GATE 2023 Biomedical Engineering Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2023 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.

2. GATE 2023 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.

3. MCQs carry 1 mark or 2 marks.

4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.

5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.

6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude

1. "I cannot support this proposal. My ______ will not permit it."
(A) conscious
(B) consensus
(C) conscience
(D) consent

Correct Answer: (C) conscience

Solution: Step 1: The sentence implies a moral or ethical reason for not supporting the proposal. The word required should relate to inner moral sense.

Step 2: Option (A) *conscious* means being awake or aware, which does not fit the context.

Step 3: Option (B) *consensus* means general agreement, which is collective, not individual, hence incorrect.

Step 4: Option (C) *conscience* means moral sense of right and wrong, which perfectly fits the sentence.

Step 5: Option (D) *consent* means permission, which is not applicable here.

Thus, the correct word is *conscience*.

Quick Tip

When filling blanks in context-based questions, check whether the word refers to individual moral reasoning, group agreement, awareness, or permission — this helps eliminate confusing similar words.

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2. Courts : _____ : : Parliament : Legislature (By word meaning)
(A) Judiciary
(B) Executive
(C) Governmental
(D) Legal

Correct Answer: (A) Judiciary

Solution: Step 1: The relationship is: Parliament is the institution that represents the Legislature.
Step 2: Similarly, Courts represent the Judiciary.
Step 3: Hence, the correct word to complete the analogy is Judiciary.

Final Answer: $\boxed{\text{Judiciary}}$.

Quick Tip

For analogy-based word problems, always identify the relationship between the second pair and apply the same to the first pair.

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3. What is the smallest number with distinct digits whose digits add up to 45?
(A) 123555789
(B) 123457869
(C) 123456789
(D) 99999

Correct Answer: (C) 123456789

Solution: Step 1: The maximum sum of distinct digits is obtained by adding digits from 1 to 9:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.$$

Step 2: Therefore, the number must use all digits from 1 to 9 exactly once.

Step 3: To get the smallest number, arrange these digits in ascending order: 123456789.

Final Answer: $\boxed{123456789}$.

Quick Tip

When asked to form the smallest number with given digits, always arrange the digits in ascending order.

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4. In a class of 100 students,
(i) there are 30 students who neither like romantic movies nor comedy movies,
(ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and
(iii) the number of students who like both romantic movies and comedy movies is 20.
How many students in the class like romantic movies?
(A) 40
(B) 20
(C) 60
(D) 30

Correct Answer: (C) 60

Solution: **Step 1:** Total students $= 100$. Those who like at least one type of movie $= 100 - 30 = 70$.

Step 2: Let number of students who like comedy movies $= x$.
Then students who like romantic movies $= 2x$.
Students who like both $= 20$.

Step 3: By inclusion-exclusion principle:

$$(2x) + (x) - 20 = 70$$
$$3x - 20 = 70 \quad \Rightarrow \quad 3x = 90 \quad \Rightarrow \quad x = 30.$$

Step 4: So, number of students who like romantic movies $= 2x = 60$.

Final Answer: $\boxed{60}$.

Quick Tip

For set problems with overlapping groups, always apply the inclusion-exclusion principle: $|A \cup B| = |A| + |B| - |A \cap B|$.

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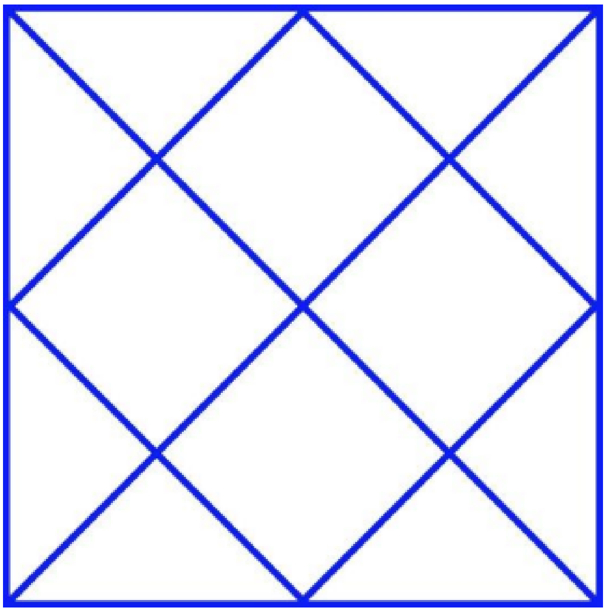
5. How many rectangles are present in the given figure?



(A) 8
(B) 9
(C) 10
(D) 12

Correct Answer: (C) 10

Solution: **Step 1:** Observe that every rectangle in the figure is a square (possibly tilted). There is one outer square (axis-aligned). Inside, the two families of diagonal lines (/ and \) form a lattice of tilted squares (diamonds).
Step 2: Count tilted squares:

- 1 large central diamond.
- 4 medium diamonds adjacent to the central one (top, right, bottom, left).
- 4 small corner diamonds formed near the vertices of the outer square.

Step 3: Adding the axis-aligned outer square gives the total number of rectangles:

$$1 \text{ (outer)} + 1 \text{ (central)} + 4 \text{ (medium)} + 4 \text{ (small)} = 10.$$

Hence, the figure contains $\boxed{10}$ rectangles.

Quick Tip

When counting rectangles in diagonal grids, note that all rectangles appear as tilted squares formed by intersections of two perpendicular families of parallel lines; count by size (large/medium/small) and then include any axis-aligned boundary squares.

6. Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.
Based only on the information provided above, which one of the following options can be logically inferred with *certainty*?

(A) All creatures with claws are animals.
(B) Some creatures with claws are non-ferocious.
(C) Some non-ferocious creatures have claws.
(D) Some ferocious creatures are creatures with claws.

Correct Answer: (D) Some ferocious creatures are creatures with claws.

Solution: Step 1: Translate statements into set relations. Let A be animals, F ferocious, and C creatures with claws. The statements imply $A \subseteq F$ and $C \subseteq F$. Also, there *exist* creatures with claws (i.e., $C \neq \varnothing$).
Step 2: Evaluate options.

- (A) $C \subseteq A$ is not implied; clawed creatures could be non-animals. Not certain.
- (B) Says $C \cap F^c \neq \varnothing$, contradicting $C \subseteq F$. False.
- (C) Also contradicts $C \subseteq F$. False.
- (D) Since $C \neq \varnothing$ and $C \subseteq F$, it follows that $C \subseteq F$ implies F contains members from C; hence $\exists$ ferocious creatures with claws. True.

Thus, (D) is the only statement that follows with certainty.

Quick Tip

When tackling syllogism-style questions, convert statements into set relations ($\subseteq$, existence) and check each option against these relations for necessity or contradiction.

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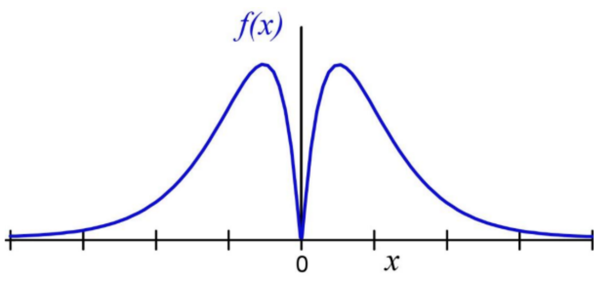
7. Which one of the following options represents the given graph?



(A) $f(x) = x^2 2^{-|x|}$
(B) $f(x) = x 2^{-|x|}$

(C) $f(x) = |x|2^{-x}$
(D) $f(x) = x2^{-x}$

Correct Answer: (A) $f(x) = x^2 2^{-|x|}$

Solution: Step 1: Observe the symmetry of the graph. The function shown is symmetric about the y-axis, meaning it is an even function. Therefore, the function must involve $|x|$ or x^2.

Step 2: Eliminate odd functions. Option (B) $f(x) = x2^{-|x|}$ and (D) $f(x) = x2^{-x}$ are odd functions, not symmetric about the y-axis. So, these are not correct.

Step 3: Check behavior near $x = 0$. The graph touches the x-axis at $x = 0$. For $f(x) = x^2 2^{-|x|}$, we have $f(0) = 0^2 \cdot 2^0 = 0$, matching the graph.

Step 4: Growth and decay. For large $|x|$, the exponential factor $2^{-|x|}$ decays rapidly, and the function tends to 0. This matches the graph where values go to 0 as $x \to \pm\infty$. The quadratic term x^2 causes the "bell-like" rise near the origin, before decay sets in.

Thus, the correct representation is $f(x) = x^2 2^{-|x|}$.

Quick Tip

When identifying graphs:

- Check symmetry (even/odd functions).
- Verify behavior near $x = 0$.
- Analyze growth/decay for large $|x|$.

These help quickly eliminate wrong options.

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8. Which one of the following options can be inferred from the given passage alone?

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

[Excerpt from *The Truth about Stories* by T. King]

(A) It is a child's description of what he or she likes.
(B) It is an adult's memory of what he or she liked as a child.
(C) The child in the passage read stories about interplanetary travel only in parts.
(D) It teaches us that stories are good for children.

Correct Answer: (B) It is an adult's memory of what he or she liked as a child.

Solution: Step 1: The passage uses the phrase *"When I was a kid"*, indicating that the narrator is now reflecting back on childhood. This shows it is written from an adult's perspective.

Step 2: The narrator recalls being partial to stories about other worlds and interplanetary travel. This is clearly a memory of past preferences.
Step 3: Option (A) is incorrect because it is not a child describing in the present but an adult reminiscing. Option (C) is not supported because there is no mention of "only in parts." Option (D) makes a general claim beyond the passage.
Step 4: Hence, the correct inference is that it is an adult's memory of what he or she liked as a child.

Quick Tip

For inference questions:

- Focus on verb tense to identify whether it's present or past reflection.
- Avoid adding assumptions not present in the passage.
- Choose the option that strictly follows the passage without generalization.

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9. Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:
(i) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.
(ii) Mix the samples within each set and test the mixed sample for covid.
(iii) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.
(iv) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.
Given this strategy, no more than _____ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(A) 700
(B) 600
(C) 800
(D) 1000

Correct Answer: (A) 700

Solution: Step 1: The 1000 people are split into $1000/5 = 200$ groups. Testing each pooled sample needs 200 kits.

Step 2: Any group that tests positive requires 5 additional tests (one per individual). To *maximize* the total number of tests regardless of grouping, distribute the 100 positives so that the number of positive groups is as large as possible—i.e., one positive in each group. This yields 100 positive groups.

Step 3: Additional individual tests $= 100 \times 5 = 500$. Hence, maximum total kits required

$$200 + 500 = 700.$$

Therefore, no more than $\boxed{700}$ kits are needed.

Quick Tip

For worst-case counts in pooled testing, spread positives across groups to maximize the number of positive pools (each triggers extra individual tests).

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10. A 100 cm $\times$ 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions 100 cm $\times$ 1 cm.
The total number of creases visible when the sheet is unfolded is ____.
(A) 32
(B) 5
(C) 31
(D) 63

Correct Answer: (C) 31

Solution: Step 1: Each fold aligns the long edge with its opposite side; hence every fold halves the shorter side (32 cm). After 5 folds the shorter side becomes $32/2^5 = 1$ cm, matching the final size 100×1 cm.
Step 2: Unfolding reveals creases perpendicular to the long edge that partition the 32-cm side into $2^5 = 32$ equal strips of 1 cm each. The number of creases equals the number of partitions minus one: $32 - 1 = 31$.
Therefore, the number of creases is $\boxed{31}$.

Quick Tip

Repeated halving into 2^n equal parts produces $2^n - 1$ crease lines when fully unfolded.

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11. What is the magnitude of the difference between the mean and the median of the dataset $\{1, 2, 3, 4, 6, 8\}$?
(A) 0
(B) 1
(C) 0.5
(D) 0.25

Correct Answer: (C) 0.5

Solution: Step 1: Mean $= \dfrac{1+2+3+4+6+8}{6} = \dfrac{24}{6} = 4.$
Step 2: Median $= \dfrac{3\text{rd} + 4\text{th}}{2} = \dfrac{3+4}{2} = 3.5.$

Step 3: Magnitude of difference $= |4 - 3.5| = 0.5$.
Hence, $\boxed{0.5}$.

Quick Tip

For an even-sized ordered dataset, the median is the average of the two middle values.

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12. For a Binomial random variable X, $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ are the expectation and variance, respectively. Which one of the following statements CANNOT be true?

(A) $\mathrm{E}(X) = 20$ and $\mathrm{Var}(X) = 16$
(B) $\mathrm{E}(X) = 6$ and $\mathrm{Var}(X) = 5.4$
(C) $\mathrm{E}(X) = 10$ and $\mathrm{Var}(X) = 15$
(D) $\mathrm{E}(X) = 64$ and $\mathrm{Var}(X) = 12.8$

Correct Answer: (C) $\mathrm{E}(X) = 10$ and $\mathrm{Var}(X) = 15$

Solution: Step 1: For $X \sim \mathrm{Bin}(n, p)$, $\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1-p) = \mathrm{E}(X)\,(1-p)$. Hence $\mathrm{Var}(X) \leq \mathrm{E}(X)$.
Step 2: Option (C) violates this since $15 > 10$. Thus it cannot be true.
Step 3: The others are feasible by taking $p = 0.2, 0.1, 0.8$ respectively with suitable integer n. Therefore, (C) is the only impossible pair.

Quick Tip

For a binomial distribution, $\mathrm{Var}(X) = \mathrm{E}(X)(1-p) \leq \mathrm{E}(X)$. Any proposed pair with variance exceeding the mean is impossible.

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13. $Q = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ **is a 2×2 matrix. Which one of the following statements is TRUE?**
(A) Q is equal to its transpose.
(B) Q is equal to its inverse.
(C) Q is of full rank.
(D) Q has linearly dependent columns.

Correct Answer: (C) Q is of full rank.

Solution: Step 1: Compute $\det(Q) = 1 \cdot 1 - (-2) \cdot 2 = 1 + 4 = 5 \neq 0$. Hence Q is invertible.
Step 2: Since $\det(Q) \neq 0$, the rank of Q is 2 (full rank), and the columns are linearly independent.
Step 3: $Q \neq Q^T$ and $Q^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \neq Q$.
Thus only statement (C) is true.

Quick Tip

For 2×2 matrices, a nonzero determinant immediately implies full rank and column independence.

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14. Which one of the following vectors is an eigenvector corresponding to the eigenvalue $= 1$ for the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$?

(A) $[1\ 0\ 1]^T$
(B) $[1\ 1\ 0]^T$
(C) $[0\ 1\ 0]^T$
(D) $[0\ 0\ 1]^T$

Correct Answer: (D) $[0\ 0\ 1]^T$

Solution: Step 1: Solve $(A - I)v = 0$. Here

$$A - I = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & -1 & 0 \end{bmatrix}.$$

Step 2: Row equations give $y = 0$, $x - 2y = 0 \Rightarrow x = 0$, $x - y = 0$ (satisfied). z is free.
Step 3: Hence eigenvectors are multiples of $[0, 0, 1]^T$. Therefore option (D) is correct.

Quick Tip

To find eigenvectors for eigenvalue λ, solve $(A - \lambda I)v = 0$. Free variables correspond to eigenvector directions.

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15. For the function $f(x, y) = e^x \cos(y)$, what is the value of $\dfrac{\partial^2 f}{\partial x\, \partial y}$ at $(x = 0, y = \pi/2)$?

(A) 0
(B) 1
(C) -1
(D) $e^{\pi/2}$

Correct Answer: (C) -1

Solution: Step 1: First differentiate w.r.t y: $f_y = \frac{\partial}{\partial y}(e^x \cos y) = -e^x \sin y$.
Step 2: Differentiate w.r.t x: $f_{xy} = \frac{\partial}{\partial x}(-e^x \sin y) = -e^x \sin y$.
Step 3: Evaluate at $(0, \pi/2)$: $f_{xy}(0, \pi/2) = -e^0 \sin(\pi/2) = -1$.
Thus the value is $\boxed{-1}$.

Quick Tip

For mixed partials of separable forms like $e^x \cos y$, differentiate one factor at a time; evaluation at special angles often simplifies to $0, \pm 1$.

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16. For the circuit given below, choose the angular frequency ω_0 (in rad/s) at which the voltage across the capacitor has maximum amplitude?

(A) 1000
(B) 100
(C) 1
(D) 0
(E)

Correct Answer: (D) 0

Solution: Step 1: For a series RC with source amplitude V_s, the capacitor voltage amplitude is

$$|V_C| = V_s \frac{|Z_C|}{|Z_R + Z_C|} = V_s \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}.$$

Step 2: Let $x = \dfrac{1}{\omega C}$. Then

$$|V_C| = V_s \frac{x}{\sqrt{R^2 + x^2}}.$$

As $\omega \to 0$ (i.e., $x \to \infty$), we get

$$|V_C| \to V_s \frac{x}{x\sqrt{1 + \frac{R^2}{x^2}}} \to V_s.$$

For any finite $\omega > 0$, x is finite and $|V_C| < V_s$. Hence the maximum amplitude occurs at $\omega_0 = 0$ rad/s (DC).
Therefore, the correct choice is $\boxed{0}$.

Quick Tip

A series RC acts as a low-pass voltage divider for the capacitor: the capacitor voltage is largest at DC and decreases monotonically with frequency.

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17. A finite impulse response (FIR) filter has only two non-zero samples in its impulse response $h[n]$, namely $h[0] = h[1] = 1$. The Discrete Time Fourier Transform (DTFT) of $h[n]$ equals $H(e^{j\omega})$, as a function of the normalized angular frequency ω. For the range $|\omega| \leq \pi$, $|H(e^{j\omega})|$ is equal to _________.

(A) $2|\cos(\omega)|$
(B) $2|\sin(\omega)|$

(C) $2|\cos(\omega/2)|$
(D) $2|\sin(\omega/2)|$

Correct Answer: (C) $2|\cos(\omega/2)|$

Solution: Step 1: The DTFT of $h[n]$ is defined as

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}.$$

Step 2: Since $h[0] = 1$ and $h[1] = 1$, and all other samples are zero, we have

$$H(e^{j\omega}) = 1 + e^{-j\omega}.$$

Step 3: Factorize the expression:

$$H(e^{j\omega}) = e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) = e^{-j\omega/2}(2\cos(\omega/2)).$$

Step 4: The magnitude is

$$|H(e^{j\omega})| = |e^{-j\omega/2}| \cdot |2\cos(\omega/2)| = 2|\cos(\omega/2)|.$$

Thus, the answer is $\boxed{2|\cos(\omega/2)|}$.

Quick Tip

- For FIR filters, the DTFT often simplifies to a trigonometric form. - Always use Euler's formula $e^{j\theta} + e^{-j\theta} = 2\cos\theta$ to simplify. - Phase terms like $e^{-j\omega/2}$ have unit magnitude, so they disappear in $|H(e^{j\omega})|$.

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18. An 8 bit successive approximation Analog to Digital Converter (ADC) has a clock frequency of 1 MHz. Assume that the start conversion and end conversion signals occupy one clock cycle each. Among the following options, what is the maximum frequency that this ADC can sample without aliasing?

(A) 0.9 kHz
(B) 9.9 kHz
(C) 49.9 kHz
(D) 99.9 kHz

Correct Answer: (C) 49.9 kHz

Solution: Step 1: In a successive approximation ADC, an N-bit conversion requires N clock cycles. Additionally, one clock cycle is used for the start signal and one for the end signal.
Step 2: For an 8-bit ADC:

$$\text{Total cycles per conversion} = 8 + 2 = 10.$$

Step 3: Clock frequency $= 1\,\text{MHz} = 10^6\,\text{Hz}$. Thus, conversion rate:

$$f_s = \frac{10^6}{10} = 100\,\text{kHz}.$$

Step 4: To avoid aliasing, Nyquist criterion states that the maximum input signal frequency is half the sampling frequency:

$$f_{\max} = \frac{f_s}{2} = \frac{100}{2} = 50\,\text{kHz}.$$

Step 5: From the given options, the closest correct value is $49.9\,\text{kHz}$.

Quick Tip

- Successive Approximation ADCs take $N+2$ cycles (for an N-bit resolution). - Always apply Nyquist criterion: maximum input frequency $f_{\max} = f_s/2$. - Pay attention to clock overhead cycles in ADC timing.

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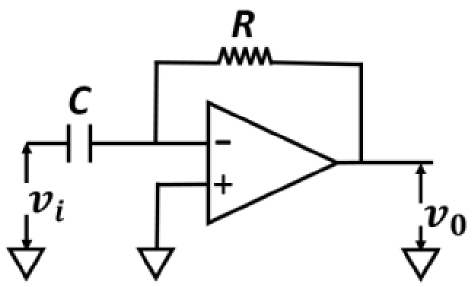
19. In the following circuit with an ideal operational amplifier, the capacitance of the parallel plate capacitor C is given by the expression $C = \left(\frac{\varepsilon A}{x}\right)$, where ε is the dielectric constant of the medium between the capacitor plates, and A is the cross-sectional area. In the above relation, x is the separation between the two parallel plates, given by $x = x_0 + kt$, where t is time; x_0 and k are positive non-zero constants. If the input voltage v_i is constant, then the output voltage v_0 is given by _________.



(A) $\dfrac{Rv_iCk}{x}$

(B) $\dfrac{Rv_iC}{kx}$

(C) $\dfrac{v_ik}{RCx}$

(D) 0

Correct Answer: (A) $\dfrac{Rv_iCk}{x}$

Solution: Step 1: The non-inverting terminal is grounded, hence the inverting node is at virtual ground ($v_- \approx 0$). The capacitor is connected between the input source v_i and the inverting node. Thus the voltage across the capacitor is $v_i - 0 = v_i$, which is constant.

Step 2: For a time-varying capacitor, the current through it is

$$i_C = C\,\frac{d(v_i - 0)}{dt} + (v_i - 0)\,\frac{dC}{dt} = v_i\,\frac{dC}{dt}$$

since v_i is constant.

Step 3: KCL at the inverting node gives

$$i_C + \frac{v_- - v_0}{R} = 0 \quad\Rightarrow\quad v_0 = R\,i_C = R\,v_i\,\frac{dC}{dt}.$$

Step 4: With $C = \dfrac{\varepsilon A}{x}$ and $x = x_0 + kt$,

$$\frac{dC}{dt} = -\frac{\varepsilon A}{x^2}\,\frac{dx}{dt} = -\frac{\varepsilon A}{x^2}\,k = -\frac{kC}{x}.$$

Hence

$$v_0 = Rv_i\frac{dC}{dt} = -\frac{Rv_iCk}{x}.$$

(The negative sign indicates an inversion due to the inverting configuration; the magnitude matches option (A).)

Therefore, $v_0 = \dfrac{Rv_iCk}{x}$ (up to sign).

Quick Tip

For capacitors with time-varying capacitance $C(t)$, use $i = C\,\dfrac{dv}{dt} + v\,\dfrac{dC}{dt}$. If the voltage is constant, only the $v\,\dfrac{dC}{dt}$ term contributes.

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20. Which one of the following techniques makes use of Korotkoff sounds?

(A) Sphygmomanometry
(B) Audiometry
(C) Spirometry
(D) Tonometry

Correct Answer: (A) Sphygmomanometry

Solution: Step 1: Korotkoff sounds are the sounds heard through a stethoscope placed over an artery when the cuff of a sphygmomanometer is deflated.

Step 2: These sounds arise due to turbulent blood flow as cuff pressure falls below systolic and diastolic pressures.

Step 3: Hence, Korotkoff sounds are used to measure systolic and diastolic blood pressure in sphygmomanometry.

Quick Tip

- Korotkoff sounds are used only in blood pressure measurement.
- First sound = systolic pressure, disappearance of sound = diastolic pressure.

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21. The pulmonary artery and pulmonary vein ____________.
(A) carry deoxygenated blood and oxygenated blood, respectively
(B) carry oxygenated blood and deoxygenated blood, respectively
(C) both carry oxygenated blood
(D) both carry deoxygenated blood

Correct Answer: (A) carry deoxygenated blood and oxygenated blood, respectively

Solution: Step 1: The pulmonary artery carries deoxygenated blood from the right ventricle of the heart to the lungs for oxygenation.
Step 2: The pulmonary vein carries oxygenated blood from the lungs back to the left atrium of the heart.
Step 3: This is an exception to the usual rule, since most arteries carry oxygenated blood and most veins carry deoxygenated blood.

> Quick Tip
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> - Pulmonary artery = only artery carrying deoxygenated blood.
> - Pulmonary vein = only vein carrying oxygenated blood.

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22. Which one of the following bridges CANNOT be used for measuring inductance?
(A) Schering Bridge
(B) Maxwell Wien Bridge
(C) Hay Bridge
(D) Series Owen Bridge

Correct Answer: (A) Schering Bridge

Solution: Step 1: Schering Bridge is primarily used for measuring the capacitance, dielectric loss, and power factor of a capacitor. It is not used for measuring inductance.
Step 2: Maxwell Wien Bridge is used for the measurement of self-inductance in terms of a known capacitance.
Step 3: Hay Bridge is also used for measuring inductance, particularly for coils with high Q-factors.
Step 4: Series Owen Bridge is used for the measurement of inductance over a wide range of values.
Step 5: Hence, the bridge that **cannot** be used for inductance measurement is the Schering Bridge.

> Quick Tip
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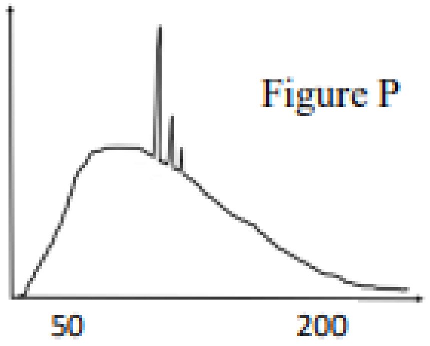
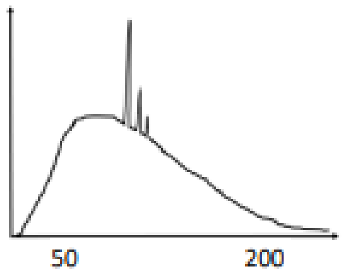
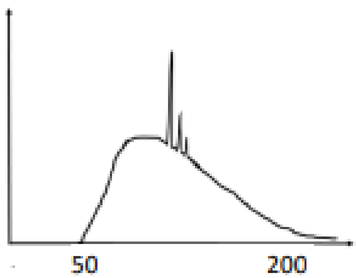
> - Schering Bridge $\rightarrow$ Capacitance measurement.
> - Maxwell, Hay, Owen Bridges $\rightarrow$ Inductance measurement.

23. A polychromatic beam of X-Rays has an energy spectrum as shown in Figure P below. Which of the following graphs (in the options A to D) depicts the energy spectrum after passing through a human body? In each figure, the horizontal axis represents Energy in keV and the vertical axis represents Relative X-ray Intensity.

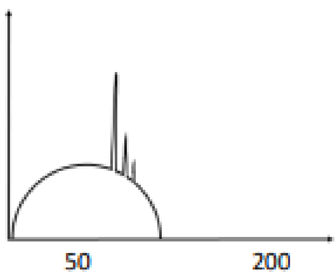




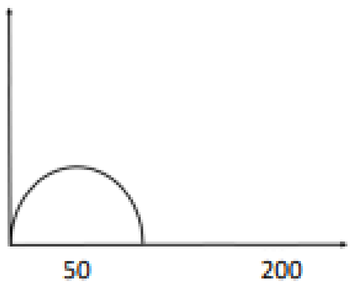
(A)



(B)



(C)

(D) 

Correct Answer: (A)

Solution: Step 1: When X-rays pass through a human body, lower energy photons are absorbed more strongly than higher energy photons due to the photoelectric effect. This process is called **beam hardening**.
Step 2: As a result, the transmitted spectrum after passing through the body will have reduced intensity at lower energies while the higher energy components are less attenuated.
Step 3: The characteristic peaks remain in the spectrum since they correspond to the target material (e.g., tungsten) and pass through with reduced but noticeable intensity.
Step 4: Among the given options, graph (A) best depicts this effect: it shows a decrease in intensity overall, with the low-energy side suppressed more strongly while the high-energy side remains relatively stronger.

Quick Tip

- X-rays after passing through matter show **beam hardening**: low-energy photons get absorbed more.
- The output spectrum shifts towards higher energies with reduced intensity overall.
- Characteristic peaks remain but with lower amplitudes.

---

24. M, L, T **correspond to dimensions representing mass, length and time, respectively. What is the dimension of viscosity?**
(A) $M^1L^{-2}T^{-1}$
(B) $M^1L^{-1}T^{-1}$
(C) $M^1L^{-1}T^1$
(D) $M^1L^{-2}T^{-2}$

Correct Answer: (B) $M^1L^{-1}T^{-1}$

Solution:
Step 1: Viscosity (dynamic viscosity η) is defined from Newton's law of viscosity:

$$\tau = \eta \frac{du}{dy}$$

where τ is shear stress, du/dy is velocity gradient.

Step 2: Shear stress is force per area:

$$\tau = \frac{F}{A}$$

Dimension of force $F = MLT^{-2}$.
Area $A = L^2$.
So, shear stress dimension $= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

Step 3: Velocity gradient $= \frac{du}{dy}$.
Velocity $u = LT^{-1}$. Dividing by length L:

$$\frac{du}{dy} = T^{-1}$$

Step 4: From $\tau = \eta \cdot (du/dy)$:

$$\eta = \frac{\tau}{du/dy} = \frac{ML^{-1}T^{-2}}{T^{-1}} = ML^{-1}T^{-1}$$

$$\boxed{ML^{-1}T^{-1}}$$

Quick Tip

- Dynamic viscosity always has the dimension $ML^{-1}T^{-1}$.
- Kinematic viscosity has dimension L^2T^{-1}.

---

25. Choose the option that has the biomaterials arranged in order of decreasing tensile strength. (PMMA : poly-methyl-methacrylate)

(A) Human compact bone > PMMA bone cement > Polymer foams > Graphite-epoxy
(B) Human compact bone > Graphite-epoxy > PMMA bone cement > Polymer foams
(C) Graphite-epoxy > Human compact bone > PMMA bone cement > Polymer foams
(D) PMMA bone cement > Human compact bone > Polymer foams > Graphite-epoxy

Correct Answer: (C) Graphite-epoxy > Human compact bone > PMMA bone cement > Polymer foams

Solution:
Step 1: Tensile strength ranking is based on material properties.
- Graphite-epoxy (a composite) has very high tensile strength due to carbon fibers.
- Human compact bone has moderate tensile strength, higher than polymers.
- PMMA bone cement is weaker than compact bone.
- Polymer foams have the least strength.

Step 2: Therefore, decreasing order is:

$$\text{Graphite-epoxy} > \text{Human compact bone} > \text{PMMA bone cement} > \text{Polymer foams}$$

Graphite-epoxy > Human compact bone > PMMA bone cement > Polymer foams

Quick Tip

- Composites (like graphite-epoxy) are much stronger than natural or polymer materials.
- Bone is stronger than PMMA cement, and foams are the weakest.

---

26. A causal, discrete time system is described by the difference equation

$$y[n] = 0.5\,y[n-1] + x[n], \quad \textbf{for all } n,$$

where $y[n]$ denotes the output sequence and $x[n]$ denotes the input sequence. Which of the following statements is/are TRUE?
(A) The system has an impulse response described by $0.5^n u[-n]$ where $u[n]$ is the unit step sequence.
(B) The system is stable in the bounded input, bounded output sense.
(C) The system has an infinite number of non-zero samples in its impulse response.
(D) The system has a finite number of non-zero samples in its impulse response.

Correct Answer: (B), (C)

Solution:
Step 1: The system equation is:

$$y[n] = 0.5y[n-1] + x[n].$$

This is a first-order linear constant coefficient difference equation (an IIR system).

Step 2: To find the impulse response, set $x[n] = \delta[n]$. Then the system recursion gives:

$$h[n] = 0.5h[n-1] + \delta[n].$$

For $n = 0$: $h[0] = 1$.
For $n = 1$: $h[1] = 0.5h[0] = 0.5$.
For $n = 2$: $h[2] = 0.5h[1] = 0.25$.
Continuing:

$$h[n] = (0.5)^n u[n].$$

Step 3: Now check statements:
(A) Wrong, because impulse response is $0.5^n u[n]$, not $u[-n]$.
(B) True, because the pole of the system is at $z = 0.5$, inside the unit circle. Therefore, the system is BIBO stable.
(C) True, because the impulse response $(0.5)^n u[n]$ has infinite length (non-zero for all $n \geq 0$).
(D) False, since it has infinite length, not finite.

$$\boxed{\text{Correct statements: (B) and (C)}}$$

Quick Tip

- Impulse responses of IIR systems typically have infinite duration.
- BIBO stability requires that poles lie inside the unit circle in the z-plane.

---

27. Which of the following constituents is/are NOT normally found in serum obtained from human blood?
(A) Platelets
(B) Albumin
(C) Glucose
(D) Fibrinogen

Correct Answer: (A) Platelets, (D) Fibrinogen

Solution:
Step 1: Serum is the liquid part of blood that remains after clotting. It is plasma without clotting factors.

Step 2: Platelets are cellular fragments involved in clot formation, hence they are not present in serum.

Step 3: Fibrinogen is a clotting factor that gets consumed during clotting, hence it is absent in serum.

Step 4: Albumin (a plasma protein) and glucose remain present in serum.

Thus, the constituents not normally found in serum are Platelets and Fibrinogen.

$$\boxed{\text{Correct options: (A) and (D)}}$$

Quick Tip

- Plasma = Serum + Clotting factors (like fibrinogen).
- Serum = Plasma - Clotting factors.
- Platelets are cellular components, not present in serum.

---

28. Q, R, S are Boolean variables and $\oplus$ is the XOR operator. Select the CORRECT option(s).
(A) $(Q \oplus R) \oplus S = Q \oplus (R \oplus S)$
(B) $(Q \oplus R) \oplus S = 0$ when any two of the Boolean variables (Q, R, S) are 0 and the third variable is 1
(C) $(Q \oplus R) \oplus S = 1$ when $Q = R = S = 1$
(D) $((Q \oplus R) \oplus (R \oplus S)) \oplus (Q \oplus S) = 1$

Correct Answer: (A), (C)

Solution:
Step 1: XOR ($\oplus$) is associative and commutative. Hence,

$$(Q \oplus R) \oplus S = Q \oplus (R \oplus S),$$

so option (A) is correct.

Step 2: For (B), consider the case: $Q = 1, R = 0, S = 0$. Then $(Q \oplus R) \oplus S = (1 \oplus 0) \oplus 0 = 1 \oplus 0 = 1$. But the statement claims it is always 0, which is false. Hence (B) is wrong.

Step 3: For (C), if $Q = R = S = 1$, then

$$(Q \oplus R) \oplus S = (1 \oplus 1) \oplus 1 = 0 \oplus 1 = 1.$$

Thus, (C) is true.

Step 4: For (D), simplify:

$$(Q \oplus R) \oplus (R \oplus S) = Q \oplus S \quad \text{(since } R \oplus R = 0\text{)}.$$

$$(Q \oplus R) \oplus (R \oplus S) = Q \oplus S,$$

then adding $(Q \oplus S)$ gives $(Q \oplus S) \oplus (Q \oplus S) = 0$ not 1. Hence, (D) is false. Final correct answers are (A) and (C) only.

Quick Tip

- XOR has properties: commutative, associative, and $X \oplus X = 0$. - Always check with truth tables for validation.

---

29. In the human pancreas, which cell types secrete insulin and glucagon?
(A) Alpha cells and delta cells, respectively
(B) Beta cells and delta cells, respectively
(C) Alpha cells and beta cells, respectively
(D) Beta cells and alpha cells, respectively

Correct Answer: (D) Beta cells and alpha cells, respectively

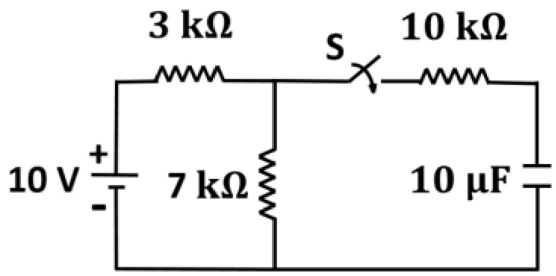
Solution:
Step 1: The pancreas contains islets of Langerhans, which have specialized hormone-secreting cells.
Step 2: Beta (β) cells produce **insulin**, which lowers blood glucose levels by promoting uptake of glucose by cells.
Step 3: Alpha (α) cells produce **glucagon**, which increases blood glucose levels by stimulating glycogen breakdown in the liver.
Step 4: Therefore, insulin is secreted by β-cells and glucagon by α-cells.

Quick Tip

- Beta cells $\Rightarrow$ Insulin (reduces blood sugar). - Alpha cells $\Rightarrow$ Glucagon (increases blood sugar). - Delta cells secrete somatostatin (regulates both insulin and glucagon).

---

30. In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed? (Round off the answer to one decimal place.)



Correct Answer: 7.0

Solution:
Step 1: For $t \to \infty$ after the switch is closed, the capacitor behaves as an open circuit (no DC current). Hence there is no current through the $10\,\text{k}\Omega$ resistor; therefore the voltage drop across it is zero and the capacitor voltage equals the node voltage at S.

Step 2: The left network is a simple divider of $3\,\text{k}\Omega$ (series) and $7\,\text{k}\Omega$ (to ground) across a 10 V source. Thus the node voltage is

$$V_S = 10\,\text{V} \times \frac{7\,\text{k}\Omega}{3\,\text{k}\Omega + 7\,\text{k}\Omega} = 10 \times \frac{7}{10} = 7\,\text{V}.$$

Step 3: Therefore, in steady state, the capacitor voltage is $V_C = V_S = \boxed{7.0 \text{ V}}$.

Quick Tip

At DC steady state, capacitors are open circuits. Replace them by open circuits and compute node voltages using simple resistive division to find final capacitor voltages.

31. For a tissue with Young's modulus of 3.6 kPa and Poisson's ratio of 0.2, what is the value of its shear modulus (in kPa)? (Round off the answer to one decimal place.)

Correct Answer: 1.5 kPa

Solution: Step 1: Use the isotropic elasticity relation between Young's modulus E, shear modulus G and Poisson's ratio ν:

$$G = \frac{E}{2(1+\nu)}.$$

Step 2: Substitute $E = 3.6$ kPa and $\nu = 0.2$:

$$G = \frac{3.6}{2(1+0.2)} = \frac{3.6}{2.4} = 1.5 \text{ kPa}.$$

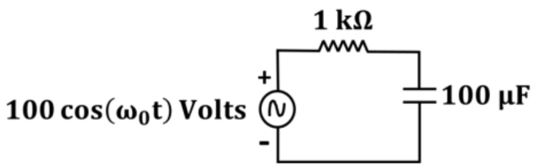
Thus, the shear modulus is $\boxed{1.5 \text{ kPa}}$.

Quick Tip

For isotropic linear materials: $E = 2G(1+\nu)$ and $E = 3K(1-2\nu)$. These let you convert among E, G (shear), K (bulk), and ν.

---

32. In the circuit shown below, the amplitudes of the voltage across the resistor and the capacitor are equal. What is the value of the angular frequency ω_0 (in rad/s)? (Round off the answer to one decimal place.)



Correct Answer: 10.0

Solution: Step 1: For a series RC circuit, the magnitudes of voltages are

$$|V_R| = IR, \qquad |V_C| = I\left|\frac{1}{j\omega C}\right| = \frac{I}{\omega C}.$$

Given $|V_R| = |V_C| \Rightarrow R = \dfrac{1}{\omega_0 C}$, hence

$$\omega_0 = \frac{1}{RC}.$$

Step 2: Substitute $R = 1\,\text{k}\Omega = 1000\,\Omega$ and $C = 100\,\mu\text{F} = 1 \times 10^{-4}\,\text{F}$:

$$\omega_0 = \frac{1}{(1000)(1\times 10^{-4})} = \frac{1}{0.1} = 10 \text{ rad/s}.$$

Rounded to one decimal place: $\boxed{10.0}$.

Quick Tip

In a series RC circuit, the resistor and capacitor have equal voltage magnitudes when the reactance equals the resistance: $|X_C| = R \Rightarrow \omega = 1/(RC)$.

---

33. A continuous time, band limited signal $x(t)$ has its Fourier transform described by $X(f) = \begin{cases} 1 - \frac{|f|}{200}, & |f| \leq 200 \text{ Hz} \\ 0, & |f| > 200 \text{ Hz} \end{cases}$. The signal is uniformly sampled at a sampling rate of 600 Hz. The Fourier transform of the sampled signal is $X_s(f)$. What is the value of $\frac{X_s(600)}{X_s(500)}$? (Round off the answer to one decimal place.)

Correct Answer: 2.0

Solution: Step 1: For uniform sampling with rate $f_s = 600$ Hz,
$X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(f - kf_s)$
(with $T = 1/f_s$). For ratios the common factor $1/T$ cancels.
Step 2: Evaluate the nonzero terms:
- $X_s(600)$: only $k = 1$ contributes since $X(600 - 600) = X(0) = 1$. Thus $X_s(600) \propto 1$.
- $X_s(500)$: only $k = 1$ contributes with $X(500 - 600) = X(-100) = 1 - \frac{100}{200} = 0.5$.
Hence $X_s(500) \propto 0.5$.
Step 3: Therefore, $\frac{X_s(600)}{X_s(500)} = \frac{1}{0.5} = 2.0$.

Quick Tip

The spectrum of a sampled signal is the periodic replication of the original spectrum with period f_s. For ratios at specific frequencies, the global scaling cancels.

---

34. At time t, the cardiac dipole is oriented at -45° (minus forty five degrees) to the horizontal axis. The magnitude of the dipole is 3 mV. Assuming Einthoven frontal plane configuration, what is the magnitude (in mV) of the electrical signal in lead II? (Round off the answer to two decimal places.)

Correct Answer: 0.78 mV

Solution: Step 1: In the Einthoven triangle, the axis of Lead II is at $+60^\circ$ with respect to the horizontal (Lead I at 0°). The measured lead voltage equals the projection of the dipole vector on the lead axis:

$$V_{\text{lead}} = M \cos(\theta_{\text{dipole}} - \theta_{\text{lead}}).$$

Step 2: Here $M = 3$ mV, $\theta_{\text{dipole}} = -45^\circ$, $\theta_{\text{lead}} = 60^\circ$. Hence

$$|V_{II}| = 3\,|\cos(60^\circ - (-45^\circ))| = 3\,|\cos(105^\circ)| = 3 \times 0.2588 \approx 0.78 \text{ mV}.$$

Therefore, the magnitude of the Lead II signal is $\boxed{0.78 \text{ mV}}$.

Quick Tip

Lead voltages in the frontal plane are simple projections of the cardiac dipole onto lead axes: $V = M\cos(\Delta\theta)$. Lead II is aligned at $+60^\circ$.

---

35. A 5 MHz ultrasound transducer is being used to measure the velocity of blood. When the transducer is placed at an angle of 45° to the direction of blood flow, a frequency shift of 200 Hz is observed in the echo. Assume that the velocity of sound is 1500 m/s. What is the velocity (in cm/s) of the blood flow? (Round off the answer to one decimal place.)

Correct Answer: 4.2 cm/s

Solution: Step 1: For Doppler ultrasound with backscattered echo, the frequency shift is

$$\Delta f = \frac{2f_0 v\cos\theta}{c},$$

where f_0 is the transmitted frequency, v the blood speed, θ the angle between beam and flow, and c the sound speed.
Step 2: Solve for v:

$$v = \frac{\Delta f\, c}{2f_0\cos\theta}.$$

Step 3: Substitute values $f_0 = 5\times10^6$ Hz, $\Delta f = 200$ Hz, $c = 1500$ m/s, $\theta = 45^\circ$ ($\cos 45^\circ = \frac{\sqrt{2}}{2}$):

$$v = \frac{200 \times 1500}{2 \times 5 \times 10^6 \times \cos 45^\circ} = \frac{300000}{10^7 \times 0.7071} \approx 0.0424 \text{ m/s} = 4.24 \text{ cm/s}.$$

Rounded to one decimal place: $\boxed{4.2 \text{ cm/s}}$.

Quick Tip

For pulsed/continuous-wave Doppler with reflections, always use the **double** Doppler shift: $\Delta f = 2f_0 v\cos\theta/c$.

---

36. The time-dependent growth of a bacterial population is governed by the equation

$$\frac{dx}{dt} = x\left(1 - \frac{x}{200}\right),$$

where x is the population size at time t. The initial population size is $x_0 = 100$ at $t = 0$. As $t \to \infty$, the population size of bacteria asymptotically approaches ____________.

(A) 150
(B) 200
(C) 300
(D) 500

Correct Answer: (B) 200

Solution: Step 1: The differential equation is of logistic form

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right),$$

with growth rate $r = 1$ and carrying capacity $K = 200$.
Step 2: In a logistic model, regardless of the initial value $x_0 > 0$, the solution $x(t)$ approaches the carrying capacity K as $t \to \infty$. Hence,

$$\lim_{t\to\infty} x(t) = K = 200.$$

Therefore, the bacterial population asymptotically approaches $\boxed{200}$.

Quick Tip

- Logistic growth has two equilibria: $x = 0$ (unstable) and $x = K$ (stable). - The parameter K is the carrying capacity—the long-term population level.

---

37. A 20 mV DC signal has been superimposed with a 10 mV RMS band-limited Gaussian noise with a flat spectrum up to 5 kHz. If an integrating voltmeter is used to measure this DC signal, what is the minimum averaging time (in seconds) required to yield a 99% accurate result with 95% certainty?

(A) 0.1
(B) 1.0
(C) 5.0
(D) 10.0

Correct Answer: (B) 1.0

Solution: Step 1: For an integrating (averaging) voltmeter of duration T, the equivalent noise bandwidth is $B_{\text{eq}} = \frac{1}{2T}$. With input white noise of one-sided bandwidth $B = 5\,\text{kHz}$ and RMS $\sigma = 10$ mV, the output noise standard deviation is

$$\sigma_{\text{avg}} = \sigma\sqrt{\frac{B_{\text{eq}}}{B}} = \sigma\sqrt{\frac{1}{2TB}} = 0.01\,\text{V}\sqrt{\frac{1}{2T \cdot 5000}} = \frac{0.1\text{ mV}}{\sqrt{T}}.$$

Step 2: "99% accurate" for a 20 mV DC means the error band is $\pm 1\% \times 20$ mV $= \pm 0.2$ mV. For 95% certainty ($\approx 1.96\sigma$ for Gaussian noise),

$$1.96\,\sigma_{\text{avg}} \le 0.2\text{ mV} \quad \Rightarrow \quad \sigma_{\text{avg}} \le 0.102\text{ mV}.$$

Step 3: Using $\sigma_{\text{avg}} = \frac{0.1}{\sqrt{T}}$ mV:

$$\frac{0.1}{\sqrt{T}} \le 0.102 \;\Rightarrow\; \sqrt{T} \ge \frac{0.1}{0.102} \approx 0.98 \;\Rightarrow\; T \gtrsim 0.96 \text{ s} \approx 1.0 \text{ s}.$$

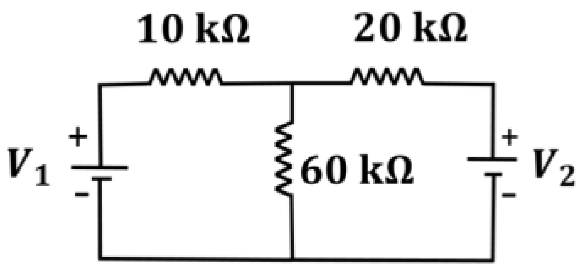
Thus the minimum averaging time is $\boxed{1.0 \text{ s}}$.

Quick Tip

For an averaging voltmeter, output noise RMS scales as $\sigma_{\text{avg}} = \sigma\sqrt{\frac{1}{2TB}}$. Combine this with the required confidence interval ($\pm 1.96\sigma$ for 95%) to size the averaging time.

---

38. In the circuit below, the two DC voltage sources have voltages of value V_1 and V_2. The expression for the power dissipated in the $60\,\text{k}\Omega$ resistor is proportional to __________.



(A) $(V_1 + V_2)^2$
(B) $(3V_1 + V_2)^2$
(C) $(2V_1 + V_2)^2$
(D) $(V_1 + 2V_2)^2$

Correct Answer: (C) $(2V_1 + V_2)^2$

Solution: Step 1: Let the voltage at the node connected to the $60\,\text{k}\Omega$ resistor (center node) be V_B. By nodal analysis:

$$\frac{V_B - V_1}{10k} + \frac{V_B - V_2}{20k} + \frac{V_B}{60k} = 0.$$

Multiplying by $60k$ gives

$$6(V_B - V_1) + 3(V_B - V_2) + V_B = 0 \Rightarrow 10V_B - 6V_1 - 3V_2 = 0.$$

Hence,

$$V_B = \frac{6}{10}V_1 + \frac{3}{10}V_2 = 0.3(2V_1 + V_2).$$

Step 2: The power in the $60\,\text{k}\Omega$ resistor is $P = \frac{V_B^2}{60k}$, which is proportional to V_B^2. Therefore,

$$P \propto \big(0.3(2V_1 + V_2)\big)^2 \propto (2V_1 + V_2)^2.$$

Thus, the required proportional expression is $\boxed{(2V_1 + V_2)^2}$.

Quick Tip

For power across a resistor to ground, first find the node voltage via nodal (superposition) analysis; the power is then proportional to the square of that node voltage.

---

39. The Laplace transform of $x_1(t) = e^{-t}u(t)$ is $X_1(s)$, where $u(t)$ is the unit step function. The Laplace transform of $x_2(t) = e^{t}u(-t)$ is $X_2(s)$. Which one of the following statements is TRUE?

(A) The region of convergence of $X_1(s)$ is $\text{Re}(s) > 0$.
(B) The region of convergence of $X_2(s)$ is confined to the left half–plane of s.
(C) The region of convergence of $X_1(s)$ is confined to the right half–plane of s.
(D) The imaginary axis in the s-plane is included in both the region of convergence of $X_1(s)$ and the region of convergence of $X_2(s)$.

Correct Answer: (D) The imaginary axis is included in both regions of convergence.

Solution: Step 1: For $x_1(t) = e^{-t}u(t)$,

$$X_1(s) = \int_0^{\infty} e^{-t}e^{-st}\,dt = \int_0^{\infty} e^{-(s+1)t}\,dt$$

converges if $\text{Re}(s) + 1 > 0 \Rightarrow \text{Re}(s) > -1$. Hence the ROC is the half-plane to the right of the vertical line $\text{Re}(s) = -1$ (not necessarily $\text{Re}(s) > 0$).
Step 2: For $x_2(t) = e^{t}u(-t)$,

$$X_2(s) = \int_{-\infty}^{0} e^{t}e^{-st}\,dt = \int_{-\infty}^{0} e^{(1-s)t}\,dt$$

converges if $\text{Re}(1-s) > 0 \Rightarrow \text{Re}(s) < 1$. Thus the ROC is the half-plane to the left of $\text{Re}(s) = 1$ (not necessarily $\text{Re}(s) < 0$).

Step 3: The imaginary axis ($\text{Re}(s) = 0$) satisfies both $\text{Re}(s) > -1$ and $\text{Re}(s) < 1$; hence it lies within both ROCs. Therefore only statement (D) is true.

Quick Tip

- For $e^{at}u(t)$, ROC: $\text{Re}(s) > -a$.
- For $e^{at}u(-t)$, ROC: $\text{Re}(s) < -a$.
- Always check whether the imaginary axis ($\text{Re}(s) = 0$) lies inside the ROC.

---

40. A circular disc of radius R (in cm) has a uniform absorption coefficient of $1\ \text{cm}^{-1}$. Consider a single ray passing through the disc in the plane of the disc.

The shortest distance from the center of the disc to the ray is t (in cm). If I_i is the intensity of the incident ray and I_o is the intensity of the transmitted ray, then $\log\left(\dfrac{I_i}{I_o}\right)$ is given by __________.

(A) $2\sqrt{R^2 - t^2}$
(B) $2R$
(C) 1
(D) $2\sqrt{R - t}$

Correct Answer: (A) $2\sqrt{R^2 - t^2}$

Solution: Step 1: Beer–Lambert law: $I_o = I_i e^{-\mu L}$, where μ is the absorption coefficient and L is the path length through the absorber. Hence

$$\log\left(\frac{I_i}{I_o}\right) = \mu L.$$

Step 2: Here $\mu = 1\ \text{cm}^{-1}$, so $\log(I_i/I_o) = L$. The ray passes through the disc along a chord whose distance from the center is t. The chord length is

$$L = 2\sqrt{R^2 - t^2}.$$

Step 3: Therefore,

$$\log\left(\frac{I_i}{I_o}\right) = 2\sqrt{R^2 - t^2}.$$

Quick Tip

- For a uniform absorber, $I = I_0 e^{-\mu L}$. - The path length through a circle for a ray at offset t is the chord length $2\sqrt{R^2 - t^2}$.

---

41. The free induction decay (FID) in the MRI of an object can be approximated as

$$s(t) = \iint m(x, y)\, e^{-j2\pi\,(K_x(t)x + K_y(t)y)}\, dx\, dy,$$

where $K_x(t) = \int_0^t G_x(\tau)\, d\tau$ and $K_y(t) = \int_0^t G_y(\tau)\, d\tau$. Here G_x and G_y are pulses of identical period and are in–phase. By changing the amplitude of the pulses, one can obtain the two–dimensional Fourier transform of the object _________.

(A) over radial lines in (K_x, K_y) space
(B) over a parabolic contour in (K_x, K_y) space
(C) along K_y only
(D) along K_x only

Correct Answer: (A) over radial lines in (K_x, K_y) space

Solution: Step 1: With G_x and G_y in–phase and of the same period, we can write $G_x(t) = a_x g(t)$ and $G_y(t) = a_y g(t)$. Then

$$K_x(t) = a_x \int_0^t g(\tau)d\tau = a_x F(t), \qquad K_y(t) = a_y F(t),$$

so $K_y/K_x = a_y/a_x = \text{constant}$.

Step 2: Hence the k–space trajectory $(K_x(t), K_y(t))$ lies on a straight line through the origin with slope a_y/a_x; changing the amplitudes (a_x, a_y) changes the slope, giving different straight lines through the origin—i.e., radial lines.

Therefore the 2D Fourier data are obtained *over radial lines* in (K_x, K_y) space.

Quick Tip

In MRI, gradients scaled versions of the same waveform create a k–space trajectory with $K_y/K_x = \text{constant}$, i.e., straight radial lines through the origin.

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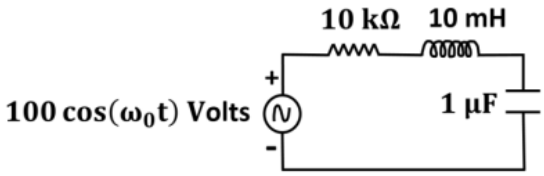
42. In the circuit shown below, it is observed that the amplitude of the voltage across the resistor is the same as the amplitude of the source voltage. What is the angular frequency ω_0 (in rad/s)?



(A) 10^4
(B) 10^3
(C) $10^3\pi$
(D) $10^4\pi$

Correct Answer: (A) 10^4

Solution: Step 1: For a series R–L–C circuit driven by V, the current amplitude is $|I| = \dfrac{|V|}{|Z|}$ with $|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

Step 2: The resistor voltage amplitude equals the source amplitude: $|V_R| = |I|R = |V| \Rightarrow |Z| = R$. Thus

$$\sqrt{R^2 + (\omega L - 1/\omega C)^2} = R \Rightarrow \omega L - \frac{1}{\omega C} = 0,$$

which is the resonance condition $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

Step 3: With $L = 10\,\text{mH} = 0.01\,\text{H}$ and $C = 1\,\mu\text{F} = 10^{-6}\,\text{F}$,

$$\omega_0 = \frac{1}{\sqrt{(0.01)(10^{-6})}} = \frac{1}{10^{-4}} = 10^4 \text{ rad/s}.$$

Quick Tip

In a series RLC circuit, if the resistor voltage equals the source voltage, the circuit is at series resonance, giving $\omega_0 = 1/\sqrt{LC}$.

---

43. In a biomaterial, formation of hydrogen bonds on alcoholic groups will lead to a ____________________.

(A) shift in the infra-red peak around 1700 cm^{-1}
(B) shift in the infra-red peak around 2800 cm^{-1}
(C) broadening of the infra-red peak around 3500 cm^{-1}
(D) disappearance of the infra-red peak around 1700 cm^{-1}

Correct Answer: (C) broadening of the infra-red peak around 3500 cm^{-1}

Solution: Step 1: Alcoholic O$-$H stretching typically appears as a broad band around $3200-3600\ \text{cm}^{-1}$.

Step 2: Hydrogen bonding strengthens intermolecular interactions and spreads the vibrational energy levels, causing the O$-$H stretch to broaden and shift to a lower wavenumber region near $\sim 3500\ \text{cm}^{-1}$.

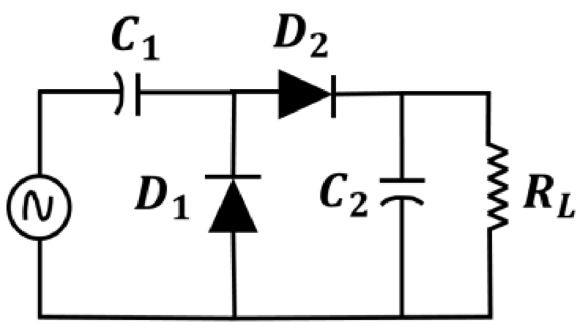
Hence the spectrum shows *broadening* around 3500 cm^{-1}.

Quick Tip

- Hydrogen bonding broadens and usually red-shifts the O$-$H stretching band in IR spectra ($3200-3600\ \text{cm}^{-1}$). - Peaks near 1700 cm^{-1} are typically C $=$ O stretches, not related to O$-$H bonding.

---

44. In the circuit shown below, the input voltage is sinusoidal and 2.5 V peak-to-peak. The capacitors are 20 μF each. Assume ideal diodes (zero knee voltage) and R_L is very large (almost infinite). Which one of the following options is *closest* to the peak-to-peak voltage across R_L, after a large number of cycles?



(A) 1.25 V
(B) 2.50 V
(C) 5.00 V
(D) 10.0 V

Correct Answer: (C) 5.00 V

Solution: Step 1: The network (C_1, D_1, D_2, C_2) is a (Greinacher/Villard) *voltage doubler*. After many cycles and with an almost open-circuit load, C_1 charges to the source peak V_p, and the subsequent half–cycle stacks the source peak with the charge on C_1 through D_2. Hence C_2 charges to approximately $2V_p$.

Step 2: The output across R_L (in steady state) therefore has a peak value $\approx 2V_p$. Consequently, the peak–to–peak value at the output is about $2 \times (2V_p) = 4V_p$. For the given input of 2.5 V peak–to–peak (i.e., $V_{pp,in} = 2V_p = 2.5 \text{ V} \Rightarrow V_p = 1.25 \text{ V}$), the doubler produces an output close to

$$V_{pp,out} \approx 2\,V_{pp,in} \approx 2 \times 2.5 = 5.0 \text{ V}.$$

(With ideal diodes and very large R_L, ripple is negligible, so the obtained value is the closest among the choices.)

Quick Tip

- A Greinacher/Villard stage ideally doubles the peak (and hence doubles the peak-to-peak) of the AC source at the high-impedance output.
- With ideal diodes and large R_L, the stored charge is retained, giving a nearly constant doubled level.

---

45. An ultrasound plane wave of amplitude P_0 hits the semi-infinite boundary of two media having acoustic impedances Z_1 and Z_2. The sum of the amplitudes of the reflected and the incident waves at the boundary is equal to _________.

(A) $\dfrac{2P_0Z_2}{(Z_1+Z_2)}$
(B) $\dfrac{P_0(Z_2-Z_1)}{(Z_1+Z_2)}$
(C) $\dfrac{P_0Z_2}{Z_1}$
(D) $\dfrac{P_0Z_1}{(Z_1+Z_2)}$

Correct Answer: (A) $\dfrac{2P_0Z_2}{(Z_1+Z_2)}$

Solution: Step 1: Let the incident, reflected and transmitted pressure amplitudes be $P_i(= P_0)$, P_r and P_t respectively. At the boundary, continuity of pressure and particle velocity gives

$$P_i + P_r = P_t, \qquad \frac{P_i - P_r}{Z_1} = \frac{P_t}{Z_2}.$$

Step 2: Eliminating P_t: $Z_2(P_i - P_r) = Z_1(P_i + P_r) \Rightarrow (Z_2 - Z_1)P_i = (Z_2 + Z_1)P_r$. Hence

$$\frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}.$$

Step 3: Therefore, the sum of the incident and reflected amplitudes at the boundary is

$$P_i + P_r = P_i\left(1 + \frac{Z_2 - Z_1}{Z_2 + Z_1}\right) = P_i\,\frac{2Z_2}{Z_2 + Z_1} = \frac{2P_0 Z_2}{Z_1 + Z_2}.$$

Quick Tip

- For normal incidence, $R_p = \dfrac{Z_2 - Z_1}{Z_2 + Z_1}$ and $T_p = \dfrac{2Z_2}{Z_2 + Z_1}$ for pressure amplitudes. - Use continuity of pressure and velocity ($u = p/Z$) at the interface.

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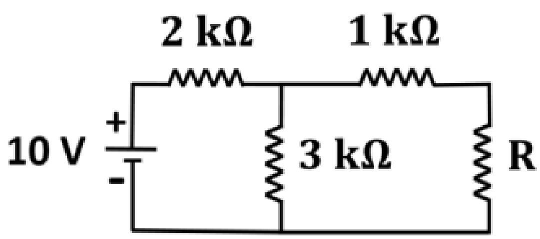
46. In the circuit given below, what should be the value of the resistance R for maximum dissipation of power in R?



(A) 1.2 kΩ
(B) 2.2 kΩ
(C) 3.2 kΩ
(D) 4.2 kΩ

Correct Answer: (B) 2.2 kΩ

Solution: Step 1: For maximum power transfer to R, set $R = R_{\text{Th}}$, where R_{Th} is the Thevenin resistance seen from the terminals of R. Replace the ideal 10 V source by a short circuit.

Step 2: With the source shorted, the 2 kΩ and 3 kΩ resistors are in parallel between the middle node and ground, and this combination is in series with the 1 kΩ resistor to the R terminal. Hence,

$$R_{\text{Th}} = 1\ \text{k}\Omega + (2\ \text{k}\Omega \parallel 3\ \text{k}\Omega) = 1 + \frac{2 \times 3}{2 + 3}\ \text{k}\Omega = 1 + 1.2 = 2.2\ \text{k}\Omega.$$

Step 3: Therefore, R should be chosen as $R_{\text{Th}} = 2.2$ kΩ for maximum power dissipation.

Quick Tip

- Maximum power is delivered to a load when the load resistance equals the Thevenin resistance seen at its terminals.
- For an ideal voltage source, short it when computing R_{Th}.

---

47. Two sequences $x_1[n]$ and $x_2[n]$ are described as follows:

$$x_1[0] = x_2[0] = 1, \quad x_1[1] = x_2[2] = 2, \quad x_1[2] = x_2[1] = 1$$

$$x_1[n] = x_2[n] = 0 \quad \text{for all } n < 0 \text{ and } n > 2$$

If $x[n]$ is obtained by convolving $x_1[n]$ with $x_2[n]$, which of the following equations is/are TRUE?

(A) $x[2] = x[3]$
(B) $x[1] = 2$
(C) $x[4] = 3$
(D) $x[2] = 5$

Correct Answer: (A) $x[2] = x[3]$, (D) $x[2] = 5$

Solution:
Step 1: Recall convolution definition.

$$x[n] = (x_1 * x_2)[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$$

Since both $x_1[n]$ and $x_2[n]$ are nonzero only for $0 \le n \le 2$, the support of $x[n]$ is $0 \le n \le 4$.

Step 2: Compute values of $x[n]$.
For $n = 0$:

$$x[0] = x_1[0]x_2[0] = 1 \cdot 1 = 1$$

For $n = 1$:

$$x[1] = x_1[0]x_2[1] + x_1[1]x_2[0] = (1)(1) + (2)(1) = 1 + 2 = 3$$

For $n = 2$:

$$x[2] = x_1[0]x_2[2] + x_1[1]x_2[1] + x_1[2]x_2[0] = (1)(2) + (2)(1) + (1)(1) = 2 + 2 + 1 = 5$$

For $n = 3$:

$$x[3] = x_1[1]x_2[2] + x_1[2]x_2[1] = (2)(2) + (1)(1) = 4 + 1 = 5$$

For $n = 4$:

$$x[4] = x_1[2]x_2[2] = (1)(2) = 2$$

Thus, the sequence is:

$$x[0] = 1,\ x[1] = 3,\ x[2] = 5,\ x[3] = 5,\ x[4] = 2$$

Step 3: Verify given statements.
- (A) $x[2] = x[3] \Rightarrow 5 = 5$, **True**
- (B) $x[1] = 2$. But actually $x[1] = 3$. **False**
- (C) $x[4] = 3$. But actually $x[4] = 2$. **False**
- (D) $x[2] = 5$. Correct. **True**

So the correct ones are (A) and (D).

Final Answer: (A) and (D)

Quick Tip

- Always remember: The convolution length for two finite sequences of length L_1 and L_2 is $L_1 + L_2 - 1$. - Carefully align terms when performing the summation to avoid missing products. - Symmetry in sequences often leads to equal values in convolution outputs.

---

48. The function $f(Z) = \dfrac{1}{Z-1}$ of a complex variable Z is integrated on a closed contour in an anti-clockwise direction. For which of the following contours, does this integral have a non-zero value?
(A) $|Z - 2| = 0.01$
(B) $|Z - 1| = 0.1$
(C) $|Z - 3| = 5$
(D) $|Z| = 2$

Correct Answer: (B), (C), (D) $\quad |Z - 1| = 0.1,\ |Z - 3| = 5,\ |Z| = 2$

Solution: Step 1: By Cauchy's integral theorem, for $f(Z) = \dfrac{1}{Z-1}$ the contour integral $\displaystyle\oint f(Z)\,dZ$ is non–zero only if the contour encloses the simple pole at $Z = 1$; then $\displaystyle\oint \frac{1}{Z-1}\,dZ = 2\pi i$ (for anticlockwise traversal).

Step 2: Check each contour:
- $|Z - 2| = 0.01$: circle centered at 2 of radius 0.01 does *not* enclose $Z = 1 \Rightarrow$ integral $= 0$.
- $|Z - 1| = 0.1$: circle centered at 1 of radius 0.1 encloses the pole $\Rightarrow$ non–zero.
- $|Z - 3| = 5$: circle centered at 3 of radius 5 also encloses $Z = 1 \Rightarrow$ non–zero.
- $|Z| = 2$: circle centered at 0 of radius 2 encloses $Z = 1 \Rightarrow$ non–zero.

Thus the integral is non-zero for options (B), (C) and (D).

Quick Tip

- The contour integral $\oint \frac{1}{z-a}\,dz$ equals $2\pi i$ if and only if the contour encloses $z = a$ (anticlockwise). Otherwise it is zero.

49. The continuous time signal $x(t)$ is described by

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following statements is/are TRUE?
(A) $y(t) = 0$ for all $t < 0$
(B) $y(t) = 0$ for all $t > 1$
(C) $y(t) = 0$ for all $t > 3$
(D) $\int_{0.1}^{0.75} \frac{dy(t)}{dt}\, dt \ne 0$

Correct Answer: (A) and (C), (D)

Solution:
Step 1: Convolution setup. We have $y(t) = x(t) * x(t)$. Since $x(t)$ is a rectangular pulse of duration 1, convolving it with itself gives a triangular signal of duration 2:

$$y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t \le 1 \\ 2 - t, & 1 < t \le 2 \\ 0, & t > 2 \end{cases}$$

Step 2: Check each option.
- (A) For $t < 0$, clearly $y(t) = 0$. **True**.
- (B) For $t > 1$, $y(t)$ is not zero immediately; in fact, for $1 < t \le 2$, $y(t) = 2 - t \ne 0$. Hence this is **False**.
- (C) For $t > 3$, of course $y(t) = 0$. But the maximum support of $y(t)$ is only up to $t = 2$. So the statement "for all $t > 3$" is trivially **true**.
- (D) Evaluate:

$$\int_{0.1}^{0.75} \frac{dy(t)}{dt} dt = y(0.75) - y(0.1)$$

Since $0 \le t \le 1$, we have $y(t) = t$. Thus $y(0.75) = 0.75,\ y(0.1) = 0.1$. So the difference is $0.75 - 0.1 = 0.65 \ne 0$. **True**.

Final Answer: (A) and (D)

> **Quick Tip**
>
> - Convolution of two rectangular pulses yields a triangular waveform. - The length of the output is the sum of the lengths of the two input signals. - Derivatives and integrals can be simplified using the Fundamental Theorem of Calculus: $\int f'(t)dt = f(b) - f(a)$.

50. Which of the following relations is/are CORRECT in terms of various lung volume measurements?

(A) Vital capacity minus expiratory reserve volume equals inspiratory capacity.
(B) Vital capacity plus expiratory reserve volume equals inspiratory capacity.
(C) Total lung capacity equals the sum of inspiratory capacity and functional residual capacity.
(D) Functional residual capacity is the difference between expiratory reserve volume and residual volume.

Correct Answer: (A) and (C)

Solution:
Step 1: Recall lung volume definitions.

- Tidal Volume (TV): Volume of air inhaled/exhaled during normal breathing.
- Inspiratory Reserve Volume (IRV): Extra volume inhaled after normal inspiration.
- Expiratory Reserve Volume (ERV): Extra volume exhaled after normal expiration.
- Residual Volume (RV): Volume left after maximal expiration.

From these we define:

$$\text{Vital Capacity (VC)} = TV + IRV + ERV$$

$$\text{Inspiratory Capacity (IC)} = TV + IRV$$

$$\text{Functional Residual Capacity (FRC)} = ERV + RV$$

$$\text{Total Lung Capacity (TLC)} = VC + RV$$

Step 2: Verify each option.
- (A) $VC - ERV = (TV + IRV + ERV) - ERV = TV + IRV = IC$. **True**.
- (B) $VC + ERV = (TV + IRV + ERV) + ERV = TV + IRV + 2ERV \neq IC$. **False**.
- (C) $TLC = VC + RV$. Also, $IC + FRC = (TV + IRV) + (ERV + RV) = TV + IRV + ERV + RV = VC + RV = TLC$. **True**.
- (D) FRC is the sum $ERV + RV$, not the difference. **False**.

Final Answer: (A) and (C)

Quick Tip

- Vital capacity represents the maximum air exhaled after maximum inhalation. - Functional residual capacity is always a sum of ERV and RV. - Total lung capacity can be expressed as $TLC = IC + FRC$.

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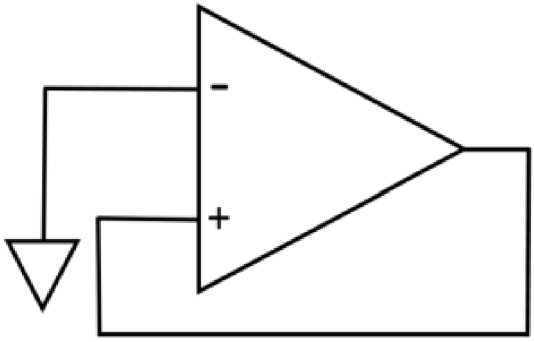
51. Assuming the operational amplifier in the circuit shown below to be ideal, which of the following properties hold(s) TRUE for the circuit?



(A) It acts as a voltage follower.
(B) It is bistable.
(C) It is astable.
(D) The output voltage is at saturation.

Correct Answer: (B), (D) It is bistable; The output voltage is at saturation

Solution: Step 1: From the figure, the op-amp output is directly fed back to the non-inverting (+) input, while the inverting (−) input is grounded. This is *positive feedback*.

Step 2: With positive feedback and no external input, an ideal op-amp's output cannot settle at zero; any infinitesimal offset is amplified and drives the output to one of the supply rails (saturation). Therefore the circuit has two stable states (bistable latch), and the output is at saturation (either $+V_{sat}$ or $-V_{sat}$).

Step 3: It is *not* a voltage follower (which requires negative feedback to the (−) input), and with only wires (no RC timing network) it is not an astable oscillator.

Quick Tip

- Negative feedback ⇒ linear operation (followers, amplifiers).
- Positive feedback with no input ⇒ saturation/bistable behavior (latches, Schmitt triggers).

---

52. A water insoluble polymeric biomaterial can become water soluble *in vivo* by which of the following mechanisms?

(A) Cleavage of crosslinks between water soluble polymer chains
(B) Cleavage of side chains leading to formation of non-polar groups
(C) Cleavage of backbone linkages between polymer repeat units leading to the formation of polar groups
(D) Enzymatic degradation of crosslinks between water soluble polymer chains

Correct Answer: (A), (C), (D) Cleavage of crosslinks; Backbone scission to polar groups; Enzymatic degradation of crosslinks

Solution: Step 1: A crosslinked network can be water-insoluble even if the individual chains are hydrophilic. Breaking the crosslinks (chemically or enzymatically) converts the network to discrete, water-soluble chains. Hence (A) and (D) are valid.

Step 2: Hydrolytic/oxidative cleavage of the *backbone* can both reduce the molecular weight and generate polar end groups (e.g., $-$COOH, $-$OH), increasing hydrophilicity and solubility. Hence (C) is also valid.

Step 3: In contrast, cleaving side chains to form *non-polar* groups would decrease hydrophilicity and does not promote solubility; therefore (B) is incorrect.

Quick Tip

- Solubility of polymers increases with **lower crosslink density**, **lower molecular weight**, and **higher polarity**.
- Enzymatically cleavable crosslinks are common triggers for *in vivo* solubilization/degradation.

---

53. For the function $f(x) = x^4 - x^2$, which of the following statements is/are TRUE?

(A) The function is symmetric about $x = 0$.
(B) The minimum value of the function is -0.5.
(C) The function has two minima.
(D) The function is an odd function.

Correct Answer: (A) and (C)

Solution:
Step 1: Check symmetry. We test $f(-x)$:

$$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x).$$

Thus, $f(x)$ is an **even function**, symmetric about $x = 0$. Hence (A) is true.

Step 2: Find critical points. Differentiate:

$$f'(x) = 4x^3 - 2x = 2x(2x^2 - 1).$$

So critical points are $x = 0$ and $x = \pm\frac{1}{\sqrt{2}}$.

Step 3: Classify critical points. Second derivative:

$$f''(x) = 12x^2 - 2.$$

- At $x = 0$: $f''(0) = -2 < 0 \Rightarrow$ local maximum. - At $x = \pm\frac{1}{\sqrt{2}}$: $f''(\pm\frac{1}{\sqrt{2}}) = 12 \cdot \frac{1}{2} - 2 = 6 - 2 = 4 > 0$. Hence local minima.

Step 4: Minimum value.

$$f\left(\pm\frac{1}{\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = -0.25$$

So the actual minimum value is -0.25, not -0.5.
Thus, statement (B) is **False**.

Step 5: Check other statements. - (C) Two minima exist at $x = \pm\frac{1}{\sqrt{2}}$. This is **True**.
- (D) Since the function is even, not odd, this is **False**.

Final Answer: (A) and (C)

> Quick Tip
>
> - Even functions satisfy $f(-x) = f(x)$, odd functions satisfy $f(-x) = -f(x)$. - Use first and second derivative tests to identify local maxima/minima. - Always substitute back into the original function to check exact minimum/maximum values.

---

54. A system is described by the following differential equation

$$0.01\frac{d^2y(t)}{dt^2} + 0.2\frac{dy(t)}{dt} + y(t) = 6x(t),$$

where time (t) is in seconds. If $x(t)$ is the unit step input applied at $t = 0$ s to this system, the magnitude of the output at $t = 1$ s is ____. (Round off the answer to two decimal places.)

Correct Answer: 6.00

Solution:
Step 1: Write transfer function. Taking Laplace transform with zero initial conditions:

$$0.01s^2Y(s) + 0.2sY(s) + Y(s) = \frac{6}{s}.$$

So the transfer function is
$$\frac{Y(s)}{X(s)} = \frac{6}{0.01s^2 + 0.2s + 1}.$$

Step 2: Normalize coefficients. Divide numerator and denominator by 0.01:

$$\frac{Y(s)}{X(s)} = \frac{600}{s^2 + 20s + 100}.$$

Step 3: Unit step input. For $x(t) = u(t)$, $X(s) = \frac{1}{s}$. Thus,

$$Y(s) = \frac{600}{s(s^2 + 20s + 100)}.$$

Step 4: Partial fraction decomposition. We factorize denominator:

$$s^2 + 20s + 100 = (s + 10)^2.$$

So,

$$Y(s) = \frac{600}{s(s+10)^2}.$$

Let

$$\frac{600}{s(s+10)^2} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2}.$$

Step 5: Solve for constants. Multiply both sides:

$$600 = A(s+10)^2 + B \cdot s(s+10) + C \cdot s.$$

- Put $s = 0$: $600 = A(100) \Rightarrow A = 6$. - Put $s = -10$: $600 = C(-10) \Rightarrow C = -60$. - Put $s = 1$: $600 = 6(11^2) + B(11) + (-60)(1)$.

$$600 = 726 + 11B - 60 \Rightarrow 600 = 666 + 11B \Rightarrow B = -6.$$

So,

$$Y(s) = \frac{6}{s} - \frac{6}{s+10} - \frac{60}{(s+10)^2}.$$

Step 6: Inverse Laplace.

$$y(t) = 6(1) - 6e^{-10t} - 60 \cdot te^{-10t}, \quad t \geq 0.$$

Step 7: Evaluate at $t = 1$.

$$y(1) = 6 - 6e^{-10} - 60e^{-10}.$$
$$= 6 - 66e^{-10}.$$

Since $e^{-10} \approx 4.54 \times 10^{-5}$,

$$y(1) \approx 6 - 66(0.0000454) \approx 6 - 0.00299 = 5.997.$$

Rounded to two decimal places: **6.00**.

Final Answer: 6.00

Quick Tip

- Always normalize coefficients before forming transfer functions. - Use partial fractions carefully for repeated roots. - For unit step response, remember $X(s) = 1/s$. - Exponentials with large negative arguments (like e^{-10}) often approximate to nearly zero.

---

55. The resistance of a thermistor is 1 kΩ at 25°C and 500 Ω at 50°C. Find the temperature coefficient of resistance (in units of °C^{-1}) at 35°C. (Round off the answer to three decimal places.)

Correct Answer: -0.028 °C^{-1}

Solution: Step 1: Model the NTC thermistor as $R(T) = Ae^{B/T}$ (with T in kelvin). From the two data points:

$$\ln\left(\frac{R_{25}}{R_{50}}\right) = B\left(\frac{1}{T_{25}} - \frac{1}{T_{50}}\right),$$

where $R_{25} = 1000\,\Omega,\ R_{50} = 500\,\Omega,\ T_{25} = 298.15\,\text{K},\ T_{50} = 323.15\,\text{K}$. Hence,

$$B = \frac{\ln(1000/500)}{\frac{1}{298.15} - \frac{1}{323.15}} = \frac{\ln 2}{\frac{25}{298.15\times 323.15}} \approx 2.672 \times 10^3\ \text{K}.$$

Step 2: The temperature coefficient is

$$\alpha(T) = \frac{1}{R}\frac{dR}{dT} = \frac{d(\ln R)}{dT} = -\frac{B}{T^2}.$$

At 35°C, $T = 308.15\,\text{K}$:

$$\alpha(35°\text{C}) = -\frac{2671.9}{(308.15)^2} \approx -2.81 \times 10^{-2}\ °\text{C}^{-1}.$$

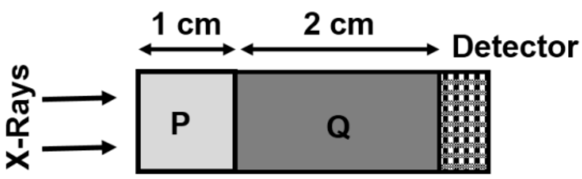
Rounded to three decimal places, $\alpha \approx -0.028\,°\text{C}^{-1}$.

Quick Tip

- For thermistors, $R(T)$ is often modeled as $R = Ae^{B/T}$; then the fractional TCR is $\alpha = -B/T^2$.
- Remember to convert temperatures to kelvin when using this model; the final units of TCR are per °C (same as per K).

---

56. A normally incident X-ray of energy 140 keV passes through a tissue phantom and is detected by the detector as shown. The phantom consists of tissue P with an absorption coefficient of $1\ \text{cm}^{-1}$ and a thickness of 1 cm, and tissue Q with an absorption coefficient of $10\ \text{cm}^{-1}$ and a thickness of 2 cm. Calculate the intensity (in μeV) detected by the detector. (Round off the answer to one decimal place.)



Correct Answer: 106.1 μeV

Solution: Step 1: X-ray intensity decays exponentially through matter:

$$I = I_0\, e^{-\mu_P d_P - \mu_Q d_Q}.$$

Here, $\mu_P = 1\ \text{cm}^{-1},\ d_P = 1$ cm and $\mu_Q = 10\ \text{cm}^{-1},\ d_Q = 2$ cm. Thus the total attenuation exponent is

$$\mu_P d_P + \mu_Q d_Q = 1 \times 1 + 10 \times 2 = 21.$$

Hence,

$$\frac{I}{I_0} = e^{-21} \approx 7.5826 \times 10^{-10}.$$

Step 2: The incident energy per photon is $I_0 = 140$ keV $= 1.4 \times 10^5$ eV. Therefore,

$$I = 1.4 \times 10^5 \text{ eV} \times 7.5826 \times 10^{-10} \approx 1.061 \times 10^{-4} \text{ eV} = 106.1\ \mu\text{eV}.$$

Quick Tip

- For layered media, multiply the linear attenuation coefficient μ by each layer thickness and add the exponents: $I = I_0 e^{-\sum \mu_i d_i}$.
- Convert energy units at the end to avoid carrying large/small numbers through intermediate steps.

---

57. A two-dimensional square plate (20 mm sides) contains a homogeneous circular inclusion of 5 mm diameter in it. A parallel beam of X-rays (beam width 30 mm) is used in a tomography system to determine the location of the inclusion. What is the minimum number of views required to approximately determine the location of the inclusion?

Correct Answer: 2

Solution: Step 1: Geometry of a disk under parallel-beam projection. In parallel-beam CT, the Radon projection at view angle θ of a circular inclusion (disk) is a *rectangular* profile of constant height whose center lies at the signed distance

$$s = x_0 \cos\theta + y_0 \sin\theta$$

from the rotation origin, where (x_0, y_0) is the center of the inclusion. The width of this rectangle equals the disk diameter (here 5 mm) and is independent of θ.

Step 2: Number of views needed to solve for the center. Each single view gives one linear equation in the unknowns (x_0, y_0) via s. Two distinct view angles θ_1 and θ_2 yield

$$\begin{cases} s_1 = x_0 \cos\theta_1 + y_0 \sin\theta_1, \\ s_2 = x_0 \cos\theta_2 + y_0 \sin\theta_2, \end{cases}$$

which can be solved uniquely for (x_0, y_0) so long as $\theta_2 \not\equiv \theta_1 \pmod{\pi}$.

Thus, the **minimum** number of distinct projection views needed to locate the inclusion is $\boxed{2}$ (e.g., orthogonal views at $0°$ and $90°$). The given beam width (30 mm) exceeds the object size (20 mm), ensuring full coverage but not affecting this minimum.

Quick Tip

- For symmetric shapes (disk), one projection gives the distance s of the center along the view normal; two non-collinear views are enough to triangulate (x_0, y_0). - Beam width and object size matter for coverage, not for the number of views required to infer position.
- Orthogonal views are a simple choice to make the two equations independent.

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58. Calculate the reciprocal of the coefficient of z^3 in the Taylor series expansion of the function $f(z) = \sin(z)$ around $z = 0$. (Provide the answer as an integer.)

Correct Answer: -6

Solution: Step 1: The Taylor series of $\sin z$ about $z = 0$ is

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$$

Hence the coefficient of z^3 is $-\frac{1}{3!} = -\frac{1}{6}$. Its reciprocal is

$$\left(-\frac{1}{6}\right)^{-1} = -6.$$

Quick Tip

- Memorize Maclaurin series of basic functions ($\sin z,\ \cos z,\ e^z$)—they save time in coefficient questions.

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59. In a cell viability experiment, 10,000 cells were cultured in the absence and presence of a compound Q for 24 h. The absorbance of a dye associated with cellular metabolic activity was measured at a wavelength of 570 nm at 24 h. The measured absorbances were 0.8 a.u. in the absence of Q, and 0.5 a.u. in its presence. If the dye gives an absorbance (at 570 nm) of 0.1 a.u. in the absence of cells, what is the percentage cell growth inhibition caused by the compound Q? (Round off the answer to one decimal place.)

Correct Answer: 42.9%

Solution:
Step 1: Subtract the dye blank (0.1 a.u.) to obtain net absorbance due to cells.
Control (no Q): $0.8 - 0.1 = 0.7$.
Treated (with Q): $0.5 - 0.1 = 0.4$.

Step 2: Viability fraction $= \dfrac{\text{treated}}{}\text{control} = \dfrac{0.4}{0.7} = 0.5714$.
Growth inhibition $= (1 - \text{viability}) \times 100 = (1 - 0.5714) \times 100 = 42.86\% \approx 42.9\%$.

Quick Tip

- Always correct for background/blank absorbance before computing viability. - % inhibition $= \left(1 - \frac{\text{treated}}{\text{control}}\right) \times 100.$

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60. The volume percentage of oxygen in inspired air is 20% and that of expired air is 16%. A person is breathing at a rate of 12 breaths per minute. Each breath is 500 ml in volume. The cardiac output is 5 liters per minute. Assuming ideal, healthy lung and cardiac conditions, what is the change in percentage of oxygen in blood over 1 minute? (Round off the answer to one decimal place.)

Correct Answer: 4.8%

Solution: Step 1: Minute ventilation $V_E = 0.5\ \text{L} \times 12 = 6\ \text{L/min}$. Oxygen uptake per minute (Fick for lungs):

$$\dot{V}_{O_2} = (F_{IO_2} - F_{EO_2})\, V_E = (0.20 - 0.16) \times 6 = 0.24\ \text{L/min}.$$

Step 2: Distribute this O_2 into the blood flow (cardiac output $Q = 5$ L/min). Increase in O_2 volume fraction in blood

$$\Delta\%O_2 = \frac{0.24}{5} \times 100\% = 4.8\%.$$

Thus, the oxygen content of blood increases by 4.8% across the lungs per minute.

Quick Tip

- Use $\dot{V}_{O_2} = (F_{IO_2} - F_{EO_2})V_E$ and distribute by cardiac output to get the blood-side percentage change. - Keep units consistent (liters per minute).

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61. The intracellular and extracellular concentrations (in mM) of three important ions are given. The relative permeability of the cell membrane to each ion is provided. Universal gas constant $R = 8.31$ J/(mol · K) and Faraday's constant $F = 96500$ C/mol. What is the *absolute value* of the resting membrane potential (in mV) across the cell membrane at 27°C? (Round off the answer to one decimal place.)

Correct Answer: 83.2 mV

Solution:
Step 1: Use the Goldman–Hodgkin–Katz equation (for cations Na^+, K^+ and anion Cl^-):

$$V_m = \frac{RT}{F} \ln\left(\frac{P_K[K^+]_o + P_{Na}[Na^+]_o + P_{Cl}[Cl^-]_i}{P_K[K^+]_i + P_{Na}[Na^+]_i + P_{Cl}[Cl^-]_o}\right).$$

At 27°C $\Rightarrow T = 300$ K, so $\frac{RT}{F} = \frac{8.31\times300}{96500} \approx 0.02582$ V $= 25.82$ mV.

Step 2: Substitute values (mM; permeabilities: $P_{Na} = 0.02$, $P_K = 1.0$, $P_{Cl} = 0.38$):

$$\text{Numerator} = 1 \cdot 3 + 0.02 \cdot 140 + 0.38 \cdot 3 = 6.94,$$

$$\text{Denominator} = 1 \cdot 140 + 0.02 \cdot 10 + 0.38 \cdot 90 = 174.4.$$

Hence

$$V_m = 25.82 \ln\left(\frac{6.94}{174.4}\right) \approx 25.82 \times (-3.224) \approx -83.2 \text{ mV}.$$

The question asks for absolute value: $|V_m| = 83.2$ mV.

Quick Tip

- For GHK, anions (e.g., Cl^-) appear with inside and outside concentrations interchanged.
- At room temperature, $RT/F \approx 25.8$ mV, useful for quick estimates.

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62. A metallic strain gauge with negligible piezoresistive effect is subjected to a strain of 50×10^{-6}. For the metal, Young's Modulus $= 80$ GPa and Poisson's ratio $= 0.42$. What is the change in resistance (in mΩ), if the unstrained resistance is $200\ \Omega$? (Round off the answer to one decimal place.)

Correct Answer: 18.4 mΩ

Solution: Step 1: With negligible piezoresistive effect, the gauge factor for a metallic gauge is

$$GF = 1 + 2\nu = 1 + 2(0.42) = 1.84.$$

Step 2: Relative resistance change:

$$\frac{\Delta R}{R} = GF \cdot \varepsilon = 1.84 \times 50 \times 10^{-6} = 9.2 \times 10^{-5}.$$

Step 3: Absolute change:

$$\Delta R = R\frac{\Delta R}{R} = 200\ \Omega \times 9.2 \times 10^{-5} = 0.0184\ \Omega = 18.4\ \text{m}\Omega.$$

Quick Tip

- For metal strain gauges without piezoresistive effect, $GF \approx 1 + 2\nu$.
- Convert microstrain ($\mu\varepsilon$) to a pure number before multiplying.

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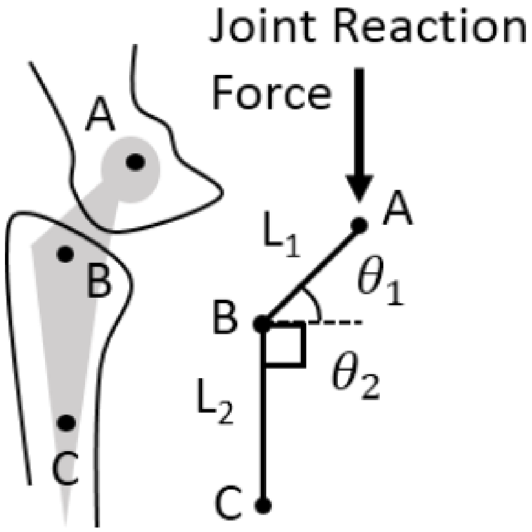
63. Consider the total hip joint prosthesis as shown in the figure. The geometric parameters of the prosthesis are such that $L_1 = 40$ mm, $L_2 = 60$ mm, $\theta_1 = 45°$,

$\theta_2 = 90°$. **Assume that, when standing symmetrically on both feet, a joint reaction force of** 400 **N is acting vertically at the femoral head (point A) due to the body weight of the subject. Calculate the magnitude of the moment (in Nm) about point C. (Round off the answer to one decimal place.)**



Correct Answer: $11.3\ \mathrm{N\cdot m}$

Solution: Step 1: The force at A is vertical; hence the moment about C equals the force magnitude times the perpendicular (horizontal) distance from C to the line of action of the force through A.
From the geometry, B is directly above C by L_2 ($\theta_2 = 90°$), and A is located from B by L_1 at an angle θ_1 to the horizontal. Therefore the horizontal distance of A from C is

$$x_A = L_1 \cos\theta_1.$$

Step 2: Compute the moment magnitude:

$$M_C = F\,x_A = 400\ \mathrm{N} \times (0.040\ \mathrm{m})\cos 45° = 400 \times 0.02828 \approx 11.3\ \mathrm{N\cdot m}.$$

Quick Tip

- When a force is vertical (or horizontal), the moment about a point reduces to the force times the horizontal (or vertical) offset of the point of application from that point.
- Keep track of units: convert mm to m before computing moments in N·m.

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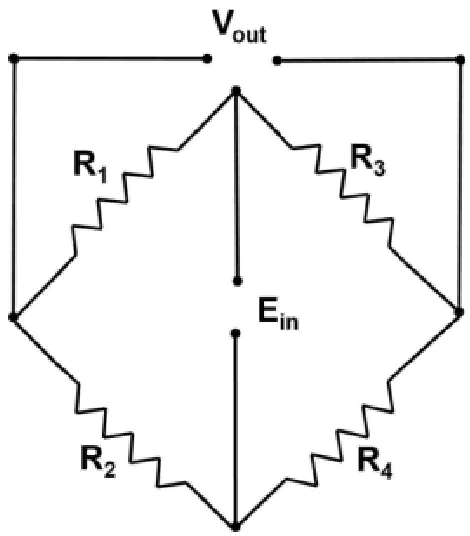
64. A Wheatstone bridge strain gauge transducer is constructed on a diaphragm in such a way that when a force is applied on the diaphragm, the resistors R_1 and R_4 will be in compression, and the resistors R_2 and R_3 will be in tension. The bridge excitation voltage (E_{in}) is 10 **V. If all the resistors have a resistance of** 200 Ω **in the absence of any force, and each resistance changes by** 20 Ω **upon application of**

a force, what is the output voltage V_{out} (in Volts) from the Wheatstone bridge? (Round off your answer to the nearest integer.)



Correct Answer: 1

Solution: Step 1: Assign the new resistances under load. Compression $\Rightarrow$ resistance decreases, tension $\Rightarrow$ increases.

$R_1 = 200-20 = 180\,\Omega, \quad R_4 = 200-20 = 180\,\Omega, R_2 = 200+20 = 220\,\Omega, \quad R_3 = 200+20 = 220\,\Omega.$

Step 2: Use the Wheatstone bridge divider relations (excitation across top and bottom nodes). Left mid-node potential:

$$V_L = E_{\text{in}}\frac{R_2}{R_1+R_2} = 10\cdot\frac{220}{180+220} = 10\cdot\frac{220}{400} = 5.5 \text{ V}.$$

Right mid-node potential:

$$V_R = E_{\text{in}}\frac{R_4}{R_3+R_4} = 10\cdot\frac{180}{220+180} = 10\cdot\frac{180}{400} = 4.5 \text{ V}.$$

Step 3: Output voltage between the two mid nodes.

$$V_{\text{out}} = V_L - V_R = 5.5 - 4.5 = 1.0 \text{ V}.$$

Rounded to the nearest integer: $\boxed{1}$.

Quick Tip

- For a Wheatstone bridge with excitation top-to-bottom, $V_{\text{out}} = E_{\text{in}}\left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)$. - Opposite changes in adjacent arms (one increases, the other decreases) maximize bridge sensitivity.

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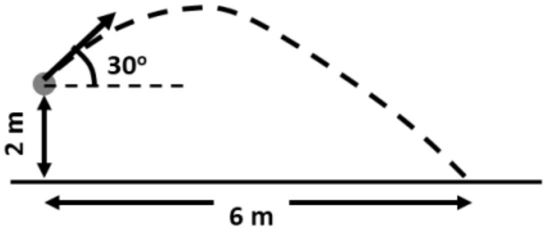
65. A stone is thrown from an elevation of 2 m above ground level, at an angle of 30° **to the horizontal axis. If the stone hits the ground at a horizontal distance of**

6 m from the point of release, at what speed (in m/s) was the stone thrown? Use $g = 10$ m/s^2 and assume that there is no air resistance. (Round off your answer to one decimal place.)



Correct Answer: 6.6 m/s

Solution:
Step 1: Define initial velocity components. Let the initial speed be u.

$$u_x = u\cos 30^\circ = \frac{\sqrt{3}}{2}u, \qquad u_y = u\sin 30^\circ = \frac{1}{2}u.$$

Step 2: Equation of vertical motion. Vertical displacement after time t:

$$y(t) = 2 + u_y t - \tfrac{1}{2}gt^2.$$

At ground hit, $y(t) = 0$:

$$0 = 2 + \tfrac{1}{2}ut - 5t^2. \tag{1}$$

Step 3: Horizontal motion condition. Horizontal displacement after time t:

$$x(t) = u_x t = \tfrac{\sqrt{3}}{2}ut = 6. \tag{2}$$

Step 4: Express t in terms of u. From (2):

$$t = \frac{12}{\sqrt{3}u} = \frac{4\sqrt{3}}{u}.$$

Step 5: Substitute in vertical equation. From (1):

$$0 = 2 + \tfrac{1}{2}u\left(\frac{4\sqrt{3}}{u}\right) - 5\left(\frac{4\sqrt{3}}{u}\right)^2.$$

$$0 = 2 + 2\sqrt{3} - \frac{240}{u^2}.$$

Step 6: Solve for u^2.

$$\frac{240}{u^2} = 2 + 2\sqrt{3}.$$

$$u^2 = \frac{240}{2(1+\sqrt{3})} = \frac{120}{1+\sqrt{3}}.$$

Multiply numerator and denominator by $(1-\sqrt{3})$:

$$u^2 = \frac{120(1-\sqrt{3})}{1-3} = -60(1-\sqrt{3}) = 60(\sqrt{3}-1).$$

Numerically: $\sqrt{3} \approx 1.732$.

$$u^2 \approx 60(0.732) = 43.92 \quad \Rightarrow \quad u \approx 6.63 \text{ m/s}.$$

Step 7: Recheck horizontal range. With $u = 6.63$, $t = \frac{4\sqrt{3}}{6.63} \approx 1.04$ s. Then $x = u_x t = (6.63 \cos 30^\circ)(1.04) \approx (5.74)(1.04) = 5.97 \approx 6$ m.✓
Thus $u \approx 6.6$ m/s.

Final Answer: 6.6 m/s

Quick Tip

- Break projectile motion into horizontal and vertical components. - Always use elimination of t to combine horizontal and vertical equations. - For problems with initial elevation, include initial height in vertical equation.
