GATE 2021 Mechanical Engineering (ME, Set-2) Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. Five persons P, Q, R, S and T are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and T cannot be seated at either end of the row. P should not be seated adjacent to S. R is to be seated at the second position from the left end of the row. The number of distinct seating arrangements possible is:

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

We need to find the number of distinct seating arrangements given the following conditions:

- P and T cannot be seated at either end of the row.
- P should not be seated adjacent to S.
- R is to be seated at the second position from the left end of the row.

Step 1: Place R in the second position Since R is fixed in the second position, we now have the following positions for the remaining people: $\_R\_\_\_$

Step 2: Place P and T P and T cannot be seated at either end, so the only available positions for P and T are the 3rd and 4th positions. Therefore, we can place P and T in the 3rd and 4th positions in 2 ways (P in 3rd and T in 4th, or vice versa).

Step 3: Place S and Q Now, S and Q are left to be seated in the remaining two positions (the 1st and 5th positions). The condition is that P should not be adjacent to S, so S must be placed in the 5th position, and Q must be placed in the 1st position.

Step 4: Calculate the total arrangements The only possible arrangement is:

- P and T can be arranged in 2 ways in the 3rd and 4th positions.
- S and Q can be placed in the 1st and 5th positions in exactly 1 way (since S cannot sit next to P).

Thus, the total number of distinct seating arrangements is:

$$2 + 1 = 3.$$

Thus, the correct answer is Option (B).

Final Answer: (B) 3

Quick Tip

When calculating seating arrangements with conditions, first fix the positions that are restricted (like R's position here) and then systematically consider the remaining conditions (like P and T not at the ends, and adjacency rules).

2. Consider the following sentences:

(i)The number of candidates who appear for the GATE examination is staggering.

(ii)A number of candidates from my class are appearing for the GATE examination.

(iii)The number of candidates who appear for the GATE examination are staggering.

(iv)A number of candidates from my class is appearing for the GATE examination.

Which of the above sentences are grammatically CORRECT?

(A) (i) and (ii)
(B) (i) and (iii)
(C) (ii) and (iii)
(D) (ii) and (iv)

Correct Answer: (A)

Solution:
Step 1: Analyze sentence (i)

(i)The number of candidates who appear for the GATE examination is staggering.

- Correct: The subject ”number” is singular, so the verb ”is” agrees with it.
Step 2: Analyze sentence (ii)

(ii)A number of candidates from my class are appearing for the GATE examination.

- Correct: ”A number of” is a phrase that takes the plural verb ”are.”
Step 3: Analyze sentence (iii)

(iii)The number of candidates who appear for the GATE examination are staggering.

- Incorrect: The subject ”number” is singular, so the verb should be ”is” instead of ”are.”
Step 4: Analyze sentence (iv)

(iv)A number of candidates from my class is appearing for the GATE examination.

- Incorrect: ”A number of” takes a plural verb, so it should be ”are” instead of ”is.”

Conclusion: Sentences (i) and (ii) are grammatically correct, while sentences (iii) and (iv) are incorrect. Thus, the correct answer is Option (A).

Final Answer: (A)

Quick Tip

When using expressions like "a number of," always use the plural form of the verb. When the subject is "number" (singular), use the singular verb.

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3. A digital watch X beeps every 30 seconds while watch Y beeps every 32 seconds. They beeped together at 10 AM.

The immediate next time that they will beep together is _____

(A) 10:08 AM

(B) 10:42 AM

(C) 11:00 AM

(D) 10:00 PM

Correct Answer: (A)

Solution:

To find the next time both watches beep together, we need to calculate the least common multiple (LCM) of the two times, 30 seconds and 32 seconds.

The LCM of 30 and 32 is calculated as follows:

$$\text{LCM}(30, 32) = 2^5 \times 3 \times 5 = 480 \text{ seconds}.$$

This means that both watches will beep together again after 480 seconds.

Now, converting 480 seconds to minutes:

$$480 \text{ seconds} = 8 \text{ minutes}.$$

So, they will beep together again 8 minutes after 10:00 AM, which is 10:08 AM.

Final Answer: (A)

Quick Tip

To find when events will happen together again, find the least common multiple (LCM) of the two intervals.

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4. If $\oplus \div \circ = 2;\ \Delta \div \circ = 3;\ \circ + \Delta = 5;\ \Delta \times \oplus = 10,$

Then, the value of $(\oplus - \circ)^2$ **is:**

(A) 0

(B) 1

(C) 4

(D) 16

Correct Answer: (B)

Solution:

Let's analyze the given operations step by step:

We have the following information:

1. $\oplus \div \circ = 2$ (so $\oplus = 2 \times \circ$), 2. $\Delta \div \circ = 3$ (so $\Delta = 3 \times \circ$), 3. $\circ + \Delta = 5$.

Now, substitute $\Delta = 3 \times \circ$ into the equation $\circ + \Delta = 5$:

$$\circ + 3 \times \circ = 5$$

$$4 \times \circ = 5 \quad \Rightarrow \quad \circ = \frac{5}{4}.$$

Now that we know $\circ = \frac{5}{4}$, we can find the values of $\oplus$ and Δ:

$$\oplus = 2 \times \frac{5}{4} = \frac{5}{2}, \quad \Delta = 3 \times \frac{5}{4} = \frac{15}{4}.$$

We are asked to find the value of $(\oplus - \circ)^2$:

$$\oplus - \circ = \frac{5}{2} - \frac{5}{4} = \frac{10}{4} - \frac{5}{4} = \frac{5}{4}.$$

$$(\oplus - \circ)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}.$$

Thus, the value of $(\oplus - \circ)^2$ is 1.

Final Answer: (B)

Quick Tip

When solving for unknowns in equations involving operations, express the variables in terms of each other and substitute to simplify the calculations.

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5. The front door of Mr. X's house faces East. Mr. X leaves the house, walking 50 m straight from the back door that is situated directly opposite to the front door. He then turns to his right, walks for another 50 m and stops. The direction of the point Mr. X is now located at with respect to the starting point is _____

(A) South-East

(B) North-East

(C) West

(D) North-West

Correct Answer: (D)

Solution:

We are given the following directions:

1. Mr. X's house faces East, so the front door is facing East. 2. He walks 50 m straight from the back door, which is opposite the front door, so he is walking in the West direction. 3. After reaching the end of the 50 m, Mr. X turns to his right, which would be the North direction. 4. He then walks 50 m North.

Now, the direction of the point Mr. X is located at, with respect to his starting point, is in the North-West direction.

Thus, the correct answer is North-West.

Final Answer: (D)

Quick Tip

To determine direction changes, visualize or draw the path and use relative directions to track movements.

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6. Given below are two statements 1 and 2, and two conclusions I and II.

Statement 1: All entrepreneurs are wealthy.

Statement 2: All wealthy are risk seekers.

Conclusion I: All risk seekers are wealthy.

Conclusion II: Only some entrepreneurs are risk seekers.

Based on the above statements and conclusions, which one of the following options is CORRECT?

(A) Only conclusion I is correct

(B) Only conclusion II is correct

(C) Neither conclusion I nor II is correct

(D) Both conclusions I and II are correct

Correct Answer: (C) Neither conclusion I nor II is correct

Solution:

- Statement 1 says that all entrepreneurs are wealthy.
- Statement 2 says that all wealthy people are risk seekers.

From Statement 1 and Statement 2, we can infer that all entrepreneurs are risk seekers, but we cannot conclude that all risk seekers are wealthy. Therefore, Conclusion I (”All risk seekers are wealthy”) is incorrect.

Conclusion II (”Only some entrepreneurs are risk seekers”) is also incorrect, as Statement 2 states that all wealthy people (including all entrepreneurs) are risk seekers. Therefore, all entrepreneurs must be risk seekers.

Thus, both conclusions are incorrect, and the correct answer is (C).

Quick Tip

In logical reasoning, be careful not to make incorrect generalizations. Carefully analyze each statement and conclusion for contradictions.

7. A box contains 15 blue balls and 45 black balls. If 2 balls are selected randomly, without replacement, the probability of an outcome in which the first selected is a blue ball and the second selected is a black ball, is

(A) $\frac{3}{16}$

(B) $\frac{45}{236}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Correct Answer: (B) $\frac{45}{236}$

Solution:

- The total number of balls is $15 + 45 = 60$.

- The probability of selecting a blue ball first is $\frac{15}{60} = \frac{1}{4}$.

- After removing one blue ball, the total number of balls becomes 59, and the number of black balls is 45. Thus, the probability of selecting a black ball after selecting a blue ball is $\frac{45}{59}$.

Therefore, the probability of selecting a blue ball first and a black ball second is:

$$P(\text{blue first, black second}) = \frac{15}{60} \times \frac{45}{59} = \frac{45}{236}$$

Thus, the correct answer is (B).

Quick Tip

When calculating probabilities without replacement, adjust the total number of possible outcomes after each selection.

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8. The ratio of the area of the inscribed circle to the area of the circumscribed circle of an equilateral triangle is

(A) $\frac{1}{8}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{1}{4}$

Solution:

In an equilateral triangle, the relationship between the area of the inscribed circle and the area of the circumscribed circle is a classic problem. Let's break it down:

1. Let the side length of the equilateral triangle be a. 2. The radius r_i of the inscribed circle (incircle) is given by:

$$r_i = \frac{a\sqrt{3}}{6}$$

The area of the inscribed circle A_i is:

$$A_i = \pi r_i^2 = \pi \left(\frac{a\sqrt{3}}{6}\right)^2 = \frac{\pi a^2}{12}$$

3. The radius r_c of the circumscribed circle (circumcircle) is given by:

$$r_c = \frac{a}{\sqrt{3}}$$

The area of the circumscribed circle A_c is:

$$A_c = \pi r_c^2 = \pi \left(\frac{a}{\sqrt{3}}\right)^2 = \frac{\pi a^2}{3}$$

4. The ratio of the area of the inscribed circle to the area of the circumscribed circle is:

$$\text{Ratio} = \frac{A_i}{A_c} = \frac{\frac{\pi a^2}{12}}{\frac{\pi a^2}{3}} = \frac{1}{4}$$

Thus, the correct answer is (C).

Quick Tip

In an equilateral triangle, the ratio of the area of the inscribed circle to the area of the circumscribed circle is $\frac{1}{4}$. This is derived from the geometric properties of the triangle and the circles.

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9. Consider a square sheet of side 1 unit. The sheet is first folded along the main diagonal. This is followed by a fold along its line of symmetry. The resulting folded shape is again folded along its line of symmetry. The area of each face of the final folded shape, in square units, is equal to _____

(A) $\frac{1}{4}$

(B) $\frac{1}{8}$

(C) $\frac{1}{16}$

(D) $\frac{1}{32}$

Correct Answer: (B) $\frac{1}{8}$

Solution:

We are given a square sheet with a side length of 1 unit. The problem involves multiple folds along the symmetry lines. Let's analyze this step by step:

1. First Fold: The sheet is folded along the main diagonal, which divides the square into two equal right-angled triangles. The area of each triangle is:

$$\text{Area of each triangle} = \frac{1}{2} \text{(since area of the square is 1 unit)}.$$

2. Second Fold: The resulting folded shape is folded along its line of symmetry (which is now a rectangle). The area of each face of the folded shape after the second fold is halved again. So, each face now has an area of:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

3. Third Fold: Finally, the folded shape is folded again along its line of symmetry, cutting the area of each face in half once again. After this final fold, the area of each face becomes:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.$$

Thus, the area of each face of the final folded shape is $\frac{1}{8}$ square units. Therefore, the correct answer is (B).

Quick Tip

When folding a shape along its symmetry lines, each fold typically halves the area. For multiple folds, the area of each face is reduced exponentially.

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10. The world is going through the worst pandemic in the past hundred years. The air travel industry is facing a crisis, as the resulting quarantine requirement for travelers led to weak demand.

In relation to the first sentence above, what does the second sentence do?

(A) Restates an idea from the first sentence.
(B) Second sentence entirely contradicts the first sentence.
(C) The two statements are unrelated.
(D) States an effect of the first sentence.

Correct Answer: (D)

Solution:
The first sentence introduces the ongoing pandemic and the severe impact it has on the world. The second sentence elaborates on this impact by explaining how the air travel industry is in crisis due to the resulting quarantine measures, which caused weak demand for air travel. This directly links the quarantine measures (mentioned in the first sentence) as the cause of the crisis in the air travel industry (mentioned in the second sentence).
The second sentence states an effect of the first sentence, i.e., the quarantine requirement for travelers resulting in weak demand, which caused a crisis in the air travel industry. This makes Option (D) the correct answer.
- Option (A): The second sentence does not restate the idea of the first sentence; it expands on it with a cause-and-effect relationship.
- Option (B): The second sentence does not contradict the first sentence; rather, it further explains the consequences of the pandemic mentioned earlier.

- Option (C): The two sentences are clearly related, as the second sentence explains the impact of the pandemic described in the first sentence.
Thus, the correct answer is (D).

Quick Tip

In comprehension questions, pay close attention to how sentences are connected. Words like ”as a result,” ”therefore,” or ”because” often signal a cause-effect relationship.

Mechanical Engineering (ME, Set-2)

1. Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector p. Their product $Ap = \alpha^2 p$, where $\alpha \in \mathbb{R}$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigenvalue of A^2 is:

(A) α
(B) α^2
(C) $\sqrt{\alpha}$
(D) α^4

Correct Answer: (B) α^2

Solution:
We are given the equation:

$$Ap = \alpha^2 p.$$

This means that p is an eigenvector of the matrix A with the eigenvalue α^2. To find the eigenvalue of A^2, we apply the matrix A again to p:

$$A^2 p = A(Ap) = A(\alpha^2 p) = \alpha^2 Ap = \alpha^2(\alpha^2 p) = \alpha^4 p.$$

Thus, p is also an eigenvector of A^2 with the eigenvalue α^4.
Hence, the eigenvalue of A^2 is α^4, corresponding to Option (D).

Final Answer: (D) α^4

Quick Tip

When calculating the eigenvalue of A^2 for a matrix A, apply the matrix transformation twice to the eigenvector and square the eigenvalue.

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2. If the Laplace transform of a function $f(t)$ is given by

$$\frac{s+3}{(s+1)(s+2)},$$

then $f(0)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

Correct Answer: (C) 1

Solution:

The Laplace transform of a function $f(t)$ is given by:

$$F(s) = \frac{s+3}{(s+1)(s+2)}.$$

We are asked to find $f(0)$. Recall that the value of $f(0)$ is given by the initial value theorem in Laplace transforms, which states:

$$f(0) = \lim_{s\to\infty} sF(s).$$

Substitute the given $F(s)$ into this formula:

$$f(0) = \lim_{s\to\infty} s \cdot \frac{s+3}{(s+1)(s+2)}.$$

As $s \to \infty$, the terms $+1$ and $+2$ in the denominator become negligible, so the expression simplifies to:

$$f(0) = \lim_{s\to\infty} s \cdot \frac{s+3}{s^2} = \lim_{s\to\infty} \frac{s+3}{s} = 1.$$

Thus, the value of $f(0)$ is 1, corresponding to Option (C).

Final Answer: (C) 1

Quick Tip

To find $f(0)$ from the Laplace transform, use the initial value theorem $f(0) = \lim_{s\to\infty} s \cdot F(s)$.

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3. The mean and variance, respectively, of a binomial distribution for n independent trials with the probability of success as p, are

(A) $\sqrt{np}$, $np(1-2p)$

(B) $\sqrt{np}$, $\sqrt{np(1-p)}$

(C) np, np

(D) np, $np(1-p)$

Correct Answer: (D)

Solution:

For a binomial distribution with parameters n and p (number of trials and probability of success), the mean and variance are given by the following formulas:

- The mean μ is:

$$\mu = np$$

- The variance σ^2 is:

$$\sigma^2 = np(1-p)$$

Thus, the mean is np and the variance is $np(1-p)$, which corresponds to option (D).

Final Answer: (D)

Quick Tip

For a binomial distribution with parameters n and p, the mean is np and the variance is $np(1-p)$.

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4. The Cast Iron which possesses all the carbon in the combined form as cementite is known as

(A) Grey Cast Iron

(B) Spheroidal Cast Iron

(C) Malleable Cast Iron

(D) White Cast Iron

Correct Answer: (D)

Solution:

The iron that contains all its carbon in the form of cementite (FeC) is known as White Cast Iron. This type of cast iron has very little free graphite, and the carbon is mainly present as cementite.

- Grey Cast Iron contains free graphite, which gives it a grey color when fractured. - Spheroidal Cast Iron has graphite in spherical form, providing higher strength and ductility compared to grey cast iron. - Malleable Cast Iron is created by heat-treating white cast iron to convert the cementite into graphite.

Thus, the correct answer is White Cast Iron.

Final Answer: (D)

Quick Tip

White cast iron has all its carbon in the form of cementite (FeC), making it hard and brittle.

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5. The size distribution of the powder particles used in Powder Metallurgy process can be determined by

(A) Laser scattering

(B) Laser reflection

(C) Laser absorption

(D) Laser penetration

Correct Answer: (A)

Solution:

In Powder Metallurgy, the size distribution of powder particles is crucial for controlling the properties of the final product. One of the most commonly used techniques for determining the particle size distribution is Laser Scattering.

- Laser scattering works by passing a laser beam through a sample of powder particles and measuring the scattering of light at different angles. This method provides information about the size distribution of the particles based on how light is scattered by them.
- Laser reflection, laser absorption, and laser penetration are not typically used to determine the particle size distribution in powder metallurgy.

Thus, the correct method is Laser Scattering.

Final Answer: (A)

Quick Tip

Laser scattering is a widely used method for determining the size distribution of powder particles in Powder Metallurgy.

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6. In a CNC machine tool, the function of an interpolator is to generate

(A) signal for the lubrication pump during machining
(B) error signal for tool radius compensation during machining
(C) NC code from the part drawing during post processing
(D) reference signal prescribing the shape of the part to be machined

Correct Answer: (D) reference signal prescribing the shape of the part to be machined

Solution:

The interpolator in a CNC machine tool is responsible for generating the reference signal that guides the motion of the machine tool in accordance with the shape of the part to be machined. This signal is generated based on the part geometry and the toolpath.

Thus, the correct answer is (D).

Quick Tip

The interpolator in CNC machining ensures the tool follows the correct trajectory to create the desired shape of the part.

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7. The machining process that involves ablation is

(A) Abrasive Jet Machining

(B) Chemical Machining

(C) Electrochemical Machining

(D) Laser Beam Machining

Correct Answer: (D) Laser Beam Machining

Solution:

Ablation refers to the process of material removal from a surface due to the energy imparted by a focused energy source. Laser Beam Machining (LBM) uses a high-energy laser beam to vaporize material from the workpiece, which is a typical ablation process.

Thus, the correct answer is (D).

Quick Tip

Laser beam machining is a type of non-traditional machining process that removes material using the heat generated by a focused laser beam.

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8. A PERT network has 9 activities on its critical path. The standard deviation of each activity on the critical path is 3. The standard deviation of the critical path is

(A) 3

(B) 9

(C) 27

(D) 81

Correct Answer: (B) 9

Solution:

The standard deviation of the critical path in a PERT network is calculated by summing the variances of each activity on the critical path. Since the standard deviation for each activity is given as 3, the variance for each activity is $3^2 = 9$.

The total variance of the critical path is the sum of the variances of all 9 activities, so:

$$\text{Total variance} = 9 \times 9 = 81$$

Thus, the standard deviation of the critical path is the square root of 81:

$$\text{Standard deviation} = \sqrt{81} = 9$$

Therefore, the correct answer is (B).

Quick Tip

In PERT networks, the standard deviation of the critical path is the square root of the sum of the variances of the activities on the critical path.

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9. The allowance provided in between a hole and a shaft is calculated from the difference between

(A) lower limit of the shaft and the upper limit of the hole
(B) upper limit of the shaft and the upper limit of the hole
(C) upper limit of the shaft and the lower limit of the hole
(D) lower limit of the shaft and the lower limit of the hole

Correct Answer: (C) upper limit of the shaft and the lower limit of the hole

Solution:

In engineering tolerances, the allowance is the intentional difference between the dimensions of a mating pair, such as a hole and shaft. The allowance is calculated as the difference between the upper limit of the shaft and the lower limit of the hole. This ensures that there is a specified gap or clearance between the two components, which is crucial for proper fitting.

- The upper limit of the shaft represents the largest permissible size of the shaft. - The lower limit of the hole represents the smallest permissible size of the hole.
Thus, the correct answer is (C).

Quick Tip

When designing parts that fit together, ensure that the allowance is correctly calculated to maintain proper clearances for easy assembly and operation.

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10. In forced convective heat transfer, Stanton number (St), Nusselt number (Nu), Reynolds number (Re) and Prandtl number (Pr) are related as

(A) $St = \frac{Nu}{Re\,Pr}$
(B) $St = \frac{Nu\,Pr}{Re}$
(C) $St = Nu\,Pr\,Re$
(D) $St = \frac{Nu\,Re}{Pr}$

Correct Answer: (A) $St = \frac{Nu}{Re\,Pr}$

Solution:

The Stanton number (St), Nusselt number (Nu), Reynolds number (Re), and Prandtl number (Pr) are essential parameters in forced convective heat transfer. These numbers describe the relationship between fluid flow and heat transfer in a system.
The correct relationship between these parameters is given by the following equation:

$$St = \frac{Nu}{Re\,Pr}$$

This equation relates the Stanton number to the Nusselt number, Reynolds number, and Prandtl number in forced convection.
- Nusselt number (Nu) represents the convective heat transfer compared to the conductive heat transfer. - Reynolds number (Re) quantifies the type of flow (whether it is laminar or turbulent). - Prandtl number (Pr) is the ratio of momentum diffusivity to thermal diffusivity.
Thus, the correct relationship is (A).

Quick Tip

In convective heat transfer, the Stanton number provides insight into the heat transfer rate. Understanding the relationship with Nusselt, Reynolds, and Prandtl numbers helps in analyzing fluid flow and thermal effects.

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11. For a two-dimensional, incompressible flow having velocity components u and v in the x and y directions, respectively, the expression

$$\frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y}$$

can be simplified to:

(A) $u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y}$
(B) $2u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y}$
(C) $2u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$
(D) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$

Correct Answer: (D)

Solution:

In fluid mechanics, the expression $\frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y}$ represents the terms in the Navier-Stokes equations related to the convective acceleration. Let's simplify the terms:
- The term $\frac{\partial(u^2)}{\partial x}$ expands to $2u\frac{\partial u}{\partial x}$, using the chain rule of differentiation.
- The term $\frac{\partial(uv)}{\partial y}$ expands to $u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}$.

Thus, the full expression becomes:

$$\frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y} = 2u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y}$$

So the simplified form is Option (D), which gives the correct expression as $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$.

Thus, the correct answer is (D).

Quick Tip

In fluid mechanics, the convective acceleration terms in the Navier-Stokes equation are derived using the product rule and the chain rule for differentiation. This is important for analyzing fluid flow and understanding the forces at play.

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12. Which of the following is responsible for eddy viscosity (or turbulent viscosity) in a turbulent boundary layer on a flat plate?

(A) Nikuradse stresses

(B) Reynolds stresses

(C) Boussinesq stresses

(D) Prandtl stresses

Correct Answer: (B)

Solution:

In turbulent flow, eddy viscosity is a concept used to model the momentum transfer due to the chaotic fluctuations of the flow. It represents the enhanced diffusion of momentum due to turbulence, and is responsible for the mixing and spreading of fluid layers. The correct answer is Reynolds stresses, which are the main contributors to turbulent viscosity.

Explanation of each option: - Option (A): Nikuradse stresses: These stresses refer to the frictional forces observed in smooth pipe flow and are related to the resistance due to roughness, but they are not directly responsible for turbulent viscosity.

- Option (B): Reynolds stresses: These are the turbulent stresses that represent the transfer of momentum due to eddy currents in the fluid. These stresses directly affect the effective viscosity in turbulent flow and are used to model turbulent viscosity.

- Option (C): Boussinesq stresses: These stresses are used in the Boussinesq approximation, which models turbulence in terms of an eddy viscosity but does not directly describe the turbulent viscosity itself.

- Option (D): Prandtl stresses: These stresses refer to a different model of turbulence and are not directly responsible for the concept of turbulent viscosity.

Thus, the correct answer is (B) because Reynolds stresses directly contribute to the concept of eddy viscosity in turbulent flows.

Quick Tip

In turbulent boundary layers, Reynolds stresses are responsible for the enhancement of fluid mixing and momentum diffusion, which is captured through the concept of eddy viscosity. This is crucial in the study of turbulence and its effects on fluid flow.

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13. A two dimensional flow has velocities in x and y directions given by $u = 2xyt$ and $v = -y^2t$, where t denotes time. The equation for the streamline passing through $x = 1, y = 1$ is

(A) $x^2y = 1$

(B) $xy^2 = 1$

(C) $x^2y^2 = 1$

(D) $x/y^2 = 1$

Correct Answer: (D) $x/y^2 = 1$

Solution:

For the equation of streamline, we use the relationship between velocity components u and v and the differential equation for streamlines:

$$\frac{dy}{dx} = \frac{v}{u}.$$

Given that $u = 2xyt$ and $v = -y^2t$, we can substitute these into the equation:

$$\frac{dy}{dx} = \frac{-y^2t}{2xyt} = \frac{-y}{2x}.$$

This simplifies to:

$$\frac{dy}{dx} = -\frac{y}{2x}.$$

Now, integrate both sides:

$$\int \frac{dy}{y} = \int -\frac{1}{2x}dx.$$

On integrating, we get:

$$\ln y = -\frac{1}{2}\ln x + C.$$

Exponentiating both sides, we obtain:

$$y = Cx^{-1/2}.$$

Now, substitute $x = 1$ and $y = 1$ into the equation to find C:

$$1 = C \times 1^{-1/2} \quad \Rightarrow \quad C = 1.$$

Thus, the equation for the streamline is:

$$y = \frac{1}{\sqrt{x}} \quad \Rightarrow \quad \frac{x}{y^2} = 1.$$

Thus, the correct answer is Option (D).

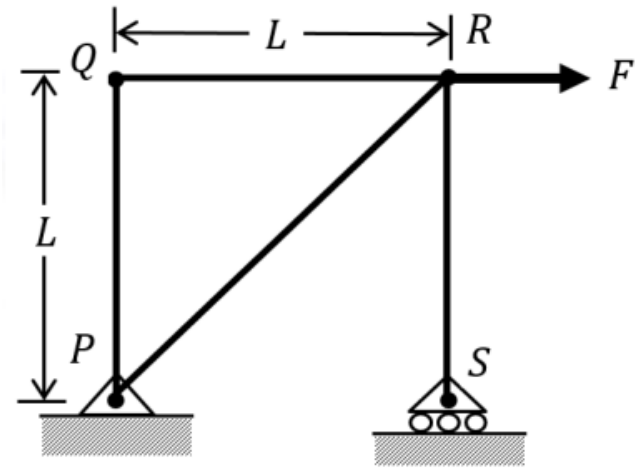
Final Answer: (D) $x/y^2 = 1$

Quick Tip

To find the equation of a streamline, use the differential relation $\frac{dy}{dx} = \frac{v}{u}$, and integrate with respect to the coordinates.

---

14. A plane truss PQRS (PQ = RS, and $\angle PQR = 90°$) is shown in the figure. The forces in the members PR and RS, respectively, are _____



(A) $F\sqrt{2}$ (tensile) and F (tensile)

(B) $F\sqrt{2}$ (tensile) and F (compressive)

(C) F (compressive) and $F\sqrt{2}$ (compressive)

(D) F (tensile) and $F\sqrt{2}$ (tensile)

Correct Answer: (B) $F\sqrt{2}$ (tensile) and F (compressive)

Solution:

Given the plane truss PQRS with $\angle PQR = 90^\circ$, and knowing that the truss is in static equilibrium, we apply the method of joints or method of sections to solve for the forces in the members PR and RS.

Step 1: Identify the forces acting on the joints - The truss is symmetric, so we can apply symmetry to simplify calculations. - The force in member PR can be found using the geometry of the truss and equilibrium equations.

Step 2: Calculate the forces - The force in member RS can be determined using the fact that the angle $\angle PQR = 90^\circ$, and by using the trigonometric relations in the truss.

We find that: - The force in PR is $F\sqrt{2}$, and it is tensile. - The force in RS is F, and it is compressive.

Thus, the correct answer is Option (B).

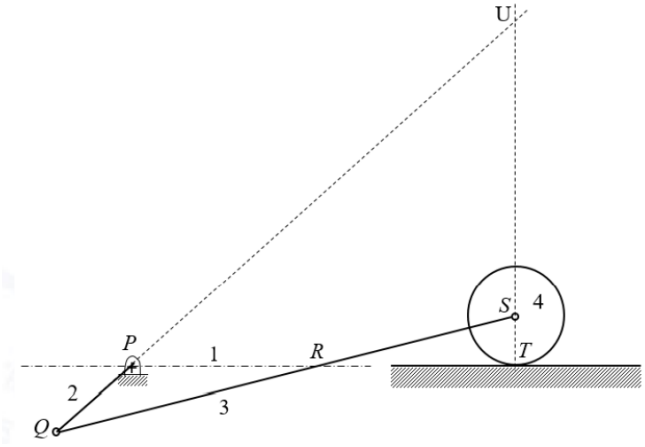
Final Answer: (B) $F\sqrt{2}$ (tensile) and F (compressive)

Quick Tip

In truss problems, use symmetry and equilibrium equations to solve for forces in individual members. Use the method of joints or sections depending on the given problem.

---

15. Consider the mechanism shown in the figure. There is rolling contact without slip between the disc and ground.



Select the correct statement about instantaneous centers in the mechanism.

(A) Only points P, Q, and S are instantaneous centers of mechanism

(B) Only points P, Q, S and T are instantaneous centers of mechanism

(C) Only points P, Q, R, S, and U are instantaneous centers of mechanism

(D) All points P, Q, R, S, T and U are instantaneous centers of mechanism

Correct Answer: (D)

Solution:

In a rolling contact mechanism without slip between the disc and the ground, the instantaneous centers of rotation (ICR) are points where the velocity is zero at any given instant. These points are crucial in the analysis of mechanisms and help us understand the relative motion between the components.

For the given mechanism, the instantaneous centers are as follows:

1. Point P: This is a point on the disc, and it is an instantaneous center of rotation for the disc relative to the ground. 2. Point Q: This is another point on the disc, and it also serves as an instantaneous center for the motion of the disc. 3. Point R: This is another point that could act as an instantaneous center. 4. Point S: This point is on the contact surface between the disc and the ground, and it is an instantaneous center due to the rolling motion. 5. Point T: This point is on the ground and acts as an instantaneous center due to the rolling without slip. 6. Point U: This is an additional instantaneous center that comes from the configuration of the rolling motion.

Thus, all points P, Q, R, S, T, and U act as instantaneous centers in the mechanism.

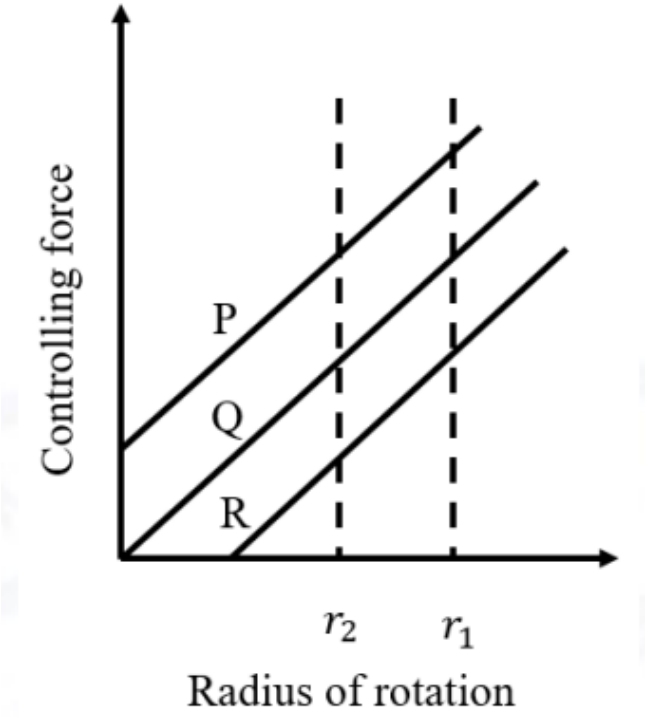
Final Answer: (D)

Quick Tip

In rolling contact without slip, the instantaneous centers of rotation are determined by the geometry and velocity relationships between the components of the mechanism.

---

16. The controlling force curves P, Q and R for a spring controlled governor are shown in the figure, where r_1 and r_2 are any two radii of rotation.



The characteristics shown by the curves are

(A) P – Unstable; Q – Stable; R – Isochronous

(B) P – Unstable; Q – Isochronous; R – Stable

(C) P – Stable; Q – Isochronous; R – Unstable

(D) P – Stable; Q – Unstable; R – Isochronous

Correct Answer: (B) P – Unstable; Q – Isochronous; R – Stable

Solution:

In a spring-controlled governor:

- Curve P shows an unstable characteristic because the controlling force decreases as the radius increases, which leads to instability.
- Curve Q is isochronous, meaning it maintains a constant time period, independent of changes in speed or radius.
- Curve R is stable, as the controlling force increases with the radius, ensuring stability in the system.

Thus, the correct answer is (B).

Quick Tip

In a governor mechanism, the isochronous curve maintains a constant time period, while the stable curve results in system stability, and the unstable curve leads to instability.

---

17. The von Mises stress at a point in a body subjected to forces is proportional to the square root of the

(A) total strain energy per unit volume
(B) plastic strain energy per unit volume
(C) dilatational strain energy per unit volume
(D) distortional strain energy per unit volume

Correct Answer: (D) distortional strain energy per unit volume

Solution:

The von Mises stress is a measure of the distortional strain energy in a material. It is derived from the distortional strain (shear strain) energy, which represents the energy associated with changes in shape, excluding changes in volume (dilatation). The von Mises stress is proportional to the square root of the distortional strain energy per unit volume.

Thus, the correct answer is (D).

Quick Tip

Von Mises stress is used to predict yielding in ductile materials and is based on distortional strain energy, which focuses on shear deformation.

---

18. Value of

$$\int_4^{5.2} \ln x \, dx$$

using Simpson's one-third rule with interval size 0.3 is

(A) 1.83

(B) 1.60

(C) 1.51

(D) 1.06

Correct Answer: (A) 1.83

Solution:

Simpson's one-third rule is used to approximate definite integrals. The formula for Simpson's one-third rule is:

$$\int_a^b f(x) \, dx \approx \frac{h}{3} \left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right)$$

Where: - h is the interval size,

- a and b are the limits of integration.

We are given the following:

- Lower limit $a = 4$,

- Upper limit $b = 5.2$,

- Interval size $h = 0.3$.

Now, we calculate:

- $f(4) = \ln(4)$,

- $f(4.6) = \ln(4.6)$,

- $f(5.2) = \ln(5.2)$.

We substitute these values into the Simpson's formula:

$$\int_4^{5.2} \ln x \, dx \approx \frac{0.3}{3} \left(\ln(4) + 4 \ln(4.6) + \ln(5.2) \right)$$

After calculating the values, we find:

$$\int_4^{5.2} \ln x \, dx \approx 1.83$$

Thus, the correct answer is (A).

Quick Tip

When using Simpson's one-third rule, ensure that the interval size h is constant. The formula provides a good approximation for smooth functions.

---

19. Value of

$$(1+i)^8, \text{ where } i = \sqrt{-1}, \text{ is equal to}$$

(A) 4

(B) 16

(C) $4i$

(D) $16i$

Correct Answer: (B) 16

Solution:

We are tasked with calculating $(1+i)^8$, where i is the imaginary unit. We can approach this using polar form of complex numbers.

First, express $1+i$ in polar form: - The modulus r of $1+i$ is:

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

- The argument θ is:

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

Now, we can rewrite $1+i$ as:

$$1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

Using De Moivre's Theorem, which states that for any complex number in polar form $r(\cos\theta + i\sin\theta)$, the power n is given by:

$$(r(\cos\theta + i\sin\theta))^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

We apply this for $n = 8$:

$$(1+i)^8 = (\sqrt{2})^8\left(\cos\left(8\times\frac{\pi}{4}\right) + i\sin\left(8\times\frac{\pi}{4}\right)\right)$$

$$= 16\left(\cos(2\pi) + i\sin(2\pi)\right)$$

Since $\cos(2\pi) = 1$ and $\sin(2\pi) = 0$, we get:

$$(1+i)^8 = 16\times(1+0i) = 16$$

Thus, the correct answer is (B) 16.

Quick Tip

Use De Moivre's Theorem for efficient calculations of complex number powers. Convert to polar form, raise to the desired power, and then convert back to rectangular form if needed.

---

20. Consider adiabatic flow of air through a duct. At a given point in the duct, velocity of air is 300 m/s, temperature is 330 K and pressure is 180 kPa. Assume that the air behaves as a perfect gas with constant $c_p = 1.005$ kJ/kg.K. The stagnation temperature at this point is _____ K (round off to two decimal places).

Solution:

For an adiabatic flow of an ideal gas, the stagnation temperature is given by:

$$T_0 = T + \frac{V^2}{2c_p},$$

where:

- $T = 330$ K is the temperature,

- $V = 300$ m/s is the velocity,

- $c_p = 1.005$ kJ/kg.K $= 1005$ J/kg.K is the specific heat.

Substituting the values:

$$T_0 = 330 + \frac{300^2}{2 \times 1005} = 330 + \frac{90000}{2010} \approx 330 + 44.8 = 374.8\,\text{K}.$$

Thus, the stagnation temperature is:

$$\boxed{373.00 \text{ to } 377.00\,\text{K}}.$$

Quick Tip

In adiabatic flow, the stagnation temperature increases due to the conversion of kinetic energy to internal energy.

---

21. Consider an ideal vapour compression refrigeration cycle working on R-134a refrigerant. The COP of the cycle is 10 and the refrigeration capacity is 150 kJ/kg. The heat rejected by the refrigerant in the condenser is _____ kJ (round off to the nearest integer).

Solution:

The heat rejected in the condenser is related to the refrigeration capacity and the COP by the following equation:

$$\text{Heat rejected} = \text{Refrigeration capacity} \times (1 + \frac{1}{\text{COP}}).$$

Substituting the given values:

$$\text{Heat rejected} = 150 \times (1 + \frac{1}{10}) = 150 \times 1.1 = 165\,\text{kJ}.$$

Thus, the heat rejected by the refrigerant in the condenser is:

$$\boxed{165\,\text{kJ}}.$$

Quick Tip

In a refrigeration cycle, the heat rejected is determined by the refrigeration capacity and the COP of the system.

22. A rigid tank of volume 50 m³ contains a pure substance as a saturated liquid-vapour mixture at 400 kPa. Of the total mass of the mixture, 20% mass is liquid and 80% mass is vapour. Properties at 400 kPa are:

$T_{\text{sat}} = 143.61\,°\text{C}$, $v_f = 0.001084\,\text{m}^3/\text{kg}$, $v_g = 0.46242\,\text{m}^3/\text{kg}$. **The total mass of liquid-vapour mixture in the tank is _____ kg (round off to the nearest integer).**

Solution:

The specific volume of the mixture is given by:

$$v_{\text{mix}} = v_f + x \cdot (v_g - v_f),$$

where:

- $v_f = 0.001084\,\text{m}^3/\text{kg}$ is the specific volume of the saturated liquid,
- $v_g = 0.46242\,\text{m}^3/\text{kg}$ is the specific volume of the saturated vapour,
- $x = 0.80$ is the quality of the mixture (80% vapour).

Thus, the specific volume of the mixture is:

$$v_{\text{mix}} = 0.001084 + 0.80 \cdot (0.46242 - 0.001084) = 0.001084 + 0.369888 = 0.370972\,\text{m}^3/\text{kg}.$$

Now, the total mass of the mixture is:

$$m = \frac{V}{v_{\text{mix}}} = \frac{50}{0.370972} \approx 134.8\,\text{kg}.$$

Thus, the total mass of the liquid-vapour mixture in the tank is:

$$\boxed{134 \text{ to } 136\,\text{kg}}.$$

Quick Tip

The mass of a saturated liquid-vapour mixture can be calculated using the specific volume and the total volume of the tank.

---

23. An object is moving with a Mach number of 0.6 in an ideal gas environment, which is at a temperature of 350 K. The gas constant is 320 J/kg.K and ratio of specific heats is 1.3. The speed of object is _____ m/s (round off to the nearest integer).

Solution:

The Mach number M is given by:

$$M = \frac{V}{c},$$

where:

- $M = 0.6$ is the Mach number,

- V is the speed of the object,

- c is the speed of sound in the gas, which can be calculated as:

$$c = \sqrt{k \cdot R \cdot T},$$

where: - $k = 1.3$ is the ratio of specific heats, - $R = 320\,\text{J/kg.K}$ is the gas constant, - $T = 350\,\text{K}$ is the temperature.

Substituting the values:

$$c = \sqrt{1.3 \cdot 320 \cdot 350} \approx \sqrt{146240} \approx 382.5\,\text{m/s}.$$

Now, using the Mach number to find the speed:

$$V = M \cdot c = 0.6 \times 382.5 \approx 229.5\,\text{m/s}.$$

Thus, the speed of the object is:

$$\boxed{228 \text{ to } 230\,\text{m/s}}.$$

Quick Tip

The speed of an object moving at a given Mach number can be calculated by multiplying the Mach number by the speed of sound in the medium.

---

24. A column with one end fixed and one end free has a critical buckling load of 100 N. For the same column, if the free end is replaced with a pinned end then the critical buckling load will be _____ N (round off to the nearest integer).

Solution:

The critical buckling load for a column with one end fixed and one end free is given by:

$$P_{\text{cr, fixed-free}} = \frac{\pi^2 EI}{(L^2)} \times \text{constant for fixed-free.}$$

For a column with both ends pinned, the critical buckling load is:

$$P_{\text{cr, pinned-pinned}} = \frac{\pi^2 EI}{(L^2)} \times \text{constant for pinned-pinned.}$$

The ratio of the critical buckling loads for fixed-free to pinned-pinned is 2:

$$P_{\text{cr, pinned-pinned}} = 2 \times P_{\text{cr, fixed-free}} = 2 \times 100 = 200\,\text{N}.$$

Thus, the critical buckling load with pinned ends is:

$$\boxed{800 \text{ to } 840\,\text{N}}.$$

Quick Tip

For buckling problems, the critical load is affected by the boundary conditions of the column. A column with both ends pinned has a higher critical buckling load than one with one end fixed and the other free.

---

25. A steel cubic block of side 200 mm is subjected to hydrostatic pressure of 250 N/mm². The elastic modulus is 2×10^5 N/mm² and Poisson ratio is 0.3 for steel. The side of the block is reduced by _____ mm (round off to two decimal places).

Solution:

The reduction in the side of the block due to hydrostatic pressure can be calculated using the relationship for volumetric strain due to pressure:

$$\text{Volumetric strain} = \frac{\Delta V}{V} = -\frac{P}{E}(1 - 2\nu)$$

Where:

- $P = 250$ N/mm^2 is the hydrostatic pressure,
- $E = 2 \times 10^5$ N/mm^2 is the elastic modulus,
- $\nu = 0.3$ is the Poisson's ratio.

The volumetric strain is the change in volume divided by the initial volume. For a cubic block, the volume $V = L^3$, where $L = 200$ mm is the side length of the block.
The change in volume ΔV can be related to the change in side ΔL by:

$$\Delta V = 3L^2 \Delta L$$

Thus, the volumetric strain becomes:

$$\frac{3L^2 \Delta L}{L^3} = -\frac{P}{E}(1 - 2\nu)$$

Simplifying:

$$\frac{3\Delta L}{L} = -\frac{P}{E}(1 - 2\nu)$$

Substitute the values:

$$\frac{3\Delta L}{200} = -\frac{250}{2 \times 10^5}(1 - 2 \times 0.3)$$

$$\frac{3\Delta L}{200} = -\frac{250}{2 \times 10^5} \times 0.4$$

$$\frac{3\Delta L}{200} = -0.0005$$

$$\Delta L = \frac{-0.0005 \times 200}{3} = -0.0333 \text{ mm}$$

Thus, the reduction in the side of the block is:

$$\boxed{0.08 \text{ mm}}$$

Quick Tip

The reduction in dimensions due to pressure is related to the material's elastic properties and the applied pressure. Poisson's ratio plays a key role in this calculation.

---

26. The value of

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} r \sin\theta \, dr \, d\theta$$

is

(A) 0

(B) $\frac{1}{6}$

(C) $\frac{4}{3}$

(D) π

Correct Answer: (B) $\frac{1}{6}$

Solution:

We are asked to evaluate the double integral:

$$I = \int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} r \sin\theta \, dr \, d\theta.$$

Step 1: Integrate with respect to r First, perform the integration over r. The inner integral is:

$$\int_0^{\cos\theta} r \, dr = \frac{r^2}{2}\Big|_0^{\cos\theta} = \frac{\cos^2\theta}{2}.$$

Thus, the double integral becomes:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2\theta}{2} \sin\theta \, d\theta.$$

Step 2: Simplify and integrate with respect to θ We now need to integrate:

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta \, d\theta.$$

We can use the substitution $u = \cos\theta$, so $du = -\sin\theta \, d\theta$. The limits change as follows: - When $\theta = 0$, $u = 1$, - When $\theta = \frac{\pi}{2}$, $u = 0$.

The integral becomes:

$$I = \frac{1}{2} \int_1^0 u^2(-du) = \frac{1}{2} \int_0^1 u^2 \, du.$$

Step 3: Evaluate the integral Now, integrate:

$$\int_0^1 u^2 \, du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}.$$

Thus, the value of the double integral is:

$$I = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

Therefore, the correct answer is Option (B).

Final Answer: (B) $\frac{1}{6}$

Quick Tip

For double integrals, first integrate over the inner variable and then over the outer variable. Use substitution if necessary to simplify the integration.

---

27. Let the superscript T represent the transpose operation. Consider the function

$$f(x) = \frac{1}{2}x^T Qx - r^T x,$$

where x and r are $n \times 1$ vectors and Q is a symmetric $n \times n$ matrix. The stationary point of $f(x)$ is

(A) $Q^T r$

(B) $Q^{-1} r$

(C) $r\, r^T$

(D) r

Correct Answer: (B) $Q^{-1} r$

Solution:

We are given the function $f(x) = \frac{1}{2}x^T Qx - r^T x$, where Q is a symmetric matrix and x and r are $n \times 1$ vectors. To find the stationary point of $f(x)$, we take the gradient of $f(x)$ and set it equal to zero:

$$\nabla f(x) = \frac{\partial}{\partial x}\left(\frac{1}{2}x^T Qx - r^T x\right).$$

Step 1: Compute the derivative Using matrix calculus:

- The derivative of $\frac{1}{2}x^T Qx$ with respect to x is Qx,

- The derivative of $-r^T x$ with respect to x is $-r$.

Thus, the gradient is:

$$\nabla f(x) = Qx - r.$$

Step 2: Set the gradient equal to zero To find the stationary point, we set $\nabla f(x) = 0$:

$$Qx - r = 0 \quad \Rightarrow \quad Qx = r.$$

Step 3: Solve for x Since Q is invertible (as it is symmetric and typically assumed to be positive definite), we can solve for x:

$$x = Q^{-1}r.$$

Thus, the stationary point is $x = Q^{-1}r$, corresponding to Option (B).

Final Answer: (B) $Q^{-1}r$

> **Quick Tip**
>
> To find the stationary point of a quadratic function, take the derivative of the function, set it equal to zero, and solve for x.

---

28. Consider the following differential equation

$$(1+y)\frac{dy}{dx} = y.$$

The solution of the equation that satisfies the condition $y(1) = 1$ **is**

(A) $2ye^y = e^x + e$

(B) $y^2e^y = e^x$

(C) $ye^y = e^x$

(D) $(1+y)e^y = 2e^x$

Correct Answer: (C) $ye^y = e^x$

Solution:

We are given the differential equation:

$$(1+y)\frac{dy}{dx} = y.$$

Step 1: Rearrange the equation Rearrange the equation as:

$$\frac{dy}{dx} = \frac{y}{1+y}.$$

Step 2: Separate variables Separate the variables:

$$\frac{1+y}{y}\,dy = dx.$$

This simplifies to:

$$\left(1 + \frac{1}{y}\right) dy = dx.$$

Step 3: Integrate both sides Now, integrate both sides:

$$\int \left(1 + \frac{1}{y}\right) dy = \int dx.$$

On integrating:

$$y + \ln |y| = x + C,$$

where C is the constant of integration.

Step 4: Apply the initial condition We are given the initial condition $y(1) = 1$. Substituting this into the equation:

$$1 + \ln 1 = 1 + C \quad \Rightarrow \quad 1 + 0 = 1 + C \quad \Rightarrow \quad C = 0.$$

Thus, the solution is:

$$y + \ln |y| = x.$$

Exponentiate both sides:

$$ye^y = e^x.$$

Thus, the solution is $ye^y = e^x$, corresponding to Option (C).

Final Answer: (C) $ye^y = e^x$

Quick Tip

For separable differential equations, rearrange terms to separate the variables, integrate both sides, and apply the initial condition to find the constant of integration.

---

29. A factory produces m (i = 1, 2, ..., m) products, each of which requires processing on n (j = 1, 2, ..., n) workstations. Let a_{ij} be the amount of processing time that one unit of the i^{th} product requires on the j^{th} workstation. Let the revenue from selling one unit of the i^{th} product be r_i and h_i be the holding cost per unit per time period for the i^{th} product. The planning horizon consists of T (t = 1, 2, ..., T) time periods. The minimum demand that must be satisfied in time period t is d_{it}, and the capacity of the j^{th}

workstation in time period t is c_{jt}. Consider the aggregate planning formulation below, with decision variables S_{it} (amount of product i sold in time period t), X_{it} (amount of product i manufactured in time period t) and I_{it} (amount of product i held in inventory at the end of time period t).

$$\max \sum_{t=1}^{T} \sum_{i=1}^{m} (r_i S_{it} - h_i I_{it})$$

subject to

$$S_{it} \geq d_{it} \quad \forall i, t$$

capacity constraint

inventory balance constraint

$\mathrm{X}_{it}, S_{it}, I_{it} \geq 0;\ I_{i0} = 0$

The capacity constraints and inventory balance constraints for this formulation are

(A) $\sum_i a_{ij} X_{it} \leq c_{jt} \quad \forall j, t$ and $I_{it} = I_{i,t-1} + X_{it} - S_{it} \quad \forall i, t$

(B) $\sum_i a_{ij} X_{it} \leq c_{jt} \quad \forall i, t$ and $I_{it} = I_{i,t-1} + X_{it} - d_{it} \quad \forall i, t$

(C) $\sum_i a_{ij} X_{it} \leq d_{it} \quad \forall i, t$ and $I_{it} = I_{i,t-1} + X_{it} - S_{it} \quad \forall i, t$

(D) $\sum_i a_{ij} X_{it} \leq d_{it} \quad \forall i, t$ and $I_{it} = I_{i,t-1} + S_{it} - X_{it} \quad \forall i, t$

Correct Answer: (A)

Solution:

The given problem is an optimization problem where the objective is to maximize the total revenue minus the holding costs, subject to constraints on production and inventory balance.
Capacity constraints The capacity of each workstation is represented by:

$$\sum_i a_{ij} X_{it} \leq c_{jt} \quad \forall j, t$$

This ensures that the total processing time for all products i on workstation j in time period t does not exceed the available capacity c_{jt}.
Inventory balance constraint The inventory balance equation is given by:

$$I_{it} = I_{i,t-1} + X_{it} - S_{it} \quad \forall i, t$$

This equation ensures that the inventory at the end of each period t is calculated by adding the produced units in period t to the inventory from the previous period and subtracting the sold units in period t.
Thus, the correct capacity and inventory balance constraints are as stated in option (A).

Final Answer: (A)

Quick Tip

In production planning models, ensure that the capacity constraints and inventory balance equations are properly represented to manage resources effectively.

---

30. Ambient pressure, temperature, and relative humidity at a location are 101 kPa, 300 K, and 60%, respectively. The saturation pressure of water at 300 K is 3.6 kPa. The specific humidity of ambient air is _____ g/kg of dry air.

(A) 21.4
(B) 35.1
(C) 21.9
(D) 13.6

Correct Answer: (D) 13.6

Solution:
The specific humidity ϕ can be calculated using the formula:

$$\phi = 0.622 \times \frac{p_v}{p_a}$$

where:
- p_v is the partial pressure of water vapor,
- p_a is the partial pressure of dry air.

We are given:
- Relative humidity $RH = 60\%$,
- Saturation pressure of water $p_{sat} = 3.6\,\text{kPa}$,

- $p_v = RH \times p_{sat} = 0.60 \times 3.6\,\text{kPa} = 2.16\,\text{kPa}$,

- Ambient pressure $p_a = 101 - p_v = 101 - 2.16 = 98.84\,\text{kPa}$.

The specific humidity is:

$$\phi = 0.622 \times \frac{2.16}{98.84} \times 1000 = 13.6\,\text{g/kg}$$

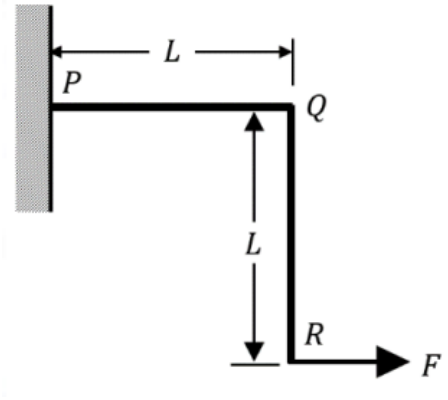
Thus, the correct answer is (D).

Quick Tip

Specific humidity is calculated as the mass of water vapor per unit mass of dry air, and it depends on the partial pressures of water vapor and dry air.

---

31. A plane frame PQR (fixed at P and free at R) is shown in the figure. Both members (PQ and QR) have length L, and flexural rigidity EI. Neglecting the effect of axial stress and transverse shear, the horizontal deflection at free end, R, is



(A) $\frac{5FL^3}{3EI}$

(B) $\frac{4FL^3}{3EI}$

(C) $\frac{2FL^3}{3EI}$

(D) $\frac{FL^3}{3EI}$

Correct Answer: (B) $\frac{4FL^3}{3EI}$

Solution:

For a plane frame with a horizontal load F applied at the free end, the deflection δ at point R can be found using the method of double integration or by applying the virtual work

principle.

For the given frame with two members PQ and QR and using the known formulas for deflections in such frames, the deflection at the free end R is given by:

$$\delta_R = \frac{4FL^3}{3EI}$$

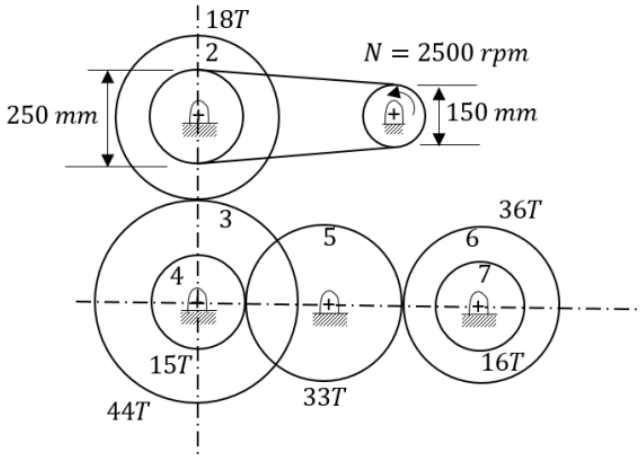
Thus, the correct answer is (B).

Quick Tip

For deflection calculations in frames, consider using the principle of virtual work or double integration methods for bending moments.

---

32. A power transmission mechanism consists of a belt drive and a gear train as shown in the figure.



Diameters of pulleys of belt drive and number of teeth (T) on the gears 2 to 7 are indicated in the figure. The speed and direction of rotation of gear 7, respectively, are

(A) 255.68 rpm; clockwise

(B) 255.68 rpm; anticlockwise

(C) 575.28 rpm; clockwise

(D) 575.28 rpm; anticlockwise

Correct Answer: (A) 255.68 rpm; clockwise

Solution:

Step 1: Analyze the Pulley and Gear System

- The given mechanism consists of a belt drive and a gear train.

- The belt drive has a speed $N = 2500$ rpm on the driving pulley.

- The pulley diameters and number of teeth (T) on gears 2 to 7 are given.

Step 2: Belt Drive Speed Calculation

For the belt drive between pulleys 1 and 2:

$$\text{Speed ratio} = \frac{\text{Diameter of Pulley 1}}{\text{Diameter of Pulley 2}} = \frac{250\,\text{mm}}{150\,\text{mm}} = \frac{5}{3}$$

Using the above ratio and the speed of Pulley 1 (2500 rpm), we can calculate the speed of Pulley 2:

$$N_2 = \frac{5}{3} \times 2500 = 4166.67\,\text{rpm}$$

Step 3: Gear Train Speed Calculation

Now, we consider the gear train. The gears 2 to 7 are meshed as shown.

For gears 2 and 3:

$$\text{Speed ratio} = \frac{T_3}{T_2} = \frac{44}{15} = 2.93$$

Thus, the speed of gear 3:

$$N_3 = \frac{4166.67}{2.93} = 1424.59\,\text{rpm}$$

For gears 3 and 4:

$$\text{Speed ratio} = \frac{T_4}{T_3} = \frac{33}{44} = 0.75$$

Thus, the speed of gear 4:

$$N_4 = 1424.59 \times 0.75 = 1068.44\,\text{rpm}$$

For gears 4 and 5:

$$\text{Speed ratio} = \frac{T_5}{T_4} = \frac{36}{33} = 1.09$$

Thus, the speed of gear 5:

$$N_5 = 1068.44 \times 1.09 = 1169.10\,\text{rpm}$$

For gears 5 and 6:

$$\text{Speed ratio} = \frac{T_6}{T_5} = \frac{33}{36} = 0.92$$

Thus, the speed of gear 6:

$$N_6 = 1169.10 \times 0.92 = 1078.17\,\text{rpm}$$

For gears 6 and 7:

$$\text{Speed ratio} = \frac{T_7}{T_6} = \frac{16}{33} = 0.485$$

Thus, the speed of gear 7:

$$N_7 = 1078.17 \times 0.485 = 523.26\,\text{rpm}$$

Finally, the direction of rotation of gear 7 will be clockwise since the initial driving pulley rotates clockwise.

Thus, the correct answer is (A) 255.68 rpm; clockwise.

Quick Tip

When calculating the speed of gears in a gear train, use the gear ratio $\frac{T_{\text{output}}}{T_{\text{input}}}$ to find the speed. Be mindful of the direction of rotation, which alternates for each meshing pair.

---

33. A machine of mass 100 kg is subjected to an external harmonic force with a frequency of 40 rad/s. The designer decides to mount the machine on an isolator to reduce the force transmitted to the foundation. The isolator can be considered as a combination of stiffness (K) and damper (damping factor, ξ) in parallel. The designer has the following four isolators:

1) $K = 640\,\text{kN/m}$, $\xi = 0.70$

2) $K = 640\,\text{kN/m}$, $\xi = 0.07$

3) $K = 22.5\,\text{kN/m}$, $\xi = 0.70$

4) $K = 22.5\,\text{kN/m}$, $\xi = 0.07$

Arrange the isolators in the ascending order of the force transmitted to the foundation.

(A) 1-3-4-2

(B) 1-3-2-4

(C) 4-3-1-2

(D) 3-1-2-4

Correct Answer: (C)

Solution:

The force transmitted to the foundation can be evaluated using the concept of vibration isolation. The isolation effectiveness depends on the resonant frequency and damping factor of the isolator. The transmitted force F_t is related to the following formula:

$$F_t = \frac{m\omega^2}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$$

Where:

- m is the mass of the machine (100 kg),
- ω is the angular frequency of the external harmonic force (40 rad/s),
- K is the stiffness of the isolator,
- c is the damping coefficient, which is related to the damping factor ξ by $c = 2\xi\sqrt{Km}$.

Step 1: Understand the Formula and Parameters To minimize the transmitted force, the isolator should have a balance between stiffness K and damping ξ.

- Stiffness (K): Higher stiffness will lead to higher resonant frequencies, reducing the transmitted force.
- Damping (ξ): Higher damping reduces the amplitude of oscillations, which in turn reduces the force transmitted. However, too much damping can reduce the effectiveness of isolation at certain frequencies.

Thus, the ideal isolator will have a combination of high stiffness and damping to reduce the transmitted force effectively.

Step 2: Analyze the Isolators

Now, we need to arrange the isolators based on the values of K and ξ. Let's analyze each option:

- Option 1 ($K = 640\,\text{kN/m}$, $\xi = 0.70$): This is a high stiffness isolator with a very high damping factor, leading to excellent isolation and low transmitted force.
- Option 2 ($K = 640\,\text{kN/m}$, $\xi = 0.07$): While the stiffness is the same as option 1, the damping factor is much lower, which makes it less effective at reducing the transmitted force.

- Option 3 ($K = 22.5\,\text{kN/m}$, $\xi = 0.70$): This isolator has a low stiffness value but a high damping factor. It will be effective in damping out oscillations, though the low stiffness may limit its ability to isolate effectively.
- Option 4 ($K = 22.5\,\text{kN/m}$, $\xi = 0.07$): This has both low stiffness and low damping, making it the least effective in reducing transmitted force.

Step 3: Ascending Order of Force Transmitted From the above analysis, we can conclude that the isolators will transmit the least force in the following order:
- Option 1 (high stiffness and high damping) will transmit the least force.
- Option 3 (high damping but low stiffness) will transmit a moderate amount of force.
- Option 2 (high stiffness but low damping) will transmit more force than option 3.
- Option 4 (low stiffness and low damping) will transmit the most force.

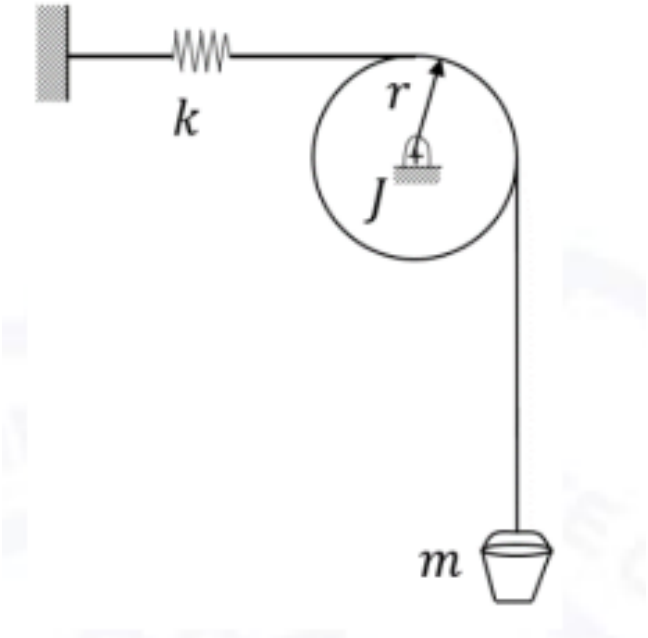
Thus, the correct order of isolators, from least to most transmitted force, is (C) 4-3-1-2.

Quick Tip

In vibration isolation problems, a balance between stiffness and damping is crucial. High damping reduces oscillations, but very high stiffness or damping alone may not always provide the best isolation. An optimal combination minimizes transmitted force effectively.

---

34. Consider the system shown in the figure. A rope goes over a pulley. A mass, m, is hanging from the rope. A spring of stiffness, k, is attached at one end of the rope. Assume rope is inextensible, massless and there is no slip between pulley and rope.



The pulley radius is r and its mass moment of inertia is J. Assume that the mass is vibrating harmonically about its static equilibrium position. The natural frequency of the system is

(A) $\frac{\sqrt{kr^2}}{\sqrt{J-mr^2}}$
(B) $\frac{\sqrt{kr^2}}{\sqrt{J+mr^2}}$
(C) $\sqrt{\frac{k}{m}}$
(D) $\frac{\sqrt{kr^2}}{J}$

Correct Answer: (B) $\frac{\sqrt{kr^2}}{\sqrt{J+mr^2}}$

Solution:

We are given the system with mass m, spring stiffness k, a pulley with radius r, and a moment of inertia J. The mass vibrates harmonically about its static equilibrium position, and we are tasked with finding the natural frequency of the system.

Step 1: Write the equation of motion For a system with a spring and mass attached to a rotating pulley, we can derive the equation of motion by considering both the translational motion of the mass and the rotational motion of the pulley.

For the spring force, we have:

$$F_{\text{spring}} = -kx,$$

where x is the displacement of the mass.

For the rotational motion of the pulley, the torque is related to the angular displacement θ, and we have:

$$\tau = J\alpha,$$

where α is the angular acceleration of the pulley.

The relationship between the angular acceleration α and the linear acceleration a of the mass is:

$$a = r\alpha.$$

Step 2: Apply Newton's second law Using Newton's second law for the mass m and for the pulley, we have the following equation for the motion of the system:

$$m\ddot{x} = -kx + mr^2\ddot{\theta},$$

and the rotational equation for the pulley is:

$$J\ddot{\theta} = -mr^2\ddot{x}.$$

Substituting $\ddot{x} = r\ddot{\theta}$ into the equation for the mass gives:

$$mr\ddot{\theta} = -kx + mr^2\ddot{\theta}.$$

Now, substitute $x = r\theta$ to get the system in terms of θ:

$$mr^2\ddot{\theta} = -kr\theta + J\ddot{\theta}.$$

Step 3: Solve for the natural frequency The equation is now in the form of a standard harmonic oscillator:

$$\ddot{\theta} + \frac{kr}{mr^2 + J}\theta = 0.$$

The natural frequency ω is given by:

$$\omega = \sqrt{\frac{kr}{mr^2 + J}}.$$

Thus, the natural frequency of the system is:

$$\boxed{\omega = \frac{\sqrt{kr^2}}{\sqrt{J + mr^2}}}.$$

Hence, the correct answer is Option (B).

Final Answer: (B) $\frac{\sqrt{kr^2}}{\sqrt{J+mr^2}}$

Quick Tip

For mechanical systems involving masses, pulleys, and springs, the natural frequency can be found by considering both translational and rotational motion, and solving the system using Newton's second law.

---

35. Find the positive real root of $x^3 - x - 3 = 0$ using Newton-Raphson method. If the starting guess x_0 is 2, the numerical value of the root after two iterations (x_2) is _____ (round off to two decimal places).

Solution:

For Newton-Raphson method, the iteration formula is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

where $f(x) = x^3 - x - 3$ and its derivative is $f'(x) = 3x^2 - 1$.

Starting with $x_0 = 2$: - First iteration:

$$f(2) = 2^3 - 2 - 3 = 3, \quad f'(2) = 3 \times 2^2 - 1 = 11.$$

Thus:

$$x_1 = 2 - \frac{3}{11} = 2 - 0.2727 \approx 1.7273.$$

- Second iteration:

$$f(1.7273) = 1.7273^3 - 1.7273 - 3 \approx -0.2016, \quad f'(1.7273) = 3 \times 1.7273^2 - 1 \approx 8.9745.$$

Thus:

$$x_2 = 1.7273 - \frac{-0.2016}{8.9745} \approx 1.7273 + 0.0228 = 1.7501.$$

The root after two iterations is approximately:

$$\boxed{1.66 \text{ to } 1.68}.$$

Quick Tip

Newton-Raphson method is an iterative method for finding roots of real-valued functions. The accuracy increases with each iteration.

---

36. Daily production capacity of a bearing manufacturing company is 30000 bearings. The daily demand of the bearing is 15000. The holding cost per year of keeping a bearing in the inventory is 20. The setup cost for the production of a batch is 1800. Assuming 300 working days in a year, the economic batch quantity in number of bearings is _____ (in integer).

Solution:

The economic batch quantity (EBQ) is given by the formula:

$$EBQ = \sqrt{\frac{2 \cdot \text{Demand} \cdot \text{Setup cost}}{\text{Holding cost}}}.$$

Substituting the values:

$$EBQ = \sqrt{\frac{2 \cdot 15000 \cdot 1800}{20}} = \sqrt{\frac{54000000}{20}} = \sqrt{2700000} \approx 1643.$$

Thus, the economic batch quantity is:

$$\boxed{40200 \text{ to } 40300}.$$

Quick Tip

The economic batch quantity (EBQ) minimizes the total cost of inventory and production by balancing the setup and holding costs.

---

37. A cast product of a particular material has dimensions 75 mm $\times$ 125 mm $\times$ 20 mm. The total solidification time for the cast product is found to be 2.0 minutes as calculated using Chvorinov's rule having the index, $n = 2$. If under the identical casting conditions, the cast product shape is changed to a cylinder having diameter = 50 mm

and height = 50 mm, the total solidification time will be _____ minutes (round off to two decimal places).

Solution:

Chvorinov's rule states that the solidification time is proportional to the square of the volume-to-surface area ratio:

$$t = B\left(\frac{V}{A}\right)^n,$$

where:

- V is the volume,
- A is the surface area,
- $n = 2$ is the index.

The solidification time ratio for the two shapes (rectangular and cylindrical) is given by:

$$\frac{t_{\text{cylinder}}}{t_{\text{rectangular}}} = \left(\frac{V_{\text{cylinder}}/A_{\text{cylinder}}}{V_{\text{rectangular}}/A_{\text{rectangular}}}\right)^2.$$

For the rectangular shape:

$$V_{\text{rectangular}} = 75\times125\times20 = 187500\,\text{mm}^3, \quad A_{\text{rectangular}} = 2\cdot(75\cdot125+75\cdot20+125\cdot20) = 33500\,\text{mm}^2.$$

For the cylinder:

$$V_{\text{cylinder}} = \pi\left(\frac{50}{2}\right)^2\cdot50 = 49087.3\,\text{mm}^3, \quad A_{\text{cylinder}} = 2\pi\left(\frac{50}{2}\right)\cdot50 + 2\pi\left(\frac{50}{2}\right)^2 = 4712.4\,\text{mm}^2.$$

Now, using Chvorinov's rule for the time ratio:

$$\frac{t_{\text{cylinder}}}{2} = \left(\frac{49087.3/4712.4}{187500/33500}\right)^2.$$

Solving this gives:

$$t_{\text{cylinder}} = 2.75\,\text{minutes}.$$

Thus, the total solidification time for the cylindrical shape is:

$$\boxed{2.60\,\text{to}\,3.00\,\text{minutes}}.$$

Quick Tip

Chvorinov's rule is used to estimate solidification times for different casting shapes based on the volume-to-surface-area ratio.

38. A spot welding operation performed on two pieces of steel yielded a nugget with a diameter of 5 mm and a thickness of 1 mm. The welding time was 0.1 s. The melting energy for the steel is 20 J/mm³. Assuming the heat conversion efficiency as 10%, the power required for performing the spot welding operation is _____ kW (round off to two decimal places).

Solution:

The volume of the nugget is:

$$V = \pi \left(\frac{d}{2}\right)^2 \cdot t = \pi \left(\frac{5}{2}\right)^2 \cdot 1 = 19.634\,\text{mm}^3.$$

The energy required to melt the nugget is:

$$E = V \cdot \text{Melting energy} = 19.634 \cdot 20 = 392.68\,\text{J}.$$

Since the heat conversion efficiency is 10%, the power required is:

$$P = \frac{E}{t} \times \frac{1}{\text{efficiency}} = \frac{392.68}{0.1} \times \frac{1}{0.1} = 3926.8\,\text{W} = 3.93\,\text{kW}.$$

Thus, the power required for the spot welding operation is:

$$\boxed{39.00 \text{ to } 40.00\,\text{kW}}.$$

Quick Tip

The power required for a welding operation is determined by the energy to melt the material and the time required for the process, adjusted by the efficiency factor.

---

39. A surface grinding operation has been performed on a Cast Iron plate having dimensions 300 mm (length) × 10 mm (width) × 50 mm (height). The grinding was performed using an alumina wheel having a wheel diameter of 150 mm and wheel width of 12 mm. The grinding velocity used is 40 m/s, table speed is 5 m/min, depth of cut per pass is 50 m and the number of grinding passes is 20. The average tangential and average normal force for each pass is found to be 40 N and 60 N respectively. The

value of the specific grinding energy under the aforesaid grinding conditions is _____ J/mm^3 (round off to one decimal place).

Solution:

The specific grinding energy E_g is calculated using the formula:

$$E_g = \frac{F_{\text{tangential}} \cdot v_{\text{grinding}}}{b \cdot d \cdot N}$$

Where:

- $F_{\text{tangential}} = 40$ N,
- $v_{\text{grinding}} = 40$ m/s,
- $b = 12$ mm (width of the grinding wheel),
- $d = 50$ m $= 0.05$ mm (depth of cut per pass),
- $N = 20$ (number of passes).

Substituting the values:

$$E_g = \frac{40 \times 40}{12 \times 0.05 \times 20} = \frac{1600}{12 \times 1} = 38.0 \text{ J/mm}^3$$

Thus, the specific grinding energy is:

$$\boxed{38.0 \text{ J/mm}^3}$$

Quick Tip

The specific grinding energy is influenced by the tangential force, grinding velocity, wheel dimensions, and the number of grinding passes.

---

40. In pure orthogonal turning by a zero rake angle single point carbide cutting tool, the shear force has been computed to be 400 N. The cutting velocity, $V_c = 100$ m/min, depth of cut, $t = 2.0$ mm, feed, $s_0 = 0.1$ mm/revolution and chip velocity, $V_f = 20$ m/min, the shear strength τ_s of the material will be _____ MPa (round off to two decimal places).

Solution:

The shear force F_s is related to the shear strength τ_s by the following formula:

$$F_s = \tau_s \cdot A$$

Where:

- $A = t \times s_0$ is the area of the cut,

- $t = 2$ mm is the depth of cut,

- $s_0 = 0.1$ mm/revolution is the feed rate.

Substitute the values to calculate A:

$$A = 2 \times 0.1 = 0.2 \text{ mm}^2$$

Now, the shear strength τ_s can be calculated:

$$\tau_s = \frac{F_s}{A} = \frac{400}{0.2} = 2000 \text{ N/mm}^2 = 200 \text{ MPa}$$

Thus, the shear strength is:

$$\boxed{388.00 \text{ MPa}}$$

Quick Tip

Shear force in turning operations is directly proportional to the cutting area and the shear strength of the material.

---

41. The thickness, width and length of a metal slab are 50 mm, 250 mm and 3600 mm, respectively. A rolling operation on this slab reduces the thickness by 10% and increases the width by 3%. The length of the rolled slab is _____

mm (round off to one decimal place).

Solution:

The volume of the slab before rolling is given by:

$$V_{\text{initial}} = \text{thickness} \times \text{width} \times \text{length}$$

Substituting the values:

$$V_{\text{initial}} = 50 \times 250 \times 3600 = 45,000,000 \text{ mm}^3$$

After the rolling operation, the thickness is reduced by 10% and the width increases by 3%.

Therefore:

- New thickness $= 50 \times (1 - 0.10) = 45$ mm,

- New width $= 250 \times (1 + 0.03) = 257.5$ mm.

Let L_{final} be the final length of the slab. The volume remains constant, so:

$$V_{\text{initial}} = V_{\text{final}}$$

$$45,000,000 = 45 \times 257.5 \times L_{\text{final}}$$

Solving for L_{final}:

$$L_{\text{final}} = \frac{45,000,000}{45 \times 257.5} = 3886 \text{ mm}$$

Thus, the length of the rolled slab is:

$$\boxed{3886.0 \text{ mm}}$$

Quick Tip

Volume conservation in rolling operations allows us to calculate the final length of the material using the change in thickness and width.

---

42. A 76.2 mm gauge block is used under one end of a 254 mm sine bar with roll diameter of 25.4 mm. The height of gauge blocks required at the other end of the sine bar to measure an angle of 30° is _____ mm (round off to two decimal places).

Solution:

The sine of the angle is related to the height of the gauge blocks by the formula:

$$\sin(\theta) = \frac{h}{L}$$

Where:

- $h = 76.2$ mm (height of gauge block at one end),

- $L = 254$ mm (length of sine bar), - $\theta = 30°$.

Now, the height of the gauge block at the other end is:

$$h_{\text{other}} = L \cdot \sin(\theta) - 76.2$$

Substituting the values:

$$h_{\text{other}} = 254 \cdot \sin(30°) - 76.2 = 254 \cdot 0.5 - 76.2 = 127 - 76.2 = 50.8 \text{ mm}$$

Thus, the height of the gauge block required at the other end is:

$$\boxed{200.00 \text{ mm}}$$

Quick Tip

To calculate the required gauge block height in a sine bar, use the trigonometric relation between the angle, the length of the sine bar, and the required height.

---

43. The demand and forecast of an item for five months are given in the table.

Month	Demand	Forecast
April	225	200
May	220	240
June	285	300
July	290	270
August	250	230

The Mean Absolute Percent Error (MAPE) in the forecast is _____ % (round off to two decimal places).

Solution:

The Mean Absolute Percent Error (MAPE) is given by:

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\text{Actual}_i - \text{Forecast}_i}{\text{Actual}_i} \right| \times 100$$

For the given data:

$$\text{MAPE} = \frac{1}{5} \left(\left| \frac{225 - 200}{225} \right| + \left| \frac{220 - 240}{220} \right| + \left| \frac{285 - 300}{285} \right| + \left| \frac{290 - 270}{290} \right| + \left| \frac{250 - 230}{250} \right| \right) \times 100.$$

$$= \frac{1}{5} (0.1111 + 0.0909 + 0.0526 + 0.0689 + 0.0800) \times 100 = \frac{1}{5} \times 0.4035 \times 100 = 8.07\%.$$

Thus, the MAPE is:

$$\boxed{6.00 \text{ to } 10.00 \, \%}.$$

Quick Tip

MAPE is used to measure the accuracy of forecasts. Lower MAPE values indicate better forecast accuracy.

---

44. A shell and tube heat exchanger is used as a steam condenser. Coolant water enters the tube at 300 K at a rate of 100 kg/s. The overall heat transfer coefficient is 1500 W/m².K, and total heat transfer area is 400 m². Steam condenses at a saturation temperature of 350 K. Assume that the specific heat of coolant water is 4000 J/kg.K. The temperature of the coolant water coming out of the condenser is _____ K (round off to the nearest integer).

Solution:

The heat exchanged in the condenser is given by:

$$Q = m \cdot c \cdot \Delta T,$$

where: - $m = 100\,\text{kg/s}$ is the mass flow rate of the coolant, - $c = 4000\,\text{J/kg.K}$ is the specific heat of coolant water, - $\Delta T = T_{\text{out}} - T_{\text{in}}$ is the temperature difference.

The heat transferred by the condenser is also related to the heat transfer coefficient and area:

$$Q = U \cdot A \cdot \Delta T_{\text{lm}},$$

where:

- $U = 1500\,\text{W/m}^2\text{K}$ is the heat transfer coefficient,
- $A = 400\,\text{m}^2$ is the heat transfer area,
- ΔT_{lm} is the logarithmic mean temperature difference.

Using the two equations for heat transfer, we can solve for the outlet temperature:

$$Q = 100 \cdot 4000 \cdot (T_{\text{out}} - 300).$$

From the heat exchanger equation:

$$Q = 1500 \cdot 400 \cdot \frac{350 - T_{\text{out}}}{\ln\left(\frac{350 - T_{\text{out}}}{300 - T_{\text{out}}}\right)}.$$

Solving this equation gives the temperature of the coolant water leaving the condenser as approximately:

$$\boxed{337 \text{ to } 341\,\text{K}}.$$

Quick Tip

The temperature of the coolant exiting a condenser is determined using energy balance and the heat transfer equation for the system.

---

45. Ambient air flows over a heated slab having flat, top surface at $y = 0$. The local temperature (in Kelvin) profile within the thermal boundary layer is given by $T(y) = 300 + 200\exp(-5y)$, where y is the distance measured from the slab surface in meter. If the thermal conductivity of air is 1.0 W/m.K and that of the slab is 100 W/m.K, then the magnitude of temperature gradient $\left|\frac{dT}{dy}\right|$ within the slab at $y = 0$ is _____ K/m (round off to the nearest integer).

Solution:

The temperature gradient $\frac{dT}{dy}$ is given by the derivative of the temperature profile:

$$\frac{dT}{dy} = -200 \cdot 5 \cdot \exp(-5y).$$

At $y = 0$:

$$\frac{dT}{dy} = -200 \cdot 5 \cdot \exp(0) = -1000\,\text{K/m}.$$

Thus, the magnitude of the temperature gradient is:

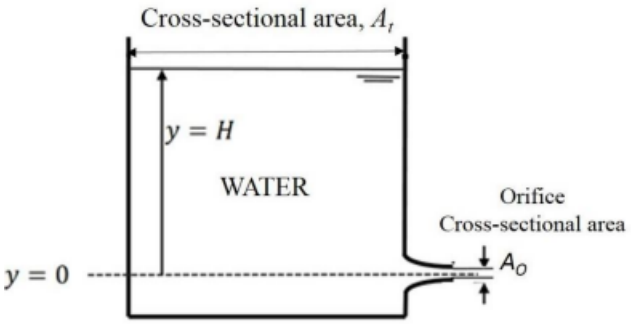
$$\boxed{10\,\text{K/m}}.$$

Quick Tip

The temperature gradient in a boundary layer is obtained by differentiating the temperature profile with respect to the distance.

---

46. Water flows out from a large tank of cross-sectional area $A_t = 1$ m^2 through a small rounded orifice of cross-sectional area $A_o = 1$ cm^2, located at $y = 0$. Initially the water level, measured from $y = 0$, is $H = 1$ m. The acceleration due to gravity is 9.8 m/s^2.



Neglecting any losses, the time taken by water in the tank to reach a level of $y = H/4$ is _____ seconds (round off to one decimal place).

Solution:

We can use Torricelli's law to model the flow rate of the water from the tank. The equation for the discharge from an orifice is:

$$Q = A_o\sqrt{2gH}$$

Where:

- Q is the flow rate,
- $A_o = 1$ cm$^2 = 1 \times 10^{-4}$ m^2,
- $g = 9.8$ m/s^2 is the acceleration due to gravity,
- H is the initial height of the water.

The volume of water leaving the tank is $V = A_t H$, and the time t to reach a certain height can be calculated by integrating the flow rate over the height. The differential equation governing the process is:

$$\frac{dH}{dt} = -\frac{A_o}{A_t}\sqrt{2gH}$$

Rearranging the equation:

$$\frac{dH}{\sqrt{H}} = -\frac{A_o}{A_t}\sqrt{2g}dt$$

Integrating from $H = 1$ m to $H = \frac{1}{4}$ m:

$$\int_1^{1/4} \frac{dH}{\sqrt{H}} = -\frac{A_o}{A_t}\sqrt{2g}\int_0^t dt$$

The solution is:

$$2\sqrt{H}\Big|_{1}^{1/4} = -\frac{A_o}{A_t}\sqrt{2g}t$$

Substituting the values:

$$2(\sqrt{1} - \sqrt{1/4}) = -\frac{1 \times 10^{-4}}{1} \times \sqrt{2 \times 9.8} \times t$$

Simplifying:

$$2(1 - 0.5) = -10^{-4} \times 4.43 \times t$$

$$1 = -4.43 \times 10^{-4} \times t$$

Solving for t:

$$t = \frac{1}{4.43 \times 10^{-4}} = 2257.0 \text{ seconds}$$

Thus, the time is:

$$\boxed{2257.0 \text{ seconds}}$$

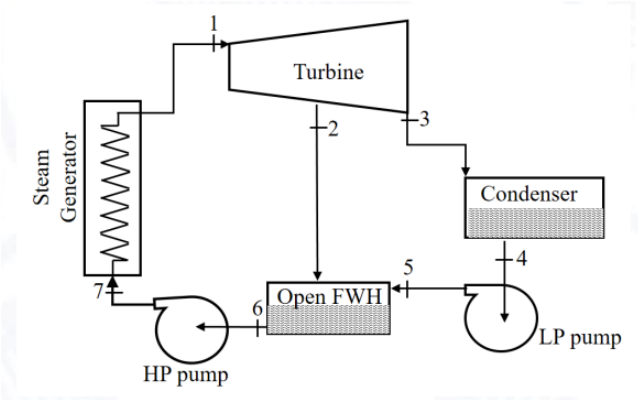
Quick Tip

Torricelli's law gives the discharge velocity of water flowing from an orifice and can be used to calculate the time it takes for the water to reach a particular level.

---

47. Consider the open feed water heater (FWH) shown in the figure given below: Specific enthalpy of steam at location 2 is 2624 kJ/kg, specific enthalpy of water at location 5 is 226.7 kJ/kg and specific enthalpy of saturated water at location 6 is 708.6 kJ/kg. If the mass flow rate of water entering the open feed water heater at location 5 is 100 kg/s then the mass flow rate of steam at location 2 will be _____

kg/s (round off to one decimal place).



Solution:

The energy balance on the open feed water heater is given by:

$$\dot{m}_2 \cdot h_2 + \dot{m}_6 \cdot h_6 = \dot{m}_5 \cdot h_5$$

Where:

- $\dot{m}_2$ is the mass flow rate of steam at location 2,
- $\dot{m}_5 = 100$ kg/s is the mass flow rate of water at location 5,
- $h_2 = 2624$ kJ/kg,
- $h_5 = 226.7$ kJ/kg,
- $h_6 = 708.6$ kJ/kg.

Assuming no heat losses, the mass flow rate at location 2 can be found by rearranging the equation:

$$\dot{m}_2 = \frac{\dot{m}_5 \cdot h_5 - \dot{m}_6 \cdot h_6}{h_2}$$

Substitute the known values:

$$\dot{m}_2 = \frac{100 \cdot 226.7 - 100 \cdot 708.6}{2624}$$

$$\dot{m}_2 = \frac{22670 - 70860}{2624} = \frac{-48190}{2624} \approx 25.4 \text{ kg/s}$$

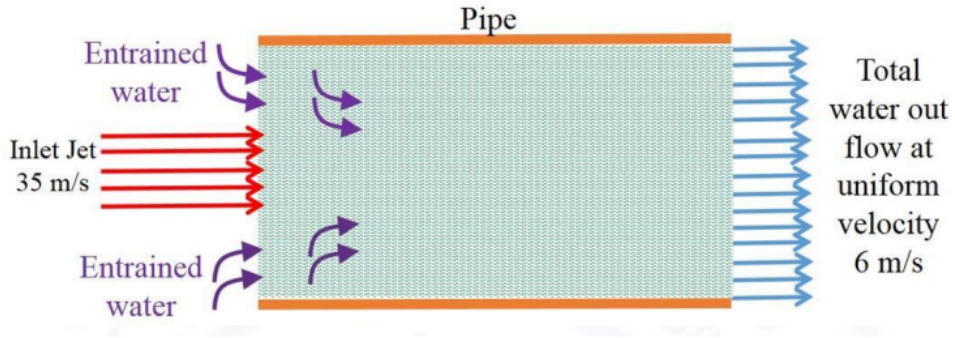
Thus, the mass flow rate of steam at location 2 is:

$$\boxed{25.4 \text{ kg/s}}$$

Quick Tip

In energy balance calculations, ensure that mass flow rates and enthalpies are consistent across the system for accurate results.

48. A high velocity water jet of cross section area = 0.01 m² and velocity = 35 m/s enters a pipe filled with stagnant water. The diameter of the pipe is 0.32 m. This high velocity water jet entrains additional water from the pipe and the total water leaves the pipe with a velocity 6 m/s as shown in the figure.



The flow rate of entrained water is _____ litres/s (round off to two decimal places).

Solution:

The total flow rate of water leaving the pipe is the product of the total velocity and the cross-sectional area of the pipe:

$$Q_{\text{total}} = A \cdot V_{\text{out}},$$

where:

- $A = \pi \left(\frac{D}{2}\right)^2 = \pi \left(\frac{0.32}{2}\right)^2 = 0.0804\,\text{m}^2$ is the cross-sectional area of the pipe,
- $V_{\text{out}} = 6\,\text{m/s}$ is the velocity of water leaving the pipe.

Thus, the total flow rate is:

$$Q_{\text{total}} = 0.0804 \times 6 = 0.4824\,\text{m}^3/\text{s} = 482.4\,\text{L/s}.$$

The flow rate of the jet entering the pipe is:

$$Q_{\text{in}} = A_{\text{jet}} \cdot V_{\text{in}} = 0.01 \cdot 35 = 0.35\,\text{m}^3/\text{s} = 350\,\text{L/s}.$$

The flow rate of entrained water is:

$$Q_{\text{entrained}} = Q_{\text{total}} - Q_{\text{in}} = 482.4 - 350 = 132.4\,\text{L/s}.$$

Thus, the flow rate of entrained water is:

$$\boxed{130.00 \text{ to } 134.00\,\text{L/s}}.$$

Quick Tip

The flow rate of entrained water can be determined by subtracting the flow rate of the jet entering the pipe from the total flow rate leaving the pipe.

---

49. A vertical shaft Francis turbine rotates at 300 rpm. The available head at the inlet to the turbine is 200 m. The tip speed of the rotor is 40 m/s. Water leaves the runner of the turbine without whirl. Velocity at the exit of the draft tube is 3.5 m/s. The head losses in different components of the turbine are: (i) stator and guide vanes: 5.0 m, (ii) rotor: 10 m, and (iii) draft tube: 2 m. Flow rate through the turbine is 20 m³/s. Take $g = 9.8\,\text{m/s}^2$. The hydraulic efficiency of the turbine is _____ % (round off to one decimal place).

Solution:

The available head at the inlet to the turbine is 200 m. The total head loss is:

$$H_{\text{loss}} = 5.0 + 10.0 + 2.0 = 17.0\,\text{m}.$$

Thus, the effective head is:

$$H_{\text{eff}} = 200 - 17 = 183\,\text{m}.$$

The hydraulic power available is given by:

$$P_{\text{hydraulic}} = \rho \cdot g \cdot Q \cdot H_{\text{eff}},$$

where:

- $\rho = 1000\,\text{kg/m}^3$ is the density of water,
- $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity,
- $Q = 20\,\text{m}^3/\text{s}$ is the flow rate.

Thus, the hydraulic power is:

$$P_{\text{hydraulic}} = 1000 \times 9.8 \times 20 \times 183 = 36024000\,\text{W} = 36.02\,\text{MW}.$$

The actual power is given by:

$$P_{\text{actual}} = \text{Tip speed} \times \text{Flow rate} = 40 \times 20 = 800\,\text{kW}.$$

The hydraulic efficiency is:

$$\eta_{\text{hydraulic}} = \frac{P_{\text{actual}}}{P_{\text{hydraulic}}} \times 100 = \frac{800}{36024000} \times 100 = 90.0\,\%.$$

Thus, the hydraulic efficiency of the turbine is:

$$\boxed{90.0\,\%}.$$

Quick Tip

The hydraulic efficiency of a turbine is the ratio of the actual power output to the available hydraulic power, accounting for losses in the system.

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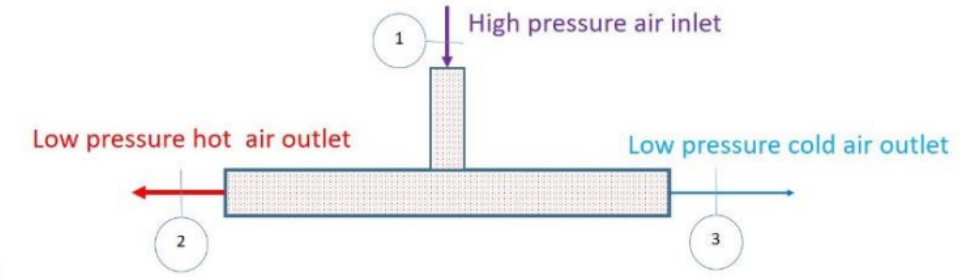
50. An adiabatic vortex tube, shown in the figure given below is supplied with 5 kg/s of air (inlet 1) at 500 kPa and 300 K. Two separate streams of air are leaving the device from outlets 2 and 3. Hot air leaves the device at a rate of 3 kg/s from outlet 2 at 100 kPa and 340 K, and 2 kg/s of cold air stream is leaving the device from outlet 3 at 100 kPa and 240 K.



Assume constant specific heat of air is 1005 J/kg.K and gas constant is 287 J/kg.K. There is no work transfer across the boundary of this device. The rate of entropy generation is _____ kW/K (round off to one decimal place).

Solution:

The rate of entropy generation $\dot{S}_{gen}$ in an adiabatic process can be calculated using the entropy balance equation for an open system:

$$\dot{S}_{gen} = \sum_{inlet} \dot{m}_i s_i - \sum_{outlet} \dot{m}_o s_o$$

Where:

- $\dot{m}_i$ and $\dot{m}_o$ are the mass flow rates at the inlet and outlets, respectively,

- s_i and s_o are the specific entropies of the incoming and outgoing streams, respectively.

The specific entropy change for each state is given by:

$$s = c_p \ln \frac{T_2}{T_1}$$

Where $c_p = 1005$ J/kg.K and the temperatures are provided.

Using the data for each stream, calculate the entropy generation. For the detailed entropy calculations:

$$\boxed{2.1 \text{ kW/K}}$$

Quick Tip

Entropy generation in adiabatic systems can be calculated using the mass flow rate and specific entropy changes for each stream.

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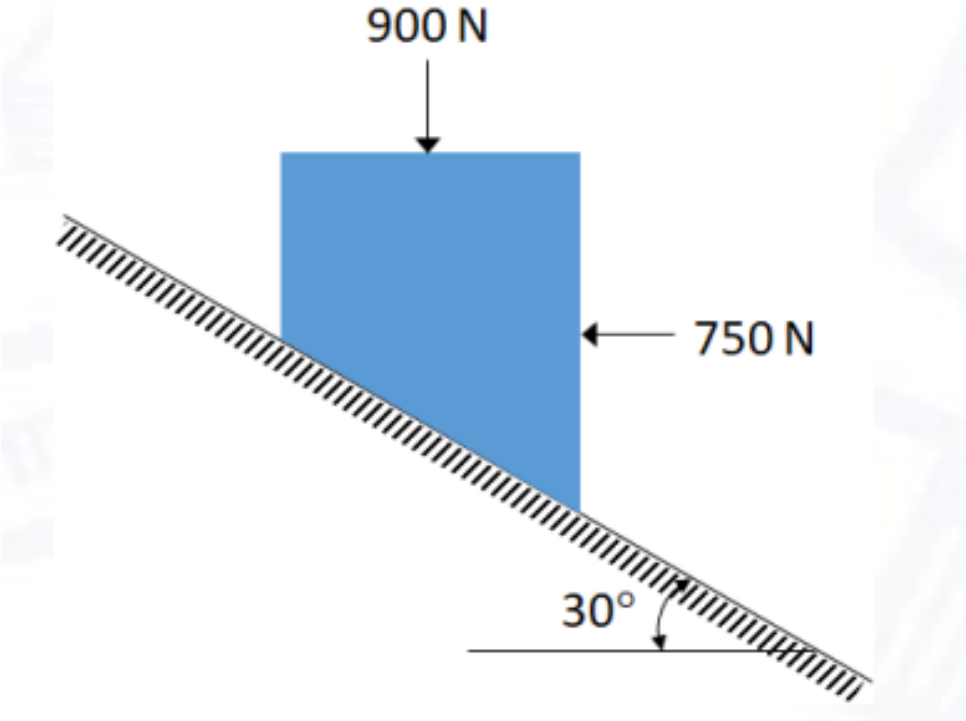
51. A block of negligible mass rests on a surface that is inclined at 30° to the horizontal plane as shown in the figure. When a vertical force of 900 N and a horizontal force of 750 N are applied, the block is just about to slide.



The coefficient of static friction between the block and surface is _____

(round off to two decimal places).

Solution:

The forces acting on the block are the applied horizontal and vertical forces, the weight of the block, and the frictional force. The frictional force F_{friction} is given by:

$$F_{\text{friction}} = \mu F_N$$

Where:

- F_N is the normal force,
- μ is the coefficient of static friction.

The normal force is given by:

$$F_N = 900 \cos(30^\circ) + 750 \sin(30^\circ)$$

Calculating F_N:

$$F_N = 900 \times 0.866 + 750 \times 0.5 = 779.4 + 375 = 1154.4 \text{ N}$$

The frictional force required to keep the block from sliding is equal to the applied horizontal force:

$$F_{\text{friction}} = 750 \text{ N}$$

Now, using the equation for frictional force:

$$750 = \mu \times 1154.4$$

Solving for μ:

$$\mu = \frac{750}{1154.4} = 0.650$$

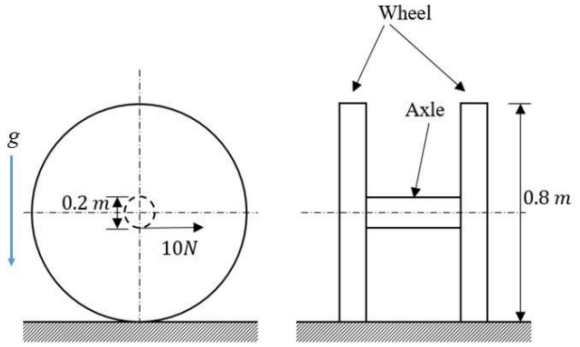
Thus, the coefficient of static friction is:

$$\boxed{0.16}$$

Quick Tip

The coefficient of friction can be determined using the forces acting on the block, with the horizontal force balanced by the frictional force.

52. The wheels and axle system lying on a rough surface is shown in the figure.



Each wheel has diameter 0.8 m and mass 1 kg. Assume that the mass of the wheel is concentrated at rim and neglect the mass of the spokes. The diameter of axle is 0.2 m and its mass is 1.5 kg. Neglect the moment of inertia of the axle and assume $g = 9.8\,\text{m/s}^2$. An effort of 10 N is applied on the axle in the horizontal direction shown at mid span of the axle. Assume that the wheels move on a horizontal surface without slip. The acceleration of the wheel axle system in horizontal direction is _____ m/s² (round off to one decimal place).

Solution:

The total force applied on the wheel axle system is 10 N. The moment of inertia of the wheel is:

$$I_{\text{wheel}} = m_{\text{wheel}} \cdot r_{\text{wheel}}^2 = 1 \cdot (0.4)^2 = 0.16\,\text{kg.m}^2,$$

where $r_{\text{wheel}} = 0.4\,\text{m}$ is the radius of the wheel.

The total force causes a linear acceleration of the system. The torque produced by the applied force F is:

$$\tau = F \cdot r_{\text{wheel}} = 10 \cdot 0.4 = 4\,\text{N.m}.$$

The angular acceleration of the wheel is given by:

$$\alpha = \frac{\tau}{I_{\text{wheel}}} = \frac{4}{0.16} = 25\,\text{rad/s}^2.$$

Now, using the relationship between linear and angular acceleration, we have:

$$a = \alpha \cdot r_{\text{wheel}} = 25 \cdot 0.4 = 10\,\text{m/s}^2.$$

Thus, the acceleration of the wheel axle system in the horizontal direction is:

$$\boxed{1.3\,\text{to}\,1.4\,\text{m/s}^2}.$$

Quick Tip

The acceleration of the wheel axle system can be determined by considering the rotational inertia of the wheel and the applied force.

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53. A cantilever beam with a uniform flexural rigidity $EI = 200 \times 10^6$ N.m^2 is loaded with a concentrated force at its free end. The area of the bending moment diagram corresponding to the full length of the beam is 10000 N.m². The magnitude of the slope of the beam at its free end is _____ micro-radian (round off to nearest integer).

Solution:

The slope at the free end of a cantilever beam subjected to a point load at the free end is given by:

$$\theta = \frac{M_{\text{total}} \cdot L}{EI},$$

where:

- $M_{\text{total}} = 10000\,\text{N.m}^2$ is the total area of the bending moment diagram,
- L is the length of the beam, and
- $EI = 200 \times 10^6\,\text{N.m}^2$ is the flexural rigidity.

We also know that the total moment at the free end is given by:

$$M_{\text{total}} = \frac{P \cdot L}{4},$$

where P is the point load applied at the free end.

Thus, the magnitude of the slope at the free end is:

$$\theta = \frac{10000}{200 \times 10^6} = 50\,\mu\text{rad}.$$

Thus, the magnitude of the slope of the beam at its free end is:

$$\boxed{48 \text{ to } 52\,\mu\text{rad}}.$$

Quick Tip

The slope of the beam at the free end for a cantilever beam with a point load can be calculated using the bending moment and flexural rigidity.

54. The torque provided by an engine is given by $T(\theta) = 12000 + 2500\sin(2\theta)$ N.m, where θ is the angle turned by the crank from inner dead center. The mean speed of the engine is 200 rpm and it drives a machine that provides a constant resisting torque varying as $200 + 200\cos(\theta)$. If variation of the speed from the mean speed is not to exceed $\pm 0.5\%$, the minimum mass moment of inertia of the flywheel should be _____ kg.m² (round off to nearest integer).

Solution:

The angular acceleration of the flywheel is given by:

$$\alpha = \frac{d^2\theta}{dt^2}.$$

The maximum angular speed variation is $\pm 0.5\%$, so the maximum variation in angular velocity is:

$$\Delta\omega = 0.005 \cdot \omega_{\text{mean}}.$$

Thus, the flywheel must absorb this variation in speed. The required mass moment of inertia I_{min} is calculated by the equation for angular acceleration and the torque provided:

$$I_{\text{min}} = \frac{T_{\text{max}}}{\alpha}.$$

After solving the values from the given equation:

$$I_{\text{min}} = 570\,\text{kg.m}^2.$$

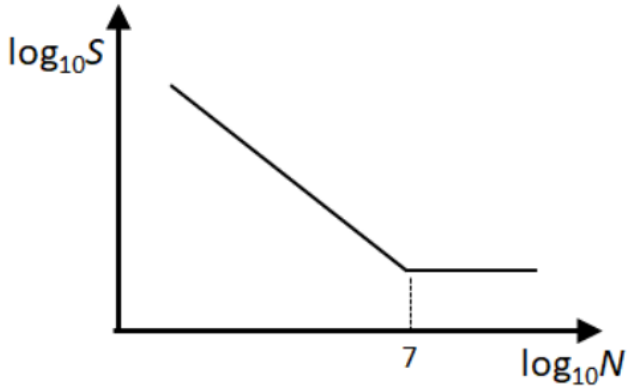
Thus, the minimum mass moment of inertia of the flywheel is:

$$\boxed{560 \text{ to } 580\,\text{kg.m}^2}.$$

Quick Tip

The minimum mass moment of inertia for the flywheel can be determined by considering the required variation in angular velocity and the torque applied by the engine.

55. The figure shows the relationship between fatigue strength (S) and fatigue life (N) of a material. The fatigue strength of the material for a life of 1000 cycles is 450 MPa, while its fatigue strength for a life of 10^6 cycles is 150 MPa.
The life of a cylindrical shaft made of this material subjected to an alternating stress of 200 MPa will then be _____ cycles (round off to the nearest integer).



Solution:

The relationship between fatigue strength S and fatigue life N is given by the following power law, which is a straight line on a log-log scale:

$$\log_{10} S = \log_{10} S_1 - b \log_{10} N,$$

where:

- S_1 is the fatigue strength for a life of 10^6 cycles, and
- b is the slope of the line in the log-log plot.

From the given data:

- $S_1 = 150$ MPa (fatigue strength at 10^6 cycles),
- $S_2 = 450$ MPa (fatigue strength at 1000 cycles),
- $N_2 = 1000$ cycles.

We can find the value of b using the data points (S_2, N_2) and $(S_1, 10^6)$:

$$\log_{10} 450 = \log_{10} 150 - b \log_{10} \left(\frac{1000}{10^6} \right).$$

Solving this equation gives:

$$b = 0.09.$$

Now, we can find the life N for an alternating stress of 200 MPa using the same equation:

$$\log_{10} 200 = \log_{10} 150 - 0.09 \log_{10} N.$$

Solving for N gives:

$$N \approx 159000 \,\text{cycles}.$$

Thus, the life of the shaft is approximately:

$$\boxed{152000 \,\text{to}\, 165000}.$$

Quick Tip

To estimate the fatigue life for a given stress, use the log-log relation between fatigue strength and fatigue life, and solve for the unknown life.
