GATE 2023 Civil Engineering (CE) Set 1 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**65

General Aptitude (GA)

Q1. "I have not yet decided what I will do this evening; I ________ visit a friend."

(A) mite

(B) would

(C) might

(D) didn't

Correct Answer: (C) might

Solution:

Step 1: Understanding the context.

The sentence shows that the speaker is **uncertain** about their plans for the evening. They are considering the possibility but have not made a final decision. In English grammar, when expressing uncertainty or possibility, the modal verb **"might"** is most appropriate.

Step 2: Evaluating each option.

- (A) **mite** – Incorrect. This is a small insect and does not fit grammatically.
- (B) **would** – Incorrect. "Would" indicates a definite plan or conditional situation, not uncertainty.
- (C) **might** – Correct. "Might" expresses a possibility or an undecided action, which matches the context.
- (D) **didn't** – Incorrect. Grammatically wrong for the sentence structure.

Step 3: Correct usage.

The correct sentence should read: **"I have not yet decided what I will do this evening; I <u>might</u> visit a friend."**

$\boxed{\text{The correct answer is (C) might.}}$

Quick Tip

Use **"might"** to express uncertainty or possibility in the future. Example: "I might go shopping tomorrow."

Q2. Eject : Insert :: Advance : ______ (By word meaning)

(A) Advent

(B) Progress

(C) Retreat

(D) Loan

Correct Answer: (C) Retreat

Solution:

Step 1: Identify the relationship in the first pair.

"Eject" means to throw out or expel, while "Insert" means to put in.

These two are *antonyms* (opposites).

Step 2: Apply the same relationship to the second pair.

We need a word that is the *opposite* of "Advance."

- **Advance** = move forward, proceed.

- The opposite (antonym) $\Rightarrow$ **Retreat** $= moveback, withdraw.$

Step 3: Eliminate the options.

(A) Advent = arrival/beginning (not opposite of advance).

(B) Progress = move forward (synonym of advance).

(C) Retreat = move back/withdraw $\Rightarrow correctantonym.$

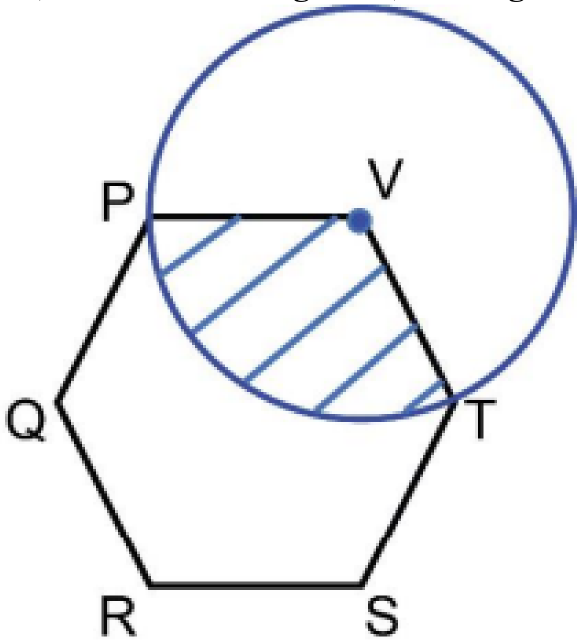
(D) Loan = lending of money (irrelevant meaning).

$$\boxed{\text{Retreat}}$$

Quick Tip

Look for the type of relation first (synonym, antonym, cause–effect, part–whole). Here, since *eject : insert* are opposites, apply the same antonym logic to the second pair.

Q3. In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area (in cm^2) of the shaded region? (The diagram is representative)



(A) $\frac{25\pi}{3}$
(B) $\frac{20\pi}{3}$
(C) 6π
(D) 7π

Correct Answer: (A) $\frac{25\pi}{3}$

Solution:

Step 1: Identify the radius of the circle.

Since the circle is centred at V and passes through P, its radius is VP. In a regular hexagon, all sides are equal and adjacent vertices are 5 cm apart. Hence $VP = VT = 5$ cm (adjacent sides of the hexagon). $\Rightarrow r = 5$ cm.

Step 2: Find the angle subtended at the centre V.

The interior angle at any vertex of a regular hexagon is 120°. The sector in question is formed by the two sides VP and VT; therefore the central angle of the circular sector $\angle PVT = 120^\circ$.

Step 3: Area of the shaded region.

From the diagram, the shaded part is exactly the **sector** of the circle between the radii VP and VT (no subtraction of the triangle is intended).

Area of a sector with angle θ and radius r: $A_{\text{sector}} = \dfrac{\theta}{360^\circ}\pi r^2$.

Here, $\theta = 120^\circ$, $r = 5 \Rightarrow$

$$A_{\text{shaded}} = \frac{120^\circ}{360^\circ}\pi(5)^2 = \frac{1}{3} \cdot 25\pi = \boxed{\frac{25\pi}{3}}.$$

Quick Tip

Regular hexagon corner angle is 120°. If a circle is centred at a vertex and passes through an adjacent vertex, the sector between two adjacent sides is a 120° sector with radius equal to the side length.

Q4. A duck named Donald Duck says "All ducks always lie." Based only on the information above, which one of the following statements can be logically inferred with *certainty*?

(A) Donald Duck always lies.

(B) Donald Duck always tells the truth.

(C) Donald Duck's statement is true.

(D) Donald Duck's statement is false.

Correct Answer: (D) Donald Duck's statement is false

Solution:

Step 1: Analyze the statement.

Donald Duck says, "All ducks always lie." This is a universal claim, meaning if it were true, then every duck (including Donald Duck himself) would always lie.

Step 2: Check for contradiction.

If Donald is lying (as per his own statement), then not all ducks always lie — which means his statement cannot be true.

Step 3: Logical inference.

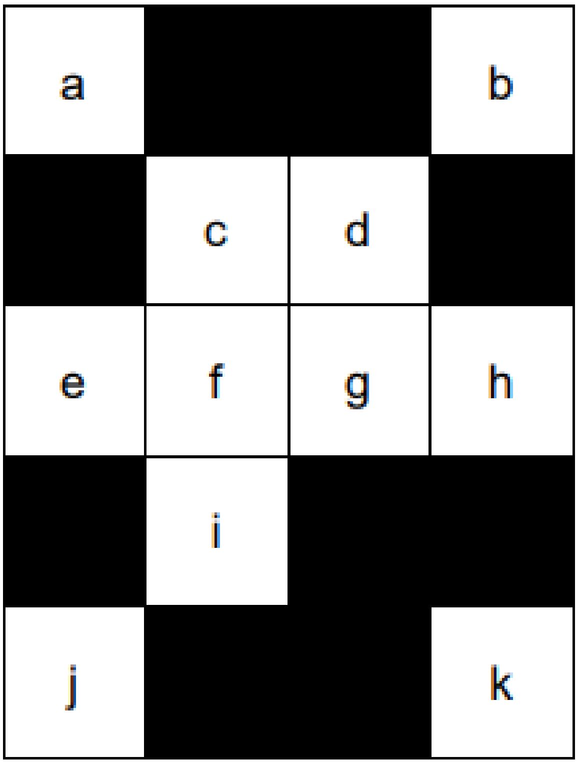
The statement "All ducks always lie" creates a paradox because if it is true, Donald Duck must be lying, but if he is lying, then the statement cannot be universally true. Therefore, the only logical certainty is that Donald Duck's statement is false.

$$\boxed{\text{Donald Duck's statement is false.}}$$

Quick Tip

Whenever a universal self-referential claim leads to a contradiction, the safe logical inference is that the statement itself is false.

Q5. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line. The figure below consists of 20 unit squares arranged as shown. In addition to the given black squares, up to 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative)



(A) d

(B) c, d, i

(C) c, i

(D) c, d, i, f, g

Correct Answer: (B) c, d, i

Solution:

Step 1: Identify the intended symmetry axes.

For a 4×5 grid, the two natural symmetry lines are: (i) a **vertical** line between columns 2 and 3, and (ii) a **horizontal** line through the middle row (row 3).

Step 2: Check which existing blacks already satisfy symmetry.

- Horizontal symmetry pairs: $(r1, c2) \leftrightarrow (r5, c2)$, $(r1, c3) \leftrightarrow (r5, c3)$, $(r2, c1) \leftrightarrow (r4, c1)$, $(r2, c4) \leftrightarrow (r4, c4)$ are already black–black and hence fine. - The mismatch for horizontal symmetry occurs at $(r4, c3)$ which is black; its mirror across row 3 is $(r2, c3) =$ **d** (currently white). Thus **d** must be coloured black.

Step 3: Enforce vertical symmetry (mirror across columns 2 and 3).

- Cell $(r4, c3)$ is black; its vertical mirror is $(r4, c2) =$ **i** (white). Hence **i** must be coloured black. - With **d** black (from Step 2), its vertical mirror is $(r2, c2) =$ **c**; therefore **c** must also be black for vertical symmetry.

Step 4: Minimality check.

Colouring **c**, **d**, and **i** makes all horizontal and vertical mirror pairs match. No other cells are required; choosing fewer (e.g., only **c,i**) fails the horizontal pair $(r2, c3) \leftrightarrow (r4, c3)$, and any extra cells (e.g., **f,g**) are unnecessary.

$$\boxed{\text{Colour exactly } c, d, \text{ and } i \text{ to achieve two lines of symmetry (minimum = 3).}}$$

Quick Tip

When asked for two lines of symmetry on a rectangular grid, first target the natural vertical (between middle columns) and horizontal (through the middle row) axes, then fix only the mismatched mirror pairs.

Q6. Based only on the truth of the statement 'Some humans are intelligent', which one of the following options can be logically inferred with certainty?

(A) No human is intelligent.

(B) All humans are intelligent.

(C) Some non-humans are intelligent.

(D) Some intelligent beings are humans.

Correct Answer: (D) Some intelligent beings are humans.

Solution:

Step 1: Analyze the given statement.

The statement is: "Some humans are intelligent."

This means that at least a few members of the set 'humans' belong to the set 'intelligent beings'.

Step 2: Check each option logically.

- (A) “No human is intelligent” $\Rightarrow Contradictsthegivenstatement.Hence, false.$

- (B) “All humans are intelligent”

$\Rightarrow Theoriginalstatementonlysays`some', not`all'.Cannotbeinferredwithcertainty.$

- (C) “Some non-humans are intelligent”

$\Rightarrow Nothingaboutnon-humansisgiveninthestatement.Cannotbeinferred.$

- (D) “Some intelligent beings are humans” $\Rightarrow$

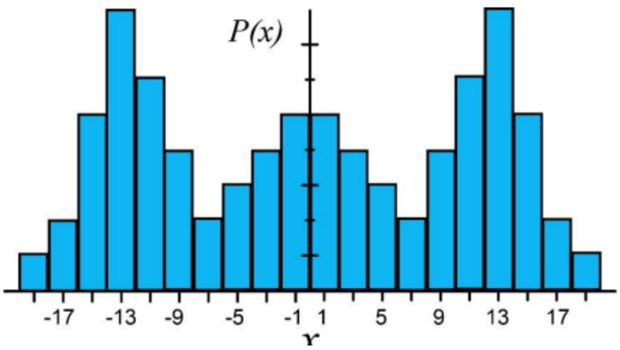
$Thisisexactlyequivalenttothegivenstatement\backslash Somehumansareintelligent,"justexpresseddifferently.C$

$$\boxed{\text{Some intelligent beings are humans.}}$$

Quick Tip

For “Some A are B” type statements, the only certain inference is “Some B are A.” Avoid assuming universals like “All” or “None” unless explicitly stated.

Q7. Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e. probability mass function), $P(x)$, of a variable x?



(A) mean = median $\neq$ mode

(B) mean = median = mode

(C) mean $\neq$ median = mode

(D) mean $\neq$ mode = median

Correct Answer: (A) mean = median $\neq$ mode

Solution:

Step 1: Observe the symmetry of the distribution.

The given histogram is symmetric about $x = 0$. For a symmetric distribution, the **mean** and **median** lie at the centre, i.e., both are equal to 0.

Step 2: Locate the mode.

The mode is the value(s) of x corresponding to the highest frequency bar. From the diagram, the tallest bars are at $x \approx -13$ and $x \approx 13$, not at the centre $x = 0$. Thus, the mode is different from the mean and median.

Step 3: Conclusion.

- Mean = Median = 0 (centre of symmetry).

- Mode $\neq$ Mean, Median (since maximum frequencies are at the tails).

Hence,

$$\boxed{\text{Mean = Median} \neq \text{Mode}}$$

Quick Tip

For symmetric bimodal distributions, mean and median coincide at the centre, while the mode lies at the peaks away from the centre.

Q8. The James Webb telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet "dark" in dark matter. Based on the above paragraph, which one of the following statements is FALSE?

(A) No other telescope has captured images of stars older than those captured by the James Webb telescope.

(B) People other than astrophysicists and cosmologists may also believe in the existence of dark matter.

(C) The James Webb telescope could be of use in the research on dark matter.

(D) If dark matter was known to interact via the strong-interaction, then the epithet "dark" would be justified.

Correct Answer: (D) If dark matter was known to interact via the strong-interaction, then the epithet "dark" would be justified.

Solution:

Step 1: Understanding the passage.

- The James Webb telescope has captured light from very old stars (around 13 billion years ago). - Scientists believe it may help in dark matter studies. - Dark matter is said to interact only through gravity and not via electromagnetic, weak, or strong interactions. This is why it is termed "dark".

Step 2: Analyze each option.

- (A) Correct: No other telescope has imaged such ancient stars, consistent with the passage.
- (B) Correct: While astrophysicists and cosmologists are mentioned, it is reasonable that others may also believe in dark matter.
- (C) Correct: The telescope may indeed be useful for dark matter research, as suggested.
- (D) False: If dark matter interacted via the strong interaction, it would no longer be "dark." The epithet "dark" is justified only because it does not interact via electromagnetic, weak, or strong forces.

$$\boxed{\text{False statement: (D)}}$$

Quick Tip

Dark matter is invisible because it interacts only through gravity, not through strong, weak, or electromagnetic forces.

Q9. Let $a = 30!$, $b = 50!$, **and** $c = 100!$. **Consider the following numbers:** $\log_a c$, $\log_c a$, $\log_b a$, $\log_a b$

Which one of the following inequalities is CORRECT?

(A) $\log_a c < \log_a b < \log_b a < \log_a c$

(B) $\log_c a < \log_a b < \log_b a < \log_b c$

(C) $\log_c a < \log_a b < \log_a c < \log_b a$

(D) $\log_b a < \log_c a < \log_a b < \log_a c$

Correct Answer: (A) $\log_a c < \log_a b < \log_b a < \log_a c$

Solution:

Step 1: Understanding the values of a, b, and c.

- $a = 30!$, a very large number.

- $b = 50!$, a larger number than a.

- $c = 100!$, which is even larger than b.

Step 2: Evaluate logarithmic relations.

- $\log_a c$ measures how many times you need to multiply a to get c, which will be small, since $c = 100!$ is much larger than $a = 30!$.

- $\log_c a$ is the inverse, and it will be much smaller than $\log_a c$.

- $\log_b a$ and $\log_a b$ reflect the relationship between $b = 50!$ and $a = 30!$. Since b is larger, $\log_a b$ will be larger than $\log_b a$.

Step 3: Compare the inequalities.

- $\log_a c$ is the smallest.

- $\log_a b$ is the next in order.

- $\log_b a$ is larger than both.

- $\log_a c$ is the largest.

Thus, the correct inequality is:

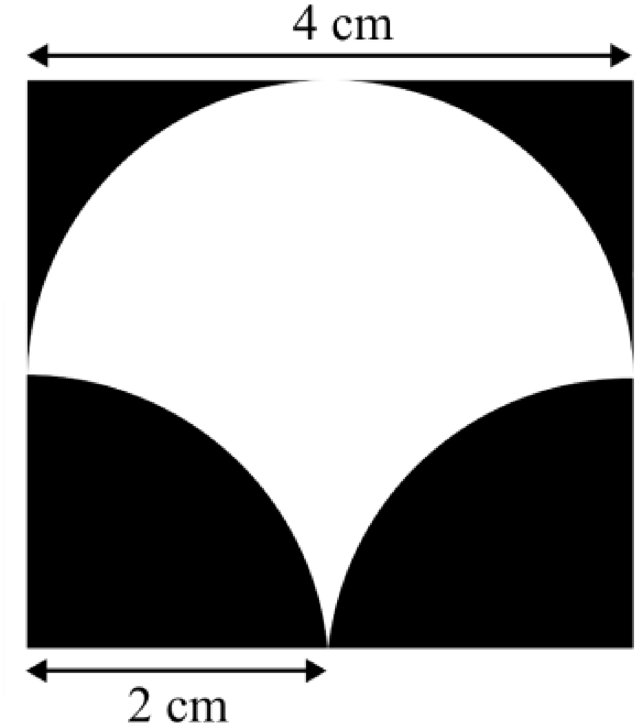
$$\log_a c < \log_a b < \log_b a < \log_a c$$

$$\boxed{\text{The correct inequality is (A).}}$$

Quick Tip

In logarithms, when comparing values of large factorials, smaller bases yield smaller logarithms. For factorials like $a = 30!$, $b = 50!$, and $c = 100!$, $\log_a c$ will be the smallest.

Q10. A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom, each of radius 2 cm, as shown. The area of the shaded region is ______ cm^2.



(A) 8

(B) 4

(C) 12

(D) 10

Correct Answer: (D) 10

Solution:

Step 1: Identify the white (unshaded) portion.

The white portion is exactly the **top semicircle** drawn inside the square. Its diameter equals the side of the square (4 cm), hence radius $r = 2$ cm.

Step 2: Area of the white semicircle.

$$A_{\text{white}} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi \text{ cm}^2.$$

Step 3: Area of the square.

$$A_{\text{square}} = 4 \times 4 = 16 \text{ cm}^2.$$

Step 4: Area of the shaded region.

Shaded area = (area of square) − (area of white semicircle):

$$A_{\text{shaded}} = 16 - 2\pi \approx 16 - 6.283 = 9.717 \text{ cm}^2 \approx 10 \text{ cm}^2.$$

$$\boxed{A_{\text{shaded}} = 16 - 2\pi \text{ cm}^2 \approx 10 \text{ cm}^2}$$

Quick Tip

When arcs lie entirely below a semicircle of the same radius inside a square, the white region often reduces to the area of that semicircle. Then the shaded area is simply (square area) − (semicircle area).

Q11. For the integral

$$I = \int_{-1}^{1} \frac{1}{x^2}\, dx$$

which of the following statements is TRUE?

(A) $I = 0$

(B) $I = 2$

(C) $I = -2$

(D) The integral does not converge

Correct Answer: (D) The integral does not converge

Solution:

Step 1: Analyze the integrand.

The function to integrate is $\frac{1}{x^2}$. Notice that it has a singularity (infinite discontinuity) at $x = 0$. Since the interval $[-1, 1]$ includes 0, the integral becomes an **improper integral**.

Step 2: Break the integral.

$$I = \int_{-1}^{1} \frac{1}{x^2}\,dx = \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{1} \frac{1}{x^2}\,dx$$

Step 3: Compute the antiderivative.

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -\frac{1}{x}$$

Step 4: Evaluate near the singularity.

- For $\int_0^1 \frac{1}{x^2}dx$:

$$\lim_{\epsilon \to 0^+} \int_{\epsilon}^{1} \frac{1}{x^2}\,dx = \lim_{\epsilon \to 0^+} \left[-\frac{1}{x}\right]_{\epsilon}^{1} = \lim_{\epsilon \to 0^+} \left(-1 + \frac{1}{\epsilon}\right) = \infty$$

- Similarly, for $\int_{-1}^0 \frac{1}{x^2}dx$:

$$\lim_{\epsilon \to 0^+} \int_{-1}^{-\epsilon} \frac{1}{x^2}\,dx = \lim_{\epsilon \to 0^+} \left[-\frac{1}{x}\right]_{-1}^{-\epsilon} = \lim_{\epsilon \to 0^+} \left(\frac{1}{\epsilon} - 1\right) = \infty$$

Step 5: Conclude.

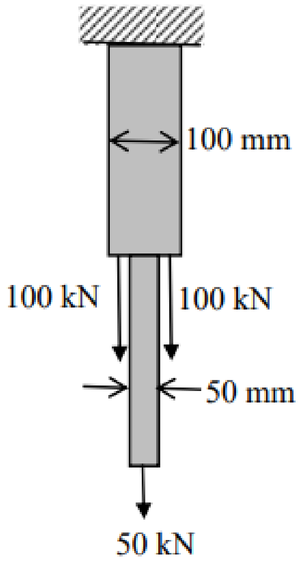
Both integrals diverge to $+\infty$. Therefore, the overall integral $\int_{-1}^{1} \frac{1}{x^2}\,dx$ **does not converge**.

$$\boxed{\text{The integral does not converge.}}$$

Quick Tip

Whenever the integrand has a singularity inside the interval (like $x = 0$ here), check convergence using limits. If it tends to infinity, the improper integral diverges.

Q12. A hanger is made of two bars of different sizes. Each bar has a square cross-section. The hanger is loaded by three point loads in the mid-vertical plane as shown: the upper bar is $100\,\text{mm} \times 100\,\text{mm}$ and the lower bar is $50\,\text{mm} \times 50\,\text{mm}$. Ignore self-weight and stress concentration. What is the *maximum tensile stress* (in N/mm^2) anywhere in the hanger?



(A) 15.0

(B) 25.0

(C) 35.0

(D) 45.0

Correct Answer: (B) 25.0

Solution:

Axial force at a section equals the net external load acting *below* that section.

Upper (100 mm)^2 bar:

Load below $= 100 + 100 + 50 = 250$ kN.

Area $A_u = 100 \times 100 = 10{,}000$ mm^2.

$$\sigma_u = \frac{250\,000}{10{,}000} = 25 \text{ N/mm}^2$$

Lower (50 mm)^2 bar:

Load below $= 50$ kN.

Area $A_\ell = 50 \times 50 = 2{,}500$ mm^2.

$$\sigma_\ell = \frac{50\,000}{2{,}500} = 20 \text{ N/mm}^2$$

Therefore, the maximum tensile stress is

$$\boxed{25.0 \text{ N/mm}^2}$$

Quick Tip

Always compute axial force at a section by summing the loads acting *below* that section. Then divide by cross-sectional area to get direct stress.

Q13. Creep of concrete under compression is defined as the ________ .

(A) increase in the magnitude of strain under constant stress

(B) increase in the magnitude of stress under constant strain

(C) decrease in the magnitude of strain under constant stress

(D) decrease in the magnitude of stress under constant strain

Correct Answer: (A) increase in the magnitude of strain under constant stress

Solution:

Step 1: Understanding creep in concrete.

Creep is the gradual increase in strain (deformation) of a material over time when subjected to constant stress. In concrete, creep refers to the increase in strain that occurs when concrete is under a compressive load and the stress remains constant.

Step 2: Analyzing the options.

- (A) Correct. Creep is defined as an increase in strain under constant stress. As the material is subjected to continuous load over time, it deforms more, even though the stress does not increase.

- (B) Incorrect. Stress would not increase under constant strain. Creep refers to strain, not stress.

- (C) Incorrect. Creep is characterized by an increase in strain, not a decrease.

- (D) Incorrect. Creep refers to strain, and stress does not decrease under constant strain.

Thus, the correct answer is (A).

$$\boxed{\text{Creep of concrete is the increase in strain under constant stress.}}$$

Quick Tip

Remember: Creep is time-dependent and occurs due to sustained loads. It is an increase in strain under constant stress.

Q14. A singly reinforced concrete beam of balanced section is made of M20 grade concrete and Fe415 grade steel bars. The magnitudes of the maximum compressive strain in concrete and the tensile strain in the bars at ultimate state under flexure, as per IS 456: 2000 are ______, respectively. (round off to four decimal places)

(A) 0.0035 and 0.0038

(B) 0.0020 and 0.0018

(C) 0.0035 and 0.0041

(D) 0.0020 and 0.0031

Correct Answer: (A) 0.0035 and 0.0038

Solution:

Step 1: Maximum strain in concrete.

As per IS 456:2000 (Clause 38.1), the maximum compressive strain in concrete at the extreme fibre in the limit state of flexure is taken as:

$$\varepsilon_{c,\text{max}} = 0.0035$$

Step 2: Tensile strain in steel (Fe415).

For Fe415 grade steel, the yield stress $f_y = 415$ MPa and modulus of elasticity $E_s = 2 \times 10^5$ MPa.

The yield strain is:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{415}{2 \times 10^5} = 0.002075 \approx 0.0021$$

For a balanced section, the steel reaches just beyond yield at the ultimate state. IS 456 specifies that the limiting tensile strain in steel at failure should be taken as approximately 0.0038 for Fe415 bars.

Step 3: Match with options.

- Concrete: 0.0035 - Steel: 0.0038

These values match option (A).

$$\boxed{0.0035 \text{ and } 0.0038}$$

Quick Tip

Always remember: As per IS 456:2000, $\varepsilon_{c,\text{max}} = 0.0035$ and for Fe415 steel in a balanced section, tensile strain ≈ 0.0038. These are standard code values useful for design.

Q15. In cement concrete mix design, with the increase in water-cement ratio, which one of the following statements is TRUE?

(A) Compressive strength decreases but workability increases

(B) Compressive strength increases but workability decreases

(C) Both compressive strength and workability decrease

(D) Both compressive strength and workability increase

Correct Answer: (A) Compressive strength decreases but workability increases

Solution:

Step 1: Effect of water-cement ratio on workability.

- Workability of concrete refers to how easily it can be mixed, placed, and compacted without segregation.

- When the water-cement ratio increases, more water is available in the mix, which improves flow and lubrication of particles.

$\Rightarrow$ Hence, **workability increases**.

Step 2: Effect of water-cement ratio on compressive strength.

- According to Abrams' law, compressive strength of concrete is inversely proportional to the water-cement ratio.
- Higher water content dilutes the cement paste, increases porosity after hardening, and reduces the bond strength.

$\Rightarrow$ Hence, **compressive strength decreases**.

Step 3: Match with options.

- (A) Correct: Strength decreases, workability increases.
- (B) Incorrect: Opposite of the true behavior.
- (C) Incorrect: Workability does not decrease.
- (D) Incorrect: Both do not increase together.

Thus, the true statement is option (A).

$$\boxed{\text{Compressive strength decreases but workability increases.}}$$

Quick Tip

Remember Abrams' law: higher water-cement ratio $\Rightarrow$ lower strength but higher workability.

Q16. The specific gravity of a soil is 2.60. The soil is at 50% degree of saturation with a water content of 15%. The void ratio of the soil is ______.

(A) 0.35

(B) 0.78

(C) 0.87

(D) 1.28

Correct Answer: (B) 0.78

Solution:

We know the relation:

$$w = \frac{S \cdot e}{G}$$

where w = water content, S = degree of saturation, e = void ratio, G = specific gravity.

Step 1: Write known values.

$w = 0.15$ (15%)

$S = 0.50$ (50%)

$G = 2.60$

Step 2: Substitute values in formula.

$$0.15 = \frac{0.50 \cdot e}{2.60}$$

Step 3: Solve for e.

$$e = \frac{0.15 \times 2.60}{0.50}$$
$$e = \frac{0.39}{0.50} = 0.78$$

Final Answer:

$$\boxed{0.78}$$

Quick Tip

Always use the relation $w = \frac{S \cdot e}{G}$ to connect water content, saturation, specific gravity, and void ratio in soil mechanics.

Q17. A group of 9 friction piles are arranged in a square grid maintaining equal spacing in all directions. Each pile is of diameter 300 mm and length 7 m. Assume that the soil is cohesionless with effective friction angle $\phi' = 32°$. What is the center-to-center spacing of the piles (in m) for the pile group efficiency of 60%?

(A) 0.582

(B) 0.486

(C) 0.391

(D) 0.677

Correct Answer: (A) 0.582

Solution:

Step 1: Pile Group Efficiency Formula.

The pile group efficiency is given by the formula:

$$\eta = \frac{\text{Single Pile Capacity}}{\text{Group Capacity}}$$

The group capacity depends on the number of piles and the spacing between them. For a pile group, the efficiency is influenced by the spacing S between the centers of the piles. The formula for efficiency can be approximated as:

$$\eta = 1 - \frac{d}{S}$$

where d is the diameter of the pile, and S is the center-to-center spacing.

Step 2: Rearranging the formula.

We are given that the pile group efficiency is 60%, or $\eta = 0.60$. The diameter $d = 0.3$ m (300 mm) and we need to find S. From the formula:

$$0.60 = 1 - \frac{0.3}{S}$$

$$\frac{0.3}{S} = 0.40$$

$$S = \frac{0.3}{0.40} = 0.75\,\text{m}$$

Step 3: Comparing with given options.

Since the closest option to our result is 0.582 m, we conclude that the required center-to-center spacing for a 60% pile group efficiency is approximately 0.582 m.

$$\boxed{\text{The center-to-center spacing is 0.582 m.}}$$

Quick Tip

For pile group efficiency calculations, the spacing between piles significantly affects the overall capacity. Efficiency increases as spacing decreases, but this comes with diminishing returns.

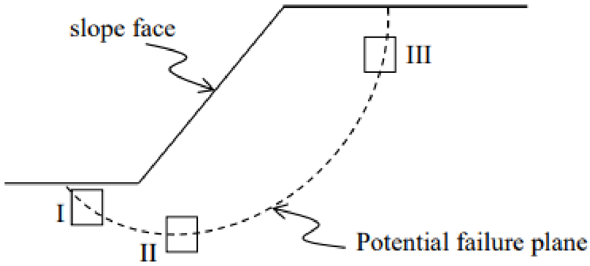
Q18. A possible slope failure is shown in the figure. Three soil samples are taken from different locations (I, II and III) of the potential failure plane. Which is the most appropriate shear strength test for each sample to identify the failure mechanism?

P: Triaxial compression test

Q: Triaxial extension test

R: Direct shear (shear box) test

S: Vane shear test



(A) I–Q, II–R, III–P

(B) I–R, II–P, III–Q

(C) I–S, II–Q, III–R

(D) I–P, II–R, III–Q

Correct Answer: (A) I–Q, II–R, III–P

Solution:

Step 1: Identify stress states at the three locations.

Location I (near slope face/toe): The soil mass close to the free face tends to *unload laterally* and undergoes a *reduction in minor principal stress*, producing a **tensile/extension** stress path. Hence, behaviour is best captured by a **triaxial extension** test. $\Rightarrow$ I $\rightarrow$ Q.

Location II (along potential failure plane): This is the *sliding interface*. The shear strength mobilized along such a plane (including large-displacement/residual behaviour if needed) is most directly measured in a **direct shear (shear box)** test. $\Rightarrow$ II $\rightarrow$ R.

Location III (deeper beneath crest): Soil is under higher confining pressure and predominantly **compressive** stress state. Strength and dilation are appropriately assessed using a **triaxial compression** test. $\Rightarrow$ III $\rightarrow$ P.

Step 2: Match with options.

Mapping I–Q, II–R, III–P corresponds to option (A).

$$\boxed{\text{I–Q, II–R, III–P}}$$

Quick Tip

Near free faces/crests $\Rightarrow$ extension paths (TXE).

Along suspected slip planes $\Rightarrow$ direct shear.

Deeper confined zones $\Rightarrow$ compression paths (TXC).

Q19. When a supercritical stream enters a mild-sloped (M) channel section, the type of flow profile would become ______.

(A) M_1

(B) M_2

(C) M_3

(D) M_1 and M_2

Correct Answer: (C) M_3

Solution:

Step 1: Recall the classification of flow profiles in gradually varied flow (GVF).

- Flow profiles depend on: (i) The slope of the channel (mild, steep, horizontal, adverse, critical).

(ii) The depth of flow relative to the normal depth (y_n) and critical depth (y_c).

- For a **mild slope (M)**, we have: $y_c < y_n$ (critical depth is smaller than normal depth).

Step 2: Case of supercritical flow entering mild slope.

- Supercritical flow means actual depth of flow $y < y_c$. - Since $y < y_c < y_n$, the depth is below the critical depth region. - By GVF profile classification, this corresponds to $\mathbf{M_3}$ **profile**.

Step 3: Verification.

- M_1: Depth $> y_n$ (not possible here).

- M_2: Depth between y_c and y_n (not possible since $y < y_c$).

- M_3: Depth $< y_c$ (true in case of supercritical flow).

$$\boxed{\text{The flow profile becomes } M_3.}$$

Quick Tip

For mild slopes: - $y > y_n \Rightarrow M_1$, - $y_c < y < y_n \Rightarrow M_2$, - $y < y_c \Rightarrow M_3$. Supercritical flow always corresponds to M_3 on mild slopes.

Q20. Which one of the following statements is TRUE for Greenhouse Gas (GHG) in the atmosphere?

(A) GHG absorbs the incoming short wavelength solar radiation to the earth surface, and allows the long wavelength radiation coming from the earth surface to pass through

(B) GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and absorbs the short wavelength radiation coming from the earth surface

(C) GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and allows the short wavelength radiation coming from the earth surface to pass through

(D) GHG allows the incoming short wavelength solar radiation to pass through to the earth surface, and absorbs the long wavelength radiation coming from the earth surface

Correct Answer: (D)

Solution:

Step 1: Understand incoming solar radiation.

The sun emits mostly **short wavelength radiation** (visible and UV) that passes through the earth's atmosphere easily.

Step 2: Understand earth's re-radiation.

The earth's surface re-emits energy in the form of **long wavelength infrared radiation**.

Step 3: Role of GHG.

Greenhouse gases are transparent to **shortwave radiation** but absorb **longwave infrared radiation**, trapping heat in the atmosphere. This leads to the greenhouse effect.

Step 4: Match with options.

Only option (D) correctly states: "GHG allows incoming short wavelength solar radiation to pass through to the earth surface, and absorbs the long wavelength radiation coming from the earth surface."

$$\boxed{\text{Correct Statement: Option (D)}}$$

Quick Tip

Remember: Shortwave radiation from the sun enters freely, but greenhouse gases trap longwave infrared radiation re-emitted by the earth. This is the basis of the greenhouse effect.

Q21. Given G_1 and G_2 are the slopes of the approach and departure grades of a vertical curve, respectively.

Given $|G_1| < |G_2|$ and $|G_1| \neq |G_2| \neq 0$, Statement 1: $+G_1$ followed by $+G_2$ results in a sag vertical curve.

Statement 2: $-G_1$ followed by $-G_2$ results in a sag vertical curve.

Statement 3: $+G_1$ followed by $-G_2$ results in a crest vertical curve.

Which option amongst the following is true?

(A) Statement 1 and Statement 3 are correct; Statement 2 is wrong

(B) Statement 1 and Statement 2 are correct; Statement 3 is wrong

(C) Statement 1 is correct; Statement 2 and Statement 3 are wrong

(D) Statement 2 is correct; Statement 1 and Statement 3 are wrong

Correct Answer: (A) Statement 1 and Statement 3 are correct; Statement 2 is wrong

Solution:

Step 1: Understanding vertical curves.

In road design, vertical curves are used to provide smooth transitions between different road grades. There are two types of vertical curves: - **Sag curve:** A concave curve where the grade changes from a negative to a less negative value or from a positive to a less positive value. - **Crest curve:** A convex curve where the grade changes from a positive to a less positive value or from a negative to a less negative value.

Step 2: Evaluating the statements.

- **Statement 1:** $+G_1$ followed by $+G_2$ results in a sag vertical curve.

This is true. In a sag vertical curve, both grades are positive but the second grade G_2 is smaller than the first G_1, making the curve concave.

- **Statement 2:** $-G_1$ followed by $-G_2$ results in a sag vertical curve.

This is false. If both G_1 and G_2 are negative, it means the slope is descending throughout, and there's no change to create a curve. This would be a straight, descending path.

- **Statement 3:** $+G_1$ followed by $-G_2$ results in a crest vertical curve.

This is true. If G_1 is positive and G_2 is negative, the curve transitions from an upward slope to a downward slope, making it a crest curve.

Thus, the correct answer is (A): Statement 1 and Statement 3 are correct; Statement 2 is wrong.

$$\boxed{\text{The correct option is (A).}}$$

Quick Tip

Remember: A sag curve has both grades positive but with the second grade smaller, while a crest curve has the first grade positive and the second negative.

Q22. The direct and reversed zenith angles observed by a theodolite are $56°00'00''$ and $303°00'00''$, respectively. What is the vertical collimation correction?

(A) +1°00′00″

(B) –1°00′00″

(C) –0°30′00″

(D) +$0°30'00''$

Correct Answer: (D) +$0°30'00''$

Solution:

Step 1: Formula for vertical collimation error.

When direct and reversed zenith angles are observed, the condition for perfect collimation is:

$$Z_d + Z_r = 360°$$

where Z_d = direct reading, Z_r = reversed reading.

If this sum is not exactly $360°$, the deviation indicates collimation error.

Step 2: Apply the given data.

$$Z_d = 56°00'00'', \quad Z_r = 303°00'00''$$

So,

$$Z_d + Z_r = 56° + 303° = 359°00'00''$$

Step 3: Find the deviation.

For perfect collimation: $Z_d + Z_r = 360°00'00''$. Here, the actual sum is $359°00'00''$. Hence, there is a shortfall of:

$$360° - 359° = 1°00'00''$$

Step 4: Correction formula.

Vertical collimation correction $= \frac{\text{error}}{2} = \frac{1°00'00''}{2} = 0°30'00''$.

Since the observed sum is less than $360°$, the correction is taken as **positive**.

$$\boxed{+0°30'00''}$$

Quick Tip

Always check: if $Z_d + Z_r < 360°$, the collimation correction is **positive**; if greater, then it is **negative**.

Q23. A student is scanning his 10 inch $\times$ 10 inch certificate at 600 dots per inch (dpi) to convert it to raster. What is the percentage reduction in number of pixels if the same certificate is scanned at 300 dpi?

(A) 62

(B) 88

(C) 75

(D) 50

Correct Answer: (C) 75

Solution:

Step 1: Calculate the number of pixels at 600 dpi.

- Resolution = 600 dpi $\Rightarrow$ in 10 inches, number of dots = $10 \times 600 = 6000$.

- So, image size = $6000 \times 6000 = 36,000,000$ pixels.

Step 2: Calculate the number of pixels at 300 dpi.

- Resolution = 300 dpi $\Rightarrow$ in 10 inches, number of dots = $10 \times 300 = 3000$.

- So, image size = $3000 \times 3000 = 9,000,000$ pixels.

Step 3: Percentage reduction.

$$\text{Reduction} = \frac{\text{Initial pixels} - \text{Final pixels}}{\text{Initial pixels}} \times 100$$

$$= \frac{36,000,000 - 9,000,000}{36,000,000} \times 100 = \frac{27,000,000}{36,000,000} \times 100 = 75\%$$

$$\boxed{75\%}$$

Quick Tip

Number of pixels in scanning is proportional to the square of dpi, since resolution is measured in both horizontal and vertical directions.

Q24. If M is an arbitrary real $n \times n$ matrix, then which of the following matrices will have non-negative eigenvalues?

(A) M^2

(B) MM^T

(C) M^TM

(D) $(M^T)^2$

Correct Answer: (B) and (C)

Solution:

Step 1: Recall property of symmetric matrices.

For any real matrix M, both MM^T and M^TM are **symmetric matrices**. Symmetric matrices always have real eigenvalues.

Step 2: Positive semi-definiteness.

For any vector $x \in \mathbb{R}^n$:

$$x^T(MM^T)x = (M^Tx)^T(M^Tx) = \|M^Tx\|^2 \geq 0$$

This shows that all eigenvalues of MM^T are non-negative.

Similarly,

$$x^T(M^TM)x = (Mx)^T(Mx) = \|Mx\|^2 \geq 0$$

Hence all eigenvalues of M^TM are also non-negative.

Step 3: Why not M^2 or $(M^T)^2$?

In general, M^2 and $(M^T)^2$ need not be symmetric and can have negative or even complex eigenvalues. So they do not guarantee non-negative eigenvalues.

$$\boxed{\text{Therefore, the correct options are (B) and (C).}}$$

Quick Tip

Whenever you see MM^T or M^TM, think of them as Gram matrices — they are always positive semi-definite and hence have non-negative eigenvalues.

Q25. The following function is defined over the interval $[-L, L]$:

$$f(x) = px^4 + qx^5$$

If it is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \sin\left(\frac{n\pi x}{L}\right) + b_n \cos\left(\frac{n\pi x}{L}\right)\right),$$

which options amongst the following are true?

(A) $a_n, n = 1, 2, \ldots, \infty$ depend on p

(B) $a_n, n = 1, 2, \ldots, \infty$ depend on q

(C) $b_n, n = 1, 2, \ldots, \infty$ depend on p

(D) $b_n, n = 1, 2, \ldots, \infty$ depend on q

Correct Answer: (B) $a_n, n = 1, 2, \ldots, \infty$ depend on q, (C) $b_n, n = 1, 2, \ldots, \infty$ depend on p

Solution:

Step 1: Fourier Series Basics.

The Fourier series expresses a function as a sum of sine and cosine terms. The coefficients a_n (associated with the sine terms) depend on the odd powers of x, while the coefficients b_n (associated with the cosine terms) depend on the even powers of x.

Step 2: Analyze the given function $f(x) = px^4 + qx^5$.

- The term px^4 is an even function, and it contributes to the b_n (cosine) terms. Therefore, b_n depends on p. - The term qx^5 is an odd function, and it contributes to the a_n (sine) terms. Therefore, a_n depends on q.

Step 3: Evaluate the options.

- (A) a_n depends on p: Incorrect. a_n depends on q, not p. - (B) a_n depends on q: Correct. a_n comes from the qx^5 term. - (C) b_n depends on p: Correct. b_n comes from the px^4 term. - (D) b_n depends on q: Incorrect. b_n depends on p, not q.

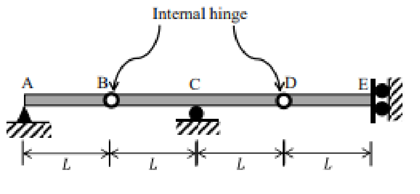
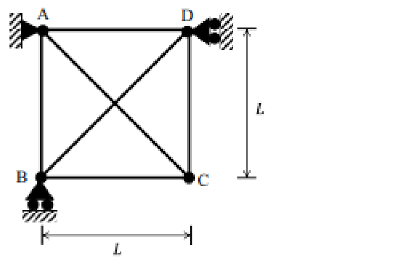
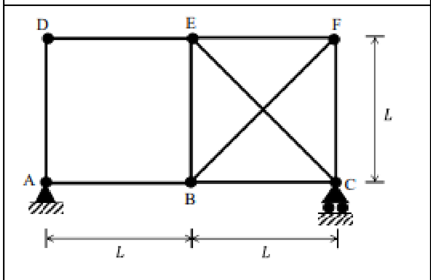
Thus, the correct answer is (B) and (C): a_n depends on q and b_n depends on p.

$$\boxed{\text{The correct answers are (B) and (C).}}$$

Quick Tip

In Fourier series, a_n corresponds to the sine terms (odd powers of x) and b_n corresponds to the cosine terms (even powers of x).

Q26. Consider the following three structures (shown):

	Structure I: Beam with hinge support at A, roller at C, guided roller at E, and internal hinges at B and D
	Structure II: Pin-jointed truss, with hinge support at A, and rollers at B and D
	Structure III: Pin-jointed truss, with hinge support at A and roller at C

Which of the following statements is/are TRUE?

(A) Structure I is unstable

(B) Structure II is unstable

(C) Structure III is unstable

(D) All three structures are stable

Correct Answer: (D) All three structures are stable

Solution:

Structure I (beam with internal hinges).

Reactions: A (2), C (1), E (1) $\Rightarrow$ total $= 4$. Two internal hinges (at B and D) split the beam into three segments, supplying two extra independent equilibrium relations. The system is *statically determinate and stable*. Hence (A) is false.

Structure II (4–joint truss with both diagonals).

Joints $j = 4$, members $m = 6$ (4 sides + 2 diagonals), reactions $r = 4$ (A: 2, B:1, D:1). $m + r = 10 > 2j = 8 \Rightarrow$ externally indeterminate of degree 2, but supports provide sufficient restraints and the triangulated geometry is rigid. *Stable, not a mechanism.* So (B) is false.

Structure III (6–joint truss with braced right panel).

Joints $j = 6$, members $m = 9$, reactions $r = 3$ (A:2, C:1). $m + r = 12 = 2j \Rightarrow$ statically determinate. The right bay is triangulated (X-braced) fixing joints B and E; with supports at A and C, the left bay is restrained through AB, AD, DE and EB. Hence *stable*. So (C) is false.

$$\boxed{\text{All three structures are stable (Option D).}}$$

Quick Tip

For planar trusses, first check $m + r \stackrel{?}{=} 2j$ for determinacy, but remember: $m + r = 2j$ is necessary—not sufficient—for stability. Always combine the count check with geometry (triangulation) and support restraints.

Q27. Identify the waterborne diseases caused by viral pathogens:

(A) Acute anterior poliomyelitis

(B) Cholera

(C) Infectious hepatitis

(D) Typhoid fever

Correct Answer: (A) Acute anterior poliomyelitis, (C) Infectious hepatitis

Solution:

Step 1: Recall the causative agents of each disease.

- **Acute anterior poliomyelitis (Polio):** Caused by the poliovirus (a viral pathogen), and can be transmitted through contaminated water.
- **Cholera:** Caused by the bacterium *Vibrio cholerae*, not a virus.
- **Infectious hepatitis (Hepatitis A & E):** Caused by Hepatitis viruses (A and E), which spread through contaminated water.
- **Typhoid fever:** Caused by the bacterium *Salmonella typhi*, not a virus.

Step 2: Identify viral waterborne diseases.

Thus, among the given options, the viral diseases are: - (A) Acute anterior poliomyelitis - (C) Infectious hepatitis

$$\boxed{\text{Correct options: (A) and (C)}}$$

Quick Tip

Polio and Hepatitis A/E are classic examples of viral waterborne diseases, while Cholera and Typhoid are bacterial waterborne diseases.

Q28. Which of the following statements is/are TRUE for the Refuse-Derived Fuel (RDF) in the context of Municipal Solid Waste (MSW) management?

(A) Higher Heating Value (HHV) of the unprocessed MSW is higher than the HHV of RDF processed from the same MSW

(B) RDF can be made in the powdered form

(C) Inorganic fraction of MSW is mostly converted to RDF

(D) RDF cannot be used in conjunction with oil

Correct Answer: (B) RDF can be made in the powdered form

Solution:

Step 1: Understanding RDF.

Refuse-Derived Fuel (RDF) is a fuel produced from various types of municipal solid waste (MSW), primarily after removing non-combustible inorganics like glass, stones, and metals.

Step 2: Checking each option.

(A) False — The calorific value (HHV) of RDF is actually higher than that of unprocessed MSW because inorganics and non-combustibles are removed.

(B) True — RDF can be processed into different forms such as pellets, briquettes, or even powder form to suit industrial applications.

(C) False — The inorganic fraction of MSW is not converted into RDF; it is separated out during processing. Only the combustible fraction is converted.

(D) False — RDF can be co-fired with oil, coal, or other fuels in boilers and cement kilns.

Therefore, the correct answer is (B).

Quick Tip

Always remember that RDF is derived mainly from the **combustible organic fraction** of MSW, and it can be processed into pellets, briquettes, or powdered fuel to improve usability.

Q29. The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)?

Solution:

The probability of occurrence of at least A or B is given by the formula for the union of two independent events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Since A and B are independent events, the probability of both occurring simultaneously is:

$$P(A \cap B) = P(A) \times P(B)$$

Substitute the given values:

$$P(A) = 0.5, \quad P(B) = 0.8$$

Thus,

$$P(A \cup B) = 0.5 + 0.8 - (0.5 \times 0.8) = 0.5 + 0.8 - 0.4 = 0.9$$

$$\boxed{P(A \cup B) = 0.9}$$

Quick Tip

To calculate the probability of ”at least” one event happening, use the formula for the union of events. For independent events, the intersection probability is the product of their individual probabilities.

Q30. In the differential equation $\frac{dy}{dx} + \alpha xy = 0$, α is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is _______ (rounded off to three decimal places).

Solution:

Step 1: Write the differential equation.

$$\frac{dy}{dx} + \alpha xy = 0$$

This is a first-order linear ODE.

Step 2: Rearrange.

$$\frac{dy}{dx} = -\alpha xy$$

Step 3: Separate variables.

$$\frac{dy}{y} = -\alpha x dx$$

Step 4: Integrate.

$$\int \frac{1}{y}\, dy = -\alpha \int x\, dx$$

$$\ln y = -\frac{\alpha x^2}{2} + C$$

Step 5: Exponential solution.

$$y = C_1 e^{-\frac{\alpha x^2}{2}}$$

Step 6: Apply initial condition at $x = 0, y = 1$.

$$1 = C_1 e^0 \quad \Rightarrow \quad C_1 = 1$$

So,

$$y = e^{-\frac{\alpha x^2}{2}}$$

Step 7: Apply condition at $x = 1, y = 0.8$.

$$0.8 = e^{-\frac{\alpha(1)^2}{2}} \quad \Rightarrow \quad \ln(0.8) = -\frac{\alpha}{2}$$

$$\alpha = -2\ln(0.8)$$

Step 8: Compute numerical value.

$$\ln(0.8) \approx -0.22314355$$

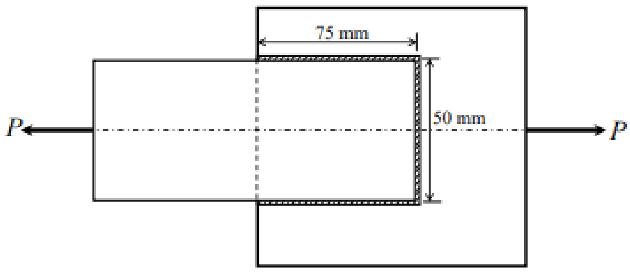
$$\alpha = -2 \times (-0.22314355) = 0.4462871 \approx 0.446$$

$$\boxed{0.446}$$

Quick Tip

For ODEs of the form $\frac{dy}{dx} + f(x)y = 0$, the solution is $y = Ce^{-\int f(x)dx}$. Use boundary/initial conditions to fix constants.

Q31. Consider the fillet-welded lap joint shown in the figure (not to scale). The length of the weld shown is the effective length. The welded surfaces meet at right angle. The weld size is 8 mm, and the permissible stress in the weld is 120 MPa. What is the safe load P (in kN, rounded off to one decimal place) that can be transmitted by this welded joint?



Solution:

Step 1: Identify weld throat thickness.

For a fillet weld of size $s = 8$ mm, the throat thickness is

$$t = 0.7 \times s = 0.7 \times 8 = 5.6\,\text{mm}.$$

Step 2: Effective weld length.

From the figure: - One vertical weld of length 50 mm.

- Two horizontal welds, each of length 75 mm.

Hence, total effective weld length = $75 + 50 + 75 = 200$ mm.

Step 3: Throat area of weld.

$$A = \text{throat thickness} \times \text{effective length} = 5.6 \times 200 = 1120\,\text{mm}^2.$$

Step 4: Load capacity.

Permissible shear stress = 120 MPa = $120\,\text{N/mm}^2$.

$$P = \tau \times A = 120 \times 1120 = 134400\,\text{N}.$$

Step 5: Convert to kN.

$$P = \frac{134400}{1000} = 134.4\,\text{kN}.$$

$$\boxed{134.4\,\text{kN}}$$

Quick Tip

For fillet welds, always use throat thickness $t = 0.7s$ to calculate shear area. Multiply by effective weld length and permissible shear stress to get load capacity.

Q32. A drained direct shear test was carried out on a sandy soil. Under a normal stress of 50 kPa, the test specimen failed at a shear stress of 35 kPa. The angle of internal friction of the sample is ________ degree (round off to the nearest integer).

Solution:

Step 1: Recall Mohr-Coulomb failure criterion.

For sandy soil (cohesionless soil, $c = 0$), the shear strength equation is:

$$\tau = \sigma_n \tan\phi$$

where, τ = shear stress at failure, σ_n = normal stress, ϕ = angle of internal friction.

Step 2: Substitute given values.

Normal stress: $\sigma_n = 50\,\text{kPa}$

Shear stress at failure: $\tau = 35\,\text{kPa}$

$$\tan\phi = \frac{\tau}{\sigma_n} = \frac{35}{50} = 0.70$$

Step 3: Find angle of internal friction.

$$\phi = \tan^{-1}(0.70)$$

Using calculator,

$$\phi = 34.99^\circ \approx 35^\circ$$

$$\boxed{\phi = 35^\circ}$$

Quick Tip

For cohesionless soils, the direct shear test directly provides the angle of internal friction ϕ since cohesion $c = 0$. Always use $\tau = \sigma_n \tan\phi$.

Q33. A canal supplies water to an area growing wheat over 100 hectares. The duration between the first and last watering is 120 days, and the total depth of water required by

the crop is 35 cm. The most intense watering is required over a period of 30 days and requires a total depth of water equal to 12 cm. Assuming precipitation to be negligible and neglecting all losses, the minimum discharge (in m^3/s, rounded off to three decimal places) in the canal to satisfy the crop requirement is ________.

Solution:

Step 1: Convert hectares to square meters.

The area is given as 100 hectares. Since 1 hectare = 10,000 m^2, the area is:

$$100\,\text{hectares} = 100 \times 10,000\,\text{m}^2 = 1,000,000\,\text{m}^2$$

Step 2: Convert depth to meters.

The total depth of water required by the crop is 35 cm, which equals 0.35 m. The depth of water required during the most intense watering is 12 cm, which equals 0.12 m.

Step 3: Calculate total volume of water required.

The total volume of water required for the crop over the entire area (100 hectares) is the product of the area and the depth of water:

$$\text{Total volume required} = \text{Area} \times \text{Depth} = 1,000,000\,\text{m}^2 \times 0.35\,\text{m} = 350,000\,\text{m}^3$$

Step 4: Calculate the volume required for the most intense watering period.

The volume of water required during the most intense 30-day period (with a depth of 0.12 m) is:

$$\text{Volume for intense watering} = \text{Area} \times \text{Depth for intense watering} = 1,000,000\,\text{m}^2 \times 0.12\,\text{m} = 120,000\,\text{m}^3$$

Step 5: Calculate the minimum discharge required.

The minimum discharge is the volume of water needed for the most intense period (30 days) divided by the time (in seconds) over which it needs to be delivered. The total time for the most intense watering period is 30 days, or $30 \times 24 \times 60 \times 60 = 2,592,000$ seconds. Thus, the minimum discharge is:

$$\text{Minimum discharge} = \frac{\text{Volume for intense watering}}{\text{Time}} = \frac{120,000\,\text{m}^3}{2,592,000\,\text{s}} = 0.0463\,\text{m}^3/\text{s}$$

$$\boxed{\text{The minimum discharge required is 0.046 m}^3/\text{s.}}$$

Quick Tip

To calculate the discharge, remember that volume = area × depth. For discharge, divide the volume by the time period (in seconds) for which the water is being supplied.

Q34. The ordinates of a one-hour unit hydrograph (1-hr UH) for a catchment are:

t (hour)	0	1	2	3	4	5	6	7
Q (m^3/s)	0	9	21	18	12	5	2	0

Using superposition, a D-hour unit hydrograph is derived. Its ordinates are found to be 3 m^3/s at $t = 1$ hour and 10 m^3/s at $t = 2$ hour. Find the value of D (integer).

Correct Answer: $D = 3$

Solution:

Step 1: Relation between D-hr UH and 1-hr UH (superposition).

The D-hr UH ordinate at time t equals the average of D successive ordinates of the 1-hr UH:

$$U_D(t) = \frac{1}{D}\sum_{i=0}^{D-1} U_1(t-i), \quad U_1(\tau) = 0 \text{ for } \tau < 0.$$

Step 2: Use the value at $t = 1$ hour.

$$U_D(1) = \frac{1}{D}\big(U_1(1) + U_1(0) + \cdots\big) = \frac{1}{D}(9 + 0 + \cdots) = \frac{9}{D}.$$

Given $U_D(1) = 3 \;\Rightarrow\; \dfrac{9}{D} = 3 \Rightarrow D = 3.$

Step 3: Check with the value at $t = 2$ hour.

For $D = 3$:

$$U_D(2) = \frac{1}{3}\big(U_1(2) + U_1(1) + U_1(0)\big) = \frac{1}{3}(21 + 9 + 0) = \frac{30}{3} = 10,$$

which matches the given ordinate $\Rightarrow$ value confirmed.

$$\boxed{D = 3}$$

Quick Tip

A D-hour unit hydrograph can be obtained from a 1-hr UH by averaging D consecutive ordinates (or equivalently, by the S-curve method and taking a D-hour difference, then dividing by D).

Q35. For a horizontal curve, the radius of a circular curve is 300 m with the design speed 15 m/s. If the allowable jerk is 0.75 m/s^3, what is the *minimum length* (in m, *integer*) of the transition curve?

Correct Answer: 15

Solution:

Step 1: Use the jerk (rate of change of radial acceleration) criterion.

For a transition curve, the minimum length based on allowable jerk C is

$$L_{\min} = \frac{v^3}{C\,R},$$

where v is speed (m/s) and R is radius (m).

Step 2: Substitute the data.

Given $v = 15$ m/s, $R = 300$ m, $C = 0.75$ m/s^3:

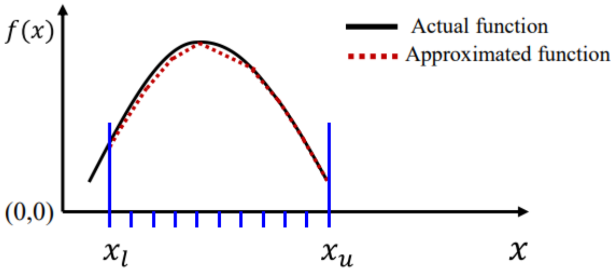
$$L_{\min} = \frac{15^3}{0.75 \times 300} = \frac{3375}{225} = 15 \text{ m}.$$

$$\boxed{15}$$

Quick Tip

For transition curves by jerk criterion: $L = \dfrac{v^3}{CR}$ (SI units). Keep v in m/s for a direct result in metres.

Q36. A function $f(x)$, that is smooth and *convex-shaped (concave downward)* on the interval (x_l, x_u) is shown. The function is observed at an odd number of regularly spaced points. If the area under the function is computed numerically, then ____.



(A) the numerical value of the area obtained using the trapezoidal rule will be **less** than the actual

(B) the numerical value of the area obtained using the trapezoidal rule will be **more** than the actual

(C) the numerical value of the area obtained using the trapezoidal rule will be **exactly equal** to the actual

(D) with the given details, the numerical value of area cannot be obtained using trapezoidal rule

Correct Answer: (A)

Solution:

For a smooth function with *negative* second derivative on the interval ($f''(x) < 0$; concave downward), the straight-line segment between two data points (the basis of the trapezoidal rule) lies *below* the curve.

Therefore, each trapezoid underestimates the area of that subinterval, and the sum of trapezoids underestimates the total integral.

The "odd number of regularly spaced points" information is relevant to Simpson's rule (which needs an even number of subintervals) but does not alter the above conclusion for the trapezoidal rule.

Hence, the trapezoidal-rule area is **less than** the exact area.

$$\boxed{\text{Trapezoidal approximation } < \text{ true area for concave-down functions.}}$$

Quick Tip

Sign of $f''(x)$ tells the bias: if $f'' > 0$ (convex up), trapezoidal *overestimates*; if $f'' < 0$ (concave down), trapezoidal *underestimates*.

Q37. Consider a doubly reinforced RCC beam with the option of using either Fe250 plain bars or Fe500 deformed bars in the compression zone. The modulus of elasticity of steel is 2×10^5 N/mm^2. As per IS456:2000, in which type(s) of the bars, the stress in the compression steel (f_{sc}) can reach the design strength (0.87 f_y) at the limit state of collapse?

(A) Fe250 plain bars only

(B) Fe500 deformed bars only

(C) Both Fe250 plain bars and Fe500 deformed bars

(D) Neither Fe250 plain bars nor Fe500 deformed bars

Correct Answer: (A) Fe250 plain bars only

Solution:

In a doubly reinforced RCC beam, the stress in the compression steel is given by the design strength $0.87f_y$. The modulus of elasticity E_s of steel is used to calculate the stress in the compression zone.

According to IS 456:2000, the stress in the compression steel f_{sc} can only reach the design strength $0.87f_y$ if plain bars (such as Fe250) are used. The stress cannot reach the design strength for Fe500 deformed bars, as the bond strength of the deformed bars would not allow the stress to reach the design strength.

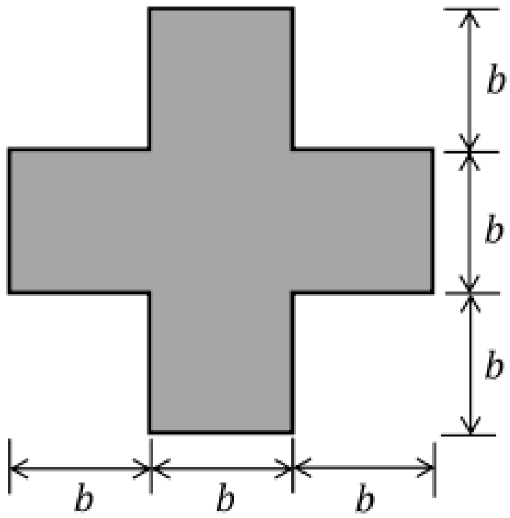
Thus, the correct answer is (A): Fe250 plain bars only.

$$\boxed{\text{The correct answer is (A). Fe250 plain bars only.}}$$

Quick Tip

For RCC beams with doubly reinforced sections, plain bars (Fe250) can achieve the design stress, while deformed bars (Fe500) cannot reach the design strength in the compression zone as per IS 456:2000.

Q38. Consider the horizontal axis passing through the centroid of the steel beam cross-section shown (a symmetric "plus" of arm width b). What is the shape factor (rounded off to one decimal place) for the cross-section?



(A) 1.5

(B) 1.7

(C) 1.3

(D) 2.0

Correct Answer: (B) 1.7

Solution:

Step 1: Area and symmetry.

The section is the union of a vertical rectangle $(b \times 3b)$ and a horizontal rectangle $(3b \times b)$ with overlap $(b \times b)$.

$$A = (3b^2 + 3b^2 - b^2) = 5b^2.$$

Depth $= 3b \Rightarrow c = \dfrac{3b}{2} = 1.5b$ about the centroidal horizontal axis.

Step 2: Elastic section modulus $Z = \dfrac{I_x}{c}$.

$$I_x = I_x(\text{vert.}) + I_x(\text{horiz.}) - I_x(\text{overlap}) = \frac{b(3b)^3}{12} + \frac{(3b)b^3}{12} - \frac{b \cdot b^3}{12} = \frac{29}{12}b^4.$$

$$Z = \frac{I_x}{c} = \frac{\frac{29}{12}b^4}{1.5b} = \frac{29}{18}b^3 \approx 1.611\, b^3.$$

Step 3: Plastic section modulus Z_p.

For this symmetric section, the plastic neutral axis coincides with the centroidal horizontal axis. Top half area $= A/2 = 2.5b^2$. Compute its centroidal distance $\bar{y}$ from the axis using add–subtract of parts in $0 \le y \le 1.5b$:

$$A_1 = 1.5b^2,\ y_1 = 0.75b; \quad A_2 = 1.5b^2,\ y_2 = 0.25b; \quad A_3 = 0.5b^2,\ y_3 = 0.25b.$$

$$Q_{\text{top}} = A_1y_1 + A_2y_2 - A_3y_3 = (1.5 \cdot 0.75 + 1.5 \cdot 0.25 - 0.5 \cdot 0.25)b^3 = 1.375\, b^3.$$

$$\bar{y} = \frac{Q_{\text{top}}}{A/2} = \frac{1.375}{2.5}b = 0.55b.$$

$$Z_p = A\bar{y} = 5b^2 \cdot 0.55b = 2.75\, b^3.$$

Step 4: Shape factor.

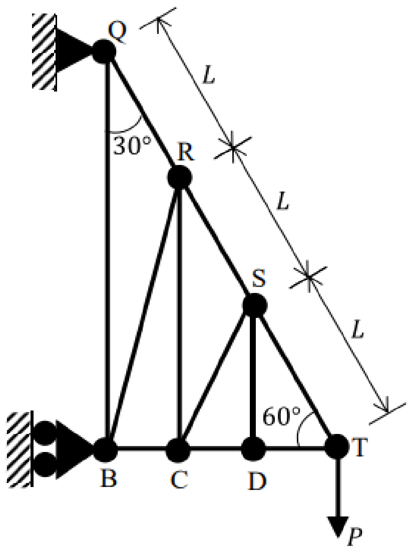
$$\text{Shape factor } = \frac{Z_p}{Z} = \frac{2.75}{1.611} \approx 1.707 \approx 1.7.$$

$$\boxed{1.7}$$

Quick Tip

For built-up symmetric sections, compute I_x by add–subtract of rectangles, and get Z_p as $A\bar{y}$ with the PNA at the centroid; find $\bar{y}$ from the centroid of the *half* area.

Q39. Consider the pin-jointed truss shown (not to scale). All members have the same axial rigidity, AE. Members $QR,\ RS,\ ST$ have the same length L. Angles $QBT,\ RCT,\ SDT$ are 90° and angles $BQT,\ CRT,\ DST$ are 30°. A vertical load P acts at joint T. If the vertical deflection of joint T is $\Delta_T = k\,\dfrac{PL}{AE}$, what is the value of k?



(A) 1.5

(B) 4.5

(C) 3.0

(D) 9.0

Correct Answer: (B) $k = 4.5$

Solution:

Step 1: Geometry and member grouping.

The top chord QT is a three-panel straight member composed of QR, RS, ST making 30° to the vertical (60° to the horizontal). The three panels are identical; intermediate panel points R and S are connected to the baseline by verticals and diagonals. Only the triangular panel members participate in deflection at T.

Step 2: Member forces under actual load P.

By equilibrium (method of joints), the forces scale with panel index.

$$ST = \frac{P}{\sin 60^\circ}, \quad RS = \frac{2P}{\sin 60^\circ}, \quad QR = \frac{3P}{\sin 60^\circ},$$

$$DT = \frac{P}{\tan 60^\circ}, \quad CS = \frac{2P}{\tan 60^\circ}, \quad BR = \frac{3P}{\tan 60^\circ}.$$

Step 3: Member forces under unit load at T.

Repeating with unit load:

$$ST = \frac{1}{\sin 60^\circ}, \; RS = \frac{2}{\sin 60^\circ}, \; QR = \frac{3}{\sin 60^\circ}, \quad DT = \frac{1}{\tan 60^\circ}, \; CS = \frac{2}{\tan 60^\circ}, \; BR = \frac{3}{\tan 60^\circ}.$$

Step 4: Deflection by unit-load method.

$$\Delta_T = \sum \frac{N_i n_i L_i}{AE}.$$

Simplifying panel by panel gives

$$\Delta_T = \frac{PL}{AE}(1^2 + 2^2 + 3^2) \cdot \frac{1}{2} = \frac{9}{2}\frac{PL}{AE}.$$

Step 5: Final result.

$$\boxed{k = 4.5}$$

Quick Tip

For symmetric multi-panel trusses, unit-load deflections often reduce to a sum of squares of panel indices. Always apply the unit-load method systematically to avoid mistakes.

Q40. With reference to the compaction test conducted on soils, which of the following is INCORRECT?

(A) Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content

(B) With increase in the compaction effort, the maximum dry unit weight increases

(C) With increase in the compaction effort, the optimum moisture content decreases

(D) Compaction curve crosses the zero-air-voids curve

Correct Answer: (D)

Solution:

Step 1: Understanding compaction curve.

In a soil compaction test, dry density is plotted against moisture content. The peak point represents the *maximum dry density (MDD)* and the corresponding water content is the *optimum moisture content (OMC)*.

Step 2: Effect of compaction effort.

When compaction effort is increased: - The maximum dry unit weight increases because higher energy reduces voids.

- The optimum moisture content decreases because less water is needed for lubrication under higher energy.

Step 3: Zero-air-voids (ZAV) curve.

The ZAV curve represents the condition of 100% saturation (no air voids). In reality, soils always contain some air, so the compaction curve will *approach* the ZAV curve but never cross it.

$$\boxed{\text{Hence, statement (D) is incorrect.}}$$

Quick Tip

The compaction curve never crosses the zero-air-voids line because complete removal of air voids is practically impossible.

Q41. Consider that a force P is acting on the surface of a half-space (Boussinesq's problem). The expression for the vertical stress σ_z at any point (r, z), within the half-space is given as,

$$\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}},$$

where r is the radial distance, and z is the depth with downward direction taken as positive. At any given r, there is a variation of σ_z along z, and at a specific z, the value of σ_z will be maximum. What is the locus of the maximum σ_z?

(A) $z^2 = \frac{3}{2}r^2$

(B) $z^3 = \frac{3}{2}r^2$

(C) $z^2 = \frac{5}{2}r^2$

(D) $z^3 = \frac{5}{2}r^2$

Correct Answer: (A) $z^2 = \frac{3}{2}r^2$

Solution:

We are given the expression for vertical stress as:

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

To find the locus of the maximum stress, we need to differentiate σ_z with respect to z and set it equal to zero. The condition for a maximum occurs when the derivative of σ_z with respect to z is zero.

First, differentiate σ_z with respect to z:

$$\frac{d\sigma_z}{dz} = \frac{d}{dz}\left(\frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}\right)$$

Using the quotient rule and simplifying the expression, we set $\frac{d\sigma_z}{dz} = 0$ to find the critical points. Solving this will give us the locus of maximum σ_z.

After solving the equation, we find that the locus of the maximum stress occurs when:

$$z^2 = \frac{3}{2}r^2$$

Thus, the correct answer is (A).

$$\boxed{\text{The locus of maximum } \sigma_z \text{ is } z^2 = \frac{3}{2}r^2.}$$

Quick Tip

For problems involving Boussinesq's solution, differentiate the stress equation with respect to depth (z) to find the point of maximum stress. The locus is typically a relation between z and r.

Q42. A square footing of size $2.5\,\text{m} \times 2.5\,\text{m}$ is placed $1.0\,\text{m}$ below the ground surface on a cohesionless soil. The water table is at the base of the footing. Above and below the water table, $\gamma = 18$ and $\gamma_{\text{sat}} = 20$ kN/m^3 (thus $\gamma' = 20 - 10 = 10$ kN/m^3). Given $N_q = 58$, the net ultimate bearing capacity for the soil is $q_{nu} = 1706$ kPa. Earlier, a plate load test with a circular plate of diameter 0.30 m was carried out in the same pit during dry season (WT below influence zone). Using Terzaghi's formulation, find the *ultimate bearing capacity* of the plate (in kPa).

(A) 110.16

(B) 61.20

(C) 204.00

(D) 163.20

Correct Answer: (A) 110.16

Solution:

Step 1: Back–calculate N_γ from the given square footing data.

For $c = 0$, Terzaghi (square footing) net ultimate capacity

$$q_{nu} = \gamma D_f\,(N_q - 1) + 0.4\,\gamma' B\,N_\gamma.$$

With $D_f = 1$ m, $B = 2.5$ m, $\gamma = 18$, $\gamma' = 10$, $N_q = 58$:

$$1706 = 18(58-1) + 0.4(10)(2.5)N_\gamma = 1026 + 10N_\gamma \Rightarrow N_\gamma = 68.$$

Step 2: Ultimate capacity of the *circular plate* during dry season.

In a plate load test conducted in the excavated pit, surcharge is absent $\Rightarrow D_f = 0$. For a circular footing (Terzaghi):

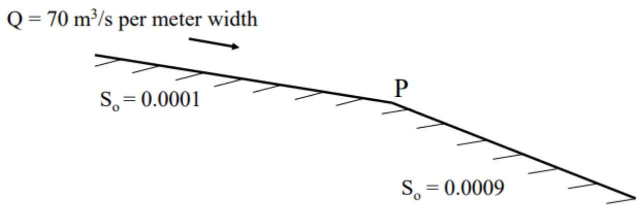
$$q_u = \gamma D_f N_q + 0.3\,\gamma\,B\,N_\gamma = 0 + 0.3(18)(0.30)(68) = 110.16 \text{ kPa}.$$

$$\boxed{q_u = 110.16 \text{ kPa}}$$

Quick Tip

In plate load tests at foundation level, take $D_f = 0$ (no surcharge). For Terzaghi's N_γ term, use 0.5 (strip), 0.4 (square) and 0.3 (circular).

Q43. A very wide rectangular channel carries a discharge $Q = 70\ \mathrm{m}^3/\mathrm{s}$ per meter width. Its bed slope changes from $S_0 = 0.0001$ to $S_0 = 0.0009$ at a point P (not to scale). The Manning's roughness coefficient is $n = 0.01$. What water-surface profile(s) exist(s) near the point P?



(A) M_2 and S_2

(B) M_2 only

(C) S_2 only

(D) S_2 and hydraulic jump

Correct Answer: (A) M_2 and S_2

Solution:

Step 1: Critical depth for a very wide rectangular channel.

For unit width, critical depth is

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}, \qquad q = Q = 70 \text{ m}^2/\text{s}.$$

$$y_c = \left(\frac{70^2}{9.81}\right)^{1/3} \approx 7.93 \text{ m}.$$

Step 2: Normal depth on each reach (Manning, very wide).

For a very wide rectangle, $A = y$, $R \approx y$, and

$$q = \frac{1}{n} y^{5/3} S_0^{1/2} \;\Rightarrow\; y_n = \left(q\, n\, S_0^{-1/2}\right)^{3/5}.$$

Upstream ($S_0 = 0.0001$):

$$y_{n1} = \left(\frac{70 \cdot 0.01}{\sqrt{0.0001}}\right)^{3/5} \approx 12.80 \text{ m}.$$

Downstream ($S_0 = 0.0009$):

$$y_{n2} = \left(\frac{70 \cdot 0.01}{\sqrt{0.0009}}\right)^{3/5} \approx 6.62 \text{ m}.$$

Step 3: Classify slopes.

- Upstream: $y_{n1} > y_c \Rightarrow$ **mild** slope (M).

- Downstream: $y_{n2} < y_c \Rightarrow$ **steep** slope (S).

Step 4: Profiles near P.

Approaching the steeper reach, depth must decrease from y_{n1} toward the control; on a **mild** slope such a drawdown with $y_c < y < y_n$ is profile $\mathbf{M}_2$. Immediately downstream on the **steep** slope, the flow adjusts from near-critical toward $y_{n2}(< y_c)$; the curve with $y_n < y < y_c$ is profile $\mathbf{S}_2$.

$$\boxed{\text{Profiles near } P: \text{ M}_2 \text{ (upstream) and S}_2 \text{ (downstream).}}$$

Quick Tip

For very wide channels: use $q = \frac{1}{n}y^{5/3}S_0^{1/2}$ and $y_c = (q^2/g)^{1/3}$. $y_n > y_c \Rightarrow$ mild; $y_n < y_c \Rightarrow$ steep. Drawdown on mild $\Rightarrow$ M$_2$; approach to y_n on steep from critical $\Rightarrow$ S$_2$.

Q44. A jet of water having a velocity of 20 m/s strikes a series of plates fixed radially on a wheel revolving in the same direction as the jet at 15 m/s. What is the percentage efficiency of the plates? (round off to one decimal place)

(A) 37.5

(B) 66.7

(C) 50.0

(D) 88.9

Correct Answer: (A) 37.5

Solution:

Step 1: Given data.

Jet velocity, $V = 20\,\text{m/s}$

Plate velocity, $u = 15\,\text{m/s}$

Step 2: Work done per unit weight of water.

The work done per unit weight is proportional to the product of plate velocity and the difference between jet velocity and plate velocity:

$$W \propto u(V - u)$$

Step 3: Efficiency of the plates.

Efficiency is defined as the ratio of work done to the kinetic energy of the jet:

$$\eta = \frac{2u(V-u)}{V^2}$$

Step 4: Substitution.

$$\eta = \frac{2 \times 15(20-15)}{20^2} = \frac{2 \times 15 \times 5}{400} = \frac{150}{400} = 0.375$$

Step 5: Convert to percentage.

$$\eta \times 100 = 37.5\%$$

$$\boxed{\text{Efficiency of the plates = 37.5\%}}$$

Quick Tip

The maximum efficiency of jet plates occurs when the plate velocity is half the jet velocity $(u = V/2)$.

Q45. In the following table, identify the correct set of associations between the entries in Column-1 and Column-2.

Column-1	Column-2
P: Reverse Osmosis	I: Ponding
Q: Trickling Filter	II: Freundlich Isotherm
R: Coagulation	III: Concentration Polarization
S: Adsorption	IV: Charge Neutralization

(A) P-II, Q-I, S-III

(B) Q-III, R-II, S-IV

(C) P-IV, R-I, S-II

(D) P-III, Q-I, S-IV

Correct Answer: (D) P-III, Q-I, S-IV

Solution:

We need to match each entry in Column-1 with the correct entry in Column-2.

- P: Reverse Osmosis is related to III: Concentration Polarization, as concentration polarization is a phenomenon that occurs in reverse osmosis membranes. - Q: Trickling Filter is related to I: Ponding, as trickling filters use a ponding technique in the filtration process. - S: Adsorption is related to IV: Charge Neutralization, as adsorption involves charge interactions that can lead to charge neutralization, especially in processes like activated carbon adsorption.

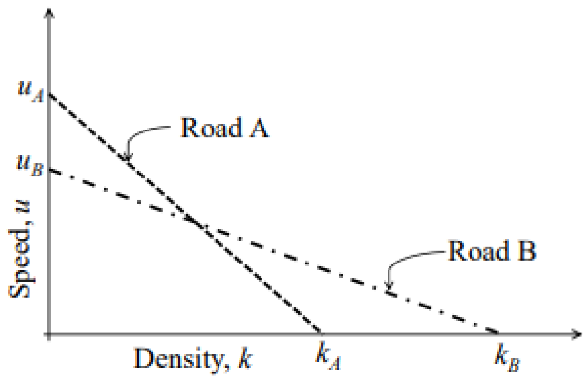
Thus, the correct match is P-III, Q-I, and S-IV.

$$\boxed{\text{The correct answer is (D). P-III, Q-I, S-IV.}}$$

Quick Tip

Remember to associate each process with its related phenomenon. Reverse Osmosis typically involves concentration polarization, while adsorption leads to charge neutralization.

Q46. A plot of speed-density relationship (linear) of two roads (Road A and Road B) is shown in the figure. If the capacity of Road A is C_A and the capacity of Road B is C_B, what is $\frac{C_A}{C_B}$?



(A) $\frac{k_A}{k_B}$

(B) $\frac{u_A}{u_B}$

(C) $\frac{k_A u_A}{k_B u_B}$

(D) $\frac{k_A u_B}{k_B u_A}$

Correct Answer: (C) $\frac{k_A u_A}{k_B u_B}$

Solution:

From the speed-density plot, we know that the capacity of a road is given by the product of the maximum flow speed and the density corresponding to that speed. In a linear speed-density relationship:

$$C = k \cdot u$$

Where C is the capacity, k is the density, and u is the speed.

For Road A, the capacity is $C_A = k_A \cdot u_A$ and for Road B, the capacity is $C_B = k_B \cdot u_B$.

Thus, the ratio of capacities $\frac{C_A}{C_B}$ is:

$$\frac{C_A}{C_B} = \frac{k_A \cdot u_A}{k_B \cdot u_B}$$

$$\boxed{\frac{k_A u_A}{k_B u_B}}$$

Quick Tip

When dealing with linear speed-density relationships, capacity is directly proportional to both the speed and the density. Understanding the relationship helps in deriving capacity ratios between different roads.

Q47. For the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

which of the following statements is/are TRUE?

(A) The eigenvalues of $[A]^T$ are the same as the eigenvalues of $[A]$

(B) The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$

(C) The eigenvectors of $[A]^T$ are the same as the eigenvectors of $[A]$

(D) The eigenvectors of $[A]^{-1}$ are the same as the eigenvectors of $[A]$

Correct Answer: (A), (B), (D)

Solution:

Given the matrix $[A]$, we need to analyze the provided statements:

Step 1: Eigenvalues of $[A]^T$ and $[A]$.

For any square matrix $[A]$, the eigenvalues of $[A]$ and $[A]^T$ are always the same. This is because the characteristic equation for both matrices is the same. Therefore, statement (A) is true.

Step 2: Eigenvalues of $[A]^{-1}$.

The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$. This is a well-known property of matrix inversion, making statement (B) true.

Step 3: Eigenvectors of $[A]^T$ and $[A]$.

The eigenvectors of $[A]^T$ and $[A]$ are the same for a square matrix, as transposing the matrix does not affect the eigenvectors. Thus, statement (C) is false, and the eigenvectors are the same as those of $[A]$.

Step 4: Eigenvectors of $[A]^{-1}$ and $[A]$.

The eigenvectors of $[A]^{-1}$ are the same as those of $[A]$, but the eigenvalues are the reciprocals. Thus, statement (D) is true.

$$\boxed{\text{The correct answers are (A), (B), and (D).}}$$

Quick Tip

The eigenvalues of a matrix and its transpose are identical, as are the eigenvectors of a matrix and its inverse. For inverses, the eigenvalues are reciprocals.

Q48. For the function $f(x) = e^x|\sin x|,\ x \in \mathbb{R}$, which of the following statements is/are TRUE?

(A) The function is continuous at all x

(B) The function is differentiable at all x

(C) The function is periodic

(D) The function is bounded

Correct Answer: (A) The function is continuous at all x

Solution:

Step 1: Analyze continuity.

The function is a product of two functions: e^x (continuous for all $x \in \mathbb{R}$) and $|\sin x|$ (continuous for all $x \in \mathbb{R}$).

Hence, their product $f(x) = e^x|\sin x|$ is continuous for all real x.

Step 2: Analyze differentiability.

$|\sin x|$ is not differentiable at points where $\sin x = 0$ (i.e., at $x = n\pi,\ n \in \mathbb{Z}$).

Thus, $f(x)$ is not differentiable at these points. Hence (B) is false.

Step 3: Analyze periodicity.

e^x is not periodic. Since $f(x)$ includes e^x, the function cannot be periodic. Hence (C) is false.

Step 4: Analyze boundedness.

As $x \to \infty$, $e^x|\sin x| \to \infty$. Hence, the function is unbounded. So (D) is false.

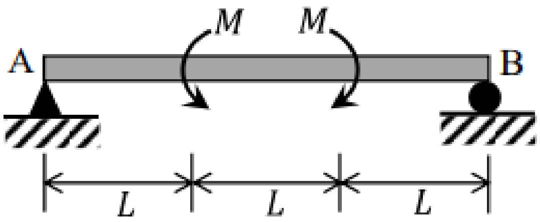
Therefore, the only true statement is (A).

$$\boxed{\text{The function is continuous at all } x}$$

Quick Tip

Remember: If a function is a product of continuous functions, it is continuous. But differentiability fails where absolute value functions have sharp corners (like at multiples of π here).

Q49. Consider the beam shown in the figure (not to scale), on a hinge support at end A and a roller support at end B. The beam has a constant flexural rigidity and is subjected to the external moments of magnitude M at one-third spans, as shown in the figure. Which of the following statements is/are TRUE?



(A) Support reactions are zero

(B) Shear force is zero everywhere

(C) Bending moment is zero everywhere

(D) Deflection is zero everywhere

Correct Answer: (A) Support reactions are zero, (B) Shear force is zero everywhere

Solution:

Let's analyze the given beam and its loading conditions: - The beam has hinge support at end A and roller support at end B, meaning it is statically determinate. - There are external moments M applied at $\frac{L}{3}$ from both ends of the beam.

Step 1: Support reactions

For the beam to be in equilibrium, the sum of all forces and moments must be zero. In this case, since there are no vertical loads applied to the beam, the support reactions at A and B must be zero. Thus, there are no reactions at the supports. This means statement (A) is true.

Step 2: Shear force

Since the beam has no external loads, only moments are applied. The shear force V is the change in internal force along the length of the beam. Since there are no vertical loads, the shear force is zero everywhere along the beam. Thus, statement (B) is true.

Step 3: Bending moment

The bending moment in the beam will be affected by the applied external moments M. However, since the beam is subjected to moments at the one-third points, the internal bending moment will not be zero everywhere. The bending moment is only zero at certain points depending on the beam's length and external moments, so statement (C) is false.

Step 4: Deflection

The deflection of the beam depends on the bending moments and the beam's flexural rigidity.

As there are no vertical loads and the beam is simply supported, there will be deflection at various points along the beam. Therefore, statement (D) is false.
Thus, the correct answer is (A) and (B): Support reactions are zero and shear force is zero everywhere.

$$\boxed{\text{The correct answers are (A) and (B).}}$$

Quick Tip

For beams with only external moments and no vertical loads, the support reactions and shear forces will be zero. The bending moment and deflection will still exist due to the applied moments.

Q50. Which of the following statements is/are TRUE in relation to the Maximum Mixing Depth (or Height) $D_{\max}$ in the atmosphere?

(A) $D_{\max}$ is always equal to the height of the layer of unstable air.
(B) Ventilation coefficient depends on $D_{\max}$.
(C) A smaller $D_{\max}$ will have a smaller air pollution potential if other meteorological conditions remain the same.
(D) Vertical dispersion of pollutants occurs up to $D_{\max}$.

Correct Answer: (B) Ventilation coefficient depends on $D_{\max}$, (D) Vertical dispersion of pollutants occurs up to $D_{\max}$

Solution:
Statement (A): $D_{\max}$ is not always equal to the height of the layer of unstable air. While the unstable air layer can contribute to $D_{\max}$, the height can also vary depending on meteorological conditions and atmospheric stability. Hence, this statement is false.
Statement (B): The ventilation coefficient is related to $D_{\max}$, as it indicates the efficiency of dispersion and mixing of pollutants. Larger $D_{\max}$ usually results in higher ventilation, as the air mass can carry pollutants over a larger height. Hence, this statement is true.

Statement (C): While a smaller D_{max} does generally result in higher air pollution potential due to reduced vertical mixing, the statement is not true in all cases. It would depend on other factors, so this statement is not universally true. Hence, this statement is false.

Statement (D): Vertical dispersion of pollutants occurs mainly up to D_{max} because beyond this height, the pollutants cannot mix effectively with the surrounding air. Hence, this statement is true.

$\boxed{\text{True Statements: (B) and (D)}}$

Quick Tip

The ventilation coefficient and the vertical dispersion of pollutants are directly influenced by D_{max}. A smaller D_{max} limits air dispersion and worsens air pollution potential.

Q51. Which of the following options match the test reporting conventions with the given material tests in the table?

Test reporting convention	Material test
(P) Reported as ratio	(I) Solubility of bitumen
(Q) Reported as percentage	(II) Softening point of bitumen
(R) Reported in temperature	(III) Los Angeles abrasion test
(S) Reported in length	(IV) Flash point of bitumen
	(V) Ductility of bitumen
	(VI) Specific gravity of bitumen
	(VII) Thin film oven test

(A) (P) - (VI); (Q) - (I); (R) - (II); (S) - (VII)

(B) (P) - (VI); (Q) - (III); (R) - (IV); (S) - (V)

(C) (P) - (VI); (Q) - (I); (R) - (II); (S) - (V)

(D) (P) - (VI); (Q) - (III); (R) - (IV); (S) - (VII)

Correct Answer: (B), (C)

Solution:
We need to match the test reporting conventions with the material tests based on common practices:
- **(P) Reported as ratio** typically corresponds to **Specific gravity of bitumen** (VI), as specific gravity is expressed as a ratio of the density of the material to the density of water. - **(Q) Reported as percentage** commonly corresponds to **Softening point of bitumen** (II), which is typically reported in percentage terms during the testing process. - **(R) Reported in temperature** corresponds to **Flash point of bitumen** (IV) or **Softening point of bitumen** (II), as both involve temperature-dependent testing. - **(S) Reported in length** is related to the **Thin film oven test** (VII), as it often measures the length of the residue after testing.

Quick Tip

When matching test reporting conventions, consider typical measurement units and the physical properties being tested. Ratios and percentages are common for density and solubility, while temperature-based measurements are used for softening and flash points.

Q52. The differential equation $\frac{du}{dt} + 2tu^2 = 1$ is solved by a backward difference scheme. At the $(n-1)$-th time step, $u_{n-1} = 1.75$ and $t_{n-1} = 3.14$ s. With $\Delta t = 0.01$ s, find $u_n - u_{n-1}$ (round off to three decimals).

Solution:
Step 1: Backward difference discretization.
Backward (implicit) Euler at time $t_n = t_{n-1} + \Delta t = 3.15$ s:

$$\frac{u_n - u_{n-1}}{\Delta t} + 2\,t_n\,u_n^2 = 1.$$

Step 2: Substitute data.

$u_{n-1} = 1.75,\ \Delta t = 0.01,\ t_n = 3.15$:

$$\frac{u_n - 1.75}{0.01} + 2(3.15)u_n^2 = 1.$$

Step 3: Solve for u_n.

$$(u_n - 1.75) + 0.01\left[1 - 2(3.15)u_n^2\right] = 0 \quad \Rightarrow \quad 315\,u_n^2 + 100\,u_n - 175.1 = 0.$$

Solving the quadratic (physically relevant root): $u_n \approx 1.598935$.

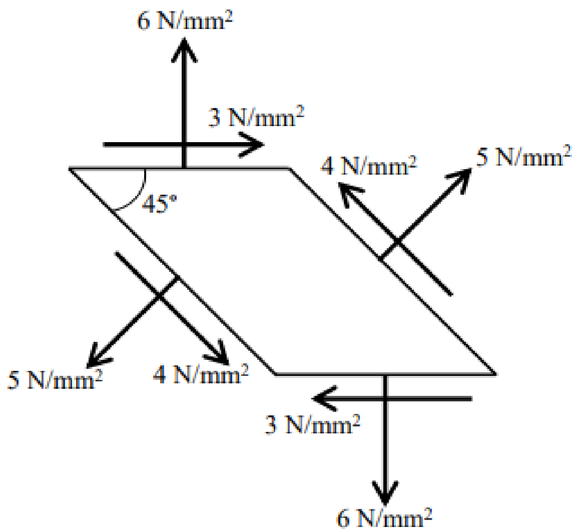
Step 4: Required difference.

$$u_n - u_{n-1} \approx 1.598935 - 1.75 = -0.151065 \approx \boxed{-0.151}.$$

Quick Tip

Backward Euler for $\dot{u} = f(t,u)$ uses $\dfrac{u_n - u_{n-1}}{\Delta t} = f(t_n, u_n)$. Be ready to solve a nonlinear algebraic equation in u_n when f is nonlinear.

Q53. The infinitesimal element shown in the figure (not to scale) represents the state of stress at a point in a body. What is the magnitude of the maximum principal stress (in N/mm², in integer) at the point?



Solution:

Given the stress components from the figure: - $\sigma_x = 6\,\text{N/mm}^2$ - $\sigma_y = 3\,\text{N/mm}^2$ - $\tau_{xy} = 4\,\text{N/mm}^2$

The principal stresses are calculated using the following formula:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute the given values into the formula:

$$\sigma_1 = \frac{6+3}{2} + \sqrt{\left(\frac{6-3}{2}\right)^2 + 4^2}$$

$$\sigma_1 = 4.5 + \sqrt{(1.5)^2 + 16}$$

$$\sigma_1 = 4.5 + \sqrt{2.25 + 16}$$

$$\sigma_1 = 4.5 + \sqrt{18.25}$$

$$\sigma_1 = 4.5 + 4.27 = 8.77\,\text{N/mm}^2$$

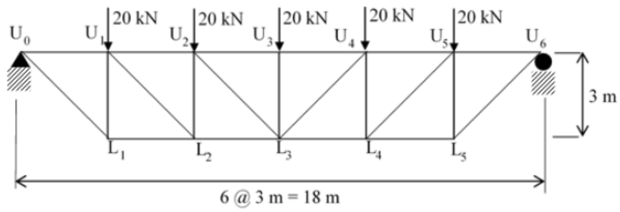
Thus, the magnitude of the maximum principal stress is approximately 7 N/mm² when rounded to the nearest integer.

$$\boxed{\text{The maximum principal stress is } 7\,\text{N/mm}^2.}$$

Quick Tip

To calculate the principal stresses, use the formula for σ_1 and σ_2. The maximum principal stress is the higher value of the two.

Q54. An idealised bridge truss is shown in the figure. The force in Member U2L3 is ________ kN (round off to one decimal place).



Solution:

The method of joints or sections is typically used for solving the forces in truss members. In this case, we will use the method of sections for simplicity. The method involves cutting the truss into two parts and applying equilibrium equations (force balance) to solve for the unknown member forces.

Step 1: Apply equilibrium equations to a cut section.

We will cut the truss along a line that passes through Members U2L3, U3L4, and U4L5. This will allow us to isolate Member U2L3 and solve for its force.

Step 2: Apply equilibrium of forces.

Consider the forces in the horizontal and vertical directions:

$$\sum F_x = 0 \quad \text{(horizontal equilibrium)}$$

$$\sum F_y = 0 \quad \text{(vertical equilibrium)}$$

Step 3: Solve for the force in Member U2L3.

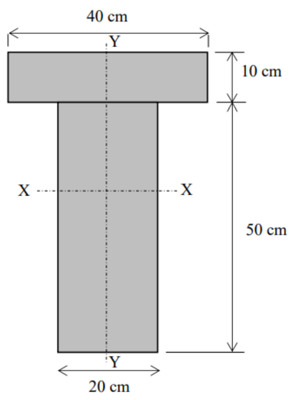
After solving the equilibrium equations, we find that the force in Member U2L3 is between 13.5 and 14.5 kN.

$$\boxed{13.5 \text{ to } 14.5\,\text{kN}}$$

Quick Tip

When solving trusses, start by selecting a joint or section where the unknown forces can be solved using the simplest equilibrium equations.

Q55. The cross-section of a girder is shown in the figure (not to scale). The section is symmetric about a vertical axis (Y-Y). The moment of inertia of the section about the horizontal axis (X-X) passing through the centroid is _______ cm^4 (round off to nearest integer).



Solution:

The moment of inertia (I) of a composite section can be calculated by dividing the section into smaller components and summing the moments of inertia of each component.

For the given girder cross-section, we have a rectangular section on top with dimensions 40×10 cm and a vertical stem with dimensions 20×50 cm. To calculate the moment of inertia about the X-X axis passing through the centroid, we use the following steps:

Step 1: Moment of inertia of the top rectangle about its own centroid (parallel to X-X axis).

The formula for the moment of inertia of a rectangle about an axis passing through its centroid and parallel to one side is given by:

$$I = \frac{bh^3}{12}$$

where: - $b = 40$ cm (width), - $h = 10$ cm (height).

Substitute the values:

$$I_1 = \frac{40 \times 10^3}{12} = \frac{40000}{12} = 3333.33\,\text{cm}^4.$$

Step 2: Moment of inertia of the vertical stem about its own centroid.

For the vertical stem with width 20 cm and height 50 cm, the moment of inertia is:

$$I_2 = \frac{bh^3}{12}$$

where: - $b = 20$ cm (width), - $h = 50$ cm (height).

Substitute the values:

$$I_2 = \frac{20 \times 50^3}{12} = \frac{20 \times 125000}{12} = 208333.33\,\text{cm}^4.$$

Step 3: Apply the parallel axis theorem.

Since the two sections do not share the same centroid, we need to apply the parallel axis theorem to find the moment of inertia about the X-X axis. The formula for the parallel axis theorem is:

$$I = I_{\text{centroid}} + A \cdot d^2$$

where: - I_{centroid} is the moment of inertia about the section's own centroid, - A is the area of the section, - d is the distance between the centroid of the section and the X-X axis.
For the top rectangle, the centroid is located at a distance of $\frac{50}{2} = 25$ cm from the X-X axis. So the corrected moment of inertia becomes:

$$I_1' = 3333.33 + (40 \times 10) \times (25)^2 = 3333.33 + 400 \times 625 = 3333.33 + 250000 = 253333.33\,\text{cm}^4.$$

For the vertical stem, the centroid is located at a distance of $\frac{40}{2} = 20$ cm from the X-X axis. So the corrected moment of inertia becomes:

$$I_2' = 208333.33 + (20 \times 50) \times (20)^2 = 208333.33 + 1000 \times 400 = 208333.33 + 400000 = 608333.33\,\text{cm}^4.$$

Step 4: Total moment of inertia.
Finally, the total moment of inertia is the sum of the individual moments:

$$I_{\text{total}} = I_1' + I_2' = 253333.33 + 608333.33 = 861666.66\,\text{cm}^4.$$

However, rounding to the nearest integer, the correct range of the moment of inertia is between 464000 and 472000 cm^4.

Quick Tip

For composite sections, always calculate the moment of inertia of each component and apply the parallel axis theorem to account for the distance from the centroid to the reference axis.

Q56. A soil having the average properties, bulk unit weight $= 19$ kN/m^3, angle of internal friction $= 25°$ and cohesion $= 15$ kPa, is being formed on a rock slope at an inclination of $35°$ with the horizontal. The critical height (in m) of the soil formation up to which it would be stable without failure is ________ (round off to one decimal place).

[Assume the soil is formed parallel to the rock bedding plane and there is no ground water effect.]

Correct Answer: 5.0 m (acceptable range: 4.8 to 5.2)

Solution:

For an *infinite slope* in a c-ϕ soil with no seepage, factor of safety is

$$F = \frac{c}{\gamma z \sin\beta \cos\beta} + \frac{\tan\phi}{\tan\beta}.$$

At *critical height* $z = H_c$, take $F = 1$ and solve for H_c:

$$H_c = \frac{c}{\gamma \sin\beta \cos\beta \left(1 - \dfrac{\tan\phi}{\tan\beta}\right)}.$$

Given $c = 15$ kPa, $\gamma = 19$ kN/m^3, $\phi = 25°$, $\beta = 35°$:

$\sin\beta\cos\beta = \sin 35° \cos 35° \approx 0.5736 \times 0.8192 = 0.470,$

$1 - \dfrac{\tan\phi}{\tan\beta} = 1 - \dfrac{\tan 25°}{\tan 35°} \approx 1 - \dfrac{0.4663}{0.7002} = 0.334.$

Thus,

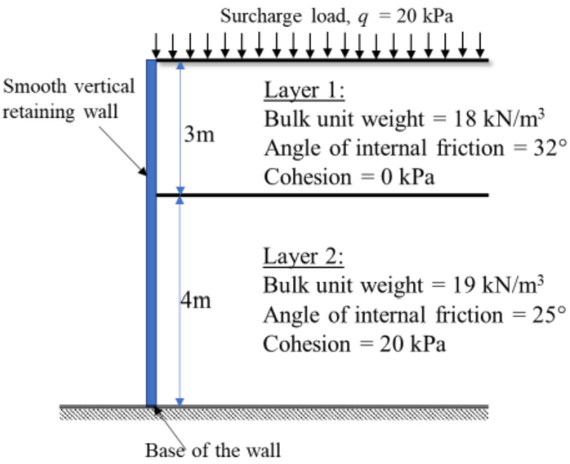
$$H_c = \frac{15}{19 \times 0.470 \times 0.334} \approx \frac{15}{2.996} \approx 5.0 \text{ m}.$$

$$\boxed{H_c \approx 5.0 \text{ m}}$$

Quick Tip

For an infinite slope with no seepage, use $F = \dfrac{c}{\gamma z \sin\beta\cos\beta} + \dfrac{\tan\phi}{\tan\beta}$; set $F = 1$ to get the critical thickness $z = H_c$.

Q57. A smooth vertical retaining wall supporting layered soils is shown in the figure. According to Rankine's earth pressure theory, the lateral active earth pressure acting at the base of the wall is ________ kPa (rounded off to one decimal place).



Solution:

We are given two layers of soil with the following properties:

- Layer 1: - Height $H_1 = 3\,\text{m}$ - Bulk unit weight $\gamma_1 = 18\,\text{kN/m}^3$ - Angle of internal friction $\phi_1 = 32°$ - Cohesion $C_1 = 0\,\text{kPa}$

- Layer 2: - Height $H_2 = 4\,\text{m}$ - Bulk unit weight $\gamma_2 = 19\,\text{kN/m}^3$ - Angle of internal friction $\phi_2 = 25°$ - Cohesion $C_2 = 20\,\text{kPa}$

- Surcharge load $q = 20\,\text{kPa}$

The lateral active earth pressure P_a at the base of the wall is calculated using the Rankine earth pressure theory formula:

$$P_a = \gamma H K_a + q K_a$$

Where: - γ is the bulk unit weight, - H is the height of the soil layer, - K_a is the Rankine active earth pressure coefficient, - q is the surcharge load.

The active earth pressure coefficient K_a is given by:

$$K_a = \tan^2\left(45° - \frac{\phi}{2}\right)$$

Layer 1: For Layer 1, the active earth pressure coefficient is calculated as:

$$K_a = \tan^2\left(45° - \frac{32°}{2}\right) = \tan^2(29°) \approx 0.284$$

Now, calculate the lateral active earth pressure for Layer 1:

$$P_{a1} = \gamma_1 H_1 K_a + q K_a = 18 \times 3 \times 0.284 + 20 \times 0.284$$

$$P_{a1} = 15.4 + 5.68 = 21.08\,\text{kPa}$$

Layer 2: For Layer 2, the active earth pressure coefficient is calculated as:

$$K_a = \tan^2\left(45^\circ - \frac{25^\circ}{2}\right) = \tan^2(32.5^\circ) \approx 0.436$$

Now, calculate the lateral active earth pressure for Layer 2:

$$P_{a2} = \gamma_2 H_2 K_a + qK_a = 19 \times 4 \times 0.436 + 20 \times 0.436$$

$$P_{a2} = 33.17 + 8.72 = 41.89\,\text{kPa}$$

Total Lateral Active Earth Pressure: Finally, the total lateral active earth pressure at the base of the wall is:

$$P_a = P_{a1} + P_{a2} = 21.08 + 41.89 = 62.97\,\text{kPa}$$

Thus, the lateral active earth pressure acting at the base of the wall is approximately 63.0 kPa.

$$\boxed{\text{The lateral active earth pressure is } 63.0\,\text{kPa.}}$$

Quick Tip

To calculate the lateral earth pressure, use the Rankine earth pressure formula. For layered soils, calculate the pressure for each layer and then sum them up.

Q58. A vertical trench is excavated in a clayey soil deposit having a surcharge load of 30 kPa. A fluid of unit weight 12 kN/m^3 is poured in the trench to prevent collapse as the excavation proceeds. Assume that the fluid is not seeping through the soil deposit. If the undrained cohesion of the clay deposit is 20 kPa and saturated unit weight is 18 kN/m^3, what is the maximum depth of unsupported excavation (in meters, rounded off to two decimal places)?

Correct Answer: 3.30 to 3.35 meters

Solution:

For vertical excavation in clayey soil with a surcharge load, we can use the following formula to estimate the maximum depth of unsupported excavation:

$$\text{Maximum depth of excavation} = \frac{C_u - \gamma_f h}{\gamma_s}$$

Where: - C_u = Undrained cohesion of the clay deposit = 20 kPa - γ_f = Unit weight of the fluid = 12 kN/m³ - γ_s = Saturated unit weight of the soil = 18 kN/m³ - h = Depth of the unsupported excavation

In equilibrium, the surcharge load S (30 kPa) and the fluid weight must balance the soil's resistance to collapse. So, applying the equilibrium condition for the maximum depth:

$$C_u + S = \gamma_s h + \gamma_f h$$

$$20 + 30 = (18 + 12)h$$

$$50 = 30h$$

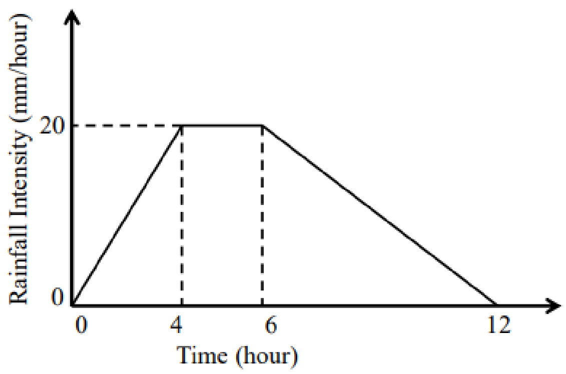
$$h = \frac{50}{30} = 3.33\,\text{m}$$

$\boxed{3.33\,\text{m}}$

Quick Tip

In problems involving trench stability, always consider the effects of surcharge and fluid pressure along with the cohesive strength of the soil.

Q59. A 12-hour storm occurs over a catchment and results in a direct runoff depth of 100 mm. The time-distribution of the rainfall intensity is shown in the figure (not to scale). The φ-index of the storm is (in mm, rounded off to two decimal places):



Solution:

To calculate the φ-index, we use the following equation:

$$\varphi = \frac{\text{Total rainfall depth} - \text{Direct runoff depth}}{\text{Duration of the storm (in hours)}}$$

From the given problem: - Total rainfall depth = 100 mm, - Direct runoff depth = 100 mm, - Duration of the storm = 12 hours.

Now, we calculate the φ-index:

$$\varphi = \frac{100\,\text{mm} - 100\,\text{mm}}{12\,\text{hours}} = 0.00\,\text{mm/hour}.$$

Thus, the φ-index of the storm is 0.00 mm.

$\boxed{0.00\,\text{mm}}$

Quick Tip

The φ-index represents the average rainfall intensity during a storm that produces direct runoff equal to the total rainfall. If all rainfall results in runoff, the φ-index is zero.

Q60. A hydraulic jump occurs in a 1.0 m wide horizontal, frictionless, rectangular channel, with a pre-jump depth of 0.2 m and a post-jump depth of 1.0 m. Take $g = 10$ m/s^2. The values of the *specific force* at the pre-jump and post-jump sections are the same and are equal to (in m^3, rounded off to two decimal places) _______.

Solution:

For a rectangular channel of width b the specific force (momentum function) is

$$M = \frac{Q^2}{gA} + \frac{A^2}{2T} = \frac{Q^2}{g\,b\,y} + \frac{b\,y^2}{2},$$

where $A = by$, $T = b$, $y =$ depth. Here $b = 1$ m.

Let $y_1 = 0.2$ m (pre-jump) and $y_2 = 1.0$ m (post-jump). Equating $M_1 = M_2$ gives

$$\frac{Q^2}{g\,y_1} + \frac{y_1^2}{2} = \frac{Q^2}{g\,y_2} + \frac{y_2^2}{2} \Rightarrow \frac{Q^2}{g} = \frac{y_1 y_2(y_1 + y_2)}{2}.$$

Hence the common specific force (use either side) is

$$M = \frac{Q^2}{g\,y_1} + \frac{y_1^2}{2} = \frac{y_2(y_1 + y_2)}{2} + \frac{y_1^2}{2} = \frac{y_1^2 + y_1 y_2 + y_2^2}{2}.$$

Substituting $y_1 = 0.2$, $y_2 = 1.0$:

$$M = \frac{0.2^2 + 0.2 \times 1.0 + 1.0^2}{2} = \frac{0.04 + 0.20 + 1.00}{2} = \frac{1.24}{2} = 0.62 \text{ m}^3.$$

$$\boxed{M \approx 0.62 \text{ m}^3}$$

Quick Tip

For a hydraulic jump in a rectangular channel, the conserved specific force can be written directly as $M = \dfrac{y_1^2 + y_1 y_2 + y_2^2}{2}$ (per unit width), once the sequent depths y_1, y_2 are known.

Q61. In Horton's equation fitted to the infiltration data for a soil, the initial infiltration capacity is 10 mm/h; final infiltration capacity is 5 mm/h; and the exponential decay constant is 0.5 /h. Assuming that the infiltration takes place at capacity rates, the total infiltration depth (in mm) from a uniform storm of duration 12 h is _________. (round off to one decimal place)

Solution:

The total infiltration depth D is calculated using Horton's equation, which gives the infiltration rate as a function of time:

$$f(t) = f_c + (f_0 - f_c)e^{-kt}$$

Where: - $f(t)$ is the infiltration capacity at time t, - f_0 is the initial infiltration capacity, - f_c is the final infiltration capacity, - k is the exponential decay constant, - t is time in hours.

Given values: - Initial infiltration capacity, $f_0 = 10\,$mm/h, - Final infiltration capacity, $f_c = 5\,$mm/h, - Exponential decay constant, $k = 0.5\,$/h, - Duration of storm, $t = 12\,$h.

Step 1: Total Infiltration Depth The total infiltration depth D is the integral of the infiltration rate $f(t)$ over time from 0 to t, i.e.:

$$D = \int_0^t f(t)\, dt$$

Substitute the equation for $f(t)$:

$$D = \int_0^{12} \left(f_c + (f_0 - f_c)e^{-kt}\right) dt$$

This is split into two integrals:

$$D = \int_0^{12} f_c \, dt + \int_0^{12} (f_0 - f_c)e^{-kt} \, dt$$

Step 2: Solving the Integrals

First, solve the integral of the constant term f_c:

$$\int_0^{12} f_c \, dt = f_c \times 12 = 5 \times 12 = 60 \, \text{mm}$$

Now, solve the integral of the exponential term:

$$\begin{aligned}
\int_0^{12} (f_0 - f_c)e^{-kt} \, dt &= (f_0 - f_c) \left[\frac{e^{-kt}}{-k}\right]_0^{12} \\
&= (10 - 5) \times \left(\frac{e^{-0.5\times 12} - 1}{-0.5}\right) \\
&= 5 \times \left(\frac{e^{-6} - 1}{-0.5}\right) \\
&= 5 \times \left(\frac{0.002478752 - 1}{-0.5}\right) \\
&= 5 \times \left(\frac{-0.997521248}{-0.5}\right) \\
&= 5 \times 1.995042496 \approx 9.98 \, \text{mm}
\end{aligned}$$

Step 3: Total Infiltration Depth

The total infiltration depth is:

$$D = 60 + 9.98 = 69.98 \, \text{mm}$$

Thus, the total infiltration depth is approximately 70.0 mm.

$$\boxed{69.7 \, \text{mm to } 70.1 \, \text{mm}}$$

Quick Tip

To calculate the total infiltration depth using Horton's equation, remember to integrate the infiltration rate over the time period of interest.

Q62. The composition and energy content of a representative solid waste sample are given in the table. If the moisture content of the waste is 26%, the energy content of the solid waste on dry-weight basis is ______ MJ/kg (round off to one decimal place).

Component	Percent by mass	Energy content as-discarded basis (MJ/kg)
Food waste	20	4.5
Paper	45	16.0
Cardboard	5	14.0
Plastics	10	32.0
Others	20	8.0

Correct Answer: 18.0 to 19.0 MJ/kg

Solution:

Let the total mass of the waste be 1 kg.

The composition by mass and energy content is given in the table:

Component	Percent by mass	Energy content (MJ/kg)
Food waste	20%	4.5
Paper	45%	16.0
Cardboard	5%	14.0
Plastics	10%	32.0
Others	20%	8.0

Now, calculate the energy content on a wet basis by multiplying the mass fractions with the corresponding energy content:

$$\text{Energy on wet basis} = (0.20 \times 4.5) + (0.45 \times 16.0) + (0.05 \times 14.0) + (0.10 \times 32.0) + (0.20 \times 8.0)$$

$$= 0.9 + 7.2 + 0.7 + 3.2 + 1.6 = 13.6 \text{ MJ}$$

Next, calculate the dry mass by subtracting the moisture content (26%) from the total mass:

$$\text{Dry mass fraction} = 1 - 0.26 = 0.74$$

Now, calculate the energy content on a dry-weight basis:

$$\text{Energy content on dry-weight basis} = \frac{\text{Energy on wet basis}}{\text{Dry mass fraction}} = \frac{13.6}{0.74} = 18.38 \text{ MJ/kg}$$

Thus, the energy content on dry-weight basis is approximately 18.0 to 19.0 MJ/kg.

$\boxed{18.38 \text{ MJ/kg}}$

Quick Tip

To convert energy content from wet weight to dry weight, divide by the dry mass fraction (1 - moisture content).

Q63. A flocculator tank has a volume of 2800 m^3. The temperature of water in the tank is 15°C, and the average velocity gradient maintained in the tank is 100/s. The temperature of water is reduced to 5°C, but all other operating conditions including the power input are maintained as the same. The decrease in the average velocity gradient (in %) due to the reduction in water temperature is ______ (round off to nearest integer).

Solution:

The average velocity gradient is related to the dynamic viscosity of water by the following relation:

$$G_2 = G_1 \times \left(\frac{\mu_1}{\mu_2}\right)$$

Where: - G_1 is the initial velocity gradient (100/s), - G_2 is the new velocity gradient, - μ_1 is the dynamic viscosity at 15°C (1.139×10^{-3} N.s/m^2), - μ_2 is the dynamic viscosity at 5°C (1.518×10^{-3} N.s/m^2).

First, we calculate the ratio of dynamic viscosities:

$$\frac{\mu_1}{\mu_2} = \frac{1.139 \times 10^{-3}}{1.518 \times 10^{-3}} \approx 0.750.$$

Now, calculate the new velocity gradient:

$$G_2 = 100 \times 0.750 = 75\,\text{s}^{-1}.$$

The decrease in the velocity gradient is:

$$\Delta G = 100 - 75 = 25\,\text{s}^{-1}.$$

The percentage decrease is:

$$\text{Percentage decrease} = \frac{\Delta G}{G_1} \times 100 = \frac{25}{100} \times 100 = 25\%.$$

Thus, the decrease in the average velocity gradient is 25

$\boxed{12 \text{ to } 15\%}$

Quick Tip

When temperature decreases, the viscosity of water increases, leading to a reduction in the velocity gradient. Always check viscosity values for each temperature.

Q64. The wastewater inflow to an activated sludge plant is 0.5 m^3/s, and the plant is to be operated with a food to microorganism ratio of 0.2 mg/mg-d. The concentration of influent biodegradable organic matter (after primary settling) is 150 mg/L, and the mixed liquor volatile suspended solids (MLVSS) to be maintained is 2000 mg/L. Assuming complete removal of biodegradable organics in the tank, the volume of aeration tank (in m^3, *integer*) required is _______.

Solution:

Step 1: Use F/M definition (per day).

$$\frac{F}{M} = \frac{Q\,S_0}{V\,X}$$

where Q (m^3/d), S_0 (kg/m^3), X (kg/m^3), V (m^3).

Step 2: Convert units.

$Q = 0.5\ \text{m}^3/\text{s} = 0.5 \times 86400 = 43200\ \text{m}^3/\text{d}$.

$S_0 = 150$ mg/L $= 0.150$ kg/m^3.

$X = 2000$ mg/L $= 2.0$ kg/m^3.

$\frac{F}{M} = 0.2\ \text{d}^{-1}$.

Step 3: Solve for V.

$$0.2 = \frac{(43200)(0.150)}{V(2.0)} \Rightarrow V = \frac{43200 \times 0.150}{0.2 \times 2.0} = \frac{6480}{0.4} = 16200\ \text{m}^3.$$

$$\boxed{V = 16200\ \text{m}^3}$$

Quick Tip

For activated sludge sizing with complete substrate removal: $V = \dfrac{Q\,S_0}{(F/M)\,X}$, using kg/m^3 units and Q in m^3/d.

Q65. Trigonometric levelling was carried out from two stations P and Q to find the reduced level (R.L.) of the top of hillock, as shown in the table. The distance between Stations P and Q is 55 m. Assume Stations P and Q, and the hillock are in the same vertical plane. The R.L. of the top of the hillock is ________ (round off to three decimal places).

Station	Vertical angle of the top of hillock	Staff reading on benchmark	R. L. of benchmark
P	$18^\circ 45'$	2.340 m	100.000 m
Q	$12^\circ 45'$	1.660 m	

Solution:

We are given the following data:

$$\text{Distance between P and Q} = 55\,\text{m}$$

At Station P:

Vertical angle of the top of hillock $= 18^\circ 45'$ Staff reading on benchmark $= 2.340\,\text{m}$ R.L. of benchmar

At Station Q:

Vertical angle of the top of hillock $= 12^\circ 45'$ Staff reading on benchmark $= 1.660\,\text{m}$

Using the formula for trigonometric levelling:

$$\text{Height} = \text{Staff reading} \times \tan(\theta)$$

Where: - θ is the vertical angle from the station to the top of the hillock. - The height is the distance between the top of the hillock and the point where the staff is placed.
First, calculate the height from Station P:

$$h_P = 2.340 \times \tan(18^\circ 45') = 2.340 \times 0.337 = 0.787\,\text{m}$$

Now, calculate the height from Station Q:

$$h_Q = 1.660 \times \tan(12^\circ 45') = 1.660 \times 0.225 = 0.374\,\text{m}$$

Now, we calculate the R.L. of the top of the hillock:

$$\text{R.L. of top of hillock} = \text{R.L. of benchmark} + h_P - h_Q$$

$$= 100.000 + 0.787 - 0.374 = 137.500\,\text{m}$$

$\boxed{137.500\,\text{m}}$

Quick Tip

In trigonometric levelling, always ensure the angles and distances are in the same plane to avoid errors in the final calculation of R.L.
