GATE 2023 Chemical Engineering Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Each GATE 2023 paper consists of a total of 100 marks. The examination is divided into two sections – General Aptitude (GA) and the Candidate's Selected Subjects. General Aptitude carries 15 marks, while the remaining 85 marks are dedicated to the candidate's chosen test paper syllabus.
2. GATE 2023 will be conducted in English as a Computer Based Test (CBT) at select centres in select cities. The duration of the examination is 3 hours.
3. MCQs carry 1 mark or 2 marks.
4. For a wrong answer in a 1-mark MCQ, 1/3 mark is deducted.
5. For a wrong answer in a 2-mark MCQ, 2/3 mark is deducted.
6. No negative marking for wrong answers in MSQ or NAT questions.

General Aptitude

Q1. "You are delaying the completion of the task. Send _____ contributions at the earliest."

(A) you are
(B) your
(C) you're
(D) yore

Correct Answer: (B) your

Solution:
Step 1: The blank is followed by the word "contributions," which is a noun. Therefore, the blank must be filled with a possessive adjective that shows ownership.
Step 2: Option (A) "you are" is grammatically incorrect here, as it cannot modify a noun directly.
Step 3: Option (B) "your" is the correct possessive adjective, making the sentence: "Send your contributions at the earliest."
Step 4: Option (C) "you're" means "you are," which is again grammatically wrong before a noun.
Step 5: Option (D) "yore" means "long ago" and is irrelevant in this context.
Hence, the correct choice is (B).

Final Answer: (B) your

Quick Tip

- Use “your” as the possessive adjective to indicate ownership.
- “You’re” is a contraction of “you are.”
- “Yore” refers to “time long past,” and is archaic.
- Always check if the blank precedes a noun: if yes, a possessive adjective is often required.

Q.2. References : _____ : Guidelines : Implement (By word meaning)

(A) Sight
(B) Site
(C) Cite
(D) Plagiarise

Correct Answer: (C) Cite

Solution:
Step 1: The relationship given is based on word meaning: ”Guidelines : Implement” means that guidelines are something you **implement**.

Step 2: Similarly, ”References : ____” means that references are something you **cite**.

Step 3: Check the options: - (A) Sight → related to vision, not correct.
- (B) Site → refers to location, not correct.
- (C) Cite → correct, as references are cited.
- (D) Plagiarise → opposite meaning, not correct.

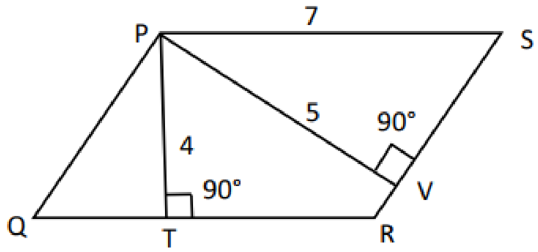
$$\therefore \text{ References : Cite :: Guidelines : Implement}$$

$$\boxed{\text{Cite}}$$

Quick Tip

- Analogies often follow the pattern: ”X is something you do with Y”.
- Always check for word meaning and correct usage.
- Watch out for homophones like sight, site, cite.

Q.3. In the given figure, PQRS is a parallelogram with $PS = 7$ cm, $PT = 4$ cm and $PV = 5$ cm. What is the length of RS in cm? (The diagram is representative.)



(A) $\frac{20}{7}$

(B) $\frac{28}{5}$

(C) $\frac{9}{2}$

(D) $\frac{35}{4}$

Correct Answer: (B) $\frac{28}{5}$

Solution: Step 1: In a parallelogram, the area is base × corresponding height. Using base PS and its perpendicular height PT:

$$\text{Area} = PS \times PT = 7 \times 4 = 28 \text{ cm}^2.$$

The same area can be written using base RS and the perpendicular distance from P to RS, which is PV:

$$\text{Area} = RS \times PV = RS \times 5.$$

Equating the areas,

$$RS \times 5 = 28 \quad \Rightarrow \quad RS = \frac{28}{5} \text{ cm}.$$

Quick Tip

- For any parallelogram, the product base × height is invariant. - Perpendicular distances from the same vertex to opposite sides can be used as heights with the corresponding bases.

Q4. In 2022, June Huh was awarded the Fields medal, which is the highest prize in Mathematics. When he was younger, he was also a poet. He did not win any medals in the International Mathematics Olympiads. He dropped out of college. Based only on the above information, which one of the following statements can be logically inferred with certainty?

(A) Every Fields medalist has won a medal in an International Mathematics Olympiad.
(B) Everyone who has dropped out of college has won the Fields medal.

(C) All Fields medalists are part-time poets.
(D) Some Fields medalists have dropped out of college.

Correct Answer: (D) Some Fields medalists have dropped out of college.

Solution:
Step 1: Analyze the given facts.
- June Huh won the Fields Medal in 2022.
- He dropped out of college.
- He did not win any medals in International Mathematics Olympiads.
- He was also a poet when younger.

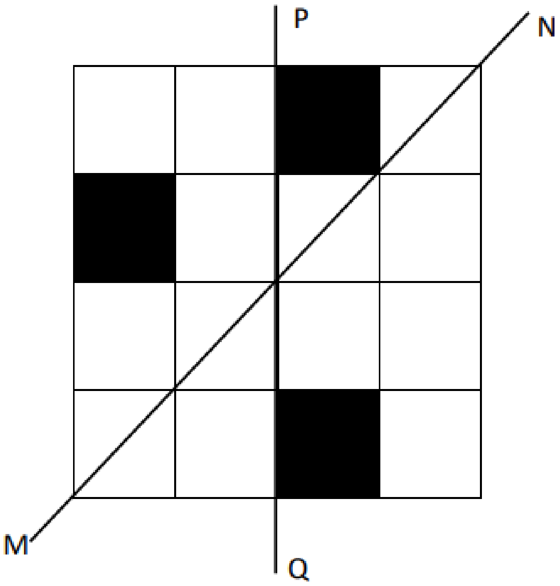
Step 2: Check each option.
- (A) False, because June Huh himself did not win any International Olympiad medal, so it cannot be true that "every" Fields medalist did.
- (B) False, because not everyone who dropped out of college won the Fields Medal; only June Huh is mentioned.
- (C) False, because June Huh was a poet, but that does not mean all Fields medalists are poets.
- (D) True, because June Huh is a Fields Medalist and he dropped out of college. Therefore, at least "some" (i.e., one) Fields Medalists have dropped out of college.

Final Answer: (D) Some Fields medalists have dropped out of college.

Quick Tip

- Always distinguish between "all," "every," and "some" in logical reasoning questions.
- One confirmed example is sufficient to prove "some." - Be careful not to overgeneralize beyond the given facts.

Q5. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line. The given figure consists of 16 unit squares arranged as shown. In addition to the three black squares, what is the minimum number of squares that must be coloured black, such that both PQ (vertical) and MN (the bottom-left to top-right diagonal) form lines of symmetry? (The figure is representative)



(A) 3
(B) 4
(C) 5
(D) 6

Correct Answer: (C) 5

Solution: Step 1: Model the 4×4 grid with coordinates (r, c) where $r = 1$ at the top row and $c = 1$ at the left column. From the figure, the initially black squares are $(1, 3), (2, 1), (4, 3)$.

Step 2: Enforce symmetry about the vertical line PQ (the line between columns 2 and 3). Reflection in PQ maps $(r, c) \mapsto (r, 5 - c)$. Therefore the current blacks force $(1, 2), (2, 4), (4, 2)$ to be black.

Step 3: Enforce symmetry about the diagonal line MN (from bottom-left to top-right). Reflection in MN maps $(r, c) \mapsto (5 - c,\ 5 - r)$. Applying this to all black squares obtained so far yields the additional required squares $(3, 1)$ and $(3, 4)$.

Step 4: The closure under both reflections is

$$\{(1, 2), (1, 3), (2, 1), (2, 4), (3, 1), (3, 4), (4, 2), (4, 3)\},$$

which contains 8 black squares in total. Since 3 were already black, the minimum number of additional squares required is $8 - 3 = 5$. $\boxed{5}$

Quick Tip

- When multiple symmetries are required, take the **orbit/closure** of given cells under all reflections—keep adding the reflected cells until no new ones appear.
- For a 4×4 grid, reflection over the mid-vertical is $(r, c) \mapsto (r, 5 - c)$ and over the anti-diagonal is $(r, c) \mapsto (5 - c, 5 - r)$.

Q6. Human beings are one among many creatures that inhabit an imagined world. In this imagined world, some creatures are cruel. If in this imagined world, it is given that the statement "Some human beings are not cruel creatures" is FALSE, then which of the following set of statement(s) can be logically inferred with certainty?

(i) All human beings are cruel creatures.
(ii) Some human beings are cruel creatures.
(iii) Some creatures that are cruel are human beings.
(iv) No human beings are cruel creatures.

(A) only (i)
(B) only (iii) and (iv)
(C) only (i) and (ii)
(D) (i), (ii) and (iii)

Correct Answer: (D) (i), (ii) and (iii)

Solution:
Step 1: The statement "Some human beings are not cruel creatures" is given as FALSE.
$\Rightarrow$ This means the opposite is TRUE: "No human beings are not cruel."
$\Rightarrow$ All human beings must be cruel creatures.

Step 2: If all human beings are cruel, then naturally:
- (i) All human beings are cruel creatures $\Rightarrow$ TRUE.

Step 3: If all human beings are cruel, then certainly some of them are cruel.
- (ii) Some human beings are cruel creatures $\Rightarrow$ TRUE.

Step 4: Since humans are part of the set of cruel creatures, some creatures that are cruel are indeed human beings.
- (iii) Some creatures that are cruel are human beings $\Rightarrow$ TRUE.

Step 5: Statement (iv) says no human beings are cruel creatures. This contradicts the result from Step 1, so it is FALSE.

Final Conclusion: The statements (i), (ii), and (iii) can be logically inferred with certainty.

$$\boxed{\text{Correct Answer: (D)}}$$

Quick Tip

- Always negate carefully: If "Some X are not Y" is FALSE, then it means "All X are Y."
- From universal truths ("all"), you can always infer particular truths ("some").
- Eliminate contradictions systematically.

Q7. To construct a wall, sand and cement are mixed in the ratio of 3:1. The cost of sand and that of cement are in the ratio of 1:2.
If the total cost of sand and cement to construct the wall is 1000 rupees, then what is the cost (in rupees) of cement used?

(A) 400
(B) 600
(C) 800
(D) 200

Correct Answer: (A) 400

Solution:
Step 1: The mixing ratio of sand : cement = 3 : 1.
This means if we take 3 units of sand, we take 1 unit of cement.

Step 2: The cost ratio of sand : cement = 1 : 2.
Let cost of 1 unit of sand = x.
Then cost of 1 unit of cement = $2x$.

Step 3: Total cost of mixture = (3 units of sand + 1 unit of cement).
$= 3 \times x + 1 \times (2x) = 3x + 2x = 5x$.

Step 4: It is given that total cost = 1000.
So, $5x = 1000 \Rightarrow x = 200$.

Step 5: Cost of cement used $= 2x = 2 \times 200 = 400$.

$$\boxed{400}$$

Quick Tip

- Always separate two ratios: the *mixing ratio* and the *cost ratio*.
- Multiply the quantity ratio by the cost ratio to find effective costs.
- Use proportional distribution when total cost is given.

Q8. The World Bank has declared that it does not plan to offer new financing to Sri Lanka, which is battling its worst economic crisis in decades, until the country has an adequate macroeconomic policy framework in place. In a statement, the World Bank said Sri Lanka needed to adopt structural reforms that focus on economic stabilisation and tackle the root causes of its crisis. The latter has starved it of foreign exchange and led to shortages of food, fuel, and medicines. The bank is repurposing resources under existing loans to help alleviate shortages of essential items such as medicine, cooking gas, fertiliser, meals for children, and cash for vulnerable households. Based only on the above passage, which one of the following statements can be inferred with *certainty*?

(A) According to the World Bank, the root cause of Sri Lanka's economic crisis is that it does not have enough foreign exchange.
(B) The World Bank has stated that it will advise the Sri Lankan government about how to tackle the root causes of its economic crisis.
(C) According to the World Bank, Sri Lanka does not yet have an adequate macroeconomic policy framework.
(D) The World Bank has stated that it will provide Sri Lanka with additional funds for essentials such as food, fuel, and medicines.

Correct Answer: (C) According to the World Bank, Sri Lanka does not yet have an adequate macroeconomic policy framework.

Solution: Step 1: The passage clearly mentions that the World Bank does not plan to offer new financing until Sri Lanka has an adequate macroeconomic policy framework in place. This means currently such a framework is absent.

Step 2: Option (A) is incorrect because the World Bank did not state that lack of foreign exchange is the root cause; rather, it said "structural reforms" are needed to tackle root causes.

Step 3: Option (B) is incorrect because the passage does not mention advising; it only mentions requiring reforms.

Step 4: Option (D) is incorrect because the World Bank is *repurposing existing loans*, not providing additional funds.

Step 5: Thus, the only statement that can be inferred with certainty is (C). $\boxed{C}$

Quick Tip

When solving inference questions:

- Focus only on what is explicitly mentioned in the passage.
- Be cautious of options that assume or add extra information not directly stated.
- Keywords like "certainty" demand strong evidence from the text.

Q9. The coefficient of x^4 in the polynomial $(x-1)^3(x-2)^3$ is equal to ______.

(A) 33
(B) -3
(C) 30
(D) 21

Correct Answer: (A) 33

Solution: Step 1: Expand $(x-1)^3 = x^3 - 3x^2 + 3x - 1$.
Step 2: Expand $(x-2)^3 = x^3 - 6x^2 + 12x - 8$.
Step 3: To find the coefficient of x^4, consider cross terms from the two cubic polynomials whose degrees add to 4.

$$\begin{aligned}
&(x^3 \cdot -6x^2) \quad \Rightarrow -6x^5 \quad \text{(too high)} \\
&(x^3 \cdot 12x) = 12x^4 \\
&(x^3 \cdot -8) = -8x^3 \quad \text{(too low)} \\
&(-3x^2 \cdot x^3) = -3x^5 \quad \text{(too high)} \\
&(-3x^2 \cdot -6x^2) = 18x^4 \\
&(3x \cdot x^3) = 3x^4 \\
&(-1 \cdot x^3) = -x^3 \quad \text{(too low)}
\end{aligned}$$

Step 4: Adding valid contributions for x^4: $12 + 18 + 3 = 33$. Wait—check carefully.

Actually, recheck carefully:

From $(x-1)^3(x-2)^3$: expand term by term:

Coefficient of x^4 arises from:

$$\begin{aligned}
&(x^3 \cdot 12x) = 12x^4 \\
&(-3x^2 \cdot -6x^2) = 18x^4 \\
&(3x \cdot x^3) = 3x^4
\end{aligned}$$

Total $= 12 + 18 + 3 = 33$.

Step 5: So coefficient of x^4 is $\boxed{33}$. Hence answer is (A).

Quick Tip

- When finding coefficients in product expansions, focus on degree combinations that add to the required power.
- Avoid expanding everything; just target terms that contribute to the desired power.

Q10. Which one of the following shapes can be used to tile (completely cover by repeating) a flat plane, extending to infinity in all directions, without leaving any empty spaces in between them? The copies of the shape used to tile are identical and are not allowed to overlap.

(A) circle
(B) regular octagon
(C) regular pentagon
(D) rhombus

Correct Answer: (D) rhombus

Solution: Step 1: For a shape to tile the plane, the interior angles must be such that they can fit around a point to sum to 360°.

Step 2: A circle cannot tile because gaps remain between circles.

Step 3: A regular octagon cannot tile alone—one needs squares to fill gaps.

Step 4: A regular pentagon cannot tile because interior angle 108° does not divide 360° evenly.

Step 5: A rhombus, being a type of parallelogram, can always tile the plane without gaps. Hence, the answer is (D). $\boxed{\text{rhombus}}$.

Quick Tip

- Parallelograms (squares, rectangles, rhombi) always tessellate the plane.
- For regular polygons, only equilateral triangles, squares, and hexagons tile by themselves.

Q.11. Which one of the following is the CORRECT value of y, as defined by the expression given below?

$$y = \lim_{x \to 0} \frac{2x}{e^x - 1}$$

(A) 1
(B) 2
(C) 0
(D) ∞

Correct Answer: (B) 2

Solution:
Step 1: As $x \to 0$, numerator $2x \to 0$ and denominator $e^x - 1 \to 0$. Thus, the limit is of the form $\frac{0}{0}$, so L'Hôpital's Rule can be applied.

Step 2: Differentiate numerator and denominator:

$$\lim_{x\to 0}\frac{2x}{e^x-1}=\lim_{x\to 0}\frac{(2)'}{(e^x-1)'}=\lim_{x\to 0}\frac{2}{e^x}.$$

Step 3: Evaluate at $x=0$:

$$\frac{2}{e^0}=\frac{2}{1}=2.$$

$$\boxed{2}$$

Quick Tip

- For limits of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, L'Hôpital's Rule is a powerful tool.
- Alternatively, use series expansion: $e^x-1\approx x$ when $x\to 0$, giving $\frac{2x}{x}=2$.

Q.12. The vector $\vec{v}$ is defined as

$$\vec{v}=zx\,\hat{i}+2xy\,\hat{j}+3yz\,\hat{k}.$$

Which one of the following is the CORRECT value of divergence of $\vec{v}$, evaluated at the point $(x,y,z)=(3,2,1)$?

(A) 0
(B) 3
(C) 14
(D) 13

Correct Answer: (D) 13

Solution: Step 1: Recall that the divergence of a vector field $\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$ is

$$\nabla\cdot\vec{v}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}.$$

Step 2: Identify the components:

$$v_x=zx,\quad v_y=2xy,\quad v_z=3yz.$$

Step 3: Compute partial derivatives.

$$\frac{\partial v_x}{\partial x}=\frac{\partial(zx)}{\partial x}=z,$$

$$\frac{\partial v_y}{\partial y}=\frac{\partial(2xy)}{\partial y}=2x,$$

$$\frac{\partial v_z}{\partial z}=\frac{\partial(3yz)}{\partial z}=3y.$$

Step 4: Add them:

$$\nabla \cdot \vec{v} = z + 2x + 3y.$$

Step 5: Substitute $(x, y, z) = (3, 2, 1)$:

$$\nabla \cdot \vec{v} = 1 + 2(3) + 3(2) = 1 + 6 + 6 = 13.$$

Step 6: Correct divergence value is $\boxed{13}$. Hence the correct option is (D).

Quick Tip

- Divergence measures the net flux density of a vector field at a point.
- Always apply $\nabla \cdot \vec{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z$.
- Carefully substitute the given point values after simplification.

Q.13. Given that $F = \frac{|z_1 + z_2|}{|z_1| + |z_2|}$, where $z_1 = 2 + 3i$ and $z_2 = -2 + 3i$ with $i = \sqrt{-1}$, which one of the following options is CORRECT?

(A) $F < 0$
(B) $F < 1$
(C) $F > 1$
(D) $F = 1$

Correct Answer: (B) $F < 1$

Solution: Step 1: Compute $z_1 + z_2 = (2 - 2) + (3 + 3)i = 6i \Rightarrow |z_1 + z_2| = |6i| = 6$.

Step 2: Compute the magnitudes $|z_1| = \sqrt{2^2 + 3^2} = \sqrt{13}$, $|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$.

Step 3: Evaluate F:

$$F = \frac{|z_1 + z_2|}{|z_1| + |z_2|} = \frac{6}{\sqrt{13} + \sqrt{13}} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}} \approx 0.832 < 1.$$

Hence $0 < F < 1$.

$$\boxed{F < 1}$$

Quick Tip

- Use $|a + bi| = \sqrt{a^2 + b^2}$.
- Often, checking whether a ratio is < 1 or > 1 is faster than computing exact decimals.
- Symmetry like $|z_1| = |z_2|$ can simplify sums in denominators.

Q.14. For a two-dimensional plane, the unit vectors $(\hat{e}_r, \hat{e}_\theta)$ of the polar coordinate system and $(\hat{i}, \hat{j})$ of the cartesian coordinate system are related by

$$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}, \qquad \hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}.$$

Which one of the following is the CORRECT value of $\dfrac{\partial(\hat{e}_r + \hat{e}_\theta)}{\partial\theta}$?

(A) 1
(B) $\hat{e}_\theta$
(C) $\hat{e}_r + \hat{e}_\theta$
(D) $-\hat{e}_r + \hat{e}_\theta$

Correct Answer: (D) $-\hat{e}_r + \hat{e}_\theta$

Solution: Step 1: Differentiate the basis vectors with respect to θ:

$$\frac{\partial\hat{e}_r}{\partial\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} = \hat{e}_\theta, \qquad \frac{\partial\hat{e}_\theta}{\partial\theta} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j} = -\hat{e}_r.$$

Step 2: Therefore,

$$\frac{\partial(\hat{e}_r + \hat{e}_\theta)}{\partial\theta} = \frac{\partial\hat{e}_r}{\partial\theta} + \frac{\partial\hat{e}_\theta}{\partial\theta} = \hat{e}_\theta - \hat{e}_r = -\hat{e}_r + \hat{e}_\theta.$$

$$\boxed{-\hat{e}_r + \hat{e}_\theta}$$

Quick Tip

- In polar coordinates: $\dfrac{\partial\hat{e}_r}{\partial\theta} = \hat{e}_\theta$ and $\dfrac{\partial\hat{e}_\theta}{\partial\theta} = -\hat{e}_r$.
- Memorizing these two relations helps in curvilinear coordinate calculus and vector differentiation problems.

Q.15. Which one of the following statements related to octane number is NOT correct?

(A) Linear alkanes with higher carbon number have higher octane number.
(B) Branching in linear alkanes increases their octane number.
(C) Catalytic reforming of hydrocarbons increases their octane number.
(D) Gasoline quality is measured in terms of octane number.

Correct Answer: (A) Linear alkanes with higher carbon number have higher octane number.

Solution: Step 1: Octane number measures the fuel's ability to resist knocking. Iso-octane (2,2,4-trimethylpentane) is defined with an octane number of 100.
Step 2: Straight-chain alkanes (n-alkanes) generally have *low* octane numbers because they

ignite easily, leading to knocking. Increasing the carbon chain length in linear alkanes *decreases*, rather than increases, their octane number.
Step 3: Branching in alkanes (Option B) increases the octane number since it enhances resistance to knocking.
Step 4: Catalytic reforming (Option C) increases octane number by converting straight-chain hydrocarbons into branched or aromatic compounds.
Step 5: Gasoline quality is indeed measured using the octane number (Option D).
Step 6: Hence, the incorrect statement is Option (A).

Quick Tip

- Straight-chain alkanes → lower octane number.
- Branched alkanes and aromatics → higher octane number.
- Octane number is a direct measure of resistance to engine knocking.

Q.16. Which one of the following options represents the major components of oleum?

(A) Sulfuric acid and nitric acid
(B) Concentrated sulfuric acid and petroleum jelly
(C) Sulfuric acid and hydrochloric acid
(D) Sulfuric acid and sulfur trioxide

Correct Answer: (D) Sulfuric acid and sulfur trioxide

Solution: Step 1: Oleum, also known as fuming sulfuric acid, is produced by dissolving sulfur trioxide (SO_3) in concentrated sulfuric acid.
Step 2: Its general composition can be expressed as:

$$\text{Oleum} = H_2SO_4 + xSO_3 \quad (x > 0).$$

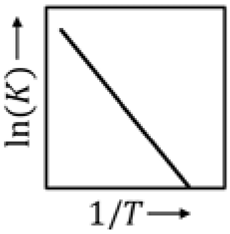
Step 3: Options (A), (B), and (C) are incorrect because nitric acid, petroleum jelly, and hydrochloric acid are not components of oleum.
Step 4: Therefore, the correct major components are sulfuric acid and sulfur trioxide (Option D).

Quick Tip

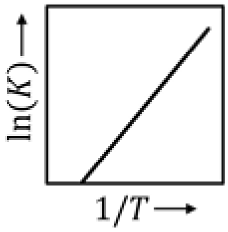
- Oleum = fuming sulfuric acid = H_2SO_4 + SO_3.
- Used in nitration reactions and in the manufacture of explosives and dyes.
- Strength of oleum is expressed as % free SO_3.

Q.17. For a reversible endothermic chemical reaction with constant heat of reaction over the operating temperature range, K is the thermodynamic equilibrium constant. Which one of the following figures shows the CORRECT dependence of K on temperature T?

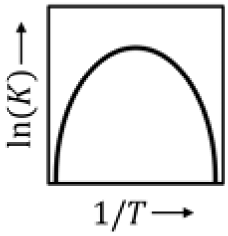
(A)



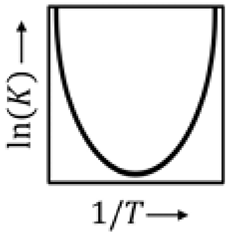
(B)



(C)



(D)



Correct Answer: (A) Linear decreasing $\ln(K)$ vs $1/T$

Solution:
Step 1: Recall the van't Hoff equation:

$$\ln K = -\frac{\Delta H^\circ}{R} \cdot \frac{1}{T} + \frac{\Delta S^\circ}{R}.$$

Step 2: For an **endothermic reaction**, $\Delta H^\circ > 0$. Thus the slope of the line in the plot of $\ln K$ vs $1/T$ is:

$$\text{slope} = -\frac{\Delta H^\circ}{R} < 0.$$

Step 3: Therefore, $\ln K$ decreases linearly as $1/T$ increases (negative slope straight line).

Step 4: This corresponds to Option (A).

$$\boxed{(A)\ \ln K \text{ decreases linearly with } 1/T}$$

Quick Tip

- Use the van't Hoff equation for equilibrium constant dependence on temperature.
- Endothermic ($\Delta H^\circ > 0$) → negative slope in $\ln K$ vs $1/T$.
- Exothermic ($\Delta H^\circ < 0$) → positive slope in $\ln K$ vs $1/T$.

Q.18. Nitrile rubber is manufactured via polymerization process. Which one of the following options is the CORRECT pair of monomers used in this process?

(A) Acrylonitrile and styrene
(B) Acrylonitrile and butadiene
(C) Butadiene and styrene
(D) Butadiene and isoprene

Correct Answer: (B) Acrylonitrile and butadiene

Solution: **Step 1:** Nitrile rubber (NBR), also called Buna-N, is a copolymer formed by emulsion polymerization of acrylonitrile (AN) and 1,3-butadiene (BD).
Step 2: The acrylonitrile content controls oil and fuel resistance; butadiene provides elasticity. Therefore, the correct monomer pair is AN + BD ⇒ Option (B).

Quick Tip

- Remember: Nitrile rubber ⇒ contains *nitrile* group from **acrylonitrile**.
- Common trade names: Buna-N, NBR; excellent oil/fuel resistance.

Q.19. John and Jane independently performed a thermodynamic experiment, in which X and Y represent the initial and final thermodynamic states of the system, respectively. John performed the experiment under reversible conditions, for which the change in entropy of the system was ΔS_{rev}. Jane performed the experiment under irreversible conditions, for which the change in entropy of the system was ΔS_{irr}. Which one of the following relationships is CORRECT?

(A) $\Delta S_{\text{rev}} = \Delta S_{\text{irr}}$
(B) $\Delta S_{\text{rev}} > \Delta S_{\text{irr}}$
(C) $\Delta S_{\text{rev}} < \Delta S_{\text{irr}}$
(D) $\Delta S_{\text{rev}} = 2\Delta S_{\text{irr}}$

Correct Answer: (A) $\Delta S_{\text{rev}} = \Delta S_{\text{irr}}$

Solution: Step 1: Entropy S is a state function. Therefore, for fixed initial and final states $\mathbf{X} \to \mathbf{Y}$, the change in entropy of the *system* depends only on the states, not on the path (reversible or irreversible).
Step 2: Hence,

$$\Delta S_{\text{system}} = S(\mathbf{Y}) - S(\mathbf{X})$$

is unique, giving

$$\boxed{\Delta S_{\text{rev}} = \Delta S_{\text{irr}}}.$$

(Note: The heat term and the inequality $\Delta S_{\text{univ}} \geq 0$ concern the *universe*, not the system's state change.)

Quick Tip

- State functions (e.g., S, U, H, G) have path-independent changes.
- Clausius inequality $\oint \frac{\delta q}{T} \leq 0$ distinguishes reversibility via entropy *generation* of the universe, not the system's ΔS between the same states.

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20. For a packed-bed comprising of uniform-sized spherical particles of diameter D_p, the pressure drop across the bed is given by the Kozeny–Carman (laminar Ergun) equation when the particle Reynolds number $(Re_p) < 1$. Under this condition, minimum fluidization velocity is proportional to D_p^n. Which one of the following is the CORRECT value of exponent n?

(A) 2
(B) -1
(C) -2
(D) 1

Correct Answer: (A) 2

Solution: Step 1: For $Re_p < 1$ (laminar regime), the Kozeny–Carman/laminar term of the Ergun equation gives the pressure drop per unit length:

$$\frac{\Delta P}{L} = \frac{150(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu\, u}{D_p^2},$$

where ε is bed voidage, μ is fluid viscosity, and u is the superficial velocity.

Step 2: At minimum fluidization, the pressure drop balances the apparent weight of solids per unit bed volume:

$$\left.\frac{\Delta P}{L}\right|_{u=u_{mf}} = (\rho_s - \rho_f)(1-\varepsilon_{mf})\, g,$$

which is independent of D_p (material and bed properties fixed).

Step 3: Equate the two expressions and solve for u_{mf}:

$$(\rho_s - \rho_f)(1-\varepsilon_{mf})\, g = \frac{150(1-\varepsilon_{mf})^2}{\varepsilon_{mf}^3} \frac{\mu\, u_{mf}}{D_p^2} \;\Rightarrow\; u_{mf} \propto D_p^2.$$

Hence, the exponent $n = 2$.

Quick Tip

- For $Re_p < 1$, only the viscous (Kozeny–Carman) term matters: $\Delta P/L \propto \mu u/D_p^2$.
- At minimum fluidization, $\Delta P/L$ equals the (constant) bed weight per volume $\Rightarrow u_{mf} \propto D_p^2$.
- Remember: laminar $u_{mf} \sim D_p^2$; if inertial effects dominated, the scaling would change.

Q21. Match the quantities in Group 1 with their units in Group 2 listed in the table below.

Group 1	**Group 2**
P) Thermal conductivity	I) W. $\text{m}^{-2}\text{K}^{-1}$
Q) Convective heat transfer coefficient	II) W. $\text{m}^{-1}\text{K}^{-1}$
R) Stefan-Boltzmann constant	III) W. K^{-1}
S) Heat capacity rate	IV) W. $\text{m}^{-2}\text{K}^{-4}$

(A) P-II, Q-I, R-IV, S-III
(B) P-I, Q-II, R-III, S-IV
(C) P-III, Q-IV, R-II, S-I
(D) P-IV, Q-I, R-III, S-II

Correct Answer: (A) P-II, Q-I, R-IV, S-III

Solution:
Step 1: Identify the units of thermal conductivity.
Thermal conductivity k has units:

$$\frac{\text{W}}{\text{m}\cdot\text{K}} = \text{W}\cdot\text{m}^{-1}\text{K}^{-1}.$$

So, $P \to II$.

Step 2: Identify the units of convective heat transfer coefficient.
Heat transfer coefficient h is defined as

$$q = hA\Delta T.$$

Thus, units are:

$$\frac{\text{W}}{\text{m}^2\cdot\text{K}} = \text{W}\cdot\text{m}^{-2}\text{K}^{-1}.$$

So, $Q \to I$.

Step 3: Identify the units of Stefan–Boltzmann constant.
The Stefan–Boltzmann law is:

$$q = \sigma T^4.$$

Here, σ has units:

$$\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4} = \mathrm{W{\cdot}m^{-2}K^{-4}}.$$

So, $R \to IV$.

Step 4: Identify the units of heat capacity rate.
Heat capacity rate is defined as:

$$C = \dot{m} c_p,$$

with units of:

$$\frac{\mathrm{J}}{\mathrm{K{\cdot}s}} = \mathrm{W{\cdot}K^{-1}}.$$

So, $S \to III$.

Hence the correct matching is: $P - II, Q - I, R - IV, S - III$.

$$\boxed{(A)\ P - II,\ Q - I,\ R - IV,\ S - III}$$

Quick Tip

- Always recall standard heat transfer units: • k (thermal conductivity) → $\mathrm{W{\cdot}m^{-1}K^{-1}}$
• h (convective coefficient) → $\mathrm{W{\cdot}m^{-2}K^{-1}}$
• σ (Stefan–Boltzmann constant) → $\mathrm{W{\cdot}m^{-2}K^{-4}}$
• Heat capacity rate → $\mathrm{W{\cdot}K^{-1}}$

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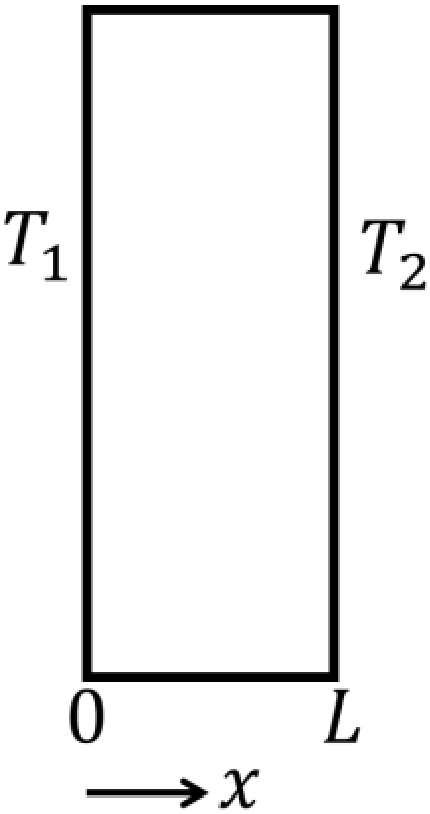
22. A slab of thickness L, as shown in the figure, has cross-sectional area A and constant thermal conductivity k. T_1 and T_2 are the temperatures at $x = 0$ and $x = L$, respectively. Which one of the following options is the CORRECT expression of the thermal resistance for steady-state one-dimensional heat conduction?



(A) $\dfrac{L}{kA}$

(B) $\dfrac{k}{LA}$

(C) $\dfrac{kA(T_1 - T_2)}{L}$

(D) $\dfrac{A}{Lk}$

Correct Answer: (A) $\dfrac{L}{kA}$

Solution: Step 1: For steady one-dimensional conduction with constant k, Fourier's law gives the heat flux

$$q_x = -k\,\frac{dT}{dx} = \frac{k\,(T_1 - T_2)}{L},$$

because the temperature profile is linear between $x = 0$ and $x = L$.

Step 2: The total heat rate is $Q = q_x A = \dfrac{kA\,(T_1 - T_2)}{L}$.

Step 3: Define thermal resistance R_{th} by $R_{th} = \dfrac{\Delta T}{Q} = \dfrac{T_1 - T_2}{Q}$. Substituting Q from Step 2,

$$R_{th} = \frac{T_1 - T_2}{\dfrac{kA\,(T_1 - T_2)}{L}} = \frac{L}{kA}.$$

Hence, the correct expression is $R_{th} = \dfrac{L}{kA}$.

Quick Tip

- Thermal resistance in conduction: $R_{cond} = L/(kA)$ (units K/W).
- Series/parallel combinations of slabs add like electrical resistances.
- If k varies with x or T, integrate $R = \int \frac{dx}{kA}$.

Q23. Spray dryers have many advantages. Which one of the following is NOT an advantage of a typical spray dryer?

(A) Has short drying time
(B) Produces hollow spherical particles
(C) Has high heat efficiency
(D) Is suitable for heat sensitive materials

Correct Answer: (C) Has high heat efficiency

Solution:
Step 1: Recall the working of spray dryers.
Spray dryers atomize liquid into fine droplets, which dry rapidly in a hot gas stream to form powder particles.

Step 2: List actual advantages of spray dryers.
- They provide very short drying time (milliseconds to seconds).
- They produce fine hollow spherical particles, useful in many industries.
- They can handle heat-sensitive materials due to rapid evaporation cooling.

Step 3: Identify the non-advantage.
Spray dryers are **not** very efficient in terms of heat usage because a large fraction of hot air exits with unused heat.

Thus, the statement “Has high heat efficiency” is NOT an advantage.

$$\boxed{(C) \text{ Has high heat efficiency}}$$

Quick Tip

- Spray dryers: short drying time, handles heat-sensitive materials, produces uniform spherical particles. - Limitation: low thermal efficiency due to exhaust air carrying unused heat.

Q24. Which one of the following quantities of a flowing fluid is measured using a rotameter?

(A) Static pressure
(B) Dynamic pressure
(C) Volumetric flow rate
(D) Viscosity

Correct Answer: (C) Volumetric flow rate

Solution:
Step 1: Recall the principle of rotameter.
A rotameter is a variable area flowmeter, consisting of a vertically tapered tube and a float.

Step 2: How it works.
- As fluid flows upward, it lifts the float.
- The float rises until the upward drag force equals the downward weight of the float.
- The float's position indicates the volumetric flow rate directly on a calibrated scale.

Step 3: Eliminate incorrect options.
- Static pressure (A) is measured by pressure gauges, not rotameter.
- Dynamic pressure (B) is measured using a Pitot tube.
- Viscosity (D) is measured using viscometers, not rotameters.

Hence, rotameter measures **volumetric flow rate**.

$$\boxed{(C)\ \text{Volumetric flow rate}}$$

Quick Tip

- Rotameter = variable area flowmeter. - Directly calibrated for volumetric flow rate. - Common in labs and industries due to simplicity and low cost.

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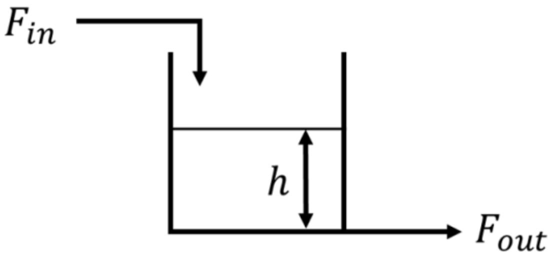
25. A liquid surge tank has F_{in} and F_{out} as the inlet and outlet flow rates respectively, as shown in the figure. F_{out} is proportional to the square root of the liquid level h. The cross-sectional area of the tank is 20 cm^2. Density of the liquid is constant everywhere in the system. At steady state, $F_{in} = F_{out} = 10$ cm^{3}s^{-1} and $h = 16$ cm. The variation of h with F_{in} is approximated as a first-order transfer function. Which one of the following is the CORRECT value of the time constant (in seconds) of this system?



(A) 20
(B) 32
(C) 64
(D) 128

Correct Answer: (C) 64

Solution: Step 1: Dynamic balance on the tank (cross-sectional area $A = 20$):

$$A\frac{dh}{dt} = F_{in} - F_{out}, \qquad F_{out} = k\sqrt{h}.$$

Use steady state $F_{in} = F_{out} = 10$ at $h = 16$ to find k:

$$10 = k\sqrt{16} = 4k \;\Rightarrow\; k = 2.5\,\frac{\text{cm}^3}{\text{s}\sqrt{\text{cm}}}.$$

Step 2: Linearize about steady state. Let deviations be $h' = h - h_{ss}$, $f'_{in} = F_{in} - F_{in,ss}$.

$$A\frac{dh'}{dt} = f'_{in} - \left.\frac{dF_{out}}{dh}\right|_{ss} h', \qquad \left.\frac{dF_{out}}{dh}\right|_{ss} = \frac{k}{2\sqrt{h_{ss}}} = \frac{2.5}{2\cdot 4} = 0.3125\,\frac{\text{cm}^3}{\text{s}\,\text{cm}}.$$

Step 3: Put into standard first-order form $A\frac{dh'}{dt} + \left.\frac{dF_{out}}{dh}\right|_{ss} h' = f'_{in}$. Hence the time constant is

$$\tau = \frac{A}{\left.\frac{dF_{out}}{dh}\right|_{ss}} = \frac{20}{0.3125} = 64 \text{ s}.$$

$\boxed{\tau = 64 \text{ s}}$.

Quick Tip

- For tanks with outflow $F_{out}(h)$, the linearized first-order time constant is $\tau = A\Big/\left(\frac{dF_{out}}{dh}\Big|_{ss}\right)$.
- Orifice-type outlets have $F_{out} \propto \sqrt{h} \Rightarrow \frac{dF_{out}}{dh} = \frac{k}{2\sqrt{h}}$.
- Use steady-state data to estimate k before linearization.

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26. A packed distillation column, with vapor having an average molecular weight of $45\ \mathrm{kg \cdot kmol^{-1}}$, density of $2\ \mathrm{kg \cdot m^{-3}}$ and a molar flow rate of $0.1\ \mathrm{kmol \cdot s^{-1}}$, has a flooding velocity of $0.15\ \mathrm{m \cdot s^{-1}}$. The column is designed to operate at 60% of the flooding velocity. Which one of the following is the CORRECT value for the column diameter (in m)?

(A) $\dfrac{5}{\sqrt{\pi}}$
(B) $5\sqrt{\pi}$
(C) 4π
(D) $\dfrac{10}{\sqrt{\pi}}$

Correct Answer: (D) $\dfrac{10}{\sqrt{\pi}}$

Solution: Step 1: Convert the given mass density to molar density using the molecular weight $M = 45\ \mathrm{kg \cdot kmol^{-1}}$:

$$C = \frac{\rho}{M} = \frac{2}{45}\ \mathrm{kmol \cdot m^{-3}}.$$

Volumetric flow rate of vapor:

$$\dot{V} = \frac{\dot{n}}{C} = \frac{0.1}{2/45} = 2.25\ \mathrm{m^3{\cdot}s^{-1}}.$$

Step 2: Design superficial vapor velocity (at 60% of flooding):

$$u = 0.6\, u_f = 0.6 \times 0.15 = 0.09\ \mathrm{m \cdot s^{-1}}.$$

Step 3: Required cross-sectional area and diameter:

$$A = \frac{\dot{V}}{u} = \frac{2.25}{0.09} = 25\ \mathrm{m^2}, \qquad D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{100}{\pi}} = \frac{10}{\sqrt{\pi}}\ \mathrm{m}.$$

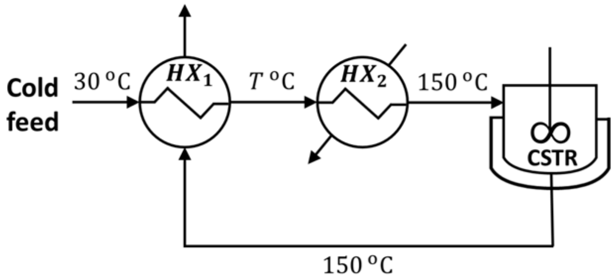
Thus, the correct diameter is $\dfrac{10}{\sqrt{\pi}}$ m.

Quick Tip

- Use $\dot{V} = \dot{n}/C$ with $C = \rho/M$ to convert molar flow to volumetric flow.
- For packed columns, size using the design superficial velocity: $A = \dot{V}/u$.
- Finally, $D = \sqrt{4A/\pi}$.

Q27. An isothermal jacketed continuous stirred tank reactor (CSTR) operating at $150^\circ\mathrm{C}$ is shown. The cold feed at $30^\circ\mathrm{C}$ is first preheated to T ($T < 150^\circ\mathrm{C}$) in heat exchanger HX_1 (using process-to-process heat recovery) and then heated to $150^\circ\mathrm{C}$ in a utility heater HX_2. Mass flow rate and heat capacity are the same for all

process streams and the overall heat transfer coefficient is independent of temperature. Which action will increase the value of T?



(A) Increase both heat transfer area of HX_1 and heat duty of HX_2.
(B) Decrease both heat transfer area of HX_1 and heat duty of HX_2.
(C) Increase the heat transfer area of HX_1 and decrease the heat duty of HX_2.
(D) Decrease the heat transfer area of HX_1 and increase the heat duty of HX_2.

Correct Answer: (C) Increase the heat transfer area of HX_1 and decrease the heat duty of HX_2

Solution:
Step 1: Write the heat recovered in the preheater HX_1.
With equal $\dot{m}c_p$ on hot and cold sides and U independent of T, the recovered heat in HX_1 is

$$Q_1 \;=\; UA_1\,\Delta T_{\text{lm}} \;=\; \dot{m}c_p\,(T - 30°\text{C}).$$

Thus, for fixed inlet temperatures, increasing A_1 (area of HX_1) $\Rightarrow$ increases Q_1 $\Rightarrow$ increases T.

Step 2: Relate utility duty in HX_2 to T.
The utility heater supplies the remaining heat to reach 150°C:

$$Q_2 \;=\; \dot{m}c_p\,(150°\text{C} - T).$$

If T increases (due to larger A_1), Q_2 must *decrease* to keep the reactor inlet at 150°C.

Step 3: Eliminate other options.
- Changing the duty of HX_2 alone *cannot* increase T because HX_2 is downstream of HX_1.
- Decreasing A_1 would reduce heat recovery and lower T, requiring higher Q_2.

Therefore, to increase T, increase the area of HX_1 and correspondingly decrease the duty of HX_2.

$$\boxed{(C)\text{ Increase } A_{HX_1} \text{ and decrease } Q_{HX_2}}$$

Quick Tip

- Upstream preheater temperature is governed by $Q_1 = UA\Delta T_{\text{lm}}$; only changes to U, A, or LMTD in that exchanger affect T.
- Any downstream heater cannot raise the upstream preheat temperature; it only supplies the remaining sensible heat to the final setpoint.

Q28. Consider a system where a Carnot engine is operating between a source and a sink. Which of the following statements about this system is/are NOT correct?

(A) This engine is reversible.
(B) The engine efficiency is independent of the source and sink temperatures.
(C) This engine has the highest efficiency among all engines that operate between the same source and sink.
(D) The total entropy of this system increases at the completion of each cycle of the engine.

Correct Answer: (B) The engine efficiency is independent of the source and sink temperatures; (D) The total entropy of this system increases at the completion of each cycle of the engine.

Solution: Step 1: Recall the Carnot efficiency. The efficiency of a Carnot engine is

$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h},$$

which *depends only* on the absolute temperatures T_h (source) and T_c (sink). Hence statement (B) claiming independence from these temperatures is **not correct**.

Step 2: Reversibility and optimality. A Carnot engine is by definition a *reversible* engine and, for given T_h and T_c, has the *maximum* possible efficiency among all engines. Therefore (A) and (C) are correct statements.

Step 3: Entropy change over a reversible cycle. For a completely reversible cycle involving the engine and the two reservoirs, the total entropy change of the *combined system* (engine + reservoirs) over a full cycle is

$$\Delta S_{\text{total}} = 0.$$

Thus the total entropy does *not* increase at the end of each cycle. Statement (D) is therefore **not correct**.

Quick Tip

- Carnot efficiency: $\eta = 1 - T_c/T_h$ — depends only on reservoir temperatures.
- Reversible cycle $\Rightarrow \Delta S_{\text{universe}} = 0$.
- Any irreversibility would make $\Delta S_{\text{universe}} > 0$ and reduce efficiency below Carnot.

Q29. For a fully developed turbulent flow of an incompressible Newtonian fluid through a pipe of constant diameter, which of the following statements is/are CORRECT?

(A) Reynolds stress, averaged over a sufficiently long time, is zero everywhere inside the pipe.
(B) Reynolds stress at the pipe wall is zero.
(C) Average velocity of the fluid is half of its center-line velocity.
(D) Average pressure gradient in the flow direction is constant.

Correct Answer: (B) Reynolds stress at the pipe wall is zero; (D) Average pressure gradient in the flow direction is constant.

Solution: Step 1: Reynolds stress distribution. In turbulent pipe flow the time-averaged Reynolds shear stress $-\rho\overline{u'v'}$ is generally nonzero across the cross-section, vanishing at the pipe centerline and at the wall (due to no-slip and no-penetration implying $u' = v' = 0$ at the wall). Hence (A) is false, while (B) is true.

Step 2: Mean vs. centerline velocity. For fully developed turbulent flow, the centerline velocity is typically about 1.2−1.3 times the mean velocity (not 2 times). Therefore statement (C) is false.

Step 3: Streamwise pressure gradient. In fully developed pipe flow (laminar or turbulent), the wall shear stress and thus the axial pressure gradient are uniform along x; hence the average pressure gradient is constant. So (D) is correct.

Quick Tip

- At the wall: $u' = v' = 0 \Rightarrow \overline{u'v'} = 0$.
- Fully developed flow: velocity profile does not change with $x \Rightarrow$ constant dp/dx.
- Mean vs. centerline velocity ratio is < 2 for turbulent pipe flow.

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30. Given that E (in W·m^{-2}) is the total hemispherical emissive power of a surface maintained at a certain temperature, which of the following statements is/are CORRECT?

(A) E does not depend on the direction of the emission.
(B) E depends on the viewfactor.
(C) E depends on the wavelength of the emission.
(D) E does not depend on the frequency of the emission.

Correct Answer: (A) E does not depend on direction; (D) E does not depend on frequency

Solution: Step 1: The total hemispherical emissive power is

$$E = \int_{\Omega=2\pi}\int_{0}^{\infty} I_\lambda(\theta,\phi)\,\cos\theta\, d\Omega\, d\lambda$$

or, for a gray surface, $E = \varepsilon\sigma T^4$. It is integrated over all directions and all wavelengths.

Step 2: Since the integration is over the full hemisphere, E is not a function of the direction $\Rightarrow$ statement (A) is true. Viewfactor pertains to radiative exchange between different surfaces, not to a single surface's emissive power $\Rightarrow$ (B) is false.

Step 3: Because E is integrated over wavelength/frequency, the *total* hemispherical E is not a function of wavelength or frequency variables themselves (though it depends on the surface spectral properties through the integral). Hence (C) is false and (D) is true.

Quick Tip

- Spectral quantities (E_λ, I_λ) depend on λ; total quantities (E) are integrals over λ and direction.
- For a blackbody, $E = \sigma T^4$; for a gray body, $E = \varepsilon\sigma T^4$.

---

31. The position $x(t)$ of a particle, at constant ω, is described by $\dfrac{d^2x}{dt^2} = -\omega^2 x$ with initial conditions $x(0) = 1$ and $\left.\dfrac{dx}{dt}\right|_{t=0} = 0$. The position of the particle at $t = \dfrac{3\pi}{\omega}$ is _______ (in integer).

Correct Answer: -1

Solution: Step 1: The ODE is simple harmonic motion; the solution satisfying $x(0) = 1,\ \dot{x}(0) = 0$ is

$$x(t) = \cos(\omega t).$$

Step 2: Evaluate at $t = \dfrac{3\pi}{\omega}$:

$$x\left(\frac{3\pi}{\omega}\right) = \cos(3\pi) = -1.$$

Quick Tip

- For $x'' + \omega^2 x = 0$, use $x = A\cos\omega t + B\sin\omega t$ and apply initial conditions.
- Remember $\cos(\pi) = -1$, $\cos(2\pi) = 1$, $\cos(3\pi) = -1$.

Q32. Burning of methane in a combustor yields carbon monoxide, carbon dioxide, and water vapor. Methane is fed to the combustor at 100 mol·hr^{-1}, of which 50% reacts. The theoretical oxygen requirement (in mol·hr^{-1}) is ______ (rounded off to one decimal place).

Correct Answer: 200.0 mol·hr^{-1}

Solution:
Step 1: Write possible combustion reactions of methane.
- Complete combustion to CO_2:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

- Incomplete combustion to CO:

$$CH_4 + 1.5O_2 \rightarrow CO + 2H_2O$$

Step 2: Define theoretical oxygen requirement.
Theoretical oxygen requirement = Oxygen needed for **complete combustion of all methane to CO_2 and H_2O**, regardless of actual extent or products.

Step 3: Calculate stoichiometric oxygen demand.
From the complete combustion equation: 1 mol CH_4 requires 2 mol O_2.

Step 4: Apply given methane flow.
Methane feed = 100 mol·hr^{-1}.
Theoretical O_2 requirement = $100 \times 2 = 200$ mol·hr^{-1}.

Step 5: Clarify the effect of 50% reaction.
Even though only 50% reacts in practice, the "theoretical oxygen requirement" is defined for **complete conversion of the entire methane feed**. Thus, still based on 100 mol·hr^{-1}.

Hence, the theoretical oxygen requirement is:

$$\boxed{200.0 \text{ mol·hr}^{-1}}$$

Quick Tip

- Theoretical oxygen requirement is always based on *complete combustion of the total fuel fed*, not the actual reacted portion. - Methane needs 2 moles of O_2 per mole for full oxidation to CO_2.

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33. The viscosity of an incompressible Newtonian fluid is measured using a capillary tube of diameter 0.5 mm and length 1.5 m. The fluid flow is laminar, steady and fully developed. For a flow rate of 1 cm^{3}s^{-1}, the pressure drop across the length of the tube is 1 MPa. If the viscosity of the fluid is $k \times 10^{-3}$ Pa·s, the value of k is __________ (rounded off to two decimal places).

Correct Answer: $k = 1.02$

Solution: Step 1: Use the Hagen–Poiseuille relation for fully developed laminar flow in a circular tube:

$$Q \;=\; \frac{\pi\,D^4}{128\,\mu\,L}\,\Delta P \quad\Rightarrow\quad \mu \;=\; \frac{\pi\,D^4\,\Delta P}{128\,L\,Q}.$$

Step 2: Convert all quantities to SI units and substitute.

$$D = 0.5\text{ mm} = 5.0\times10^{-4}\text{ m},\quad L = 1.5\text{ m},$$

$$Q = 1\text{ cm}^3/\text{s} = 1.0\times10^{-6}\text{ m}^3/\text{s},\quad \Delta P = 1\text{ MPa} = 1.0\times10^{6}\text{ Pa}.$$

$$\mu \;=\; \frac{\pi(5.0\times10^{-4})^4(1.0\times10^{6})}{128\,(1.5)\,(1.0\times10^{-6})} \;=\; 1.02265\times10^{-3}\text{ Pa}\cdot\text{s}.$$

Step 3: Express μ as $k\times10^{-3}$ Pa · s and round k to two decimals:

$$k \;=\; 1.02265 \;\approx\; \boxed{1.02}.$$

Quick Tip

- For laminar, fully developed flow in a circular tube, use Hagen–Poiseuille: $Q = \dfrac{\pi D^4}{128\mu L}\Delta P$.
- Always convert mm → m, $\text{cm}^3 \to \text{m}^3$, and MPa → Pa before substituting.
- If an answer is given as $k\times10^{-3}$, compute μ in Pa·s and then set $k = \mu/(10^{-3})$.

Q.34. A liquid L containing a dissolved gas S is stripped in a countercurrent operation using a pure carrier gas V. The liquid phase inlet and outlet mole fractions of S are 0.1 and 0.01, respectively. The equilibrium distribution of S between V and L is governed by $y_e = x_e$, where y_e and x_e are the mole fractions of S in V and L, respectively. The molar feed rate of the carrier gas stream is twice that of the liquid stream. Under dilute solution conditions, the minimum number of ideal stages required is __________ (in integer).

Correct Answer: 3

Solution: Step 1: Under dilute conditions with a linear equilibrium $y^* = mx$ and $m = 1$, define the *stripping factor*

$$S \;=\; \frac{V}{mL} \;=\; \frac{V}{L} \;=\; 2$$

since the carrier gas molar flow rate is twice the liquid flow rate.

Step 2: For countercurrent *stripping* with solute-free entering gas ($y_{N+1} = 0$) and linear equilibrium, the Kremser relation for the exiting liquid composition after N ideal stages is

$$x_1 \;=\; \frac{x_{N+1}}{1+S+S^2+\cdots+S^{N-1}}.$$

Here $x_{N+1} = 0.1$ (feed, rich liquid) and $x_1 = 0.01$ (exit, lean liquid). Hence

$$\frac{x_1}{x_{N+1}} = \frac{0.01}{0.1} = \frac{1}{10} = \frac{1}{1 + S + S^2 + \cdots + S^{N-1}}.$$

Step 3: With $S = 2$,

$$1 + 2 + 2^2 + \cdots + 2^{N-1} = 2^N - 1 = 10 \Rightarrow 2^N = 11 \Rightarrow N = \log_2 11 \approx 3.46.$$

Step 4: Since N must be an integer number of ideal stages, the minimum integer satisfying the required reduction is

$$N_{\text{min}} = 3.$$

Quick Tip

- For dilute absorption/stripping with linear equilibrium $y^* = mx$, use the Kremser factors: $A = \dfrac{L}{mV}$ (absorption) and $S = \dfrac{V}{mL}$ (stripping).
- With pure entering gas for stripping, $x_1 = \dfrac{x_{N+1}}{\sum_{k=0}^{N-1} S^k}$. A geometric sum makes computations quick.
- A larger S (more carrier gas) or more stages both lower the exit liquid concentration.

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35. In a binary gas-liquid system, $N_{A,EMD}$ is the molar flux of a gas A for equimolar counter diffusion with a liquid B. $N_{A,UMD}$ is the molar flux of A for steady one-component diffusion through stagnant B. Using the mole fraction of A in the bulk of the gas phase as 0.2 and that at the gas-liquid interface as 0.1 for both the modes of diffusion, the ratio of $N_{A,UMD}$ to $N_{A,EMD}$ is equal to _____ (rounded off to two decimal places).

Correct Answer: 1.18

Solution:
Step 1: For equimolar counter diffusion (EMD) in a binary gas at constant T, P,

$$N_{A,EMD} = \frac{D_{AB}C}{L}(x_{A1} - x_{A2}).$$

Given $x_{A1} = 0.2$ (bulk gas) and $x_{A2} = 0.1$ (interface).
Step 2: For one-component diffusion of A through stagnant B (UMD),

$$N_{A,UMD} = \frac{D_{AB}C}{L}\ln\left(\frac{1 - x_{A2}}{1 - x_{A1}}\right).$$

Step 3: Take the ratio (note that D_{AB}, C, and L cancel):

$$\frac{N_{A,UMD}}{N_{A,EMD}} = \frac{\ln\left(\dfrac{1 - x_{A2}}{1 - x_{A1}}\right)}{x_{A1} - x_{A2}} = \frac{\ln\left(\dfrac{0.9}{0.8}\right)}{0.2 - 0.1}.$$

Step 4: Evaluate numerically:

$$\ln\left(\frac{0.9}{0.8}\right) = \ln(1.125) \approx 0.11778, \quad \Rightarrow \quad \frac{N_{A,UMD}}{N_{A,EMD}} \approx \frac{0.11778}{0.1} = 1.1778 \approx \boxed{1.18}.$$

Quick Tip

- EMD uses a Δx_A (linear) driving force; diffusion through stagnant B introduces a logarithmic correction $\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$.
- Ratios are convenient—common factors like D_{AB}, C, and L cancel.

Q.36. An exhibition was held in a hall on 15 August 2022 between 3 PM and 4 PM during which any person was allowed to enter only once. Visitors who entered before 3:40 PM exited the hall exactly after 20 minutes from their time of entry. Visitors who entered at or after 3:40 PM, exited exactly at 4 PM. The probability distribution of the arrival time of any visitor is uniform between 3 PM and 4 PM. Two persons X and Y entered the exhibition hall independent of each other. Which one of the following values is the probability that their visits to the exhibition overlapped with each other?

(A) $\frac{5}{9}$

(B) $\frac{4}{9}$

(C) $\frac{2}{9}$

(D) $\frac{7}{9}$

Correct Answer: (A) $\frac{5}{9}$

Solution: Step 1: Let the entry times (in minutes after 3 PM) be $x, y \sim \text{Unif}[0, 60]$. Each visitor stays on $[t,\, e(t)]$ where

$$e(t) = \begin{cases} t + 20, & 0 \le t < 40, \\ 60, & 40 \le t \le 60. \end{cases}$$

Two visits do *not* overlap iff $e(x) \le y$ or $e(y) \le x$.

Step 2: For a fixed $x \in [0, 40)$: $e(x) = x + 20$. Non-overlap occurs for $y \ge x + 20$ or (if $y < 40$) $y \le x - 20$. Hence the measure in y is

$$L(x) = \begin{cases} 40 - x, & 0 \le x < 20, \\ 20, & 20 \le x < 40. \end{cases}$$

Step 3: For $x \in [40, 60)$: $e(x) = 60$. Non-overlap requires $e(y) \le x \Rightarrow y \le x - 20$ with $y < 40$. Thus $L(x) = x - 20$, $40 \le x < 60$.

Step 4: Compute the total non-overlap area in the $[0,60]\times[0,60]$ square:

$$A_{\text{no}} = \int_0^{20}(40-x)\,dx + \int_{20}^{40} 20\,dx + \int_{40}^{60}(x-20)\,dx = 600+400+600 = 1600.$$

Hence

$$\mathbb{P}(\text{no overlap}) = \frac{1600}{60^2} = \frac{4}{9}, \quad \mathbb{P}(\text{overlap}) = 1-\frac{4}{9} = \frac{5}{9}.$$

Quick Tip

- Convert time windows to intervals and use geometric probability on the unit square.
- When stay-time saturates at the end time, split cases by the threshold (here, 40 minutes).

Q.37. Simpson's one-third rule is used to estimate the definite integral $I = \int_{-1}^{1}\sqrt{1-x^2}\,dx$ with an interval length of 0.5. Which one of the following is the CORRECT estimate of I obtained using this rule?

(A) $\frac{1}{3} - \frac{1}{\sqrt{3}}$

(B) $\frac{1}{3} + \frac{2}{\sqrt{3}}$

(C) $\frac{1}{3} + \frac{1}{\sqrt{3}}$

(D) $\frac{1}{3} - \frac{2}{\sqrt{3}}$

Correct Answer: (B) $\frac{1}{3} + \frac{2}{\sqrt{3}}$

Solution: Step 1: Composite Simpson's 1/3 rule with $h = 0.5$ on $[-1,1]$ uses the nodes

$$x_0 = -1,\ x_1 = -0.5,\ x_2 = 0,\ x_3 = 0.5,\ x_4 = 1.$$

Let $f(x) = \sqrt{1-x^2}$. Then

$$f(\pm 1) = 0, \quad f(0) = 1, \quad f(\pm 0.5) = \sqrt{1-\tfrac{1}{4}} = \frac{\sqrt{3}}{2}.$$

Step 2: Apply the formula

$$I \approx \frac{h}{3}\Big[f(x_0)+f(x_4)+4\big(f(x_1)+f(x_3)\big)+2f(x_2)\Big] = \frac{0.5}{3}\Big[0+4\big(\sqrt{3}\big)+2\Big] = \frac{2\sqrt{3}+1}{3}.$$

Since $\frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$, the estimate equals $\boxed{\frac{1}{3}+\frac{2}{\sqrt{3}}}$.

Quick Tip

- For composite Simpson: $I \approx \frac{h}{3}\left[f_0 + f_n + 4\sum f_{\text{odd}} + 2\sum f_{\text{even}}\right]$ with even number of subintervals.
- Recognize symmetry: values at $\pm x$ are equal for even $f(x)$, simplifying computations.

---

38. Match the products in Group 1 with the manufacturing processes in Group 2 listed in the table below.

Group 1	Group 2
P) Acetaldehyde	I) Sulfate process
Q) Sulfuric acid	II) Electric furnace process
R) Pulp	III) Wacker process
S) Phosphorus	IV) Contact process

(A) P-III, Q-IV, R-I, S-II
(B) P-III, Q-I, R-IV, S-II
(C) P-IV, Q-I, R-II, S-III
(D) P-I, Q-IV, R-II, S-III

Correct Answer: (A) P-III, Q-IV, R-I, S-II

Solution:
Step 1: Match Acetaldehyde (P). Acetaldehyde is industrially produced by the **Wacker process**, where ethylene is oxidized to acetaldehyde using $PdCl_2$ and $CuCl_2$.

$$P \rightarrow III$$

Step 2: Match Sulfuric acid (Q). Sulfuric acid is manufactured by the **Contact process**, where SO_2 is oxidized to SO_3 and absorbed in water.

$$Q \rightarrow IV$$

Step 3: Match Pulp (R). Wood pulp for paper is produced by the **Sulfate process** (also known as the Kraft process).

$$R \rightarrow I$$

Step 4: Match Phosphorus (S). Elemental phosphorus is manufactured by the **Electric furnace process**, reducing phosphate rock with coke and silica at high temperatures.

$$S \rightarrow II$$

Step 5: Final Combination:

$$P - III, \quad Q - IV, \quad R - I, \quad S - II$$

This matches with option (A).

Correct Answer: (A)

Quick Tip

- Wacker process ⇒ Acetaldehyde.
- Contact process ⇒ Sulfuric acid.
- Sulfate (Kraft) process ⇒ Pulp.
- Electric furnace ⇒ Phosphorus.

Q.39. Match the reactions in Group 1 with the catalysts in Group 2 listed in the table below.

Group 1	**Group 2**
P) $C_6H_6 + Cl_2 \longrightarrow$ Chlorobenzene + HCl	I) Mixed oxide of Mo and Fe
Q) $H_2C = CH_2 + \frac{1}{2}O_2 \longrightarrow$ Ethylene oxide	II) V_2O_5
R) $CH_3OH + \frac{1}{2}O_2 \longrightarrow$ Formaldehyde + H_2O	III) $FeCl_3$
S) Naphthalene + $\frac{9}{2}O_2 \longrightarrow$ Phthalic Anhydride + $2\mathrm{H}_2O + 2CO_2$	IV) Ag_2O

(A) P-III, Q-IV, R-II, S-I
(B) P-III, Q-IV, R-I, S-II
(C) P-IV, Q-II, R-I, S-III
(D) P-IV, Q-III, R-I, S-II

Correct Answer: (B) P-III, Q-IV, R-I, S-II

Solution:
Step 1: Identify catalysts for each reaction

- **P)** Chlorination of benzene is an electrophilic substitution that requires a Lewis acid catalyst such as $\mathrm{FeCl_3}$. Hence $P \rightarrow$ III.
- **Q)** Partial oxidation of ethylene to ethylene oxide is industrially carried out on silver catalysts; represented here as $\mathrm{Ag_2O}$. Hence $Q \rightarrow$ IV.
- **R)** Oxidation of methanol to formaldehyde (Formox process) uses a mixed oxide catalyst of molybdenum and iron (e.g., $\mathrm{Fe_2(MoO_4)_3}$). Hence $R \rightarrow$ I.
- **S)** Oxidation of naphthalene to phthalic anhydride proceeds over $\mathrm{V_2O_5}$-based catalysts. Hence $S \rightarrow$ II.

Step 2: Consolidate the matching

$$P - III, \quad Q - IV, \quad R - I, \quad S - II,$$

which corresponds to option (B).

Quick Tip

- Remember common industrial pairs:
  - Benzene halogenation → Lewis acids ($FeCl_3$/$AlCl_3$).
  - Ethylene → ethylene oxide → silver.
  - Methanol → formaldehyde → Fe–Mo mixed oxides (Formox).
  - Naphthalene/o-xylene → phthalic anhydride → V_2O_5.

---

40. Water in a container at 290 K is exposed to air containing 3 % CO_2 by volume. Air behaves like an ideal gas and is maintained at 100 kPa pressure. The liquid phase comprising of dissolved CO_2 in water behaves like an ideal solution. Use Henry's constant of CO_2 dissolved in water at 290 K as 12 MPa. Under equilibrium conditions, which one of the following is the CORRECT value of the mole fraction of CO_2 dissolved in water?

(A) 2.9×10^{-4}
(B) 0.9×10^{-4}
(C) 2.5×10^{-4}
(D) 0.5×10^{-4}

Correct Answer: (C) 2.5×10^{-4}

Solution: Step 1: Find the partial pressure of CO_2 in the gas: $p_{CO_2} = y_{CO_2}P = 0.03 \times 100\text{ kPa} = 3\text{ kPa}$.

Step 2: Apply Henry's law in the form $x_{CO_2} = \dfrac{p_{CO_2}}{H}$ with $H = 12\text{ MPa} = 12000\text{ kPa}$:

$$x_{CO_2} = \frac{3}{12000} = 2.5 \times 10^{-4}.$$

Hence option (C).

Quick Tip

- When H is given in pressure units, use $x_A = p_A/H$; keep units consistent (kPa–kPa or Pa–Pa).
- For gas mixtures behaving ideally, $p_A = y_A P$.

---

41. The enthalpy (H, in J·mol^{-1}) of a binary liquid system at constant temperature and pressure is given as

$$H = 40x_1 + 60x_2 + x_1x_2(4x_1 + 2x_2),$$

where x_1 and x_2 represent the mole fractions of species 1 and 2 in the liquid, respectively. Which one of the following is the CORRECT value of the partial molar enthalpy of species 1 at infinite dilution, $\overline{H}_1^{\infty}$ (in J·mol^{-1})?

(A) 100
(B) 42
(C) 64
(D) 40

Correct Answer: (B) 42

Solution: Step 1: Eliminate x_2 using $x_2 = 1 - x_1$:

$$H(x_1) = 40x_1 + 60(1 - x_1) + x_1(1 - x_1)\big(4x_1 + 2(1 - x_1)\big) = 60 - 18x_1 - 2x_1^3.$$

Step 2: Use the binary relation for a molar property H:

$$\overline{H}_1 = H + (1 - x_1)\frac{dH}{dx_1}.$$

Compute $\dfrac{dH}{dx_1} = -18 - 6x_1^2$. Hence

$$\overline{H}_1 = \big(60 - 18x_1 - 2x_1^3\big) + (1 - x_1)\big(-18 - 6x_1^2\big) = 42 - 6x_1^2 + 4x_1^3.$$

Step 3: At infinite dilution of 1, $x_1 \to 0$:

$$\overline{H}_1^{\infty} = \lim_{x_1 \to 0} \overline{H}_1 = 42 \text{ J mol}^{-1}.$$

Thus option (B).

Quick Tip

- For any molar property $M(x_1)$ of a binary mixture: $\overline{M}_1 = M + (1 - x_1)\dfrac{dM}{dx_1}$ and $\overline{M}_2 = M - x_1\dfrac{dM}{dx_1}$.
- "Infinite dilution of 1" means $x_1 \to 0$ (and $x_2 \to 1$).

Q.42. Which one of the following represents the CORRECT effects of concentration polarization in a reverse osmosis process?

(A) Reduced water flux and reduced solute rejection
(B) Increased water flux and increased solute rejection
(C) Reduced water flux and increased solute rejection
(D) Increased water flux and reduced solute rejection

Correct Answer: (A) Reduced water flux and reduced solute rejection

Solution:

Step 1: Understanding concentration polarization

In reverse osmosis (RO), water permeates through the membrane while solutes are rejected. However, solutes accumulate near the membrane surface, creating a concentrated boundary layer — this phenomenon is called *concentration polarization.*

Step 2: Effect on driving force (water flux)

Due to solute accumulation, the effective osmotic pressure at the membrane surface increases. Since net driving pressure is

$$\Delta P_{\text{net}} = \Delta P - \Delta \pi,$$

where $\Delta \pi$ is osmotic pressure difference, the higher osmotic pressure reduces the net driving force. Thus, water flux decreases.

Step 3: Effect on solute rejection

At higher solute concentration near the membrane surface, more solute tends to diffuse through the membrane. This reduces the overall solute rejection capability of the process.

Step 4: Conclusion

Hence, concentration polarization leads to:

$$\text{Reduced water flux and reduced solute rejection.}$$

Quick Tip

- Concentration polarization is one of the main operational challenges in RO systems.
- It can be minimized by enhancing turbulence at the membrane surface (e.g., using spacers, crossflow operation).
- Reduced flux and rejection lower both productivity and quality of RO systems.

---

43. CO and H_2 participate in a catalytic reaction. The partial pressures (in atm) of the reacting species CO and H_2 in the feed stream are p_{CO} and p_{H_2}, respectively. While CO undergoes molecular adsorption, H_2 adsorbs via dissociative adsorption (as H atoms). The equilibrium constants (in atm^{-1}) corresponding to adsorption of CO and H_2 to the catalyst sites are K_{CO} and K_{H_2}, respectively. Total molar concentration of active sites per unit mass of the catalyst is C_t (in mol·(g cat)$^{-1}$). Both the adsorption steps are at equilibrium. Which one of the following expressions is the CORRECT ratio of the concentration of catalyst sites occupied by CO to that by hydrogen atoms?

(A) $\dfrac{K_{CO}\, p_{CO}}{\sqrt{K_{H_2}\, p_{H_2}}}$

(B) $\dfrac{K_{CO}}{\sqrt{K_{H_2}}}$

(C) $\dfrac{p_{CO}}{\sqrt{p_{H_2}}}$

(D) $\dfrac{K_{CO}\, p_{CO}}{K_{H_2}\, p_{H_2}}$

Correct Answer: (A) $\dfrac{K_{CO}\,p_{CO}}{\sqrt{K_{H_2}\,p_{H_2}}}$

Solution: Step 1: Write equilibrium relations using free site concentration $[S]$.

$$\text{Molecular adsorption: } CO(g) + S \rightleftharpoons CO^*, \quad K_{CO} = \frac{[CO^*]}{p_{CO}[S]} \Rightarrow [CO^*] = K_{CO}\,p_{CO}\,[S].$$

$$\text{Dissociative adsorption: } H_2(g) + 2S \rightleftharpoons 2H^*, \quad K_{H_2} = \frac{[H^*]^2}{p_{H_2}[S]^2} \Rightarrow [H^*] = \sqrt{K_{H_2}\,p_{H_2}}\,[S].$$

Step 2: Take the required ratio of site concentrations:

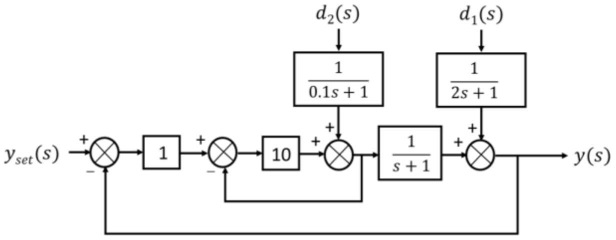
$$\frac{[CO^*]}{[H^*]} = \frac{K_{CO}\,p_{CO}\,[S]}{\sqrt{K_{H_2}\,p_{H_2}}\,[S]} = \boxed{\frac{K_{CO}\,p_{CO}}{\sqrt{K_{H_2}\,p_{H_2}}}}.$$

(The free-site concentration cancels; C_t is not needed for this ratio.)

Quick Tip

- Molecular adsorption: $K = \dfrac{\theta_A}{p_A(1-\theta)} \approx \dfrac{[A^*]}{p_A[S]}$.
- Dissociative adsorption: $K = \dfrac{[A^*]^2}{p_A[S]^2} \Rightarrow [A^*] \propto \sqrt{p_A}$.
- Ratios often cancel the unknown free-site concentration $[S]$.

Q.44. A cascade control strategy is shown in the figure. The transfer function between the output (y) and the secondary disturbance (d_2) is defined as $G_{d2}(s) = \dfrac{y(s)}{d_2(s)}$. Which one of the following is the CORRECT expression for $G_{d2}(s)$?



(A) $\dfrac{1}{(11s+21)(0.1s+1)}$

(B) $\dfrac{1}{(s+1)(0.1s+1)}$

(C) $\dfrac{s+1}{(s+2)(0.1s+1)}$

(D) $\dfrac{s+1}{(s+1)(0.1s+1)}$

Correct Answer: (A) $\dfrac{1}{(11s+21)(0.1s+1)}$

Solution:

Step 1: Loop reduction up to the point where d_2 enters

Let $G_1(s) = \dfrac{1}{0.1s+1}$ and $G_2(s) = \dfrac{1}{s+1}$. From the diagram, the signal entering the d_2-summer comes from a unity gain followed by a controller of gain 10, and there is an outer unity feedback from y to the first summer (error $= y_{\text{set}} - y$). With $y_{\text{set}} = 0$ (for disturbance transfer) the path from y back to the d_2-summer through the controller forms a feedback around the plant. The equivalent forward block from the d_2-summer to y is G_1G_2 with a feedback path of gain 10.

Step 2: Closed-loop due to the controller feedback

Writing the closed-loop relation at the d_2-injection node (call its signal u):

$$y \;=\; G_1G_2\big(d_2 - 10\,y\big) \quad\Rightarrow\quad y\big(1 + 10G_1G_2\big) = G_1G_2\,d_2.$$

Hence

$$\frac{y}{d_2} = \frac{G_1G_2}{1+10G_1G_2} = \frac{1}{(0.1s+1)(s+1)+10}.$$

Step 3: Rearrangement to the factored form

$$(0.1s+1)(s+1)+10 = 0.1s^2 + 1.1s + 11 = (0.1s+1)(11s+21).$$

Therefore

$$G_{d2}(s) = \frac{1}{(11s+21)(0.1s+1)}.$$

This matches option (A).

Quick Tip

- For disturbance transfer functions, set the set-point input to zero and keep only the disturbance path.
- If the disturbance enters inside a feedback loop, include the loop gain in the denominator $1 + LG$.
- After obtaining a polynomial denominator, factor it to compare with given options.

---

45. Level (h) in a steam boiler is controlled by manipulating the flow rate (F) of the make-up water using a proportional (P) controller. The transfer function is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}.$$

The measurement and valve transfer functions are both 1. A process engineer wants the closed-loop response to give decaying oscillations under servo mode. Which one of the following is the CORRECT value of the controller gain?

(A) 0.25
(B) 2

(C) 4
(D) 6

Correct Answer: (B) 2

Solution:
Step 1: Open-loop transfer function with proportional controller gain K_c:

$$G(s) = K_c \cdot \frac{0.25(1-s)}{s(2s+1)}.$$

Step 2: Closed-loop characteristic equation:

$$1 + G(s) = 0 \quad \Rightarrow \quad 1 + \frac{0.25K_c(1-s)}{s(2s+1)} = 0.$$

Multiply through:

$$s(2s+1) + 0.25K_c(1-s) = 0.$$

Step 3: Simplify:

$$2s^2 + s + 0.25K_c - 0.25K_c s = 0,$$
$$2s^2 + \big(1 - 0.25K_c\big)s + 0.25K_c = 0.$$

Step 4: For decaying oscillations, system must be underdamped ⇒ complex roots with negative real part:

$$\text{Discriminant} < 0, \quad b^2 - 4ac < 0.$$

Here $a = 2,\ b = (1 - 0.25K_c),\ c = 0.25K_c$.

$$\Delta = (1 - 0.25K_c)^2 - 4(2)(0.25K_c).$$
$$= (1 - 0.25K_c)^2 - 2K_c.$$

Step 5: Condition $\Delta < 0$. Test given options:
- $K_c = 0.25 :\ \Delta = (1 - 0.0625)^2 - 0.5 = 0.8789 - 0.5 > 0 \Rightarrow$ no oscillations.
- $K_c = 2 :\ \Delta = (1 - 0.5)^2 - 4 = 0.25 - 4 = -3.75 < 0 \Rightarrow$ decaying oscillations.
- $K_c = 4 :\ \Delta = (1 - 1)^2 - 8 = -8 < 0 \Rightarrow$ oscillations but may be unstable real part check: coefficient $b = (1 - 1) = 0$, real part=0, sustained oscillations (not decaying).
- $K_c = 6 :\ \Delta = (1 - 1.5)^2 - 12 = 0.25 - 12 = -11.75 < 0$, coefficient $b = (1 - 1.5) = -0.5$, negative real part? $a = 2 > 0$, so roots with positive real parts ⇒ unstable.
Thus correct is $K_c = 2$.

$$\boxed{2}$$

Quick Tip

- For stability, check sign of coefficients and discriminant.
- Decaying oscillations require underdamped roots (complex with negative real part).
- Servo mode means reference tracking response.

46. Which of the following statements is/are CORRECT?

(A) Bond number includes surface tension.
(B) Jakob number includes latent heat.
(C) Prandtl number includes liquid-vapor density difference.
(D) Biot number includes gravity.

Correct Answer: (A), (B)

Solution:
Step 1: Bond number. $\mathrm{Bo} = \frac{\Delta\rho g L^2}{\sigma}$. Includes surface tension σ. So (A) is correct.
Step 2: Jakob number. $\mathrm{Ja} = \frac{c_p(T - T_{sat})}{h_{fg}}$. Denominator is latent heat. So (B) is correct.
Step 3: Prandtl number. $\mathrm{Pr} = \frac{c_p\mu}{k}$. Contains specific heat, viscosity, and thermal conductivity, not density difference. So (C) is false.
Step 4: Biot number. $\mathrm{Bi} = \frac{hL}{k}$. Involves heat transfer coefficient, length scale, and thermal conductivity, not gravity. So (D) is false.

$$\boxed{\text{Correct: (A), (B)}}$$

Quick Tip

- Always recall definitions of dimensionless numbers.
- Bond number compares gravity to surface tension.
- Jakob number involves latent heat, important in boiling/condensation.
- Prandtl relates momentum vs thermal diffusivity.
- Biot compares internal vs external resistance in heat transfer.

Q.47. If a matrix M is defined as $M = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$, the sum of all the eigenvalues of M^3 is equal to ________ (in integer).

Correct Answer: 4160

Solution:
Step 1: Find eigenvalues of M
For a 2×2 symmetric matrix $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$, eigenvalues are $a + b$ and $a - b$. Here $a = 10,\ b = 6$. So

$$\lambda_1 = 10 + 6 = 16, \quad \lambda_2 = 10 - 6 = 4.$$

Step 2: Eigenvalues of M^3
If λ is an eigenvalue of M, then λ^3 is an eigenvalue of M^3. Thus eigenvalues of M^3 are:

$$16^3 = 4096, \quad 4^3 = 64.$$

Step 3: Sum of eigenvalues

$$4096 + 64 = 4160.$$

So the sum of all eigenvalues of M^3 is $\boxed{4160}$.

Quick Tip

- Sum of eigenvalues of a matrix equals its trace.
- For powers of matrices, eigenvalues are raised to that power.

Q.48. The first derivative of the function

$$U(r) = 4\left[\left(\frac{1}{r}\right)^{12} - \left(\frac{1}{r}\right)^{6}\right]$$

evaluated at $r = 1$ is _______ (in integer).

Correct Answer: -24

Solution:
Step 1: Simplify function

$$U(r) = 4\left(r^{-12} - r^{-6}\right).$$

Step 2: Differentiate w.r.t r

$$U'(r) = 4\left(-12r^{-13} + 6r^{-7}\right) = -48r^{-13} + 24r^{-7}.$$

Step 3: Evaluate at $r = 1$

$$U'(1) = -48(1)^{-13} + 24(1)^{-7} = -48 + 24 = -24.$$

Thus the derivative at $r = 1$ is $\boxed{-24}$.

Quick Tip

- Always convert reciprocal powers into r^{-n} before differentiation.
- For $U(r) = Ar^{-n}$, derivative is $-nAr^{-n-1}$.

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49. Wet air containing 10 mole percent water vapor is dried by continuously passing it through a column of $CaCl_2$ pellets. The pellets remove 50 percent of water from wet air entering the column. The mole percent of water vapor in the product stream exiting the column is __________ (rounded off to two decimal places).

Correct Answer: 5.26 %

Solution: Step 1: Assume a basis of 100 mol wet air entering. Water = 10 mol; dry air = 90 mol.
Step 2: The pellets remove 50% of inlet water: removed water $= 0.5 \times 10 = 5$ mol.
Exit water $= 10 - 5 = 5$ mol; dry air unchanged = 90 mol.
Step 3: Total exit moles $= 90 + 5 = 95$ mol. Mole percent water at exit:

$$\text{mol } \%\ H_2O = \frac{5}{95} \times 100 = 5.263\% \approx \boxed{5.26\ \%}.$$

Quick Tip

- In drying/absorption problems, dry-gas moles remain unchanged if only the condensable component is removed.
- Use a convenient 100 mol basis to convert between mole fraction and mole percent quickly.

---

50. Orsat analysis of a stack gas on a dry basis is: $N_2 = 65\%$, $CO_2 = 15\%$, $CO = 10\%$, $O_2 = 10\%$. **The measured mole fraction of** H_2O **in the stack gas (wet basis) is** 0.07. **The mole fraction of** N_2 **on a wet basis is ______ (rounded off to two decimal places).**

Correct Answer: 0.60

Solution: Step 1: On a wet basis, the water mole fraction is $y_{H_2O} = 0.07$.
Hence the dry-gas fraction in the wet mixture is $1 - y_{H_2O} = 0.93$.
Step 2: The dry-basis mole fraction of N_2 is $x_{N_2,\text{dry}} = 0.65$.
Therefore the wet-basis mole fraction is

$$y_{N_2} = x_{N_2,\text{dry}}\,(1 - y_{H_2O}) = 0.65 \times 0.93 = 0.6045 \approx \boxed{0.60}.$$

Quick Tip

- Convert from dry to wet basis by multiplying dry-basis fractions by $(1 - y_{H_2O})$.
- Orsat analysis is always reported on a dry basis (water removed before analysis).

Q.51. A pump draws water (density = $1000\ \mathrm{kg\,m^{-3}}$) at a steady rate of $10\ \mathrm{kg\,s^{-1}}$. The pressures at the suction and discharge sides of the pump are -20 kPa (gauge) and 350 kPa (gauge), respectively. The pipe diameters at the suction and discharge side are 70 mm and 50 mm, respectively. The suction and discharge lines are at the same elevation, and the pump operates at an efficiency of 80%. Neglecting frictional losses in the system, the power (in kW) required to drive the pump is _____ (rounded off to two decimal places).

Correct Answer: 4.74

Solution:

Step 1: Convert mass flow to volumetric flow and compute velocities

$$\dot{m} = 10\ \mathrm{kg\,s^{-1}}, \quad \rho = 1000\ \mathrm{kg\,m^{-3}} \Rightarrow Q = \frac{\dot{m}}{\rho} = 0.01\ \mathrm{m^3\,s^{-1}}.$$

Areas:

$$A_s = \frac{\pi(0.07)^2}{4} = 3.848 \times 10^{-3}\ \mathrm{m}^2, \quad A_d = \frac{\pi(0.05)^2}{4} = 1.963 \times 10^{-3}\ \mathrm{m}^2.$$

Velocities:

$$V_s = \frac{Q}{A_s} = 2.598\ \mathrm{m\,s^{-1}}, \quad V_d = \frac{Q}{A_d} = 5.093\ \mathrm{m\,s^{-1}}.$$

Step 2: Specific energy added to the fluid (Bernoulli with pump)

Elevation change $= 0$. Pressure rise:

$$\Delta p = 350 - (-20) = 370\ \mathrm{kPa} = 3.70 \times 10^5\ \mathrm{Pa}.$$

Specific energy added:

$$e = \frac{\Delta p}{\rho} + \frac{V_d^2 - V_s^2}{2} = \frac{3.70 \times 10^5}{1000} + \frac{5.093^2 - 2.598^2}{2} = 379.59\ \mathrm{J\,kg^{-1}}.$$

Step 3: Hydraulic power and shaft power

$$P_{\text{hyd}} = \dot{m}\,e = 10 \times 379.59 = 3795.93\ \mathrm{W}.$$

With efficiency $\eta = 0.80$,

$$P_{\text{shaft}} = \frac{P_{\text{hyd}}}{\eta} = \frac{3795.93}{0.80} = 4744.91\ \mathrm{W} = 4.74\ \mathrm{kW}\ \text{(to two decimals)}.$$

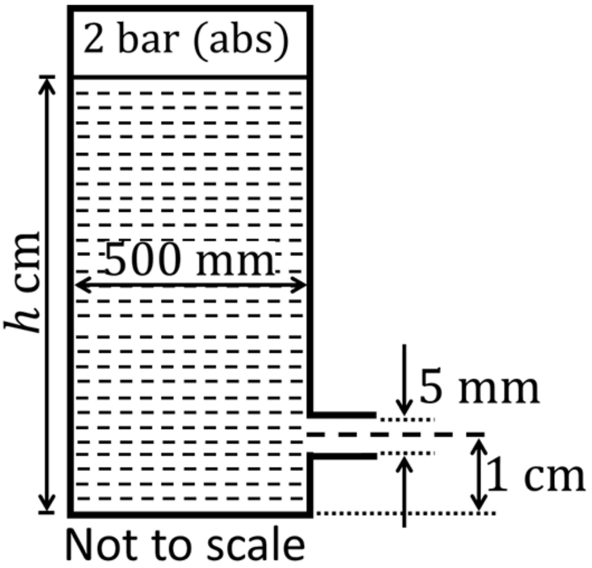
Quick Tip

- For pump power with negligible losses and equal elevation:

$$P_{\text{shaft}} = \frac{\dot{m}}{\eta}\left(\frac{\Delta p}{\rho} + \frac{V_d^2 - V_s^2}{2}\right).$$

- Use gauge pressures directly for Δp when both are gauge values.

52. A cylindrical tank with a diameter of 500 mm contains water (density = 1 g $\cdot$ cm^{-3}) up to a height h. A 5 mm diameter round nozzle, whose center is 1 cm above the base of the tank, has its exit open to the atmosphere. The pressure above the water level in the tank is maintained at 2 bar (abs). Neglect all frictional and entry/exit losses. Take $g = 10$ m s^{-2} and atmospheric pressure = 1 bar. The absolute value of initial $\frac{dh}{dt}$ (in mm s^{-1}) when $h = 51$ cm is ______ (rounded off to two decimal places).



Correct Answer: 1.45 mm s^{-1}

Solution:
Step 1: Head available at the nozzle. Apply Bernoulli between the liquid surface (pressure = 2 bar, velocity $\approx$ 0, elevation $z = h$) and the nozzle exit (pressure = 1 bar, elevation $z = 0.01$ m). Pressure head difference:

$$\frac{\Delta p}{\rho g} = \frac{(2-1)\times 10^5}{1000 \times 10} = 10 \text{ m}.$$

Static head difference:

$$h - z = 0.51 - 0.01 = 0.50 \text{ m}.$$

Hence total head = 10 + 0.50 = 10.5 m.

Step 2: Exit velocity and volumetric flow rate.

$$v = \sqrt{2g(10.5)} = \sqrt{2 \times 10 \times 10.5} = \sqrt{210} = 14.49 \text{ m s}^{-1}.$$

Nozzle area $A_n = \frac{\pi d^2}{4} = \frac{\pi (0.005)^2}{4} = 1.963 \times 10^{-5}$ m^2.

$$Q = A_n v = (1.963 \times 10^{-5})(14.49) = 2.84 \times 10^{-4} \text{ m}^3/\text{s}.$$

Step 3: Tank area and rate of level drop. Tank cross-sectional area $A_t = \pi(0.25)^2 = 0.19635\ \mathrm{m}^2$.

$$\frac{dh}{dt} = -\frac{Q}{A_t} = -\frac{2.84 \times 10^{-4}}{0.19635} = -1.447 \times 10^{-3}\ \mathrm{m\ s^{-1}} = -1.447\ \mathrm{mm\ s^{-1}}.$$

Absolute value (initial):

$$\boxed{1.45\ \mathrm{mm\ s^{-1}}}.$$

Quick Tip

- With pressurized tanks, add the pressure head $\Delta p/(\rho g)$ to the static head difference between liquid level and exit.
- For level change: $\dfrac{dh}{dt} = -\dfrac{Q}{A_{\mathrm{tank}}}$.

Q.53. A large tank is filled with water (density $= 1\ \mathrm{g\,cm^{-3}}$) up to a height of 5 m. A $100\ \mu$m diameter solid spherical particle (density $= 0.8\ \mathrm{g\,cm^{-3}}$) is released at the bottom of the tank. The particle attains its terminal velocity (v_t) after traveling to a certain height in the tank. Use acceleration due to gravity as $10\ \mathrm{m\,s^{-2}}$ and water viscosity as $10^{-3}\ \mathrm{Pa\,s}$. Neglect wall effects on the particle. If Stokes' law is applicable, the absolute value of v_t (in $\mathrm{mm\,s^{-1}}$) is ________ (rounded off to two decimal places).

Correct Answer: 1.11

Solution: Step 1: Apply Stokes' terminal velocity for a sphere
For creeping flow,

$$v_t = \frac{(\rho_p - \rho_f) g d^2}{18\mu}.$$

Given $\rho_p = 0.8\ \mathrm{g\,cm^{-3}} = 800\ \mathrm{kg\,m^{-3}}$, $\rho_f = 1000\ \mathrm{kg\,m^{-3}}$, $g = 10\ \mathrm{m\,s^{-2}}$, $d = 100\ \mu\mathrm{m} = 1 \times 10^{-4}$ m, $\mu = 10^{-3}\ \mathrm{Pa\,s}$.

Step 2: Compute magnitude

$$|v_t| = \frac{|800 - 1000| \times 10 \times (1 \times 10^{-4})^2}{18 \times 10^{-3}} = \frac{200 \times 10 \times 10^{-8}}{18 \times 10^{-3}} = \frac{2 \times 10^{-5}}{1.8 \times 10^{-2}} = 1.111 \cdots \times 10^{-3}\ \mathrm{m\,s^{-1}}.$$

Step 3: Convert to $\mathrm{mm\,s^{-1}}$ and round

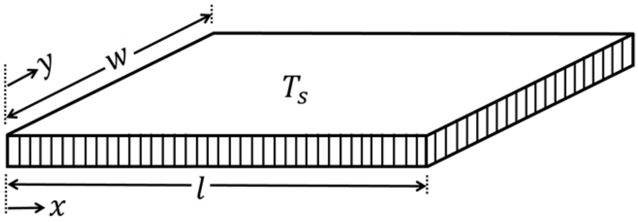
$$|v_t| = 1.111\ldots\ \mathrm{mm\,s^{-1}} \approx \boxed{1.11\ \mathrm{mm\,s^{-1}}}.$$

Quick Tip

- Stokes' law holds for very small Reynolds number $Re = \rho_f v_t d/\mu \ll 1$.
- Use absolute density difference $|\rho_p - \rho_f|$; sign only indicates direction (here, upward since $\rho_p < \rho_f$).

---

54. A fluid is flowing steadily under laminar conditions over a thin rectangular plate at temperature T_s as shown. The free-stream velocity and temperature are u_∞ and T_∞, respectively. When the fluid flow is only in the x-direction, h_x is the local heat transfer coefficient. Similarly, when the fluid flow is only in the y-direction, h_y is the corresponding local heat transfer coefficient. Use the correlation $\mathrm{Nu} = 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$ for the local heat transfer coefficient, where Nu, Re and Pr are the appropriate Nusselt, Reynolds and Prandtl numbers. The average heat transfer coefficients are defined as $\overline{h}_l = \dfrac{1}{l}\displaystyle\int_0^l h_x\,dx$ and $\overline{h}_w = \dfrac{1}{w}\displaystyle\int_0^w h_y\,dy$. If $w = 1$ m and $l = 4$ m, the value of the ratio $\overline{h}_w/\overline{h}_l$ is ______ (in integer).



Correct Answer: 2

Solution: Step 1: For laminar flow over an isothermal flat plate, the local correlation is

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} = 0.332\left(\frac{u_\infty x}{\nu}\right)^{1/2}\mathrm{Pr}^{1/3}.$$

Thus

$$h_x = \frac{k\,\mathrm{Nu}_x}{x} = 0.332\,k\,\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu}\right)^{1/2} x^{-1/2} \equiv C\,x^{-1/2},$$

where $C = 0.332\,k\,\mathrm{Pr}^{1/3}\left(\dfrac{u_\infty}{\nu}\right)^{1/2}$ (independent of x). Similarly for y-direction flow: $h_y = C\,y^{-1/2}$.

Step 2: Compute the area-averaged coefficients.

$$\overline{h}_l = \frac{1}{l}\int_0^l Cx^{-1/2}dx = \frac{1}{l}\left[2Cx^{1/2}\right]_0^l = \frac{2C}{\sqrt{l}}, \quad \overline{h}_w = \frac{1}{w}\int_0^w Cy^{-1/2}dy = \frac{2C}{\sqrt{w}}.$$

Step 3: Take the ratio:

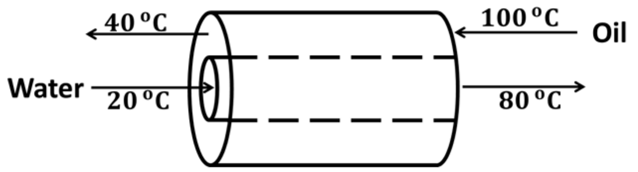
$$\frac{\overline{h}_w}{\overline{h}_l} = \frac{2C/\sqrt{w}}{2C/\sqrt{l}} = \sqrt{\frac{l}{w}} = \sqrt{\frac{4}{1}} = 2.$$

$$\boxed{2}$$

Quick Tip

- With $\mathrm{Nu}_x \propto \mathrm{Re}_x^{1/2}$, the local h varies as $x^{-1/2}$ for laminar boundary layers over flat plates.
- Averages over a length scale L then scale as $1/\sqrt{L}$; ratios reduce to square roots of the respective lengths.

Q.55. A perfectly insulated, concentric tube countercurrent heat exchanger is used to cool lubricating oil using water as a coolant. Oil enters the outer annulus at a mass flow rate of $2\ \mathrm{kg\,s^{-1}}$ with a temperature of $100°\mathrm{C}$ and leaves at $40°\mathrm{C}$. Water enters the inner tube at $1\ \mathrm{kg\,s^{-1}}$ with a temperature of $20°\mathrm{C}$ and leaves at $80°\mathrm{C}$. Specific heats: $c_{p,\mathbf{oil}} = 2089\ \mathrm{J\,kg^{-1}K^{-1}}$, $c_{p,\mathbf{water}} = 4178\ \mathrm{J\,kg^{-1}K^{-1}}$. No phase change. Under steady state, the number of transfer units (NTU) is ____ (in integer).



Correct Answer: 3

Solution:

Step 1: Capacity rates and heat duty

$$C_h = \dot{m}_h c_{p,h} = 2 \times 2089 = 4178\ \mathrm{W\,K^{-1}}, \qquad C_c = \dot{m}_c c_{p,c} = 1 \times 4178 = 4178\ \mathrm{W\,K^{-1}}.$$

Thus $C_{\min} = C_{\max} = 4178 \Rightarrow C_r = C_{\min}/C_{\max} = 1$. Actual heat transfer:

$$Q = C_h\,(T_{h,in} - T_{h,out}) = 4178(100-40) = 4178 \times 60.$$

Step 2: Effectiveness

$$Q_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = 4178(100-20) = 4178 \times 80,$$

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{4178 \times 60}{4178 \times 80} = \frac{60}{80} = 0.75.$$

Step 3: NTU from countercurrent relation (for $C_r = 1$)

For a countercurrent exchanger with $C_r = 1$,

$$\varepsilon = \frac{\mathrm{NTU}}{1+\mathrm{NTU}}.$$

Hence

$$0.75 = \frac{\mathrm{NTU}}{1+\mathrm{NTU}} \quad \Rightarrow \quad \mathrm{NTU} = 3.$$

Quick Tip

- When $C_h = C_c$ (i.e., $C_r = 1$), the countercurrent effectiveness simplifies to $\varepsilon = \text{NTU}/(1 + \text{NTU})$.
- Always compute $\varepsilon = Q/Q_{\max}$ first; it avoids using the overall heat transfer coefficient explicitly.

---

56. Partially saturated air at 1 bar and $50°$C is contacted with water in an adiabatic saturator. The air is cooled and humidified to saturation, and exits at $25°$C with an absolute humidity of 0.02 kg water per kg dry air. Use latent heat of vaporization of water as 2450 kJ kg^{-1}, and average specific heats $c_{p,a} = 1.01$ kJ kg^{-1}K^{-1} (dry air) and $c_{p,w} = 4.18$ kJ kg^{-1}K^{-1} (liquid water). If the absolute humidity of air entering the adiabatic saturator is $\mathcal{H} \times 10^{-3}$ kg water per kg dry air, the value of $\mathcal{H}$ is ______ (rounded off to two decimal places).

Correct Answer: 9.69

Solution: Step 1: Perform an energy balance for the adiabatic saturator on a basis of 1 kg dry air. With water introduced at the exit temperature T_2, the adiabatic-saturation relation reduces to

$$c_{p,a}(T_1 - T_2) = (w_2 - w_1)\, h_{fg}(T_2),$$

where w_1 and w_2 are inlet and saturated exit humidities (kg/kg dry air).

Step 2: Substitute data: $T_1 = 50°\text{C}$, $T_2 = 25°\text{C}$, $w_2 = 0.02$, $c_{p,a} = 1.01$ kJ kg^{-1}K^{-1}, $h_{fg} = 2450$ kJ kg^{-1}.

$$w_1 = w_2 - \frac{c_{p,a}(T_1 - T_2)}{h_{fg}} = 0.02 - \frac{1.01 \times 25}{2450} = 0.02 - 0.01031 = 0.00969 \text{ kg/kg dry air}.$$

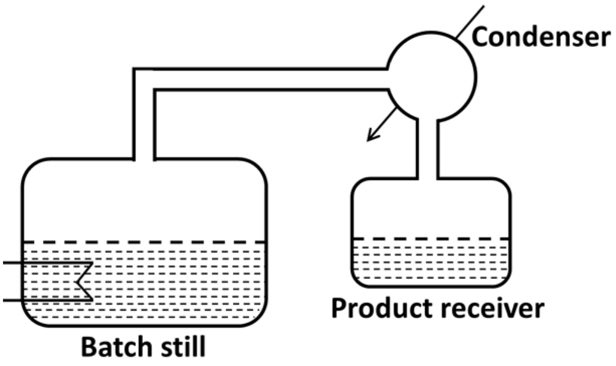
Step 3: Express as $\mathcal{H} \times 10^{-3}$: $w_1 = \mathcal{H} \times 10^{-3} \Rightarrow \mathcal{H} = 9.69$.

$$\boxed{\mathcal{H} = 9.69}.$$

Quick Tip

- For an adiabatic saturator with water at exit temperature, $c_{p,w}$ does not appear and the key relation is $c_{p,a}(T_1 - T_2) = (w_2 - w_1)h_{fg}$.
- Keep units in kJ/kg consistently when using h_{fg} and $c_{p,a}$.

Q.57. Distillation of a non-reactive binary mixture with components A and B is carried out in a batch still as shown. The initial charge in the still is 1 kmol. The initial and final amounts of A in the still are 0.1 kmol and 0.01 kmol, respectively. The relative volatility is constant at $\alpha = 4.5$. The mole fraction of B remaining in



the vessel is _____ (rounded off to three decimal places).

Correct Answer: 0.982

Solution: Step 1: Define compositions and data
Initial total moles $W_0 = 1$ kmol. Initial moles of A: $n_{A0} = 0.1 \Rightarrow x_0 = \frac{n_{A0}}{W_0} = 0.1$. Final moles of A: $n_{Af} = 0.01$. Let the final total be W_f and liquid mole fraction x_f. Then

$$x_f = \frac{n_{Af}}{W_f} = \frac{0.01}{W_f}.$$

Step 2: Rayleigh equation for differential (batch) distillation
With constant relative volatility α, the equilibrium vapor composition of A is

$$y^* = \frac{\alpha x}{1 + (\alpha - 1)x}.$$

Rayleigh equation:

$$\ln\left(\frac{W_f}{W_0}\right) = \int_{x_0}^{x_f} \frac{dx}{y^* - x} = \int_{x_0}^{x_f} \frac{dx}{\frac{\alpha x}{1 + (\alpha - 1)x} - x}.$$

Step 3: Solve for W_f and x_f
Using $\alpha = 4.5$, $x_0 = 0.1$ and the relation $x_f = 0.01/W_f$, numerical evaluation of the integral and root-finding gives

$$W_f \approx 0.5495 \text{ kmol}, \qquad x_f = \frac{0.01}{0.5495} \approx 0.01820.$$

Step 4: Mole fraction of B remaining

$$x_{B,f} = 1 - x_f \approx 1 - 0.01820 = 0.98180 \approx \boxed{0.982}.$$

Quick Tip

- For batch distillation with constant α, use the Rayleigh equation with $y^* = \frac{\alpha x}{1 + (\alpha - 1)x}$.
- Use a component balance ($x_f W_f = n_{Af}$) to couple the Rayleigh equation to the final composition.

---

58. Fresh catalyst is loaded into a reactor for the reaction $A \to$ products. The catalyst deactivates with time. The instantaneous activity $a(t)$ is defined as the ratio of the rate $-r'_A(t)$ (with deactivated catalyst) to the rate with fresh catalyst. Experimental correlation:

$$-r'_A(t) = -0.5\,t + 10 \quad (\textbf{mol}\,(\textbf{g cat})^{-1}\,\textbf{hr}^{-1}),$$

with t in hours. The activity of the catalyst at $t = 10$ hr is _____ (rounded off to one decimal place).

Correct Answer: 0.5

Solution: Step 1: Activity is the rate relative to the fresh rate: $a(t) = \dfrac{-r'_A(t)}{-r'_A(0)}$.

Step 2: Evaluate the rates from the correlation:

$$-r'_A(0) = 10, \qquad -r'_A(10) = -0.5(10) + 10 = 5.$$

Step 3: Compute activity:

$$a(10) = \frac{5}{10} = 0.5 \;\Rightarrow\; \boxed{0.5}.$$

Quick Tip

- Catalyst activity is commonly normalized to the initial (fresh) rate: $a(t) = r(t)/r(0)$.
- With linear deactivation, read rates directly from the given line and take the ratio.

---

59. A unimolecular, irreversible liquid-phase reaction $A \to P$ is carried out in an ideal batch reactor at temperature T. The rate of reaction $(-r_A)$ (in mol m^{-3} s^{-1}) measured at different conversions X_A is given:

X_A	0	0.1	0.2	0.4	0.6	0.8
$-r_A$	0.45	0.35	0.31	0.18	0.11	0.05

The reaction is also carried out in an ideal CSTR at the same T with a feed concentration $C_{A0} = 1$ mol m^{-3} under steady state. For a conversion of 0.8, the space time (in s) of the CSTR is ______ (in integer).

Correct Answer: 16

Solution: Step 1: For a steady-state CSTR,

$$\tau = \frac{C_{A0}X_A}{-r_A\big|_{\text{exit}}}.$$

Step 2: At the desired conversion $X_A = 0.8$, the exit rate from the table is $(-r_A) = 0.05\ \mathrm{mol\,m^{-3}\,s^{-1}}$.
Step 3: Substitute $C_{A0} = 1\ \mathrm{mol\,m^{-3}}$:

$$\tau = \frac{1 \times 0.8}{0.05} = 16\ \mathrm{s}.$$

$$\boxed{16}$$

Quick Tip

- For CSTR design at a specified conversion, use the exit rate at that conversion: $\tau = C_{A0}X_A/(-r_A)_{\mathrm{exit}}$.
- The batch data table directly provides $(-r_A)$ vs. X_A, so interpolation at the target X_A (if needed) gives the required exit rate.

Q.60. An irreversible liquid-phase second-order reaction $A \xrightarrow{k} B$ with rate constant $k = 0.2\ \mathrm{L\,mol^{-1}\,min^{-1}}$ is carried out isothermally in a non-ideal reactor. A tracer test gives the residence time distribution $E(t)$ shown: three rectangles (i), (ii), (iii) of equal areas; rectangles (i) and (ii) span $t = 5$–10 min (stacked), and (iii) spans $t = 10$–15 min. Pure A of concentration $C_{A0} = 1.5\ \mathrm{mol\,L^{-1}}$ is fed. Using the segregated model, determine the percentage conversion of A at the exit (rounded to the nearest integer).

Correct Answer: 72%

Solution: Step 1: Determine $E(t)$
Areas of (i), (ii), (iii) are equal. Let the common height be a for rectangles (i) and (iii), and the lower rectangle (ii) also has height a. Total area of $E(t)$ must be 1:

$$\int_0^\infty E(t)\,dt = 5(a+a) + 5(a) = 15a = 1 \Rightarrow a = \frac{1}{15}\ \mathrm{min}^{-1}.$$

Hence

$$E(t) = \begin{cases} \dfrac{2}{15}, & 5 \le t \le 10, \\ \dfrac{1}{15}, & 10 < t \le 15, \\ 0, & \text{otherwise.} \end{cases}$$

Step 2: Concentration for a fluid element (segregated model)
For a 2nd-order batch element:

$$\frac{dC_A}{dt} = -kC_A^2 \;\Rightarrow\; C_A(t) = \frac{C_{A0}}{1 + kC_{A0}t}.$$

Here $kC_{A0} = 0.2 \times 1.5 = 0.3\ \mathrm{min}^{-1}$.

Step 3: Mixed effluent concentration

$$C_{A,\text{out}} = \int_0^\infty C_A(t)\, E(t)\, dt = C_{A0}\left[\frac{2}{15}\int_5^{10} \frac{dt}{1+0.3t} + \frac{1}{15}\int_{10}^{15} \frac{dt}{1+0.3t}\right].$$

Using $\int \frac{dt}{1+0.3t} = \frac{1}{0.3}\ln(1+0.3t)$,

$$\frac{C_{A,\text{out}}}{C_{A0}} = \frac{2}{15}\cdot\frac{1}{0.3}\ln\frac{4}{2.5} + \frac{1}{15}\cdot\frac{1}{0.3}\ln\frac{5.5}{4} = \frac{4}{9}\ln(1.6) + \frac{2}{9}\ln(1.375) \approx 0.2797.$$

Thus $C_{A,\text{out}} \approx 1.5 \times 0.2797 = 0.4195\ \mathrm{mol\,L^{-1}}$.

Step 4: Overall conversion

$$X = 1 - \frac{C_{A,\text{out}}}{C_{A0}} \approx 1 - 0.2797 = 0.720\ \Rightarrow\ \boxed{72\%}.$$

Quick Tip

- In the segregated model, compute $C_{\text{out}} = \int C(t)E(t)\,dt$; then $X = 1 - C_{\text{out}}/C_0$.
- For an n-th order batch element, use the batch solution $C(t)$ with residence time t.

---

61. The outlet concentration C_A of a plug flow reactor (PFR) is controlled by manipulating the inlet concentration C_{A0}. The transfer function is

$$\frac{C_A(s)}{C_{A0}(s)} = \exp\left[-\left(\frac{V}{F}\right)(k+s)\right],$$

with $V = 1\ \mathrm{m^3}$, $F = 0.1\ \mathrm{m^3 min^{-1}}$, $k = 0.5\ \mathrm{min^{-1}}$. Measurement and valve transfer functions are 1. Find the *ultimate gain* (proportional controller gain for sustained oscillations).

Correct Answer: 148.4

Solution: Step 1: Rewrite the process as a pure dead-time with static gain:

$$\frac{C_A}{C_{A0}} = K_p\, e^{-\theta s}, \quad K_p = e^{-(V/F)k} = e^{-5}, \qquad \theta = \frac{V}{F} = 10\ \text{min}.$$

Step 2: For proportional control of a dead-time process, the marginal (sustained) oscillation condition from $1 + K_c K_p e^{-\theta s} = 0$ at $s = j\omega$ gives

$$|K_c K_p| = 1\ \Rightarrow\ K_u = \frac{1}{|K_p|} = e^5 = 148.413\ldots$$

Rounded to one decimal place: $\boxed{148.4}$.

Quick Tip

For a pure delay process, the ultimate gain is simply $K_u = 1/|K_p|$; the delay sets the oscillation frequency $\omega = (2n+1)\pi/\theta$ but not K_u.

---

62. A measuring instrument has transfer function

$$G_m(s) = \frac{1.05}{2s+1}\, e^{-s}.$$

At $t = 0$, a step of $+1$ unit is applied. Find the time taken for the output to increase by 1 unit.

Correct Answer: 7.09

Solution: Step 1: The step response of a FOPDT model $K\,\dfrac{1}{\tau s+1}e^{-Ls}$ is

$$y(t) = K\left[1 - e^{-(t-L)/\tau}\right] u(t-L).$$

Here $K = 1.05,\ \tau = 2,\ L = 1.$
Step 2: Set $y(t) = 1$ and solve for $t \geq L$:

$$1 = 1.05\left(1 - e^{-(t-1)/2}\right) \Rightarrow e^{-(t-1)/2} = 1 - \frac{1}{1.05} = \frac{1}{21}.$$

$$\frac{t-1}{2} = \ln 21 \Rightarrow t = 1 + 2\ln 21 = 1 + 2(3.044522) = 7.089 \text{ time units.}$$

Rounded to two decimals: $\boxed{7.09}$.

Quick Tip

For a FOPDT device, the output reaches a value y^* at time $t = L - \tau\ln(1 - y^*/K)$ (for a unit input), remembering to include the pure time delay L.

---

63. A design engineer needs to purchase a membrane module (M) for a plant. Two options are available: M1 and M2. The overall plant life is 7 years. Interest rate is 8% per annum, compounded annually. The table gives purchase cost (lakhs of rupees) and expected life (years):

	M1	**M2**
Purchase cost (lakhs)	10	5
Expected life (years)	5	3

Find the difference in the net present value (NPV) of these two options over the 7-year horizon (lakhs of rupees).

Correct Answer: 4.7

Solution: Step 1: Over 7 years, replacements are needed to keep the module available. For M1 (life 5 y): buy at $t = 0$ and again at $t = 5$ y. For M2 (life 3 y): buy at $t = 0, 3, 6$ y. Salvage is ignored.

Step 2: Present values with $i = 0.08$.

$$\text{NPV}_{\text{M1}} = 10 + \frac{10}{(1.08)^5} = 10 + \frac{10}{1.4693} = 16.81.$$

$$\text{NPV}_{\text{M2}} = 5 + \frac{5}{(1.08)^3} + \frac{5}{(1.08)^6} = 5 + \frac{5}{1.2597} + \frac{5}{1.5869} = 12.12.$$

Step 3: Difference:

$$\Delta\text{NPV} = \text{NPV}_{\text{M1}} - \text{NPV}_{\text{M2}} = 16.81 - 12.12 = 4.69 \approx \boxed{4.7}.$$

Quick Tip

- For a finite project horizon, count each replacement at its purchase time and discount to present using $(1+i)^{-t}$.
- If salvage is not specified, ignore it; otherwise include discounted salvage at the horizon.

---

64. A distillation column has a purchase cost of Rs. 10 lakhs with an installation factor 5.8. The installed capital is annualized over 6 years at 5% per annum, compounded annually. Find the annual cost (in lakhs of rupees) of the installed capital.

Correct Answer: 11.4

Solution: Step 1: Installed capital:

$$C_0 = 10 \times 5.8 = 58 \text{ lakhs}.$$

Step 2: Use capital recovery factor (CRF):

$$\text{CRF} = \frac{i(1+i)^n}{(1+i)^n - 1}, \quad i = 0.05,\ n = 6.$$

$$(1+i)^n = 1.05^6 = 1.3401,\ \text{CRF} = \frac{0.05 \times 1.3401}{1.3401 - 1} = 0.1971.$$

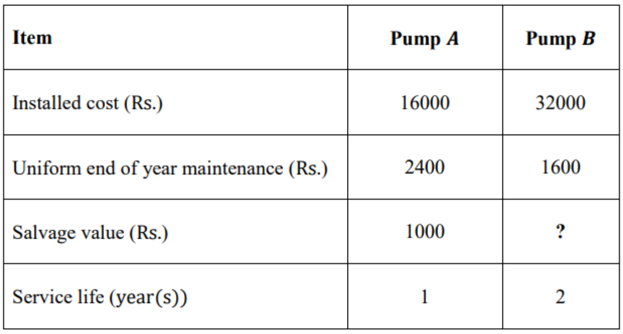
Step 3: Annual cost:

$$A = C_0 \times \text{CRF} = 58 \times 0.1971 = 11.42 \text{ lakhs} \approx \boxed{11.4}.$$

Quick Tip

- Annualizing a capital cost uses $A = C_0 \times \text{CRF}$ with $\text{CRF} = \dfrac{i(1+i)^n}{(1+i)^n - 1}$.
- Installation factor converts purchase price to total installed cost before annualization.

Q.65. Pumps A and B are being considered for purchase in a chemical plant. Cost details are: Pump A — installed cost 16000, uniform end-of-year maintenance 2400, salvage value 1000, service life 1 year; Pump B — installed cost 32000, uniform end-of-year maintenance 1600, salvage value S (unknown), service life 2 years. The interest rate is 10% per annum, compounded annually. For both pumps to have the same capitalized cost, find the salvage value S of pump B (in Rs., rounded to the nearest integer).



Item	**Pump *A***	**Pump *B***
Installed cost (Rs.)	16000	32000
Uniform end of year maintenance (Rs.)	2400	1600
Salvage value (Rs.)	1000	**?**
Service life (year(s))	1	2

Correct Answer: 2180

Solution: Step 1: Capitalized cost (perpetual replacement)
For a unit with life n, installed cost P, uniform end-of-year maintenance A, salvage F at year n, and interest i, the present worth of one cycle is

$$PW_{\text{cycle}} = P + A\sum_{k=1}^{n}\frac{1}{(1+i)^k} - \frac{F}{(1+i)^n}.$$

Capitalized cost (perpetual repetitions every n years) is

$$K = \frac{PW_{\text{cycle}}}{1-(1+i)^{-n}}.$$

Step 2: Pump A ($n = 1$)

$$PW_A = 16000 + \frac{2400}{1.1} - \frac{1000}{1.1} = 17272.7273, \quad K_A = \frac{PW_A}{1-1/1.1} = 190000.$$

Step 3: Pump B ($n = 2$) and equality of capitalized costs

$$PW_B = 32000 + \frac{1600}{1.1} + \frac{1600}{1.1^2} - \frac{S}{1.1^2}, \qquad K_B = \frac{PW_B}{1-1/1.1^2}.$$

Set $K_B = K_A$ and solve for S:

$$\frac{32000 + \frac{1600}{1.1} + \frac{1600}{1.1^2} - \frac{S}{1.1^2}}{1-1/1.1^2} = 190000 \;\Rightarrow\; S \approx 2179.999 \text{ Rs.}$$

Step 4: Rounded answer

$$S \approx \boxed{2180 \text{ Rs}}.$$

Quick Tip

- For comparing alternatives with different lives, either use capitalized cost (perpetual service) or equivalent annual cost; both give the same decision at a fixed interest rate.
- Capitalized cost equals the present worth of one full cycle divided by $1 - (1+i)^{-n}$.