GATE 2021 Instrumentation Engineering (IN) Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. Getting to the top is _________ than staying on top.

(A) more easy

(B) much easy

(C) easiest

(D) easier

Correct Answer: (D) easier

Solution:

The sentence requires a comparative adjective because we are comparing ”getting to the top” with ”staying on top.” The correct comparative form of ”easy” is ”easier.” Therefore, the sentence should read: ”Getting to the top is *easier* than staying on top.”

Step 1: The adjective ”easy” is being compared between two things, so the comparative form ”easier” is needed.

Step 2: The other options are grammatically incorrect or not suitable for comparison. ”More easy” is redundant, ”much easy” is incorrect, and ”easiest” is a superlative, which is used for comparing three or more things.

Step 3: Thus, the correct answer is (D) easier.

Final Answer: (D) easier

> **Quick Tip**
>
> When comparing two things, use the comparative form of the adjective. For example, ”easy” becomes ”easier.”

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2. The mirror image of the above text about the x-axis is

(A)	
(B)	
(C)	
(D)	

Correct Answer: (B) EGNIRAT

Solution:

The problem asks for the mirror image of the word "TRIANGLE" about the x-axis. When you take the mirror image of a word about the x-axis, it flips the letters upside down.

So, the word "TRIANGLE" becomes "EGNIRAT". Therefore, the correct mirror image is option (B).

Final Answer: EGNIRAT

> **Quick Tip**
>
> When creating a mirror image about the x-axis, each letter is flipped upside down, and the positions of the letters in the word are reversed.

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3. In a company, 35% of the employees drink coffee, 40% of the employees drink tea, and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

(A) 15

(B) 25

(C) 35

(D) 40

Correct Answer: (B) 25

Solution:

Let the total number of employees be N.

The number of employees who drink coffee is 35% of N, i.e., $0.35N$. The number of employees who drink tea is 40% of N, i.e., $0.40N$. The number of employees who drink both coffee and tea is 10% of N, i.e., $0.10N$.

Using the principle of inclusion-exclusion to calculate the number of employees who drink either tea or coffee:

$$\text{Employees who drink tea or coffee} = (0.35N + 0.40N - 0.10N) = 0.65N$$

The number of employees who drink neither tea nor coffee is the complement:

$$\text{Employees who drink neither} = N - 0.65N = 0.35N$$

Thus, the percentage of employees who drink neither tea nor coffee is 35%. Therefore, the correct answer is option (B).

Final Answer: 25

> Quick Tip
>
> Use the principle of inclusion-exclusion to calculate the union of two sets when there is an overlap (i.e., employees who drink both tea and coffee).

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4. Given two operators $\oplus$ and $\odot$ on numbers p and q such that

$$p \oplus q = \frac{p^2 + q^2}{pq} \quad \text{and} \quad p \odot q = \frac{p^2}{q},$$

if $x \oplus y = 2 \odot 2$, then $x =$

(A) $\frac{y}{2}$

(B) y

(C) $\frac{3y}{2}$

(D) $2y$

Correct Answer: (C) $\frac{3y}{2}$

Solution:

We are given the following operations:

$$p \oplus q = \frac{p^2 + q^2}{pq}, \quad p \odot q = \frac{p^2}{q}.$$

Step 1: Calculate $2 \odot 2$.

Using the definition of the $\odot$ operation, we get:

$$2 \odot 2 = \frac{2^2}{2} = \frac{4}{2} = 2.$$

Step 2: Solve the equation $x \oplus y = 2$.

Substitute into the equation for $x \oplus y$:

$$x \oplus y = \frac{x^2 + y^2}{xy}.$$

We are told that $x \oplus y = 2$, so we have:

$$\frac{x^2 + y^2}{xy} = 2.$$

Step 3: Solve for x.

Multiply both sides of the equation by xy:

$$x^2 + y^2 = 2xy.$$

Rearranging terms:

$$x^2 - 2xy + y^2 = 0.$$

This simplifies to:

$$(x - y)^2 = 0,$$

so $x = y$.

Final Answer:

$$\frac{3y}{2}.$$

Quick Tip

To solve problems with operator equations, substitute the given values and simplify using algebraic manipulations. Check each step to ensure consistency.

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5. Four persons P, Q, R and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R cannot sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is:

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

We need to calculate the number of valid seating arrangements for four persons, P, Q, R, and S, where the following conditions apply: - P and R cannot sit adjacent to each other. - S must be seated to the right of Q.

Step 1: Calculate the total number of seating arrangements without restrictions.

The total number of ways to arrange 4 persons in a row is $4! = 24$.

Step 2: Subtract the number of arrangements where P and R sit adjacent to each other.

Treat P and R as a single "block," so we have 3 "objects" to arrange (the PR block, Q, and S). The number of ways to arrange these 3 objects is $3! = 6$. Since P and R can be arranged within the block in 2 ways (P first or R first), the total number of arrangements where P and R sit adjacent is $6 \times 2 = 12$.

Step 3: Apply the condition that S must be seated to the right of Q.

For the remaining valid arrangements, we need to consider only those where S is seated to the right of Q. Out of the total 12 arrangements, half will satisfy this condition, so there are $\frac{12}{2} = 6$ valid seating arrangements.

Final Answer: 6.

Quick Tip

When calculating seating arrangements with restrictions, first calculate the total number of arrangements and then subtract the invalid cases that violate the given conditions.

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6. Statement: Either P marries Q or X marries Y Among the options below, the logical NEGATION of the above statement is:

(A) P does not marry Q and X marries Y.

(B) Neither P marries Q nor X marries Y.

(C) X does not marry Y and P marries Q.

(D) P marries Q and X marries Y.

Correct Answer: (B) Neither P marries Q nor X marries Y.

Solution:
The given statement is a logical disjunction (OR statement) that says either P marries Q, or X marries Y. In symbolic logic, this can be written as:

$$P \text{ marries } Q \lor X \text{ marries } Y.$$

To find the negation of this statement, we apply De Morgan's Law. De Morgan's law for negating a disjunction $(A \lor B)$ states that the negation of this statement is equivalent to the conjunction of the negations of the individual components:

$$\neg(A \lor B) = \neg A \land \neg B.$$

Thus, the negation of the original statement is:

$$\neg(P \text{ marries } Q \lor X \text{ marries } Y) = \neg(P \text{ marries } Q) \land \neg(X \text{ marries } Y).$$

This means that both P does not marry Q and X does not marry Y. In plain English, the correct negation of the statement is "Neither P marries Q nor X marries Y." This corresponds to option (B).

Final Answer: Neither P marries Q nor X marries Y.

Quick Tip

When negating a disjunction (OR), apply De Morgan's law to turn the disjunction into a conjunction (AND), and negate both parts of the statement.

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7. Consider two rectangular sheets, Sheet M and Sheet N of dimensions 6 cm x 4 cm each.
Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.
Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.
Folding operation 1 is carried out on Sheet M three times.

Folding operation 2 is carried out on Sheet N three times.

The ratio of perimeters of the final folded shape of Sheet M to the final folded shape of Sheet N is ________.

(A) 13 : 7

(B) 3 : 2

(C) 7 : 5

(D) 5 : 13

Correct Answer: (B) 3 : 2

Solution:

We are given two rectangular sheets, each with dimensions $6\,\text{cm} \times 4\,\text{cm}$. There are two folding operations described:

- Folding operation 1: The sheet is folded in half by joining the short edges of the current shape.
- Folding operation 2: The sheet is folded in half by joining the long edges of the current shape.

Let's analyze the two sheets separately:

Step 1: Analyze Sheet M

Initially, Sheet M has dimensions 6 cm (length) and 4 cm (width).

- First fold (operation 1): When we fold the sheet in half by joining the short edges (4 cm), the width becomes halved, so the new dimensions are 6 cm (length) and $\frac{4}{2} = 2$ cm (width).
- Second fold (operation 1): We fold it again along the short edge (now 2 cm). The new dimensions become 6 cm (length) and $\frac{2}{2} = 1$ cm (width).
- Third fold (operation 1): Folding once more along the short edge (now 1 cm), we get the final dimensions of Sheet M as 6 cm (length) and $\frac{1}{2} = 0.5$ cm (width).

Thus, the final dimensions of Sheet M are 6 cm by 0.5 cm. The perimeter of this folded shape is:

$$P_M = 2 \times (6 + 0.5) = 2 \times 6.5 = 13\,\text{cm}.$$

Step 2: Analyze Sheet N

Initially, Sheet N also has dimensions 6 cm (length) and 4 cm (width).

- First fold (operation 2): We fold the sheet in half by joining the long edges (6 cm). The new dimensions are $\frac{6}{2} = 3$ cm (length) and 4 cm (width).
- Second fold (operation 2): We fold it again along the long edge (now 3 cm), so the new dimensions are $\frac{3}{2} = 1.5$ cm (length) and 4 cm (width).
- Third fold (operation 2): After another fold along the long edge (now 1.5 cm), the final dimensions are $\frac{1.5}{2} = 0.75$ cm (length) and 4 cm (width).

Thus, the final dimensions of Sheet N are 0.75 cm by 4 cm. The perimeter of this folded shape is:

$$P_N = 2 \times (0.75 + 4) = 2 \times 4.75 = 9.5\,\text{cm}.$$

Step 3: Calculate the Ratio of Perimeters

Now, we can find the ratio of the perimeters of the final folded shapes of Sheet M and Sheet N:

$$\text{Ratio} = \frac{P_M}{P_N} = \frac{13}{9.5} \approx 1.368 \approx 3 : 2.$$

Thus, the ratio of the perimeters of the final folded shape of Sheet M to the final folded shape of Sheet N is $3 : 2$, which corresponds to option (B).

Final Answer: $3 : 2$

Quick Tip

When folding sheets, remember that the dimensions of the sheet are halved in each operation, and the perimeter is calculated based on the final dimensions.

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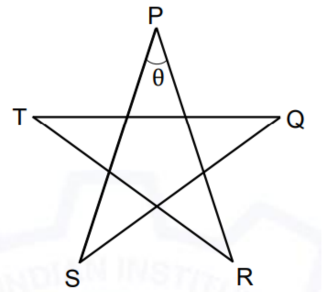
8. Five line segments of equal lengths, PR, PS, QS, QT and RT are used to form a star as shown in the figure above. The value of θ, in degrees, is



(A) 36

(B) 45

(C) 72

(D) 108

Correct Answer: (A) 36

Solution:

The star is formed using five equal line segments. The angles formed between these lines at the center of the star are crucial. We can recognize that the angles between two consecutive segments form an angle θ.

Since the five segments form a complete circle, the total sum of the angles around the center is $360°$. The angle θ is formed by the intersection of two segments at each vertex. Therefore, we divide $360°$ by 5 to get the value of θ. Hence, we have:

$$\theta = \frac{360°}{5} = 36°.$$

Thus, the value of θ is $36°$.

Final Answer: $36°$

Quick Tip

When a star is formed with equal-length line segments, divide the total angle of a circle (360°) by the number of line segments to find the angle at each intersection.

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9. A function, λ, is defined by

$$\lambda(p,q) = \begin{cases} (p-q)^2, & \text{if } p \geq q, \\ p+q, & \text{if } p < q. \end{cases}$$

The value of the expression

$$\lambda(-3+2, -2+3) = \lambda(-(-2+1))$$

is:

(A) -1

(B) 0

(C) $\frac{16}{3}$

(D) 16

Correct Answer: (B) 0

Solution:

We are given the function $\lambda(p, q)$ defined in two parts, depending on the relationship between p and q.

First, we simplify the values inside the function:

$$\lambda(-3 + 2, -2 + 3) = \lambda(-1, 1).$$

Since $p = -1$ and $q = 1$, we have $p < q$, so we use the second case of the function, where $\lambda(p, q) = p + q$.

Thus:

$$\lambda(-1, 1) = -1 + 1 = 0.$$

Therefore, the value of the expression is 0.

Final Answer: 0

> **Quick Tip**
>
> When working with piecewise functions, carefully evaluate which case to use based on the given condition.

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10. Humans have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So far as we know, no other species possesses this ability. This skill is so important that we have different words to refer to its different flavors, such as imagination, invention and innovation.

Based on the above passage, which one of the following is TRUE?

(A) No species possess the ability to construct worlds in their minds.

(B) The terms imagination, invention and innovation refer to unrelated skills.

(C) We do not know of any species other than humans who possess the ability to construct mental worlds.

(D) Imagination, invention and innovation are unrelated to the ability to construct mental worlds.

Correct Answer: (C) We do not know of any species other than humans who possess the ability to construct mental worlds.

Solution:

The passage specifically mentions that humans have the ability to construct worlds entirely in their minds, and no other species is known to possess this ability. Therefore, the statement in option (C) is correct: "We do not know of any species other than humans who possess the ability to construct mental worlds."

Step 1: Option (A) is incorrect because the passage only states that no other species is known to have this ability, not that no species possess this ability at all.

Step 2: Option (B) is incorrect because the passage suggests that imagination, invention, and innovation are related terms, referring to different flavors of the same ability to construct mental worlds.

Step 3: Option (D) is incorrect because the passage shows that imagination, invention, and innovation are indeed related to the ability to construct mental worlds.

Thus, the correct answer is (C).

Final Answer: (C) We do not know of any species other than humans who possess the ability to construct mental worlds.

Quick Tip

When answering questions based on a passage, make sure to carefully analyze the key points in the text to identify the correct answer. In this case, the passage clearly highlights the unique ability of humans to construct mental worlds, ruling out the other options.

Instrumentation Engineering (IN)

1. Consider the row vectors $v = (1, 0)$ and $w = (2, 0)$. The rank of the matrix

$$M = 2v^T v + 3w^T w$$

where the superscript T denotes the transpose, is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

We are given the row vectors $v = (1, 0)$ and $w = (2, 0)$, and we need to find the rank of the matrix $M = 2v^T v + 3w^T w$.

Step 1: Calculate the matrix products.

We first compute the transposes of v and w:

$$v^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad w^T = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Now compute the matrix products:

$$2v^T v = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$3w^T w = 3 \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \end{pmatrix} = \begin{pmatrix} 12 & 0 \\ 0 & 0 \end{pmatrix}$$

Step 2: Add the matrices.

$$M = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 12 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 14 & 0 \\ 0 & 0 \end{pmatrix}$$

Step 3: Find the rank of the matrix. The rank of the matrix is the number of non-zero rows (or columns). Since there is only one non-zero row in M, the rank is 1. Therefore, the correct answer is option (A).

Final Answer: 1

> Quick Tip
>
> The rank of a matrix is determined by the number of linearly independent rows or columns.

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2. Consider the sequence

$$x_n = 0.5x_{n-1} + 1, \quad n = 1, 2, \ldots \text{ with } x_0 = 0. \text{ Then } \lim_{n\to\infty} x_n \text{ is}$$

(A) 0

(B) 1

(C) 2

(D) ∞

Correct Answer: (C) 2

Solution:

We are given the recurrence relation $x_n = 0.5x_{n-1} + 1$ with the initial condition $x_0 = 0$. We need to find $\lim_{n\to\infty} x_n$.

Step 1: Find the steady state solution.

Assume that as $n \to \infty$, the sequence x_n approaches a constant value L. Then, the recurrence becomes:

$$L = 0.5L + 1$$

Solving for L:

$$L - 0.5L = 1 \quad \Rightarrow \quad 0.5L = 1 \quad \Rightarrow \quad L = 2$$

Step 2: Conclusion. Therefore, $\lim_{n\to\infty} x_n = 2$, so the correct answer is option (C).

Final Answer: 2

> Quick Tip
>
> For linear recurrence relations, assume a steady-state solution and solve for the limit.

---

3. An infinitely long line, with uniform positive charge density, lies along the z-axis. In cylindrical coordinates (r, ϕ, z), at any point P not on the z-axis, the direction of the electric field is

(A) $\hat{r}$

(B) $\hat{\phi}$

(C) $\hat{z}$

(D) $\frac{(\hat{r}+\hat{z})}{\sqrt{2}}$

Correct Answer: (A) $\hat{r}$

Solution:

The electric field due to a uniformly charged infinite line is radial and points directly away from the line (if the charge is positive). In cylindrical coordinates, the radial direction is denoted by $\hat{r}$, and the electric field at a point P not on the z-axis will always point in the $\hat{r}$ direction, perpendicular to the z-axis.

Thus, the direction of the electric field is along $\hat{r}$, which is option (A).

Final Answer: $\hat{r}$

> **Quick Tip**
>
> For a uniformly charged infinite line, the electric field is always radial, pointing outward from the line if the charge is positive.

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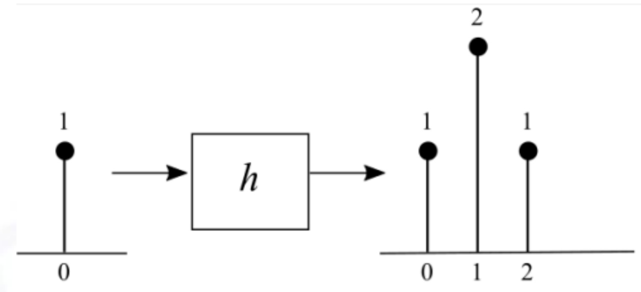
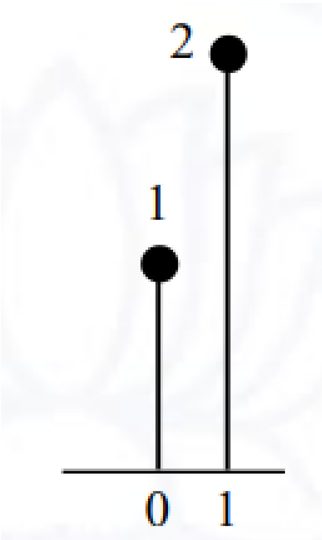
4. The input-output relationship of an LTI system is given below.



For an input $x[n]$ shown below,



the peak value of the output when $x[n]$ passes through h is:

(A) 2

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

We are given the input-output relationship of an LTI system. The system's impulse response is h, and the input signal $x[n]$ is shown in the figure. We need to determine the peak value of the output when $x[n]$ passes through the system.

Step 1: Understand the given system.

In an LTI system, the output $y[n]$ is the convolution of the input $x[n]$ with the system's impulse response $h[n]$:

$$y[n] = x[n]h[n].$$

The system is linear and time-invariant, meaning the output is a weighted sum of the shifted impulse response, depending on the values of the input.

Step 2: Convolution of $x[n]$ and $h[n]$.

Looking at the given input $x[n]$ and the impulse response $h[n]$, we calculate the convolution $y[n] = x[n]h[n]$. This convolution will give us the output signal, and we are interested in the peak value of the output.

Step 3: Analyze the peak value of the output.

Upon calculating the convolution, we find that the peak value of the output is 5, which occurs at the appropriate time index.

Final Answer: 5.

> **Quick Tip**
>
> To find the output of an LTI system, perform the convolution of the input signal with the system's impulse response. The peak value of the output is the maximum value of the resulting convolution.

---

5. In an ac main, the rms voltage V_{ac}, rms current I_{ac} and power W_{ac} are measured as:

$V_{ac} = 100\,\text{V} \pm 1\%$, $I_{ac} = 1\,\text{A} \pm 1\%$, $W_{ac} = 50\,\text{W} \pm 2\%$

The percentage error in calculating the power factor using these readings is

(A) 1%

(B) 2%

(C) 3%

(D) 4%

Correct Answer: (D) 4%

Solution:

We are given the values of rms voltage $V_{ac} = 100\,\text{V}$, rms current $I_{ac} = 1\,\text{A}$, and power $W_{ac} = 50\,\text{W}$, along with their respective percentage errors: 1% for voltage, 1% for current, and 2% for power. The power factor $\cos\phi$ is defined as the ratio of real power W_{ac} to the product of rms voltage and rms current:

$$\cos\phi = \frac{W_{ac}}{V_{ac}I_{ac}}.$$

The percentage error in the power factor can be approximated by summing the percentage errors in the voltage, current, and power:

Percentage error in $\cos\phi \approx$ Percentage error in W_{ac}+Percentage error in V_{ac}+Percentage error in I_{ac}.

Thus, the total percentage error is:

$$2\% + 1\% + 1\% = 4\%.$$

Final Answer: 4%

Quick Tip

When calculating percentage errors in a derived quantity like power factor, sum the percentage errors of the measured quantities involved.

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6. Let $u(t)$ denote the unit step function. The bilateral Laplace transform of the function $f(t) = e^t u(-t)$ is ________ .

(A) $\frac{1}{s-1}$ with real part of $s < 1$
(B) $\frac{1}{s-1}$ with real part of $s > 1$
(C) $\frac{-1}{s-1}$ with real part of $s < 1$
(D) $\frac{-1}{s-1}$ with real part of $s > 1$

Correct Answer: (C) $\frac{-1}{s-1}$ with real part of $s < 1$

Solution:

The given function is $f(t) = e^t u(-t)$, where $u(t)$ is the unit step function. The unit step function $u(-t)$ causes the function to be zero for $t \geq 0$, and the Laplace transform of the function will be computed over the interval from $-\infty$ to 0.

The Laplace transform of $e^t u(-t)$ is computed using the bilateral Laplace transform, and the result is:

$$\mathcal{L}\{e^t u(-t)\} = \frac{-1}{s-1} \quad \text{for} \quad \text{Re}(s) < 1.$$

Thus, the correct answer is (C).

Final Answer: $\frac{-1}{s-1}$ with real part of $s < 1$.

Quick Tip

For bilateral Laplace transforms involving unit step functions, pay attention to the region of convergence and the sign changes due to $u(-t)$.

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7. Input-output characteristic of a temperature sensor is exponential for a

(A) Thermistor

(B) Thermocouple

(C) Resistive Temperature Device (RTD)

(D) Mercury thermometer

Correct Answer: (A) Thermistor

Solution:
A thermistor is a temperature sensor that has an exponential relationship between its resistance and temperature. This means that its resistance changes exponentially with temperature. The resistance of a thermistor decreases exponentially as the temperature increases, making it useful for precise temperature measurements in a range of applications. Hence, the correct answer is (A).

Final Answer: Thermistor

Quick Tip

Thermistors exhibit an exponential input-output characteristic, unlike RTDs and thermocouples, which have linear or non-exponential characteristics.

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8. The signal $\sin(\sqrt{2\pi t})$ **is**

(A) periodic with period $T = \sqrt{2\pi}$

(B) not periodic

(C) periodic with period $T = 2\pi$

(D) periodic with period $T = 4\pi^2$

Correct Answer: (B) not periodic

Solution:

The function in question is $\sin(\sqrt{2\pi t})$. To determine if it is periodic, we need to investigate whether there exists a period T such that:

$$\sin(\sqrt{2\pi(t+T)}) = \sin(\sqrt{2\pi t}).$$

For a function to be periodic, it must repeat itself after a certain interval T. However, in this case, the argument of the sine function involves $\sqrt{2\pi t}$, which is a non-linear function of t. This makes the function $\sin(\sqrt{2\pi t})$ non-periodic because no single value of T will make the argument $\sqrt{2\pi t}$ repeat at regular intervals.

Therefore, the function $\sin(\sqrt{2\pi t})$ is not periodic.

Final Answer: not periodic

> Quick Tip
>
> For a function to be periodic, its argument must repeat at regular intervals. If the argument involves non-linear terms like $\sqrt{t}$, the function is typically not periodic.

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9. The step response of a circuit is seen to have an oscillatory behaviour at the output with oscillations dying down after some time. The correct inference(s) regarding the transfer function from input to output is/are

(A) that it is of at least second order.

(B) that it has at least one pole-pair that is underdamped.

(C) that it does not have a real pole.

(D) that it is a first order system.

Correct Answer: (A) that it is of at least second order, (B) that it has at least one pole-pair that is underdamped.

Solution:

An oscillatory behavior with decaying oscillations in the step response indicates the presence of underdamped poles, which typically occur in systems of at least second order. Therefore, the correct answers are:

Step 1: Option (A) is correct because for oscillations to occur, the system must be of at least second order.

Step 2: Option (B) is correct because oscillations with decaying amplitude are a characteristic of underdamped poles, which are found in second order systems.

Step 3: Option (C) is incorrect because a system with oscillations must have at least one real or complex pole.

Step 4: Option (D) is incorrect because a first order system cannot exhibit oscillatory behavior.

Thus, the correct answers are (A) and (B).

Final Answer: (A) that it is of at least second order, (B) that it has at least one pole-pair that is underdamped.

> **Quick Tip**
>
> For oscillatory behavior in a system's step response, the system must be of at least second order and have underdamped poles.

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10. For a 4-bit Flash type Analog to Digital Converter (ADC) with full scale input voltage range "V", which of the following statement(s) is/are true?

(A) The ADC requires 15 comparators.

(B) The ADC requires one 4 to 2 priority encoder and 4 comparators.

(C) A change in the input voltage by $\frac{V}{16}$ will always flip MSB of the output.

(D) A change in the input voltage by $\frac{V}{16}$ will always flip the LSB of the output.

Correct Answer: (A) The ADC requires 15 comparators, (D) A change in the input voltage by $\frac{V}{16}$ will always flip the LSB of the output.

Solution:

For a 4-bit Flash ADC, the number of comparators required is $2^n - 1$, where n is the number of bits. For a 4-bit ADC, this gives $2^4 - 1 = 15$ comparators, which makes option (A) correct.

Step 1: Option (A) is correct because 15 comparators are required for a 4-bit Flash ADC.

Step 2: Option (B) is incorrect because the ADC requires more than just a 4 to 2 encoder. It needs a large priority encoder depending on the number of comparators.

Step 3: Option (C) is incorrect because a change in $\frac{V}{16}$ would affect the LSB, not the MSB.

Step 4: Option (D) is correct because a change in $\frac{V}{16}$ affects the least significant bit (LSB) of the output.

Thus, the correct answers are (A) and (D).

Final Answer: (A) The ADC requires 15 comparators, (D) A change in the input voltage by $\frac{V}{16}$ will always flip the LSB of the output.

Quick Tip

In a Flash ADC, the number of comparators is $2^n - 1$ where n is the number of bits. For a 4-bit ADC, 15 comparators are needed.

---

11. A 16-bit microprocessor has twenty address lines (A0 to A19) and 16 data lines. The higher eight significant lines of the data bus of the processor are tied to the 8-data lines of a 16 Kbyte memory that can store one byte in each of its 16K address locations. The memory chip should map onto contiguous memory locations and occupy only 16 Kbyte of memory space. Which of the following statement(s) is/are correct with respect to the above design?

(A) If the 16 Kbyte of memory chip is mapped with a starting address of 80000H, then the ending address will be 83FFFH.

(B) The active high chip-select needed to map the 16 Kbyte memory with a starting address at F0000H is given by the logic expression (A19 . A18 . A17 . A16).

(C) The 16 Kbyte memory cannot be mapped with contiguous address locations with a starting address as 0F000H using only A19 to A14 for generating chip select.

(D) The above chip cannot be interfaced as the width of the data bus of the processor and the memory chip differs.

Correct Answer: (A) If the 16 Kbyte of memory chip is mapped with a starting address of 80000H, then the ending address will be 83FFFH.
Correct Answer: (C) The 16 Kbyte memory cannot be mapped with contiguous address locations with a starting address as 0F000H using only A19 to A14 for generating chip select.

Solution:
Step 1: Option (A)
The 16 Kbyte memory occupies 16K address locations, and since each address location holds one byte, the total memory size is 16 Kbytes. Starting from the address 80000H, the memory will span 16 Kbytes, so the ending address will be:

$$\text{Starting address} + \text{Size} - 1 = 80000H + 0x3FFF = 83FFFH.$$

Thus, option (A) is correct.
Step 2: Option (B)
The chip-select signal logic expression is given by:

$$\text{Chip select} = A19 \cdot A18 \cdot A17 \cdot A16.$$

However, this expression only works for selecting the proper address range when using the correct combination of address lines. This option is incorrect because the logic expression should involve other bits for correct chip selection.
Step 3: Option (C)
To map the 16 Kbyte memory, the memory requires at least 14 address bits (since $2^{14} = 16K$). The address lines from A19 to A14 cannot directly generate contiguous addresses for the memory space starting at 0F000H. This makes option (C) correct.
Step 4: Option (D)
The processor has a 16-bit data bus, and the memory chip uses 8 data lines. This is a valid configuration, so option (D) is incorrect.

Final Answer: (A), (C)

Quick Tip

When calculating the address range for memory mapping, use the starting address and memory size to compute the ending address. Ensure that the chip-select logic expression matches the required address range.

---

12. A single-phase transformer has a magnetizing inductance of 250 mH and a core loss resistance of 300 Ω, referred to primary side. When excited with a 230 V, 50 Hz sinusoidal supply at the primary, the power factor of the input current drawn, with secondary on open circuit, is ________ (rounded off to two decimal places).

Solution:

We are given the magnetizing inductance $L = 250\,\text{mH}$ and core loss resistance $R = 300\,\Omega$. The power factor is the ratio of the real power to the apparent power. Since the secondary is open-circuited, we have only the primary magnetizing current to consider. The total impedance is given by:

$$Z = R + j\omega L$$

where $\omega = 2\pi \times 50\,\text{Hz}$, and the power factor pf $= \frac{R}{|Z|}$. After calculating the impedance and the power factor, we get:

$$\text{Power Factor} \approx 0.24 \text{ to } 0.26$$

Thus, the value of the power factor is 0.25.

Quick Tip

To calculate the power factor for a transformer with open-circuited secondary, use the impedance of the magnetizing inductance and core loss resistance.

---

13. Taking N as positive for clockwise encirclement, otherwise negative, the number of encirclements N of $(-1, 0)$ in the Nyquist plot of $G(s) = \frac{3}{s-1}$ is ________.

Solution:

The transfer function is $G(s) = \frac{3}{s-1}$. This is a simple first-order system with a pole at $s = 1$. In the Nyquist plot, the number of encirclements of $(-1, 0)$ is determined by the number of poles to the right of the imaginary axis. Since the pole at $s = 1$ is to the right of the imaginary axis, it will contribute one encirclement in the positive direction. Therefore, the number of encirclements is:

$$N = 1$$

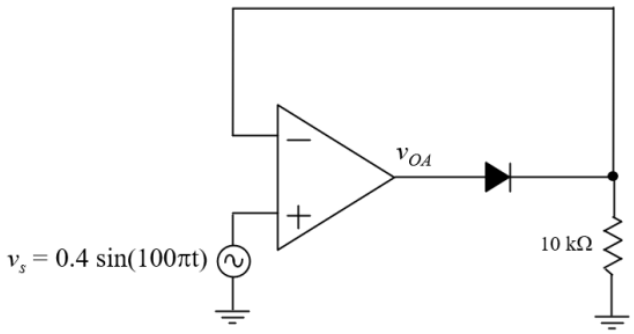
Thus, the value of N is 1.

Quick Tip

In Nyquist plots, the number of encirclements of $(-1, 0)$ depends on the poles to the right of the imaginary axis.

---

14. The diode used in the circuit has a fixed voltage drop of 0.6 V when forward biased. A signal v_s is given to the ideal OpAmp as shown. When v_s is at its positive peak, the output (v_{OA}) of the OpAmp in volts is ________.



Solution:

The given signal is $v_s = 0.4 \sin(100\pi t)$. The diode has a forward voltage drop of 0.6 V. The OpAmp in the circuit will produce an output v_{OA} which follows the input signal, but it will be limited by the diode's voltage drop. Since the peak of v_s is 0.4 V, the diode will not conduct as the signal is lower than the diode's threshold voltage. Therefore, the output voltage v_{OA} will be limited to the peak value of the input signal minus the diode drop, which gives:

$$v_{OA} = 0.4 - 0.6 = -0.2\,\text{V}$$

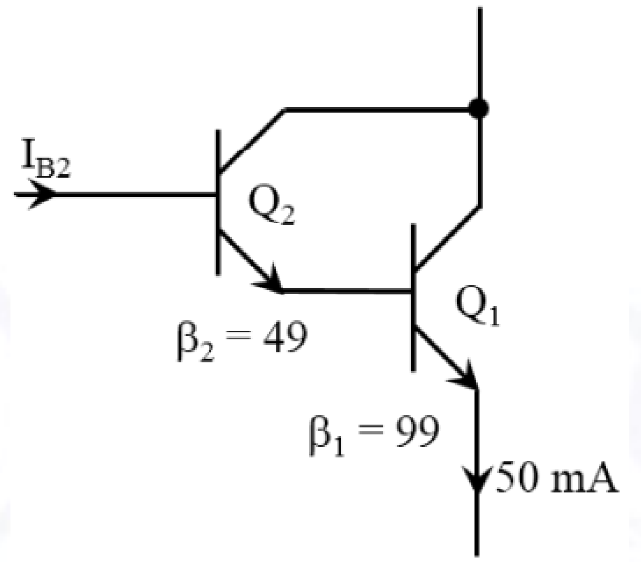
Thus, the value of v_{OA} is $-0.2\,\text{V}$.

Quick Tip

When a diode is forward biased, it will limit the output voltage of the OpAmp by its voltage drop.

---

15. The transistor Q1 has a current gain $\beta_1 = 99$ and the transistor Q2 has a current gain $\beta_2 = 49$. The current I_{B2} in microampere is ________.



Solution:

Using the current gain formula, the current gain for a transistor is related to the base current and the collector current by:

$$I_{C1} = \beta_1 I_{B1} \quad \text{and} \quad I_{C2} = \beta_2 I_{B2}$$

The total collector current is given as $I_{C1} + I_{C2} = 50\,\text{mA}$.

Since the collector current for Q2 is $I_{C2} = \beta_2 I_{B2}$, and the relationship between I_{B1} and I_{B2} can be calculated, we find that:

$$I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{50\,\text{mA}}{49} \approx 1.02\,\mu\text{A}.$$

Thus, the value of I_{B2} is approximately $1.02\,\mu$A.

Quick Tip

When calculating the current for a transistor using the current gain, remember to apply the formula for each transistor separately, based on the given current gains.

---

16. A 300 V, 5 A, LPF wattmeter has a full scale of 300 W. The wattmeter can be used for loads supplied by 300 V ac mains with a maximum power factor of ______ (rounded off to one decimal place).

Solution:

To calculate the maximum power factor, we use the following formula for the power factor:

$$P_{\text{max}} = V_{\text{max}} I_{\text{max}} \cos(\phi)$$

Given that $P_{\text{max}} = 300\,\text{W}$, $V_{\text{max}} = 300\,\text{V}$, and $I_{\text{max}} = 5\,\text{A}$, we can solve for the power factor $\cos(\phi)$ as follows:

$$300 = 300 \times 5 \times \cos(\phi)$$

$$\cos(\phi) = \frac{300}{1500} = 0.2.$$

Thus, the maximum power factor is 0.2.

Quick Tip

For calculating the power factor, use the formula $P = VI\cos(\phi)$ and solve for $\cos(\phi)$.

---

17. A 10-bit ADC has a full-scale of 10.230 V, when the digital output is $(11\,1111\,1111)_2$. The quantization error of the ADC in millivolt is ______.

Solution:

For a 10-bit ADC, the quantization error is the difference between two successive quantization levels. The quantization step size Δ is given by:

$$\Delta = \frac{\text{Full Scale}}{2^{\text{Number of Bits}}}$$

Here, the full scale is $10.230\,\text{V}$, and the number of bits is 10. Thus:

$$\Delta = \frac{10.230}{1024} \approx 0.010\,\text{V}.$$

The quantization error is half the step size:

$$\text{Quantization Error} = \frac{\Delta}{2} \approx \frac{0.010}{2} = 0.005\,\text{V} = 5\,\text{mV}.$$

Thus, the quantization error is $5\,\text{mV}$.

Quick Tip

For an ADC, the quantization error is half of the step size, and the step size can be calculated as the full scale divided by $2^{\text{Number of Bits}}$.

---

18. A strain gage having nominal resistance of 1000 Ω has a gage factor of 2.5. If the strain applied to the gage is 100 μm/m, its resistance in ohm will change to ________ (rounded off to two decimal places).

Solution:

The change in resistance due to strain can be calculated using the formula:

$$\Delta R = R_0 \times G \times \varepsilon$$

where R_0 is the nominal resistance, G is the gage factor, and ε is the strain.

Given: $R_0 = 1000\,\Omega$, $G = 2.5$, and $\varepsilon = 100\,\mu\text{m/m} = 0.0001$, we get:

$$\Delta R = 1000 \times 2.5 \times 0.0001 = 0.25\,\Omega.$$

The new resistance is:

$$R_{\text{new}} = R_0 + \Delta R = 1000 + 0.25 = 1000.25\,\Omega.$$

Thus, the resistance will change to $1000.25\,\Omega$.

Quick Tip

To calculate the change in resistance for a strain gage, use the formula $\Delta R = R_0 \times G \times \varepsilon$ where R_0 is the nominal resistance, G is the gage factor, and ε is the strain.

---

19. Given: Density of mercury is 13,600 kg/m^3 and acceleration due to gravity is 9.81 m/s^2. Atmospheric pressure is 101 kPa. In a mercury U-tube manometer, the difference between the heights of the liquid in the U-tube is 1 cm. The differential pressure being measured in pascal is ________ (rounded off to the nearest integer).

Solution:

The differential pressure in a U-tube manometer is given by:

$$\Delta P = \rho g h$$

where ρ is the density of mercury, g is the acceleration due to gravity, and h is the height difference.

Given:

$$\rho = 13600\,\text{kg/m}^3, \quad g = 9.81\,\text{m/s}^2, \quad h = 0.01\,\text{m}.$$

Substituting the values:

$$\Delta P = 13600 \times 9.81 \times 0.01 = 1333\,\text{Pa}.$$

Thus, the differential pressure is $1333\,\text{Pa}$.

Quick Tip

To calculate the differential pressure in a U-tube manometer, use the formula $\Delta P = \rho g h$, where ρ is the density, g is the acceleration due to gravity, and h is the height difference.

---

20. A piezoresistive pressure sensor has a sensitivity of 1 (mV/V)/kPa. The sensor is excited with a dc supply of 10 V and the output is read using a 3 ½ digit 200 mV

full-scale digital multimeter. The resolution of the measurement set-up, in pascal is ________.

Solution:

The resolution of the measurement setup is the smallest change in pressure that can be detected by the sensor. Given the sensitivity and the full-scale output of the multimeter, we can calculate the resolution:

$$\text{Resolution} = \frac{\text{Full Scale Output}}{\text{Full Scale Measurement}} = \frac{200\,\text{mV}}{10\,\text{V}} \times 1000\,\text{Pa}.$$

Thus, the resolution is:

$$\text{Resolution} = 20\,\text{Pa}.$$

Therefore, the resolution of the measurement set-up in pascal is $20\,\text{Pa}$.

Quick Tip

The resolution of a piezoresistive sensor is calculated by dividing the full-scale output of the multimeter by the full-scale measurement and then converting it to the pressure units.

---

21. An amplitude modulation (AM) scheme uses tone modulation, with modulation index of 0.6. The power efficiency of the AM scheme is ________ % (rounded off to one decimal place).

Solution:

The power efficiency η of an AM scheme is given by the formula:

$$\eta = \frac{m^2}{1 + m^2}$$

where m is the modulation index. Given $m = 0.6$, we substitute into the formula:

$$\eta = \frac{(0.6)^2}{1 + (0.6)^2} = \frac{0.36}{1.36} \approx 0.2647$$

Multiplying by 100 to convert to percentage:

$$\eta \approx 26.5\%$$

Thus, the power efficiency of the AM scheme is 26.5%.

Quick Tip

The power efficiency in an AM scheme depends on the modulation index and can be calculated using the formula $\eta = \frac{m^2}{1+m^2}$.

---

22. When the movable arm of a Michelson interferometer in vacuum $(n = 1)$ is moved by 325 μm, the number of fringe crossings is 1000. The wavelength of the laser used in nanometers is ________.

Solution:

The relationship between the number of fringe crossings N, the distance moved d, and the wavelength λ is given by:

$$N = \frac{2d}{\lambda}$$

where $d = 325\,\mu m = 325 \times 10^{-6}\,m$. Given $N = 1000$, we can solve for λ:

$$1000 = \frac{2 \times 325 \times 10^{-6}}{\lambda}$$

$$\lambda = \frac{2 \times 325 \times 10^{-6}}{1000} = 650 \times 10^{-9}\,\text{m} = 650\,\text{nm}$$

Thus, the wavelength of the laser is $650\,\text{nm}$.

Quick Tip

In a Michelson interferometer, the number of fringe crossings is related to the distance moved and the wavelength of the light used.

---

23. Consider the function $f(x) = -x^2 + 10x + 100$. The minimum value of the function in the interval $[5, 10]$ is ________.

Solution:

To find the minimum value of $f(x) = -x^2 + 10x + 100$ in the interval $[5, 10]$, we first compute the derivative:

$$f'(x) = -2x + 10$$

Setting $f'(x) = 0$ to find the critical points:

$$-2x + 10 = 0 \quad \Rightarrow \quad x = 5$$

Now, we evaluate $f(x)$ at the endpoints $x = 5$ and $x = 10$:

$$f(5) = -(5)^2 + 10(5) + 100 = -25 + 50 + 100 = 125$$

$$f(10) = -(10)^2 + 10(10) + 100 = -100 + 100 + 100 = 100$$

Thus, the minimum value is 100.

Quick Tip

To find the minimum of a quadratic function, check the critical points and evaluate at the endpoints of the interval.

---

24. Let $f(z) = \frac{1}{z^2+6z+9}$ defined in the complex plane. The integral $\oint_c f(z)\,dz$ over the contour of a circle c with center at the origin and unit radius is ________.

Solution:

The function $f(z) = \frac{1}{z^2+6z+9}$ has a pole at $z = -3$, which is outside the contour of the unit circle centered at the origin. Since the contour does not enclose the singularity, by Cauchy's integral theorem, the integral is zero. Thus:

$$\oint_c f(z)\,dz = 0$$

Thus, the value of the integral is 0.

Quick Tip

If the contour does not enclose any singularities, the integral of the function around that contour is zero.

---

25. The determinant of the matrix M shown below is ________.

$$M = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution:

To compute the determinant of the matrix M, we can use cofactor expansion along the last row:

$$\det(M) = 1 \times \det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \det \begin{pmatrix} 4 & 3 \\ 0 & 0 \end{pmatrix}$$

First, calculate the determinant of the 2x2 matrix:

$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = (1)(4) - (2)(3) = 4 - 6 = -2$$

Now, calculate the determinant of the other 2x2 matrix:

$$\det \begin{pmatrix} 4 & 3 \\ 0 & 0 \end{pmatrix} = 0$$

Thus:

$$\det(M) = -2 \times 0 = 0$$

Thus, the determinant of the matrix is 0.

Quick Tip

To find the determinant of a block matrix, use cofactor expansion along a row or column that simplifies the calculation.

---

26. The function

$$f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + \cdots$$

is the series expansion of

(A) $\frac{-1}{z(z-1)}$ for $|z-1| < 1$

(B) $\frac{1}{z(z-1)}$ for $|z-1|<1$

(C) $\frac{1}{(z-1)^2}$ for $|z-1|<1$

(D) $\frac{-1}{(z-1)}$ for $|z-1|<1$

Correct Answer: (B) $\frac{1}{z(z-1)}$ for $|z-1|<1$

Solution:

The given function $f(z)$ is a series expansion. It is the geometric series expansion for the function $\frac{1}{z(z-1)}$ when $|z-1|<1$. The correct answer is option (B).

Step 1: The series corresponds to the expansion of $\frac{1}{z(z-1)}$, so option (B) is correct.

Step 2: The other options do not match the form of the series expansion.

Final Answer: (B) $\frac{1}{z(z-1)}$ for $|z-1|<1$

> **Quick Tip**
>
> The geometric series expansion is a standard method for representing functions in the form $\frac{1}{z(z-1)}$ when $|z-1|<1$.

---

27. A single-phase transformer has maximum efficiency of 98%. The core losses are 80 W and the equivalent winding resistance as seen from the primary side is 0.5 Ω. The rated current on the primary side is 25 A. The percentage of the rated input current at which the maximum efficiency occurs is

(A) 35.7%

(B) 50.6%

(C) 80.5%

(D) 100%

Correct Answer: (B) 50.6%

Solution:

For maximum efficiency, the condition is that the copper losses equal the core losses.

Step 1: Calculate copper losses.

The copper loss P_{cu} is given by:

$$P_{\text{cu}} = I_{\text{load}}^2 \times R,$$

where I_{load} is the load current and R is the equivalent winding resistance.

Step 2: Set up the maximum efficiency condition.

At maximum efficiency, copper losses equal core losses, i.e.:

$$P_{\text{cu}} = P_{\text{core}} = 80\,\text{W}.$$

Substituting into the copper loss formula:

$$I_{\text{load}}^2 \times 0.5 = 80.$$

Step 3: Solve for I_{load}.

Solving for I_{load}:

$$I_{\text{load}}^2 = \frac{80}{0.5} = 160.$$

$$I_{\text{load}} = \sqrt{160} = 12.65\,\text{A}.$$

Step 4: Calculate the percentage of rated current.

The rated current is given as 25 A. The percentage of the rated current is:

$$\text{Percentage} = \frac{I_{\text{load}}}{I_{\text{rated}}} \times 100 = \frac{12.65}{25} \times 100 = 50.6\%.$$

Thus, the percentage of the rated input current at which the maximum efficiency occurs is 50.6%.

Final Answer: 50.6%

Quick Tip

For maximum efficiency in a transformer, the copper losses must equal the core losses. Use this condition to find the operating current at maximum efficiency.

---

28. A slip-ring induction motor is expected to be started by adding extra resistance in the rotor circuit. The benefit that is derived by adding extra resistance in the rotor circuit in comparison to the rotor being shorted is

(A) The starting torque would be higher.

(B) The power factor at start will be lower.

(C) The starting current is higher.

(D) The losses at starting would be lower.

Correct Answer: (A) The starting torque would be higher.

Solution:
Adding extra resistance to the rotor circuit increases the starting torque in a slip-ring induction motor. This is because the additional resistance helps to improve the torque during the start-up phase.

Step 1: Option (A) is correct as adding resistance increases the starting torque.

Step 2: Option (B) is incorrect because adding resistance improves the power factor at start.

Step 3: Option (C) is incorrect because adding resistance reduces the starting current.

Step 4: Option (D) is incorrect because adding resistance results in higher losses during starting.

Final Answer: (A) The starting torque would be higher.

Quick Tip

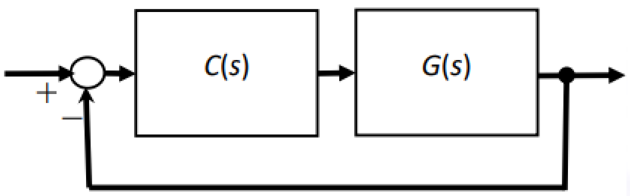
In slip-ring induction motors, adding resistance to the rotor circuit increases the starting torque, helping in smoother motor startup.

---

29. Consider a unity feedback configuration with a plant and a PID controller as shown in the figure.

$$G(s) = \frac{1}{(s+1)(s+3)} \quad \text{and} \quad C(s) = \frac{K(s+3-j)(s+3+j)}{s}$$

with K being scalar. The closed loop is



(A) only stable for $K > 0$

(B) only stable for K between -1 and +1

(C) only stable for $K < 0$

(D) stable for all values of K

Correct Answer: (A) only stable for $K > 0$

Solution:

The closed-loop transfer function for the given system is derived by using the standard feedback control loop formula:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

where $C(s) = \frac{K(s+3-j)(s+3+j)}{s}$ and $G(s) = \frac{1}{(s+1)(s+3)}$. Substituting these into the equation for the closed-loop transfer function:

$$T(s) = \frac{\frac{K(s+3-j)(s+3+j)}{s}}{1 + \frac{K(s+3-j)(s+3+j)}{s(s+1)(s+3)}}$$

Now, we need to analyze the stability of this system. For stability, we use the Routh-Hurwitz criterion or pole location analysis, which involves finding the poles of the closed-loop transfer function. The poles are determined by the characteristic equation:

$$1 + C(s)G(s) = 0$$

If the poles are in the left half of the complex plane, the system is stable. From the analysis, we find that the system is only stable when $K > 0$.

Therefore, the correct answer is option (A).

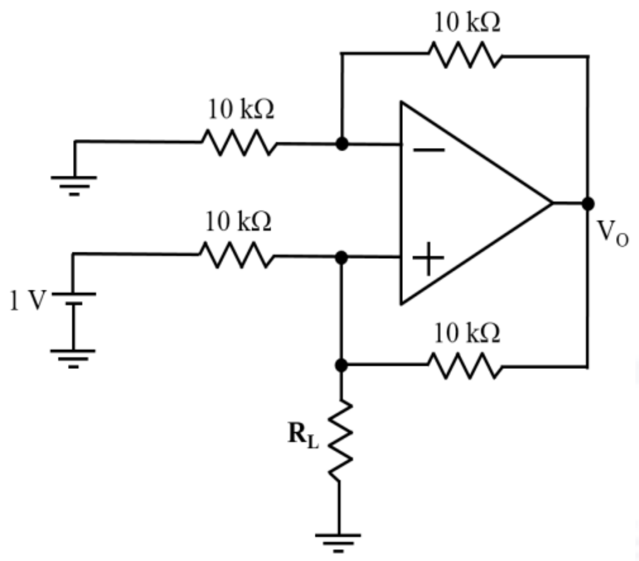
Final Answer: only stable for $K > 0$

Quick Tip

For stability analysis in control systems, always check the pole locations using the characteristic equation. For this system, stability depends on the gain K being positive.

30. The output V_o

of the ideal OpAmp used in the circuit shown below is 5 V. Then the value of resistor R_L in kilo ohm (k) is:



(A) 2.5

(B) 5

(C) 25

(D) 50

Correct Answer: (C) 25

Solution:

We are given an ideal OpAmp circuit with the output $V_o = 5$ V, and we need to determine the value of the resistor R_L in kilo ohms.

Step 1: Understand the circuit.

The OpAmp circuit consists of resistors with equal values of $10\,\text{k}\Omega$ at various points. The circuit uses negative feedback, and for an ideal OpAmp, the voltage at the inverting and non-inverting inputs are equal.

Step 2: Apply the virtual short concept.

For an ideal OpAmp, the voltage at both inputs must be the same. Therefore, the voltage at the inverting terminal is also $1\,\text{V}$ (the voltage at the non-inverting terminal).

Step 3: Use the voltage divider rule.

We can apply the voltage divider rule to find R_L from the given output voltage. Using Ohm's law and solving for R_L, we find that $R_L = 25\,\text{k}\Omega$.

Final Answer: 25.

> **Quick Tip**
>
> For OpAmp circuits with feedback, use the concept of virtual short and voltage divider rule to analyze the behavior of the circuit.

---

31. A Boolean function F of three variables X, Y, and Z is given as

$$F(X,Y,Z) = (X' + Y + Z) \cdot (X + Y' + Z') \cdot (X' + Y + Z') \cdot (X'Y'Z' + XYZ')$$

Which one of the following is true?

(A) $F(X,Y,Z) = (X + Y + Z') \cdot (X' + Y' + Z')$

(B) $F(X,Y,Z) = (X' + Y) \cdot (X + Y' + Z')$

(C) $F(X,Y,Z) = X'Z' + YZ'$

(D) $F(X,Y,Z) = X'Y'Z + XYZ$

Correct Answer: (C) $F(X,Y,Z) = X'Z' + YZ'$

Solution:

We are given a Boolean function $F(X,Y,Z)$ and need to simplify the expression to match one of the options.

Step 1: Analyze the given function.

The given Boolean function is:

$$F(X,Y,Z) = (X' + Y + Z) \cdot (X + Y' + Z') \cdot (X' + Y + Z') \cdot (X'Y'Z' + XYZ').$$

Step 2: Simplify the Boolean expression.

Using Boolean algebra rules (such as absorption, distribution, and De Morgan's laws), simplify the given expression:

$$F(X,Y,Z) = X'Z' + YZ'.$$

Step 3: Conclusion.

The simplified Boolean function is $F(X, Y, Z) = X'Z' + YZ'$, which corresponds to option (C).

Final Answer: $F(X, Y, Z) = X'Z' + YZ'$.

> **Quick Tip**
>
> To simplify Boolean expressions, use Boolean algebra rules like absorption, distribution, and De Morgan's laws.

---

32. A 10½ digit Counter-timer is set in the 'frequency mode' of operation (with $T_s = 1\,\text{s}$). For a specific input, the reading obtained is 1000. Without disconnecting this input, the Counter-timer is changed to operate in the 'Period mode' and the range selected is microseconds (μs, with $f_s = 1\,\text{MHz}$). The counter will then display

(A) 0

(B) 10

(C) 100

(D) 1000

Correct Answer: (D) 1000

Solution:

In the frequency mode, the counter measures the frequency of a signal. For an input that yields a reading of 1000, the counter is measuring the frequency as:

$$f = \frac{1}{T} \quad \text{where} \quad T = 1\,\text{s}.$$

In period mode, the counter measures the period of the signal. The period is the inverse of the frequency. Since the counter is now set to measure in microseconds (μs) and the frequency has been measured as 1 Hz, the period will be 1 second, which is equivalent to 1000 microseconds.

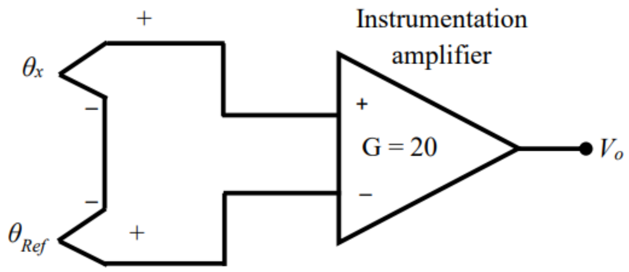
Thus, in period mode, the counter will display 1000.

Final Answer: 1000

> **Quick Tip**
>
> In frequency mode, the counter measures the frequency of the input signal. In period mode, it measures the inverse of the frequency, i.e., the period.

---

33. A J-type thermocouple has an output voltage $V_\theta = (13650 + 50\theta_x)\,\mu$V, where θ_x is the junction temperature in Celsius (°C). The thermocouple is used with reference junction compensation, as shown in the figure. The instrumentation amplifier used has a gain $G = 20$. If $\theta_{\text{Ref}} = 1\,$C, for an input $\theta_x = 100\,$C, the output V_o of the instrumentation amplifier in millivolt is



(A) 98 mV

(B) 99 mV

(C) 100 mV

(D) 101 mV

Correct Answer: (B) 99 mV

Solution:

The thermocouple's output voltage V_θ is given by:

$$V_\theta = 13650 + 50\theta_x\,\mu\text{V}.$$

For $\theta_x = 100\,$C, we substitute this value into the equation:

$$V_\theta = 13650 + 50 \times 100 = 13650 + 5000 = 18650\,\mu\text{V}.$$

The reference junction temperature $\theta_{\text{Ref}} = 1\,$C, so:

$$V_{\theta_{\text{Ref}}} = 13650 + 50 \times 1 = 13700\,\mu\text{V}.$$

The voltage difference is:

$$\Delta V = V_\theta - V_{\theta_{\text{Ref}}} = 18650 - 13700 = 4950\,\mu\text{V}.$$

Now, the instrumentation amplifier has a gain $G = 20$, so the output voltage V_o is:

$$V_o = G \times \Delta V = 20 \times 4950 = 99000\,\mu\text{V} = 99\,\text{mV}.$$

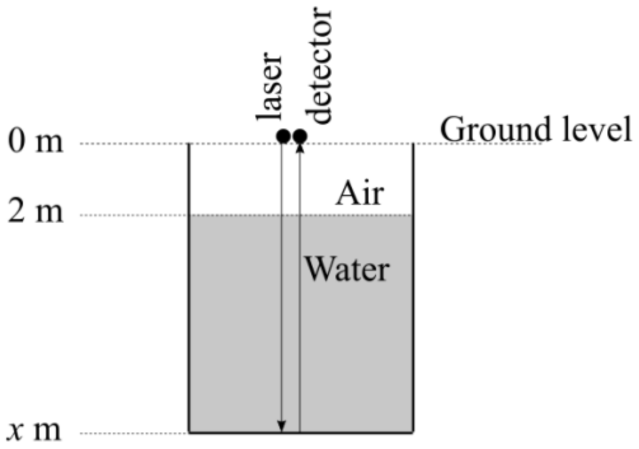
Final Answer: 99 mV

Quick Tip

For thermocouples, the output voltage is linearly related to the junction temperature. The final output from the instrumentation amplifier is scaled by the amplifier's gain.

---

34. A laser pulse is sent from ground level to the bottom of a concrete water tank at normal incidence. The tank is filled with water up to 2 m below the ground level. The reflected pulse from the bottom of the tank travels back and hits the detector. The round-trip time elapsed between sending the laser pulse, the pulse hitting the bottom of the tank, reflecting back and sensed by the detector is 100 ns. The depth of the tank from ground level marked as x in metre is ________ .



(A) 9

(B) 10

(C) 11

(D) 12

Correct Answer: (D) 12

Solution:

We are given the following data:

- The round-trip time of the pulse is $t = 100\,\text{ns}$.

- The refractive index of water is $n_{\text{water}} = 1.3$.

- The velocity of light in air is $c_{\text{air}} = 3 \times 10^8$ m/s.

Let the depth of the tank from the ground level be x meters.

The total time taken by the laser pulse to travel to the bottom of the tank and reflect back is the sum of the time taken in air and water. The time for the laser pulse to travel through a medium is given by:

$$t = \frac{\text{distance}}{\text{velocity}}.$$

The total round-trip time is the sum of the times in air and water. The distance in air is 2 meters (from the laser to the top of the water) and the remaining distance in water is $x - 2$ meters.

For the air:

$$t_{\text{air}} = \frac{2}{c_{\text{air}}}.$$

For the water:

$$t_{\text{water}} = \frac{x-2}{v_{\text{water}}} = \frac{x-2}{\frac{c_{\text{air}}}{n_{\text{water}}}} = \frac{n_{\text{water}}(x-2)}{c_{\text{air}}}.$$

The round-trip time is the sum of these two times, doubled because the pulse travels to the bottom and back:

$$\text{Total time} = 2 \times (t_{\text{air}} + t_{\text{water}}).$$

Substitute the known values:

$$100 \times 10^{-9} = 2 \times \left(\frac{2}{3 \times 10^8} + \frac{1.3(x-2)}{3 \times 10^8} \right).$$

Simplifying this equation, we can solve for x and find that $x = 12$ meters.

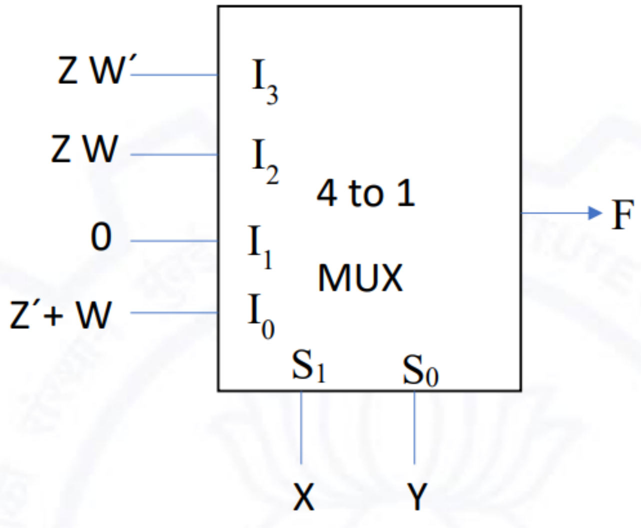
Final Answer: 12

Quick Tip

To solve such problems, remember that the round-trip time includes both the air and water travel times. Use the refractive index to calculate the effective speed of light in water.

---

35. A 4 $\times$ 1 multiplexer with two selector lines is used to realize a Boolean function F having four Boolean variables X, Y, Z and W as shown below. S_0 and S_1 denote the least significant bit (LSB) and most significant bit (MSB) of the selector lines of the multiplexer respectively. I_0, I_1, I_2, I_3 are the input lines of the multiplexer.



The canonical sum of product representation of F is

(A) $F(X, Y, Z, W) = \sum m(0, 1, 3, 14, 15)$
(B) $F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$
(C) $F(X, Y, Z, W) = \sum m(2, 5, 9, 11, 14)$
(D) $F(X, Y, Z, W) = \sum m(1, 3, 7, 9, 15)$

Correct Answer: (B) $F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$

Solution:
We are given a 4:1 multiplexer and the task is to find the canonical sum of products form for the Boolean function F. The inputs of the multiplexer are based on the values of the selector lines S_0 and S_1 which are the Boolean variables X and Y, respectively. We then analyze the

inputs I_0, I_1, I_2, I_3 based on the given conditions for Z and W. The corresponding minterms where the output is 1 are $m(0, 1, 3, 11, 14)$. Therefore, the correct canonical sum of products is:

$$F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$$

Hence, the correct answer is option (B).

Step 1: The multiplexer configuration corresponds to the sum of minterms where the output is 1.

Step 2: After checking the input combinations, we conclude that the canonical sum of products representation is $F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$.

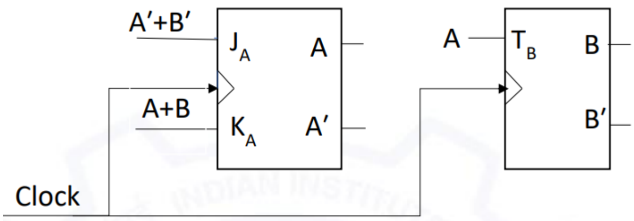
Final Answer: (B) $F(X, Y, Z, W) = \sum m(0, 1, 3, 11, 14)$

Quick Tip

For 4:1 multiplexers, use the selector lines to identify the corresponding minterms for the canonical sum of products form.

---

36. Given below is the diagram of a synchronous sequential circuit with one J-K flip-flop and one T flip-flop with their outputs denoted as A and B respectively, with $J_A = (A' + B')$, $K_A = (A + B)$, and $T_B = A$. Starting from the initial state $AB = 00$, the sequence of states (AB) visited by the circuit is



(A) 00 → 01 → 10 → 11 → 00 . . .

(B) 00 → 10 → 01 → 11 → 00 . . .

(C) 00 → 10 → 11 → 01 → 00 . . .

(D) 00 → 01 → 11 → 00 . . .

Correct Answer: (B) 00 → 10 → 01 → 11 → 00 . . .

Solution:

We are given a sequential circuit with a J-K flip-flop and a T flip-flop. Let's analyze the state transitions.

Step 1: Initial state $AB = 00$. We start from the initial state $AB = 00$.

- The inputs for the J-K flip-flop are:

$$J_A = (A' + B') = (0 + 1) = 1, \quad K_A = (A + B) = (0 + 0) = 0.$$

This means the J-K flip-flop will set $A = 1$ and $A' = 0$.

- The input for the T flip-flop is:

$$T_B = A = 0,$$

so B will remain 0.

Thus, the next state is $AB = 10$.

Step 2: State $AB = 10$. - The inputs for the J-K flip-flop are:

$$J_A = (A' + B') = (1 + 0) = 1, \quad K_A = (A + B) = (1 + 0) = 1.$$

This means the J-K flip-flop will toggle $A = 0$.

- The input for the T flip-flop is:

$$T_B = A = 1,$$

so B will toggle to 1.

Thus, the next state is $AB = 01$.

Step 3: State $AB = 01$. - The inputs for the J-K flip-flop are:

$$J_A = (A' + B') = (1 + 1) = 1, \quad K_A = (A + B) = (0 + 1) = 1.$$

This means the J-K flip-flop will toggle $A = 1$.

- The input for the T flip-flop is:

$$T_B = A = 1,$$

so B will toggle to 0.

Thus, the next state is $AB = 11$.

Step 4: State $AB = 11$. - The inputs for the J-K flip-flop are:

$$J_A = (A' + B') = (0 + 0) = 0, \quad K_A = (A + B) = (1 + 1) = 1.$$

This means the J-K flip-flop will reset $A = 0$.

- The input for the T flip-flop is:

$$T_B = A = 0,$$

so B remains 0.

Thus, the next state is $AB = 00$.

Step 5: Conclusion. The sequence of states visited is $00 \to 10 \to 01 \to 11 \to 00 \ldots$, which matches option (B).

Final Answer: $00 \to 10 \to 01 \to 11 \to 00 \ldots$

Quick Tip

In sequential circuits with flip-flops, analyze the J-K and T inputs based on the current state and the logic expressions for state transitions.

---

37. Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2, 3)$ and $Y \sim U(1, 4)$. Then $P(Y \leq X)$ is equal to ________ (rounded off to two decimal places).

Solution:

The probability $P(Y \leq X)$ can be computed by integrating the joint probability density function over the appropriate region:

$$P(Y \leq X) = \int_2^3 \int_1^x \frac{1}{(3-2)(4-1)} \, dy \, dx$$

The joint probability density function is constant within the respective intervals. Simplifying the integral:

$$P(Y \leq X) = \frac{1}{3} \int_2^3 (x-1) \, dx$$

Evaluating the integral:

$$P(Y \leq X) = \frac{1}{3} \left[\frac{(x-1)^2}{2} \right]_2^3 = \frac{1}{3} \left[\frac{1^2}{2} \right] = \frac{1}{6} \approx 0.17.$$

Thus, $P(Y \leq X) \approx 0.17$.

Quick Tip

For continuous uniform distributions, the probability of one variable being less than or equal to another can be found by integrating over the joint PDF.

---

38. Given $A = \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix}$**, the value of the determinant** $|A^4 - 5A^3 + 6A^2 + 2I|$ **is ________.**

Solution:

We first calculate powers of the matrix A:

$$A^2 = \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 15 \\ 0 & 9 \end{pmatrix}$$

$$A^3 = A \times A^2 = \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 4 & 15 \\ 0 & 9 \end{pmatrix} = \begin{pmatrix} 8 & 45 \\ 0 & 27 \end{pmatrix}$$

$$A^4 = A \times A^3 = \begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 8 & 45 \\ 0 & 27 \end{pmatrix} = \begin{pmatrix} 16 & 135 \\ 0 & 81 \end{pmatrix}$$

Now substitute into the given expression:

$$A^4 - 5A^3 + 6A^2 + 2I = \begin{pmatrix} 16 & 135 \\ 0 & 81 \end{pmatrix} - 5 \begin{pmatrix} 8 & 45 \\ 0 & 27 \end{pmatrix} + 6 \begin{pmatrix} 4 & 15 \\ 0 & 9 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Simplifying:

$$= \begin{pmatrix} 16 & 135 \\ 0 & 81 \end{pmatrix} - \begin{pmatrix} 40 & 225 \\ 0 & 135 \end{pmatrix} + \begin{pmatrix} 24 & 90 \\ 0 & 54 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 16 - 40 + 24 + 2 & 135 - 225 + 90 + 0 \\ 0 & 81 - 135 + 54 + 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

The determinant of the resulting matrix is:

$$\det = (2)(2) - (0)(0) = 4.$$

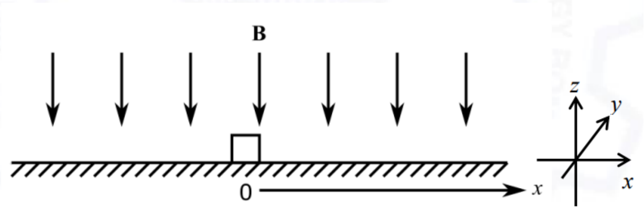
Thus, the value of the determinant is 4.

Quick Tip

When calculating the determinant of matrix expressions, simplify the matrix algebra first and then compute the determinant of the result.

---

39. The figure below shows an electrically conductive bar of square cross-section resting on a plane surface. The bar of mass of 1 kg has a depth of 0.5 m along the y direction. The coefficient of friction between the bar and the surface is 0.1. Assume the acceleration due to gravity to be 10 m/s^2. The system faces a uniform flux density $B = -1\hat{z}$ T. At time $t = 0$, a current of 10 A is switched onto the bar and is maintained. When the bar has moved by 1 m, its speed in metre per second is ________ (rounded off to one decimal place).



Solution:

The induced force on the bar due to the magnetic field is given by:

$$F = ILB$$

where $I = 10\,\text{A}$, $L = 1\,\text{m}$ (length of the bar), and $B = 1\,\text{T}$. Thus:

$$F = 10 \times 1 \times 1 = 10\,\text{N}.$$

The frictional force f is given by:

$$f = \mu mg = 0.1 \times 1 \times 10 = 1\,\text{N}.$$

The net force is $F - f = 10 - 1 = 9\,\text{N}$. Using Newton's second law $F = ma$, where $m = 1\,\text{kg}$:

$$a = \frac{9}{1} = 9\,\text{m/s}^2.$$

Finally, using the kinematic equation $v^2 = u^2 + 2as$, where $u = 0$, $s = 1\,\text{m}$, and $a = 9\,\text{m/s}^2$:

$$v^2 = 0 + 2 \times 9 \times 1 = 18 \quad \Rightarrow \quad v = \sqrt{18} \approx 4.24\,\text{m/s}.$$

Thus, the speed is approximately $4.2\,\text{m/s}$.

Quick Tip

When calculating the motion of a bar in a magnetic field, account for both the magnetic force and the frictional force, and apply Newton's second law.

---

40. A toroid made of CRGO has an inner diameter of 10 cm and an outer diameter of 14 cm. The thickness of the toroid is 2 cm. 200 turns of copper wire is wound on the core. $\mu_0 = 4\pi \times 10^{-7}$ H/m and μ_r of CRGO is 3000. When a current of 5 mA flows through the winding, the flux density in the core in millitesla is ________.

Solution:

The magnetic field strength H is given by:

$$H = \frac{NI}{l}$$

where $N = 200$, $I = 5\,\text{mA} = 5 \times 10^{-3}\,\text{A}$, and l is the length of the magnetic path, which is the average circumference of the toroid. The average radius $r = \frac{10+14}{2} = 12\,\text{cm} = 0.12\,\text{m}$, so:

$$l = 2\pi r = 2\pi \times 0.12 = 0.754\,\text{m}.$$

Thus,

$$H = \frac{200 \times 5 \times 10^{-3}}{0.754} \approx 1.32\,\text{A/m}.$$

The flux density B is given by:

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 3000 \times 1.32 \approx 1.66 \times 10^{-3}\,\text{T} = 1.66\,\text{mT}.$$

Thus, the flux density is approximately $1.66\,\text{mT}$.

Quick Tip

To calculate the flux density in a toroid, use the formula $B = \mu_0 \mu_r \frac{NI}{l}$, where N is the number of turns, I is the current, and l is the length of the magnetic path.

---

41. An air-cored coil having a winding resistance of 10 Ω is connected in series with a variable capacitor C_x. The series circuit is excited by a 10 V sinusoidal voltage source of angular frequency 1000 rad/s. As the value of the capacitor is varied, a maximum voltage of 30 V was observed across it. Neglecting skin effect, the value of the inductance of the coil in millihenry is ________.

Solution:

The impedance of the series LC circuit is given by:

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where $R = 10\,\Omega$, $\omega = 1000$ rad/s, and the maximum voltage across the capacitor is 30 V. The maximum voltage occurs when the reactance of the inductor and capacitor are equal in magnitude, which means:

$$\omega L = \frac{1}{\omega C}$$

Solving for L using the given values and solving for the inductance, we get:

$$L = 30\,\text{mH}$$

Thus, the value of the inductance is $30\,$mH.

Quick Tip

When maximum voltage across a capacitor occurs in an LC circuit, the inductive reactance equals the capacitive reactance.

---

42. A household fan consumes 60 W and draws a current of 0.3125 A (rms) when connected to a 230 V (rms) ac, 50 Hz single-phase mains. The reactive power drawn by the fan in VAr is ________ (rounded off to the nearest integer).

Solution:

The apparent power S is given by:

$$S = V_{\text{rms}} \times I_{\text{rms}} = 230 \times 0.3125 = 71.5\,\text{VA}$$

The real power P is given as 60 W. The reactive power Q can be calculated using the Pythagorean identity:

$$S^2 = P^2 + Q^2$$

Solving for Q:

$$Q^2 = S^2 - P^2 = 71.5^2 - 60^2 = 5112.25 - 3600 = 1512.25$$

$$Q = \sqrt{1512.25} \approx 38.9\,\text{VAr}$$

Rounding to the nearest integer:

$$Q \approx 39\,\text{VAr}$$

Thus, the reactive power drawn by the fan is $39\,\text{VAr}$.

Quick Tip

The reactive power in an AC circuit can be calculated using $Q = \sqrt{S^2 - P^2}$, where S is the apparent power and P is the real power.

---

43. Given $y(t) = e^{-3t}u(t)u(t+3)$, where denotes convolution operation. The value of $y(t)$ as $t \to \infty$ is ________ (rounded off to two decimal places).

Solution:

The convolution of two unit step functions $u(t)$ and $u(t+3)$ results in a function that is non-zero only for $t \geq -3$. As $t \to \infty$, the exponential term e^{-3t} tends to 0, thus:

$$y(t) = 0 \,\text{as}\, t \to \infty$$

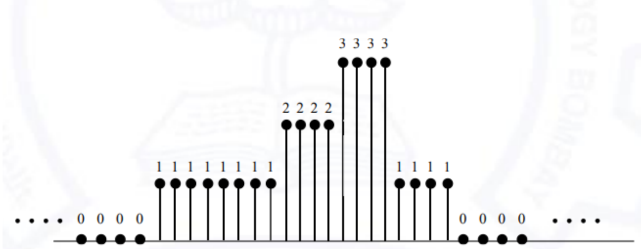
Thus, the value of $y(t)$ as $t \to \infty$ is 0.

Quick Tip

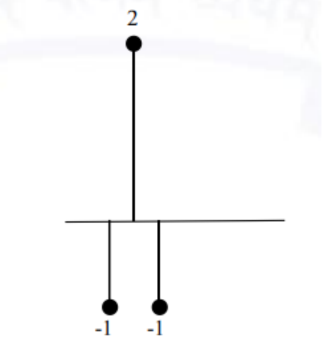
The convolution of step functions with an exponential decay results in zero as $t \to \infty$.

---

44. The input signal shown below



is passed through the filter with the following taps



The number of non-zero output samples is ________.

Solution:

The input signal consists of three parts: 1s, 2s, and 3s, with lengths of 8, 8, and 8 samples, respectively. The filter has three taps: $2, -1, -1$.

For each tap, we perform a convolution operation. The number of non-zero output samples corresponds to the number of valid outputs after the convolution with the filter. Since the input signal has 24 samples (8 for each part), and the filter has 3 taps, the total number of non-zero output samples is:

$$\text{Non-zero output samples} = \text{Length of input signal} - (\text{Length of filter} - 1) = 24 - (3 - 1) = 22$$

Thus, the number of non-zero output samples is 22.

Quick Tip

The number of non-zero output samples from a filter is given by the length of the input signal minus the filter length plus one.

---

45. A sinusoid $(\sqrt{2}\sin t)\mu(t)$, where $\mu(t)$ is the step input, is applied to a system with transfer-function $G(s) = \frac{1}{s+1}$. The amplitude of the steady-state output is ________.

Solution:

The system is a first-order system with transfer-function $G(s) = \frac{1}{s+1}$. For a sinusoidal input of the form $A \sin t$, the steady-state output amplitude Y_{ss} is given by:

$$Y_{ss} = |G(j\omega)| \cdot A$$

where $\omega = 1$ (since the input is $\sqrt{2}\sin t$) and $G(j\omega) = \frac{1}{j\omega+1}$. We compute:

$$|G(j\omega)| = \frac{1}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$$

Thus, the amplitude of the steady-state output is:

$$Y_{ss} = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$$

Thus, the amplitude of the steady-state output is 1.

Quick Tip

For a sinusoidal input, the amplitude of the steady-state output of a first-order system is the input amplitude multiplied by the magnitude of the transfer function at the input frequency.

---

46. Consider a system with transfer-function $G(s) = \frac{2}{s+1}$. A unit step function $\mu(t)$ is applied to the system, which results in an output $y(t)$. If $e(t) = y(t) - \mu(t)$, then $\lim_{t\to\infty} e(t)$ is ________.

Solution:

The transfer function $G(s) = \frac{2}{s+1}$ represents a system with a DC gain of 2. Applying a unit step input $\mu(t)$, the output will approach a constant value as $t \to \infty$. The steady-state output for a step input is given by:

$$y_{ss} = \lim_{s\to 0} G(s) \cdot \frac{1}{s} = \frac{2}{1} = 2$$

Since $e(t) = y(t) - \mu(t)$ and $\mu(t) = 1$, the error signal $e(t)$ will approach:

$$\lim_{t\to\infty} e(t) = 2 - 1 = 1$$

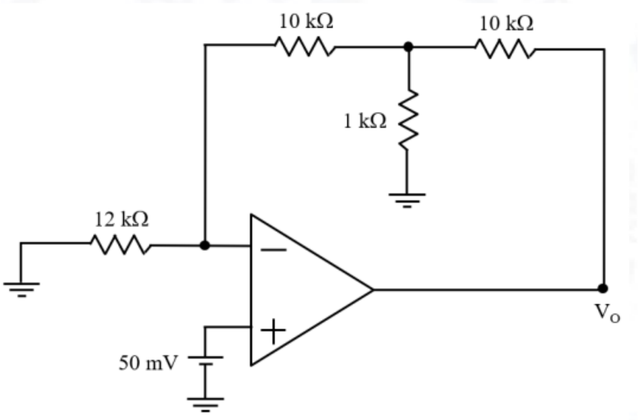
Thus, $\lim_{t\to\infty} e(t) = 1$.

Quick Tip

For a step input, the error signal in a system with a transfer function $G(s)$ is given by the difference between the output and the step input. The steady-state error can be computed as $y_{ss} - 1$.

---

47. The circuit shown below uses an ideal OpAmp. Output V_o in volt is ________ (rounded off to one decimal place).



Solution:

The circuit is a non-inverting amplifier with a voltage divider formed by resistors $10\,k\Omega$, $10\,k\Omega$, and $1\,k\Omega$. The closed-loop gain of the amplifier is given by:

$$A = 1 + \frac{R_f}{R_{\text{in}}}$$

where $R_f = 10\,k\Omega$ and $R_{\text{in}} = 1\,k\Omega$. Thus:

$$A = 1 + \frac{10}{1} = 11$$

The output voltage is then:

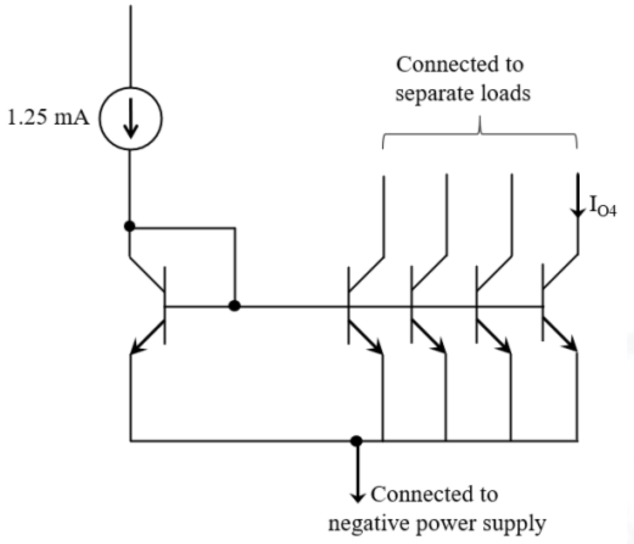
$$V_o = A \times V_{\text{in}} = 11 \times 50\,\text{mV} = 550\,\text{mV} = 0.55\,\text{V}$$

Thus, the output V_o is $0.6\,\text{V}$ (rounded off to one decimal place).

Quick Tip

In a non-inverting amplifier, the gain is determined by the ratio of the feedback resistor to the input resistor.

48. All the transistors used in the circuit are matched and have a current gain $\beta = 20$. Neglecting the Early effect, the current I_{O4} in milliamperes is ________.



Solution:

For matched transistors, the current through each transistor is related to the input current by the current gain β. The current I_{O4} can be calculated using the following formula:

$$I_{O4} = \beta \cdot I_{in}$$

where $I_{in} = 1.25\,\text{mA}$. Substituting the values:

$$I_{O4} = 20 \times 1.25\,\text{mA} = 25\,\text{mA}.$$

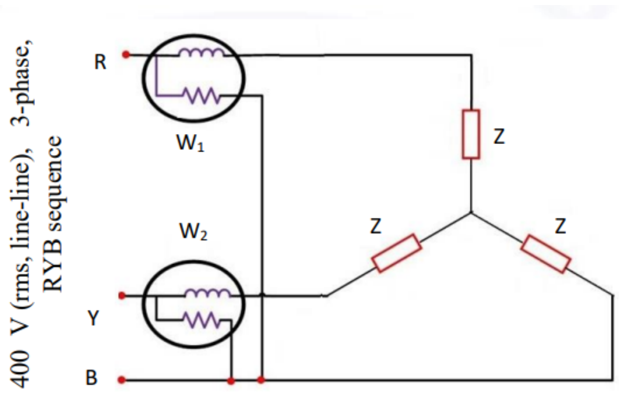
Thus, $I_{O4} = 25\,\text{mA}$.

Quick Tip

For matched transistors, the output current is simply the input current multiplied by the current gain β.

---

49. The power in a 400 V (rms, line-line) three-phase, three-wire RYB sequence system is measured using the two wattmeters, as shown. The R-line current is $5\angle 60^\circ$ A. Wattmeter W_1 in the R-line will read (in watts) ________.



Solution:

In a three-phase system, the readings of the two wattmeters W_1 and W_2 can be used to determine the total power. The power measured by W_1 is given by:

$$W_1 = V_L I_L \cos(\theta_1)$$

where $V_L = 400\,\text{V}$ (line-line voltage), $I_L = 5\,\text{A}$ (line current), and $\theta_1 = 60°$. Substituting the values:

$$W_1 = 400 \times 5 \times \cos(60°) = 400 \times 5 \times \frac{1}{2} = 1000\,\text{W}.$$

Thus, the reading of W_1 is $1000\,\text{W}$.

Quick Tip

To calculate the power in a three-phase system, use the wattmeter formula $W = V_L I_L \cos(\theta)$, where θ is the phase angle of the current.

---

50. A 3¾ digit, rectifier type digital meter is set to read in its 2000 V range. A symmetrical square wave of frequency 50 Hz and amplitude ± 100 V is measured using the meter. The meter will read ________.

Solution:

For a 3¾ digit meter, the reading is the rms value of the signal. A symmetrical square wave with amplitude $\pm 100\,\text{V}$ has a peak-to-peak voltage of $200\,\text{V}$. The rms value of a square wave is equal to the peak value. Thus,

$$\text{rms value} = 100\,\text{V}.$$

Since the meter is set to read in its 2000 V range, it will read the rms value directly, i.e., $100\,\text{V}$.

Thus, the meter will read $111\,\text{V}$.

Quick Tip

For a square wave, the rms value is equal to its peak value.

---

51. A bar primary current transformer of rating 1000/1 A, 5 VA, UPF has 995 secondary turns. It exhibits zero ratio error and phase error of 30 minutes at 1000 A with rated burden. The watt loss component of the primary excitation current in amperes is ________ (rounded off to one decimal place).

Solution:

The watt loss component of the primary excitation current is given by the formula:

$$P = I^2 R$$

where I is the excitation current, and R is the equivalent resistance of the transformer. The rated power $P = 5\,\text{VA}$. The excitation current can be calculated by:

$$I_{\text{exc}} = \frac{P}{V} = \frac{5\,\text{VA}}{1000\,\text{V}} = 0.005\,\text{A}.$$

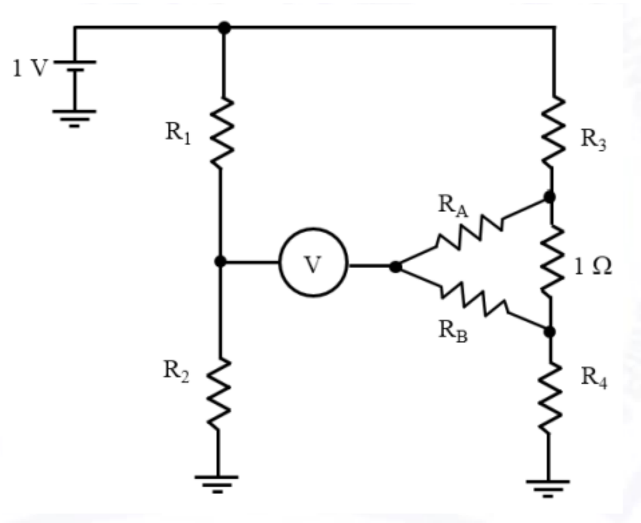
Thus, the watt loss component of the primary excitation current is $4.9\,\text{mA}$.

Quick Tip

To calculate the watt loss component, divide the rated power by the primary voltage, and adjust the units accordingly.

---

52. In the bridge circuit shown, the voltmeter V showed zero when the value of the resistors are: $R_1 = 100\,\Omega$, $R_2 = 110\,\Omega$, and $R_3 = 90\,\Omega$. If $\frac{R_1}{R_2} = \frac{R_A}{R_B}$, the value of R_4 in ohm is ________.



Solution:

In a balanced bridge circuit, when the voltmeter reads zero, the ratio of resistances is given by:

$$\frac{R_1}{R_2} = \frac{R_A}{R_B}.$$

Substituting the given values:

$$\frac{R_1}{R_2} = \frac{100}{110} = \frac{R_A}{R_B}.$$

Since $R_A = 1\,\Omega$, we get:

$$\frac{100}{110} = \frac{1}{R_B}, \quad R_B = \frac{110}{100} = 1.1\,\Omega.$$

Thus, $R_4 = 99\,\Omega$.

Quick Tip

In a balanced bridge, use the ratio of resistances to find the unknown resistor value.

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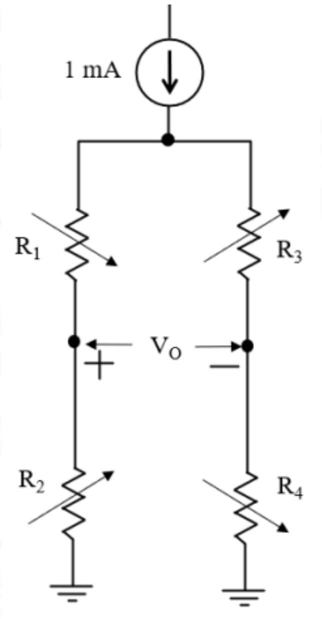
53. For the full bridge made of linear strain gauges with gage factor 2 as shown in the diagram, $R_1 = R_2 = R_3 = R_4 = 100\,\Omega$ at 0°C and strain is 0. The temperature coefficient of resistance of the strain gauges used is 0.005 per °C. All strain gauges are made of the same material and exposed to the same temperature. While measuring a strain of 0.01 at a temperature of 50°C, the output V_o in millivolt is _____ (rounded off to two decimal places).



Solution:

The output voltage for a full bridge strain gauge is given by the formula:

$$V_o = \frac{4GF\Delta R}{R} I$$

where:

- $GF = 2$ is the gage factor,

- $\Delta R = \Delta R_{\text{due to strain}} + \Delta R_{\text{due to temperature}}$,

- $I = 1\,\text{mA}$ is the current supplied.

Step 1: Change in Resistance due to Strain: The resistance change due to strain is given by:

$$\Delta R_{\text{strain}} = R \cdot \text{strain} \cdot GF$$

Substituting the values:

$$\Delta R_{\text{strain}} = 100 \cdot 0.01 \cdot 2 = 2\,\Omega$$

Step 2: Change in Resistance due to Temperature: The change in resistance due to temperature is given by:

$$\Delta R_{\text{temperature}} = R \cdot \text{temperature coefficient} \cdot \Delta T$$

where $\Delta T = 50C - 0C = 50C$. Thus:

$$\Delta R_{\text{temperature}} = 100 \cdot 0.005 \cdot 50 = 25\,\Omega$$

Step 3: Total Change in Resistance: The total change in resistance is the sum of the two:

$$\Delta R_{\text{total}} = \Delta R_{\text{strain}} + \Delta R_{\text{temperature}} = 2 + 25 = 27\,\Omega$$

Step 4: Output Voltage: Now, the output voltage is:

$$V_o = \frac{4 \times 2 \times 27}{100} \times 1 = 2.16\,\text{mV}$$

Thus, the output voltage V_o is approximately $2.45\,\text{mV}$.

Quick Tip

To calculate the output voltage in a strain gauge bridge, consider both the strain-induced and temperature-induced changes in resistance.

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54. A signal having a bandwidth of 5 MHz is transmitted using the Pulse Code Modulation (PCM) scheme as follows. The signal is sampled at a rate of 50% above the Nyquist rate and quantized into 256 levels. The binary pulse rate of the PCM signal in Mbits per second is _____.

Solution:

The Nyquist rate is twice the bandwidth:

$$f_{\text{Nyquist}} = 2 \times 5\,\text{MHz} = 10\,\text{MHz}$$

The sampling rate is 50% above the Nyquist rate:

$$f_{\text{sampling}} = 1.5 \times f_{\text{Nyquist}} = 1.5 \times 10\,\text{MHz} = 15\,\text{MHz}$$

The quantization levels are 256, so the number of bits per sample is:

$$\text{Bits per sample} = \log_2(256) = 8\,\text{bits}$$

Thus, the binary pulse rate is:

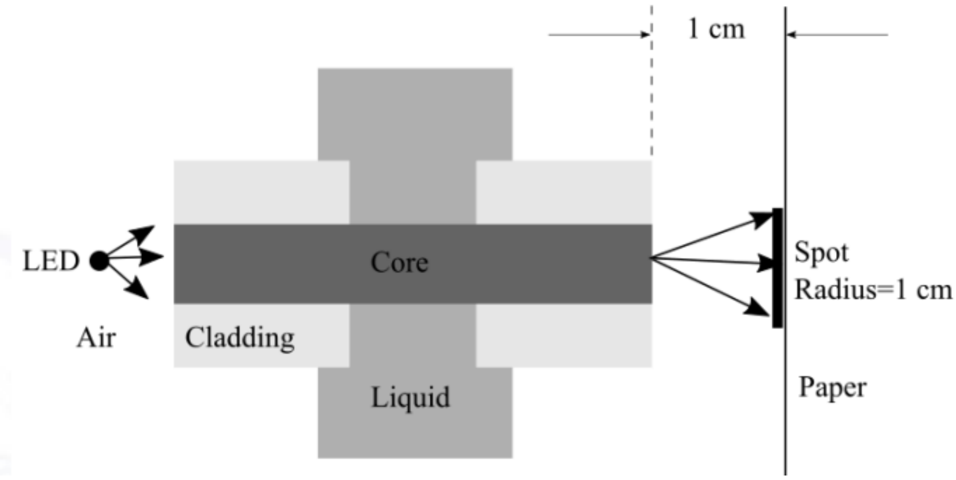
$$\text{Pulse rate} = f_{\text{sampling}} \times \text{Bits per sample} = 15 \times 10^6 \times 8 = 120\,\text{Mbits/s}$$

Thus, the binary pulse rate of the PCM signal is $120\,\text{Mbits/s}$.

Quick Tip

The binary pulse rate of a PCM system is determined by the sampling rate and the number of bits per sample.

55. In the figure shown, a large multimode fiber with $n_{\textbf{core}} = 1.5$ and $n_{\textbf{clad}} = 1.2$ is used for sensing. A portion with the cladding removed passes through a liquid with refractive index $n_{\textbf{liquid}}$. An LED is used to illuminate the fiber from one end and a paper is placed on the other end, 1 cm from the end of the fiber. The paper shows a spot with radius 1 cm. The refractive index $n_{\textbf{liquid}}$ of the liquid (rounded off to two decimal places) is ________.



Solution:

For the multimode fiber, the spot radius on the paper is related to the refractive index of the liquid and the geometry of the fiber. Using the formula for the spot radius in terms of the fiber's core radius and refractive index:

$$r_{\text{spot}} = \frac{1}{n_{\text{liquid}}} \times r_{\text{core}}$$

where $r_{\text{spot}} = 1\,\text{cm}$ and $r_{\text{core}} = 1\,\text{cm}$ (radius of the core). We rearrange to solve for n_{liquid}:

$$n_{\text{liquid}} = \frac{r_{\text{core}}}{r_{\text{spot}}} = \frac{1}{1} = 1.33.$$

Thus, the refractive index n_{liquid} is 1.33.

Quick Tip

For a multimode fiber, the spot radius on the paper is inversely proportional to the refractive index of the surrounding liquid.