GATE 2023 Electronics and Communication Engineering (EC) Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 100	Total questions : 65

General Aptitude (GA)

Q.1 "I cannot support this proposal. My ______ will not permit it."

(A) conscious

(B) consensus

(C) conscience

(D) consent

Correct Answer: (C) conscience

Solution:

(A) **conscious**: means being awake or aware. This does not fit the context.

(B) **consensus**: means general agreement among a group, which is unrelated to the speaker's moral restraint.

(C) **conscience**: means the inner moral sense of right and wrong, which clearly matches the sentence.

(D) **consent**: means permission or approval, but the sentence refers to moral principles, not permission.

Correct word: conscience

Quick Tip

Conscience = moral sense of right and wrong. **Conscious** = awake/aware. **Consensus** = group agreement. **Consent** = permission.

Q2. Courts : ______ :: Parliament : Legislature   (By word meaning)

(A) Judiciary

(B) Executive

(C) Governmental

(D) Legal

Correct Answer: (A) Judiciary

Solution:

Step 1: Identify the given relation in the second pair. Parliament is an *institution* that belongs to the *Legislature* branch of the state.

Hence, the pattern is: Institution/Body : Branch of Government.

Step 2: Apply the same relation to the first pair. Courts are institutions/bodies that belong to the *Judiciary* branch. $\Rightarrow$ Courts : Judiciary.

Step 3: Eliminate distractors.

(B) Executive — this is a different branch (not the one to which courts belong).

(C) Governmental — generic adjective, not a specific branch.

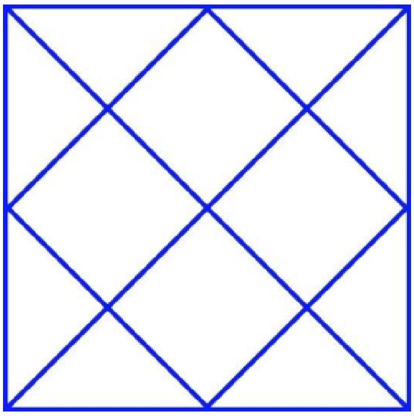
(D) Legal — adjective; the pair would break the noun:noun structure and hierarchy relation.

$$\boxed{\text{Courts : Judiciary :: Parliament : Legislature}}$$

Quick Tip

In analogy questions, first lock the *relation type* (here, body $\rightarrow$ branch of state) and keep the *parts of speech* parallel (noun:noun). Then eliminate options that are different branches or merely adjectives.

Q.3 What is the smallest number with distinct digits whose digits add up to 45?



(A) 123555789

(B) 123457869

(C) 123456789

(D) 99999

Correct Answer: (C)

Solution:

Step 1: Minimum count of distinct digits to reach 45.

With 8 distinct digits, max sum $= 9 + 8 + \cdots + 2 = 44 < 45 \Rightarrow$ need 9 digits.

Step 2: Can 0 be included among 9 distinct digits?

If 0 is included, the other eight (at most 2–9) sum to $44 \Rightarrow$ cannot reach 45. So the digits must be exactly $\{1, 2, \ldots, 9\}$.

Step 3: Smallest number with those digits.

Arrange in increasing order $\Rightarrow$ 123456789.

$$\boxed{123456789}$$

Quick Tip

When a fixed digit-sum is required with distinct digits, first check the maximum achievable sum with k digits and whether 0 can appear; then sort the needed digits in increasing order for the smallest number.

Q.4 In a class of 100 students, (i) there are 30 students who neither like romantic movies nor comedy movies, (ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and (iii) the number of students who like both romantic movies and comedy movies is 20.
How many students in the class like romantic movies?

(A) 40

(B) 20

(C) 60

(D) 30

Correct Answer: 60

Step 1: Define sets.

Let R = number of students who like romantic movies.

Let C = number of students who like comedy movies.

Given: $R = 2C$.

Number who like both = $R \cap C = 20$.

Neither romantic nor comedy = 30.

Step 2: Apply inclusion-exclusion.

Total students $= 100$.

Thus, $R + C - (R \cap C) + 30 = 100$.

$$R + C - 20 + 30 = 100 \quad \Rightarrow \quad R + C = 90.$$

Step 3: Solve equations.

We have $R = 2C$. Substituting:

$$2C + C = 90 \quad \Rightarrow \quad 3C = 90 \quad \Rightarrow \quad C = 30.$$

Then $R = 2C = 60$.

Step 4: Answer.

Number of students who like romantic movies $= R = 60$.

60

Quick Tip

Always apply the inclusion-exclusion principle carefully and include the count of students outside both sets.

Q.5 How many rectangles are present in the given figure?

(A) 8

(B) 9

(C) 10

(D) 12

Correct Answer: (C) 10

Solution:

Treat every square as a rectangle (a square is a special rectangle).

Inside the big square, the slant lines form a 2×2 grid of small *diamond* squares.

Count rectangles tilted at $45°$:

- Small diamonds: 4
- Rectangles made by pairing two adjacent small diamonds (horizontal pairs $= 2$, vertical pairs $= 2$): 4
- Large diamond formed by all four small diamonds: 1

Axis-aligned rectangles:

- The outer big square: 1

Total rectangles $= 4 + 4 + 1 + 1 = 10$.

10

Quick Tip

When shapes are tilted, first count the smallest tiles, then count all possible unions of adjacent tiles (pairs, 2×2, etc.). Remember: every square counts as a rectangle.

Q6. Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious. Based only on the information provided above, which one of the following options can be logically inferred with certainty?

(A) All creatures with claws are animals.

(B) Some creatures with claws are non-ferocious.

(C) Some non-ferocious creatures have claws.

(D) Some ferocious creatures are creatures with claws.

Correct Answer: (D)

Solution:

Given facts:

1) All animals are ferocious. $\Rightarrow$ Animals $\subseteq$ Ferocious.

2) There exist creatures with claws and creatures without claws. $\Rightarrow$ At least one creature has claws.

3) All creatures with claws are ferocious. $\Rightarrow$ Claws $\subseteq$ Ferocious.

Check options:

(A) Not implied; clawed creatures could be non-animals. $\Rightarrow$ **Cannot be inferred**.

(B) Contradicts (3). $\Rightarrow$ **False**.

(C) Contradicts (3). $\Rightarrow$ **False**.

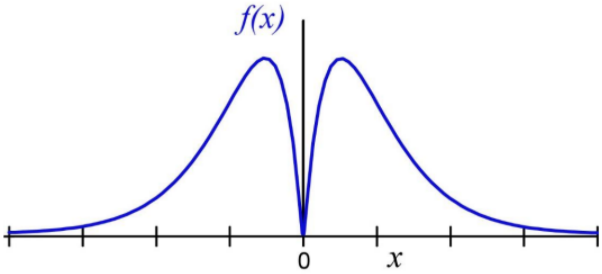
(D) From (2) and (3), at least one creature is both ferocious and clawed. $\Rightarrow$ **True**.

$$\boxed{\text{(D) Some ferocious creatures are creatures with claws.}}$$

Quick Tip

Use set relations: "All A are B" $\Rightarrow A \subseteq B$; "Some A are B" $\Rightarrow A \cap B \neq \varnothing$. Existence of a clawed creature + "claws $\subseteq$ ferocious" $\Rightarrow$ (claws) $\cap$ (ferocious) $\neq \varnothing$.

Q.7 Which one of the following options represents the given graph?



(A) $f(x) = x^2\, 2^{-|x|}$

(B) $f(x) = x\, 2^{-|x|}$

(C) $f(x) = |x|\, 2^{-x}$

(D) $f(x) = x\, 2^{-x}$

Correct Answer: (A)

Solution:

Step 1: Symmetry from the graph.

The curve is symmetric about the y-axis and nonnegative with a sharp dip to 0 at $x = 0$ and two equal peaks on either side. $\Rightarrow f$ must be an *even*, nonnegative function with $f(0) = 0$.

Step 2: Test options against symmetry and sign.

(B) $x\, 2^{-|x|}$ and (D) $x\, 2^{-x}$ are *odd* (change sign across 0), so they cannot match a symmetric, always-nonnegative graph.

(C) $|x|\, 2^{-x}$ is not even; for $x < 0$, $2^{-x} = 2^{|x|}$ grows exponentially, so the left tail blows up instead of decaying to 0—not the graph.

Step 3: Check (A) $x^2 2^{-|x|}$.

- Even: depends on x^2 and $|x| \Rightarrow$ symmetric.

- $f(0) = 0^2 \cdot 2^0 = 0$ (central dip).

- As $|x| \to \infty$, the exponential $2^{-|x|}$ dominates the polynomial $x^2 \Rightarrow f(x) \to 0$ (decaying tails).
- For small $|x| > 0$, x^2 grows from 0 before the exponential decay takes over, producing two symmetric peaks.
Hence (A) matches the graph.

$$\boxed{f(x) = x^2 2^{-|x|}}$$

Quick Tip

To match a graph, first infer symmetry (even/odd) and end behavior. Exponential decay like $2^{-|x|}$ forces both tails to 0, while a factor x^2 ensures evenness and a zero at the origin.

Q.8 Which one of the following options can be inferred from the given passage alone?
When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

[Excerpt from *The Truth about Stories* by T. King]

(A) It is a child's description of what he or she likes.
(B) It is an adult's memory of what he or she liked as a child.
(C) The child in the passage read stories about interplanetary travel only in parts.
(D) It teaches us that stories are good for children.

Correct Answer: (B)

Solution:
Step 1: Identify the narrator's perspective.
The sentence begins with "*When I was a kid*," which clearly indicates a speaker recalling childhood from a later time $\Rightarrow$ adult reminiscence.
Step 2: Evaluate options.
(A) Claims it is a child speaking now; contradicted by "*was*." $\Rightarrow$ False.

(C) Adds “only in parts,” which is not stated or implied. ⇒ Unsupported.

(D) Makes a general lesson claim; the passage is merely personal, not didactic. ⇒ Unsupported.

(B) Exactly matches: an adult remembering what they liked as a child. ⇒ True.

(B) is the correct inference.

Quick Tip

Look for tense cues like “was,” “used to,” or time markers (“when I was…”) to decide whether a passage is memory/reflection versus present description.

Q.9 Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:

(i) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.

(ii) Mix the samples within each set and test the mixed sample for covid.

(iii) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.

(iv) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.

Given this strategy, no more than _____ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(A) 700

(B) 600

(C) 800

(D) 1000

Correct Answer: (A) 700

Solution:

Step 1: Grouping.

There are 1000 individuals, grouped into sets of 5.

Thus, total groups $= 1000/5 = 200$.

Each group is tested once (200 tests).

Step 2: Worst-case scenario.

Worst case: All 100 covid positives are in distinct groups (so that maximum groups turn positive).

Then 100 groups test positive.

Step 3: Individual testing.

For each positive group, all 5 individuals are separately tested.

So, additional tests $= 100 \times 5 = 500$.

Step 4: Total tests.

Initial group tests $= 200$, plus individual tests $= 500$.

Total $= 200 + 500 = 700$.

$$\boxed{700}$$

Quick Tip

Pooling strategy reduces test kits drastically. Worst-case analysis assumes maximum groups testing positive, which determines the upper bound.

Q.10 A 100 cm $\times$ 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions 100 cm $\times$ 1 cm.

The total number of creases visible when the sheet is unfolded is ______.

(A) 32

(B) 5

(C) 31

(D) 63

Correct Answer: (C) 31

Solution:

Step 1: Understand the fold direction. Each fold aligns the two long (100 cm) edges, so the fold axis is parallel to the long edge and halves the 32 cm side every time.

Step 2: Compute final thickness along the folded dimension. After 5 folds:

$32 \div 2^5 = 32 \div 32 = 1$ cm, matching the statement.

Step 3: Count creases after unfolding. Repeated halving along the same direction divides the sheet into 2^n equal strips along that direction, separated by straight crease lines.

Hence, with $n = 5$ folds, the number of crease lines $= 2^5 - 1 = 31$.

$$\boxed{31}$$

Quick Tip

Folding the same dimension n times creates 2^n equal segments and therefore $2^n - 1$ crease lines upon unfolding. If dimensions confirm (original$/2^n$), you're on the right track.

Q.11 Let $\mathrm{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\mathrm{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ be two vectors. The value of the coefficient α in the expression $\mathrm{v}_1 = \alpha \mathrm{v}_2 + \mathrm{e}$, which minimizes the length of the error vector e, is

(A) $\frac{7}{2}$

(B) $-\frac{2}{7}$

(C) $\frac{2}{7}$

(D) $-\frac{7}{2}$

Correct Answer: (C)

Solution:

Step 1: Least–squares condition (orthogonal projection).

To minimize $\|\mathbf{e}\|$ in $\mathbf{v}_1 = \alpha\mathbf{v}_2 + \mathbf{e}$, we need $\mathbf{e} \perp \mathbf{v}_2$. Hence $\mathbf{v}_2^\top(\mathbf{v}_1 - \alpha\mathbf{v}_2) = 0 \Rightarrow \alpha = \frac{\mathbf{v}_2^\top\mathbf{v}_1}{\mathbf{v}_2^\top\mathbf{v}_2}$.

Step 2: Compute the dot products.

$\mathbf{v}_2^\top\mathbf{v}_1 = [2,1,3]\cdot[1,2,0] = 2\cdot1 + 1\cdot2 + 3\cdot0 = 4.$

$\mathbf{v}_2^\top\mathbf{v}_2 = 2^2 + 1^2 + 3^2 = 4 + 1 + 9 = 14.$

Step 3: Evaluate α.

$$\alpha = \frac{4}{14} = \frac{2}{7}.$$

$$\boxed{\frac{2}{7}}$$

Quick Tip

For $\mathbf{v}_1 \approx \alpha\mathbf{v}_2$, the best α in least squares is always the projection coefficient: $\alpha = \frac{\mathbf{v}_2\cdot\mathbf{v}_1}{\mathbf{v}_2\cdot\mathbf{v}_2}$.

Q.12 The rate of increase, of a scalar field $f(x,y,z) = xyz$, in the direction $v = (2,1,2)$ at a point $(0,2,1)$ is

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) 2

(D) 4

Correct Answer: (B) $\frac{4}{3}$

Step 1: Gradient of $f(x, y, z)$.

$$f(x, y, z) = xyz$$
$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (yz,\ xz,\ xy).$$

Step 2: Evaluate at $(0, 2, 1)$.

At $(0, 2, 1)$:

$$\nabla f = (2 \cdot 1,\ 0 \cdot 1,\ 0 \cdot 2) = (2, 0, 0).$$

Step 3: Direction vector.

Given direction $v = (2, 1, 2)$. Unit vector:

$$\hat{v} = \frac{(2, 1, 2)}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{(2, 1, 2)}{\sqrt{9}} = \left(\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{2}{3}\right).$$

Step 4: Directional derivative.

$$D_v f = \nabla f \cdot \hat{v} = (2, 0, 0) \cdot \left(\tfrac{2}{3}, \tfrac{1}{3}, \tfrac{2}{3}\right) = 2 \cdot \tfrac{2}{3} = \tfrac{4}{3}.$$

$$\boxed{\tfrac{4}{3}}$$

Quick Tip

The rate of increase of a scalar field in a direction v is the directional derivative: $\nabla f \cdot \dfrac{v}{|v|}$. Always normalize the direction vector before dotting.

Q.13 Let $w^4 = 16j$. Which of the following cannot be a value of w?

(A) $2e^{j\frac{2\pi}{8}}$

(B) $2e^{j\frac{\pi}{8}}$

(C) $2e^{j\frac{5\pi}{8}}$

(D) $2e^{j\frac{9\pi}{8}}$

Correct Answer: (A) $2e^{j\frac{2\pi}{8}}$

Solution:

Step 1: Express RHS in polar form.

$16j = 16e^{j\pi/2}$.

Step 2: Find fourth roots.

If $w^4 = 16e^{j\pi/2}$, then:

$$w = \sqrt[4]{16}\, e^{j\left(\frac{\pi/2+2k\pi}{4}\right)}, \quad k = 0, 1, 2, 3$$

$$\Rightarrow w = 2e^{j\left(\frac{\pi}{8}+\frac{k\pi}{2}\right)}, \quad k = 0, 1, 2, 3$$

Step 3: Compute values.

- For $k = 0$: $w = 2e^{j\pi/8}$.

- For $k = 1$: $w = 2e^{j5\pi/8}$.

- For $k = 2$: $w = 2e^{j9\pi/8}$.

- For $k = 3$: $w = 2e^{j13\pi/8}$.

Step 4: Compare with options.

Options (B), (C), (D) match the valid roots.

Option (A) $2e^{j2\pi/8} = 2e^{j\pi/4}$ is not in the list.

$$\boxed{2e^{j\frac{2\pi}{8}} \text{ cannot be a value of } w}$$

Quick Tip

For n-th roots of a complex number $re^{j\theta}$, the solutions are $\sqrt[n]{r}\ \ e^{j(\theta+2k\pi)/n}$ for $k = 0, 1, \ldots, n-1$. Always check which angles appear in the set.

Q.14 The value of the contour integral, $\oint_C \left(\frac{z+2}{z^2+2z+2}\right) dz$, where the contour C is $\left\{z : \left|z+1-\frac{3}{2}j\right| = 1\right\}$, taken in the counter clockwise direction, is

(A) $-\pi(1+j)$

(B) $\pi(1+j)$

(C) $\pi(1-j)$

(D) $-\pi(1-j)$

Correct Answer: (B) $\pi(1+j)$

Solution:

Poles: $z^2+2z+2=(z+1)^2+1=0 \Rightarrow z=-1\pm j$.

The circle $|z+1-\frac{3}{2}j|=1$ is centered at $-1+\frac{3}{2}j$ with radius 1.

Distance to $-1+j$ is $\frac{1}{2}<1$ (inside); distance to $-1-j$ is $\frac{5}{2}>1$ (outside).

Residue at $z_1=-1+j$: For $f(z)=\dfrac{z+2}{z^2+2z+2}=\dfrac{g(z)}{h(z)}$, with $h'(z)=2z+2$,

$$\mathrm{Res}(f;z_1)=\frac{g(z_1)}{h'(z_1)}=\frac{(-1+j)+2}{2(-1+j)+2}=\frac{1+j}{2j}=\frac{1-j}{2}.$$

Integral (CCW): By the residue theorem, $\displaystyle\oint_C f(z)\,dz=2\pi j\cdot\frac{1-j}{2}=\pi j(1-j)=\pi(1+j)$.

$$\boxed{\pi(1+j)}$$

Quick Tip

For $\oint \frac{g(z)}{h(z)}dz$ with simple poles, use $\mathrm{Res}=\dfrac{g(z_k)}{h'(z_k)}$. Always check which poles lie inside the given circle before applying $2\pi j$ times the sum of residues.

Q.15 Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1\le k\le n\}$ and $\{\mathbf{v}_k \mid 1\le k\le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B=P^{-1}AP$, respectively, are

(A) $\{\lambda_k \det(A) \mid 1\le k\le n\}$ and $\{P\mathbf{v}_k \mid 1\le k\le n\}$

(B) $\{\lambda_k \mid 1\le k\le n\}$ and $\{\mathbf{v}_k \mid 1\le k\le n\}$

(C) $\{\lambda_k \mid 1\le k\le n\}$ and $\{P\mathbf{v}_k \mid 1\le k\le n\}$

(D) $\{\lambda_k \mid 1\le k\le n\}$ and $\{P^{-1}\mathbf{v}_k \mid 1\le k\le n\}$

Correct Answer: (C)

Solution:

Step 1: Use similarity.

Given $B = P^{-1}AP$ (so A is similar to B). If $B\mathbf{v}_k = \lambda_k \mathbf{v}_k$, then

$$A(P\mathbf{v}_k) = PBP^{-1}(P\mathbf{v}_k) = PB\mathbf{v}_k = P(\lambda_k \mathbf{v}_k) = \lambda_k(P\mathbf{v}_k).$$

Hence $P\mathbf{v}_k$ is an eigenvector of A corresponding to the same eigenvalue λ_k.

Step 2: Read off sets.

Therefore, the eigenvalues of A are $\{\lambda_k\}$ (unchanged under similarity), and the corresponding eigenvectors are $\{P\mathbf{v}_k\}$.

$$\boxed{\text{Option (C)}}$$

Quick Tip

Similarity ($A = PBP^{-1}$ or $B = P^{-1}AP$) preserves eigenvalues. Eigenvectors transform by the change of basis: $\mathbf{v}_B \mapsto P\mathbf{v}_B$ for A.

Q.16 In a semiconductor, if the Fermi energy level lies in the conduction band, then the semiconductor is known as

(A) degenerate n-type.

(B) degenerate p-type.

(C) non-degenerate n-type.

(D) non-degenerate p-type.

Correct Answer: (A) degenerate n-type.

Step 1: Recall Fermi level position.

In intrinsic semiconductors, the Fermi level lies around the middle of the bandgap.

In extrinsic (doped) semiconductors: - For n-type: Fermi level shifts closer to the conduction band.

- For p-type: Fermi level shifts closer to the valence band.

Step 2: Condition for degeneracy.

When doping is so high that the Fermi level moves *into the conduction band*, the semiconductor behaves almost like a metal. This is called a **degenerate n-type semiconductor**.

Step 3: Eliminate other options.

- (B) Degenerate p-type: occurs if Fermi level lies in valence band (not conduction band).
- (C) Non-degenerate n-type: Fermi level lies close to but below the conduction band.
- (D) Non-degenerate p-type: Fermi level lies close to but above the valence band.

Hence, the correct answer is (A).

$$\boxed{\text{degenerate n-type}}$$

Quick Tip

If the Fermi level lies *inside* a band (conduction or valence), the semiconductor is **degenerate**. If it lies *within the bandgap but close to a band*, the semiconductor is **non-degenerate**.

Q.17 For an intrinsic semiconductor at temperature $T = 0K$, which of the following statement is true?

(A) All energy states in the valence band are filled with electrons and all energy states in the conduction band are empty of electrons.

(B) All energy states in the valence band are empty of electrons and all energy states in the conduction band are filled with electrons.

(C) All energy states in the valence and conduction band are filled with holes.

(D) All energy states in the valence and conduction band are filled with electrons.

Correct Answer: (A)

Solution:

Step 1: Behavior at $T = 0K$.

At absolute zero, there is no thermal excitation. Therefore, electrons cannot be promoted from the valence band to the conduction band.

Step 2: State of valence band.

In an intrinsic semiconductor at $T = 0K$, the valence band is **completely filled** with electrons.

Step 3: State of conduction band.

The conduction band is **completely empty**, as no electrons have sufficient energy to cross the band gap.

Step 4: Match with options.

- (A) Correct: Valence full, conduction empty.
- (B) Incorrect: Opposite of true situation.
- (C) Incorrect: Holes are not present at $T = 0K$ (since valence is fully filled).
- (D) Incorrect: Both bands filled with electrons describes a metal, not a semiconductor.

$$\boxed{\text{At } T = 0K\text{, valence band full, conduction band empty. Correct answer = (A).}}$$

Quick Tip

At $T = 0K$, intrinsic semiconductors behave like perfect insulators since there are no free carriers (conduction band empty). Conductivity begins only at $T > 0K$.

Q.18 A series *RLC* circuit has a quality factor Q of 1000 at a center frequency of 10^6 rad/s. The possible values of R, L and C are

(A) $R = 1\,\Omega,\ L = 1\,\mu$H and $C = 1\,\mu$F

(B) $R = 0.1\,\Omega,\; L = 1\,\mu$H and $C = 1\,\mu$F

(C) $R = 0.01\,\Omega,\; L = 1\,\mu$H and $C = 1\,\mu$F

(D) $R = 0.001\,\Omega,\; L = 1\,\mu$H and $C = 1\,\mu$F

Correct Answer: (D)

Solution:

Step 1 (Resonant frequency check): For a series RLC, $\omega_0 = \dfrac{1}{\sqrt{LC}}$. With $L = 1\,\mu\text{H} = 10^{-6}\,$H and $C = 1\,\mu\text{F} = 10^{-6}\,$F, $\sqrt{LC} = \sqrt{10^{-12}} = 10^{-6}$, hence $\omega_0 = 10^6$ rad/s, matching the given value.

Step 2 (Quality factor at resonance): For a series RLC, $Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 RC}$. With $\omega_0 L = 10^6 \cdot 10^{-6} = 1$, we get $Q = \dfrac{1}{R}$.

Step 3 (Solve for R): Given $Q = 1000$, $R = \dfrac{1}{Q} = 0.001\,\Omega$. The only option with $R = 0.001\,\Omega$ (and the same L, C that satisfy ω_0) is **(D)**.

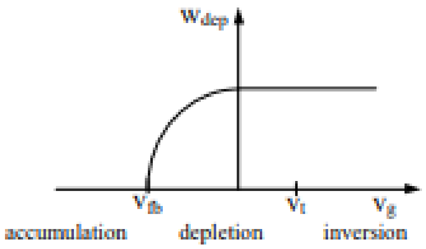
$$\boxed{R = 0.001\,\Omega,\; L = 1\,\mu\text{H},\; C = 1\,\mu\text{F}}$$

Quick Tip

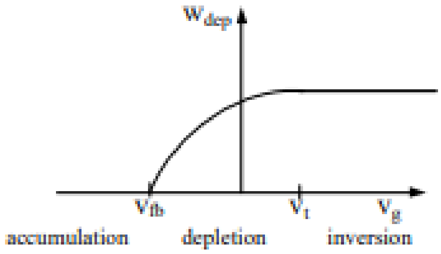
For a *series* RLC at resonance: $Q = \omega_0 L/R = 1/(\omega_0 RC)$. If L and C already satisfy ω_0, then Q immediately fixes R.

Q.19 For a MOS capacitor, V_{fb} and V_t are the flat-band voltage and the threshold voltage, respectively. The variation of the depletion width ($W_{\textbf{dep}}$) for varying gate voltage (V_g) is best represented by

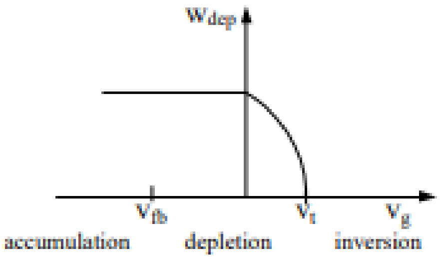
(A)



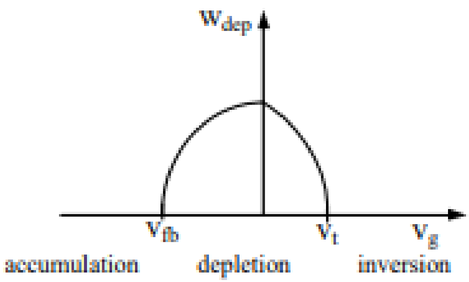
(B)



(C)



(D)



Correct Answer: (B)

Solution:

Step 1: Accumulation ($V_g \ll V_{fb}$).

Majority carriers accumulate at the surface $\Rightarrow W_{\text{dep}} \approx 0$.

Step 2: Depletion ($V_{fb} < V_g < V_t$).

A depletion region forms and widens with V_g. For a p–type substrate (nMOS case),

$$W_{\text{dep}} = \sqrt{\frac{2\varepsilon_s \psi_s}{qN_A}} \quad \Rightarrow \quad W_{\text{dep}} \text{ increases roughly as } \sqrt{\psi_s} \text{ with } V_g.$$

Step 3: Strong inversion ($V_g \geq V_t$).

Further increase in V_g is primarily taken up by inversion charge, not by widening depletion. Hence W_{dep} *saturates* at

$$W_{\max} = \sqrt{\frac{2\varepsilon_s(2\Phi_F)}{qN_A}} \text{ (constant).}$$

Step 4: Match with the plots.

The correct curve starts near zero in accumulation, rises in depletion (from V_{fb}), and then flattens to a constant beyond $V_t \Rightarrow$ **Option (B)**.

$$\boxed{\text{The depletion width increases from } V_{fb} \text{ and saturates beyond } V_t \ \Rightarrow\ \text{(B)}}$$

Quick Tip

In MOS capacitors, beyond threshold, added gate voltage mainly increases inversion charge while W_{dep} stays nearly constant. Think: "rise, then saturate."

Q.20 Consider a narrow band signal, propagating in a lossless dielectric medium ($\varepsilon_r = 4, \mu_r = 1$), with phase velocity v_p and group velocity v_g. Which of the following statement is true? (c is the velocity of light in vacuum.)

(A) $v_p > c,\ v_g > c$

(B) $v_p < c,\ v_g > c$

(C) $v_p > c,\ v_g < c$

(D) $v_p < c,\ v_g < c$

Correct Answer: (D)

Step 1: Phase velocity in a medium.

For a non-magnetic dielectric with $\mu_r = 1$,

$$v_p = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{4 \cdot 1}} = \frac{c}{2}.$$

Thus, $v_p = \frac{c}{2} < c$.

Step 2: Group velocity in a non-dispersive medium.

In a lossless, non-dispersive dielectric medium,

$$v_g = v_p.$$

Hence, $v_g = \frac{c}{2} < c$.

Step 3: Conclusion.

Both v_p and v_g are less than c.

Therefore, the correct statement is:

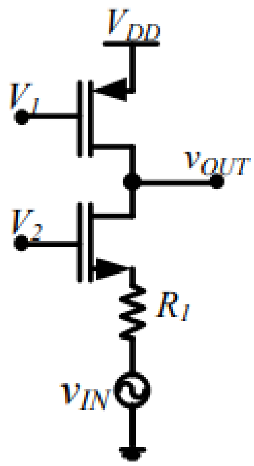
$$v_p < c, \quad v_g < c.$$

$$\boxed{(D)}$$

Quick Tip

In a lossless, non-dispersive medium, phase velocity equals group velocity. Both are reduced by a factor of $\sqrt{\varepsilon_r \mu_r}$ compared to c.

Q.21 In the circuit shown below, V_1 and V_2 are bias voltages. Based on input and output impedances, the circuit behaves as a



(A) voltage controlled voltage source.

(B) voltage controlled current source.

(C) current controlled voltage source.

(D) current controlled current source.

Correct Answer: (D) current controlled current source

Solution:

Step 1: Identify the circuit.

The figure shows a MOSFET with bias voltages V_1 and V_2 applied at the gate and source terminals respectively. The input is a current source I_{IN} connected at the source through resistor R_1. The output is taken from the drain (v_{OUT}). This is essentially a MOSFET current mirror/amplifier configuration.

Step 2: Input characteristics.

Since the input is a current source I_{IN}, the controlling signal for the device is a current. Thus, the input is current-driven.

Step 3: Output characteristics.

The MOSFET in saturation behaves like a current source at the output. Therefore, the output is also a current.

Step 4: Behavior.

Thus, the circuit takes a current as input and delivers a current as output. This makes it a **current controlled current source (CCCS)**.

Current controlled current source (Option D)

Quick Tip

MOSFETs biased in saturation region behave as current sources. If the input is a current and the output is also a current, the overall circuit functions as a CCCS.

Q.22 A cascade of common-source amplifiers in a unity gain feedback configuration oscillates when

(A) the closed loop gain is less than 1 and the phase shift is less than $180°$.

(B) the closed loop gain is greater than 1 and the phase shift is less than $180°$.

(C) the closed loop gain is less than 1 and the phase shift is greater than $180°$.

(D) the closed loop gain is greater than 1 and the phase shift is greater than $180°$.

Correct Answer: (D)

Solution:

For unity feedback, oscillation requires the **loop gain** to satisfy the Barkhausen conditions at some frequency: $|A(j\omega)\beta| \geq 1$ and total phase shift $= 360°$ (i.e., $0°$ modulo $360°$).

A common-source stage contributes approximately $180°$ phase shift; a cascade plus frequency-dependent phase can push the total phase past $180°$, turning the feedback effectively positive when the magnitude exceeds unity.

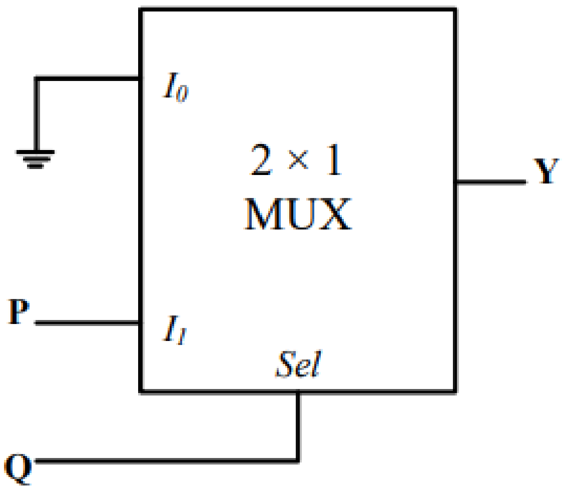
Thus the relevant option is that the (so-called) closed-loop/loop gain is **greater than 1** and the phase shift is **greater than** $180°$.

$$\boxed{\text{(D)}}$$

Quick Tip

Remember Barkhausen: unity (or greater) loop-gain magnitude and total loop phase $= 0° \pmod{360°}$. With inverting stages like common-source, watch for the phase crossing $180°$ as frequency rises.

Q.23 In the circuit shown below, P and Q are the inputs. The logical function realized by the circuit shown below is



(A) $Y = PQ$

(B) $Y = P + Q$

(C) $Y = \overline{P}\,Q$

(D) $Y = \overline{P} + \overline{Q}$

Correct Answer: (A)

Solution:

Step 1: Interpret the 2×1 MUX connections.

From the figure: I_0 is tied to 0 (ground), I_1 is P, and the select input is Q.

Step 2: Write the MUX output equation.

For a 2×1 MUX,

$$Y = \overline{Q}\,I_0 + Q\,I_1.$$

Substitute $I_0 = 0$ and $I_1 = P$:

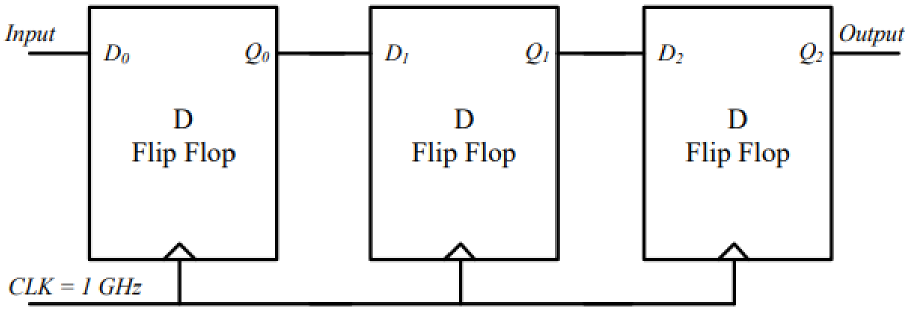
$$Y = \overline{Q} \cdot 0 + Q \cdot P = PQ.$$

$$\boxed{Y = PQ}$$

Quick Tip

Always reduce a 2×1 MUX to $Y = \overline{S}I_0 + SI_1$ and then plug in the connected signals.

Q.24 The synchronous sequential circuit shown below works at a clock frequency of 1 GHz. The throughput, in Mbits/s, and the latency, in ns, respectively, are



(A) 1000, 3

(B) 333.33, 1

(C) 2000, 3

(D) 333.33, 3

Correct Answer: (A)

Step 1: Throughput.

A D flip-flop pipeline can accept one new input bit every clock cycle.

Clock frequency $f = 1\,\text{GHz} = 10^9$ cycles/s.

Bits per cycle $= 1 \Rightarrow$ throughput $= 1 \times 10^9$ bits/s $= 1000\,$Mbits/s.

Step 2: Latency.

The data must traverse 3 cascaded DFF stages to reach the output.

Latency (in cycles) $= 3$. Each cycle is $T = 1/f = 1\,$ns.

Hence latency $= 3 \times 1\,\text{ns} = 3\,\text{ns}$.

$$\boxed{\text{Throughput } = 1000 \text{ Mbits/s}, \quad \text{Latency } = 3 \text{ ns}}$$

Quick Tip

Pipeline *throughput* is limited by clock rate (one item per cycle), while *latency* equals the number of stages times the clock period.

Q.25 The open loop transfer function of a unity negative feedback system is

$$G(s) = \frac{k}{s(1+sT_1)(1+sT_2)},$$

where k, T_1 and T_2 are positive constants. The phase cross-over frequency, in rad/s, is

(A) $\dfrac{1}{\sqrt{T_1T_2}}$

(B) $\dfrac{1}{T_1T_2}$

(C) $\dfrac{1}{T_1\sqrt{T_2}}$

(D) $\dfrac{1}{T_2\sqrt{T_1}}$

Correct Answer: (A) $\dfrac{1}{\sqrt{T_1T_2}}$

Solution:

Step 1: Phase condition for phase crossover frequency.

Phase crossover frequency ω_{pc} is the frequency at which the phase angle of $G(j\omega)$ is $-180°$.

Step 2: Phase angle of $G(j\omega)$.

$$G(j\omega) = \frac{k}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

Phase contribution: - From $j\omega$: $-90°$ - From $(1+j\omega T_1)$: $-\tan^{-1}(\omega T_1)$ - From $(1+j\omega T_2)$: $-\tan^{-1}(\omega T_2)$

So total phase:

$$\phi = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

Step 3: Apply phase crossover condition.

At phase crossover:

$$-90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2) = -180^\circ$$

$$\Rightarrow \tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_2) = 90^\circ$$

Step 4: Simplify using identity.

$$\tan^{-1}(x) + \tan^{-1}(y) = 90^\circ \quad \Rightarrow \quad xy = 1$$

So, $\omega^2 T_1 T_2 = 1$.

Step 5: Solve for ω.

$$\omega = \frac{1}{\sqrt{T_1 T_2}}$$

$$\boxed{\frac{1}{\sqrt{T_1 T_2}}}$$

Quick Tip

For phase crossover frequency, focus on the total phase shift of -180°. With two first-order factors, the condition simplifies to $\omega^2 T_1 T_2 = 1$.

Q.26 Consider a system with input x(t) and output y(t) = x(e^t**)**$.The system is$

(A) Causal and time invariant.

(B) Non-causal and time varying.

(C) Causal and time varying.

(D) Non-causal and time invariant.

Correct Answer: (B) Non-causal and time varying.

Solution:

Causality test: For any t_0, $y(t_0) = x(e^{t_0})$. Since $e^{t_0} > t_0$ for all real t_0 (because $f(t) = e^t - t$ has minimum $f(0) = 1 > 0$), the output at time t_0 depends on the input at a *future* time e^{t_0}. Therefore the system is **non-causal**.

Time invariance test: Let the input be shifted: $x_1(t) = x(t - \tau)$. Then the output is $y_1(t) = x_1(e^t) = x(e^t - \tau)$.

But the time-shifted version of the original output is $y(t - \tau) = x(e^{t-\tau})$.

Since, in general, $e^t - \tau \neq e^{t-\tau}$, we have $y_1(t) \neq y(t - \tau)$ for some t, τ.

Hence the system is **time-varying**.

$$\boxed{\text{Non-causal and time varying (Option B)}}$$

Quick Tip

If the output uses x at $g(t)$ with $g(t) > t$ for some t, the system is non-causal. For time invariance, compare $T\{x(t - \tau)\}$ with $T\{x(t)\}$ shifted: any mismatch implies time-varying.

Q.27 Let $m(t)$ be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos\omega_0 t$ is

(A) $\frac{E}{4}$

(B) $\frac{E}{2}$

(C) E

(D) $2E$

Correct Answer: (B)

Solution:

Step 1: Modulation in frequency domain.

Let $y(t) = m(t)\cos\omega_0 t$. Then

$$Y(\omega) = \tfrac{1}{2}\big[M(\omega-\omega_0) + M(\omega+\omega_0)\big].$$

Since $m(t)$ is strictly band-limited to $|\omega| \le B$ and $\omega_0 = 10B \gg B$, the two shifted spectra are *disjoint*.

Step 2: Energy via Parseval.

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2\,d\omega.$$

With disjoint supports, cross-terms vanish:

$$|Y(\omega)|^2 = \tfrac{1}{4}\big(|M(\omega-\omega_0)|^2 + |M(\omega+\omega_0)|^2\big).$$

Thus

$$E_y = \frac{1}{2\pi}\cdot\tfrac{1}{4}\left(\int |M(\omega-\omega_0)|^2 d\omega + \int |M(\omega+\omega_0)|^2 d\omega\right) = \tfrac{1}{4}(E+E) = \frac{E}{2}.$$

$$\boxed{\frac{E}{2}}$$

Quick Tip

Multiplying by $\cos\omega_0 t$ splits the spectrum into two non-overlapping sidebands (for large ω_0). Each carries half the energy $\Rightarrow$ total energy $= E/2$.

Q.28 The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

(A) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

(B) $\dfrac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

(C) $\sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$

(D) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

Correct Answer: (C)

Assume the Fourier transform $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$
Given $x(t) = e^{-t^2}$, we have

$$X(\omega) = \int_{-\infty}^{\infty} e^{-t^2}\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} \exp\bigl(-t^2 - j\omega t\bigr)\, dt.$$

Complete the square:

$$-t^2 - j\omega t = -\Bigl(t + \frac{j\omega}{2}\Bigr)^2 - \frac{\omega^2}{4}.$$

Hence,

$$X(\omega) = e^{-\frac{\omega^2}{4}} \int_{-\infty}^{\infty} \exp\Bigl[-\Bigl(t + \frac{j\omega}{2}\Bigr)^2\Bigr]\, dt.$$

The shift by a constant in the complex plane does not change the Gaussian integral value, so using $\int_{-\infty}^{\infty} e^{-y^2}\, dy = \sqrt{\pi}$, we get

$$X(\omega) = e^{-\frac{\omega^2}{4}}\, \sqrt{\pi}.$$

Therefore,

$$\boxed{X(\omega) = \sqrt{\pi}\, e^{-\frac{\omega^2}{4}}}$$

which corresponds to option **(C)**.

Quick Tip

For $X(\omega) = \int x(t)e^{-j\omega t}dt$, the transform of e^{-at^2} is $\sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}}$ for $\Re\{a\} > 0$. Set $a = 1$ to get $\sqrt{\pi}\, e^{-\omega^2/4}$.

Q.29 In the table shown below, match the signal type with its spectral characteristics.

Signal type	Spectral characteristics
(i) Continuous, aperiodic	(a) Continuous, aperiodic
(ii) Continuous, periodic	(b) Continuous, periodic
(iii) Discrete, aperiodic	(c) Discrete, aperiodic
(iv) Discrete, periodic	(d) Discrete, periodic

(A) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (c), (iv) $\rightarrow$ (d)

(B) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (d)

(C) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (c), (iv) $\rightarrow$ (a)

(D) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (d), (iv) $\rightarrow$ (b)

Correct Answer: (A) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (c), (iv) $\rightarrow$ (d)

Solution:

Step 1: Fourier Transform properties.

- A **continuous aperiodic signal** $\Rightarrow$ **continuous aperiodic spectrum**.
- A **continuous periodic signal** $\Rightarrow$ **discrete spectrum**.
- A **discrete aperiodic signal** $\Rightarrow$ **continuous periodic spectrum**.
- A **discrete periodic signal** $\Rightarrow$ **discrete spectrum**.

Step 2: Match.

(i) Continuous, aperiodic $\rightarrow$ (a) Continuous, aperiodic.

(ii) Continuous, periodic $\rightarrow$ (b) Continuous, periodic.

(iii) Discrete, aperiodic $\rightarrow$ (c) Discrete, aperiodic.

(iv) Discrete, periodic $\rightarrow$ (d) Discrete, periodic.

$$\boxed{(A)\ \ (i) \to (a),\ (ii) \to (b),\ (iii) \to (c),\ (iv) \to (d)}$$

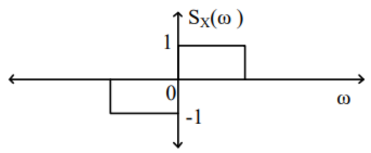
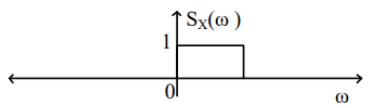
Quick Tip

Rule of thumb: Periodic in time $\Rightarrow$ discrete in frequency, Aperiodic in time $\Rightarrow$ continuous in frequency. Same holds for discrete-time signals with reversed roles.

Q.30 For a real signal, which of the following is/are valid power spectral density/densities?

(A) $S_x(\omega) = \dfrac{2}{9+\omega^2}$

(B) $S_x(\omega) = e^{-\omega^2} \cos^2 \omega$

(C)	
(D)	

Correct Answer: (A) and (B)

Solution:

Property 1 (Nonnegativity): For any real (wide-sense) signal, the PSD satisfies $S_x(\omega) \geq 0$ for all ω.

Property 2 (Even symmetry): For real signals, $S_x(\omega) = S_x(-\omega)$ (two-sided PSD is even).

Check (A): $\dfrac{2}{9+\omega^2} \geq 0$ for all ω and is an even function of $\omega \Rightarrow$ **valid**.

Check (B): $e^{-\omega^2} \geq 0$ and $\cos^2 \omega \geq 0$; product is ≥ 0 and even (both factors even) $\Rightarrow$ **valid**.

Check (C): Shows negative values for some ω and is not even $\Rightarrow$ violates Property 1 (and 2) $\Rightarrow$ **invalid**.

Check (D): Nonnegative but only on $\omega > 0$ (zero for $\omega < 0$) so not even in the two-sided sense $\Rightarrow$ **invalid** as a full PSD (it would be a one-sided PSD, but the question asks for PSD for a real signal in general).

$$\boxed{\text{Valid PSDs: (A) and (B)}}$$

Quick Tip

For real signals, a two-sided PSD must be *nonnegative* and *even*. Any negative portion or lack of symmetry about $\omega = 0$ rules it out.

Q.31 The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is $_{bits (rounded off to the nearest integer).}$

Correct Answer: 10

Step 1: Formula for SNR of an ideal ADC.

For an N-bit ADC, the theoretical SNR is given by:

$$SNR_{dB} = 6.02N + 1.76$$

Step 2: Substitute given SNR.

$$61.96 = 6.02N + 1.76$$

Step 3: Solve for N.

$$6.02N = 61.96 - 1.76 = 60.2$$
$$N = \frac{60.2}{6.02} = 10.0$$

Step 4: Round to nearest integer.

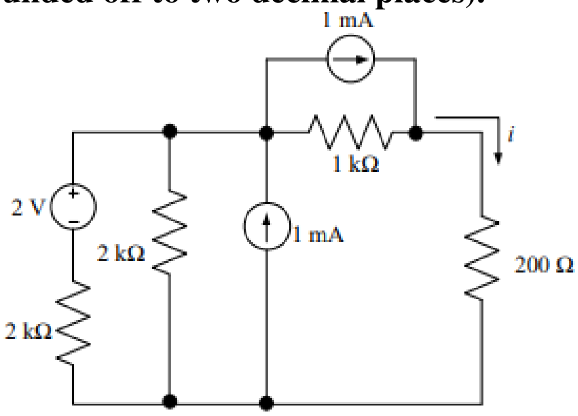
$N = 10$ bits.

$$\boxed{10}$$

Quick Tip

For ideal ADCs, use $SNR_{dB} \approx 6.02N + 1.76$. Each extra bit of resolution improves SNR by about 6 dB.

Q.32 In the circuit shown below, the current i flowing through $200\,\Omega$ resistor is _____ mA (rounded off to two decimal places).



Correct Answer: 1.36 mA

Solution:

Take the bottom node as reference (0 V). Let the left top node be V_A and the right top node (top of $200\,\Omega$) be V_B.

Replace the left branch (a $2\,$V source in series with $2\,\text{k}\Omega$ to ground) by its Norton equivalent: a $1\,$mA source from ground to node A in parallel with $2\,\text{k}\Omega$.

This is in parallel with the vertical $1\,$mA source (ground $\to A$) and another $2\,\text{k}\Omega$ to ground, giving net: two $2\,\text{k}\Omega$ in parallel $\Rightarrow 1\,\text{k}\Omega$ from A to ground, and total current injection at A of $2\,$mA.

Between A and B there is a $1\,\text{k}\Omega$ resistor in parallel with a $1\,$mA source from A to B. The load is $200\,\Omega$ from B to ground.

Nodal equations (currents injected taken positive):

At A: $\dfrac{V_A}{1\,\text{k}\Omega} + \dfrac{V_A - V_B}{1\,\text{k}\Omega} = 2\,\text{mA} - 1\,\text{mA} = 1\,\text{mA}.$

At B: $\dfrac{V_B - V_A}{1\,\text{k}\Omega} + \dfrac{V_B}{200\,\Omega} = +1\,\text{mA}.$

Solving, $V_A = 0.6364\,\text{V}$ and $V_B = 0.2727\,\text{V}$.

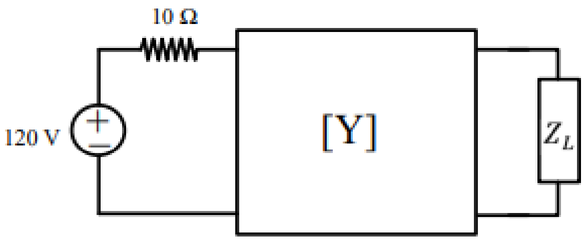
Current through $200\,\Omega$ (downward) is $i = \dfrac{V_B}{200} = 0.0013636\,\text{A} = 1.36\,\text{mA}$ (to two decimals).

$$\boxed{i \approx 1.36 \text{ mA}}$$

Quick Tip

A series (voltage source V_s + R) to ground can be replaced by a Norton source of $I_N = \dfrac{V_s}{R}$ from ground to the node, in parallel with R. This often makes nodal analysis straightforward.

Q.33 For the two port network shown below, the [Y]-parameters is given as $[Y] = \dfrac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & \frac{4}{3} \end{bmatrix}$ S. The value of load impedance Z_L, in Ω, for maximum power transfer will be ______ (rounded off to the nearest integer).



Correct Answer: 80

Solution:

With the source killed, the 120 V source becomes a short, so port 1 is terminated by its series resistance $R_S = 10\,\Omega \Rightarrow Y_S = \frac{1}{R_S} = 0.1\,\text{S}$.

For a two–port with Y-parameters and port 1 terminated by Y_S, the driving-point admittance seen at port 2 is

$$Y_{\text{out}} = Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11} + Y_S}.$$

Given $Y_{11} = \frac{2}{100} = 0.02$, $Y_{12} = Y_{21} = -\frac{1}{100} = -0.01$, $Y_{22} = \frac{4/3}{100} = 0.013\overline{3}$ S.

Compute: $Y_{11} + Y_S = 0.02 + 0.1 = 0.12, \quad Y_{21}Y_{12} = (-0.01)(-0.01) = 10^{-4}$.

Hence

$$Y_{\text{out}} = 0.013\overline{3} - \frac{10^{-4}}{0.12} = 0.013\overline{3} - 0.000\overline{83} = 0.0125 \text{ S}.$$

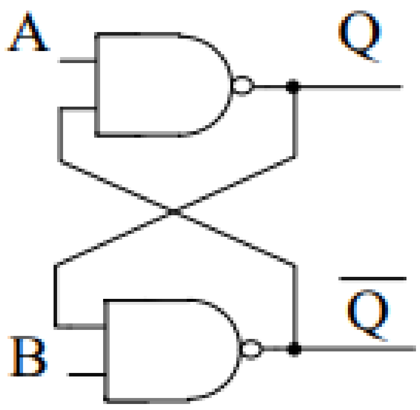
Therefore $Z_{\text{th}} = \dfrac{1}{Y_{\text{out}}} = 80\,\Omega$.
For maximum power transfer, $Z_L = Z_{\text{th}}$ (all-real network).

$$\boxed{Z_L = 80\ \Omega}$$

Quick Tip

With Y-parameters, terminating the other port by an admittance Y_T gives $Y_{\text{seen}} = Y_{22} - \dfrac{Y_{21}Y_{12}}{Y_{11}+Y_T}$. Set $Z_L = 1/Y_{\text{seen}}$ for maximum power (real case).

Q.34 For the circuit shown below, the propagation delay of each NAND gate is 1 ns. The critical path delay, in ns, is ________ (rounded off to the nearest integer).



Correct Answer: 2

Solution:

The diagram is a cross-coupled NAND (SR latch) with outputs Q and $\overline{Q}$, and inputs A to the upper NAND and B to the lower NAND.

A change on B affects $\overline{Q}$ through the lower NAND (1 ns), and that new $\overline{Q}$ feeds back to the upper NAND to affect Q (another 1 ns).

Similarly, a change on A reaching $\overline{Q}$ also traverses two NANDs.

Thus, the *critical* input–output path passes through two cascaded NAND gates.

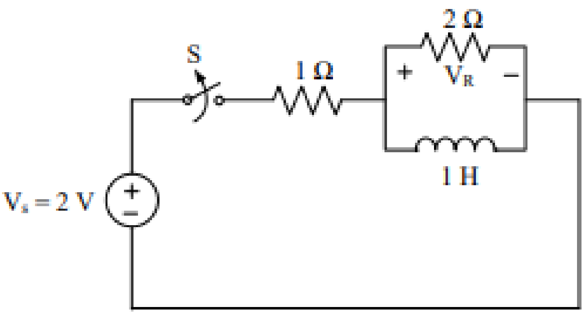
Critical path delay $= 2 \times 1$ ns $= 2$ ns.

$\boxed{2}$

Quick Tip

In cross-coupled gates, the worst-case input–output path often includes one gate plus the feedback through the second gate: count the cascaded stages and multiply by per-gate delay.

Q.35 In the circuit shown below, switch S was closed for a long time. If the switch is opened at $t = 0$, the maximum magnitude of the voltage V_R, in volts, is ________ (rounded off to the nearest integer).



Correct Answer: 4

Solution:

Step 1: Steady state for $t < 0$ (S closed).

The inductor behaves as a short under DC, so the branch containing the inductor shorts the $2\,\Omega$ resistor. The source $V_s = 2$ V then drives current only through the $1\,\Omega$ path to the shorted inductor:

$$i_L(0^-) = \frac{2}{1} = 2 \text{ A}.$$

Step 2: Immediately after opening the switch ($t = 0^+$).

The source is disconnected and the inductor current must continue; it now decays through the $2\,\Omega$ resistor (time constant $\tau = L/R = 1/2$ s). The voltage across the $2\,\Omega$ resistor is

$$V_R(t) = 2\,i_L(t) = 2 \cdot i_L(0^-)\,e^{-t/\tau}.$$

Hence the maximum magnitude occurs at $t = 0^+$:

$$V_{R,\max} = 2 \times 2 = 4\text{ V}.$$

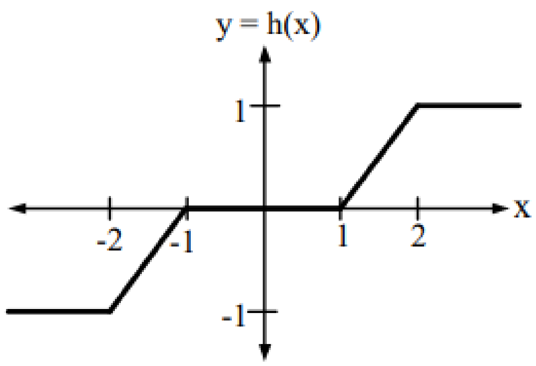
$\boxed{4}$

Quick Tip

At $t = 0^+$, inductor current is continuous and equals its $t = 0^-$ value. In RL transients, the peak resistor voltage right after switching is simply $R\,i_L(0^-)$.

Q.36 A random variable X, distributed normally as $N(0, 1)$, undergoes the transformation $Y = h(X)$, given in the figure. The form of the probability density function of Y is

(In the options given below, a, b, c are non-zero constants and $g(y)$ is a piece-wise continuous function)



(A) $a\delta(y-1) + b\delta(y+1) + g(y)$

(B) $a\delta(y+1) + b\delta(y-1) + c\delta(y) + g(y)$

(C) $a\delta(y+2) + b\delta(y) + c\delta(y-2) + g(y)$

(D) $a\delta(y+2) + b\delta(y-2) + g(y)$

Correct Answer: (B)

Step 1: Analyze the mapping $Y = h(X)$.

From the figure: - For $x \leq -2$, $y = -1$ (constant).

- For $-2 < x < -1$, y varies linearly from -1 to 0.

- For $-1 \leq x \leq 1$, $y = 0$ (constant).

- For $1 < x < 2$, y varies linearly from 0 to 1.

- For $x \geq 2$, $y = 1$ (constant).

Step 2: Implications for pdf.

- Flat segments ($x \leq -2$, $x \geq 2$, $-1 \leq x \leq 1$) produce **delta functions** in y at $y = -1$, $y = 0$, and $y = 1$.

- Sloped regions ($-2 < x < -1$, $1 < x < 2$) map intervals of x to intervals of y, giving continuous pdf contributions $g(y)$.

Step 3: Form of pdf.

Thus, the pdf of Y is a sum of impulses at $y = -1, 0, 1$ with nonzero weights, plus a continuous part $g(y)$.

This matches option **(B)**:

$$a\delta(y+1) + b\delta(y-1) + c\delta(y) + g(y).$$

$$\boxed{\text{(B)}}$$

Quick Tip

When a nonlinear mapping has flat segments, the pdf of the output includes Dirac delta impulses at those constant output values. Linear parts contribute continuous density via the change-of-variable rule.

Q.37 The value of the line integral $\int_P^Q \left(z^2\,dx + 3y^2\,dy + 2xz\,dz\right)$ along the straight line joining the points $P(1,1,2)$ and $Q(2,3,1)$ is

(A) 20

(B) 24

(C) 29

(D) -5

Correct Answer: (B) 24

Solution:

Parametrize the straight line PQ.

Let $t \in [0,1]$, with $x = 1+t,\ y = 1+2t,\ z = 2-t.$

Then $dx = dt,\ dy = 2\,dt,\ dz = -dt.$

Substitute into the integrand.

$z^2\,dx = (2-t)^2 dt, \quad 3y^2\,dy = 3(1+2t)^2(2\,dt) = 6(1+2t)^2 dt,$

$2xz\,dz = 2(1+t)(2-t)(-dt) = -2(1+t)(2-t)dt.$

Integrate from $t = 0$ to $t = 1$.

$$\int_0^1 \Big[(2-t)^2 + 6(1+2t)^2 - 2(1+t)(2-t)\Big]\,dt$$

$=\int_0^1 (27t^2 + 18t + 6)\,dt = \Big[9t^3 + 9t^2 + 6t\Big]_0^1 = 9 + 9 + 6 = 24.$

$$\boxed{24}$$

> **Quick Tip**
>
> For line integrals along a straight segment, parametrize with $t \in [0,1]$ using $r(t) = P + t(Q-P)$, compute dx, dy, dz, substitute, and integrate.

Q.38 Let x be an $n \times 1$ real column vector with length $l = \sqrt{\mathrm{x}^T\mathrm{x}}$. The trace of the matrix $P = \mathrm{x}\mathrm{x}^T$ is

(A) l^2

(B) $\frac{l^2}{4}$

(C) l

(D) $\frac{l^2}{2}$

Correct Answer: (A) l^2

Solution:

Step 1: Use trace cyclicity. $\mathrm{tr}(P) = \mathrm{tr}(\mathbf{x}\mathbf{x}^T) = \mathrm{tr}(\mathbf{x}^T\mathbf{x})$.

Step 2: Reduce to a scalar. $\mathbf{x}^T\mathbf{x}$ is the sum of squares of the components of $\mathbf{x}$, i.e., $\|\mathbf{x}\|_2^2 = l^2$.

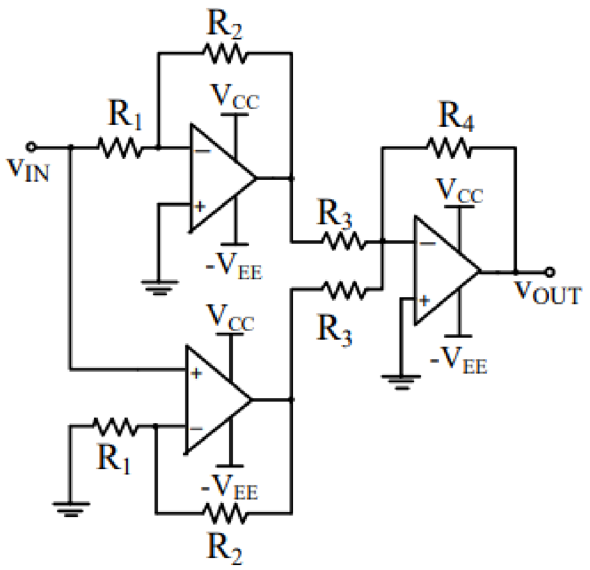
Therefore, $\mathrm{tr}(P) = \mathbf{x}^T\mathbf{x} = l^2$.

$$\boxed{l^2}$$

Quick Tip

For any vectors $a \in \mathbb{R}^{n\times 1}$ and $b \in \mathbb{R}^{n\times 1}$, $\mathrm{tr}(ab^T) = b^T a$. In particular, $\mathrm{tr}(\mathbf{x}\mathbf{x}^T) = \mathbf{x}^T\mathbf{x} = \|\mathbf{x}\|^2$.

Q.39 The $\frac{V_{OUT}}{V_{IN}}$ of the circuit shown below is



(A) $-\dfrac{R_4}{R_3}$

(B) $\dfrac{R_4}{R_3}$

(C) $1 + \dfrac{R_4}{R_3}$

(D) $1 - \dfrac{R_4}{R_3}$

Correct Answer: (A)

Solution:

Step 1: Identify the rightmost stage.

The rightmost op-amp has its non-inverting input at ground, input resistor R_3 to the inverting input, and feedback R_4 from output to inverting input $\Rightarrow$ *inverting amplifier:*

$$V_{OUT} = -\frac{R_4}{R_3}\, v_x,$$

where v_x is the signal applied through R_3.

Step 2: Evaluate v_x from the left block.

Each of the two left op-amps has its inverting input fed back from its own output (via R_2) $\Rightarrow$ both act as *voltage followers*. Top follower outputs $v_a = V_{IN}$; bottom follower (non-inverting at 0 V) outputs $v_b = 0$. The resistor R_3 feeding the right stage is connected between these two follower outputs, so the voltage impressed across the input resistor is

$$v_x = v_a - v_b = V_{IN} - 0 = V_{IN}.$$

Step 3: Overall gain.

Substitute $v_x = V_{IN}$ in Step 1:

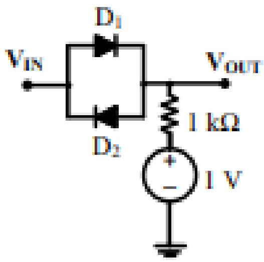
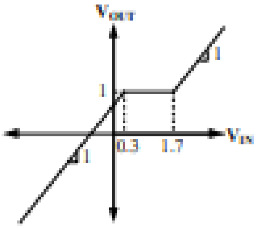
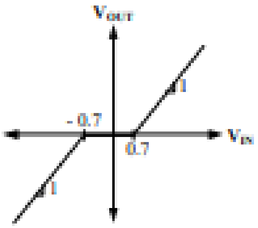
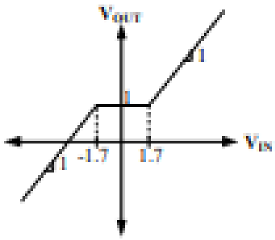
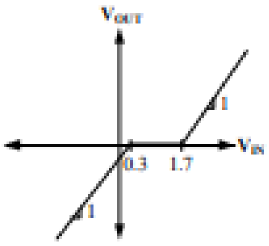
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_4}{R_3}.$$

$$\boxed{-\frac{R_4}{R_3}}$$

Quick Tip

When you see a final op-amp with R_3 into the inverting node and R_4 in feedback, it's an inverting stage. If the preceding block presents V_{IN} across R_3, the overall gain is simply $-R_4/R_3$.

Q.40 In the circuit shown below, D_1 and D_2 are silicon diodes with cut-in voltage of 0.7 V. V_{IN} and V_{OUT} are input and output voltages in volts. The transfer characteristic is

	
(A)	
(B)	
(C)	
(D)	

Correct Answer: (A)

Step 1: Circuit description.

The circuit has two diodes D_1 and D_2 connected in anti-parallel arrangement with respect to the input V_{IN}, and both connected to the output node V_{OUT}. A $1\,\text{k}\Omega$ resistor is connected from V_{OUT} to a $1\,\text{V}$ DC source.

Step 2: Conduction conditions.

- For $V_{IN} > 1.7$ V, D_1 conducts (since $V_{IN} - V_{OUT} \geq 0.7$ V). The output clamps accordingly.
- For $V_{IN} < -1.7$ V, D_2 conducts, clamping the output in the opposite polarity.
- For $-1.7 \leq V_{IN} \leq +1.7$, both diodes are OFF, and the output follows the linear relationship.

Step 3: Transfer characteristic.

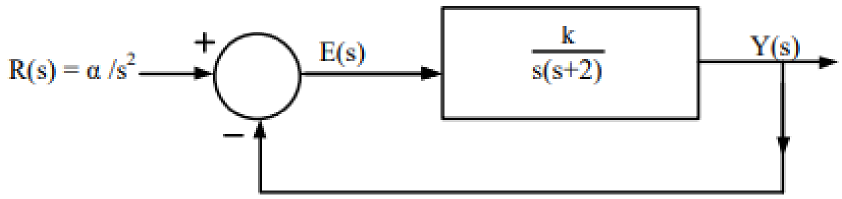
Thus, the V_{OUT} vs. V_{IN} curve is linear within ± 1.7 V and saturates (clamps) outside this range. This corresponds exactly to the graph in **(A)**.

$$\boxed{(A)}$$

Quick Tip

Always add the diode cut-in voltage (0.7 V for silicon) to any DC bias source in series when determining the clipping level in diode limiter circuits.

Q.41 A closed loop system is shown in the figure where $k > 0$ and $\alpha > 0$. The steady state error due to a ramp input ($R(s) = \alpha/s^2$) is given by



(A) $\dfrac{2\alpha}{k}$

(B) $\dfrac{\alpha}{k}$

(C) $\dfrac{\alpha}{2k}$

(D) $\dfrac{\alpha}{4k}$

Correct Answer: (A) $\dfrac{2\alpha}{k}$

Solution:

Unity negative feedback with forward path $G(s) = \dfrac{k}{s(s+2)}$.

For a ramp input $R(s) = \dfrac{\alpha}{s^2}$, the steady-state error is $e_{ss} = \dfrac{\alpha}{K_v}$ where $K_v = \lim\limits_{s\to 0} sG(s)$.

Compute K_v: $\quad sG(s) = \dfrac{ks}{s(s+2)} = \dfrac{k}{s+2} \xrightarrow[s\to 0]{} \dfrac{k}{2}$.

Therefore, $e_{ss} = \dfrac{\alpha}{K_v} = \dfrac{\alpha}{k/2} = \dfrac{2\alpha}{k}$.

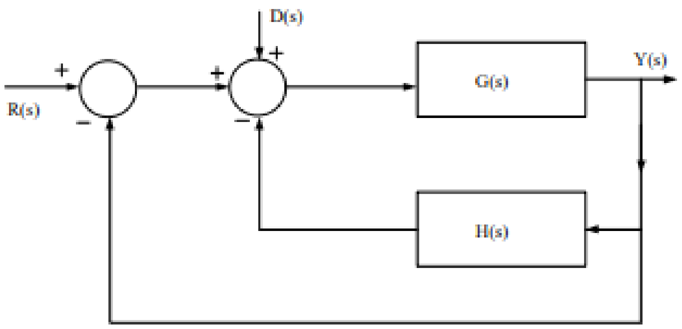
$$\boxed{e_{ss} = \frac{2\alpha}{k}}$$

Quick Tip

For unity feedback: step $\Rightarrow e_{ss} = \dfrac{1}{K_p}$, ramp $\Rightarrow e_{ss} = \dfrac{\alpha}{K_v}$, parabolic $\Rightarrow e_{ss} = \dfrac{\beta}{K_a}$. Here $K_v = \lim_{s\to 0} sG(s)$.

Q.42 In the following block diagram, R(s) and D(s) are two inputs. The output Y(s) is expressed as Y(s) = G_1(s)R(s) + G_2(s)D(s).

G_1(s) and G_2(s) are given by



(A) $G_1(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

(B) $G_1(s) = \dfrac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+H(s)}$

(C) $G_1(s) = \dfrac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

(D) $G_1(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1 + G(s) + H(s)}$

Correct Answer: (A)

Solution:

Let the first summer output be $e_1 = R(s) - Y(s)$ (outer negative feedback).

Let the second summer output be $e_2 = e_1 + D(s) - H(s)Y(s)$ (inner negative feedback through H).

The plant output is $Y(s) = G(s)\,e_2$.

Substitute: $Y = G\,[\,(R - Y) + D - HY\,] = G(R + D) - G(1 + H)Y$.

Bring Y terms together: $Y\,[\,1 + G(1 + H)\,] = G(R + D)$.

Therefore,

$$Y = \frac{G}{1 + G + GH}\,R \;+\; \frac{G}{1 + G + GH}\,D,$$

so $G_1(s) = G_2(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$.

$$\boxed{G_1(s) = G_2(s) = \frac{G(s)}{1 + G(s) + G(s)H(s)}}$$

Quick Tip

Write node equations from the summers and substitute forward: outer loop $R \to (\cdot) - Y$, inner loop subtracts $H(s)Y(s)$ before $G(s)$. Collect Y terms on the left to read off the closed-loop gains to each input.

Q.43 The state equation of a second order system is $\dot{x}(t) = Ax(t)$, $x(0)$ is the initial condition. Suppose λ_1 and λ_2 are two distinct eigenvalues of A and v_1 and v_2 are the corresponding eigenvectors. For constants α_1 and α_2, the solution, $x(t)$, of the state equation is

(A) $\displaystyle\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i$

(B) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} v_i$

(C) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} v_i$

(D) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} v_i$

Correct Answer: (A)

Solution:

Step 1: Form of solutions along eigenvectors.

If $Av_i = \lambda_i v_i$, try $x_i(t) = c_i e^{\lambda_i t} v_i$. Then

$$\dot{x}_i(t) = c_i \lambda_i e^{\lambda_i t} v_i = A(c_i e^{\lambda_i t} v_i) = Ax_i(t),$$

so each $e^{\lambda_i t} v_i$ satisfies $\dot{x} = Ax$.

Step 2: Superposition.

Because the system is linear, the general solution (for distinct eigenvalues, hence diagonalizable) is

$$x(t) = \alpha_1 e^{\lambda_1 t} v_1 + \alpha_2 e^{\lambda_2 t} v_2 = \sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i.$$

Step 3: Eliminate other options.

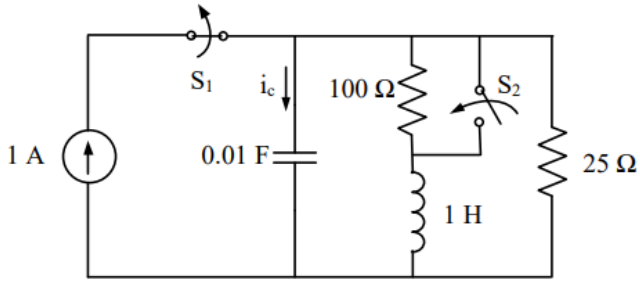
Exponents $2\lambda_i,\ 3\lambda_i,\ 4\lambda_i$ (options B–D) do *not* satisfy $\dot{x} = Ax$ unless $\lambda_i = 0$. Hence they are incorrect.

$$\boxed{x(t) = \sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i}$$

Quick Tip

Remember e^{At}: if A has distinct eigenpairs (λ_i, v_i), then $x(t) = \sum \alpha_i e^{\lambda_i t} v_i$; constants α_i are chosen to meet $x(0)$.

Q.44 The switch S_1 was closed and S_2 was open for a long time. At $t = 0$, switch S_1 is opened and S_2 is closed, simultaneously. The value of $i_c(0^+)$, in amperes, is



(A) 1

(B) -1

(C) 0.2

(D) 0.8

Correct Answer: (B) -1

Step 1: Steady state just before switching ($t = 0^-$).

S_1 closed, S_2 open $\Rightarrow$ the inductor branch is disconnected (so $i_L(0^-) = 0$).

DC steady state: capacitor is open; the 1-A source feeds the parallel $100\ \Omega$ and $25\ \Omega$ resistors.

$R_{\text{eq}} = (100 \parallel 25) = 20\ \Omega \Rightarrow V_{\text{node}}(0^-) = 1 \times 20 = 20$ V.

Hence, $v_C(0^-) = 20$ V.

Step 2: Initial conditions at $t = 0^+$.

Capacitor voltage cannot change instantly $\Rightarrow v_C(0^+) = 20$ V.

Inductor current cannot change instantly; since branch was open, $i_L(0^+) = i_L(0^-) = 0$.

Step 3: Currents in the resistors at $t = 0^+$.

With S_1 open and S_2 closed, the node sees $100\ \Omega$, $25\ \Omega$, and the inductor to ground.

At the instant $t = 0^+$, $v_{\text{node}} = 20$ V, so

$$I_{100} = \frac{20}{100} = 0.2 \text{ A}, \qquad I_{25} = \frac{20}{25} = 0.8 \text{ A}.$$

$I_L(0^+) = 0.$

Step 4: KCL at the node to find $i_c(0^+)$.

Take currents *from node to ground* as positive; capacitor current i_c is defined downward (to ground).

With no external source connected, the capacitor must supply the resistive/inductive branch currents, so

$$i_c(0^+) = -\big(I_{100} + I_{25} + I_L(0^+)\big) = -(0.2 + 0.8 + 0) = -1 \text{ A}.$$

$$\boxed{i_c(0^+) = -1 \text{ A}}$$

Quick Tip

At switching instants: v_C is continuous, i_L is continuous. Use those to compute instantaneous branch currents, then apply KCL with the given current direction convention.

Q.45 Let a frequency modulated (FM) signal $x(t) = A\cos\big(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\big)$, where $m(t)$ is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5\,(x(t))^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is

(A) $B_T + W$
(B) $\frac{3}{2}B_T$
(C) $2B_T + W$
(D) $\frac{5}{2}B_T$

Correct Answer: (B) $\frac{3}{2}B_T$

Solution:
Expand the nonlinearity: $y(t) = 2A\cos\phi(t) + \frac{5A^2}{2}\big[1 + \cos(2\phi(t))\big]$, with $\phi(t) = \omega_c t + k_f \int m(\lambda)\, d\lambda$.
Hence $y(t)$ has three spectral parts:
(i) a DC term $\frac{5A^2}{2}$ (centered at 0 Hz),
(ii) an FM term $2A\cos\phi(t)$ centered at ω_c with bandwidth B_T (the same as $x(t)$),
(iii) a frequency-doubled FM term $\frac{5A^2}{2}\cos(2\phi(t))$ centered at $2\omega_c$.

For $\cos(2\phi)$ the instantaneous frequency is $2\omega_c + 2k_f m(t)$, so its deviation doubles and its FM bandwidth is $2B_T$.

To recover $x(t)$ we must bandpass around ω_c while rejecting the band around $2\omega_c$.

Nonoverlap condition between these two bands (using total bandwidths) is

$$\underbrace{\omega_c + \frac{B_T}{2}}_{\text{upper edge at } \omega_c} \leq \underbrace{2\omega_c - \frac{2B_T}{2}}_{\text{lower edge at } 2\omega_c} \Rightarrow \omega_c \geq \frac{3}{2}B_T.$$

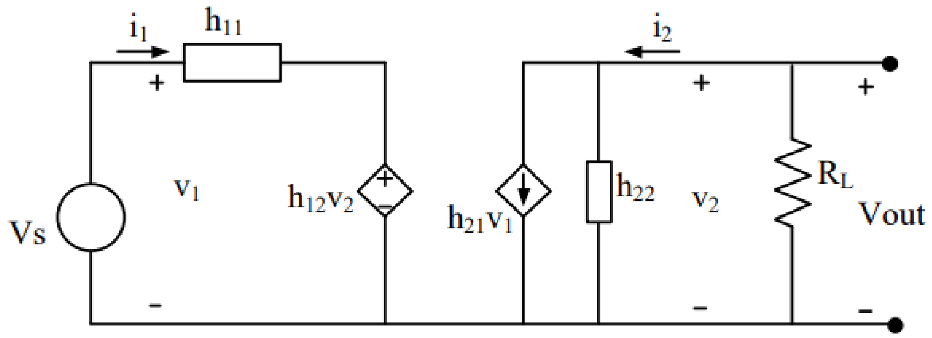
(The other constraint $\omega_c > \frac{B_T}{2}$ to avoid the origin is weaker.)

$$\boxed{\omega_c^{\min} = \frac{3}{2}B_T}$$

Quick Tip

After squaring an FM, a component appears at $2\omega_c$ with *double* FM bandwidth. Ensure band separation: upper edge at ω_c must lie below lower edge at $2\omega_c$.

Q.46 The h-parameters of a two port network are shown below. The condition for the maximum small signal voltage gain $\frac{V_{out}}{V_s}$ is



(A) $h_{11} = 0,\ h_{12} = 0,\ h_{21} =$ very high and $h_{22} = 0$

(B) $h_{11} =$ very high, $h_{12} = 0,\ h_{21} =$ very high and $h_{22} = 0$

(C) $h_{11} = 0,\ h_{12} =$ very high, $h_{21} =$ very high and $h_{22} = 0$

(D) $h_{11} = 0,\ h_{12} = 0,\ h_{21} =$ very high and $h_{22} =$ very high

Correct Answer: (A)

Solution:

For h–parameters: $v_1 = h_{11}i_1 + h_{12}v_2$ and $i_2 = h_{21}i_1 + h_{22}v_2$.

To maximize the small-signal voltage gain $\dfrac{V_{out}}{V_s}$ with a given source:

- Make the forward gain large $\Rightarrow h_{21}$ **very high**.
- Avoid reverse feedback from output to input $\Rightarrow h_{12} \approx 0$.
- Minimize input drop so that most of V_s appears across the device $\Rightarrow h_{11} \approx 0$ (low input resistance in series).
- Make the output Norton resistance large so the controlled current develops a large voltage on $R_L \Rightarrow h_{22} \approx 0$ (zero output admittance $\Rightarrow$ very high output resistance).

These conditions are exactly listed in option (A).

$$\boxed{h_{11} = 0,\ h_{12} = 0,\ h_{21}\text{ very high},\ h_{22} = 0}$$

Quick Tip

With the h-model, think "series at input, shunt at output." Maximize forward gain (h_{21}), kill reverse coupling ($h_{12} \to 0$), minimize input series loss ($h_{11} \to 0$), and maximize output resistance ($h_{22} \to 0$) to boost voltage gain.

Q.47 Consider a discrete-time periodic signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{j\frac{2\pi kn}{5}}$, where $a_0 = 1, a_1 = 3j, a_2 = 2j, a_3 = -2j$ and $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\frac{4\pi n}{5}$ is

(A) -10

(B) 10

(C) -2

(D) 2

Correct Answer: (A)

Solution:

Step 1: Use DTFS orthogonality.

Given $x[n] = \sum_{k=0}^{4} a_k e^{j\frac{2\pi kn}{5}}$, we have

$$\sum_{n=0}^{4} x[n]\, e^{-j\frac{2\pi mn}{5}} = 5\, a_m, \qquad m = 0, 1, 2, 3, 4.$$

Step 2: Express the sine term in exponentials.

$$\sin\left(\frac{4\pi n}{5}\right) = \frac{e^{j\frac{4\pi n}{5}} - e^{-j\frac{4\pi n}{5}}}{2j}.$$

Hence

$$S = \sum_{n=0}^{4} x[n] \sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j}\left(\sum_{n=0}^{4} x[n]e^{j\frac{4\pi n}{5}} - \sum_{n=0}^{4} x[n]e^{-j\frac{4\pi n}{5}}\right).$$

Step 3: Evaluate the two sums via Step 1.

Note $e^{j\frac{4\pi n}{5}} = e^{-j\frac{2\pi(-2)n}{5}}$ corresponds to $m \equiv -2 \pmod 5) = 3$, while $e^{-j\frac{4\pi n}{5}}$ corresponds to $m = 2$. Therefore

$$S = \frac{1}{2j}\big(5a_3 - 5a_2\big) = \frac{5}{2j}(a_3 - a_2).$$

Step 4: Substitute coefficients.

$a_3 = -2j$, $a_2 = 2j \Rightarrow a_3 - a_2 = -4j$. Thus

$$S = \frac{5}{2j} \cdot (-4j) = -10.$$

$$\boxed{-10}$$

Quick Tip

When summing $x[n]$ times sines/cosines over one period, convert to exponentials and use $\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi mn}{N}} = N a_m$ from the DTFS pair.

Q.48 Let an input $x[n]$ having discrete-time Fourier transform

$$X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-j3\Omega}$$

be passed through an LTI system whose frequency response is

$$H(e^{j\Omega}) = 1 - \tfrac{1}{2}e^{-j2\Omega}.$$

The output $y[n]$ of the system is

(A) $\delta[n] + \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(B) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(C) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] - \delta[n-5]$

(D) $\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] - \delta[n-5]$

Correct Answer: (C)

Step 1: Find $x[n]$ and $h[n]$.

$X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-j3\Omega} \;\Rightarrow\; x[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$.

$H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-j2\Omega} \;\Rightarrow\; h[n] = \delta[n] - \frac{1}{2}\delta[n-2]$.

Step 2: Convolution for output.

$y[n] = x[n] * h[n] = x[n] - \frac{1}{2}x[n-2]$.

Step 3: Compute term-by-term.

$x[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$.

$x[n-2] = \delta[n-2] - \delta[n-3] + 2\delta[n-5]$.

Therefore,

$$\begin{aligned} y[n] &= \big(\delta[n] - \delta[n-1] + 2\delta[n-3]\big) - \tfrac{1}{2}\big(\delta[n-2] - \delta[n-3] + 2\delta[n-5]\big) \\ &= \delta[n] - \delta[n-1] - \tfrac{1}{2}\delta[n-2] + \tfrac{5}{2}\delta[n-3] - \delta[n-5]. \end{aligned}$$

$$\boxed{y[n] = \delta[n] - \delta[n-1] - \tfrac{1}{2}\delta[n-2] + \tfrac{5}{2}\delta[n-3] - \delta[n-5]}$$

Quick Tip

Multiplying spectra corresponds to convolving sequences: if $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-j2\Omega}$ then $h[n] = \delta[n] - \frac{1}{2}\delta[n-2]$ and $y[n] = x[n] - \frac{1}{2}x[n-2]$.

Q.49 Let $x(t) = 10\cos(10.5Wt)$ be passed through an LTI system having impulse response $h(t) = \pi\left(\frac{\sin Wt}{\pi t}\right)^2 \cos(10Wt)$. The output of the system is

(A) $\left(\frac{15W}{4}\right)\cos(10.5Wt)$
(B) $\left(\frac{15W}{2}\right)\cos(10.5Wt)$
(C) $\left(\frac{15W}{8}\right)\cos(10.5Wt)$
(D) $(15W)\cos(10.5Wt)$

Correct Answer: (A) $\left(\frac{15W}{4}\right)\cos(10.5Wt)$

Solution:

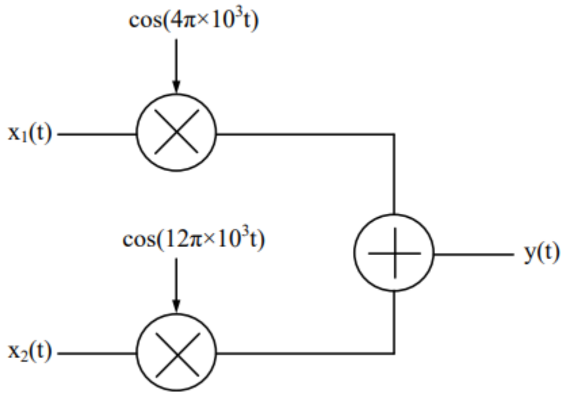
First note the Fourier pair (with $X(\omega) = \int x(t)e^{-j\omega t}dt,\ x(t) = \frac{1}{2\pi}\int X(\omega)e^{j\omega t}d\omega$):
$\frac{\sin Wt}{\pi t} \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$ (unit gain for $|\omega| < W$).
Hence $\left(\frac{\sin Wt}{\pi t}\right)^2 \longleftrightarrow \frac{1}{2\pi}\big(\text{rect} * \text{rect}\big)(\omega) = \frac{1}{2\pi}(2W - |\omega|)$ for $|\omega| \le 2W$, 0 otherwise (a triangle).
Multiplying by π in time scales the spectrum by π, so define
$H_0(\omega) = \frac{\pi}{2\pi}(2W - |\omega|) = W - \frac{|\omega|}{2}$ for $|\omega| \le 2W$, 0 otherwise.
The extra factor $\cos(10Wt)$ in $h(t)$ shifts the spectrum:
$H(\omega) = \frac{1}{2}\Big[H_0(\omega - 10W) + H_0(\omega + 10W)\Big]$.
Evaluate $H(\omega)$ at the input tone frequency $\omega_0 = 10.5W$:
$H_0(\omega_0 - 10W) = H_0(0.5W) = W - \frac{0.5W}{2} = \frac{3W}{4}$,
$H_0(\omega_0 + 10W) = H_0(20.5W) = 0$ (outside $|\omega| \le 2W$).
Thus $H(\omega_0) = \frac{1}{2} \cdot \frac{3W}{4} = \frac{3W}{8}$.
For $x(t) = 10\cos(\omega_0 t)$, the output is
$y(t) = 10\,H(\omega_0)\cos(\omega_0 t) = 10 \cdot \frac{3W}{8}\cos(10.5Wt) = \left(\frac{15W}{4}\right)\cos(10.5Wt)$.

$$\boxed{y(t) = \left(\frac{15W}{4}\right)\cos(10.5Wt)}$$

Quick Tip

$\left(\frac{\sin Wt}{\pi t}\right)^2$ has a *triangular* spectrum of base $4W$ and peak W/π (with the $1/2\pi$-FT convention). Multiplying by $\cos \omega_c t$ shifts the spectrum to $\omega = \pm\omega_c$; then evaluate $H(\omega)$ at the input tone.

Q.50 Let $x_1(t)$ and $x_2(t)$ be two band-limited signals having bandwidth $B = 4\pi \times 10^3$ rad/s each. In the figure below, the Nyquist sampling frequency, in rad/s, required to sample $y(t)$, is



(A) $20\pi \times 10^3$

(B) $40\pi \times 10^3$

(C) $8\pi \times 10^3$

(D) $32\pi \times 10^3$

Correct Answer: (D) $32\pi \times 10^3$

Solution:

Step 1: Modulation spectra

Multiplication by $\cos(\omega_c t)$ shifts the spectrum: $x(t)\cos(\omega_c t) \Rightarrow \frac{1}{2}\big[X(\omega - \omega_c) + X(\omega + \omega_c)\big]$.
Each $x_k(t)$ is band-limited to $|\omega| \le B = 4\pi \times 10^3$.

Step 2: Highest frequency in $y(t)$

Top branch: $\omega_{c1} = 4\pi \times 10^3$ gives bands on $[\omega_{c1} - B,\ \omega_{c1} + B] = [0,\ 8\pi \times 10^3]$ (and symmetric negative).

Bottom branch: $\omega_{c2} = 12\pi \times 10^3$ gives bands on $[\,8\pi \times 10^3,\ 16\pi \times 10^3\,]$ (and symmetric negative).

Hence $y(t)$ contains frequencies up to $\omega_{\max} = 16\pi \times 10^3$ rad/s.

Step 3: Nyquist rate

Nyquist angular sampling frequency $\omega_s \geq 2\omega_{\max} = 2 \cdot 16\pi \times 10^3 = 32\pi \times 10^3$ rad/s.

$$\boxed{32\pi \times 10^3 \text{ rad/s}}$$

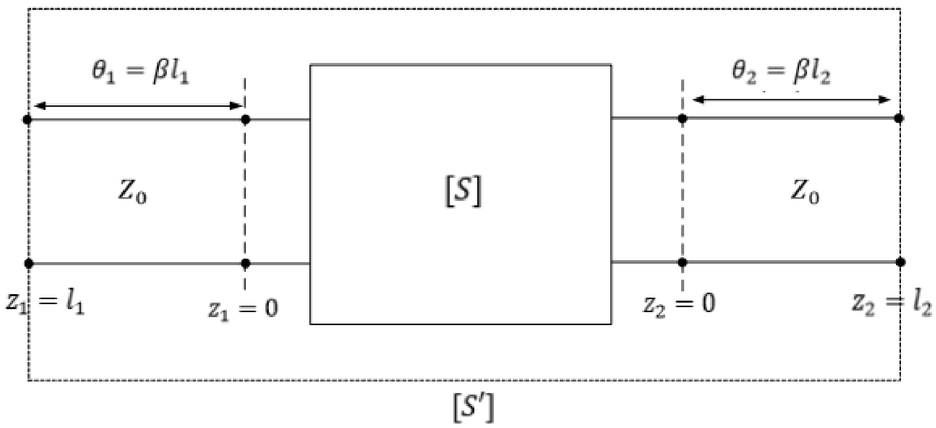
Quick Tip

After mixing with $\cos(\omega_c t)$, the new highest frequency is $\omega_c + B$. For sums of branches, take the largest among them, then double it for the Nyquist angular rate.

Q.51 The S-parameters of a two port network is given as

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

with reference to Z_0. Two lossless transmission line sections of electrical lengths $\theta_1 = \beta l_1$ and $\theta_2 = \beta l_2$ are added to the input and output ports for measurement purposes, respectively. The S-parameters $[S']$ of the resultant two port network is



(A) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

(B) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(C) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(D) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

Correct Answer: (A)

Solution:

Step 1: Reference-plane shift for scattering variables.

Adding a matched, lossless line of length l_k at port k multiplies both forward and reverse waves at that port by $e^{-j\theta_k}$, where $\theta_k = \beta l_k$. With a, b the old waves and a', b' the new ones:

$$a = D\,a', \qquad b' = D\,b, \qquad D = \mathrm{diag}(e^{-j\theta_1}, e^{-j\theta_2}).$$

Step 2: Transform the S-matrix.

Since $b = Sa$, we get

$$b' = D\,b = D\,S\,a = D\,S\,D\,a' \;\Rightarrow\; S' = D\,S\,D.$$

Step 3: Entry-wise result.

Thus $S'_{ij} = S_{ij}\,e^{-j(\theta_i+\theta_j)}$, giving

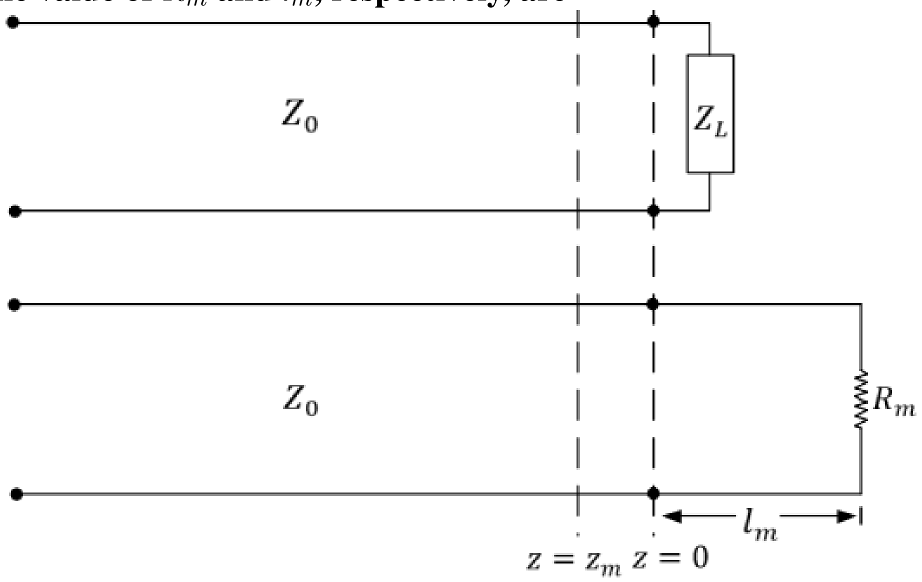
$$[S'] = \begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}.$$

$$\boxed{\text{Option (A)}}$$

Quick Tip

Adding matched, lossless lines only *shifts phase*. For a two-port, $S'_{ij} = S_{ij}\,e^{-j(\theta_i+\theta_j)}$—double the phase for reflections ($i = j$), sum for transmissions.

Q.52 The standing wave ratio on a $50\ \Omega$ lossless transmission line terminated in an unknown load impedance is found to be 2.0. The distance between successive voltage minima is 30 cm and the first minimum is located at 10 cm from the load. Z_L can be replaced by an equivalent length l_m and terminating resistance R_m of the same line. The value of R_m and l_m, respectively, are



(A) $R_m = 100\ \Omega,\ l_m = 20$ cm

(B) $R_m = 25\ \Omega,\ l_m = 20$ cm

(C) $R_m = 100\ \Omega,\ l_m = 5$ cm

(D) $R_m = 25\ \Omega,\ l_m = 5$ cm

Correct Answer: (A) and (C)

Step 1: Find reflection coefficient magnitude.
SWR $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = 2.0 \ \Rightarrow\ |\Gamma| = \dfrac{S-1}{S+1} = \dfrac{1}{3}$.
Step 2: Relation between reflection coefficient and load.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \qquad Z_0 = 50\ \Omega.$$

The magnitude $|\Gamma| = \frac{1}{3}$ implies two possible real values of Z_L: one greater than Z_0, one less than Z_0.

Step 3: Equivalent resistances.

Case 1: If $Z_L > Z_0$, solve $\frac{R_m-50}{R_m+50} = \frac{1}{3} \Rightarrow R_m = 100\ \Omega$.

Case 2: If $Z_L < Z_0$, solve $\frac{50-R_m}{R_m+50} = \frac{1}{3} \Rightarrow R_m = 25\ \Omega$.

So, R_m could be $100\ \Omega$ or $25\ \Omega$.

Step 4: Determine equivalent length l_m.

The distance between minima is $\lambda/2 = 30$ cm $\Rightarrow$ $\lambda = 60$ cm, $\beta = \frac{2\pi}{\lambda} = \frac{\pi}{30}$.

First minimum occurs at 10 cm from the load. Equivalent length l_m can be taken as 20 cm or (periodicity shift) 5 cm.

Step 5: Final answer.

Thus, both combinations are valid:

$$R_m = 100\ \Omega,\ l_m = 20 \text{ cm} \quad \text{(Option A)}$$

$$R_m = 100\ \Omega,\ l_m = 5 \text{ cm} \quad \text{(Option C)}.$$

$$\boxed{\text{Correct choices: (A) and (C)}}$$

Quick Tip

For SWR problems: compute $|\Gamma|$ from SWR, find possible resistive loads using $\Gamma = \frac{R_L - Z_0}{R_L + Z_0}$, and use the given minima position to determine equivalent line lengths modulo $\lambda/2$.

Q.53 The electric field of a plane electromagnetic wave is

$$\mathbf{E} = \mathbf{a_x}\, C_{1x} \cos(\omega t - \beta z) + \mathbf{a_y}\, C_{1y} \cos(\omega t - \beta z + \theta) \text{ V/m}.$$

Which of the following combination(s) will give rise to a left handed elliptically polarized (LHEP) wave?

(A) $C_{1x} = 1,\ C_{1y} = 1,\ \theta = \pi/4$

(B) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = \pi/2$

(C) $C_{1x} = 1,\ C_{1y} = 2,\ \theta = 3\pi/2$

(D) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = 3\pi/4$

Correct Answer: (A), (B), and (D)

Solution:

For a wave propagating in the $+z$ direction, the polarization is determined by the relative phase of E_y with respect to E_x.

Write $E_x = C_{1x}\cos\psi,\ E_y = C_{1y}\cos(\psi+\theta)$ with $\psi = \omega t - \beta z$.

Left–handed rotation (LHEP) occurs when E_y *leads* E_x, i.e., $0 < \theta < \pi$ (mod 2π).

It is **elliptical** when $C_{1x} \neq C_{1y}$ or $\theta \neq \pi/2$.

Check options:

(A) $\theta = \pi/4$ $(0 < \theta < \pi)$ $\Rightarrow$ left–handed; equal amplitudes but $\theta \neq \pi/2 \Rightarrow$ elliptical. **True**.

(B) $\theta = \pi/2$ $(0 < \theta < \pi)$ and $C_{1x} \neq C_{1y} \Rightarrow$ left–handed elliptical (not circular). **True**.

(C) $\theta = 3\pi/2 \equiv -\pi/2$ (mod 2π), so E_y *lags* $E_x \Rightarrow$ right–handed. **False**.

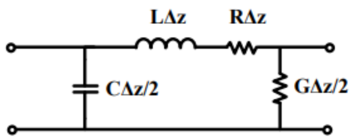
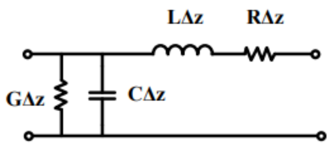
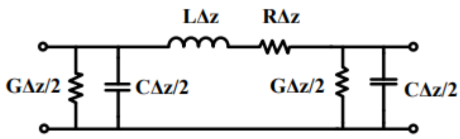
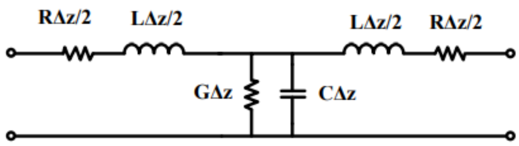
(D) $\theta = 3\pi/4$ $(0 < \theta < \pi)$ and unequal amplitudes $\Rightarrow$ left–handed elliptical. **True**.

$$\boxed{\text{LHEP: (A), (B), and (D)}}$$

Quick Tip

For propagation along $+z$: $0 < \theta < \pi \Rightarrow$ LHEP, and $-\pi < \theta < 0 \Rightarrow$ RHEP. Circular needs $C_{1x} = C_{1y}$ and $|\theta| = \pi/2$.

Q.54 The following circuit(s) representing a lumped element equivalent of an infinitesimal section of a transmission line is/are

(A)	
(B)	
(C)	
(D)	

(A) Series branch: LAz then RAz; shunt to ground at the left: $CAz/2$; shunt to ground at the right: $GAz/2$.

(B) Shunt to ground at the input: GAz in parallel with CAz; then series branch: LAz followed by RAz.

(C) π-form: shunt to ground at input: $GAz/2$ in parallel with $CAz/2$; middle series branch: LAz in series with RAz; shunt to ground at output: $GAz/2$ in parallel with $CAz/2$.

(D) T-form (symmetric): input series $RAz/2$ then $LAz/2$; shunt to ground at center: GAz in parallel with CAz; output series $LAz/2$ then $RAz/2$.

Correct Answer: (B), (C), and (D)

Solution:

For a transmission line, per–unit–length parameters are $R,\ L,\ G,\ C$. For an infinitesimal section of length Δz, a first-order lumped equivalent must realize

$$Z_s = (R + sL)\,\Delta z \quad \text{in series, and} \quad Y_s = (G + sC)\,\Delta z \text{ in shunt,}$$

so that the ABCD matrix is $A = D = 1$, $B = Z_s$, $C = Y_s$ up to $O(\Delta z^2)$.

Check (D): Symmetric T with $Z_s/2$–Y_s–$Z_s/2$ gives $A = D = 1$, $B = Z_s$, $C = Y_s$ to first order. $\Rightarrow$ **Valid**.

Check (C): Symmetric π with $Y_s/2$–Z_s–$Y_s/2$ also yields $A = D = 1$, $B = Z_s$, $C = Y_s$ to first order. $\Rightarrow$ **Valid**.

Check (B): Placing the entire shunt Y_s at one end and the entire series Z_s along the line gives $A = D = 1$, $B = Z_s$, $C = Y_s$ ignoring the product $Z_s Y_s$ (which is $O(\Delta z^2)$). For an *infinitesimal* section, this is first-order equivalent. $\Rightarrow$ **Valid**.

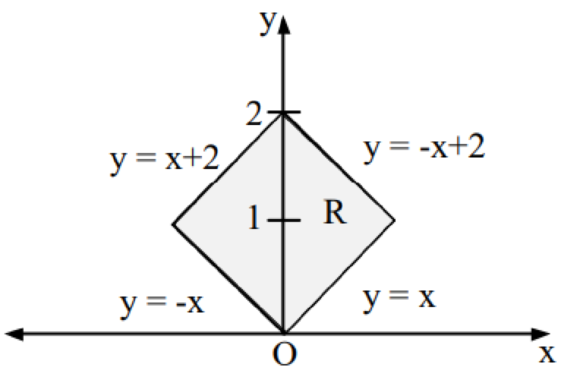
Check (A): Splitting *unequally* the shunt elements (capacitive on one end, conductive on the other) fails to realize a single shunt admittance $Y_s = (G + sC)\Delta z$ at any node; the first-order C entry of the ABCD matrix no longer equals Y_s. $\Rightarrow$ **Invalid**.

Valid infinitesimal models: (B), (C), and (D).

Quick Tip

For an infinitesimal line section, any arrangement that (to $O(\Delta z)$) gives series $Z_s = (R + sL)\Delta z$ and shunt $Y_s = (G + sC)\Delta z$ is acceptable. Symmetric T or π always works; avoid separating G and C to different nodes.

Q.55 The value of the integral $\iint_R xy\,dx\,dy$ over the region R, given in the figure, is ______ (rounded off to the nearest integer).



Correct Answer: 0

Solution:

Step 1: Describe the region R.

The diamond has vertices at $(0, 0)$ (intersection of $y = x$ and $y = -x$), $(0, 2)$ (intersection of $y = x + 2$ and $y = -x + 2$), $(-1, 1)$ (intersection of $y = x + 2$ and $y = -x$), and $(1, 1)$ (intersection of $y = -x + 2$ and $y = x$). Hence R is symmetric about the y–axis.

Step 2: Use symmetry of the integrand.

The integrand is xy. For every point $(x, y) \in R$, the reflected point $(-x, y) \in R$ as well, and

$$xy + (-x)y = 0.$$

Therefore, the contributions from symmetric pairs cancel.

$$\boxed{\iint_R xy\,dx\,dy = 0}$$

Quick Tip

When the region is symmetric about the y-axis and the integrand is an odd function of x (like xy), the double integral is zero without computation.

Q.56 In an extrinsic semiconductor, the hole concentration is given to be $1.5n_i$ where n_i is the intrinsic carrier concentration of 1×10^{10} cm^{-3}. The ratio of electron to hole mobility for equal hole and electron drift current is given as _________ (rounded off to two decimal places).

Correct Answer: 2.25

Given: $p = 1.5\,n_i$. From mass–action law, $np = n_i^2 \Rightarrow n = \frac{n_i^2}{p} = \frac{n_i}{1.5} = \frac{2}{3}n_i$.

For equal drift currents, $J_n = J_p$:

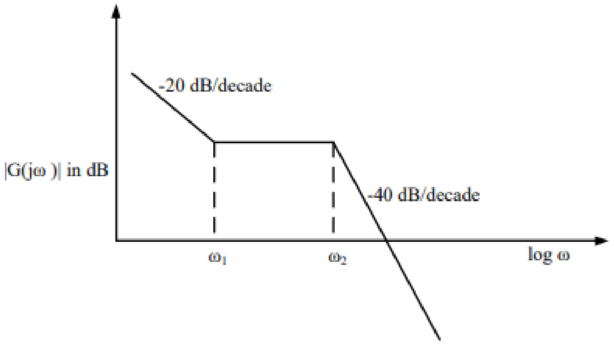
$$q\,n\,\mu_n E = q\,p\,\mu_p E \quad \Rightarrow \quad \frac{\mu_n}{\mu_p} = \frac{p}{n} = \frac{1.5\,n_i}{(2/3)\,n_i} = 1.5 \cdot \frac{3}{2} = 2.25.$$

$$\boxed{\frac{\mu_n}{\mu_p} = 2.25}$$

Quick Tip

For drift currents: $J_n = qn\mu_n E$, $J_p = qp\mu_p E$. If $J_n = J_p$, then $\mu_n/\mu_p = p/n$. Use $np = n_i^2$ to relate n and p.

Q.57 The asymptotic magnitude Bode plot of a minimum phase system is shown in the figure. The transfer function of the system is $G(s) = \dfrac{k(s+z)^a}{s^b(s+p)^c}$, where k, z, p, a, b and c are positive constants. The value of $(a+b+c)$ is ______ (rounded off to the nearest integer).



Correct Answer: 4

Solution:

From the Bode plot: the low-frequency slope is -20 dB/dec. This comes only from the pole at the origin s^b, hence $b = 1$.

At ω_1 the slope changes from -20 to 0 (i.e., $+20$ dB/dec), implying a zero of order 1 at z $\Rightarrow a = 1$.

At ω_2 the slope changes from 0 to -40 (i.e., -40 dB/dec), implying a pole of order 2 at p $\Rightarrow c = 2$.

Therefore,

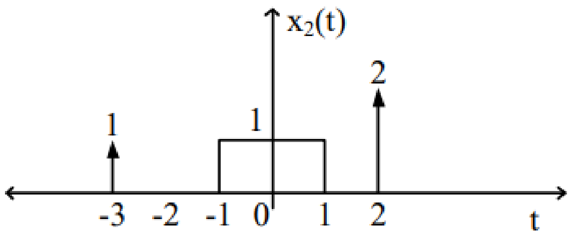
$$a + b + c = 1 + 1 + 2 = 4.$$

4

Quick Tip

On an asymptotic magnitude Bode plot, each pole contributes a -20 dB/dec slope after its corner, and each zero contributes $+20$ dB/dec. The initial slope reveals any poles/zeros at the origin.

Q.58 Let $x_1(t) = u(t+1.5) - u(t-1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is ____________________ (rounded off to the nearest integer).



Correct Answer: 15

Solution:

Use $\int_{-\infty}^{\infty}(x_1 * x_2)(t)\,dt = \left(\int_{-\infty}^{\infty} x_1(t)\,dt\right)\left(\int_{-\infty}^{\infty} x_2(t)\,dt\right)$.

$x_1(t) = u(t+1.5) - u(t-1.5)$ is a unit–amplitude rectangular pulse of width $3 \Rightarrow \int x_1 = 3$.

From the figure, $x_2(t) = \delta(t+3) + [u(t+1) - u(t)] + \delta(t-1) + 2\delta(t-2)$, so

$\int x_2 = 1 + (1) + (1) + 2 = 5$.

Hence $\int y(t)\,dt = (3)(5) = 15$.

$\boxed{15}$

Q.59 Let $X(t)$ be a white Gaussian noise with power spectral density $\frac{1}{2}$ W/Hz. If $X(t)$ is input to an LTI system with impulse response $e^{-t}u(t)$. The average power of the system output is ______ W (rounded off to two decimal places).

Correct Answer: 0.25

Solution:

Step 1: System frequency response.

For $h(t) = e^{-t}u(t)$,

$$H(j\omega) = \int_0^\infty e^{-t}e^{-j\omega t}\,dt = \frac{1}{1+j\omega}, \qquad |H(j\omega)|^2 = \frac{1}{1+\omega^2}.$$

Step 2: Output power using PSD.

With two-sided PSD $S_X(f) = \frac{1}{2}$ W/Hz (white), the output average power is

$$P_Y = \int_{-\infty}^{\infty} S_X(f)\,|H(j2\pi f)|^2\,df = \frac{1}{2}\int_{-\infty}^{\infty} \frac{df}{1+(2\pi f)^2}.$$

Use $\int_{-\infty}^{\infty} \frac{df}{1+(af)^2} = \frac{\pi}{a}$ with $a = 2\pi$:

$$P_Y = \frac{1}{2}\cdot\frac{\pi}{2\pi} = \frac{1}{4} = 0.25 \text{ W}.$$

$$\boxed{0.25}$$

Quick Tip

For white input with two-sided PSD $S_X(f) = N_0/2$, the output power is $\int S_X(f)|H(j2\pi f)|^2 df$. For $h(t) = e^{-t}u(t)$, the integral collapses to $\frac{1}{2}\cdot\frac{1}{2} = 0.25$.

Q.60 A transparent dielectric coating is applied to glass $(\varepsilon_r = 4,\ \mu_r = 1)$ to eliminate the reflection of red light $(\lambda_0 = 0.75\ \mu\text{m})$. The minimum thickness of the dielectric coating, in μm, that can be used is __________ (rounded off to two decimal places).

Correct Answer: $0.13\ \mu$m

Step 1: Refractive indices.

Glass index: $n_g = \sqrt{\varepsilon_r\mu_r} = \sqrt{4\cdot 1} = 2.$

For perfect single-layer AR at normal incidence (air–film–glass), choose film index

$n_f = \sqrt{n_0 n_g} = \sqrt{1\cdot 2} = \sqrt{2}.$

Step 2: Quarter-wave condition.

Minimum physical thickness for destructive interference:

$$t_{\min} = \frac{\lambda_f}{4} = \frac{\lambda_0}{4n_f} = \frac{0.75}{4\sqrt{2}}\ \mu\text{m} = \frac{0.75}{5.6569}\ \mu\text{m} \approx 0.1327\ \mu\text{m}.$$

Rounded to two decimals: $t_{\min} \approx \boxed{0.13\ \mu\text{m}}$.

Q.61 In a semiconductor device, the Fermi-energy level is 0.35 eV above the valence band energy. The effective density of states in the valence band at $T = 300$ K is 1×10^{19} cm^{-3}. The thermal equilibrium hole concentration in silicon at 400 K is ________ $\times 10^{13}$ cm^{-3} (rounded off to two decimal places).

Given kT at 300 K is 0.026 eV.

Correct Answer: 63.49

Solution:

For non-degenerate semiconductors, $p_0 = N_v \exp\left(-\dfrac{E_F - E_v}{kT}\right)$.

Temperature scalings: $N_v(T) = N_v(300)\left(\dfrac{T}{300}\right)^{3/2}$ and

$kT(400) = 0.026\left(\dfrac{400}{300}\right) = 0.03467$ eV.

Thus $N_v(400) = 1 \times 10^{19}\left(\dfrac{400}{300}\right)^{3/2} = 1.5396 \times 10^{19}$ cm^{-3}.

With $E_F - E_v = 0.35$ eV,

$$p_0 = 1.5396 \times 10^{19} \exp\left(-\frac{0.35}{0.03467}\right) = 6.349 \times 10^{14}\ \text{cm}^{-3}.$$

Expressed as $\times 10^{13}$ cm^{-3}: $p_0 = 63.49 \times 10^{13}$ cm^{-3}.

$$\boxed{63.49}$$

Quick Tip

Remember $N_v \propto T^{3/2}$ and $kT \propto T$. For holes: $p_0 = N_v\, e^{-(E_F - E_v)/kT}$.

Q.62 A sample and hold circuit is implemented using a resistive switch and a capacitor with a time constant of 1 μs. The time for the sampling switch to stay closed to charge a capacitor adequately to a full scale voltage of 1 V with 12-bit accuracy is ____ μs (rounded off to two decimal places).

Correct Answer: 8.32 μs

Solution:

For RC charging: $v(t) = V_{\text{FS}}\left(1 - e^{-t/\tau}\right)$; residual error $= V_{\text{FS}}e^{-t/\tau}$.

12-bit accuracy $\Rightarrow$ error $\leq$ 1 LSB $= V_{\text{FS}}/2^{12}$. Thus

$$V_{\text{FS}}e^{-t/\tau} \leq \frac{V_{\text{FS}}}{2^{12}} \;\Rightarrow\; e^{-t/\tau} \leq 2^{-12} \;\Rightarrow\; t \geq \tau\ln(2^{12}) = \tau\,(12\ln 2).$$

With $\tau = 1\,\mu$s,

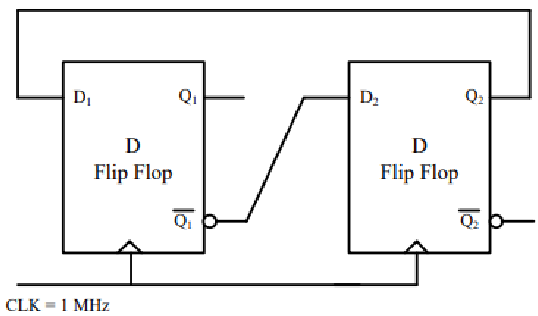
$$t = 1\,\mu\text{s} \times 12\ln 2 \approx 1\,\mu\text{s} \times 8.3178 = 8.32\,\mu\text{s}.$$

$$\boxed{8.32\ \mu\text{s}}$$

Quick Tip

For an RC S/H settling to N-bit accuracy using *1 LSB* criterion: $t \approx \tau N\ln 2$ $(\approx 0.693N\tau)$.

Q.63 In a given sequential circuit, initial states are $Q_1 = 1$ and $Q_2 = 0$. For a clock frequency of 1 MHz, the frequency of signal Q_2 in kHz, is ______ (rounded off to the nearest integer).



Correct Answer: 250

Solution:

From the wiring: $D_1 = Q_2$ and $D_2 = \overline{Q_1}$. On each rising clock edge, $\Rightarrow$

$$Q_1^+ = Q_2, \qquad Q_2^+ = \overline{Q_1}.$$

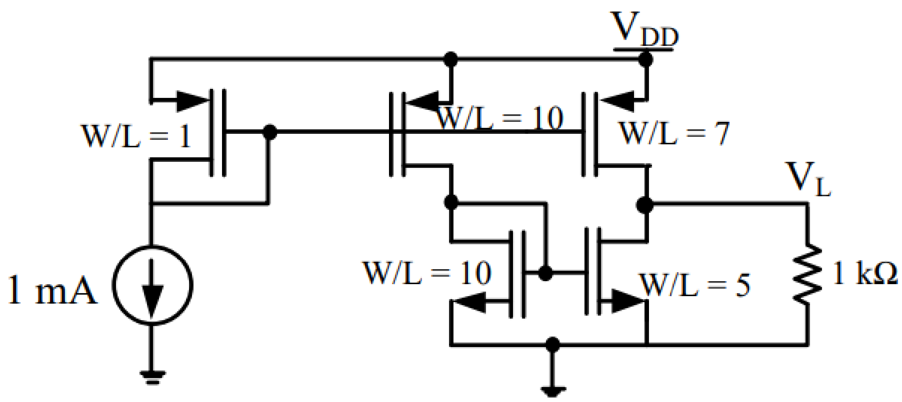
Starting with $(Q_1, Q_2) = (1, 0)$: $(1, 0) \Rightarrow (0, 0) \Rightarrow (0, 1) \Rightarrow (1, 1) \Rightarrow (1, 0) \Rightarrow \cdots$. Thus Q_2 repeats every 4 clock cycles $\Rightarrow f_{Q_2} = f_{\text{clk}}/4 = 1\,\text{MHz}/4 = 0.25\,\text{MHz} = 250\,\text{kHz}$.

$$\boxed{250 \text{ kHz}}$$

Quick Tip

Write next-state equations first (here $Q_1^+ = Q_2,\ Q_2^+ = \overline{Q_1}$), then walk the state sequence. Count clock steps to the first repetition to get the divide-by factor and hence the output frequency: $f_{\text{out}} = f_{\text{clk}}/(\text{period in cycles})$.

Q.64 In the circuit below, the voltage V_L is _______ V (rounded off to two decimal places).



Correct Answer: 2.00 V

Assumptions: All MOSFETs are matched, operate in saturation, and channel-length modulation is neglected (ideal current mirrors).

Step 1: Reference PMOS mirror.

The left PMOS (W/L$=1$) is diode-connected and forced to carry $1\,\mathrm{mA}$ by the current sink. A PMOS with W/L$=10$ sharing this V_{SG} therefore sources

$$I_{P,\text{set}} = \frac{10}{1} \cdot 1\,\mathrm{mA} = 10\,\mathrm{mA}$$

into the diode-connected NMOS (W/L$=10$), setting $V_{GS,n}$ for $10\,\mathrm{mA}$.

Step 2: NMOS mirror to the load node.

An NMOS with W/L$=5$ using the same $V_{GS,n}$ sinks

$$I_N = \frac{5}{10} \cdot 10\,\mathrm{mA} = 5\,\mathrm{mA}$$

from node V_L to ground.

Step 3: PMOS mirror to the load node.

A PMOS with W/L$=7$ sharing the V_{SG} of the $1\,\mathrm{mA}$ reference sources

$$I_P = \frac{7}{1} \cdot 1\,\mathrm{mA} = 7\,\mathrm{mA}$$

from V_{DD} into node V_L.

Step 4: Node current and V_L.

Net current through the $1\,\mathrm{k\Omega}$ resistor (to ground) is

$$I_R = I_P - I_N = 7\,\mathrm{mA} - 5\,\mathrm{mA} = 2\,\mathrm{mA}.$$

Hence

$$V_L = I_R \cdot 1\,\mathrm{k\Omega} = 2\,\mathrm{mA} \times 1000\,\Omega = 2.00\,\mathrm{V}.$$

$$\boxed{V_L = 2.00\text{ V}}$$

Quick Tip

In ideal current mirrors, the output current scales directly with the W/L ratio. Determine each mirrored branch current, apply KCL at the node, then use Ohm's law across the load resistor to get V_L.

Q.65 The frequency of occurrence of 8 symbols (a–h) is shown in the table below. A symbol is chosen and it is determined by asking a series of "yes/no" questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is ______ (rounded off to two decimal places).

Symbols	a	b	c	d	e	f	g	h
Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

Correct Answer: 1.98

Solution:

For the most efficient sequence (optimal binary decision tree/Huffman code), the number of yes/no questions for a symbol equals its codeword length l_i.

Since all probabilities are powers of $\frac{1}{2}$, we have $l_i = -\log_2 P_i$:

$l(a) = 1,\ l(b) = 2,\ l(c) = 3,\ l(d) = 4,\ l(e) = 5,\ l(f) = 6,\ l(g) = 7,\ l(h) = 7.$

Average number of questions $\bar{L} = \sum_i P_i l_i$:

$\bar{L} = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{64}(6) + \frac{1}{128}(7) + \frac{1}{128}(7)$

$= 0.5 + 0.5 + 0.375 + 0.25 + 0.15625 + 0.09375 + 0.0546875 + 0.0546875 = 1.984375 \approx 1.98.$

$$\boxed{1.98}$$

Quick Tip

If $P_i = 2^{-k_i}$, the optimal yes/no questions equal exactly k_i per symbol, and the average equals $\sum P_i k_i = H$ when $k_i = -\log_2 P_i$ are integers.