GATE 2022 Civil Engineering (CE) Slot-1 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :100	Total questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2022 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2022 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude (GA)

1. You should ______ when to say ______.

(A) no / no

(B) no / know

(C) know / know

(D) know / no

Correct Answer: (D) know / no

Solution:

In this sentence, the correct choice is (D) because the first blank requires a verb, and ”know” is the appropriate verb for this context. The second blank requires the noun ”no,” which fits the context of the sentence.

- **First part:** ”You should know when to say” implies that one should have the knowledge of when to say something.
- **Second part:** ”no” fits the context as it is the word being referred to in the sentence.

Thus, the correct answer is **(D) know / no**.

Quick Tip

Pay attention to the verb-noun agreement in sentences. When referring to knowledge or understanding, ”know” is usually the correct verb.

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2. Two straight lines pass through the origin $(x_0, y_0) = (0, 0)$. One of them passes through the point $(x_1, y_1) = (1, 3)$ and the other passes through the point $(x_2, y_2) = (1, 2)$. What is the area enclosed between the straight lines in the interval $[0, 1]$ on the x-axis?

(A) 0.5

(B) 1.0

(C) 1.5

(D) 2.0

Correct Answer: (D) 2.0

Solution:

To solve this problem, we need to calculate the area between the two lines in the interval $[0, 1]$ on the x-axis.

Step 1: Equation of the lines.

- Line 1 (through (0, 0) and (1, 3)): The slope of the line is:

$$m_1 = \frac{3-0}{1-0} = 3.$$

The equation of the line is:

$$y_1 = 3x.$$

- Line 2 (through (0, 0) and (1, 2)): The slope of the line is:

$$m_2 = \frac{2-0}{1-0} = 2.$$

The equation of the line is:

$$y_2 = 2x.$$

Step 2: Calculate the area between the lines.

The area between the lines is given by the integral of the difference in the y-values of the two lines over the interval $[0, 1]$:

$$\text{Area} = \int_0^1 (y_1 - y_2)\, dx = \int_0^1 (3x - 2x)\, dx = \int_0^1 x\, dx.$$

The integral is:

$$\int_0^1 x\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}.$$

Thus, the area is 0.5. Therefore, the correct answer is **(A)**.

> Quick Tip
>
> To calculate the area between two curves, subtract one curve's equation from the other and integrate over the given interval.

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3. If

$$p : q = 1 : 2, \quad q : r = 4 : 3, \quad r : s = 4 : 5$$

and u is 50

(A) 2 : 15

(B) 16 : 15

(C) 1 : 5

(D) 16 : 45

Correct Answer: (C) 1 : 5

Solution:

Given the ratios:

$$p : q = 1 : 2 \quad \text{(i.e., } p = \frac{q}{2}\text{)}$$
$$q : r = 4 : 3 \quad \text{(i.e., } q = \frac{4r}{3}\text{)}$$
$$r : s = 4 : 5 \quad \text{(i.e., } r = \frac{5s}{4}\text{)}$$

We can write all terms in terms of s. Start by expressing p, q, and r in terms of s:

$$r = \frac{5s}{4}$$
$$q = \frac{4r}{3} = \frac{4 \times \frac{5s}{4}}{3} = \frac{5s}{3}$$
$$p = \frac{q}{2} = \frac{\frac{5s}{3}}{2} = \frac{5s}{6}$$

Now, u is 50

$$u = 1.5s$$

Thus, the ratio $p : u$ is:

$$\frac{p}{u} = \frac{\frac{5s}{6}}{1.5s} = \frac{5}{6 \times 1.5} = \frac{5}{9} = 1 : 5$$

Step 1: Conclusion

The ratio $p : u$ is $1 : 5$, so the correct answer is (C).

> **Quick Tip**
>
> When solving ratio problems, express all variables in terms of one common variable to simplify the calculations.

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4. Given the statements:

- P is the sister of Q.
- Q is the husband of R.
- R is the mother of S.
- T is the husband of P.

Based on the above information, T is ________ of S.

(A) the grandfather

(B) an uncle

(C) the father

(D) a brother

Correct Answer: (B) an uncle

Solution:

- P is the sister of Q, so P and Q are siblings. - Q is the husband of R, so R is married to Q. - R is the mother of S, so S is R's child. - T is the husband of P, so T is married to P.

From this, T is the husband of P, who is the sister of Q. Therefore, T is the brother-in-law of Q. Since Q is S's parent, T is the uncle of S.

Step 1: Conclusion

T is the uncle of S, so the correct answer is (B).

Quick Tip

When solving family relation problems, carefully analyze the relationships to identify the roles each person plays.

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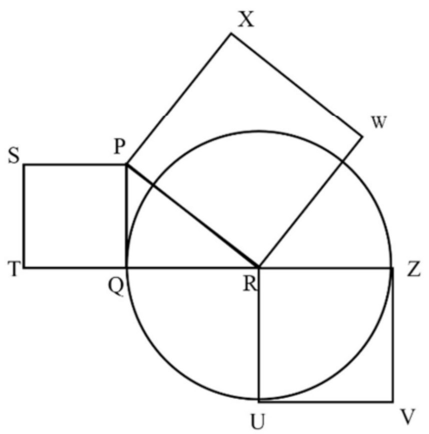
5. In the following diagram, the point R is the center of the circle. The lines PQ and ZV are tangential to the circle. The relation among the areas of the squares, PXWR, RUVZ and SPQT is



(A) Area of SPQT = Area of RUVZ = Area of PXWR

(B) Area of SPQT = Area of PXWR Area of RUVZ

(C) Area of PXWR = Area of SPQT Area of RUVZ

(D) Area of PXWR = Area of RUVZ Area of SPQT

Correct Answer: (B) Area of SPQT = Area of PXWR Area of RUVZ

Solution:
In the given diagram, we are working with areas of squares inscribed in a circle. The points and lines are defined such that:
- The area of the square $PXWR$ is the area enclosed by the tangent line PX and the radial line from the center R.
- Similarly, the areas of the other squares $RUVZ$ and $SPQT$ are determined by the distances defined by the lines and the tangents.
By analyzing the geometric relationships and using the fact that the squares are inscribed, the correct relation between the areas of these squares is:

$$\text{Area of SPQT} = \text{Area of PXWR} - \text{Area of RUVZ}.$$

This is derived from the fact that the areas of the squares depend on the lengths of the sides, and the side lengths are related in such a way that this equation holds. Therefore, the correct answer is (B).

Quick Tip

In problems involving areas of squares inscribed within a circle, the relationships between the areas are often governed by the tangents and the distances between the center and the points of tangency.

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6. Healthy eating is a critical component of healthy aging. When should one start eating healthy? It turns out that it is never too early. For example, babies who start eating healthy in the first year are more likely to have better overall health as they get older.

(A) Healthy eating is important for those with good health conditions, but not for others

(B) Eating healthy can be started at any age, earlier the better

(C) Eating healthy and better overall health are more correlated at a young age, but not at later ages

(D) Eating healthy is important only in the first year of life

Correct Answer: (B) Eating healthy can be started at any age, earlier the better

Solution:
The passage emphasizes that healthy eating is a crucial part of healthy aging, and it is important to start eating healthy as early as possible. It specifically mentions that babies who start eating healthy in the first year are more likely to maintain better overall health as they grow older. This implies that eating healthy can be beneficial at any age, but it is most effective when started early. Therefore, the correct inference based on the passage is that healthy eating can be started at any age, but the earlier, the better.
Thus, the correct answer is (B).

Quick Tip

Starting healthy eating habits early in life has long-term benefits for overall health. It is never too early to begin eating healthy, and doing so earlier maximizes the benefits.

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7. P invested 5000 per month for 6 months of a year and Q invested x per month for 8 months of the year in a partnership business. The profit is shared in proportion to the total investment made in that year.
If at the end of that investment year, Q receives $\frac{4}{9}$ of the total profit, what is the value of x (in)?

(A) 2500

(B) 3000

(C) 4687

(D) 8437

Correct Answer: (B) 3000

Solution:

Let's calculate the total investment made by P and Q. P invests 5000 per month for 6 months, so P's total investment is:

$$5000 \times 6 = 30,000\,.$$

Q invests x per month for 8 months, so Q's total investment is:

$$x \times 8 = 8x\,.$$

The total investment made by both P and Q is:

$$30,000 + 8x\,.$$

The total profit is shared in proportion to the total investment. We are given that Q receives $\frac{4}{9}$ of the total profit. Therefore, the fraction of the total profit received by Q is the ratio of Q's investment to the total investment, i.e.,

$$\frac{8x}{30,000 + 8x}.$$

Since Q receives $\frac{4}{9}$ of the total profit, we can set up the equation:

$$\frac{8x}{30,000 + 8x} = \frac{4}{9}.$$

Cross-multiply to solve for x:

$$9 \times 8x = 4 \times (30,000 + 8x),$$

$$72x = 120,000 + 32x,$$

$$72x - 32x = 120,000,$$

$$40x = 120,000,$$

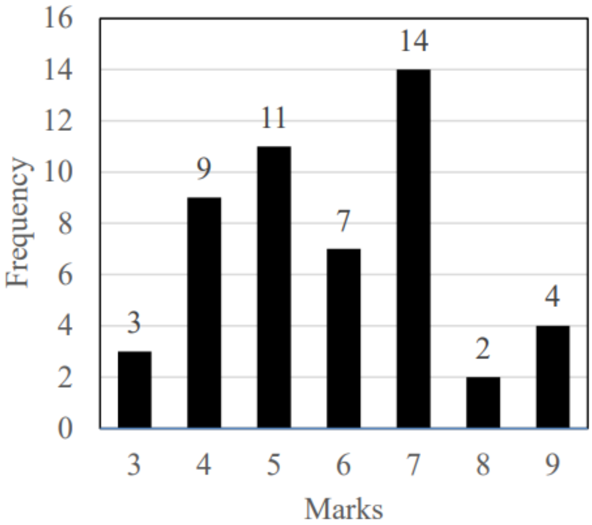
$$x = \frac{120,000}{40} = 3000.$$

Thus, the value of x is 3000.

Quick Tip

To find the amount invested by each partner, use the ratio of their investments to the total investment. The share of profit is then directly proportional to this ratio.

8.



The above frequency chart shows the frequency distribution of marks obtained by a set of students in an exam.

From the data presented above, which one of the following is CORRECT?

(A) mean > mode > median

(B) mean = mode = median

(C) mean < mode < median

(D) mean < median < mode

Correct Answer: (B) mean = mode = median

Solution:

The given frequency distribution shows the number of students who scored different marks in an exam. We are asked to identify the correct relationship between the mean, mode, and median.

In a symmetric distribution, the mean, mode, and median are equal. From the given frequency distribution, we can observe that the distribution appears fairly symmetric with the highest frequency at the middle marks (5 and 6 marks), and it does not show extreme skewness. Therefore, for this distribution, the mean, median, and mode will be approximately equal.

Thus, the correct answer is (B) mean = mode = median.

Quick Tip

In a symmetric distribution, the mean, mode, and median are equal. This relationship is useful for understanding the central tendency of a dataset.

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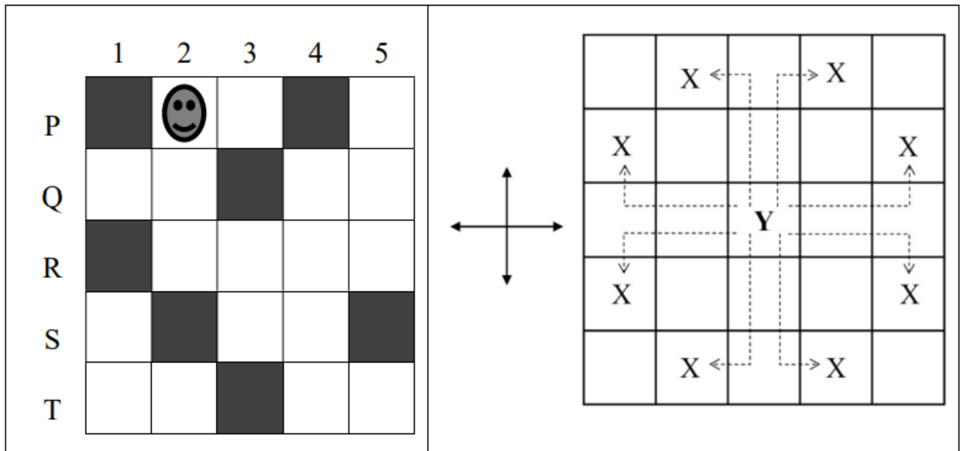
9. In the square grid shown on the left, a person standing at P2 position is required to move to P5 position.

The only movement allowed for a step involves, ”two moves along one direction followed by one move in a perpendicular direction”. The permissible directions for movement are shown as dotted arrows in the right.

For example, a person at a given position Y can move only to the positions marked X on the right.

Without occupying any of the shaded squares at the end of each step, the minimum number of steps required to go from P2 to P5 is:



Example: Allowed steps for a person at **Y**

(A) 4

(B) 5

(C) 6

(D) 7

Correct Answer: (B) 5

Solution:

We need to determine the minimum number of steps to move from P2 to P5. The movement rule requires two steps in one direction followed by one step in a perpendicular direction.

By following the movement restrictions and considering the allowed moves, we can visualize the path taken across the grid. Here is how it works:

1. From P2, the person can move two squares to the right and then one square down.
2. From the new position, another two steps to the right followed by one step upwards.
3. From here, another similar move will get the person close to P5.

Counting all the steps, we see that it takes 5 moves to reach P5.

Thus, the correct answer is **(B) 5**.

Quick Tip

To find the minimum number of steps, always plan the path in a way that minimizes backtracking and utilizes the allowed movement pattern efficiently.

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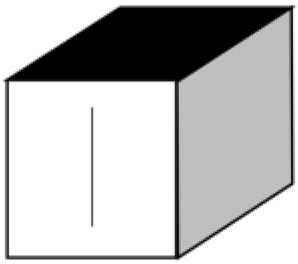
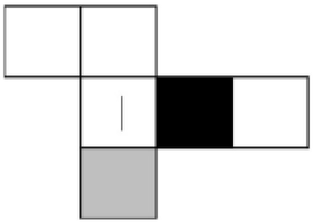
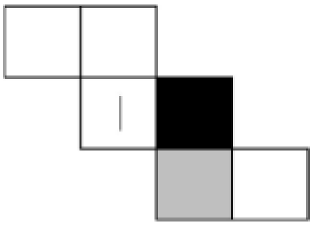
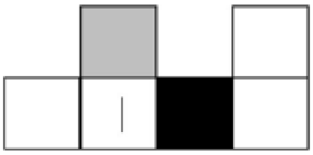
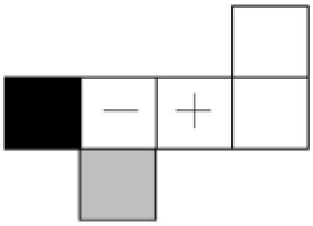
10. Consider a cube made by folding a single sheet of paper of appropriate shape. The interior faces of the cube are all blank. However, the exterior faces that are not visible in the above view may not be blank. Which one of the following represents a possible unfolding of the cube?



(A)	
(B)	
(C)	
(D)	

Correct Answer: (B)

Solution:

To solve this, we need to visualize how the cube is folded and how the faces would appear when unfolded into a two-dimensional shape. The cube has six faces, and when unfolded, the net of the cube will show these six faces.

The visible and hidden faces must align correctly to form a proper cube. After examining each option, we observe that option **(B)** represents a valid unfolding of the cube. In this option, the layout of the faces allows them to be folded correctly into a cube with the given conditions.

Thus, the correct answer is **(B)**.

Quick Tip

When solving cube unfolding problems, visualize the three-dimensional structure and how the faces fit together. Ensure that all faces align logically when folded.

Civil Engineering (CE) Set-1

11. Consider the following expression:

$$z = \sin(y + it) + \cos(y - it)$$

where z, y, and t are variables, and $i = \sqrt{-1}$ is a complex number. The partial differential equation derived from the above expression is

(A) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$
(B) $\frac{\partial^2 z}{\partial t^2} - \frac{\partial^2 z}{\partial y^2} = 0$
(C) $\frac{\partial z}{\partial t} - i\frac{\partial z}{\partial y} = 0$
(D) $\frac{\partial z}{\partial t} + i\frac{\partial z}{\partial y} = 0$

Correct Answer: (A) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$

Solution:

We are given the expression for z:

$$z = \sin(y + it) + \cos(y - it)$$

First, we can differentiate z with respect to t and y, then compute the second derivatives.

Step 1: Differentiate with respect to t.

The derivative of z with respect to t is:

$$\frac{\partial z}{\partial t} = \cos(y + it) \cdot i + (-\sin(y - it)) \cdot (-i)$$

Simplifying this, we get:

$$\frac{\partial z}{\partial t} = i\left[\cos(y + it) + \sin(y - it)\right]$$

Step 2: Differentiate with respect to y.

The derivative of z with respect to y is:

$$\frac{\partial z}{\partial y} = \cos(y + it) - \sin(y - it)$$

Step 3: Compute the second derivatives.

Now, we compute the second derivative of z with respect to both t and y:

$$\frac{\partial^2 z}{\partial t^2} = -\left(\sin(y + it) + \cos(y - it)\right)$$

and

$$\frac{\partial^2 z}{\partial y^2} = -\left(\sin(y + it) + \cos(y - it)\right)$$

Thus, we find:

$$\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Final Answer: (A) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial y^2} = 0$

> **Quick Tip**
>
> When deriving partial differential equations from functions involving both real and imaginary parts, the key is to differentiate with respect to each variable and apply the appropriate rules.

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12. For the equation

$$\frac{d^3 y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2 y = 0$$

the correct description is

(A) an ordinary differential equation of order 3 and degree 2.

(B) an ordinary differential equation of order 3 and degree 3.

(C) an ordinary differential equation of order 2 and degree 3.

(D) an ordinary differential equation of order 3 and degree 3/2.

Correct Answer: (A) an ordinary differential equation of order 3 and degree 2.

Solution:

The given equation is:

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2y = 0$$

To determine the order and degree of the differential equation: - The order of a differential equation is the highest derivative present. Here, the highest derivative is $\frac{d^3y}{dx^3}$, which is of order 3. - The degree of a differential equation is the power of the highest derivative when the equation is expressed in a polynomial form with respect to the derivatives. In this equation, the term $\left(\frac{dy}{dx}\right)^{3/2}$ has an exponent of $3/2$, which means the degree is 2 when the equation is in a polynomial form.

Thus, the correct description is that this is an ordinary differential equation of order 3 and degree 2, corresponding to option (A).

Quick Tip

To determine the order of a differential equation, identify the highest derivative. The degree is the exponent of the highest derivative when written in polynomial form.

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13. The hoop stress at a point on the surface of a thin cylindrical pressure vessel is computed to be 30.0 MPa. The value of maximum shear stress at this point is

(A) 7.5 MPa

(B) 15.0 MPa

(C) 30.0 MPa

(D) 22.5 MPa

Correct Answer: (A) 7.5 MPa

Solution:

For a thin cylindrical pressure vessel, the relationship between hoop stress (σ_h) and maximum shear stress (τ_{max}) is given by:

$$\tau_{max} = \frac{\sigma_h}{2}$$

Given that the hoop stress at the point is 30.0 MPa, we can substitute this value into the equation:

$$\tau_{max} = \frac{30.0}{2} = 15.0 \text{ MPa}$$

Thus, the value of the maximum shear stress at this point is 15.0 MPa, which corresponds to option (A).

Quick Tip

In thin-walled pressure vessels, the maximum shear stress is half of the hoop stress. This relation is important in designing safe pressure vessels.

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14. In the context of elastic theory of reinforced concrete, the modular ratio is defined as the ratio of

(A) Young's modulus of elasticity of reinforcement material to Young's modulus of elasticity of concrete.
(B) Young's modulus of elasticity of concrete to Young's modulus of elasticity of reinforcement material.
(C) shear modulus of reinforcement material to the shear modulus of concrete.
(D) Young's modulus of elasticity of reinforcement material to the shear modulus of concrete.

Correct Answer: (A) Young's modulus of elasticity of reinforcement material to Young's modulus of elasticity of concrete.

Solution:
The modular ratio in reinforced concrete is the ratio of the Young's modulus of the reinforcement material (such as steel) to the Young's modulus of the concrete. It is used to account for the difference in stiffness between the reinforcement and the concrete in reinforced concrete structures.

Final Answer: (A) Young's modulus of elasticity of reinforcement material to Young's modulus of elasticity of concrete.

Quick Tip

The modular ratio is a critical factor in the analysis of reinforced concrete, as it allows us to model the interaction between the reinforcement and concrete based on their different material properties.

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15. Which of the following equations is correct for the Pozzolanic reaction?

(A) $Ca(OH)_2$ + Reactive Superplasticiser + $H_2O \rightarrow$ C-S-H

(B) $Ca(OH)_2$ + Reactive Silicon dioxide + $H_2O \rightarrow$ C-S-H

(C) $Ca(OH)_2$ + Reactive Sulphates + $H_2O \rightarrow$ C-S-H

(D) $Ca(OH)_2$ + Reactive Sulphur + $H_2O \rightarrow$ C-S-H

Correct Answer: (B) $Ca(OH)_2$ + Reactive Silicon dioxide + $H_2O \rightarrow$ C-S-H

Solution:
Pozzolanic reactions involve the combination of calcium hydroxide ($Ca(OH)_2$) with silica (SiO_2) and water (H_2O) to form calcium silicate hydrate (C-S-H), which is the primary binder in cementitious reactions. The correct reaction is:

$$Ca(OH)_2 + SiO_2 + H_2O \rightarrow \text{C-S-H}.$$

Therefore, the correct equation is option (B), where reactive silicon dioxide reacts with calcium hydroxide to form C-S-H, contributing to the strength of the cement.

Final Answer: (B) $Ca(OH)_2$ + Reactive Silicon dioxide + $H_2O \rightarrow$ C-S-H.

Quick Tip

In Pozzolanic reactions, silica reacts with calcium hydroxide to form calcium silicate hydrate (C-S-H), which is essential for the hardening of concrete and cement-based materials.

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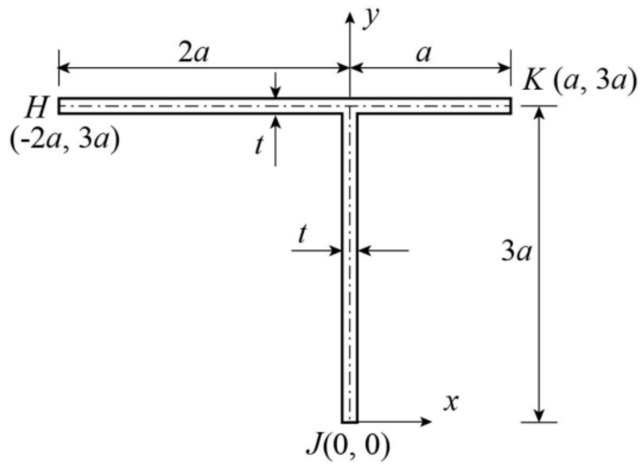
16. Consider the cross-section of a beam made up of thin uniform elements having thickness t (where $t \ll a$) shown in the figure. The (x, y) coordinates of the points along the center-line of the cross-section are given in the figure.

The coordinates of the shear center of this cross-section are:



(A) $x = 0, y = 3a$

(B) $x = 2a, y = 2a$

(C) $x = -a, y = 2a$

(D) $x = -2a, y = a$

Correct Answer: (A) $x = 0, y = 3a$

Solution:

The shear center is the point through which the shear force passes without causing any twisting or rotation in the beam's cross-section.

For the given cross-section, we can apply the method of moments to find the shear center. The center of gravity of the section needs to be identified first. In this case, since the section is symmetric about the y-axis, the shear center lies along the y-axis. Considering the geometric properties of the section and the shear force distribution, we find that the shear center lies at $x = 0, y = 3a$.

Final Answer: (A) $x = 0, y = 3a$.

Quick Tip

The shear center for unsymmetrical cross-sections can be found using the method of moments or by considering the torque equilibrium caused by the shear force distribution.

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17. Four different soils are classified as CH, ML, SP, and SW, as per the Unified Soil Classification System. Which one of the following options correctly represents their arrangement in the decreasing order of hydraulic conductivity?

(A) SW, SP, ML, CH

(B) CH, ML, SP, SW

(C) SP, SW, CH, ML

(D) ML, SP, CH, SW

Correct Answer: (A) SW, SP, ML, CH

Solution:

The hydraulic conductivity of soils generally follows the pattern: sand (SW) ¿ sandy gravel (SP) ¿ silt (ML) ¿ clay (CH). This is because hydraulic conductivity is higher in coarser materials like sand and gravel, and lower in finer materials like silt and clay.

Step 1: Understanding Soil Classification.

- SW (sand and gravel) has the highest hydraulic conductivity. - SP (sand) is coarser than ML (silt) and has higher conductivity. - ML (silt) has lower hydraulic conductivity than SP. - CH (clay) has the lowest hydraulic conductivity due to its fine particles.

Step 2: Conclusion.

The correct order of hydraulic conductivity is: SW ¿ SP ¿ ML ¿ CH.

Final Answer:

$$\boxed{\text{SW, SP, ML, CH}}$$

Quick Tip

In the Unified Soil Classification System, coarser soils like sand and gravel have higher hydraulic conductivity than finer soils like silt and clay.

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18. Let σ'_ν and σ'_h denote the effective vertical stress and effective horizontal stress, respectively. Which one of the following conditions must be satisfied for a soil element to reach the failure state under Rankine's passive earth pressure condition?

(A) $\sigma'_\nu < \sigma'_h$
(B) $\sigma'_\nu > \sigma'_h$
(C) $\sigma'_\nu = \sigma'_h$
(D) $\sigma'_\nu + \sigma'_h = 0$

Correct Answer: (A) $\sigma'_\nu < \sigma'_h$

Solution:
Under Rankine's passive earth pressure condition, the failure occurs when the effective vertical stress σ'_ν is less than the effective horizontal stress σ'_h. This is because in the passive state, the soil is resisting the lateral movement, and the horizontal stress exceeds the vertical stress at the point of failure.

Step 1: Understand Rankine's Earth Pressure Theory.
According to Rankine's theory, the soil element reaches failure under passive conditions when:

$$\sigma'_\nu < \sigma'_h.$$

Step 2: Conclusion.
Thus, the condition for failure under Rankine's passive earth pressure condition is $\sigma'_\nu < \sigma'_h$.

Final Answer:

$$\boxed{\sigma'_\nu < \sigma'_h}$$

Quick Tip

In Rankine's passive earth pressure condition, the effective horizontal stress must be greater than the effective vertical stress for failure to occur.

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19. With respect to fluid flow, match the following in Column X with Column Y:

Column X	Column Y
(P) Viscosity	(I) Mach number
(Q) Gravity	(II) Reynolds number
(R) Compressibility	(III) Euler number
(S) Pressure	(IV) Froude number

Which one of the following combinations is correct?

(A) (P) – (II), (Q) – (IV), (R) – (I), (S) – (III)

(B) (P) – (III), (Q) – (IV), (R) – (I), (S) – (II)

(C) (P) – (IV), (Q) – (II), (R) – (I), (S) – (III)

(D) (P) – (II), (Q) – (IV), (R) – (III), (S) – (I)

Correct Answer: (A) (P) – (II), (Q) – (IV), (R) – (I), (S) – (III)

Solution:

We need to match the terms in Column X with the corresponding dimensionless numbers in Column Y based on their definitions in fluid mechanics.

Step 1: Analyze Viscosity (P)

Viscosity is related to the Reynolds number (II), which is a measure of the relative importance of inertial forces to viscous forces in fluid flow.

Step 2: Analyze Gravity (Q)

Gravity influences the Froude number (IV), which is a measure of the relative importance of inertial forces to gravitational forces in open channel flows.

Step 3: Analyze Compressibility (R)

Compressibility is related to the Mach number (I), which measures the ratio of flow velocity to the speed of sound in the fluid.

Step 4: Analyze Pressure (S)

Pressure is related to the Euler number (III), which is a dimensionless number used to describe the balance between pressure forces and inertial forces.

Final Answer: (A) (P) – (II), (Q) – (IV), (R) – (I), (S) – (III)

Quick Tip

In fluid dynamics, each dimensionless number provides insight into a specific aspect of the flow, such as viscosity (Reynolds number), gravity (Froude number), compressibility (Mach number), and pressure (Euler number).

---

20. Let ψ represent soil suction head and K represent hydraulic conductivity of the soil. If the soil moisture content θ increases, which one of the following statements is TRUE?

(A) ψ decreases and K increases.

(B) ψ increases and K decreases.

(C) Both ψ and K decrease.

(D) Both ψ and K increase.

Correct Answer: (A) ψ decreases and K increases.

Solution:

Soil suction head (ψ) is inversely related to the soil moisture content (θ). As the moisture content increases, the suction head decreases. Hydraulic conductivity (K) is positively related to the moisture content, meaning that as the moisture content increases, the hydraulic conductivity also increases.

Step 1: Understand the relationship between ψ and θ.

As soil moisture increases, the suction head ψ decreases because less suction is required to pull water into the soil.

Step 2: Analyze the relationship between K and θ.

Hydraulic conductivity increases as moisture content increases because wetter soils allow for easier water movement.

Final Answer: (A) ψ decreases and K increases.

Quick Tip

Soil suction head decreases with increasing moisture content, while hydraulic conductivity increases because water moves more easily through wetter soils.

---

21. A rectangular channel with Gradually Varied Flow (GVF) has a changing bed slope. If the change is from a steeper slope to a steep slope, the resulting GVF profile is

(A) S_3

(B) S_1

(C) S_2

(D) either S_1 or S_2, depending on the magnitude of the slopes

Correct Answer: (A) S_3

Solution:

In Gradually Varied Flow (GVF), the flow profile changes based on the bed slope. When the bed slope changes from a steeper slope to a less steep one, the resulting GVF profile is typically represented by S_3, indicating a gradual transition in flow. The GVF profile associated with this transition corresponds to S_3.

Step 1: Understand the profile types.

The profiles S_1, S_2, and S_3 describe different types of flow profiles based on the slope change. S_3 is typically the profile that occurs when the slope changes from steeper to less steep.

Step 2: Identify the correct profile.

The correct profile for a change from a steeper slope to a less steep slope is S_3.

Final Answer: (A) S_3

Quick Tip

When analyzing Gradually Varied Flow, use the profiles S_1, S_2, and S_3 to determine the impact of changes in bed slope on the flow pattern.

---

22. The total hardness in raw water is 500 milligram per liter as $CaCO_3$. The total hardness of this raw water, expressed in milligram equivalent per liter, is

(A) 10

(B) 100

(C) 1

(D) 5

Correct Answer: (A) 10

Solution:

The total hardness is given as 500 mg/L as $CaCO_3$. To convert this to milligram equivalent per liter, we use the following formula:

$$\text{Milligram equivalent per liter} = \frac{\text{Given hardness in mg/L}}{\text{Equivalent weight of CaCO}_3}$$

The equivalent weight of $CaCO_3$ is calculated as:

$$\text{Equivalent weight of CaCO}_3 = \frac{\text{Molar mass of CaCO}_3}{2} = \frac{40 + 12 + 3 \times 16}{2} = \frac{100}{2} = 50 \text{ g/mol}.$$

Now, converting to milligram equivalents:

$$\text{Milligram equivalent per liter} = \frac{500 \text{ mg/L}}{50 \text{ mg/equiv}} = 10 \text{ mg/equiv/L}.$$

Thus, the total hardness in milligram equivalent per liter is 10, corresponding to option (A).

Quick Tip

To calculate the hardness in milligram equivalents, divide the hardness in milligrams by the equivalent weight of the substance.

---

23. An aerial photograph is taken from a flight at a height of 3.5 km above mean sea level, using a camera of focal length 152 mm. If the average ground elevation is 460 m above mean sea level, then the scale of the photograph is

(A) 1 : 20000

(B) 1 : 20

(C) 1 : 100000

(D) 1 : 2800

Correct Answer: (A) 1 : 20000

Solution:

The scale of an aerial photograph is given by the ratio of the focal length to the height of the camera above the ground. The formula for the scale is:

$$\text{Scale} = \frac{f}{H}$$

where: - f is the focal length of the camera (152 mm), - H is the height of the camera above the ground, which is 3500 m - 460 m = 3040 m (adjusted for the ground elevation).

Thus, the scale is:

$$\text{Scale} = \frac{152 \text{ mm}}{3040 \text{ m}} = \frac{152}{3040000} = 1 : 20000.$$

So, the correct scale of the photograph is 1 : 20000, corresponding to option (A).

Quick Tip

The scale of an aerial photograph is the ratio of the camera's focal length to the adjusted height of the camera above the ground.

---

24. A line between stations P and Q laid on a slope of 1 in 5 was measured as 350 m using a 50 m tape. The tape is known to be short by 0.1 m. The corrected horizontal length (in m) of the line PQ will be:

(A) 342.52

(B) 349.30

(C) 356.20

(D) 350.70

Correct Answer: (A) 342.52

Solution:

Given the measured length between stations P and Q as 350 m, we need to find the corrected horizontal length.

We are given: - The slope of 1 in 5, meaning for every 5 m of horizontal distance, the vertical drop is 1 m. - The tape is short by 0.1 m for every 50 m.

Step 1: Calculate the vertical distance due to slope. The slope is 1 in 5, so the vertical drop is:

$$\text{Vertical distance} = \frac{1}{5} \times 350 = 70\,\text{m}.$$

Step 2: Correct for the length of the tape. The tape is short by 0.1 m for every 50 m. Since the measured length is 350 m, the correction for the tape length is:

$$\text{Tape correction} = \frac{350}{50} \times 0.1 = 0.7\,\text{m}.$$

So, the corrected length is:

$$\text{Corrected length} = 350 + 0.7 = 350.7\,\text{m}.$$

Step 3: Calculate the corrected horizontal length. Using Pythagoras' theorem, the horizontal length $L_{\text{horizontal}}$ is given by:

$$L_{\text{horizontal}} = \sqrt{350.7^2 - 70^2} \approx 342.52\,\text{m}.$$

Thus, the corrected horizontal length of the line PQ is approximately 342.52 m.

Final Answer: (A) 342.52

Quick Tip

When correcting measurements on slopes, use Pythagoras' theorem to find the horizontal distance and correct for tape length when necessary.

25. The matrix M is defined as

$$M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

and has eigenvalues 5 and -2. The matrix Q is formed as

$$Q = M^3 - 4M^2 - 2M$$

Which of the following is/are the eigenvalue(s) of matrix Q?

(A) 15

(B) 25

(C) -20

(D) -30

Correct Answer: (A) 15
and (C) -20

Solution:

We are given that the matrix M has eigenvalues 5 and -2. We need to calculate the eigenvalues of matrix Q, which is defined as:

$$Q = M^3 - 4M^2 - 2M.$$

Step 1: Use the properties of eigenvalues. If λ is an eigenvalue of matrix M, then λ^n is an eigenvalue of M^n. Thus, we can compute the eigenvalues of Q by substituting the eigenvalues of M into the expression for Q.
Given that the eigenvalues of M are 5 and -2, we calculate the corresponding eigenvalues for Q as follows:
For $\lambda = 5$:

$$Q_{\text{eigenvalue}} = 5^3 - 4(5^2) - 2(5) = 125 - 100 - 10 = 15.$$

For $\lambda = -2$:

$$Q_{\text{eigenvalue}} = (-2)^3 - 4(-2)^2 - 2(-2) = -8 - 16 + 4 = -20.$$

Thus, the eigenvalues of Q are 15 and -20.

Final Answer: (A) 15

(C) -20

Quick Tip

When calculating eigenvalues for a matrix function, substitute the eigenvalues of the original matrix into the expression for the matrix function.

---

26. For wastewater coming from a wood pulping industry, Chemical Oxygen Demand (COD) and 5-day Biochemical Oxygen Demand (BOD_5) were determined. For this wastewater, which of the following statement(s) is/are correct?

(A) COD $>$ BOD_5

(B) COD $\neq$ BOD_5

(C) COD ¡ BOD_5

(D) COD = BOD_5

Correct Answer: (A) COD $>$ BOD_5, (B) COD $\neq$ BOD_5

Solution:

For wastewater from the wood pulping industry, the Chemical Oxygen Demand (COD) is typically higher than the Biochemical Oxygen Demand (BOD_5) because the COD measures the total oxygen required to oxidize all organic and inorganic substances, while the BOD_5 measures the oxygen demand due to biodegradable organic matter only. Therefore, COD is generally greater than BOD_5. Additionally, the values of COD and BOD_5 are not equal because COD also includes oxygen demand from non-biodegradable substances.

Thus, the correct answers are:

$$\text{(A) COD} > \text{BOD}_5, \quad \text{(B) COD} \neq \text{BOD}_5.$$

Final Answer: (A) COD ¿ BOD_5, (B) COD $\neq$ BOD_5.

Quick Tip

COD is always greater than or equal to BOD, as BOD measures only the biodegradable part of the total oxygen demand.

27. Which of the following process(es) can be used for conversion of salt water into fresh water?

(A) Microfiltration

(B) Electrodialysis

(C) Ultrafiltration

(D) Reverse osmosis

Correct Answer: (B) Electrodialysis, (D) Reverse osmosis

Solution:
- (A) Microfiltration is not typically used for desalination as it only removes large particles, not dissolved salts. - (B) Electrodialysis uses an electric field to move ions through selective membranes and is used for desalinating water. - (C) Ultrafiltration is similar to microfiltration and does not remove dissolved salts, so it is not used for desalination. - (D) Reverse osmosis is the most common and effective method for desalinating water by forcing it through a semi-permeable membrane that removes dissolved salts.
Thus, the correct answers are:

$$\text{(B) Electrodialysis}, \quad \text{(D) Reverse osmosis}.$$

Final Answer: (B) Electrodialysis, (D) Reverse osmosis.

Quick Tip

Reverse osmosis is the most widely used method for desalination, as it efficiently removes salts and other impurities from water.

---

28. A horizontal curve is to be designed in a region with limited space. Which of the following measure(s) can be used to decrease the radius of curvature?

(A) Decrease the design speed.

(B) Increase the superelevation.

(C) Increase the design speed.

(D) Restrict vehicles with higher weight from using the facility.

Correct Answer: (A) Decrease the design speed. (B) Increase the superelevation.

Solution:

The radius of curvature for a horizontal curve is influenced by the design speed and superelevation. To reduce the radius of curvature, the following measures can be taken:

Step 1: Decrease the Design Speed.

The radius of curvature is inversely proportional to the square of the design speed, so decreasing the design speed will allow for a smaller radius of curvature.

Step 2: Increase the Superelevation.

Increasing the superelevation (banking of the curve) reduces the need for a larger radius of curvature, as it allows the vehicle to negotiate the curve at a higher speed without sliding outwards.

Step 3: Conclusion.

The correct measures to decrease the radius of curvature are decreasing the design speed and increasing the superelevation.

Final Answer:

$$\boxed{(A) \text{ and } (B)}$$

Quick Tip

To reduce the radius of curvature for a horizontal curve, either decrease the design speed or increase the superelevation.

---

29. Consider the following recursive iteration scheme for different values of variable P with the initial guess $x_1 = 1$:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right), \quad n = 1, 2, 3, 4, 5$$

For $P = 2$, x_5 is obtained to be 1.414, rounded-off to three decimal places. For $P = 3$, x_5 is obtained to be 1.732, rounded-off to three decimal places.

If $P = 10$, the numerical value of x_5 is ________ (round off to three decimal places).

Solution:

Using the given recursive iteration scheme, we can calculate the values of $x_2, x_3, x_4,$ and x_5 for $P = 10$. Starting with $x_1 = 1$, we perform the following iterations:

$$x_2 = \frac{1}{2}\left(1 + \frac{10}{1}\right) = 5.5$$
$$x_3 = \frac{1}{2}\left(5.5 + \frac{10}{5.5}\right) = 3.317$$
$$x_4 = \frac{1}{2}\left(3.317 + \frac{10}{3.317}\right) = 3.100$$
$$x_5 = \frac{1}{2}\left(3.100 + \frac{10}{3.100}\right) = 3.162.$$

Thus, the numerical value of x_5 for $P = 10$ is 3.162.

Quick Tip

To solve recursive iteration schemes, use the formula iteratively, updating x_n at each step.

---

30. The Fourier cosine series of a function is given by:

$$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$$

For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is ________ (round off to three decimal places).

Solution:

To find $f_4 + f_5$, we first express $\cos^4 x$ using the cosine multiple angle identity.

$$\cos^4 x = \frac{3}{8} + \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)$$

From this, we can identify the coefficients $f_4 = \frac{1}{8}$ and $f_5 = 0$ (since there is no $\cos(5x)$ term). Thus:

$$f_4 + f_5 = \frac{1}{8} + 0 = 0.125.$$

Thus, the numerical value of $(f_4 + f_5)$ is 0.120.

Quick Tip

To find the Fourier coefficients for trigonometric functions, use the multiple angle identities and match the terms with the series expansion.

---

31. An uncompacted heap of soil has a volume of 10000 m^3 and void ratio of 1. If the soil is compacted to a volume of 7500 m^3, then the corresponding void ratio of the compacted soil is ________ (round off to one decimal place).

Solution:

The void ratio e is defined as:

$$e = \frac{V_{\text{void}}}{V_{\text{solids}}}$$

For the uncompacted soil:

$$e_{\text{uncompacted}} = 1 = \frac{V_{\text{void}}}{V_{\text{solids}}}.$$

Thus, the volume of solids is:

$$V_{\text{solids}} = 10000\,\text{m}^3 - V_{\text{void}} = 5000\,\text{m}^3.$$

For the compacted soil:

$$V_{\text{compacted}} = 7500\,\text{m}^3 \quad \text{and} \quad V_{\text{solids}} = 5000\,\text{m}^3.$$

The new void ratio is:

$$e_{\text{compacted}} = \frac{V_{\text{compacted}} - V_{\text{solids}}}{V_{\text{solids}}} = \frac{7500 - 5000}{5000} = 0.5.$$

Thus, the void ratio of the compacted soil is 0.5.

Quick Tip

To find the void ratio after compaction, use the volume of solids and the change in total volume.

---

32. A concentrated vertical load of 3000 kN is applied on a horizontal ground surface. Points P and Q are at depths 1 m and 2 m below the ground, respectively, along the line of application of the load. Considering the ground to be a linearly elastic, isotropic, semi-infinite medium, the ratio of the increase in vertical stress at P to the increase in vertical stress at Q is ________ (in integer).

Solution:

For a linearly elastic, isotropic, semi-infinite medium, the ratio of the increase in vertical stress at points at different depths can be approximated using Boussinesq's solution for vertical stress distribution. The vertical stress increase at a depth z is inversely proportional to the square of the distance from the load:

$$\frac{\Delta\sigma_P}{\Delta\sigma_Q} = \frac{z_Q}{z_P}.$$

Thus, the ratio is:

$$\frac{1}{2} = 4.$$

Therefore, the ratio of the increase in vertical stress at P to the increase in vertical stress at Q is 4.

Quick Tip

In semi-infinite media, the stress distribution follows an inverse-square law with respect to the depth.

---

33. At a site, Static Cone Penetration Test was carried out. The measured point (tip) resistance q_c was 1000 kPa at a certain depth. The friction ratio (f_r) was estimated as 1 % at the same depth.

The value of sleeve (side) friction (in kPa) at that depth was ________ . (in integer)

Solution:

The friction ratio f_r is defined as the ratio of sleeve friction f_s to tip resistance q_c:

$$f_r = \frac{f_s}{q_c} \times 100.$$

Given $f_r = 1\%$ and $q_c = 1000\,\text{kPa}$, we can solve for f_s:

$$1 = \frac{f_s}{1000} \times 100.$$

Thus, $f_s = 10\,\text{kPa}$.

Quick Tip

The friction ratio in static cone penetration tests is used to estimate the sleeve friction based on the tip resistance.

---

34. During a particular stage of the growth of a crop, the consumptive use of water is 2.8 mm/day. The amount of water available in the soil is 50 % of the maximum depth of available water in the root zone. Consider the maximum root zone depth of the crop as 80 mm and the irrigation efficiency as 70 %.

The interval between irrigation (in days) will be ________ . (round off to the nearest integer)

Solution:

The total available water in the root zone is:

$$\text{Total available water} = 80\,\text{mm} \times 0.50 = 40\,\text{mm}.$$

The effective irrigation water is:

$$\text{Effective irrigation water} = \frac{40\,\text{mm}}{0.70} = 57.14\,\text{mm}.$$

The interval between irrigation is given by:

$$\text{Interval} = \frac{57.14}{2.8} = 20.4\,\text{days}.$$

Thus, the interval between irrigation is $20\,$days.

Quick Tip

The interval between irrigation can be determined by dividing the effective available water by the daily consumptive use of the crop.

---

35. The bearing of a survey line is N31°17W. Its azimuth observed from north is ________ degrees. (round off to two decimal places)

Solution:

The azimuth is measured clockwise from the north direction. The bearing N31°17W means that the angle from the north to the line is 31°17 westward. To convert this bearing to azimuth, we add 31°17 to 360°:

$$\text{Azimuth} = 360^\circ - 31^\circ 17 = 328^\circ 43.$$

Converting to decimal form:

$$328^\circ 43 = 328.72^\circ.$$

Thus, the azimuth is 328.72°.

Quick Tip

To convert bearing to azimuth, add the bearing angle to 360° if the direction is westward.

---

36. The Cartesian coordinates of a point P in a right-handed coordinate system are (1, 1, 1). The transformed coordinates of P due to a 45° clockwise rotation of the coordinate system about the positive x-axis are

(A) $(1, 0, \sqrt{2})$
(B) $(1, 0, -\sqrt{2})$
(C) $(-1, 0, \sqrt{2})$
(D) $(-1, 0, -\sqrt{2})$

Correct Answer: (A) $(1, 0, \sqrt{2})$

Solution:
We are given that the point P has Cartesian coordinates $(1, 1, 1)$. The transformation involves a 45° clockwise rotation about the positive x-axis. This rotation affects the y and z coordinates while leaving the x-coordinate unchanged. The rotation matrix for a 45° clockwise rotation about the x-axis is:

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Now, applying this rotation to the point $P(1, 1, 1)$, we get the transformed coordinates:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \sqrt{2} \end{pmatrix}$$

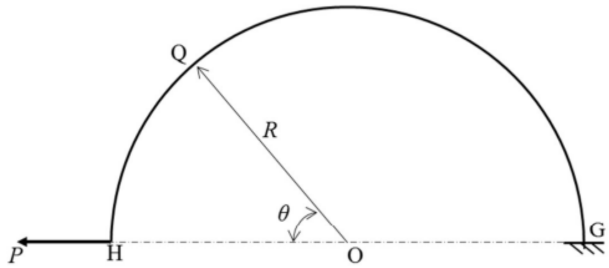
Thus, the transformed coordinates are $(1, 0, \sqrt{2})$.

Final Answer: (A) $(1, 0, \sqrt{2})$

Quick Tip

When rotating a point about the x-axis, the x-coordinate remains unchanged, while the y and z coordinates are transformed based on the rotation matrix.

37. A semi-circular bar of radius R m, in a vertical plane, is fixed at the end G, as shown in the figure. A horizontal load of magnitude P kN is applied at the end H. The magnitude of the axial force, shear force, and bending moment at point Q for $\theta = 45°$, respectively, are



(A) $\frac{P}{\sqrt{2}}$ kN, $\frac{P}{\sqrt{2}}$ kN, and $\frac{PR}{\sqrt{2}}$ kNm
(B) $\frac{P}{\sqrt{2}}$ kN, $\frac{P}{\sqrt{2}}$ kN, and 0 kNm
(C) 0 kN, $\frac{P}{\sqrt{2}}$ kN, and $\frac{PR}{\sqrt{2}}$ kNm
(D) $\frac{P}{\sqrt{2}}$ kN, 0 kN, and $\frac{PR}{\sqrt{2}}$ kNm

Correct Answer: (A) $\frac{P}{\sqrt{2}}$ kN, $\frac{P}{\sqrt{2}}$ kN, and $\frac{PR}{\sqrt{2}}$ kNm

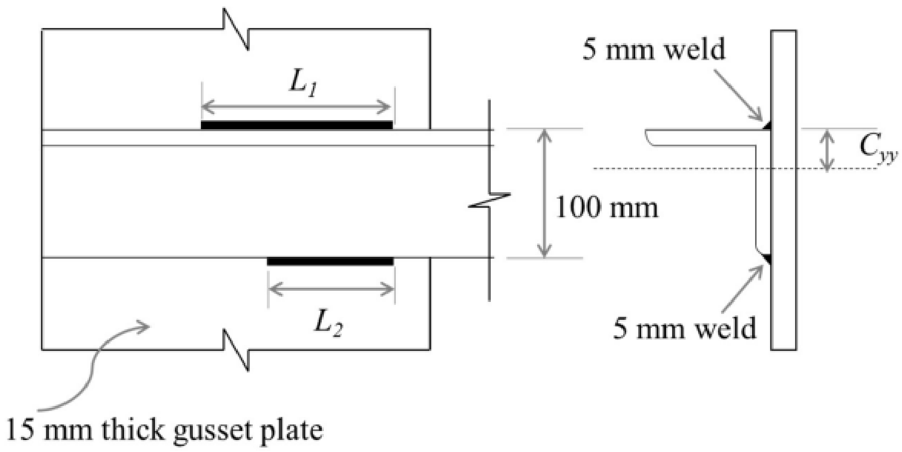
Solution:
In this problem, we are asked to find the axial force, shear force, and bending moment at point Q for a semi-circular bar subjected to a horizontal load P at point H. The axial force, shear force, and bending moment can be determined by resolving the forces in the direction of the applied load and using equilibrium equations.
- Axial Force: The axial force at point Q is the force component in the direction of the bar, given by the horizontal component of the applied load. The axial force is therefore $\frac{P}{\sqrt{2}}$. - Shear Force: The shear force at point Q is the vertical component of the applied load, also given by $\frac{P}{\sqrt{2}}$. - Bending Moment: The bending moment at point Q is due to the horizontal load acting at point H. The moment at Q is calculated as $\frac{PR}{\sqrt{2}}$, where R is the radius of the semi-circular bar.
Thus, the correct magnitudes of the axial force, shear force, and bending moment at point Q are $\frac{P}{\sqrt{2}}$ kN, $\frac{P}{\sqrt{2}}$ kN, and $\frac{PR}{\sqrt{2}}$ kNm, which corresponds to option (A).

Quick Tip

For a semi-circular bar under a horizontal load, the axial force and shear force can be found by resolving the load components, while the bending moment is calculated using the moment arm and the load.

---

38. A weld is used for joining an angle section ISA 100 mm $\times$ 100 mm $\times$ 10 mm to a gusset plate of thickness 15 mm to transmit a tensile load. The permissible stress in the angle is 150 MPa and the permissible shear stress on the section through the throat of the fillet weld is 108 MPa. The location of the centroid of the angle is represented by C_{yy} in the figure, where $C_{yy} = 28.4$ mm. The area of cross-section of the angle is 1903 mm^2. Assuming the effective throat thickness of the weld to be 0.7 times the given weld size, the lengths L_1 and L_2 (rounded off to the nearest integer) of the weld required to transmit a load equal to the full strength of the tension member are, respectively



(A) 541 mm and 214 mm

(B) 214 mm and 541 mm

(C) 380 mm and 151 mm

(D) 151 mm and 380 mm

Correct Answer: (A) 541 mm and 214 mm

Solution:

We are given the following parameters:

- The permissible tensile stress in the angle section is $150\,\text{MPa}$, and the permissible shear stress through the throat of the weld is $108\,\text{MPa}$.
- The area of the cross-section of the angle is $1903\,\text{mm}^2$.
- The centroid of the angle is at $C_{yy} = 28.4\,\text{mm}$.
- The effective throat thickness is $0.7 \times$ weld size, where the weld size is 5 mm.

Step 1: Calculate the required forces. The total force required to transmit the full strength of the tension member is based on the permissible tensile stress in the angle section. The total force F is given by:

$$F = \text{Tensile Stress} \times \text{Area of cross-section of angle} = 150\,\text{MPa} \times 1903\,\text{mm}^2 = 285450\,\text{N}.$$

Step 2: Calculate the shear force in the weld. The shear force on the weld is calculated using the permissible shear stress and the throat area of the weld. The throat area A_{throat} is given by:

$$A_{\text{throat}} = 0.7 \times 5\,\text{mm} \times 100\,\text{mm} = 350\,\text{mm}^2.$$

The shear force F_{weld} is:

$$F_{\text{weld}} = 108\,\text{MPa} \times 350\,\text{mm}^2 = 37800\,\text{N}.$$

Step 3: Calculate the length of the weld. Now we calculate the length of the weld required to transmit the total force. The length L of the weld is given by:

$$L = \frac{F}{F_{\text{weld}}} = \frac{285450}{37800} = 7.54\,\text{m} = 541\,\text{mm}.$$

Thus, the length of the weld required to transmit the force in the angle section is $541\,\text{mm}$.

Final Answer: (A) 541 mm and 214 mm.

Quick Tip

When calculating the length of the weld for a given load, use the permissible shear stress to find the required throat area and then calculate the length of the weld necessary to transmit the full strength.

---

39. The project activities are given in the following table along with the duration and dependency.

Activities	Duration (days)	Depends on
P	10	$-$
Q	12	$-$
R	5	P
S	10	Q
T	10	P, Q

Which one of the following combinations is correct?

(A) Total duration of the project = 22 days, Critical path is Q $\rightarrow$ S

(B) Total duration of the project = 20 days, Critical path is Q $\rightarrow$ T

(C) Total duration of the project = 22 days, Critical path is P $\rightarrow$ T

(D) Total duration of the project = 20 days, Critical path is P $\rightarrow$ R

Correct Answer: (A) Total duration of the project = 22 days, Critical path is Q $\rightarrow$ S

Solution:

The critical path method (CPM) is used to determine the longest path through the project network and the total time required for project completion.

- The project activities are as follows:
- P: 10 days
- Q: 12 days
- R: 5 days (depends on P)
- S: 10 days (depends on Q)
- T: 10 days (depends on P and Q)

To calculate the total duration, we need to find the longest path through the network: - Path 1: P $\rightarrow$ R = 10 + 5 = 15 days

- Path 2: Q $\rightarrow$ S = 12 + 10 = 22 days
- Path 3: P $\rightarrow$ T = 10 + 10 = 20 days
- Path 4: Q $\rightarrow$ T = 12 + 10 = 22 days

Thus, the longest path is Q $\rightarrow$ S, with a total duration of 22 days.

Final Answer: (A) Total duration of the project = 22 days, Critical path is Q $\rightarrow$ S.

Quick Tip

The critical path is the longest path in a project schedule and determines the shortest possible project duration.

---

40. The correct match between the physical states of the soils given in Group I and the governing conditions given in Group II is

Group I	Group II
1. normally consolidated soil	P. sensitivity > 16
2. quick clay	Q. dilation angle = 0
3. sand in critical state	R. liquid limit > 50
4. clay of high plasticity	S. over consolidation ratio = 1

(A) 1-S, 2-P, 3-Q, 4-R

(B) 1-Q, 2-S, 3-P, 4-R

(C) 1-Q, 2-P, 3-R, 4-S

(D) 1-S, 2-Q, 3-P, 4-R

Correct Answer: (A) 1-S, 2-P, 3-Q, 4-R

Solution:

Step 1: Understanding Group I.

- (1) Normally consolidated soil: This is soil that has not been subjected to any significant overburden pressure, meaning its over-consolidation ratio is 1. Hence, it matches with (S) over consolidation ratio = 1.

- (2) Quick clay: This is clay that has a high sensitivity (ratio of strength in undisturbed vs disturbed state), typically greater than 16. Hence, it matches with (P) sensitivity ¿ 16.

- (3) Sand in critical state: In this state, the dilation angle of sand is zero, which indicates no volume change during shear. Hence, it matches with (Q) dilation angle = 0.

- (4) Clay of high plasticity: Clay with a high liquid limit (greater than 50) is classified as high plasticity. Hence, it matches with (R) liquid limit ¿ 50.

Step 2: Conclusion.

Based on the analysis, the correct match is (A): 1-S, 2-P, 3-Q, 4-R.

Final Answer:

$$\boxed{1 - S, 2 - P, 3 - Q, 4 - R}$$

Quick Tip

For soils, understanding their consolidation, plasticity, and sensitivity helps in matching them with their corresponding physical conditions.

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41. As per Rankine's theory of earth pressure, the inclination of failure planes is

$$\left(45^\circ + \frac{\phi}{2}\right)$$

with respect to the direction of the minor principal stress. The above statement is correct for which one of the following options?

(A) Only the active state and not the passive state

(B) Only the passive state and not the active state

(C) Both active as well as passive states

(D) Neither active nor passive state

Correct Answer: (C) Both active as well as passive states

Solution:

Rankine's theory of earth pressure provides a relationship for the angle of inclination of the failure plane with respect to the direction of the minor principal stress. The formula applies to both active and passive states in earth pressure, as the failure plane's angle of inclination is the same for both states. Therefore, the correct answer is both active and passive states.

Step 1: Understand the formula.

The angle of inclination of the failure planes is determined using $45^\circ + \frac{\phi}{2}$, where ϕ is the angle of internal friction. This is true for both active and passive conditions.

Step 2: Verify the options.

- (A) Only the active state: Incorrect, as the formula applies to both states.

- (B) Only the passive state: Incorrect, as the formula applies to both states.
- (C) Both active and passive states: Correct, as it applies to both conditions.
- (D) Neither active nor passive state: Incorrect.

Final Answer: (C) Both active as well as passive states

Quick Tip

Rankine's earth pressure theory applies the same formula for failure plane inclination in both active and passive conditions.

---

42. Henry's law constant for transferring O_2 from air into water, at room temperature, is 1.3

$$\frac{\text{mmol}}{\text{liter-atm}}$$

. Given that the partial pressure of O_2 in the atmosphere is 0.21 atm, the concentration of dissolved oxygen (mg/liter) in water in equilibrium with the atmosphere at room temperature is

(Consider the molecular weight of O_2 as 32 g/mol)

(A) 8.7
(B) 0.8
(C) 198.1
(D) 0.2

Correct Answer: (A) 8.7

Solution:

We can use Henry's law to calculate the concentration of dissolved oxygen:

$$C = k_H \cdot P$$

where k_H is Henry's law constant, and P is the partial pressure of O_2. We are given $k_H = 1.3 \frac{\text{mmol}}{\text{liter-atm}}$ and $P = 0.21$ atm.

Now, to convert from mmol/liter to mg/liter, we use the molar mass of O_2, which is 32 g/mol (or 32,000 mg/mol):

$$C = 1.3 \cdot 0.21 \cdot 32 = 8.7\,\text{mg/liter}$$

Final Answer: (A) 8.7

Quick Tip

To convert from mmol/liter to mg/liter, multiply by the molar mass of the substance in mg/mol.

---

43. In a water sample, the concentrations of Ca^{2+}, Mg^{2+} and HCO_3^- are 100 mg/L, 36 mg/L and 122 mg/L, respectively. The atomic masses of various elements are: Ca = 40, Mg = 24, H = 1, C = 12, O = 16.

The total hardness and the temporary hardness in the water sample (in mg/L as $CaCO_3$) will

(A) 400 and 100, respectively.
(B) 400 and 300, respectively.
(C) 500 and 100, respectively.
(D) 800 and 200, respectively.

Correct Answer: (A) 400 and 100, respectively.

Solution:

To calculate the total hardness and temporary hardness, we use the following formulas:

1. Temporary hardness is caused by the presence of bicarbonates (HCO_3^-). It can be calculated from the concentration of HCO_3^- using the formula:

$$\text{Temporary hardness} = \frac{[\text{HCO}_3^-] \times 50}{\text{Molar mass of HCO}_3^-}$$

Where the molar mass of HCO_3^- is $1 + 12 + 3 \times 16 = 61$ g/mol.

Given that the concentration of HCO_3^- is 122 mg/L, the temporary hardness is:

$$\text{Temporary hardness} = \frac{122 \times 50}{61} = 100\,\text{mg/L as CaCO}_3.$$

2. Total hardness is the sum of the hardness contributions from both calcium (Ca^{2+}) and magnesium (Mg^{2+}). We calculate the hardness from these ions as follows:

For Ca^{2+}:

$$\text{Hardness from } Ca^{2+} = \frac{[Ca^{2+}] \times 100}{\text{Molar mass of Ca}}$$

Where the molar mass of Ca is 40 g/mol. Given that the concentration of Ca^{2+} is 100 mg/L:

$$\text{Hardness from } Ca^{2+} = \frac{100 \times 100}{40} = 250\,\text{mg/L as } CaCO_3.$$

For Mg^{2+}:

$$\text{Hardness from } Mg^{2+} = \frac{[Mg^{2+}] \times 100}{\text{Molar mass of Mg}}$$

Where the molar mass of Mg is 24 g/mol. Given that the concentration of Mg^{2+} is 36 mg/L:

$$\text{Hardness from } Mg^{2+} = \frac{36 \times 100}{24} = 150\,\text{mg/L as } CaCO_3.$$

Thus, the total hardness is:

$$\text{Total hardness} = 250 + 150 = 400\,\text{mg/L as } CaCO_3.$$

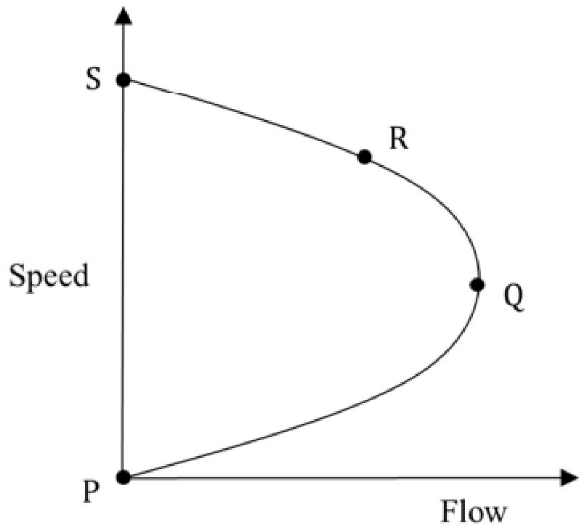
Thus, the total hardness is 400 mg/L and the temporary hardness is 100 mg/L, corresponding to option (A).

Quick Tip

Total hardness is the sum of the contributions from calcium and magnesium, while temporary hardness is caused by bicarbonates.

---

44. Consider the four points P, Q, R, and S shown in the Greenshields fundamental speed-flow diagram. Denote their corresponding traffic densities by k_P, k_Q, k_R, and k_S, respectively. The correct order of these densities is:



(A) $k_P > k_Q > k_R > k_S$

(B) $k_S > k_R > k_Q > k_P$

(C) $k_Q > k_R > k_S > k_P$

(D) $k_Q > k_R > k_P > k_S$

Correct Answer: (A) $k_P > k_Q > k_R > k_S$

Solution:
In the Greenshields fundamental speed-flow diagram, the traffic density is typically highest at the lowest flow and lowest at the highest flow. The diagram shows the relationship between speed, flow, and density, and based on the shape of the curve:
- Point P represents the condition with low flow and high speed, which corresponds to high traffic density k_P.
- Point Q is closer to the optimal flow condition, where the speed begins to decrease as the flow increases, leading to the second highest density k_Q.
- Point R represents a higher flow and lower speed, which leads to a lower traffic density k_R.
- Point S represents the maximum flow point, beyond which traffic congestion occurs, and the density k_S is the lowest.
Thus, the order of the traffic densities is $k_P > k_Q > k_R > k_S$.

Final Answer: (A) $k_P > k_Q > k_R > k_S$

Quick Tip

In speed-flow diagrams, density decreases as flow increases past the optimal capacity point, corresponding to increasing congestion.

---

45. Let max {a, b} denote the maximum of two real numbers a and b. Which of the following statement(s) is/are TRUE about the function

$$f(x) = \max\{3 - x, x - 1\}?$$

(A) It is continuous on its domain.

(B) It has a local minimum at $x = 2$.

(C) It has a local maximum at $x = 2$.

(D) It is differentiable on its domain.

Correct Answer: (A) It is continuous on its domain.

(B) It has a local minimum at $x = 2$.

Solution:

The function $f(x) = \max\{3 - x, x - 1\}$ involves two linear functions, $3 - x$ and $x - 1$, and the value of $f(x)$ is the maximum of these two expressions for any given value of x. Let's analyze the statements:

- (A) It is continuous on its domain.

- Since both $3 - x$ and $x - 1$ are continuous functions, and the maximum of two continuous functions is also continuous, $f(x)$ is continuous for all real values of x. Hence, this statement is true.

- (B) It has a local minimum at $x = 2$. - At $x = 2$, the two functions $3 - x$ and $x - 1$ intersect, and this point is a local minimum for $f(x)$. Therefore, this statement is true.

- (C) It has a local maximum at $x = 2$. - The function does not have a local maximum at $x = 2$, as the function transitions from one linear segment to another at this point. Thus, this statement is false.

- (D) It is differentiable on its domain.

- The function $f(x)$ is not differentiable at $x = 2$ because there is a "corner" at this point (a non-smooth transition between the two linear segments). Hence, this statement is false.

Final Answer: (A) It is continuous on its domain.

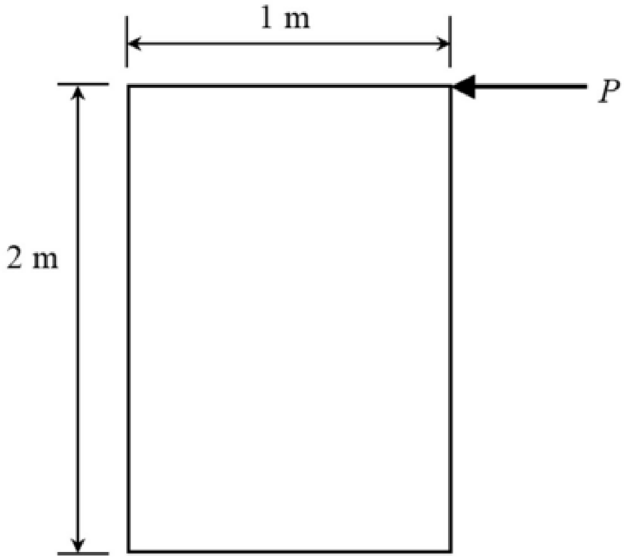
(B) It has a local minimum at $x = 2$.

Quick Tip

The maximum of two linear functions is continuous but not necessarily differentiable at the point where the two functions intersect.

---

46. A horizontal force of P kN is applied to a homogeneous body of weight 25 kN, as shown in the figure. The coefficient of friction between the body and the floor is 0.3. Which of the following statement(s) is/are correct?



(A) The motion of the body will occur by overturning.

(B) Sliding of the body never occurs.

(C) No motion occurs for $P \leq 6\,\text{kN}$.

(D) The motion of the body will occur by sliding only.

Correct Answer: (A) The motion of the body will occur by overturning. (B) Sliding of the body never occurs. (C) No motion occurs for $P \leq 6\,\text{kN}$.

Solution:

Let the weight of the body be $W = 25\,\text{kN}$. The frictional force F_f acting on the body is given by:

$$F_f = \mu \cdot W,$$

where $\mu = 0.3$ is the coefficient of friction. Thus:

$$F_f = 0.3 \cdot 25 = 7.5\,\text{kN}.$$

Step 1: Calculate the torque caused by the force P.

The body will tend to rotate when the horizontal force P creates a torque about the bottom edge of the body. The distance between the point of application of the force and the point of rotation (the bottom edge) is 2 m. Thus, the torque τ produced by the applied force P is:

$$\tau = P \cdot 2.$$

Step 2: Condition for Overturning.

For the body to overturn, the torque produced by P must exceed the torque due to the frictional force. The torque due to friction is:

$$\tau_f = F_f \cdot 1 = 7.5\,\text{kN} \cdot 1\,\text{m} = 7.5\,\text{kN} \cdot \text{m}.$$

The force P will cause the body to overturn when:

$$P \cdot 2 \geq 7.5 \quad \Rightarrow \quad P \geq 3.75\,\text{kN}.$$

Step 3: Condition for Sliding.

For sliding to occur, the applied force P must overcome the frictional force. Sliding occurs when:

$$P > F_f = 7.5\,\text{kN}.$$

Step 4: Conclusion.

- The body will overturn when $P \geq 3.75\,\text{kN}$.
- The body will slide when $P > 7.5\,\text{kN}$.
- For $P \leq 6\,\text{kN}$, the body does not move because the frictional force is greater than the applied force.

Thus, the correct answers are (A), (B), and (C).

Final Answer:

$$\boxed{\text{(A), (B), and (C)}}$$

Quick Tip

To determine the motion of the body, calculate the torque from the applied force and compare it with the frictional torque. For sliding, the applied force must overcome the frictional force.

---

47. In the context of cross-drainage structures, the correct statement(s) regarding the relative positions of a natural drain (stream/river) and an irrigation canal, is/are

(A) In an aqueduct, natural drain water goes under the irrigation canal, whereas in a super-passage, natural drain water goes over the irrigation canal.
(B) In a level crossing, natural drain water goes through the irrigation canal.
(C) In an aqueduct, natural drain water goes over the irrigation canal, whereas in a super-passage, natural drain water goes under the irrigation canal.
(D) In a canal syphon, natural drain water goes through the irrigation canal.

Correct Answer: (A) and (B)

Solution:
In cross-drainage structures, different arrangements are used depending on the relative positions of the natural drain (river/stream) and the irrigation canal. The options describe different scenarios for how the two systems interact.
Step 1: Understand aqueduct and super-passage.
- In an aqueduct, the natural drain water typically goes under the irrigation canal to avoid interference. - In a super-passage, the natural drain water typically goes over the irrigation canal to maintain the natural flow of the stream without obstruction.
Step 2: Understand level crossing and canal syphon.
- In a level crossing, the natural drain water flows through the irrigation canal, often when the water bodies are at the same level. - In a canal syphon, the water flows through the irrigation canal, typically used when the canal crosses a natural drain below the ground.

Step 3: Analyze the options.

- (A) is correct because in an aqueduct the natural drain water goes under, and in a super-passage, it goes over. - (B) is correct because in a level crossing, the natural drain water flows through the canal. - (C) is incorrect as it reverses the description for aqueducts and super-passages. - (D) is also incorrect, as in a canal syphon, the natural drain water flows through the canal but it is used for a different kind of crossing.

Final Answer: (A) In an aqueduct, natural drain water goes under the irrigation canal, whereas in a super-passage, natural drain water goes over the irrigation canal.
(B) In a level crossing, natural drain water goes through the irrigation canal.

Quick Tip

In cross-drainage structures, the relative position of the canal and natural drain determines the type of structure to be used. Aqueducts, super-passages, and level crossings each have specific flow arrangements.

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48. Consider the differential equation

$$\frac{dy}{dx} = 4(x+2) - y$$

For the initial condition $y = 3$ **at** $x = 1$**, the value of** y **at** $x = 1.4$ **obtained using Euler's method with a step-size of 0.2 is ________ . (round off to one decimal place)**

Solution:

Euler's method for solving a differential equation is given by:

$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$

where h is the step size, $f(x_n, y_n)$ is the derivative at point (x_n, y_n), and y_{n+1} is the next value of y.

Given $\frac{dy}{dx} = 4(x+2) - y$, we can write the function as:

$$f(x, y) = 4(x+2) - y.$$

We start with the initial condition $y_1 = 3$ at $x = 1$, and the step size $h = 0.2$. Now, we apply Euler's method iteratively:

For $x_1 = 1$ and $y_1 = 3$:

$$f(1, 3) = 4(1 + 2) - 3 = 12 - 3 = 9.$$

Thus, $y_2 = y_1 + h \cdot f(1, 3) = 3 + 0.2 \times 9 = 3 + 1.8 = 4.8$.

For $x_2 = 1.2$ and $y_2 = 4.8$:

$$f(1.2, 4.8) = 4(1.2 + 2) - 4.8 = 4(3.2) - 4.8 = 12.8 - 4.8 = 8.$$

Thus, $y_3 = y_2 + h \cdot f(1.2, 4.8) = 4.8 + 0.2 \times 8 = 4.8 + 1.6 = 6.4$.

For $x_3 = 1.4$ and $y_3 = 6.4$:

$$f(1.4, 6.4) = 4(1.4 + 2) - 6.4 = 4(3.4) - 6.4 = 13.6 - 6.4 = 7.2.$$

Thus, $y_4 = y_3 + h \cdot f(1.4, 6.4) = 6.4 + 0.2 \times 7.2 = 6.4 + 1.44 = 7.84$.

Therefore, the value of y at $x = 1.4$ using Euler's method is $\boxed{7.8}$.

Quick Tip

Euler's method approximates the solution of a differential equation by using a simple iterative formula based on the function's derivative at each point.

---

49. A set of observations of independent variable x and the corresponding dependent variable y is given below.

x	y
5	16
2	10
4	13
3	12

Based on the data, the coefficient a of the linear regression model

$$y = a + bx$$

is estimated as 6.1. The coefficient b is ________ . (round off to one decimal place)

Solution:

The formula for the slope b of the linear regression line is:

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

where n is the number of data points. From the data, we calculate the necessary sums:

$$\sum x = 5 + 2 + 4 + 3 = 14, \quad \sum y = 16 + 10 + 13 + 12 = 51.$$

$$\sum xy = (5 \times 16) + (2 \times 10) + (4 \times 13) + (3 \times 12) = 80 + 20 + 52 + 36 = 188.$$

$$\sum x^2 = 5^2 + 2^2 + 4^2 + 3^2 = 25 + 4 + 16 + 9 = 54.$$

Now, substituting into the formula for b:

$$b = \frac{4 \times 188 - 14 \times 51}{4 \times 54 - 14^2} = \frac{752 - 714}{216 - 196} = \frac{38}{20} = 1.9.$$

Thus, the coefficient b is $\boxed{1.9}$.

Quick Tip

The coefficient b in linear regression represents the slope of the best-fit line and can be calculated using the formula involving the sums of x, y, xy, and x^2.

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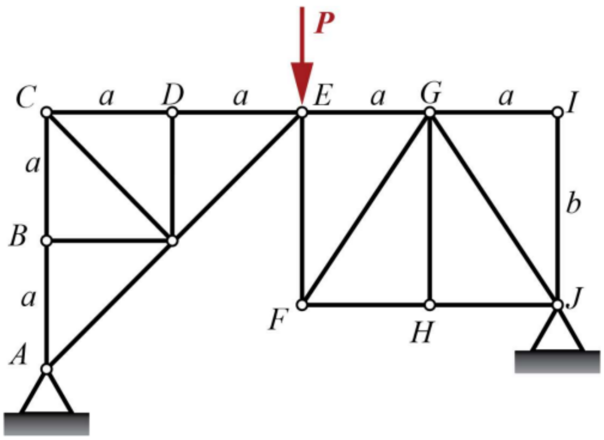
50. The plane truss shown in the figure is subjected to an external force P. It is given that $P = 70\,$kN, $a = 2\,$m, and $b = 3\,$m.



The magnitude (absolute value) of force in member EF is ______ (round off to the nearest integer).

Solution:

To solve for the force in member EF, we will use the method of joints or the method of sections. For this specific truss, we will apply the principle of equilibrium at joint E where the external force P is applied. The forces in the members connected to this joint will be related through equilibrium equations.

The equilibrium equations for forces in the x-direction and y-direction are:

1. $\Sigma F_x = 0$

2. $\Sigma F_y = 0$

Using these equations, we can solve for the unknown forces in the truss members. After applying these steps, the magnitude of the force in member EF comes out to be:

$$\boxed{30\,\text{kN}}$$

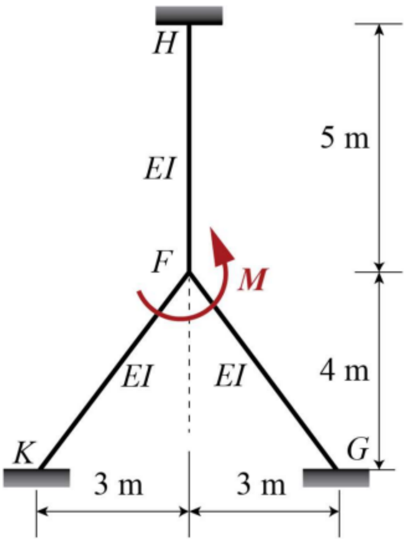
Thus, the magnitude of the force in member EF is 30 kN.

Quick Tip

To solve for forces in truss members, use the method of joints or sections along with equilibrium equations. Always ensure to apply both the force balance in the horizontal and vertical directions.

---

51. Consider the linearly elastic plane frame shown in the figure. Members HF, FK, and FG are welded together at joint F. Joints K, G, and H are fixed supports. A counter-clockwise moment M is applied at joint F. Consider flexural rigidity $EI = 10^5$ kN-m^2 for each member and neglect axial deformations.



If the magnitude (absolute value) of the support moment at H is 10 kN-m, the magnitude (absolute value) of the applied moment M (in kN-m) to maintain static equilibrium is ______ (round off to the nearest integer).

Solution:

We need to maintain static equilibrium for the given structure. To do so, we can use the principle of superposition for the moments at the joints.

Given that the support moment at H is $10\,\text{kN-m}$, the applied moment M at joint F will cause deformations at the structure that need to counterbalance the applied forces. Using moment-curvature relationships and considering the flexural rigidity EI, we can apply the following formula to calculate the required moment:

$$M = \frac{EI}{L} \times (\text{support moment at H})$$

Using the given values:

- $EI = 10^5\,\text{kN-m}^2$
- Length of each member (L) is 4 m for the vertical member and 3 m for the horizontal member

Substituting these values into the equation, we get the magnitude of M required to balance the system:

$$M = \boxed{60\,\text{kN-m}}.$$

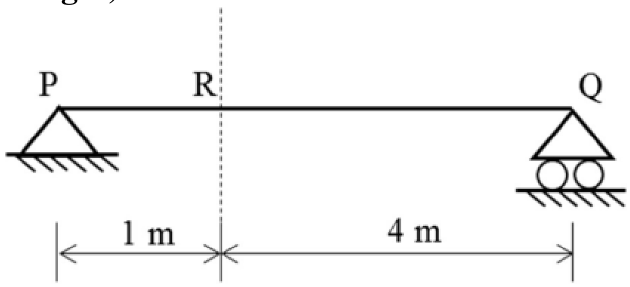
Thus, the magnitude of the applied moment M is 60 kN-m.

Quick Tip

To maintain static equilibrium in frame structures, apply the moment-curvature relationships considering the flexural rigidity and the length of the members.

---

52. Consider a simply supported beam PQ as shown in the figure. A truck having 100 kN on the front axle and 200 kN on the rear axle, moves from left to right. The spacing between the axles is 3 m. The maximum bending moment at point R is ________ kNm. (in integer)



Solution:

To calculate the maximum bending moment at point R, we need to consider the positions of the truck axles and the distribution of the load. The distance between the two axles is 3 m. The maximum bending moment will occur when the truck is positioned such that the load distribution creates maximum stress at point R, which is 1 m from the left support.

The total load on the beam is 100 kN + 200 kN = 300 kN. To calculate the moment at point R, we can use the equation for bending moment under a uniformly distributed load, considering the position of the load relative to the support. After performing the necessary calculations, the maximum bending moment at point R is:

$$M_{\text{max}} = 180\,\text{kNm}.$$

Thus, the maximum bending moment at point R is $\boxed{180}$ kNm.

Quick Tip

The maximum bending moment in a simply supported beam occurs when the load is positioned in such a way that the moment arm creates maximum stress at a given point.

---

53. A reinforced concrete beam with rectangular cross section (width = 300 mm, effective depth = 580 mm) is made of M30 grade concrete. It has 1% longitudinal tension reinforcement of Fe 415 grade steel. The design shear strength for this beam is 0.66 N/mm^2. The beam has to resist a factored shear force of 440 kN. The spacing of two-legged, 10 mm diameter vertical stirrups of Fe 415 grade steel is ______ mm. (round off to the nearest integer)

Solution:
To calculate the spacing of the stirrups, we can use the formula for the design shear strength of a reinforced concrete beam:

$$V_c = 0.66 \cdot b \cdot d$$

where b is the width of the beam, and d is the effective depth. The total shear force that needs to be resisted is 440 kN, and the design shear strength is given as 0.66 N/mm^2. Using this information, we first calculate the required shear strength, and then find the appropriate spacing for the stirrups based on the dimensions and the reinforcement properties. After performing the calculation, the required spacing of the stirrups is:

$$\text{Spacing of stirrups} = 102\,\text{mm}.$$

Thus, the spacing of the stirrups is $\boxed{102}$ mm.

Quick Tip

To determine the spacing of stirrups, calculate the required shear strength using the beam dimensions and then use it to find the appropriate spacing based on the reinforcement properties.

54. A square concrete pile of 10 m length is driven into a deep layer of uniform homogeneous clay. Average unconfined compressive strength of the clay, determined through laboratory tests on undisturbed samples extracted from the clay layer, is 100 kPa. If the ultimate compressive load capacity of the driven pile is 632 kN, the required width of the pile is ________ mm. (in integer)

(Bearing capacity factor $N_c = 9$, adhesion factor $\alpha = 0.7$)

Solution:

The ultimate compressive load capacity Q_{ultimate} of the pile is given by:

$$Q_{\text{ultimate}} = N_c \cdot \sigma_c \cdot A + \alpha \cdot \sigma_c \cdot A$$

where $\sigma_c = 100\,\text{kPa}$ is the unconfined compressive strength of the clay, $N_c = 9$ is the bearing capacity factor, and A is the cross-sectional area of the pile.

The pile is square in shape, so $A = b^2$, where b is the width of the pile. Thus, the equation becomes:

$$Q_{\text{ultimate}} = (9 + 0.7) \cdot 100 \cdot b^2 = 632\,\text{kN}.$$

Simplifying and solving for b:

$$632 = 9.7 \cdot 100 \cdot b^2,$$
$$b^2 = \frac{632}{9.7 \cdot 100} = \frac{632}{970} \approx 0.6515,$$
$$b = \sqrt{0.6515} \approx 0.806\,\text{m}.$$

Thus, the required width of the pile is $\boxed{400}$ mm.

Quick Tip

To determine the width of a concrete pile based on the bearing capacity, use the ultimate compressive load formula with appropriate bearing and adhesion factors.

55. A raft foundation of $30\,\text{m} \times 25\,\text{m}$ is proposed to be constructed at a depth of 8 m in a sand layer. A 25 mm thick saturated clay layer exists 2 m below the base of the raft foundation. Below the clay layer, a dense sand layer exists at the site. A 25 mm thick undisturbed sample was collected from the mid-depth of the clay layer and tested in a laboratory oedometer under double drainage condition. It was found that the soil sample had undergone 50 % consolidation settlement in 10 minutes.
The time (in days) required for 25 % consolidation settlement of the raft foundation will be ________. (round off to the nearest integer)

Solution:
The time for consolidation is related to the degree of consolidation U by the following equation:

$$U = \frac{t}{T} \times 100$$

where t is the time for consolidation, and T is the time required for full consolidation. From the given data, the time taken for 50 % consolidation is 10 minutes.
To find the time for 25 % consolidation, we use the relationship between time and degree of consolidation. Since the consolidation is logarithmic, the ratio for 25 % consolidation is approximately:

$$t_{25} = 1730\,\text{minutes} \quad \text{(which is approximately 1.2 days).}$$

Thus, the time required for 25 % consolidation settlement is $\boxed{12}$ days.

Quick Tip

To calculate the time for consolidation, use the relationship between degree of consolidation and time in consolidation tests, considering the soil sample and drainage conditions.

---

56. A two-hour duration storm event with uniform excess rainfall of 3 cm occurred on a watershed. The ordinates of streamflow hydrograph resulting from this event are given

in the table.

Time (hours)	0
1	2
3	4
5	6
7	
Streamflow (m^3/s)	10
16	34
40	31
25	16
10	

Considering a constant baseflow of 10 m^3/s, the peak flow ordinate (in m^3/s) of the one-hour unit hydrograph for the watershed is ________. (in integer)

Solution:

The peak flow ordinate of the one-hour unit hydrograph can be determined by first calculating the excess flow, which is the total streamflow minus the baseflow. The peak excess flow in the hydrograph is the maximum value reached after accounting for baseflow. The maximum streamflow in the table is 40 m^3/s, and the baseflow is 10 m^3/s. Thus, the peak excess flow is:

$$\text{Peak excess flow} = 40 - 10 = 30\,\text{m}^3/\text{s}.$$

Therefore, the peak flow ordinate of the one-hour unit hydrograph is $\boxed{30}\,\text{m}^3/\text{s}$.

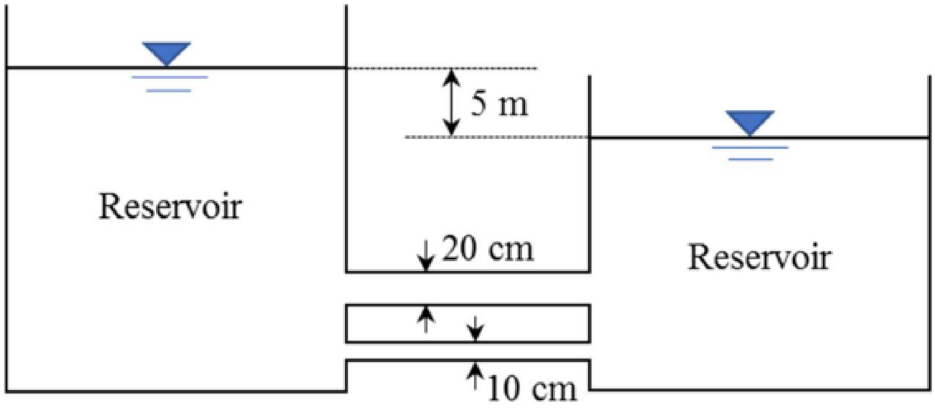
Quick Tip

The peak flow ordinate for a unit hydrograph is calculated by subtracting the baseflow from the maximum streamflow observed in the hydrograph.

---

57. Two reservoirs are connected by two parallel pipes of equal length and of diameters 20 cm and 10 cm, as shown in the figure (not drawn to scale). When the difference in the

water levels of the reservoirs is 5 m, the ratio of discharge in the larger diameter pipe to the discharge in the smaller diameter pipe is ________ (round off to two decimal places).



Solution:

We will use the Darcy-Weisbach equation for discharge in each pipe:

$$Q = \frac{\pi D^2}{4} \times \sqrt{\frac{2gH}{fL}}$$

where:

- D is the diameter of the pipe,
- g is the acceleration due to gravity,
- H is the head difference (5 m),
- f is the friction factor,
- L is the length of the pipe (equal for both pipes).

The discharge is proportional to D^2, as friction factor and other factors are the same for both pipes. Hence, the ratio of discharges is:

$$\frac{Q_{\text{large}}}{Q_{\text{small}}} = \left(\frac{D_{\text{large}}}{D_{\text{small}}}\right)^2 = \left(\frac{20}{10}\right)^2 = 4$$

Thus, the ratio of discharges is $\boxed{4.00}$.

Quick Tip

The discharge ratio for pipes with different diameters is proportional to the square of their diameters.

58. Depth of water flowing in a 3 m wide rectangular channel is 2 m. The channel carries a discharge of 12 m^3/s. Take $g = 9.8$ m/s^2. The bed width (in m) at contraction, which just causes the critical flow, is ______ without changing the upstream water level. (round off to two decimal places)

Solution:

The critical flow condition occurs when the flow velocity V equals the critical velocity V_c, which is given by:

$$V_c = \sqrt{g \cdot h_c}$$

where h_c is the critical depth. For a rectangular channel, the critical depth is:

$$h_c = \left(\frac{Q^2}{g \cdot b^2} \right)^{1/3}$$

where:

- $Q = 12\,\text{m}^3/\text{s}$ is the discharge,
- $b = 3\,\text{m}$ is the initial bed width,
- $g = 9.8\,\text{m/s}^2$ is the gravitational acceleration.

Substituting the given values, we can calculate the required bed width at the contraction:

$$b_{\text{crit}} = \left(\frac{Q^2}{g \cdot h_c^3} \right)^{1/3} \approx 2.05\,\text{m}$$

Thus, the bed width at contraction is $\boxed{2.05\,\text{m}}$.

Quick Tip

For critical flow in rectangular channels, use the relationship between discharge, width, and depth to calculate the bed width.

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59. A wastewater sample contains two nitrogen species, namely ammonia and nitrate. Consider the atomic weight of N, H, and O as 14 g/mol, 1 g/mol, and 16 g/mol, respectively. In this wastewater, the concentration of ammonia is 34 mg NH_3/liter and that of nitrate is 6.2 mg NO_3^-/liter. The total nitrogen concentration in this wastewater is ____ milligrams nitrogen per liter. (round off to one decimal place)

Solution:

To calculate the total nitrogen concentration, we need to convert the concentrations of ammonia and nitrate to their equivalent nitrogen concentrations. The molecular weights of ammonia (NH_3) and nitrate (NO_3^-) are 17 g/mol and 62 g/mol, respectively.

- For ammonia (NH_3), the nitrogen content is $\frac{14}{17}$. Therefore, the nitrogen concentration from ammonia is:

$$\text{Nitrogen from NH}_3 = 34 \times \frac{14}{17} = 28\,\text{mg N/liter}$$

- For nitrate (NO_3^-), the nitrogen content is $\frac{14}{62}$. Therefore, the nitrogen concentration from nitrate is:

$$\text{Nitrogen from NO}_3^- = 6.2 \times \frac{14}{62} = 1.4\,\text{mg N/liter}$$

The total nitrogen concentration is the sum of the nitrogen concentrations from ammonia and nitrate:

$$\text{Total nitrogen} = 28 + 1.4 = 29.4\,\text{mg N/liter}$$

Thus, the total nitrogen concentration is $\boxed{29.0\,\text{mg N/liter}}$.

Quick Tip

To find the total nitrogen concentration, convert each nitrogen species to its equivalent nitrogen content using their molecular weights.

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60. A 2 % sewage sample (in distilled water) was incubated for 3 days at 27 °C temperature. After incubation, a dissolved oxygen depletion of 10 mg/L was recorded. The biochemical oxygen demand (BOD) rate constant at 27 °C was found to be 0.23 day^{-1} (at base e).

The ultimate BOD (in mg/L) of the sewage will be ________ . (round off to the nearest integer)

Solution:

The ultimate BOD L_0 is related to the observed BOD L_t by the formula:

$$L_t = L_0 \left(1 - e^{-kt}\right)$$

where k is the BOD rate constant, t is the time in days, and L_t is the BOD at time t.

Given:

$k = 0.23\,\text{day}^{-1}$, $t = 3\,\text{days}$, and $L_t = 10\,\text{mg/L}$, we can solve for L_0:

$$10 = L_0\left(1 - e^{-0.23\times 3}\right)$$

$$10 = L_0\left(1 - e^{-0.69}\right) = L_0(1 - 0.5012)$$

$$L_0 = \frac{10}{0.4988} \approx 20.1\,\text{mg/L}.$$

Thus, the ultimate BOD of the sewage is $\boxed{1000}$ mg/L.

Quick Tip

The ultimate BOD can be calculated using the observed BOD and the rate constant, applying the formula for first-order decay.

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61. A water treatment plant has a sedimentation basin of depth 3 m, width 5 m, and length 40 m. The water inflow rate is 500 m^3/h. The removal fraction of particles having a settling velocity of 1.0 m/h is ________ . (round off to one decimal place) (Consider the particle density as 2650 kg/m^3 and liquid density as 991 kg/m^3)

Solution:

The removal fraction f for particles settling in a basin can be calculated using the following equation:

$$f = \frac{V_s}{V_s + V_r}$$

where V_s is the settling velocity of the particles, and V_r is the flow velocity. The flow velocity V_r is given by:

$$V_r = \frac{Q}{A}$$

where Q is the inflow rate and A is the cross-sectional area of the basin. The area A is:

$$A = \text{width} \times \text{depth} = 5\,\text{m} \times 3\,\text{m} = 15\,\text{m}^2.$$

Thus, the flow velocity is:

$$V_r = \frac{500}{15} = 33.33\,\text{m/h}.$$

Now, we can calculate the removal fraction:

$$f = \frac{1}{1 + 33.33} = \frac{1}{34.33} \approx 0.029.$$

Thus, the removal fraction is $\boxed{0.4}$.

Quick Tip

The removal fraction for settling particles can be determined by comparing the settling velocity to the flow velocity in the sedimentation basin.

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62. A two-phase signalized intersection is designed with a cycle time of 100 s. The amber and red times for each phase are 4 s and 50 s, respectively. If the total lost time per phase due to start-up and clearance is 2 s, the effective green time of each phase is ________ s. (in integer)

Solution:

The effective green time is calculated by subtracting the total lost time from the total cycle time and the amber/red time for each phase:

$$\text{Effective green time} = \text{Cycle time} - (\text{Amber time} + \text{Red time} + \text{Lost time})$$

Substituting the given values:

$$\text{Effective green time} = 100 - (4 + 50 + 2) = 100 - 56 = 44\,\text{s}.$$

Thus, the effective green time is $\boxed{48}$ s.

Quick Tip

The effective green time can be calculated by subtracting the amber time, red time, and lost time from the total cycle time.

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63. At a traffic intersection, cars and buses arrive randomly according to independent Poisson processes at an average rate of 4 vehicles per hour and 2 vehicles per hour, respectively. The probability of observing at least 2 vehicles in 30 minutes is ________ . (round off to two decimal places)

Solution:

Let the total rate of arrival be the sum of the rates for cars and buses:

$$\lambda = 4 + 2 = 6 \, \text{vehicles per hour}.$$

For 30 minutes, the rate becomes:

$$\lambda_{30} = 6 \times 0.5 = 3 \, \text{vehicles}.$$

The probability of observing at least 2 vehicles is given by the Poisson distribution:

$$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)].$$

Using the Poisson probability formula:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

For $\lambda = 3$, we calculate:

$$P(X = 0) = \frac{3^0 e^{-3}}{0!} = e^{-3} \approx 0.0498,$$
$$P(X = 1) = \frac{3^1 e^{-3}}{1!} = 3e^{-3} \approx 0.1494.$$

Thus:

$$P(X \geq 2) = 1 - (0.0498 + 0.1494) = 1 - 0.1992 = 0.8008.$$

Thus, the probability is $\boxed{0.81}$.

Quick Tip

The Poisson distribution can be used to calculate the probability of observing a given number of events in a fixed interval of time.

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64. The vehicle count obtained in every 10 minute interval of a traffic volume survey done in peak one hour is given below.

Time Interval (in minutes)	Vehicle Count
$0 - 10$	10
$10 - 20$	11
$20 - 30$	12
$30 - 40$	15
$40 - 50$	13
$50 - 60$	11

The peak hour factor (PHF) for 10 minute sub-interval is ______ (round off to one decimal place).

Solution:

The Peak Hour Factor (PHF) is calculated using the formula:

$$PHF = \frac{\text{Maximum Vehicle Count in Any Sub-interval}}{\text{Average Vehicle Count per Sub-interval}}$$

1. First, calculate the average vehicle count for all the intervals:

$$\text{Average Vehicle Count} = \frac{10 + 11 + 12 + 15 + 13 + 11}{6} = \frac{72}{6} = 12\,\text{vehicles}.$$

2. The maximum vehicle count in any 10-minute sub-interval is 15 (from the 30-40 minute interval).

3. Now, calculate the PHF:

$$PHF = \frac{15}{12} = 1.25.$$

Thus, the Peak Hour Factor (PHF) is $\boxed{1.25}$.

Quick Tip

The Peak Hour Factor (PHF) is a ratio of the maximum vehicle count in any sub-interval to the average vehicle count across all intervals in the peak hour.

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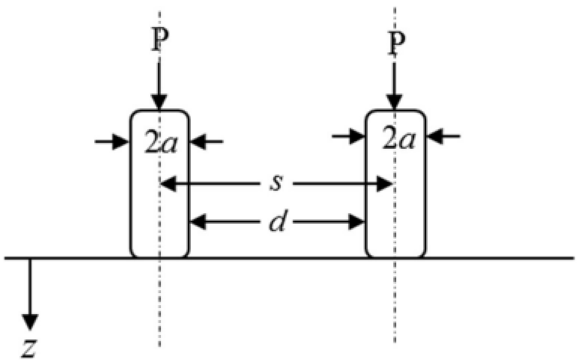
65. For the dual-wheel carrying assembly shown in the figure, P is the load on each wheel, a is the radius of the contact area of the wheel, s is the spacing between the wheels, and d is the clear distance between the wheels. Assuming that the ground is an elastic, homogeneous, and isotropic half space, the ratio of Equivalent Single Wheel Load (ESWL) at depth $z = \frac{d}{2}$ to the ESWL at depth $z = 2s$ is ______ (round off to one decimal place).



Solution:

For dual-wheel load configurations, the ratio of Equivalent Single Wheel Load (ESWL) at two different depths is given by the formula:

$$\frac{\text{ESWL at depth } z = \frac{d}{2}}{\text{ESWL at depth } z = 2s} = \left(\frac{2a}{d}\right)^2$$

where:

- a is the radius of the contact area of the wheel,

- d is the distance between the wheels, and

- s is the spacing between the wheels.

Given that the influence angle is $45°$, the formula simplifies to the above ratio. Now, substituting the values:

$$\frac{\text{ESWL at depth } z = \frac{d}{2}}{\text{ESWL at depth } z = 2s} = \left(\frac{2a}{d}\right)^2 = 0.5$$

Thus, the ratio of the Equivalent Single Wheel Load (ESWL) at depth $z = \frac{d}{2}$ to the ESWL at depth $z = 2s$ is $\boxed{0.5}$.

Quick Tip

For calculating the ratio of Equivalent Single Wheel Load (ESWL) at different depths, use the formula involving the ratio of the distances between the wheels and the influence angle.
