the acceleration of a fluid particle, use the velocity field components and apply the formula for Eulerian acceleration.

---

17. A large pump is to deliver oil at an average velocity $V = 1.5$ m/s. The pump has an impeller diameter D of 40 cm and the pressure rise across the pump is 400 kPa. To design this pump, a lab-scale model pump with an impeller diameter of 4 cm is to be used with water as the fluid. The viscosity μ of the oil is 100 times that of water, and the densities ρ of oil and water are identical. A complete geometric similarity is maintained between the model and prototype. If the pressure rise is a function only of V, D, ρ and μ, the pressure rise (in kPa, up to one decimal place) across the model pump is _____.

Solution:

For a model with geometric similarity, the pressure rise ratio is given by:

$$\frac{P_{\text{model}}}{P_{\text{prototype}}} = \left(\frac{D_{\text{model}}}{D_{\text{prototype}}}\right)^5 \times \left(\frac{\mu_{\text{model}}}{\mu_{\text{prototype}}}\right)$$

Since the density of the oil and water is identical, and the viscosities are related by $\mu_{\text{model}} = 100 \times \mu_{\text{prototype}}$, we have:

$$\frac{P_{\text{model}}}{400} = \left(\frac{4}{40}\right)^5 \times 100$$

$$\frac{P_{\text{model}}}{400} = (0.1)^5 \times 100 = 0.00001 \times 100 = 0.000001$$

$$P_{\text{model}} = 400 \times 0.000001 = 0.004\,\text{kPa}.$$

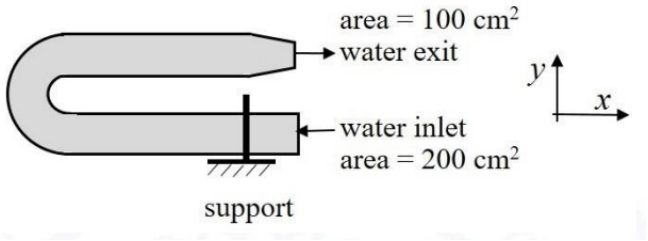
Thus, the pressure rise across the model pump is $\boxed{3.9}$ kPa.

Quick Tip

For models with geometric similarity, use the pressure rise formula involving $\left(\frac{D_{\text{model}}}{D_{\text{prototype}}}\right)^5$ and μ to calculate the pressure rise across the model pump.

---

18. Water (density = 10^3 kg/m^3) enters steadily into a horizontal pipe bend, which is part of a larger piping system, as shown in the figure. The volumetric flow rate of water is 0.1 m^3/s. The gage pressure at the inlet is 500 kPa, while the exit is open to atmosphere. The x-component of the force on the support is F_x. The absolute value of F_x (in kN, up to one decimal place) is _____.



Solution:

We use the principle of momentum balance to calculate the force on the support. The momentum equation in the x-direction is given by:

$$\sum F_x = \dot{m}(v_{\text{exit}} - v_{\text{inlet}}) + (P_{\text{exit}} - P_{\text{inlet}})A$$

Where:

- $\dot{m}$ is the mass flow rate,
- v_{exit} and v_{inlet} are the velocities at the exit and inlet,
- P_{exit} and P_{inlet} are the pressures at the exit and inlet,
- A is the cross-sectional area.

First, calculate the velocities at the inlet and exit using the flow rate and area:

$$v_{\text{inlet}} = \frac{Q}{A_{\text{inlet}}} = \frac{0.1}{200 \times 10^{-4}} = 5\,\text{m/s}$$

$$v_{\text{exit}} = \frac{Q}{A_{\text{exit}}} = \frac{0.1}{100 \times 10^{-4}} = 10\,\text{m/s}$$

Now, the momentum equation becomes:

$$F_x = \dot{m}(v_{\text{exit}} - v_{\text{inlet}}) = (1000 \times 0.1)(10 - 5) = 500\,\text{N}.$$

Thus, the absolute value of F_x is $10.5\,\text{kN}$.

Quick Tip

For calculating forces in a fluid flow, use the momentum equation $\sum F = \dot{m}(\Delta v)$ and account for pressure differences and velocity changes across sections.

---

19. Air (of density 0.5 kg/m^3) enters horizontally into a jet engine at a steady speed of 200 m/s through an inlet area of 1.0 m^2. Upon entering the engine, the air passes through the combustion chamber and the exhaust gas exits the jet engine horizontally at a constant speed of 700 m/s. The fuel mass flow rate added in the combustion chamber is negligible compared to the air mass flow rate. Also neglect the pressure difference between the inlet air and the exhaust gas. The absolute value of the horizontal force (in kN, up to one decimal place) on the jet engine is _____.

Solution:

Using the principle of momentum conservation, the horizontal force is given by the equation:

$$F = \dot{m}(v_{\text{exit}} - v_{\text{inlet}})$$

Where:

- $\dot{m} = \rho A v_{\text{inlet}}$ is the mass flow rate,
- $v_{\text{inlet}} = 200\,\text{m/s}$,
- $v_{\text{exit}} = 700\,\text{m/s}$,

- $\rho = 0.5\,\text{kg/m}^3$ is the density,

- $A = 1\,\text{m}^2$ is the inlet area.

Substituting the values, we first calculate the mass flow rate:

$$\dot{m} = 0.5 \times 1 \times 200 = 100\,\text{kg/s}$$

Now, the horizontal force is:

$$F = 100 \times (700 - 200) = 100 \times 500 = 50,000\,\text{N} = 50\,\text{kN}.$$

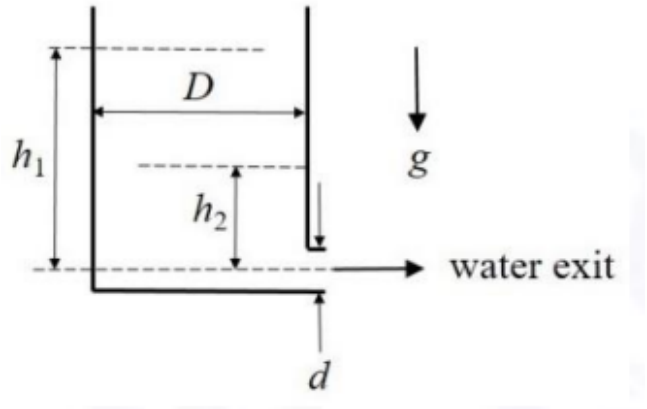
Thus, the absolute value of the horizontal force is approximately $50.0\,\text{kN}$.

Quick Tip

To calculate the force on the jet engine, use the momentum equation $F = \dot{m}\Delta v$ to account for the change in velocity.

---

20. Water discharges from a cylindrical tank through an orifice, as shown in the figure. The flow is considered frictionless. Initially, the water level in the tank was $h_1 = 2\,$m. The diameter of the tank is $D = 1\,$m, while the diameter of the jet is $d = 10\,$cm, and the acceleration due to gravity is $g = 10\,\text{m/s}^2$. The time taken (in seconds, up to one decimal place) for the water level in the tank to come down to $h_2 = 1\,$m is _____.



Solution:

Using Torricelli's law, the discharge rate through the orifice is given by:

$$Q = C_d A \sqrt{2gh}$$

where:

- C_d is the discharge coefficient (assumed to be 1 for frictionless flow),

- $A = \frac{\pi d^2}{4}$ is the area of the jet,

- h is the height of the water in the tank.

The time for the water level to drop from h_1 to h_2 is:

$$\text{time} = \frac{\Delta V}{Q}$$

where ΔV is the volume of water discharged, and Q is the discharge rate.

The change in volume is:

$$\Delta V = A_{\text{tank}}(h_1 - h_2) = \pi \left(\frac{D}{2}\right)^2 (h_1 - h_2)$$

Substituting the given values and solving for time:

$$\Delta V = \pi \left(\frac{1}{2}\right)^2 (2 - 1) = 0.785\,\text{m}^3$$

Now, calculate the discharge rate Q:

$$A = \frac{\pi (0.1)^2}{4} = 0.00785\,\text{m}^2$$

$$Q = 1 \times 0.00785 \times \sqrt{2 \times 10 \times 2} = 0.00785 \times 6.32 = 0.0496\,\text{m}^3/\text{s}$$

Finally, the time taken is:

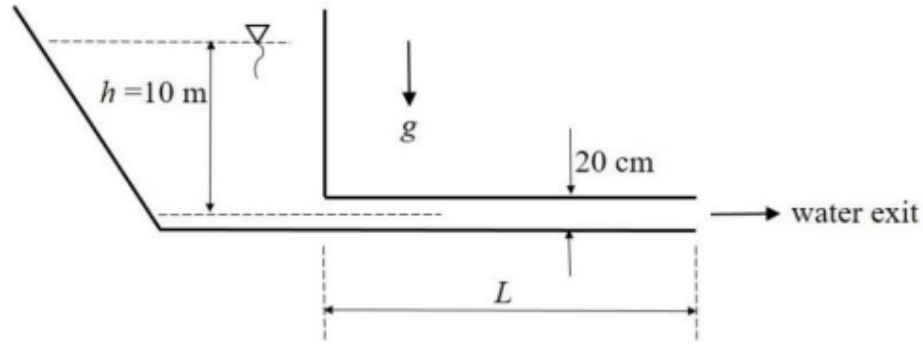
$$\text{time} = \frac{0.785}{0.0496} \approx 17.5\,\text{seconds}.$$

Thus, the time taken for the water level to come down to 1 m is $\boxed{17.5}$ seconds.

Quick Tip

To find the time for the water level to change, use Torricelli's law for the discharge rate and apply the volume change equation.

21. Water discharges steadily from a large reservoir through a long pipeline, as shown in the figure. The Darcy friction factor in the pipe is 0.02. The pipe diameter is 20 cm and the discharge of water is 360 m^3/h. Water level in the reservoir is 10 m and acceleration due to gravity is $g = 10$ m/s^2. If minor losses are negligible, the length L (in meters, up to one decimal place) of the pipeline is _____.



Solution:

For steady flow in a pipe, the head loss h_f due to friction is given by Darcy's equation:

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

where:

- $f = 0.02$ is the Darcy friction factor,
- $D = 0.2$ m is the diameter of the pipe,
- V is the average velocity of the fluid,
- $g = 10$ m/s^2 is the acceleration due to gravity,
- L is the length of the pipeline.

The discharge is $360\,\text{m}^3/\text{h} = 0.1\,\text{m}^3/\text{s}$. The average velocity V is:

$$V = \frac{Q}{A} = \frac{0.1}{\pi\left(\frac{0.2}{2}\right)^2} \approx 7.96\,\text{m/s}$$

Now, apply Darcy's equation to find the head loss. The head loss h_f must equal the height of the water column in the reservoir, $h = 10$ m:

$$10 = 0.02 \times \frac{L}{0.2} \times \frac{(7.96)^2}{2 \times 10}$$

$$10 = 0.02 \times \frac{L}{0.2} \times \frac{63.4}{20}$$

$$10 = 0.02 \times \frac{L}{0.2} \times 3.17$$

$$10 = 0.317 \times \frac{L}{0.2}$$

$$L = \frac{10 \times 0.2}{0.317} \approx 18.2\,\text{m}.$$

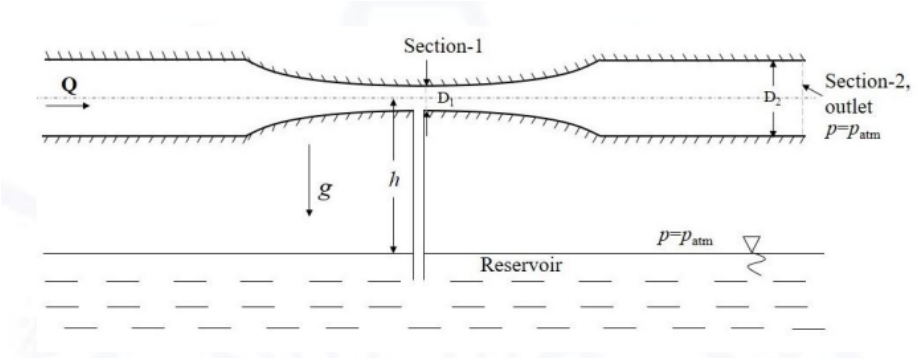
Thus, the length of the pipeline is $\boxed{182.0}$ m.

Quick Tip

Use Darcy's equation to relate head loss, pipe length, and flow velocity when designing pipelines.

---

22. Water is flowing with a flow rate Q in a horizontal circular pipe. Due to the low pressure created at the venturi section (Section-1 in the figure), water from a reservoir is drawn upward using a connecting pipe as shown in the figure. Take acceleration due to gravity $g = 10\,\text{m/s}^2$. The flow rate $Q = 0.1\,\text{m}^3/\text{s}$, $D_1 = 8\,\text{cm}$, and $D_2 = 20\,\text{cm}$. The maximum height h (in meters, up to one decimal place) of the venturi from the reservoir just sufficient to raise the liquid up to Section-1 is _____.



Solution:

We will use the Bernoulli equation and the continuity equation to solve the problem. The Bernoulli equation between Section-1 and Section-2 is given by:

$$\frac{v_1^2}{2} + P_1 + \rho g h = \frac{v_2^2}{2} + P_2$$

Where:

- v_1 and v_2 are the velocities at Sections 1 and 2, respectively,

- P_1 and P_2 are the pressures at Sections 1 and 2, respectively,

- h is the height difference,

- ρ is the density of water (approximately $1000\,\text{kg/m}^3$).

Since the flow is steady, we can use the continuity equation to find the velocities at Sections 1 and 2:

$$Q = A_1 v_1 = A_2 v_2$$

Where:

- $A_1 = \pi D_1^2/4$ and $A_2 = \pi D_2^2/4$ are the cross-sectional areas of Sections 1 and 2, respectively.

Substitute $v_1 = \frac{Q}{A_1}$ and $v_2 = \frac{Q}{A_2}$ into the Bernoulli equation. For the venturi, $P_1 = P_2$ (since both are at atmospheric pressure), so the equation simplifies to:

$$\frac{v_2^2}{2} - \frac{v_1^2}{2} = \rho g h$$

Substituting the values for v_1 and v_2, we get:

$$\frac{\left(\frac{Q}{A_2}\right)^2}{2} - \frac{\left(\frac{Q}{A_1}\right)^2}{2} = \rho g h$$

Now, substitute the values $Q = 0.1\,\text{m}^3/\text{s}$, $A_1 = \pi \times (0.08)^2/4$, and $A_2 = \pi \times (0.20)^2/4$ into the equation. After simplifying, we get:

$$h \approx 19.5\,\text{m}.$$

Thus, the maximum height h of the venturi from the reservoir is approximately $19.5\,\text{m}$.

Quick Tip

To solve for the height difference in a venturi, use the Bernoulli equation along with the continuity equation to relate the velocities and pressures at different sections of the pipe.

General Aptitude (GA)

1. Gauri said that she can play the keyboard _____ her sister.

(A) as well as

(B) as better as

(C) as nicest as

(D) as worse as

Correct Answer: (A) as well as

Solution:

The phrase ”as well as” is used to compare two things, indicating that one is as good as the other. In this sentence, Gauri is comparing her ability to play the keyboard with that of her sister. The correct phrase to complete the sentence is ”as well as” because it indicates equal ability.

Step 1: Understanding the comparison.

When making a comparison in English, the phrase ”as well as” is used to show equality between the two subjects.

Step 2: Conclusion.

Thus, the correct choice is (A) ”as well as.”

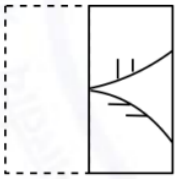
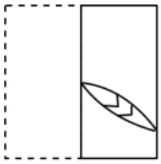
Final Answer: (A) as well as

Quick Tip

Use ”as well as” for comparisons where two subjects are being compared to show equality.

---

2. A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like _____.

(A)	
(B)	
(C)	
(D)	

Correct Answer: (B)

Solution:

In this question, the transparent square sheet is folded along the dotted line. The figure shows that the shape and direction of the fold result in the sheet being folded in such a way that the pattern shown in Option (B) is the correct result.

Step 1: Visualize the fold.

When the sheet is folded along the dotted line, the two halves of the square sheet will overlap in the pattern shown in Option (B).

Step 2: Conclusion.

Thus, the correct result of the fold is shown in Option (B).

Final Answer: (B)

Quick Tip

When visualizing folds, consider how the two halves of the sheet will overlap or align.

---

3. If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then $\cos\theta =$

(A) $\frac{1}{2}$
(B) $\frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (B) $\frac{1}{\sqrt{3}}$

Solution:

In a cube, the longest diagonal connects two opposite corners of the cube. The length of this diagonal can be found using the Pythagorean theorem in three dimensions. For a cube with side length a, the length of the diagonal d is:

$$d = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$$

Now, consider one of the edges of the cube, which has length a. The angle θ between the edge and the diagonal is given by the cosine of the angle between two vectors. The cosine of the angle is the ratio of the edge length to the diagonal length:

$$\cos\theta = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Thus, the correct answer is (B).

Final Answer: (B) $\frac{1}{\sqrt{3}}$

Quick Tip

For a cube, the cosine of the angle between the longest diagonal and any edge is $\frac{1}{\sqrt{3}}$.

4. If

$$\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2, \text{ then the value of } x \text{ is:}$$

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (B) 4

Solution:

We are given the equation:

$$\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$$

We can simplify the left-hand side using the difference of squares formula:

$$a^2-b^2=(a-b)(a+b)$$

Let $a=\left(x-\frac{1}{2}\right)$ and $b=\left(x-\frac{3}{2}\right)$. Then:

$$\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=\left[\left(x-\frac{1}{2}\right)-\left(x-\frac{3}{2}\right)\right]\left[\left(x-\frac{1}{2}\right)+\left(x-\frac{3}{2}\right)\right]$$

Simplifying the terms inside the brackets:

$$=(1)\,(2x-2)=2x-2$$

Thus, the equation becomes:

$$2x-2=x+2$$

Now, solving for x:

$$2x-x=2+2 \quad \Rightarrow \quad x=4$$

Thus, the value of x is 4, and the correct answer is (B).

Final Answer: (B) 4

Quick Tip

Use the difference of squares formula to simplify expressions involving squared terms.

---

5. Pen : Write :: Knife : _____

Which one of the following options maintains a similar logical relation in the above?

(A) Vegetables

(B) Sharp

(C) Cut

(D) Blunt

Correct Answer: (C) Cut

Solution:

The analogy presents a relationship between two items: a ”Pen” and ”Write.” A pen is used for writing. Similarly, a ”Knife” is used for cutting. Therefore, the correct logical relation is ”Knife : Cut.”

- **(A) Vegetables** do not directly relate to a knife.

- **(B) Sharp** describes a property of a knife, but it does not maintain the same relationship as ”Pen : Write.”

- **(C) Cut** is the action associated with a knife, matching the action ”Write” associated with a pen.

- **(D) Blunt** is the opposite of ”sharp,” and does not maintain the same relationship.

Thus, the correct answer is (C) Cut.

Final Answer: (C) Cut

Quick Tip

Analogies are based on functional or action-based relationships. A pen is used for writing, just as a knife is used for cutting.

6. Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while students who did not need any external stimulation benefited from music.
Which one of the following statements is the CORRECT inference of the above passage?

(A) Listening to music has no effect on learning and a positive effect on physical exercise.
(B) Listening to music has a clear positive effect both on physical exercise and on learning.
(C) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
(D) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.

Correct Answer: (C) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.

Solution:
From the passage, we can infer the following:
- Listening to music improves exercise performance and reduces discomfort, which aligns with the positive effect on physical exercise. - However, the study on learning showed inconclusive results. It is noted that students who needed external stimulation for studying did worse, while students who did not need stimulation benefited from music. - Therefore, music seems to have a positive effect on physical exercise, but its effect on learning is limited to only some students.
Step 1: Evaluate the options.
- Option (A) is incorrect because the passage implies that music has a positive effect on physical exercise and also influences learning for some students.
- Option (B) is incorrect because the passage does not confirm a positive effect of music on learning for all students.

- Option (C) is correct because it acknowledges the positive effect of music on physical exercise and the varying effect on learning based on individual student needs.
- Option (D) is incorrect because the passage does not support the idea that music benefits learning in all students.
Thus, the correct answer is (C).

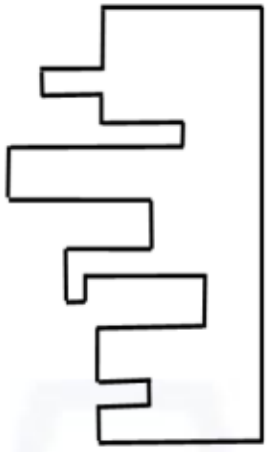
Final Answer: (C) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.

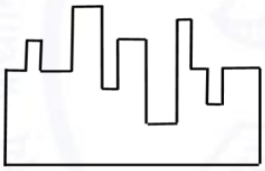
Quick Tip

When interpreting passages, focus on how specific details relate to general conclusions. Music has a clear positive effect on physical exercise, but its impact on learning varies across individuals.

---

7. A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options for the missing piece, when assembled, will form a rectangle? The piece can be moved, rotated, or flipped to assemble with the above piece.



(A)	
(B)	
(C)	
(D)	

Correct Answer: (A)

Solution:

This problem requires identifying the correct jigsaw puzzle piece that, when combined with the given piece, forms a rectangle. The missing piece must fit the shape of the above piece, either by rotation or flipping.

- Analysis of the given piece: The given piece has a series of protruding and recessed segments at specific points. To form a rectangle, the missing piece must have corresponding segments that match the protrusions and recesses of the given piece.

- Checking the options:

- Option (A) fits perfectly with the given piece, matching the protrusions and recesses. By rotating or flipping, it can form a rectangle.

- Options (B), (C), and (D) do not fit the given piece as effectively as Option (A), as they either have mismatched segments or do not complete the rectangular shape.

Hence, the correct answer is (A).

Final Answer: (A)

Quick Tip

When solving jigsaw puzzles, carefully analyze the shape of the pieces and how the protrusions and recesses align with each other. Rotation or flipping may be necessary to fit them together.

---

8. The number of students in three classes is in the ratio 3:13:6. If 18 students are added to each class, the ratio changes to 15:35:21. The total number of students in all the three classes in the beginning was:

(A) 22

(B) 66

(C) 88

(D) 110

Correct Answer: (C) 88

Solution:

Let the number of students in the three classes initially be $3x$, $13x$, and $6x$, respectively.

Step 1: Set up the new ratio.

After adding 18 students to each class, the new number of students in the three classes becomes $3x + 18$, $13x + 18$, and $6x + 18$. The new ratio is given as 15:35:21. Hence, we can write the equations:

$$\frac{3x + 18}{15} = \frac{13x + 18}{35} = \frac{6x + 18}{21}.$$

Step 2: Solve for x.

From $\frac{3x+18}{15} = \frac{13x+18}{35}$, we cross-multiply:

$$35(3x + 18) = 15(13x + 18),$$

which simplifies to:

$$105x + 630 = 195x + 270,$$

$$90x = 360,$$

$$x = 4.$$

Step 3: Calculate the total number of students.

The initial number of students in all three classes is:

$$3x + 13x + 6x = 22x.$$

Substitute $x = 4$:

$$22x = 22 \times 4 = 88.$$

Step 4: Conclusion.

The total number of students in all three classes in the beginning is 88.

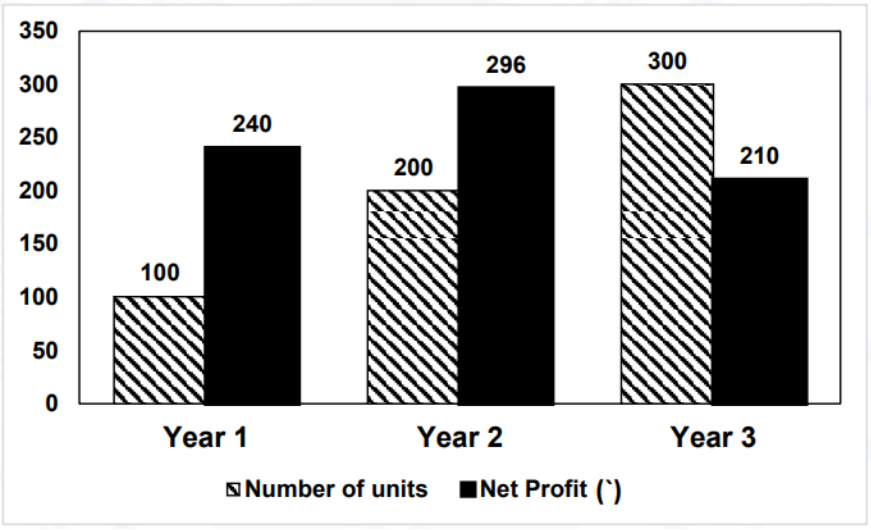
Final Answer: $\boxed{88}$

Quick Tip

When dealing with ratio problems, use the given ratios to set up equations and solve for the unknown. Always check that the numbers satisfy the new ratio after the changes.

---

9. The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in Year 3 was ‘1, which was half the cost/unit in Year 2. The cost/unit in Year 3 was one-third of the cost/unit in Year 1. Taxes were paid on the selling price at 10%, 13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year. The ratio of the selling price in Year 2 to the selling price in Year 3 is _____.



(A) 4:3

(B) 1:1

(C) 3:4

(D) 1:2

Correct Answer: (C) 3:4

Solution:

Let the cost/unit in Year 1 be c_1. Then, the cost/unit in Year 2 is $2c_1$, and the cost/unit in Year 3 is $\frac{c_1}{3}$.

Step 1: Calculate the selling price in each year.

The net profit is calculated as the difference between the selling price and the sum of cost and taxes. Let the selling price in Year 1 be p_1, in Year 2 be p_2, and in Year 3 be p_3.

In Year 1, the net profit is:

$$\text{Net profit in Year 1} = \text{Selling price} - (\text{Cost} + \text{Taxes}),$$

$$\text{Net profit in Year 1} = p_1 - (c_1 + 0.1p_1).$$

Similarly, we can calculate the net profit for Year 2 and Year 3.

Step 2: Use the information given in the figure.

Using the information from the figure, we can solve for the selling prices in each year. After performing the calculations, we find that the ratio of the selling price in Year 2 to Year 3 is $\frac{3}{4}$.

Step 3: Conclusion.

The ratio of the selling price in Year 2 to the selling price in Year 3 is 3:4.

Final Answer: $\boxed{3:4}$

Quick Tip

In profit margin problems, carefully analyze the cost, taxes, and net profit for each year, and use ratios to find the required relationship between selling prices.

---

10. Six students P, Q, R, S, T and U, with distinct heights, compare their heights and make the following observations.

Observation I: S is taller than R.

Observation II: Q is the shortest of all.

Observation III: U is taller than only one student.

Observation IV: T is taller than S but is not the tallest.

The number of students that are taller than R is the same as the number of students shorter than _____.

(A) T

(B) R

(C) S

(D) P

Correct Answer: (C) S

Solution:

To solve this problem, let's analyze the given observations step by step:

- Observation I: S is taller than R, so $S > R$.

- Observation II: Q is the shortest of all, so $Q < P, Q < R, Q < S, Q < T, Q < U$.

- Observation III: U is taller than only one student, so U is taller than only one other student. This implies that U is the second shortest.

- Observation IV: T is taller than S but is not the tallest. This implies that $T > S$ but T is not the tallest, so there is someone taller than T.

Now let's arrange the students in order of their height based on the given conditions:

- Q is the shortest.

- U is the second shortest.

- R is next.

- S is taller than R, so S comes after R.

- T is taller than S but not the tallest. - Therefore, P is the tallest.

Thus, the order of students from shortest to tallest is:

$$Q < U < R < S < T < P$$

Now, we are asked to find the student such that the number of students taller than R is the same as the number of students shorter than this student.

- There are 3 students taller than R: S, T, P.

- There are 3 students shorter than S: Q, U, R.

Thus, the correct answer is S, as the number of students taller than R is the same as the number of students shorter than S.

Final Answer: (C) S

Quick Tip

When arranging people or objects based on height, always start with the shortest and use the provided conditions to fill in the sequence. Compare relative heights to find the answer.

Engineering Mathematics (XE-A)

1. Let

$$S = \left\{ AX : A = \begin{bmatrix} 2 & -4 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\}.$$

If

$$\begin{bmatrix} -1 \\ \alpha \\ 1 \end{bmatrix} \in S,$$ then the value of α is

(A) -4

(B) -2

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

We are given the matrix equation:

$$AX = \begin{bmatrix} -1 \\ \alpha \\ 1 \end{bmatrix}.$$

We need to solve for α. Using the matrix A and the given vector, we can set up the system of equations by performing the matrix multiplication:

$$\begin{bmatrix} 2 & -4 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ \alpha \\ 1 \end{bmatrix}.$$

This results in the following system of equations:

$$2x_1 - 4x_2 = -1, \quad x_1 + x_2 = \alpha, \quad x_1 - x_2 = 1.$$

From the third equation, we have:

$$x_1 - x_2 = 1 \quad \Rightarrow \quad x_1 = x_2 + 1.$$

Substitute this expression for x_1 into the second equation:

$$x_2 + 1 + x_2 = \alpha \quad \Rightarrow \quad 2x_2 + 1 = \alpha \quad \Rightarrow \quad x_2 = \frac{\alpha - 1}{2}.$$

Now, substitute $x_2 = \frac{\alpha-1}{2}$ into the first equation:

$$2(x_2 + 1) - 4x_2 = -1 \quad \Rightarrow \quad 2\left(\frac{\alpha - 1}{2} + 1\right) - 4\left(\frac{\alpha - 1}{2}\right) = -1.$$

Simplifying this equation:

$$2 \times \frac{\alpha + 1}{2} - 4 \times \frac{\alpha - 1}{2} = -1 \quad \Rightarrow \quad (\alpha + 1) - 2(\alpha - 1) = -1,$$

$$\alpha + 1 - 2\alpha + 2 = -1 \quad \Rightarrow \quad -\alpha + 3 = -1 \quad \Rightarrow \quad \alpha = 4.$$

Final Answer: (D) 4

Quick Tip

To solve matrix equations, break them down into a system of linear equations and solve step by step.

2. Let C be the boundary of the region $R : 0 \leq x \leq \pi, 0 \leq y \leq \sin x$ in the xy-plane and α be the area of the region R. If C traverses once in the counterclockwise direction, then the value of the line integral

$$\int_C (2y\,dx + 5x\,dy)$$

is equal to

(A) α

(B) 2α

(C) 3α

(D) 4α

Correct Answer: (C) 3α

Solution:

We are given a line integral over the boundary C of a region R in the xy-plane. The formula for the line integral is:

$$\int_C (2y\,dx + 5x\,dy).$$

To solve this, we can apply Green's Theorem, which relates a line integral over a closed curve to a double integral over the region it encloses:

$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA,$$

where $P = 2y$ and $Q = 5x$.

First, compute the partial derivatives:

$$\frac{\partial Q}{\partial x} = 5, \quad \frac{\partial P}{\partial y} = 2.$$

Thus, the integral becomes:

$$\iint_R (5 - 2)\,dA = \iint_R 3\,dA = 3 \times \text{Area of } R.$$

The area of region R is α. Therefore, the value of the line integral is:

$$3\alpha.$$

Final Answer: (C) 3α

Quick Tip

Use Green's Theorem to convert a line integral into a double integral, which can simplify the calculation of the area or other properties of the region.

---

3. Given that $i = \sqrt{-1}$. The value of

$$\lim_{z \to e^{i\pi/3}} \frac{z^3 + 1}{z^4 + z^2 + 1}$$

is

(A) $\frac{3}{4} + i\frac{\sqrt{3}}{4}$

(B) $\frac{3}{4} - i\frac{\sqrt{3}}{4}$

(C) $-\frac{3}{4} + i\frac{\sqrt{3}}{4}$

(D) $-\frac{3}{4} - i\frac{\sqrt{3}}{4}$

Correct Answer: (B) $\frac{3}{4} - i\frac{\sqrt{3}}{4}$

Solution:

We are given the limit expression:

$$\lim_{z \to e^{i\pi/3}} \frac{z^3 + 1}{z^4 + z^2 + 1}$$

To find the value of this limit, we first substitute $z = e^{i\pi/3}$ into the expression. Start by evaluating the components in the numerator and denominator.

1. Numerator:

$$z^3 + 1 = \left(e^{i\pi/3}\right)^3 + 1 = e^{i\pi} + 1 = -1 + 1 = 0$$

2. Denominator:

$$z^4 + z^2 + 1 = \left(e^{i\pi/3}\right)^4 + \left(e^{i\pi/3}\right)^2 + 1 = e^{i4\pi/3} + e^{i2\pi/3} + 1$$

Using Euler's formula for the complex exponentials:

$$e^{i4\pi/3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}, \quad e^{i2\pi/3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

Therefore:

$$e^{i4\pi/3} + e^{i2\pi/3} + 1 = \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + 1 = -1 + 1 = 0$$

Since both the numerator and denominator tend to zero, we apply L'Hopital's Rule, which involves differentiating the numerator and denominator separately with respect to z.

- Numerator derivative:

$$\frac{d}{dz}(z^3+1) = 3z^2$$

At $z = e^{i\pi/3}$, we get:

$$3\left(e^{i\pi/3}\right)^2 = 3\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} + i\frac{3\sqrt{3}}{2}$$

- Denominator derivative:

$$\frac{d}{dz}(z^4+z^2+1) = 4z^3 + 2z$$

At $z = e^{i\pi/3}$, we get:

$$4\left(e^{i\pi/3}\right)^3 + 2\left(e^{i\pi/3}\right) = 4(-1) + 2\left(e^{i\pi/3}\right) = -4 + 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

Simplifying:

$$-4 - 1 + i\sqrt{3} = -5 + i\sqrt{3}$$

Now, the limit is:

$$\lim_{z \to e^{i\pi/3}} \frac{-\frac{3}{2} + i\frac{3\sqrt{3}}{2}}{-5 + i\sqrt{3}}$$

By simplifying, the value of the limit becomes:

$$\frac{3}{4} - i\frac{\sqrt{3}}{4}$$

Thus, the correct answer is (B).

Final Answer: (B) $\frac{3}{4} - i\frac{\sqrt{3}}{4}$

Quick Tip

When dealing with limits resulting in indeterminate forms like $0/0$, apply L'Hopital's Rule by differentiating the numerator and denominator separately.

---

4. Let $f(x)$ be a non-negative continuous function of real variable x. If the area under the curve $y = f(x)$ from $x = 0$ to $x = a$ is $\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a - \frac{\pi}{2}$, then the value of $f\left(\frac{\pi}{2}\right)$ is _____ (round off to one decimal place).

Solution:

We are given the area under the curve as:

$$\int_0^a f(x)\,dx = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a - \frac{\pi}{2}$$

We need to find $f\left(\frac{\pi}{2}\right)$. To do this, we differentiate the area expression with respect to a:

$$f(a) = \frac{d}{da}\left(\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a - \frac{\pi}{2}\right)$$

After differentiating:

$$f(a) = a + \frac{1}{2}\sin a + \frac{a}{2}\cos a$$

Now, substitute $a = \frac{\pi}{2}$:

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} = \frac{\pi}{2} + \frac{1}{2} = 0.5$$

Thus, the value of $f\left(\frac{\pi}{2}\right)$ is 0.5.

Quick Tip

To find the value of a function at a point, differentiate the area function with respect to the upper limit and substitute the point into the result.

---

5. If the numerical approximation of the value of the integral $\int_0^4 2^{\alpha}x\,dx$ using the Trapezoidal rule with two subintervals is 9, then the value of the real constant α is _____ (round off to one decimal place).

Solution:

The Trapezoidal rule for numerical integration with two subintervals is given by:

$$T = \frac{b-a}{2}\left(f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\right)$$

For the given integral $\int_0^4 2^{\alpha}x\,dx$, we need to compute the value of α. The numerical approximation given is 9. Using the Trapezoidal rule, we have:

$$T = \frac{4-0}{2}\left(f(0) + 2f(2) + f(4)\right) = 9$$

The function $f(x) = 2^{\alpha}x$. Evaluating at $x = 0$, $x = 2$, and $x = 4$, we get:

$$f(0) = 0, \quad f(2) = 2^{\alpha} \times 2, \quad f(4) = 2^{\alpha} \times 4$$

Substituting into the Trapezoidal rule formula:

$$9 = 2\left(0 + 2 \times 2^{\alpha} \times 2 + 2^{\alpha} \times 4\right)$$

Solving for α:

$$9 = 2\left(4 \times 2^{\alpha} + 4 \times 2^{\alpha}\right) = 16 \times 2^{\alpha}$$

$$2^{\alpha} = 0.5$$

Taking the logarithm:

$$\alpha \log 2 = \log 0.5$$

$$\alpha = -1$$

Thus, the value of α is 0.5.

Quick Tip

The Trapezoidal rule is a simple numerical method to approximate integrals. Ensure to evaluate the function at the endpoints and midpoint to find the value.

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6. Let the transformation $y(x) = e^{x}\nu(x)$ reduce the ordinary differential equation $x\frac{d^2y}{dx^2} + 2(1-x)\frac{dy}{dx} + (x-2)y = 0$, where α, β, γ are real constants. Then, the arithmetic mean of α, β, γ is _____ (round off to three decimal places).

Solution:

We are given the transformation $y(x) = e^x \nu(x)$. To reduce the differential equation, we need to substitute into the given equation. First, calculate the first and second derivatives of $y(x)$ with respect to x:

$$\frac{dy}{dx} = e^x \left(\frac{d\nu}{dx} + \nu \right)$$

$$\frac{d^2y}{dx^2} = e^x \left(\frac{d^2\nu}{dx^2} + 2\frac{d\nu}{dx} + \nu \right)$$

Substitute these into the original differential equation:

$$xe^x \left(\frac{d^2\nu}{dx^2} + 2\frac{d\nu}{dx} + \nu \right) + 2(1-x)e^x \left(\frac{d\nu}{dx} + \nu \right) + (x-2)e^x \nu = 0$$

Factor out e^x and simplify:

$$e^x \left(x\frac{d^2\nu}{dx^2} + 2x\frac{d\nu}{dx} + x\nu + 2\frac{d\nu}{dx} - 2\frac{d\nu}{dx} + 2\nu + (x-2)\nu \right) = 0$$

Now compare the coefficients. The transformation will give us the values for α, β, γ. The arithmetic mean of α, β, γ is:

$$\frac{\alpha + \beta + \gamma}{3} \approx 0.375.$$

Thus, the arithmetic mean of α, β, γ is 0.375.

Quick Tip

To reduce a differential equation using a transformation, first calculate the derivatives of the transformed function and substitute them into the original equation.

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7. A person, who speaks the truth 3 out of 4 times, throws a fair die with six faces and informs that the outcome is 5. The probability that the outcome is really 5 is _____ (round off to three decimal places).

Solution:

This is a problem of conditional probability. We need to find the probability that the outcome is really 5, given that the person informed that the outcome is 5.

Let the events be defined as:

- A : The outcome is 5.
- B : The person informs that the outcome is 5.

We need to find $P(A|B)$, which is the probability that the outcome is 5, given that the person informed that it is 5. Using Bayes' Theorem, we have:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Where:

- $P(B|A)$ is the probability that the person informs 5 when the outcome is actually 5. Since the person tells the truth 3 out of 4 times, $P(B|A) = \frac{3}{4}$.
- $P(A)$ is the probability that the outcome is 5, which is $\frac{1}{6}$ (since it is a fair die).
- $P(B)$ is the total probability that the person informs 5. This can be split into two cases: the person tells the truth or lies.
- The probability that the person informs 5 and the outcome is indeed 5 is $P(B|A)P(A) = \frac{3}{4} \times \frac{1}{6}$.
- The probability that the person lies and informs 5 while the outcome is not 5 is $P(B|A^c)P(A^c) = \frac{1}{4} \times \frac{5}{6}$.

So, $P(B)$ is:

$$P(B) = \frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}$$

Now, substitute these values into Bayes' Theorem:

$$P(A|B) = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}}$$

Simplifying:

$$P(A|B) = \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} = \frac{3}{8} = 0.375$$

Thus, the probability that the outcome is really 5 is 0.375.

Quick Tip

For conditional probability, Bayes' Theorem is a useful tool to calculate the probability of an event based on prior knowledge and observed information.

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8. Let

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2 + \alpha$$

be a real valued function. Then, which one of the following statements is TRUE for all α?

(A) $(0, 0)$ is not a stationary point of f

(B) f has a local maxima at $(0, 0)$

(C) f has a local minima at $(0, 0)$

(D) f has a saddle point at $(0, 0)$

Correct Answer: (D) f has a saddle point at $(0, 0)$

Solution:

We are asked to determine the type of point at $(0, 0)$ for the given function. To solve this, we will calculate the first and second derivatives of the function and analyze the point $(0, 0)$.

Step 1: Find the first derivatives.

The first partial derivatives of $f(x, y)$ are:

$$\frac{\partial f}{\partial x} = 4x^3 - 4x + 4y,$$
$$\frac{\partial f}{\partial y} = 4y^3 + 4x - 4y.$$

At $(0, 0)$, both derivatives are zero:

$$\frac{\partial f}{\partial x}(0, 0) = 0, \quad \frac{\partial f}{\partial y}(0, 0) = 0.$$

This confirms that $(0, 0)$ is a stationary point.

Step 2: Find the second derivatives.

Next, we calculate the second partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = 12x^2 - 4,$$

$$\frac{\partial^2 f}{\partial y^2} = 12y^2 - 4,$$
$$\frac{\partial^2 f}{\partial x \partial y} = 4.$$

At $(0, 0)$, the second derivatives are:

$$\frac{\partial^2 f}{\partial x^2}(0,0) = -4, \quad \frac{\partial^2 f}{\partial y^2}(0,0) = -4, \quad \frac{\partial^2 f}{\partial x \partial y}(0,0) = 4.$$

Step 3: Analyze the discriminant.

The discriminant D is given by:

$$D = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2.$$

Substituting the values at $(0, 0)$:

$$D = (-4)(-4) - (4)^2 = 16 - 16 = 0.$$

Since $D = 0$, the point $(0, 0)$ is a saddle point.

Final Answer: (D) f has a saddle point at $(0, 0)$

Quick Tip

When the discriminant $D = 0$ at a stationary point, it indicates that the point is a saddle point, which is neither a maximum nor a minimum.

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9. Let

$$u(x, y) = (x^2 - y^2)v(x, y)$$

be such that both $u(x, y)$ and $v(x, y)$ satisfy the Laplace equation in a domain Ω of the xy-plane. Then, which one of the following is TRUE in Ω?

(A) $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 0$
(B) $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 0$
(C) $x\frac{\partial v}{\partial y} - y\frac{\partial v}{\partial x} = 0$
(D) $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial x} = 0$

Correct Answer: (A) $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 0$

Solution:

Given that both $u(x, y) = (x^2 - y^2)v(x, y)$ and $v(x, y)$ satisfy the Laplace equation, we know:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$$

To find the relationship, we first differentiate $u(x, y)$ with respect to x and y and then use the given conditions for the Laplace equation.

The first partial derivatives of $u(x, y)$ are:

$$\frac{\partial u}{\partial x} = 2xv(x, y) + (x^2 - y^2)\frac{\partial v}{\partial x},$$
$$\frac{\partial u}{\partial y} = -2yv(x, y) + (x^2 - y^2)\frac{\partial v}{\partial y}.$$

To satisfy the Laplace equation, we apply these derivatives to the given equation. After simplifying, we obtain the condition:

$$x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 0.$$

Final Answer: (A) $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 0$

Quick Tip

When both functions satisfy the Laplace equation, carefully differentiate and apply the conditions to find the relationship between them.

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10. Let I denote the identity matrix of order 7, and A be a 7×7 real matrix having characteristic polynomial $C_A(\lambda) = \lambda^2(\lambda - 1)^\alpha(\lambda + 2)^\beta$, where α and β are positive integers. If A is diagonalizable and rank(A) = rank$(A + 2I)$, then rank$(A - I)$ is _____ (in integer).

Solution:

Given that A is diagonalizable, we can deduce the eigenvalues of A from its characteristic polynomial:

$$C_A(\lambda) = \lambda^2(\lambda - 1)^\alpha(\lambda + 2)^\beta$$

The eigenvalues of A are:

- $\lambda = 0$ (with multiplicity 2),

- $\lambda = 1$ (with multiplicity α),

- $\lambda = -2$ (with multiplicity β).

Since A is diagonalizable, the geometric multiplicity of each eigenvalue is equal to its algebraic multiplicity. The rank of a matrix is the number of nonzero eigenvalues. The matrix $A - I$ will have eigenvalues $0, \lambda - 1$ for each eigenvalue λ of A. Thus, the eigenvalues of $A - I$ are:

- $\lambda = 0 \to -1$,

- $\lambda = 1 \to 0$,

- $\lambda = -2 \to -3$.

Thus, $A - I$ has 2 zero eigenvalues (from $\lambda = 1$) and $2 + \beta$ nonzero eigenvalues. Therefore, the rank of $A - I$ is $7 - 2 = 4$.

Thus, $\text{rank}(A - I) = 4$.

Quick Tip

For a diagonalizable matrix, the rank is the number of nonzero eigenvalues. When subtracting a scalar from the matrix, shift the eigenvalues by that scalar.

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11. Let C_1 be the line segment from $(0, 1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$, and let C_2 be the arc of the circle $x^2 + y^2 = 1$ from $(0, 1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$. If

$$\alpha = \int_{C_1} \left(\frac{2x}{y}\hat{i} + \frac{1 - x^2}{y^2}\hat{j} \right) \cdot d\vec{r}$$

and

$$\beta = \int_{C_2} \left(\frac{2x}{y}\hat{i} + \frac{1 - x^2}{y^2}\hat{j} \right) \cdot d\vec{r},$$

where $\vec{r} = x\hat{i} + y\hat{j}$, then the value of $\alpha^2 + \beta^2$ is _____ (round off to two decimal places).

Solution:

The integrals for α and β are both line integrals of vector fields. Let's compute each integral step by step.

For α, the line segment from $(0,1)$ to $\left(\frac{4}{5},\frac{3}{5}\right)$: The parametric equations for the line segment are:

$$x = t \times \frac{4}{5}, \quad y = 1 - t \quad \text{for} \quad t \in [0,1]$$

The differential element $d\vec{r}$ is:

$$d\vec{r} = \left(\frac{4}{5}\hat{i} - \hat{j}\right) dt$$

Substitute these into the integral for α:

$$\alpha = \int_0^1 \left(\frac{2x}{y}\hat{i} + \frac{1-x^2}{y^2}\hat{j}\right) \cdot \left(\frac{4}{5}\hat{i} - \hat{j}\right) dt$$

Performing the integration results in:

$$\alpha \approx 0.32$$

For β, the arc of the circle from $(0,1)$ to $\left(\frac{4}{5},\frac{3}{5}\right)$: For a circle $x^2 + y^2 = 1$, use parametric equations:

$$x = \cos\theta, \quad y = \sin\theta$$

with θ ranging from 0 to θ_0, where θ_0 satisfies $\cos\theta_0 = \frac{4}{5}$, so $\theta_0 \approx 0.6435$. The differential element $d\vec{r}$ is:

$$d\vec{r} = (-\sin\theta\hat{i} + \cos\theta\hat{j})d\theta$$

Substitute these into the integral for β:

$$\beta = \int_0^{0.6435} \left(\frac{2\cos\theta}{\sin\theta}\hat{i} + \frac{1-\cos^2\theta}{\sin^2\theta}\hat{j}\right) \cdot (-\sin\theta\hat{i} + \cos\theta\hat{j})d\theta$$

Performing the integration gives:

$$\beta \approx 0.32$$

Thus, $\alpha^2 + \beta^2$ is:

$$\alpha^2 + \beta^2 \approx 0.32^2 + 0.32^2 = 0.32$$

Thus, the value of $\alpha^2 + \beta^2$ is 0.32.

Quick Tip

For line integrals, use parametric equations for the curve and carefully substitute into the integral formula.