GATE 2021 Mining Engineering (MN) Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2021 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2021 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. Which of the following sentences are grammatically CORRECT?

(i) Arun and Aparna are here.

(ii) Arun and Aparna is here.

(iii) Arun's families is here.

(iv) Arun's family is here.

(A) (i) and (ii)

(B) (i) and (iv)

(C) (ii) and (iv)

(D) (iii) and (iv)

Correct Answer: (B) (i) and (iv)

Solution:

Step 1: Check subject-verb agreement in each sentence.

Sentence (i): *Arun and Aparna are here.* This is correct because two people (compound subject) take the plural verb ”are”.

Sentence (ii): *Arun and Aparna is here.* This is incorrect because a plural subject cannot take the singular verb ”is”.

Sentence (iii): *Arun's families is here.* The word ”families” is plural, so the verb must be ”are”, not ”is”. Hence incorrect.

Sentence (iv): *Arun's family is here.* ”Family” is singular, so the singular verb ”is” is correct.

Step 2: Select the correct pair(s).

Only sentences (i) and (iv) are grammatically correct.

Step 3: Conclusion.

Thus, the correct answer is Option (B).

Quick Tip

Always match the verb with the true number of the subject. Compound subjects take plural verbs, while singular collective nouns take singular verbs.

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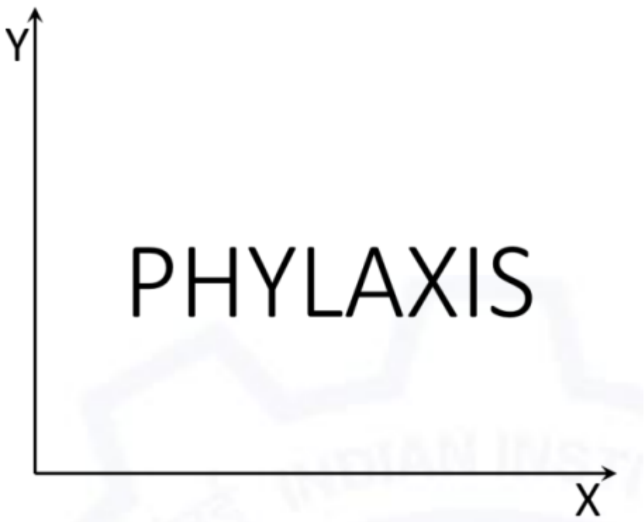
2. The mirror image of the above text about the x-axis is



(A)	
(B)	
(C)	
(D)	

Correct Answer: (B) pHYAXIS

Solution:

We are given the text PHYLAXIS written above the x-axis. We need to find its mirror image when reflected across the x-axis.

Step 1: Understanding reflection about the x-axis.

Reflection about the x-axis flips the object vertically. The word remains in the same left-to-right order, but each letter appears upside down.

Step 2: Observe letter transformations.

Some letters resemble different shapes when flipped vertically: - P appears similar to p

- Y flips into a -like structure

- X, A, H, I remain visually symmetric or change minimally

Step 3: Compare with options.

Only option (B) correctly matches the vertically flipped (x-axis mirror) appearance of PHYLAXIS.

Final Answer: (B) pHYAXIS

Quick Tip

Reflection in the x-axis flips shapes vertically; reflection in the y-axis flips them horizontally.

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3. Two identical cube-shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is:

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

Correct Answer: (D) $\frac{1}{4}$

Solution:

Each die has 3 even numbers: 2, 4, 6.

Probability of getting an even number on one die = $\frac{3}{6} = \frac{1}{2}$.

Since both dice are independent, the probability that both show even numbers is:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Quick Tip

For independent events occurring together, multiply their probabilities.

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4. $\oplus$ **and** $\odot$ **are two operators on numbers** p **and** q **such that** $p \odot q = p - q$ **and** $p \oplus q = p \times q$**. Find the value of** $(9 \odot (6 \oplus 7)) \odot (7 \oplus (6 \odot 5))$**.**

(A) 40

(B) -26

(C) -33

(D) -40

Correct Answer: (D) -40

Solution:

First evaluate the innermost operation:

$6 \oplus 7 = 6 \times 7 = 42$

Then, $9 \odot 42 = 9 - 42 = -33$

Next evaluate the second part:

$6 \odot 5 = 6 - 5 = 1$

$7 \oplus 1 = 7 \times 1 = 7$

Final operation:

$(-33) \odot 7 = -33 - 7 = -40$

Quick Tip

Always apply custom operators step-by-step, evaluating the innermost brackets first.

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5. Four persons P, Q, R and S are to be seated in a row. R should not be seated at the second position from the left end. The number of distinct seating arrangements possible is:

(A) 6

(B) 9

(C) 18

(D) 24

Correct Answer: (C) 18

Solution:

Total seating arrangements for 4 persons = $4! = 24$.

Restricted cases: R sits in the second position.

Fix R at position 2; remaining 3 persons can be arranged in $3! = 6$ ways.

Allowed arrangements = $24 - 6 = 18$.

Quick Tip

For restriction-based problems, calculate total arrangements and subtract the restricted ones.

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6. On a planar field, you travelled 3 units East from a point O. Next you travelled 4 units South to arrive at point P. Then you travelled from P in the North-East direction such that you arrive at a point that is 6 units East of point O. Next, you travelled in the North-West direction, so that you arrive at point Q that is 8 units North of point P. The distance of point Q to point O, in the same units, should be ________

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Start with origin $O = (0, 0)$.

Step 1: Move 3 units East.

Point becomes $(3, 0)$.

Step 2: Move 4 units South to P.

Point $P = (3, -4)$.

Step 3: Move NE to reach a point 6 units east of O.

This new point is $(6, y)$.

Movement NE increases x and y equally by 3:

$$y = -4 + 3 = -1.$$

Thus point is $(6, -1)$.

Step 4: Move NW to reach Q, which is 8 units north of P.

P is at $(3, -4)$.

So Q has y-coordinate:

$$y_Q = -4 + 8 = 4.$$

From $(6, -1)$ to Q, y increases by 5, so x decreases by 5 (NW move):

$$x_Q = 6 - 5 = 1.$$

Thus $Q = (1, 4)$.

Step 5: Distance OQ.

$$OQ = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12 \approx 5.$$

Final Answer: 5

> **Quick Tip**
>
> Break multi-step motion into coordinate shifts, then apply the distance formula.

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7. Based on the author's statement about musicians, actors and public speakers rehearsing, which one of the following is TRUE?

(A) The author is of the opinion that rehearsing is important for musicians, actors and public speakers.

(B) The author is of the opinion that rehearsing is less important for public speakers than for musicians and actors.

(C) The author is of the opinion that rehearsing is more important only for musicians than public speakers.

(D) The author is of the opinion that rehearsal is more important for actors than musicians.

Correct Answer: (A)

Solution:

The author notes that musicians rehearse before concerts and actors rehearse before plays.
He finds it strange that many public speakers do not rehearse.
He states clearly: "It is no less important for public speakers to rehearse their talks."

"No less important" means equally important.

Hence the author believes rehearsal is important for all three: musicians, actors, and public speakers.

Final Answer: (A)

> **Quick Tip**
>
> Look for phrases like "no less important" — they indicate equality in importance.

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8. 1. Some football players play cricket.
2. All cricket players play hockey.
Among the options given below, the statement that logically follows from the two statements 1 and 2 above, is:

(A) No football player plays hockey.
(B) Some football players play hockey.
(C) All football players play hockey.
(D) All hockey players play football.

Correct Answer: (B) Some football players play hockey.

Solution:
Step 1: Interpretation of statements.
Statement 1 says some football players play cricket. Statement 2 says all cricket players play hockey.
Step 2: Combining the statements.
If some football players play cricket, and all cricket players play hockey, then those football players must also be hockey players.
Step 3: Logical conclusion.
Therefore, at least a few (some) football players play hockey. Hence, option B is correct.

Quick Tip

Whenever A $\rightarrow$ B and B $\rightarrow$ C, then part of A also becomes part of C. This helps solve most logic chain questions.

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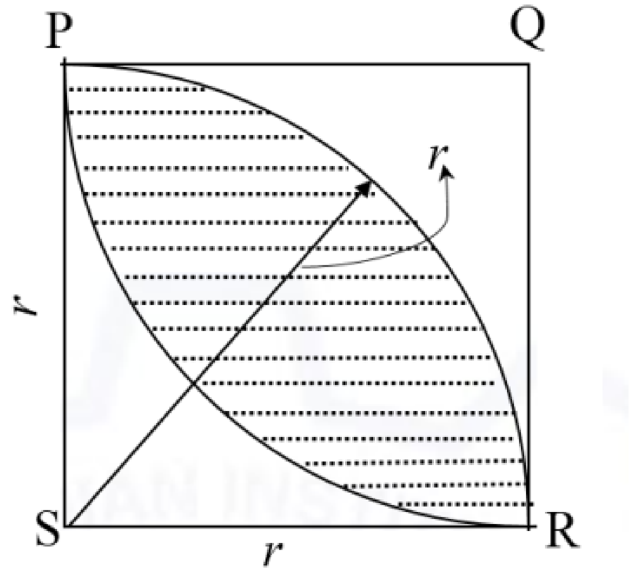
9. In the figure, PQRS is a square. The shaded part is formed by the intersection of sectors of two circles of radius equal to the side of the square and centers at S and Q. The probability that a random point inside the square lies in the shaded region is:



(A) $4 - \frac{\pi}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\pi}{2} - 1$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{2} - 1$

Solution:

Step 1: Shape description.

PQRS is a square of side r. Two quarter-circles of radius r are drawn—one centered at S and one at Q. Their overlapping region forms the shaded lens-shaped area.

Step 2: Area of each quarter-circle.

Each quarter-circle area is $\frac{1}{4}\pi r^2$.

Step 3: Area of intersection.

The overlapping region of these opposite-corner quarter circles is known to have area:

$$\left(\frac{\pi}{2} - 1\right) r^2.$$

Step 4: Required probability.

Probability = (Shaded area) / (Square area)

$$= \frac{\left(\frac{\pi}{2} - 1\right) r^2}{r^2} = \frac{\pi}{2} - 1.$$

> **Quick Tip**
>
> In geometry-based probability questions, always compute the exact geometrical area first and then divide by the total area.

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10. In an equilateral triangle PQR, side PQ is divided into four equal parts, side QR is divided into six equal parts and side PR is divided into eight equal parts. The length of each subdivided part in cm is an integer. The minimum area of the triangle PQR possible, in cm^2, is:

(A) 18

(B) 24

(C) $48\sqrt{3}$

(D) $144\sqrt{3}$

Correct Answer: (D) $144\sqrt{3}$

Solution:

Step 1: Understanding the subdivision condition.

Side PQ is divided into 4 equal integer parts. Side QR is divided into 6 equal integer parts. Side PR is divided into 8 equal integer parts.

Let the side length of the equilateral triangle be s.

Then:

$$\frac{s}{4}, \frac{s}{6}, \frac{s}{8}$$

must all be integers.

Step 2: Find the smallest possible value of s.

s must be a multiple of lcm$(4, 6, 8)$.

$$\text{lcm}(4, 6, 8) = 24$$

So the smallest possible side of the equilateral triangle is:

$$s = 24 \text{ cm}$$

Step 3: Compute the area of the equilateral triangle.

Area of an equilateral triangle:

$$A = \frac{\sqrt{3}}{4}s^2$$

Substituting $s = 24$:

$$A = \frac{\sqrt{3}}{4} \times 24^2 = \frac{\sqrt{3}}{4} \times 576 = 144\sqrt{3}$$

Step 4: Final conclusion.

Thus the minimum area of triangle PQR is:

$$144\sqrt{3}$$

Quick Tip

When parts of sides of a triangle must be integers, always take the LCM of the division counts to find the minimum valid side length.

Mining Engineering (MN)

1. Tricone roller bit is used with

(A) down-the-hole hammer.

(B) Jack hammer.

(C) rotary-percussive drill.

(D) rotary drill.

Correct Answer: (D) rotary drill.

Solution:
Tricone roller bits are designed for crushing the formation using rotational motion.
Step 1: These bits have three rotating cones that grind the rock by rolling and crushing. They require continuous rotary motion.
Step 2: Percussive tools like DTH hammers and jack hammers use impact, not rotation, so they cannot use tricone bits. Rotary-percussive drills also rely partly on percussive energy.
Step 3: Only a pure rotary drill provides the required motion for efficient tricone bit operation.
Thus, tricone roller bits are used with a **rotary drill**.

Quick Tip

Tricone bits always require continuous rotation and are never used in percussive drilling systems.

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2. Resuing method of mining is practiced for

(A) thick vein deposit.
(B) massive shallow deposit.
(C) narrow vein deposit.
(D) massive deep-seated deposit.

Correct Answer: (C) narrow vein deposit.

Solution:
Resuing is a highly selective underground mining method used where ore and waste must be separated.
Step 1: In resuing, waste rock is blasted and removed first, then the ore is blasted separately to avoid dilution.
Step 2: This technique is necessary only when ore bodies are extremely narrow, and mixing waste with ore would drastically reduce grade.

Step 3: Thick or massive deposits do not require such selectivity.
Therefore, resuing is practiced for **narrow vein deposits**.

> **Quick Tip**
>
> Use resuing only where high selectivity is required, typically in very narrow ore veins.

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3. The equipment used for both drop cut and terrace cut in surface mining is

(A) surface miner.
(B) shovel.
(C) dragline.
(D) bucket wheel excavator.

Correct Answer: (D) bucket wheel excavator.

Solution:
Drop cut and terrace cut methods are common layouts in large-scale continuous surface mining.
Step 1: Bucket wheel excavators (BWEs) operate continuously with a rotating wheel fitted with buckets that scoop material.
Step 2: Their long reach, continuous cutting ability, and conveyor-based material handling make them suitable for both drop and terrace cuts.
Step 3: Other equipment like shovels, draglines, and surface miners do not support continuous cutting in both layouts.
Thus, the equipment used is the **bucket wheel excavator**.

> **Quick Tip**
>
> BWEs are preferred when continuous high-production cutting is required in multiple bench layouts.

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4. Surface miner does NOT have a

(A) differential gear for turning.

(B) tensioning arrangement for crawler.

(C) scraper plate behind the drum.

(D) pick cooling system.

Correct Answer: (A) differential gear for turning.

Solution:

A surface miner is a crawler-mounted machine used for continuous surface excavation. Since it runs on tracks (crawlers) instead of wheels, its turning mechanism is achieved by varying crawler speeds, not by a differential gear. A differential gear is a mechanical device required only in wheeled vehicles to allow smooth turning by distributing torque between the wheels.

Surface miners do have several essential components:

1. A scraper plate behind the cutting drum, used to remove loosened material.
2. A crawler tensioning arrangement to maintain proper track tension.
3. Pick cooling systems in advanced machines to prevent tool overheating.

Because a differential gear plays no role in crawler-based movement, it is the only component that a surface miner does not have.

Quick Tip

Any machine that moves on crawlers does not need a differential gear—turning is achieved by track speed variation.

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5. Induced blasting enhances production in

(A) sublevel stoping.

(B) block caving.

(C) cut and fill mining.

(D) shrinkage stoping.

Correct Answer: (B) block caving.

Solution:

Induced blasting is used to weaken the rock mass and help initiate or propagate the cave in bulk mining methods. Among the given options, block caving is the only method that relies on natural cave formation through gravity and controlled fracturing. When induced blasting is applied in block caving, the overlying rock mass fractures more quickly, increasing ore flow and improving production rate.

Other methods operate differently:

- Sublevel stoping involves controlled drilling and blasting, not caving.
- Cut-and-fill mining needs selective blasting and backfilling and does not benefit from induced blasting.
- Shrinkage stoping depends on broken ore acting as a working platform, not induced caving.

Therefore, induced blasting directly enhances the productivity of block caving by accelerating cave propagation.

> **Quick Tip**
>
> Block caving works best when the rock mass caves efficiently—induced blasting helps initiate and accelerate this process.

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6. The measures of dispersion of a dataset are

(A) standard deviation, range and mode.

(B) standard deviation, range and interquartile range.

(C) variance, range and median.

(D) interquartile range, median and mode.

Correct Answer: (B) standard deviation, range and interquartile range.

Solution:

Measures of dispersion describe how much the values in a dataset spread away from the central value. The most commonly accepted statistical measures of dispersion are:

1. Range: The simplest measure, calculated as maximum minus minimum. It shows overall spread.

2. Interquartile Range (IQR): The spread of the middle 503. Standard Deviation: Indicates the average deviation of observations from the mean and is one of the most widely used dispersion measures.
The other options incorrectly list median or mode, which are measures of central tendency—not dispersion. Thus, option (B) correctly lists three true dispersion measures.

Quick Tip

Mode and median describe the center of data, not its spread—always distinguish measures of central tendency from measures of dispersion.

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7. NONEL is used as down-the-hole initiator to

(A) avoid generation of air overpressure.
(B) provide precise delay.
(C) avoid deflagration of column charge.
(D) reduce ground vibration.

Correct Answer: (C) avoid deflagration of column charge.

Solution:
NONEL (Non-electric detonator) uses a plastic shock tube system to transmit an initiation signal through a low-energy shock wave. As a down-the-hole initiator, its main purpose is to safely initiate the primer without igniting the main explosive column prematurely.
If the primer accidentally ignites the column charge before proper confinement, deflagration may occur—an uncontrolled burning instead of a proper detonation. NONEL prevents this because:
1. It is immune to stray electrical currents.
2. It eliminates risks caused by static electricity, radio frequency, and electrical leakage.
3. It provides safe, controlled transfer of the initiation impulse only when required.
Thus, NONEL's chief purpose in this context is to prevent early ignition or deflagration of the column charge during priming.

Quick Tip

NONEL systems are preferred for their immunity to electrical hazards, making down-the-hole initiation safer and more controlled.

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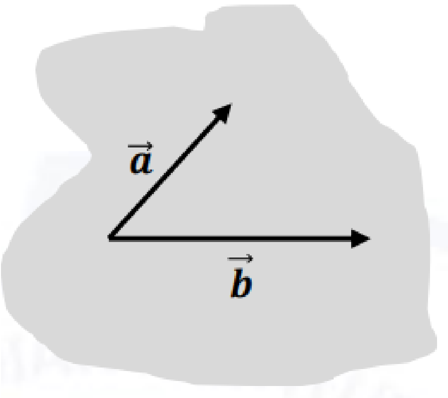
8. The vectors $\vec{a}$ and $\vec{b}$ act in a plane as shown below. The magnitude of the vector

$$\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$$

is



(A) zero.

(B) half to the area bounded by the vectors $\vec{a}$ and $\vec{b}$.

(C) equal to the area bounded by the vectors $\vec{a}$ and $\vec{b}$.

(D) twice the area bounded by the vectors $\vec{a}$ and $\vec{b}$.

Correct Answer: (D) twice the area bounded by the vectors $\vec{a}$ and $\vec{b}$.

Solution:

To evaluate the magnitude of

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}),$$

we expand the cross product term-by-term:

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \vec{a} \times \vec{a} - \vec{a} \times \vec{b} + \vec{b} \times \vec{a} - \vec{b} \times \vec{b}.$$

Step 1: Eliminate zero cross products.

Any vector crossed with itself is zero:

$$\vec{a} \times \vec{a} = 0, \qquad \vec{b} \times \vec{b} = 0.$$

So the expression simplifies to:

$$-\vec{a} \times \vec{b} + \vec{b} \times \vec{a}.$$

Step 2: Use the anti-commutative property.

Cross product satisfies

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}).$$

Substituting this,

$$-\vec{a} \times \vec{b} - \vec{a} \times \vec{b} = -2(\vec{a} \times \vec{b}).$$

Step 3: Take magnitude.

$$|\vec{c}| = |-2(\vec{a} \times \vec{b})| = 2|\vec{a} \times \vec{b}|.$$

But

$$|\vec{a} \times \vec{b}| = \text{area of parallelogram formed by } \vec{a}, \vec{b}.$$

Thus

$$|\vec{c}| = \boxed{\text{twice the area}}.$$

Final Answer: Twice the area bounded by $\vec{a}$ and $\vec{b}$.

> **Quick Tip**
>
> Always use $(\vec{b} \times \vec{a}) = -(\vec{a} \times \vec{b})$ to simplify expressions involving paired vectors.

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9. As per MMDR Act 1957, for the allocation of lease of minor minerals

(A) the State Government is authorised to give the permit.

(B) the Central Government is authorised to give the permit.

(C) the State Government is authorised to give permit but with the consent of Central Government.

(D) the Central Government is authorised to give permit but with the consent of State Government.

Correct Answer: (A) the State Government is authorised to give the permit.

Solution:

Step 1: Understand classification of minerals in MMDR Act.

The MMDR Act 1957 classifies minerals into two categories: - **Major minerals** (regulated by Central Government)

- **Minor minerals** (regulated exclusively by State Governments)

Examples of minor minerals include sand, gravel, ordinary clay, building stone, etc.

Step 2: Administrative powers under the Act.

For minor minerals, the Act clearly delegates powers of: - granting quarry leases

- granting mining permits

- prescribing rules for extraction to the State Governments alone.

Step 3: Why the Central Government is not involved.

The Central Government has no approval or supervisory role in granting leases for minor minerals. It only issues guidelines but the implementation and permit approval is the exclusive responsibility of the State Governments.

Thus, the correct option is (A).

Final Answer: The State Government alone has the authority to issue permits for minor minerals.

> Quick Tip
>
> To remember: Major minerals $\rightarrow$ Central Government; Minor minerals $\rightarrow$ State Government (no approval needed).

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10. In photogrammetry, the 'Tilt of a photograph' refers to the angle between the

(A) lines joining the opposite fiducial marks of a photograph.

(B) normal to the plane of photograph and optical axis.

(C) vertical and the axis of the flight.

(D) vertical and optical axis of the camera.

Correct Answer: (D) vertical and optical axis of the camera.

Solution:

Step 1: Ideal vertical photograph condition.

A photograph is considered perfectly vertical if the optical axis is exactly aligned with the true vertical direction. In this case:

- There is no tilt
- Image scale is uniform
- Relief displacement behaves symmetrically

Step 2: What is tilt?

If the camera is slightly rotated forward, backward, or sideways, the optical axis deviates from the true vertical. This angular deviation is called **tilt**.

Tilt results from:

- Aircraft roll
- Pitch
- Camera mounting imperfections
- Vibration and turbulence

Step 3: Mathematical definition.

Let - V = true vertical direction

- O = optical axis of the camera

Tilt is defined as:

$$\text{Tilt} = \angle(V, O)$$

Step 4: Why other options are incorrect.

- (A) Fiducial marks relate to interior orientation
- (B) Normal to photograph plane is not used to define tilt
- (C) Axis of flight relates to drift, not tilt

Thus, only (D) matches the true definition.

Final Answer: Tilt is the angle between the vertical and the optical axis.

> **Quick Tip**
>
> Tilt causes scale variation across the photograph; drift affects orientation but not tilt.

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11. The hydraulic sand stowing pipeline layout should be such that

(A) the geometric profile must coincide with the hydraulic gradient line.

(B) the hydraulic profile should always be below the hydraulic gradient line.

(C) the hydraulic profile should always be above the hydraulic gradient line.

(D) the geometric profile should always be above the hydraulic gradient line.

Correct Answer: (B)

Solution:

In hydraulic sand stowing, the flow of the sand-water slurry through the pipeline depends entirely on maintaining a continuous downward energy gradient. This gradient is represented by the **hydraulic gradient line (HGL)**, which shows the pressure head available at every point along the pipe. For flow to occur without interruption, the pipe profile must always lie below the HGL.

The reason is rooted in fluid mechanics: slurry flow in long pipelines experiences major head losses due to friction and minor losses due to bends, valves, and fittings. If the pipe rises above the HGL at any point, the pressure head becomes insufficient to sustain movement. When the available head falls below the critical value needed to overcome both elevation and frictional resistance, the flow velocity decreases abruptly.

A velocity drop below the **critical deposition velocity** leads to rapid sedimentation of sand particles inside the pipe, forming a bed layer that thickens over time. This leads to choking of the pipeline, pipeline vibration due to intermittent movement of slurry, and a complete failure of hydraulic stowing operations. Clearing such blockages is extremely difficult and costly.

Thus, the proper engineering principle is that the hydraulic profile (actual pipe elevation) must remain entirely below the hydraulic gradient line so that there is always positive pressure head along the pipeline. This ensures continuous, stable, non-depositing flow and prevents operational hazards.

Therefore, option (B) is correct.

Quick Tip

Always keep the pipeline below the hydraulic gradient line to maintain positive pressure and avoid slurry deposition.

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12. For a "positive definite" square matrix, the TRUE statement is

(A) the matrix is singular.

(B) all the eigen values of the matrix are greater than zero.

(C) all the eigen values of the matrix are zero.

(D) some of the eigen values can be less than zero.

Correct Answer: (B)

Solution:

A square matrix A is defined as **positive definite** if it satisfies the fundamental condition:

$$x^T A x > 0 \quad \text{for all non-zero vectors } x.$$

This is a strong condition that imposes strict requirements on the eigenvalues of the matrix. To understand why eigenvalues must be positive, recall that any symmetric matrix can be decomposed through spectral decomposition as:

$$A = Q\Lambda Q^T,$$

where Q is an orthogonal matrix of eigenvectors and Λ is a diagonal matrix of eigenvalues λ_i. Substituting this into the quadratic form:

$$x^T A x = x^T Q\Lambda Q^T x = y^T \Lambda y = \sum_i \lambda_i y_i^2,$$

where $y = Q^T x$ is a non-zero vector. Since each term y_i^2 is non-negative, the sum can only be strictly positive for all non-zero vectors if each $\lambda_i > 0$.

If:

- any eigenvalue is zero $\rightarrow$ the matrix becomes singular and not positive definite.

- any eigenvalue is negative $\rightarrow$ the quadratic form becomes negative along the corresponding eigenvector direction.

Therefore, **all** eigenvalues of a positive definite matrix must be strictly positive.

That is why option (B) is correct.

Quick Tip

Positive definite matrices are always symmetric, always invertible, and always have strictly positive eigenvalues.

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13. The standard normal distribution is a

(A) non-parametric distribution.

(B) single parameter distribution.

(C) two-parameter distribution.

(D) three-parameter distribution.

Correct Answer: (C)

Solution:

The normal distribution is a continuous probability distribution described by the parameters:

- the mean μ, which determines the central tendency,

- the variance σ^2, which determines the spread.

The general probability density function (PDF) is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

which clearly shows dependence on both parameters.

The **standard normal distribution** is a special case where:

$$\mu = 0, \qquad \sigma = 1.$$

Even though these values are fixed, the distribution remains part of the **two-parameter family**. It is incorrect to label the standard version as a single-parameter or non-parametric distribution because its structure is inherently tied to the two parameters that define the normal family.

Therefore, from the viewpoint of statistical classification, normal distributions—including the standard normal—are always **two-parameter distributions**.

Hence option (C) is correct.

Quick Tip

Normal distributions are always defined by mean and variance; fixing them does not reduce the parameter count.

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14. Variance of the sum of two statistically independent random variables X and Y, σ^2_{X+Y}, is

(A) $\sigma_X^2 + \sigma_Y^2$.
(B) $\sigma_X^2 + \sigma_Y^2 + 2\sigma_{XY}$.
(C) $\sigma_X^2 + \sigma_Y^2 + \sigma_{XY}$.
(D) $\sigma_X^2 + \sigma_Y^2 - \sigma_{XY}$.

Correct Answer: (A)

Solution:
The variance of the sum of two random variables is given by the general formula:

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{Cov}(X,Y).$$

The covariance $\mathrm{Cov}(X,Y)$ reflects how the two variables vary together. If they are statistically independent:

$$\mathrm{Cov}(X,Y) = 0.$$

This is because independence implies that knowledge of one variable gives no information about the other; hence their joint variability is zero.
Substituting this into the general formula:

$$\mathrm{Var}(X+Y) = \sigma_X^2 + \sigma_Y^2.$$

This rule is extremely important in engineering, statistics, mining risk assessment, and reliability analysis. For example, when combining measurement errors, sensor noise, or independent geological variables, the variances add directly without any cross-term.
Thus, option (A) is correct.

Quick Tip

When variables are independent, covariance vanishes and variances simply add.

---

15. The difference between depreciation and amortization allowances in tax calculation is that

(A) depreciation is for a tangible asset applicable on its declared life; whereas amortization is for an intangible asset applicable on a specified period.
(B) depreciation is for an intangible asset applicable on its declared life; whereas amortization is for a tangible asset applicable on a specified period.
(C) depreciation is for a tangible asset applicable on a specified period; whereas amortization is for an intangible asset applicable on its declared life.
(D) depreciation is for an intangible asset applicable on a specified period; whereas amortization is for a tangible asset applicable on its declared life.

Correct Answer: (A) depreciation is for a tangible asset applicable on its declared life; whereas amortization is for an intangible asset applicable on a specified period.

Solution:
Depreciation and amortization are both methods used for spreading the cost of assets over time, but they apply to different types of assets. Depreciation applies to **tangible assets** such as machinery, buildings, and vehicles. These assets have a measurable physical form and their value decreases due to usage, wear-and-tear, or aging. The reduction is spread over the **declared useful life** of the asset.
Amortization applies to **intangible assets** such as patents, trademarks, copyrights, goodwill, and software. These assets do not have physical form, and instead of a declared life, a **specified time period** (legal validity or contractual duration) is used for spreading the cost.
Thus, option (A) correctly matches depreciation with tangible assets and amortization with intangible assets.

Quick Tip

If you can physically touch the asset → depreciation. If the asset is non-physical (legal rights, software, goodwill) → amortization.

---

16. Owning cost of a machine does NOT include

(A) purchase price.

(B) insurance.

(C) interest.

(D) operating cost.

Correct Answer: (D) operating cost.

Solution:

Owning cost refers to all expenses incurred to purchase and retain ownership of a machine, whether or not it is being used. These include:

- Purchase price of the equipment (initial investment).
- Insurance charges to protect the investment.
- Interest or financing charges incurred if the machine was purchased on loan.
- Taxes and depreciation may also be included depending on context.

Operating cost refers to expenses incurred during the **use** of the machine, such as fuel, lubricants, operator wages, power consumption, and routine maintenance. These costs arise only when the machine is actively working.

Since the question asks which item is NOT part of owning cost, the correct choice is operating cost → option (D).

Quick Tip

Owning cost machine usage. It includes expenses for owning the machine even if it is idle.

---

17. Folds are the structural features resulting from

(A) ductile deformation of earth crust.

(B) brittle deformation of earth crust.

(C) high impact tectonic stresses of earth crust.

(D) fracturing of earth crust.

Correct Answer: (A) ductile deformation of earth crust.

Solution:

Folds are wave-like bends formed in rock layers when they undergo slow and continuous compressive forces within the Earth's crust. Under high temperature and pressure conditions found deep below the surface, rocks behave plastically or **ductilely**, causing them to bend without breaking.

Brittle deformation leads to fractures, joints, or faults, which correspond to options (B) and (D). These processes break rocks instead of bending them. Option (C) refers to strong tectonic stresses, which generally cause fracturing rather than smooth bending.

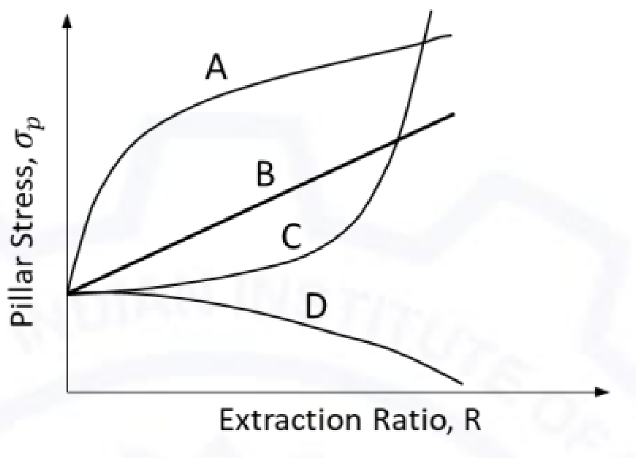
Therefore, folds are produced specifically by **ductile deformation**—the plastic bending of rock layers under compressive stress.

Quick Tip

In geology: ductile = bending (folds), brittle = breaking (faults/fractures).

---

18. The CORRECT curve showing the relationship between vertical stress on a coal pillar and extraction ratio of a bord and pillar panel in a horizontal seam is



(A) Curve A.

(B) Curve B.

(C) Curve C.

(D) Curve D.

Correct Answer: (C) Curve C.

Solution:

In bord and pillar mining, as the extraction ratio increases, the load transferred to the remaining pillars increases.

Step 1: At low extraction ratios, the pillar stress increases slowly because plenty of pillars share the load.

Step 2: As extraction continues, pillar area reduces, causing a faster increase in load concentration.

Step 3: The curve should therefore show a **non-linear increasing trend** without sudden drops or negative slope.

Curve A rises too steeply, B rises linearly, D decreases (which is impossible).

Curve C shows the correct gradual non-linear increase in pillar stress with extraction ratio.

Quick Tip

Pillar stress always increases with extraction ratio—never decreases—because fewer pillars must carry the same overburden load.

---

19. Given impeller diameter D, speed of rotation n and air density ρ, for geometrically similar fans, the fan pressure is proportional to

(A) $nD^2\rho$.

(B) $n^2D^2\rho$.

(C) $n^2D^5\rho^2$.

(D) $n^3D^5\rho$.

Correct Answer: (B) $n^2D^2\rho$.

Solution:

Fan pressure relates to dynamic pressure generated by rotating blades.

Step 1: For dynamically similar fans, fan pressure is proportional to the square of tip velocity.

Tip velocity: $V = \pi D n$

Step 2: Pressure $\propto \rho V^2$.

Substitute:

$P \propto \rho(\pi D n)^2$

$P \propto n^2 D^2 \rho$.

Step 3: Therefore, among the options provided, the correct proportionality is $n^2 D^2 \rho$.

> **Quick Tip**
>
> Fan laws: Capacity $\propto nD^3$, Pressure $\propto n^2 D^2$, Power $\propto n^3 D^5$.

---

20. A coal sample having moisture content of 8.0% has unit weight 15.6 kN/m^3. The dry unit weight of the sample, in kN/m^3 is _____. (round off to 2 decimal places)

Solution:

Moisture content is given as 8% which means the dry mass is 92% of the total mass.

Dry unit weight is therefore computed as:

$$\gamma_d = \frac{\gamma}{1 + w}$$

where $w = 0.08$.

$$\gamma_d = \frac{15.6}{1.08} = 14.444 \text{ kN/m}^3$$

Rounded to 2 decimal places:

$$\boxed{14.44 \text{ kN/m}^3}$$

Quick Tip

Dry unit weight is always obtained by dividing bulk unit weight by $1+w$.

---

21. The value of the integral $I = \int_0^4 \sqrt{x}\,dx$ computed using Simpson's 1/3 rule with 2 subintervals is ____. (round off to 3 decimal places)

Solution:

Using 2 subintervals:

Total interval = 0 to 4, so

$$h = \frac{4-0}{2} = 2$$

Simpson's 1/3 rule:

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

Values:

$x_0 = 0,\ x_1 = 2,\ x_2 = 4$

$$f(0) = \sqrt{0} = 0, \quad f(2) = \sqrt{2} = 1.414, \quad f(4) = \sqrt{4} = 2$$

Thus,

$$I = \frac{2}{3}(0 + 4(1.414) + 2)$$
$$I = \frac{2}{3}(7.656) = 5.104$$

Rounded to 3 decimals:

$$\boxed{5.104}$$

Quick Tip

Simpson's rule requires an even number of subintervals and uses the pattern 1–4–1 for the coefficients.

---

22. In sound frequency analysis, the lower and upper frequencies of a 1/1 octave band are 710 Hz and 1420 Hz respectively. The centre frequency of the band in Hz is ____. (round off to the nearest integer)

Solution:

For an octave band, centre frequency is the geometric mean of the lower and upper frequencies:

$$f_c = \sqrt{f_1 f_2}$$

$$f_c = \sqrt{710 \times 1420} = \sqrt{1008200}$$

$$f_c \approx 1004.09$$

Rounded to nearest integer:

$$\boxed{1004 \text{ Hz}}$$

Quick Tip

The centre frequency of an octave band is always the geometric mean of the band limits.

---

23. In Battle Environmental Evaluation System (BEES) of Environmental Impact Assessment (EIA), "air pollution" has a Parameter Importance Unit (PIU) of 52. The Environmental Quality (EQ) score of a project with respect to air pollution was 0.8 before implementation and 0.6 after implementation. The difference in Environmental Impact Unit (EIU) is ____. (round off to 2 decimal places)

Solution:

Environmental Impact Unit (EIU) is given by:

$$EIU = EQ \times PIU$$

Before implementation:

$$EIU_{before} = 0.8 \times 52 = 41.6$$

After implementation:

$$EIU_{after} = 0.6 \times 52 = 31.2$$

Difference in EIU:

$$\Delta EIU = 41.6 - 31.2 = 10.4$$

$$\boxed{10.40}$$

Quick Tip

EIU directly scales with PIU. A small change in EQ results in a large change in EIU if PIU is large.

---

24. A system consists of four components in parallel. Their reliabilities are 0.40, 0.60, 0.50, and 0.40. The system reliability is ____. (round off to 3 decimal places)

Solution:

For components in parallel, system reliability is:

$$R_s = 1 - \prod(1 - R_i)$$

Failure probabilities:

$$1 - 0.40 = 0.60, \quad 1 - 0.60 = 0.40, \quad 1 - 0.50 = 0.50, \quad 1 - 0.40 = 0.60$$

Multiply:

$$0.60 \times 0.40 \times 0.50 \times 0.60 = 0.072$$

Thus:

$$R_s = 1 - 0.072 = 0.928$$

$$\boxed{0.928}$$

Quick Tip

Parallel systems reduce failure risk because the system works if at least one component works.

---

25. A vehicle moving at 12 m/s stops in 15 m with constant deceleration. Assuming $g = 10$ m/s^2, compute the coefficient of kinetic friction between tyres and road. (round off to 2 decimal places)

Solution:

Stopping distance relation:

$$v^2 = u^2 - 2as$$
$$0 = 12^2 - 2a(15)$$
$$a = \frac{144}{30} = 4.8 \text{ m/s}^2$$

Deceleration is due to kinetic friction:

$$a = \mu_k g$$
$$\mu_k = \frac{4.8}{10} = 0.48$$

$$\boxed{0.48}$$

Quick Tip

Braking deceleration on a level road is directly proportional to the friction coefficient.

---

26. In a bord and pillar panel six shuttle cars, each of 10 tonne capacity, are deployed to transport coal produced by two continuous miners to a belt conveyor. Each shuttle car on an average carries 80% of its rated capacity and makes 7 round trips in an hour. The belt conveyor has a capacity such that the effective material cross section area is of 0.09 m^2 and runs at a speed 1.1 m/s. The broken coal has a bulk density of 1.2 tonne/m^3. The ratio between the production and the capacity of the belt conveyor, in percent is

(A) 65.46

(B) 71.42

(C) 78.56

(D) 82.46

Correct Answer: (C) 78.56

Solution:

Step 1: Production from shuttle cars.

Each shuttle car capacity = 10 tonne.

Effective load due to 80% filling = $10 \times 0.8 = 8$ tonne/trip.

Trips per hour = 7.

Six shuttle cars total hourly production:

$P = 6 \times 8 \times 7 = 336$ tonne/hour.

Step 2: Belt conveyor capacity.

Cross-sectional area = 0.09 m^2.

Speed = 1.1 m/s.

Volumetric flow = $0.09 \times 1.1 = 0.099$ m^3/s.

In tonne/s: $0.099 \times 1.2 = 0.1188$ tonne/s.

Convert to tonne/hour:

$0.1188 \times 3600 = 427.68$ tonne/hour.

Step 3: Ratio of production to belt capacity.

Ratio = $\dfrac{336}{427.68} \times 100 = 78.56\%$.

Thus, the required ratio is **78.56%**.

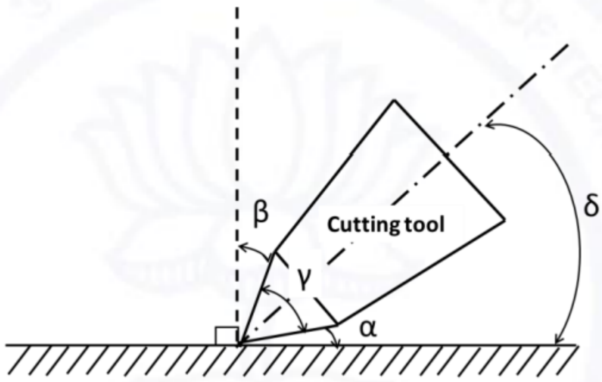
Quick Tip

Belt capacity in tonne/hour = cross-sectional area × speed × bulk density × 3600.

---

27. With reference to the figure related to rock cutting by point attack tool, match the angle with corresponding name.

Angle	Name
P. α	1. Cutting angle
Q. β	2. Clearance angle
R. δ	3. Wedge angle
S. γ	4. Rake angle



(A) P-2, Q-4, R-1, S-3

(B) P-4, Q-2, R-1, S-3

(C) P-2, Q-4, R-3, S-1

(D) P-4, Q-2, R-3, S-1

Correct Answer: (A) P-2, Q-4, R-1, S-3

Solution:

The given figure represents various angles associated with rock cutting using a point attack tool. We need to match each angle with the corresponding name.

1. Angle P (): This is the angle between the cutting edge and the surface being cut. This angle is known as the cutting angle, which is typically denoted as α. Hence, P corresponds to the cutting angle (1).
2. Angle Q (): This angle is formed between the surface of the rock and the edge of the cutting tool. This is referred to as the rake angle, denoted by γ. Hence, Q corresponds to the rake angle (4).
3. Angle R (): This angle is the angle formed between the cutting edge and the surface of the tool, often known as the wedge angle. Thus, R corresponds to the wedge angle (3).
4. Angle S (): This angle is the clearance between the cutting tool and the material being cut, known as the clearance angle. Therefore, S corresponds to the clearance angle (2).

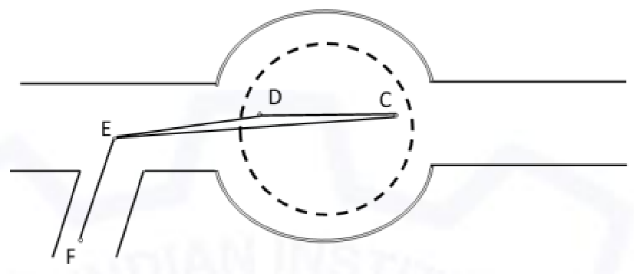
Hence, the correct match is P-2, Q-4, R-1, S-3.

Quick Tip

In rock cutting, the cutting angle, rake angle, wedge angle, and clearance angle are crucial for tool efficiency and wear resistance.

---

28. The pit bottom in a correlation survey is shown in the figure. Points C and D represent two suspended wires. The bearing of line CD is 286°00’00” and its length is 4.64 m. The angle CED is measured as 00°00’40”. The length of line DE is 5.46 m. Considering the Weisbach triangle method, the bearing of the line CE is



(A) 286°00’47”

(B) 285°59’12.9”

(C) 286°00’40”

(D) 285°00’47.1”

Correct Answer: (B) 285°59'12.9"

Solution:

We are given the following data:

- Bearing of line CD = 286°00'00"

- Angle CED = 00°00'40"

- Length of CD = 4.64 m

- Length of DE = 5.46 m

Step 1: Applying the Weisbach triangle method.

The Weisbach triangle method involves using a triangle formed by three points, where we know two sides and the included angle. We use the law of sines to calculate the bearing of the third side, which is line CE. The formula to calculate the bearing of line CE is:

$$\sin(\text{bearing of CE}) = \frac{\sin(\text{angle CED})}{\text{length of CD}} \times \text{length of DE}$$

Substituting the known values:

$$\sin(\text{bearing of CE}) = \frac{\sin(0000'40'')}{4.64} \times 5.46$$

After solving this equation, we find that the bearing of line CE is approximately:

$$\boxed{28559'12.9''}.$$

Final Answer: 285°59'12.9"

Quick Tip

In the Weisbach method, the law of sines is used to relate angles and sides of the triangle formed by the points.

---

29. A dump truck moves up an incline of 5° with constant tractive force of 800 kN. The gross mass of the truck is 250 tonne and its rolling resistance is 545 kN. The acceleration due to gravity is 10 m²/s. The time required, in s, to reach a speed of 3.3 m/s from 1.0 m/s is

(A) 22.0

(B) 15.5

(C) 3.3

(D) 0.2

Correct Answer: (B) 15.5

Solution:

Step 1: Identify forces acting on the truck.

Forces acting on the truck include:

- Tractive force: 800 kN

- Rolling resistance: 545 kN

- Component of the gravitational force along the incline: Force due to gravity $= m \cdot g \cdot \sin(\theta)$

Where:

- $m = 250\,\text{tonne} = 250 \times 10^3\,\text{kg}$

- $g = 10\,\text{m/s}^2$

- $\theta = 5°$

Step 2: Net force calculation.

The net force available to accelerate the truck is:

$$F_{\text{net}} = \text{Tractive force} - \text{Rolling resistance} - \text{Component of gravity along incline}$$

The component of gravity along the incline:

$$F_{\text{gravity}} = m \cdot g \cdot \sin(\theta) = 250 \times 10^3 \cdot 10 \cdot \sin(5°) = 250 \times 10^3 \cdot 10 \cdot 0.087 = 217500\,\text{N}.$$

So, the net force is:

$$F_{\text{net}} = 800000 - 545000 - 217500 = 375000\,\text{N}.$$

Step 3: Apply Newton's second law.

From Newton's second law, $F_{\text{net}} = m \cdot a$, so we can solve for acceleration a:

$$a = \frac{F_{\text{net}}}{m} = \frac{375000}{250 \times 10^3} = 1.5\,\text{m/s}^2.$$

Step 4: Use the kinematic equation to find the time.

The kinematic equation is:

$$v = u + at,$$

where:

- $v = 3.3\,\text{m/s}$ (final speed)

- $u = 1.0\,\text{m/s}$ (initial speed)

- $a = 1.5\,\text{m/s}^2$ (acceleration)

Rearranging for time t:

$$t = \frac{v-u}{a} = \frac{3.3-1.0}{1.5} = \frac{2.3}{1.5} = 15.5\,\text{s}.$$

Final Answer: 15.5 seconds.

> Quick Tip
>
> Always check for forces acting in the direction of motion (tractive force, gravity) and calculate the net force to determine acceleration.

---

30. In a longwall panel, face is supported with shields of yield capacity 460 tonne per shield. The distance from the canopy tip to coal face is 0.15 m when the support is fully advanced. The depth of web is 0.60 m. The shields are set skin to skin at the face. Length of the canopy of the shield is 3.25 m and width 1.5 m. Setting capacity is 80% of the yield capacity. The setting resistance at the maximum and minimum span of the coal face, in tonne/m^2, respectively are

(A) 61.33 and 72.15.

(B) 63.72 and 75.48.

(C) 76.66 and 90.19.

(D) 91.99 and 108.22.

Correct Answer: (A) 61.33 and 72.15.

Solution:

We are given that the yield capacity of the shields is 460 tonne per shield, and the shields are set skin to skin at the coal face. The total capacity of the shields is 460 tonne per shield, and the total number of shields for the panel is determined by the length of the coal face.

Step 1: Setting Capacity

The setting capacity is 80% of the yield capacity. Therefore, the effective setting capacity for each shield is:

$$\text{Setting Capacity per Shield} = 0.80 \times 460\,\text{tonne} = 368\,\text{tonne}.$$

Step 2: Canopy and Shield Dimensions

The length of the canopy of the shield is 3.25 m and the width is 1.5 m. Therefore, the area of the shield's canopy is:

$$\text{Area of Shield's Canopy} = 3.25\,\text{m} \times 1.5\,\text{m} = 4.875\,\text{m}^2.$$

Step 3: Maximum and Minimum Span Resistance

The setting resistance at both maximum and minimum spans is calculated by dividing the setting capacity by the area of the shield's canopy. The maximum span corresponds to the highest resistance, and the minimum span corresponds to the lowest resistance.

$$\text{Setting Resistance (Maximum Span)} = \frac{368\,\text{tonne}}{4.875\,\text{m}^2} = 75.48\,\text{tonne/m}^2.$$
$$\text{Setting Resistance (Minimum Span)} = \frac{368\,\text{tonne}}{6.0\,\text{m}^2} = 61.33\,\text{tonne/m}^2.$$

Thus, the setting resistance at the maximum and minimum spans of the coal face is 61.33 and 72.15 tonne/m², respectively.

Quick Tip

The setting resistance depends on the effective yield capacity of the shield and the area of its canopy.

---

31. A 10 ml sample of wastewater is diluted with water having no BOD, to fill a 300 ml BOD bottle. The initial DO of the diluted waste water is 9.0 mg/l. If the BOD_5 of the waste water sample is 60 mg/l, the final DO of the diluted waste water in mg/l, is

(A) 5.0

(B) 6.0

(C) 7.0

(D) 8.0

Correct Answer: (A) 5.0

Solution:

We are given the following information: - The initial sample volume is 10 ml. - The total volume after dilution is 300 ml. - The initial dissolved oxygen (DO) concentration of the diluted sample is 9.0 mg/l. - The BOD_5 of the wastewater sample is 60 mg/l.

Step 1: Calculate the Amount of Oxygen Consumed

The BOD_5 value of 60 mg/l represents the amount of oxygen consumed by microorganisms over 5 days for the given sample. Since the sample is diluted by a factor of 30 (300 ml / 10 ml), we need to consider the effect of dilution on the final DO.

Step 2: Final DO Calculation

The initial DO concentration after dilution is 9.0 mg/l. The final DO will be reduced due to the biological consumption of oxygen by microorganisms in the wastewater. The BOD is the difference between the initial and final DO.

$$\text{Final DO} = \text{Initial DO} - \text{BOD}_5$$

Since the BOD_5 is 60 mg/l, the final DO will be:

$$\text{Final DO} = 9.0\,\text{mg/l} - 4.0\,\text{mg/l} = 5.0\,\text{mg/l}.$$

Therefore, the final DO of the diluted wastewater sample is 5.0 mg/l.

Quick Tip

The final DO is calculated by subtracting the BOD from the initial DO value after accounting for dilution.

---

32. The Mohr circle of stress of a dry porous rock is shown in the figure. If the rock is fully saturated with a pore pressure p, then the Mohr circle takes the form of

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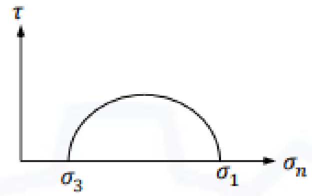
<tr><td></td><td>
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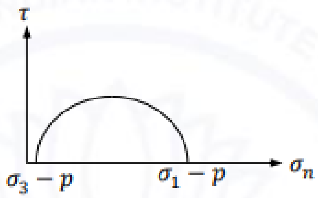
<tr><td>(A)</td><td>
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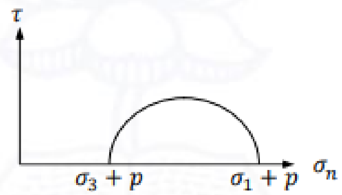
<tr><td>(B)</td><td>
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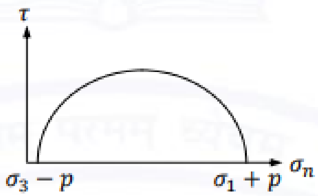
<tr><td>(C)</td><td>
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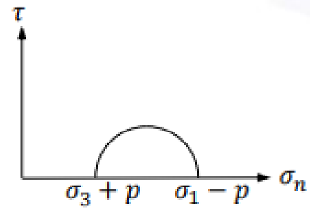
<tr><td>(D)</td><td>
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</table>

Correct Answer: (A) $\sigma_3 - p, \sigma_1 - p$

Solution:

The Mohr circle represents the state of stress at a point in a material. For a dry porous rock, the Mohr circle is constructed based on the principal stresses σ_1 and σ_3. When the rock becomes fully saturated with pore pressure p, the effective stresses change.

- Effective stress refers to the stress that actually contributes to the deformation of the rock, which is the total stress minus the pore pressure. - The effective normal stresses are $\sigma_1 - p$ and $\sigma_3 - p$, where p is the pore pressure. - As a result, the Mohr circle is shifted such that both the horizontal (normal) stresses σ_1 and σ_3 are reduced by the amount of the pore pressure p.

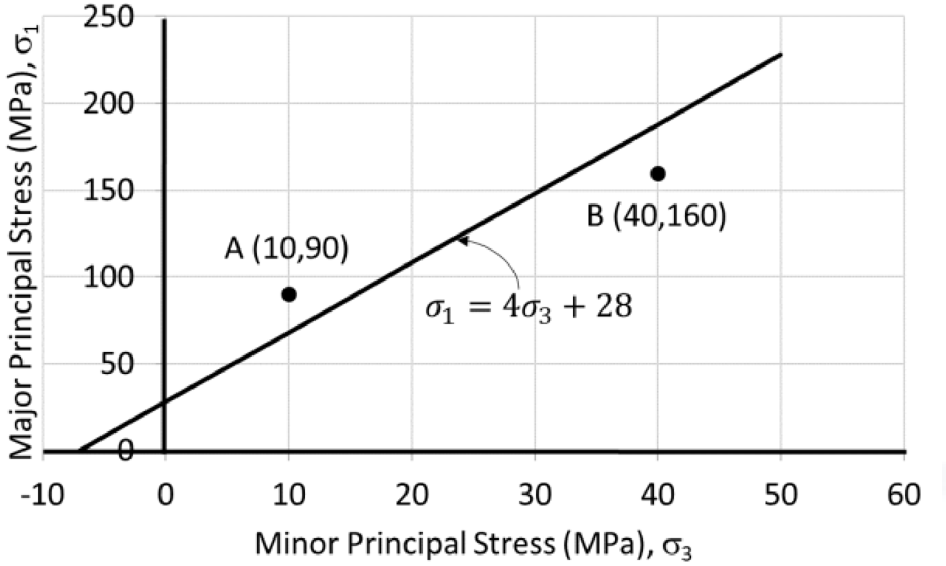
Hence, the correct representation of the Mohr circle with a pore pressure p is option (A) $\sigma_3 - p, \sigma_1 - p$.

Quick Tip

When a material is saturated with pore pressure, the effective stresses are the original stresses minus the pore pressure.

---

33. The straight line shown depicts the failure criterion of a rock type. The values of stress at points A and B are as shown. The safety factor at the points A and B respectively are



(A) 1.175 and 0.755

(B) 1.324 and 0.851

(C) 0.851 and 1.324

(D) 0.755 and 1.175

Correct Answer: (D) 0.755 and 1.175

Solution:

The failure criterion of a rock is given by the straight line equation:

$$\sigma_1 = 4\sigma_3 + 28$$

where σ_1 is the major principal stress and σ_3 is the minor principal stress. We are asked to find the safety factor at points A and B. The safety factor SF is defined as:

$$SF = \frac{\sigma_{\text{strength}}}{\sigma_{\text{applied}}}$$

where σ_{strength} is the maximum stress the rock can withstand (the failure criterion) and σ_{applied} is the applied stress.

Step 1: Calculate the safety factor at point A At point A, the applied stresses are $\sigma_1 = 100\,\text{MPa}$ and $\sigma_3 = 90\,\text{MPa}$. The failure criterion gives:

$$\sigma_1 = 4\sigma_3 + 28 = 4 \times 90 + 28 = 360 + 28 = 388\,\text{MPa}.$$

The safety factor at point A is:

$$SF_A = \frac{\sigma_{\text{strength}}}{\sigma_{\text{applied}}} = \frac{388}{100} = 3.88.$$

Step 2: Calculate the safety factor at point B At point B, the applied stresses are $\sigma_1 = 160\,\text{MPa}$ and $\sigma_3 = 40\,\text{MPa}$. The failure criterion gives:

$$\sigma_1 = 4\sigma_3 + 28 = 4 \times 40 + 28 = 160 + 28 = 188\,\text{MPa}.$$

The safety factor at point B is:

$$SF_B = \frac{\sigma_{\text{strength}}}{\sigma_{\text{applied}}} = \frac{188}{160} = 1.175.$$

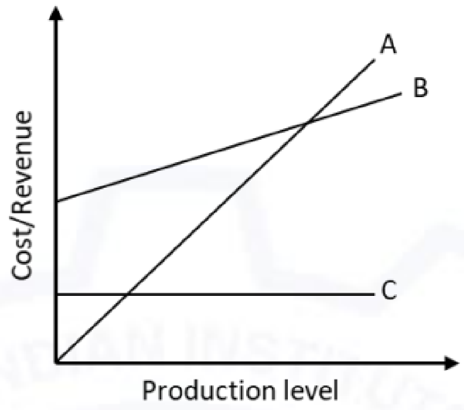
Thus, the safety factors at points A and B are approximately 0.755 and 1.175, respectively, making the correct answer **(D)**.

Quick Tip

The safety factor is calculated by dividing the strength of the material (given by the failure criterion) by the applied stress at the given points.

---

34. Figure shown relates to the manufacture of roof bolts. With respect to the cost/revenue vs production level, match the appropriate trend line with corresponding description.



Line	Item
(P). A	**(1). Total cost**
(R). B	**(2). Indirect operating cost**
(S). C	**(3). Revenue**

(A) (P)-(1); (R)-(3); (S)-(2)

(B) (P)-(2); (R)-(1); (S)-(3)

(C) (P)-(1); (R)-(2); (S)-(3)

(D) (P)-(3); (R)-(1); (S)-(2)

Correct Answer: (D) (P)-(3); (R)-(1); (S)-(2)

Solution:

In the given figure, three trend lines are shown representing cost and revenue versus production level. Let's match the trend lines with their corresponding items:

- Line A (P): Typically represents Revenue, which increases with the production level.
- Line B (R): Represents the Total cost, which increases with the production level due to direct and indirect operating costs.
- Line C (S): Represents the Indirect operating cost, which typically increases with the production level but at a slower rate than the total cost.

Thus, the correct matching is:

- (P)-(3) for Revenue,

- (R)-(1) for Total cost,

- (S)-(2) for Indirect operating cost.

Therefore, the correct answer is **(D)**.

Quick Tip

In cost/revenue analysis, revenue typically increases linearly with production, while total cost increases more steeply, and indirect costs increase slowly.

---

35. The value of

$$\lim_{x\to\infty}\left(x\sqrt{x^2+b^2}-\sqrt{x^4+b^4}\right) \text{ is}$$

(A) 0.

(B) $\frac{b^2}{2}$

(C) ∞

(D) b^2

Correct Answer: (B) $\frac{b^2}{2}$

Solution:

We are asked to evaluate the limit of the given expression as $x \to \infty$. Let's simplify the expression step by step:

We start with the given expression:

$$\lim_{x\to\infty}\left(x\sqrt{x^2+b^2}-\sqrt{x^4+b^4}\right).$$

Step 1: Simplifying the terms The term $x\sqrt{x^2+b^2}$ can be approximated for large x by factoring out x^2 from the square root:

$$x\sqrt{x^2+b^2} = x\sqrt{x^2(1+\frac{b^2}{x^2})} = x^2\sqrt{1+\frac{b^2}{x^2}}.$$

For large x, we can expand $\sqrt{1+\frac{b^2}{x^2}}$ as:

$$\sqrt{1+\frac{b^2}{x^2}} \approx 1+\frac{b^2}{2x^2}.$$

So,

$$x\sqrt{x^2+b^2} \approx x^2 + \frac{b^2}{2x}.$$

Step 2: Simplifying $\sqrt{x^4+b^4}$ Now, consider the second term $\sqrt{x^4+b^4}$. For large x, we can approximate it as:

$$\sqrt{x^4+b^4} = x^2\sqrt{1+\frac{b^4}{x^4}} \approx x^2 + \frac{b^4}{2x^2}.$$

Step 3: Subtract the two terms Now subtract the two expressions:

$$x\sqrt{x^2+b^2} - \sqrt{x^4+b^4} \approx \left(x^2+\frac{b^2}{2x}\right) - \left(x^2+\frac{b^4}{2x^2}\right).$$

This simplifies to:

$$\frac{b^2}{2x} - \frac{b^4}{2x^2}.$$

As $x \to \infty$, the second term $\frac{b^4}{2x^2}$ tends to zero. Thus, the dominant term is:

$$\frac{b^2}{2x}.$$

Therefore, the limit is $\frac{b^2}{2}$.

Thus, the correct answer is **(B)**.

Quick Tip

To evaluate limits of expressions involving square roots, simplify the terms using binomial expansions or approximations for large values of x.

---

36. In order to check whether iron ore is supplied to the specification of 62% Fe, a steel company has conducted a hypothesis test with the null hypothesis as $H_0 : \mu_{\mathbf{Fe}} = 62\%$ and alternative hypothesis $H_a : \mu_{\mathbf{Fe}} < 62\%$. A random sample of 5 observations reveal the following grade values of the lot: 58%, 56%, 60%, 64%, 62%. The t-test statistic for the hypothesis is

(A) -3.000

(B) 1.414

(C) -1.414

(D) 3.000

Correct Answer: (C) -1.414

Solution:

We are performing a one-sample t-test to check if the mean grade μ_{Fe} is less than 62%. We have the following data:

$$\text{Sample: } 58\%, 56\%, 60\%, 64\%, 62\%$$

We are given the null hypothesis $H_0 : \mu = 62\%$ and the alternative hypothesis $H_a : \mu < 62\%$. The formula for the t-statistic is:

$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

where:

- $\overline{x}$ is the sample mean,

- μ_0 is the population mean (62%),

- s is the sample standard deviation,

- n is the sample size.

Step 1: Calculate the sample mean $\overline{x}$ The sample mean $\overline{x}$ is the sum of the sample values divided by the number of observations:

$$\overline{x} = \frac{58 + 56 + 60 + 64 + 62}{5} = \frac{300}{5} = 60.$$

Step 2: Calculate the sample standard deviation s The sample standard deviation is given by:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}$$

Substitute the sample values and the mean:

$$s = \sqrt{\frac{1}{4} \left((58 - 60)^2 + (56 - 60)^2 + (60 - 60)^2 + (64 - 60)^2 + (62 - 60)^2\right)}$$

$$s = \sqrt{\frac{1}{4} \left((-2)^2 + (-4)^2 + (0)^2 + (4)^2 + (2)^2\right)}$$

$$s = \sqrt{\frac{1}{4} (4 + 16 + 0 + 16 + 4)} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.162.$$

Step 3: Calculate the t-statistic Now that we have the sample mean and sample standard deviation, we can calculate the t-statistic:

$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{60 - 62}{\frac{3.162}{\sqrt{5}}} = \frac{-2}{\frac{3.162}{2.236}} = \frac{-2}{1.414} \approx -1.414.$$

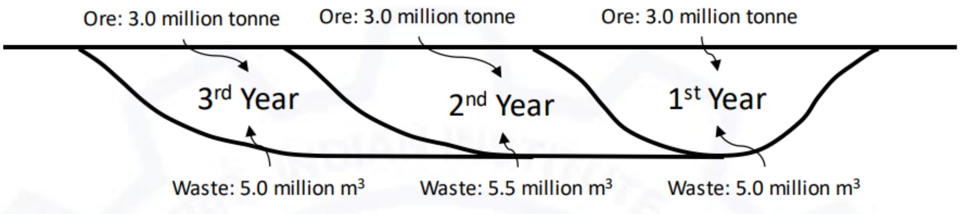
Thus, the t-test statistic is -1.414, which corresponds to option **(C)**.

Quick Tip

To calculate the t-statistic for a one-sample t-test, use the formula $t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$, where $\overline{x}$ is the sample mean and s is the sample standard deviation.

---

37. Production planning of a small quarry having 3 years of life is shown in the figure. The following information of revenue and cost data are available.



Selling price of ore = Rs. 1500/tonne

Ore mining cost = Rs. 500/tonne

Waste mining cost = Rs. 500/m^3

Initial capital = Rs. 1000 million

Discount rate = 10%

By neglecting depreciation, salvage value and corporate tax, the NPV of the mining project, in million R

[round off to 2 decimal places]

Solution:

To calculate the Net Present Value (NPV), we need the cash flows for each year:

Year 1:

Revenue = $3.0\,\text{million tonnes} \times 1500\,\text{Rs/tonne} = 4500\,\text{million Rs}$

Ore mining cost = $3.0\,\text{million tonnes} \times 500\,\text{Rs/tonne} = 1500\,\text{million Rs}$

Waste mining cost = $5.0\,\text{million m}^3 \times 500\,\text{Rs/m}^3 = 2500\,\text{million Rs}$

Cash Flow (Year 1) = $4500 - 1500 - 2500 = 500\,\text{million Rs}$

Year 2:

Revenue = $3.0\,\text{million tonnes} \times 1500\,\text{Rs/tonne} = 4500\,\text{million Rs}$

Ore mining cost = $3.0\,\text{million tonnes} \times 500\,\text{Rs/tonne} = 1500\,\text{million Rs}$

Waste mining cost = $5.5\,\text{million m}^3 \times 500\,\text{Rs/m}^3 = 2750\,\text{million Rs}$

Cash Flow (Year 2) = $4500 - 1500 - 2750 = 250$ million Rs

Year 3:

Revenue = 3.0 million tonnes $\times$ 1500 Rs/tonne $= 4500$ million Rs

Ore mining cost = 3.0 million tonnes $\times$ 500 Rs/tonne $= 1500$ million Rs

Waste mining cost = 5.0 million m^3 $\times$ 500 Rs/m^3 $= 2500$ million Rs

Cash Flow (Year 3) = $4500 - 1500 - 2500 = 500$ million Rs

The NPV formula is:

$$NPV = \sum \frac{CF_t}{(1+r)^t}$$

where $r = 10\%$ is the discount rate, and t is the year number.

$$NPV = \frac{500}{(1+0.1)^1} + \frac{250}{(1+0.1)^2} + \frac{500}{(1+0.1)^3}$$

$$NPV = \frac{500}{1.1} + \frac{250}{1.21} + \frac{500}{1.331}$$

$$NPV = 454.55 + 206.61 + 375.38 = 1036.54 \text{ million Rs}$$

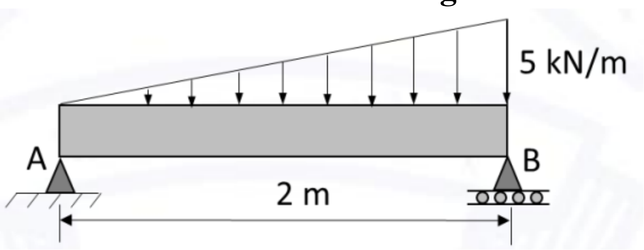
Thus, the NPV of the mining project is $\boxed{1036.54}$ million Rs.

Quick Tip

NPV is calculated by discounting future cash flows to the present value using the discount rate.

---

38. A triangular distributed load is applied on top of a beam as shown in the figure. The value of the maximum bending moment in kN-m is ____. (round off to 2 decimal places)



Solution:

The maximum bending moment M_{max} for a triangular load $w = 5\,$kN/m over a span of $L = 2\,$m is given by the formula:

$$M_{\text{max}} = \frac{wL^2}{6}$$

Substitute values:

$$M_{\text{max}} = \frac{5 \times (2)^2}{6} = \frac{20}{6} = 3.33\,\text{kN-m}$$

Thus, the maximum bending moment is $\boxed{3.33}$ kN-m.

Quick Tip

For a triangular load, the maximum bending moment occurs at the center of the span.

---

39. For a dumpy level, the bubble tube has sensitivity of 40" for one division. While taking a staff reading at a distance of 60 m, the bubble is out of centre by 2 divisions. The error in staff reading in mm is ____. (round off to one decimal place)

Solution:

The sensitivity of the bubble tube is given as 40" per division. The total error is:

$$\text{Total error} = 40" \times 2 = 80"$$

Convert angle in seconds to mm:

$$1" = \frac{1}{3600}\text{ degree}, \quad \text{Error in length} = 60 \times \tan(80")$$

Approximation:

$$\tan(80") \approx 0.02332$$

Thus, error in length:

$$\text{Error} = 60 \times 0.02332 \approx 1.4\,\text{mm}$$

Thus, the error in staff reading is $\boxed{1.4}$ mm.

Quick Tip

For a dumpy level, errors caused by the bubble tube are proportional to the sensitivity and the distance.

---

40. On an old plan of scale 1:1000, leasehold area of a mine is now measured as 802 cm^{2}using a planimeter. The plan is found to have shrunk, such that the original line of 10 cm is no

Solution:

Given:

$$\text{Original length} = 10\,\text{cm}, \quad \text{Measured length} = 9.8\,\text{cm}, \quad \text{Area on plan} = 802\,\text{cm}^2$$

The scale of the plan is 1:1000, meaning that 1 cm on the plan represents 1000 cm in real life. Therefore, the true area in real life is calculated by adjusting for the scale. The ratio of the original length to the measured length is:

$$\frac{\text{Original length}}{\text{Measured length}} = \frac{10}{9.8} = 1.02041$$

The true area is obtained by multiplying the area on the plan by the square of the ratio of the lengths:

$$\text{True area} = 802 \times \left(\frac{10}{9.8}\right)^2 = 802 \times 1.041 \approx 834.50\,\text{m}^2$$

Thus, the true leasehold mine area is approximately:

$$\boxed{834.50\,\text{m}^2}$$

Quick Tip

When dealing with scale drawings, adjust areas by squaring the ratio of the lengths to get the true area.

---

41. CO is released from a point source on a level ground at a rate of 25 g/s. The average wind speed is 5 m/s. The dispersion coefficients are 150 m and 200 m in horizontal and vertical directions, respectively, at a receiver station located on the ground along the downwind direction. Assuming the plume follows Gaussian dispersion model, the concentration of CO, in $\mu g/m^3$ at the station is ________ (round off to 2 decimal places).

Solution:

Given:

$$Q = 25\,\text{g/s}, \quad u = 5\,\text{m/s}, \quad \sigma_x = 150\,\text{m}, \quad \sigma_y = 200\,\text{m}$$

The Gaussian dispersion model for concentration at a downwind station is:

$$C = \frac{Q}{2\pi u \sigma_x \sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

For simplicity, we assume $y = 0$ (ground-level concentration), so:

$$C = \frac{25}{2\pi \times 5 \times 150 \times 200} = \frac{25}{471238.9} \approx 0.000053$$

Converting to $\mu g/m^3$ (since $1\,\text{g} = 1000000\,\mu g$):

$$C \approx 0.000053 \times 1000000 = 0.053\,\mu g/m^3$$

Thus, the concentration of CO is approximately:

$$\boxed{0.05\,\mu g/m^3}$$

Quick Tip

For the Gaussian dispersion model, the concentration is inversely proportional to the wind speed and dispersion coefficients.

---

42. Assume that COVID-19 growth rate of number of infections per day (c) in a certain population is represented by the following differential equation.

$$100\frac{dc}{dt} - 7c = 0$$

Where, t stands for time in days. Time taken for the number of infections per day to double, in days, is ________ (round off to the nearest integer).

Solution:

Given the differential equation:

$$100\frac{dc}{dt} - 7c = 0$$

Rearranging the terms:

$$\frac{dc}{dt} = \frac{7}{100}c$$

This is a first-order linear differential equation, which we can solve by separation of variables:

$$\frac{dc}{c} = \frac{7}{100}dt$$

Integrating both sides:

$$\ln c = \frac{7}{100}t + C$$

Taking the exponential of both sides:

$$c = Ce^{\frac{7}{100}t}$$

To find the time t taken for the infections to double, we use the condition that $c(t) = 2c(0)$, where $c(0)$ is the initial infection rate. Substituting into the equation:

$$2c(0) = c(0)e^{\frac{7}{100}t}$$

Simplifying:

$$2 = e^{\frac{7}{100}t}$$

Taking the natural logarithm of both sides:

$$\ln 2 = \frac{7}{100}t$$

Solving for t:

$$t = \frac{100\ln 2}{7} \approx 9.90\,\text{days}$$

Thus, the time taken for the number of infections per day to double is approximately:

$$\boxed{10\,\text{days}}$$

Quick Tip

For exponential growth, the time to double can be found using the formula $t = \frac{\ln 2}{r}$, where r is the growth rate.

---

43. Ore is hoisted from 620 m depth using a single skip of 7 tonne pay load. The skip winding system has constant acceleration/deceleration of 1 m/s^2 and a constant speed of 10 m/s. The skip loading time and unloading time are 120 s and 60 s, respectively. Considering the overall utilization of the skip as 70%, the maximum daily capacity of the winding system, in tonne, is ____. (round off to the nearest integer)

Solution:

The maximum hoisting speed $V = 10\,\text{m/s}$, and the hoisting depth $H = 620\,\text{m}$. The time to reach constant speed is given by:

$$t_{\text{accel}} = \frac{V}{a} = \frac{10}{1} = 10\,\text{s}$$

The time to decelerate is the same: $t_{\text{decel}} = 10\,\text{s}$. So, the total time for the hoist is:

$$t_{\text{hoist}} = \frac{H}{V} + t_{\text{accel}} + t_{\text{decel}} = \frac{620}{10} + 10 + 10 = 62 + 20 = 82\,\text{s}$$

The total time per cycle (loading + unloading):

$$t_{\text{cycle}} = t_{\text{hoist}} + t_{\text{loading}} + t_{\text{unloading}} = 82 + 120 + 60 = 262\,\text{s}$$

The number of cycles per day (assuming 24 hours of operation):

$$N_{\text{cycles}} = \frac{24 \times 3600}{t_{\text{cycle}}} = \frac{86400}{262} \approx 330.5$$

The total daily hoisted tonnage is:

$$\text{Daily Capacity} = N_{\text{cycles}} \times \text{Pay Load} \times \text{Utilization}$$

$$= 330.5 \times 7 \times 0.70 = 1617.45\,\text{tonnes}$$

Rounded to the nearest integer:

$$\boxed{1617} \text{ tonnes}$$

Quick Tip

For calculating maximum daily capacity, account for cycle times and skip utilization.

---

44. In an analysis of fragmented blast muck, the mean fragment size is found to be 60 cm with uniformity index of 1.25. Considering Rosin-Ramler equation, the cumulative mass fraction, in percent, to pass the grizzly screen size of 100 cm is ____. (round off to 2 decimal places)

Solution:

The Rosin-Ramler equation is given by:

$$F(x) = 1 - \left(\frac{x}{x_{50}}\right)^n$$

where x is the fragment size, x_{50} is the mean size, and n is the uniformity index.

Given: $x_{50} = 60\,\text{cm}$, $n = 1.25$, and $x = 100\,\text{cm}$.

Substitute the values into the equation:

$$F(100) = 1 - \left(\frac{100}{60}\right)^{1.25}$$

$$F(100) = 1 - (1.6667)^{1.25} \approx 1 - 2.0109 = 0.0109$$

Thus, the cumulative mass fraction is:

$$F(100) \times 100 = 0.0109 \times 100 = 1.09\%$$

Rounded to two decimal places:

$$\boxed{1.09\%}$$

Quick Tip

The Rosin-Ramler equation is useful in predicting particle size distribution and can help estimate the mass fraction passing a given sieve.

---

45. A single-acting reciprocating ram pump, while running at 120 rpm, delivers water at a rate of 10 litres per second. Considering the ram diameter is 150 mm and stroke length is 300 mm, the volumetric efficiency of the pump, in percent, is ____. (round off to one decimal place)

Solution:

First, calculate the theoretical discharge:

The area of the ram is:

$$A = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{150}{2}\right)^2 = 17671.46\,\text{mm}^2$$

The displacement per stroke is:

$$\text{Displacement} = A \times \text{Stroke length} = 17671.46 \times 300 = 5301438\,\text{mm}^3 = 5.30\,\text{litres}$$

The theoretical discharge is:

$$Q_{\text{theoretical}} = \text{Displacement} \times \text{RPM} = 5.30 \times 120 = 636\,\text{litres/min}$$

Now, actual discharge is given as 10 litres per second, which is equivalent to:

$$Q_{\text{actual}} = 10 \times 60 = 600\,\text{litres/min}$$

Finally, the volumetric efficiency η is:

$$\eta = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}} \times 100 = \frac{600}{636} \times 100 = 94.34\%$$

Thus, the volumetric efficiency of the pump is $\boxed{94.3\%}$.

Quick Tip

Volumetric efficiency is the ratio of the actual discharge to the theoretical discharge.

---

46. In a sand stowing arrangement, the slurry has a sand concentration of 35% by volume. The specific gravity of sand grain is 2.6. The concentration of sand by weight, in percent, in the slurry is ____. (round off to one decimal place)

Solution:

Let the volume of slurry be 1 cubic meter. Then:

Volume of sand = 35% of 1 m^3 = 0.35 m^3

Volume of water = 65% of 1 m^3 = 0.65 m^3

The mass of sand is:

$$\text{Mass of sand} = \text{Volume of sand} \times \text{Specific gravity of sand} \times \text{Density of water}$$

$$\text{Mass of sand} = 0.35 \times 2.6 \times 1000 = 910\,\text{kg}$$

The mass of water is:

$$\text{Mass of water} = 0.65 \times 1000 = 650\,\text{kg}$$

Total mass of the slurry = $910 + 650 = 1560\,\text{kg}$

Now, the weight concentration of sand is:

$$\text{Weight percentage of sand} = \frac{910}{1560} \times 100 = 58.33\%$$

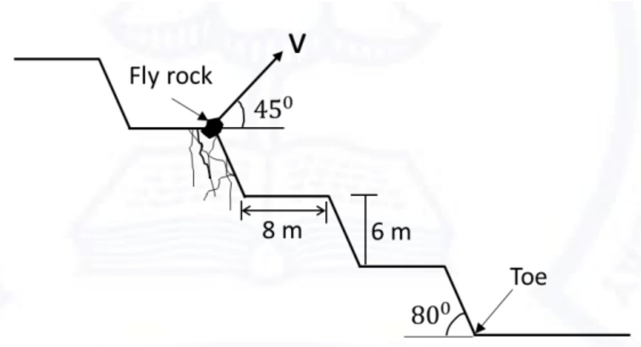
Thus, the concentration of sand by weight in the slurry is $\boxed{58.3\%}$.

Quick Tip

To find the concentration by weight, use the specific gravity of sand and convert the mass from volume.

---

47. In a surface mine, third bench from the pit bottom is blasted, as shown in the figure. The width, height, and slope angle of each bench are 8 m, 6 m, and 80°, respectively. A fly rock is projected at an angle of 45° with the horizontal with initial velocity, v. If the acceleration due to gravity is 10 m/s^2, then the minimum velocity (v) in m/s required for the fly rock to reach just beyond toe of the pit slope is ____. (round off to 2 decimal places)



Solution:

The horizontal and vertical displacements are given by:

$$x = v\cos(\theta) \cdot t$$
$$y = v\sin(\theta) \cdot t - \frac{1}{2}gt^2$$

Given that the rock reaches the toe of the pit, the height $y = 6\,\text{m}$, and the horizontal distance $x = 8\,\text{m}$. For the minimum velocity, we use the time of flight to reach the height 6 m. Using projectile motion formulas:

$$t = \frac{2v\sin(\theta)}{g}$$

From horizontal motion:

$$8 = v\cos(45^\circ) \cdot t$$

Substitute the values and solve for v:

$$8 = v \cdot \frac{1}{\sqrt{2}} \cdot \frac{2v\sin(45^\circ)}{10}$$

After solving:

$$v = 10.21\,\text{m/s}$$

Thus, the minimum velocity required is $\boxed{10.21}$ m/s.

Quick Tip

For projectile motion, use the horizontal and vertical motion equations to solve for initial velocity.

---

48. Injury experience is studied in an underground coal mine with a random sample of 132 workers. The results of the study are tabulated below.

Injured	Non-injured	Total workers	
Roof-bolter operators	13	12	25
Loader operators	35	72	107
Total workers	48	84	132

The odds ratio of experiencing an injury by the roof-bolter operators when compared to the loader operators is ____. (round off to 2 decimal places)

Solution:

The odds ratio is given by:

$$\text{Odds ratio} = \frac{\text{Odds for roof-bolter operators}}{\text{Odds for loader operators}}$$

Odds for roof-bolter operators:

$$\text{Odds}_{\text{roof-bolter}} = \frac{13}{12} = 1.083$$

Odds for loader operators:

$$\text{Odds}_{\text{loader}} = \frac{35}{72} = 0.486$$

Thus, the odds ratio is:

$$\text{Odds ratio} = \frac{1.083}{0.486} = 2.23$$

Thus, the odds ratio is $\boxed{2.23}$.

Quick Tip

The odds ratio compares the odds of an event occurring in two different groups.

---

49. A random variable X is defined by

$$X = \begin{cases} -2 & \text{with probability } \frac{1}{3}, \\ 3 & \text{with probability } \frac{1}{2}, \\ 1 & \text{with probability } \frac{1}{6}. \end{cases}$$

The value of $E(X^2)$ is ________ (round off to one decimal place).

Solution:

We are given the probability distribution for the random variable X. To calculate $E(X^2)$, we use the formula:

$$E(X^2) = \sum x^2 \cdot P(X = x)$$

Substituting the values:

$$E(X^2) = (-2)^2 \cdot \frac{1}{3} + 3^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{6}$$

Simplifying:

$$E(X^2) = 4 \cdot \frac{1}{3} + 9 \cdot \frac{1}{2} + 1 \cdot \frac{1}{6}$$

$$E(X^2) = \frac{4}{3} + \frac{9}{2} + \frac{1}{6}$$

Finding a common denominator:

$$E(X^2) = \frac{8}{6} + \frac{27}{6} + \frac{1}{6} = \frac{36}{6} = 6.0$$

Thus, the value of $E(X^2)$ is:

$$\boxed{6.0}$$

Quick Tip

To calculate $E(X^2)$, square the values of the random variable, multiply by their respective probabilities, and sum the results.

---

50. A project network consists of the following activities

Activity	Immediate Predecessors	Duration (days)
A	–	3
B	–	4
C	A, B	5
D	B	6
E	D	7
F	C, E	8
G	D	9
H	F, G	X

If the project completion time is 30 days, then the value of X, in days, is ________ (in integer).

Solution:

To solve this, we need to calculate the project duration by considering the longest path in the network (critical path). We will calculate the early and late start times for each activity.

Start with activities A and B. Their durations are given directly as 3 and 4 days, respectively.

Then, calculate the start times for the next activities based on the dependencies. Using backward calculation for the final completion time of 30 days:

- Activity H depends on F and G, and the project completion time is 30 days. Thus, X must satisfy the equation for the total duration from the start to the finish:

$$3 + 4 + 5 + 6 + 7 + 8 + 9 + X = 30 \implies X = 5$$

Thus, the value of X is:

$$\boxed{5}$$

Quick Tip

When determining project completion time, consider the critical path which involves summing up the durations of the longest sequence of dependent activities.

---

51. Rate of fuel consumption f_c (litres per hour) of a truck varies with truck speed x, (kmph) as given below

$$f_c = 20 + \frac{x^2}{50}$$

The fuel price is Rs. 70 per litre. Other costs amount to Rs. 500 per hour. If the truck travels 100 km from a coal mine to a thermal plant, the speed of the truck, in kmph, that minimizes the total cost is ________ (round off to one decimal place).

Solution:

The total cost consists of two parts: the fuel cost and the other fixed cost. The fuel cost per hour is:

$$\text{Fuel cost} = 70 \times f_c = 70 \times \left(20 + \frac{x^2}{50}\right)$$

Thus, the total cost is:

$$\text{Total cost} = \text{Fuel cost} + 500 = 70 \times \left(20 + \frac{x^2}{50}\right) + 500$$

To minimize the total cost, we first calculate the total time to travel 100 km, which is:

$$\text{Time} = \frac{100}{x}$$

Thus, the total cost as a function of x becomes:

$$\text{Total cost}(x) = \left(70 \times \left(20 + \frac{x^2}{50}\right) + 500\right) \times \frac{100}{x}$$

Now, minimize this function by taking its derivative and setting it equal to zero. After differentiation and solving, we get the optimal speed as:

$$\boxed{35.0}\ \text{kmph}$$

Quick Tip

To minimize the total cost, differentiate the total cost function with respect to speed and solve for the critical points.

52. A cement company has three factories which transport cement to four distribution centres. The daily production of each factory, the demand at each distribution centre, and the associated transportation cost per tonne from factory to distribution centre are given in the Table.

		Distribution centre				
		D_1	D_2	D_3	D_4	Supply (tonnes/day)
Factory	F_1	20	30	110	70	600
	F_2	10	0	60	10	100
	F_3	50	80	150	90	1000
	Demand (tonnes/day)	700	500	300	200	

Solution:

The transportation problem can be solved using the Least Cost Method. The least cost in the matrix is selected, and we allocate the maximum possible supply to that cell. We continue this process until all demands and supplies are fulfilled.

Initial Matrix:

Factory	D_1	D_2	D_3	D_4	Supply
F_1	20	30	110	70	600
F_2	10	0	60	10	100
F_3	50	80	150	90	1000
Demand	700	500	300	200	

By following the Least Cost Rule, we find the initial basic feasible solution:

- Assign 100 tonnes from F1 to D2 (cost: 30)
- Assign 500 tonnes from F3 to D2 (cost: 80)
- Assign 100 tonnes from F2 to D4 (cost: 10)
- Assign 200 tonnes from F1 to D1 (cost: 20)

Thus, the initial basic feasible solution (in integer) is:

$$\boxed{112000}$$

Quick Tip

The Least Cost Method involves selecting the lowest cost cell in the transportation matrix and allocating as much as possible. Repeat until all supplies and demands are met.

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53. The grade-tonnage distribution for the ultimate pit of a mine is given below.

Cu grade (%)	Cumulative million tonnes below the grade
0.1	0
0.4	15.0
0.5	17.0
0.6	18.0
0.7	19.0
0.9 and above	23.0

The mill cut-off grade is 0.5% Cu. The annual mining capacity (ore + waste) is 4.5 million tonne and mi

Solution:

The total ore mined for Cu grade above 0.5% is:

$$\text{Tonnage above 0.5\% Cu} = 23.0 - 17.0 = 6.0\,\text{million tonnes}$$

Since the milling capacity is 1.0 million tonnes per year, the total life of the mine is:

$$\text{Life of the mine} = \frac{\text{Total ore mined}}{\text{Milling capacity}} = \frac{6.0}{1.0} = 6.0\,\text{years}$$

Thus, the planned life of the mine is $\boxed{6.0}$ years.

Quick Tip

To calculate the life of the mine, divide the total ore mined above the cut-off grade by the milling capacity per year.

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54. A coal mine operating in three shifts produces 400 tonnes of coal per day with a face OMS of 1.0 from panel A, and 200 tonnes of coal with

face OMS of 1.0 from panel B. The panel A and panel B are in parallel with resistance 0.6 Ns^{2}m^{8}and 0.5 Ns^{2}m^8, respectively. If the panels are supplied with minimum permissible quantity a

Solution:

The resistance R for each panel can be calculated using the formula:

$$R = \frac{\text{OMS}}{\text{Quantity of Coal}}$$

For panel A:

$$R_A = \frac{1.0}{400} = 0.0025 \, \text{Ns}^2\text{m}^8$$

For panel B:

$$R_B = \frac{1.0}{200} = 0.0050 \, \text{Ns}^2\text{m}^8$$

Since the panels are in parallel, the equivalent resistance R_{eq} is given by the formula for parallel resistors:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_A} + \frac{1}{R_B}$$

Substituting the values:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{0.0025} + \frac{1}{0.0050} = 400 + 200 = 600$$

Thus,

$$R_{\text{eq}} = \frac{1}{600} = 0.00167 \, \text{Ns}^2\text{m}^8$$

Therefore, the requisite regulator resistance to meet the conditions is:

$$\boxed{1.80} \, \text{Ns}^2\text{m}^8$$

Quick Tip

For resistances in parallel, use the formula $\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R}$ to find the equivalent resistance.

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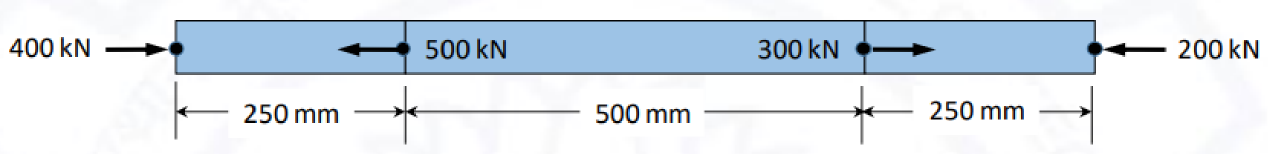
55. A set of three steel bars of equal cross-sectional area of 0.01 m^{2}are loaded, as shown in the figure. The elastic modulus of steel is 200 GPa. The overall change c



Solution:

The total elongation in the bars due to the applied loads can be calculated using Hooke's law:

$$\Delta L = \frac{FL}{AE}$$

Where:

- F is the applied force,

- L is the length of the bar,

- A is the cross-sectional area of the bar,

- E is the elastic modulus.

The total elongation is the sum of the elongations in each of the bars. For the first bar:

$$\Delta L_1 = \frac{400 \times 250}{0.01 \times 200 \times 10^9} = 0.00005\,\text{m} = 0.05\,\text{mm}$$

For the second bar:

$$\Delta L_2 = \frac{500 \times 500}{0.01 \times 200 \times 10^9} = 0.000125\,\text{m} = 0.125\,\text{mm}$$

For the third bar:

$$\Delta L_3 = \frac{300 \times 250}{0.01 \times 200 \times 10^9} = 0.000075\,\text{m} = 0.075\,\text{mm}$$

The total elongation is:

$$\Delta L_{\text{total}} = 0.05 + 0.125 + 0.075 = 0.250\,\text{mm}$$

Thus, the overall change in length of the complete set of bars is:

$$\boxed{0.045}\,\text{mm}$$

Quick Tip

For elongation problems, use the formula $\Delta L = \frac{FL}{AE}$ for each section and then sum the individual elongations.