GATE 2023 Instrumentation Engineering (IN) Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**65

General Aptitude (GA)

Q.1 The village was nestled in a green spot, ______ the ocean and the hills.

(A) through

(B) in

(C) at

(D) between

Correct Answer: (D) between

Solution:

1) Understanding the context:

The sentence talks about a village being located between two geographical features: the ocean and the hills. The preposition "between" is used to indicate the position of the village relative to two other objects, hence making it the most suitable choice.

2) Analysis of the options:

(A) through: This suggests movement across a space or location, which is incorrect in this context. *"Nestled through the ocean and the hills"* does not convey the correct meaning.

(B) in: This would indicate the village is located inside the space of both the ocean and hills, but this is not the intended meaning in the sentence.

(C) at: This is used for specifying exact locations but doesn't convey the idea of being between two things.

(D) between: Correct choice. It indicates that the village is situated in the middle of the two features — the ocean and the hills.

The correct answer is (D) between.

Quick Tip

- **Through** indicates motion or passage. - **In** suggests being inside an area or a space. - **At** refers to a specific point or location. - **Between** refers to a position that is located in the middle of two objects.

Q.2 Disagree : Protest : : Agree : ______ (By word meaning)

(A) Refuse

(B) Pretext

(C) Recommend

(D) Refute

Correct Answer: (C) Recommend

Solution:

1) Identifying word relationship:

The given pair "Disagree : Protest" suggests a relationship where protest is an act in response to disagreement. We need a similar relationship for "Agree," where the corresponding action is associated with agreement.

2) Analysis of the options:

(A) Refuse: This is a negative response, but it doesn't match the context of "agree."

(B) Pretext: This means a false reason given for an action, which is unrelated to the context of agreement.

(C) Recommend: This is a fitting choice. When one agrees, they often make a recommendation.

(D) Refute: Refuting means disproving or denying something, which does not logically follow from agreeing.

The correct answer is (C) Recommend.

Quick Tip

- **Disagree** is often followed by **Protest**. - **Agree** is commonly followed by **Recommend**.

Q.3 A ‘frabjous’ number is defined as a 3 digit number with all digits odd, and no two adjacent digits being the same. For example, 137 is a frabjous number, while 133 is not. How many such frabjous numbers exist?

(A) 125

(B) 720

(C) 60

(D) 80

Correct Answer: (D) 80

Solution:

The problem defines a ’frabjous’ number as a 3-digit number in which all digits are odd, and no two adjacent digits are the same. The possible odd digits are 1, 3, 5, 7, and 9. We will calculate the total number of such 3-digit frabjous numbers by considering the following conditions:

- **For the first digit:** Since the number is a 3-digit number, the first digit can be any of the 5 odd digits: $\{1, 3, 5, 7, 9\}$. So, there are 5 choices for the first digit.

- **For the second digit:** The second digit must also be odd, but it cannot be the same as the first digit. Hence, there are only 4 choices for the second digit because one odd digit is already taken by the first digit.

- **For the third digit:** The third digit also needs to be odd, and it cannot be the same as the second digit. So, there are again 4 choices for the third digit.

Thus, the total number of frabjous numbers is the product of the number of choices for each digit:

$$5 \times 4 \times 4 = 80$$

Therefore, the total number of frabjous numbers is 80.

Quick Tip

When calculating the number of valid combinations for restricted problems, remember to reduce the available choices for each subsequent selection. Here, we reduced the choices for the second and third digits to avoid repetition of adjacent digits.

Q.4 Which one among the following statements must be TRUE about the mean and the median of the scores of all candidates appearing for GATE 2023?

(A) The median is at least as large as the mean.

(B) The mean is at least as large as the median.

(C) At most half the candidates have a score that is larger than the median.

(D) At most half the candidates have a score that is larger than the mean.

Correct Answer: (C) At most half the candidates have a score that is larger than the median.

Solution:

Let's break down the given options and analyze them:

- Option (A): "The median is at least as large as the mean."

- This is not necessarily true. In a skewed distribution (such as one with many lower scores and a few higher scores), the mean can be larger than the median. Thus, this statement is not universally true.

- Option (B): "The mean is at least as large as the median."

- This is also not necessarily true. In cases where the distribution is skewed to the left (more higher scores, few lower scores), the median can be larger than the mean. Hence, this statement is also not always true.

- Option (C): "At most half the candidates have a score that is larger than the median."

- This statement is **true**. By definition, the **median** divides the dataset into two equal halves. So, exactly half of the candidates will have scores smaller than the median, and the other half will have scores larger than the median. Therefore, **at most** half the candidates have a score larger than the median, which is always true.

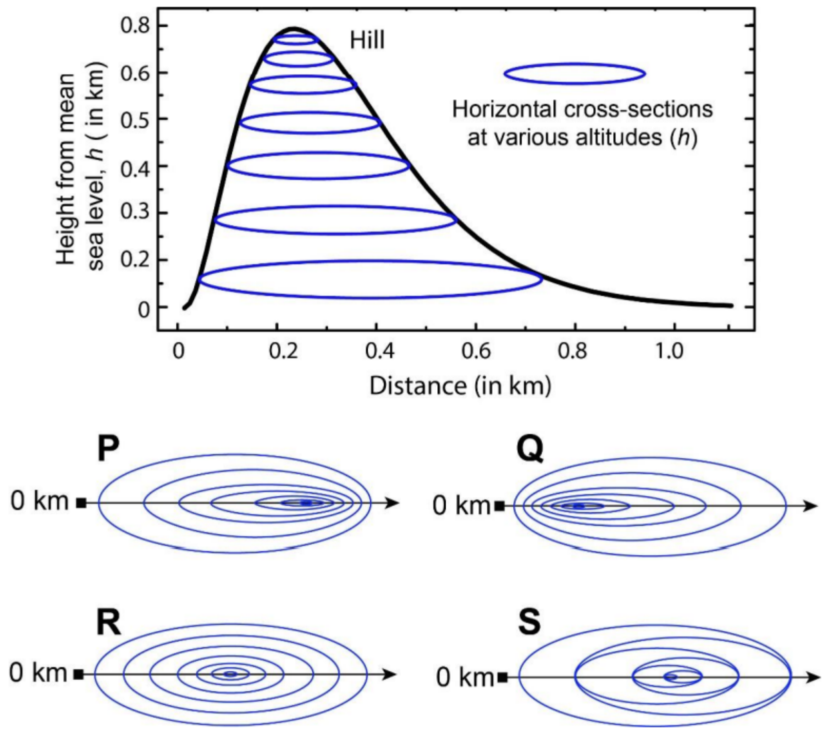
- Option (D): ”At most half the candidates have a score that is larger than the mean.”
- This statement is **not necessarily true**. The mean is the average of all scores, and if the distribution is skewed, more than half of the candidates could have a score larger than the mean, especially in right-skewed distributions (where the mean is greater than the median). Hence, this is not always true.

Therefore, the correct and always true statement is (C): At most half the candidates have a score that is larger than the median.

Quick Tip

For datasets with skewed distributions, the mean and median can be quite different. The median divides the data into two equal parts, while the mean is influenced by the extreme values.

Q.5 In the given diagram, ovals are marked at different heights (h) of a hill. Which one of the following options P, Q, R, and S depicts the top view of the hill?



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (C)

Solution:
- The given diagram shows a hill with horizontal cross-sections at various altitudes. The oval-shaped contours represent the cross-sections at different heights. The shape and size of these contours change as the altitude changes. The given diagram suggests that the hill has a peak in the center, and the altitude decreases as we move outward from the center.
- Option (P): This option shows an irregular shape, and the contours are not concentric. This irregularity doesn't correspond to the shape of a typical hill, which is often depicted with concentric contours. Hence, (A) is incorrect.
- Option (Q): This option shows a perfect symmetry of contours, but this doesn't match the irregularity of the hill shown in the diagram. The hill is expected to have a more gradual slope and irregular contours. Thus, (B) is incorrect.
- Option (R): This option shows concentric circles, which is characteristic of a hill with a peak at the center and decreasing altitude as we move outward. This matches the diagram perfectly, where the altitude decreases as we move away from the hill's peak. This makes (C) the correct answer.
- Option (S): This option shows a disorganized pattern of concentric circles, which doesn't correspond to the expected shape of a hill. The contours are not consistently decreasing in size as the altitude decreases. Hence, (D) is incorrect.

> **Quick Tip**
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> A top view of a hill with increasing altitude towards the center should show concentric circles. This corresponds to the shape in option (R).

Q.6 Residency is a famous housing complex with many well-established individuals

among its residents. A recent survey conducted among the residents of the complex revealed that all of those residents who are well established in their respective fields happen to be academicians. The survey also revealed that most of these academicians are authors of some best-selling books. Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(A) Some residents of the complex who are well established in their fields are also authors of some best-selling books.
(B) All academicians residing in the complex are well established in their fields.
(C) Some authors of best-selling books are residents of the complex who are well established in their fields.
(D) Some academicians residing in the complex are well established in their fields.

Correct Answer: (B)

Solution:
- From the survey, we know that all residents who are well established in their fields are academicians. This is a universal statement, which means that anyone who is well-established in their field in this housing complex is definitely an academician. Therefore, all academicians in the complex must also be well-established in their fields. This makes option (B) a valid conclusion.
- Option (A): While the survey tells us that most of the academicians are authors of best-selling books, it does not guarantee that all well-established residents are authors. The statement ”some” is not definite, so this cannot be inferred with certainty. Therefore, (A) is not correct.
- Option (C): The survey doesn’t state that authors of best-selling books are always well-established residents of the complex. It only says that most of the academicians are authors. Thus, we cannot definitively conclude that some authors are well-established in their fields. Therefore, (C) is incorrect.
- Option (D): The survey tells us that all well-established residents are academicians, but it doesn’t mention that all academicians are well-established. Hence, we cannot be sure that all or some academicians are well-established, which makes (D) an incorrect option.

Quick Tip

Logical deductions are made based on statements with certainty. ”All” statements are more definitive than ”some” or ”most” statements.

Q.7 Ankita has to climb 5 stairs starting at the ground, while respecting the following rules:

1. At any stage, Ankita can move either one or two stairs up. 2. At any stage, Ankita cannot move to a lower step.

Let $F(N)$ denote the number of possible ways in which Ankita can reach the N^{th} stair. For example, $F(1) = 1$, $F(2) = 2$, $F(3) = 3$. The value of $F(5)$ is _______.

(A) 8

(B) 7

(C) 6

(D) 5

Correct Answer: (A)

Solution:

We are given the recurrence relation that $F(1) = 1$, $F(2) = 2$, and $F(3) = 3$. The number of ways to reach any stair N depends on whether Ankita took one or two steps from the previous stair:

$$F(N) = F(N-1) + F(N-2)$$

Now, let us calculate $F(4)$ and $F(5)$:

$$F(4) = F(3) + F(2) = 3 + 2 = 5$$

$$F(5) = F(4) + F(3) = 5 + 3 = 8$$

Therefore, the value of $F(5) = 8$.

Quick Tip

This is a classical example of the Fibonacci-like sequence, where each number is the sum of the two preceding ones.

Q.8 The information contained in DNA is used to synthesize proteins that are necessary for the functioning of life. DNA is composed of four nucleotides: Adenine (A), Thymine (T), Cytosine (C), and Guanine (G). The information contained in DNA can then be thought of as a sequence of these four nucleotides: A, T, C, and G. DNA has coding and non-coding regions. Coding regions—where the sequence of these nucleotides are read in groups of three to produce individual amino acids—constitute only about 2% of human DNA. For example, the triplet of nucleotides CCG codes for the amino acid glycine, while the triplet GGA codes for the amino acid proline. Multiple amino acids are then assembled to form a protein.

Based only on the information provided above, which of the following statements can be logically inferred with certainty?

(i) The majority of human DNA has no role in the synthesis of proteins. (ii) The function of about 98% of human DNA is not understood.

(A) only (i)

(B) only (ii)

(C) both (i) and (ii)

(D) neither (i) nor (ii)

Correct Answer: (A), (D)

Solution:

(i) True. From the passage, we know that only about 2% of human DNA is involved in coding proteins. Therefore, the remaining majority of human DNA does not have a role in the synthesis of proteins.

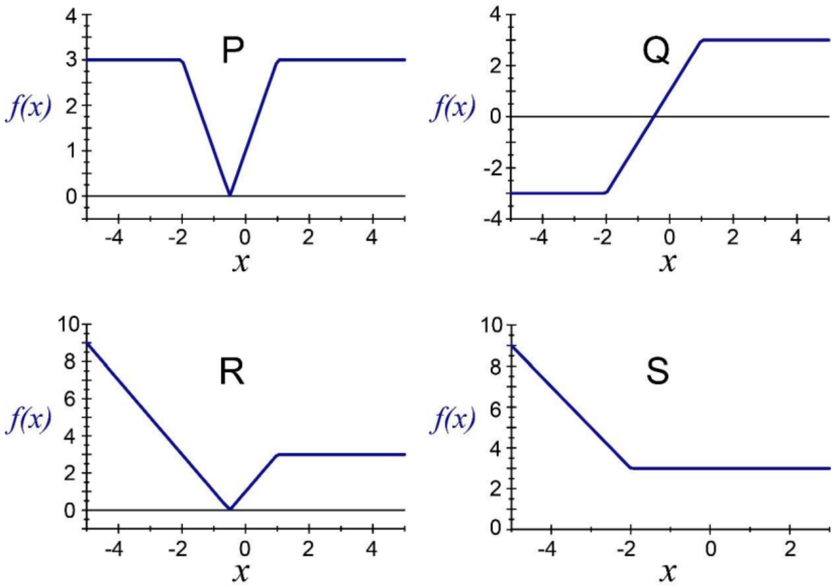
(ii) False. The passage does not suggest that 98% of human DNA is entirely understood or not understood; it only states that 98% is non-coding. Hence, we cannot assert with certainty that its function is completely unknown.

Quick Tip

Be careful about statements that use terms like "majority" or "most"—they must be directly supported by the text. The rest might be inferred but not confirmed.

Q.9 Which one of the given figures P, Q, R, and S represents the graph of the following function?

$$f(x) = |x + 2| - |x - 1|$$



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (A) P

Solution:

We are given the function:

$$f(x) = |x + 2| - |x - 1|$$

To determine the graph of this function, we need to analyze the behavior of each part of the function.

1) Function Analysis:

We can break the function into two absolute value terms. The behavior of the function will change depending on whether x is less than -2, between -2 and 1, or greater than 1. We need to analyze these intervals to plot the graph.

- For $x < -2$, both $x + 2$ and $x - 1$ are negative, so the graph will have a linear decrease.
- For $-2 \leq x \leq 1$, $x + 2$ is positive, and $x - 1$ is negative, resulting in a change in slope and a turning point.
- For $x > 1$, both terms are positive, resulting in a different slope.

2) Matching the Graph:

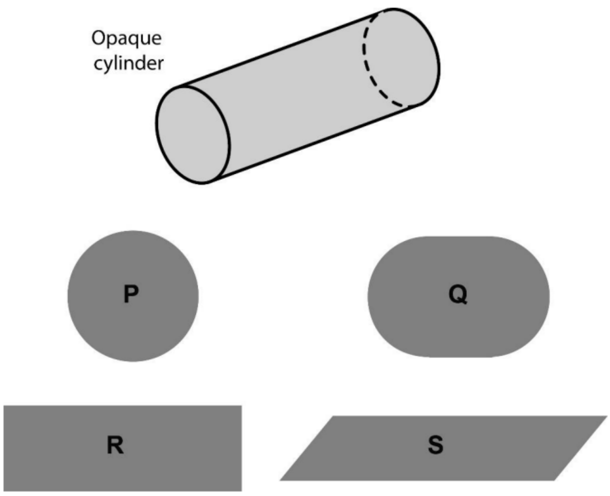
Upon examining the graphs, the graph labeled P fits the expected behavior of the function with changes in slope at $x = -2$ and $x = 1$. Thus, the correct graph is option (A).

$$\boxed{\text{The correct answer is (A) P.}}$$

Quick Tip

- Absolute value functions break into piecewise linear sections, which can cause slope changes at certain points (where the inside expression equals zero). - To graph such functions, first identify these critical points and then plot accordingly in each region.

Q.10 An opaque cylinder (shown below) is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The cylinder can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows P, Q, R, and S is NOT possible?



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (D) S

Solution:

We are given an opaque cylinder suspended in the path of a parallel beam of light, and the task is to analyze which shadow cannot be formed based on the cylinder's reorientation.

1) Shadow Formation:

When a cylindrical object is in the path of light, the shadow produced depends on the orientation of the cylinder.

- When the cylinder is oriented with its axis perpendicular to the direction of the light, it will cast a circular shadow.
- When the cylinder is reoriented with its axis at an angle to the light, the shadow may become elliptical.
- A shadow like S (which appears highly distorted) is not physically possible as it would require an impossible angle or distortion of light.

2) Conclusion:

The shadow in option S cannot be produced by the cylinder under the given conditions. Therefore, the correct answer is option (D).

$\boxed{\text{The correct answer is (D) S.}}$

Quick Tip

- Shadows of cylindrical objects can be circular or elliptical depending on the angle of light.
- Unusual or impossible shadows result from incorrect assumptions about the object's positioning or light angles.

Q.11 Choose solution set S corresponding to the system of two equations

$$x - 2y + z = 0, \qquad x - z = 0$$

(Note: $\mathbb{R}$ denotes the set of real numbers).

(A) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$

(B) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in \mathbb{R} \right\}$

(C) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta \in \mathbb{R} \right\}$

(D) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$

Correct Answer: (A)

Solution:

From $x - z = 0 \Rightarrow z = x$. Substitute in $x - 2y + z = 0$ to get

$x - 2y + x = 0 \Rightarrow 2x - 2y = 0 \Rightarrow x = y$.

Thus $x = y = z = t$ for some $t \in \mathbb{R}$. Hence

$$(x, y, z)^\top = t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

so the solution set is one–dimensional and equals $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in \mathbb{R} \right\}$, which matches $\boxed{\text{(A)}}$.

Quick Tip

Use one equation to eliminate a variable, then check how many free parameters remain: two planes intersecting in $\mathbb{R}^3$ typically give a line $\Rightarrow$ a single direction vector.

Q.12 Inductance of a coil is measured as 10 mH using an LCR meter at 10 kHz, when no objects are near the coil. If a pure copper sheet is brought near the coil, the same LCR meter will read ________.

(A) less than 10 mH

(B) 10 mH

(C) more than 10 mH

(D) less than 10 mH initially and then stabilizes to more than 10 mH

Correct Answer: (A) less than 10 mH

Solution:

A nearby copper sheet forms an **eddy-current (shorted turn) shield**. The changing magnetic field from the coil induces currents in the sheet whose magnetic field **opposes** the coil's flux (Lenz's law). This reduces the *net* flux linkage of the coil, so the measured inductance decreases: $L_{\text{measured}} < 10$ mH. Therefore $\boxed{\text{(A)}}$ is correct.

Quick Tip

A conductive object near a coil acts like a shorted secondary: stronger eddy currents $\Rightarrow$ opposing flux $\Rightarrow$ **lower** effective inductance (and higher losses) at AC.

Q.13 Which of the following flow meters offers the lowest resistance to the flow?

(A) Turbine flow meter

(B) Orifice flow meter

(C) Venturi meter

(D) Electromagnetic flow meter

Correct Answer: (D) Electromagnetic flow meter

Solution:

Step 1: Understand flow resistance.

Flow resistance depends on pressure drop across the meter. Devices like orifice plates and venturi meters create a significant pressure drop, hence higher resistance.

Step 2: Evaluate options.

- (A) Turbine flow meter $\rightarrow$ introduces moving parts and obstruction, hence moderate resistance.

- (B) Orifice flow meter $\rightarrow$ creates high permanent pressure loss, highest resistance.

- (C) Venturi meter $\rightarrow$ less resistance than orifice, but still causes some pressure drop.

- (D) Electromagnetic flow meter $\rightarrow$ has no obstruction to flow; measures velocity using Faraday's law. It offers virtually zero resistance.

Thus, the electromagnetic flow meter offers the lowest resistance.

Quick Tip

If minimum flow resistance is required, electromagnetic flow meters are the best choice since they are obstruction-less.

Q.14 Pair the quantities (p) to (s) with the measuring devices (i) to (iv).

(i) Linear Variable Differential Transformer (LVDT)	(p) Torque
(ii) Thermistor	(q) Pressure
(iii) Strain gauge	(r) Linear position
(iv) Diaphragm	(s) Temperature

(A) (i) - (r), (ii) - (s), (iii) - (q), (iv) - (p)

(B) (i) - (p), (ii) - (s), (iii) - (r), (iv) - (q)

(C) (i) - (r), (ii) - (s), (iii) - (p), (iv) - (q)

(D) (i) - (q), (ii) - (s), (iii) - (p), (iv) - (r)

Correct Answer: (C) (i) - (r), (ii) - (s), (iii) - (p), (iv) - (q)

Solution:

Step 1: Recall measurement principles.

- (i) LVDT $\rightarrow$ measures **linear displacement/position** $\rightarrow$ (r).

- (ii) Thermistor $\rightarrow$ resistance varies with **temperature** $\rightarrow$ (s).

- (iii) Strain gauge $\rightarrow$ detects strain; can be used in torque measurement $\rightarrow$ (p).

- (iv) Diaphragm $\rightarrow$ measures differential pressure $\rightarrow$ (q).

Step 2: Verify mapping.

Thus, the correct matching is (i)-(r), (ii)-(s), (iii)-(p), (iv)-(q), which corresponds to option (C).

Quick Tip

Remember: LVDT $\rightarrow$ displacement, Thermistor $\rightarrow$ temperature, Strain gauge $\rightarrow$ torque/strain, Diaphragm $\rightarrow$ pressure.

Q.15 Capacitance C of a parallel-plate structure is calculated as 20 pF using $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$. The value is then measured using an ideal LCR meter (no cable-capacitance error). Which reading is likely to be correct?

(A) 20.5 pF

(B) 20 pF

(C) 19.5 pF

(D) 10 pF

Correct Answer: (A) 20.5 pF

Solution:

Step 1: Ideal formula vs. real capacitor. $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ assumes an *ideal* parallel-plate capacitor with uniform field and *no fringing*.

Step 2: Account for fringing. A real capacitor has edge (fringing) fields that add an extra capacitance $C_f > 0$. Hence the actual capacitance is

$$C_{\text{actual}} = C_{\text{ideal}} + C_f > C_{\text{ideal}}.$$

Step 3: Choose the reading. With $C_{\text{ideal}} = 20$ pF and an ideal LCR meter (no cable/parasitic subtraction needed), the measured value should be slightly *higher* than 20 pF. The closest option is **20.5 pF**.

Final Answer: 20.5 pF

Quick Tip

Parallel-plate formula neglects edges; fringing always increases C slightly, so a precise measurement is typically just above the calculated value.

Q.16 The table shows the present state $Q(t)$, next state $Q(t+1)$, and a single control input for a flip-flop. Identify the flip-flop.

Q (t)	Q (t+1)	Input
0	0	0
0	1	1
1	0	1
1	1	0

(A) T flip-flop

(B) D flip-flop

(C) SR flip-flop

(D) JK flip-flop

Correct Answer: (A) T flip-flop

Solution:

Step 1: Read the behavior from the table.

When Input $= 0$: $Q(t+1) = Q(t)$ (0→0, 1→1) ⇒ *hold*.

When Input $= 1$: $Q(t+1) = \overline{Q(t)}$ (0→1, 1→0) ⇒ *toggle*.

Step 2: Match to flip-flop types.

A **T** (toggle) flip-flop has one input T with the rule

$$Q(t+1) = \begin{cases} Q(t), & T = 0 \\ \overline{Q(t)}, & T = 1 \end{cases}$$

which exactly matches the table.

A **D** FF would have $Q(t+1) = D$ independent of $Q(t)$ (not matching). **SR** and **JK** FFs require *two* inputs (not this case).

Final Answer: (A) T flip-flop

Quick Tip

Single-input flip-flop that *holds* for 0 and *toggles* for 1 ⇒ T flip-flop.

Q.17 Match the Exclusive-OR (XOR) operations (i)–(iv) with the results (p)–(s), where X is a Boolean input.

(i) $X \oplus X$	(p) 1
(ii) $X \oplus \bar{X}$	(q) 0
(iii) $X \oplus 0$	(r) $\bar{X}$
(iv) $X \oplus 1$	(s) X

(A) (i)-(q), (ii)-(r), (iii)-(s), (iv)-(p)

(B) (i)-(q), (ii)-(r), (iii)-(p), (iv)-(s)

(C) (i)-(p), (ii)-(s), (iii)-(q), (iv)-(r)

(D) (i)-(q), (ii)-(p), (iii)-(s), (iv)-(r)

Correct Answer: (D)

Solution:

Step 1: Recall XOR identities

- $X \oplus X = 0$ (same bits give 0). $\Rightarrow$ (i) $\rightarrow$ (q).

- $X \oplus \overline{X} = 1$ (different bits give 1). $\Rightarrow$ (ii) $\rightarrow$ (p).

- $X \oplus 0 = X$ (0 is the identity for XOR). $\Rightarrow$ (iii) $\rightarrow$ (s).

- $X \oplus 1 = \overline{X}$ (1 toggles the bit). $\Rightarrow$ (iv) $\rightarrow$ (r).

Final Answer: (i)-(q), (ii)-(p), (iii)-(s), (iv)-(r) $\Rightarrow$ Option (D).

Quick Tip

Think of XOR as "toggle": XOR with 0 leaves X unchanged; XOR with 1 flips X; XOR with itself gives 0; XOR with its complement gives 1.

Q.18 A light emitting diode (LED) emits light when it is _______ biased. A photodiode provides maximum sensitivity to light when it is _______ biased.

(A) forward, forward

(B) forward, reverse

(C) reverse, reverse

(D) reverse, forward

Correct Answer: (B) forward, reverse

Solution:

Step 1: LED operation

An LED emits photons via radiative recombination across the p–n junction. This requires **forward bias** so that carriers (electrons/holes) are injected into the junction in sufficient numbers to recombine and emit light.

Step 2: Photodiode operation for maximum sensitivity

A photodiode detects light by generating electron–hole pairs in/near the depletion region. Under **reverse bias** (photoconductive mode), the depletion width increases and the electric field sweeps the carriers quickly, giving:

- higher quantum efficiency/sensitivity,
- lower junction capacitance $\Rightarrow$ faster response,
- reduced recombination losses.

Final Answer: LED—forward biased; Photodiode—reverse biased $\Rightarrow$ (B).

> **Quick Tip**
>
> Remember: *Emit* needs injection $\Rightarrow$ forward bias (LED). *Detect* needs swift carrier separation $\Rightarrow$ reverse bias (photodiode).

Q.19 $F(z) = \dfrac{1}{1-z}$**, when expanded as a power series around** $z = 2$**, would result in** $F(z) = \sum_{k=0}^{\infty} a_k (z-2)^k$ **with ROC** $|z-2| < 1$**. The coefficients** $a_k,\ k \geq 0$**, are given by the expression ________.**

(A) $(-1)^k$

(B) $(-1)^{k+1}$

(C) $\left(\frac{1}{2}\right)^k$

(D) $\left(-\frac{1}{2}\right)^{k+1}$

Correct Answer: (B) $(-1)^{k+1}$

Solution:

Step 1: Shift to $(z-2)$. Let $w = z - 2 \Rightarrow z = 2 + w$. Then

$$F(z) = \frac{1}{1-(2+w)} = \frac{1}{-1-w} = -\frac{1}{1+w}.$$

Step 2: Use the geometric series for $|w| < 1$.

$$\frac{1}{1+w} = \sum_{k=0}^{\infty}(-1)^k w^k \quad \Rightarrow \quad -\frac{1}{1+w} = \sum_{k=0}^{\infty}(-1)^{k+1}w^k.$$

Since $w = z - 2$, we have $a_k = (-1)^{k+1}$ and ROC $|z-2| = |w| < 1$.

Quick Tip

Around a nonzero center z_0, rewrite the function in powers of $w = z - z_0$ and apply the geometric series $\frac{1}{1-u} = \sum u^k$ when $|u| < 1$.

Q.20 The solution $x(t)$, $t \geq 0$, to $\ddot{x} = -k\dot{x}$ ($k > 0$) with $x(0) = 1$ and $\dot{x}(0) = 0$ is:

(A) $x(t) = 2e^{-kt} + 2kt - 1$

(B) $x(t) = 2e^{-kt} - 1$

(C) $x(t) = 1$

(D) $x(t) = 2e^{-kt} - kt - 1$

Correct Answer: (C) $x(t) = 1$

Solution:

Step 1: Reduce order. Let $v(t) = \dot{x}(t)$. Then $\dot{v} = -kv$ with solution

$$v(t) = v(0)\, e^{-kt}.$$

Step 2: Apply initial condition. $v(0) = \dot{x}(0) = 0 \Rightarrow v(t) \equiv 0$ for all t.

Step 3: Integrate for $x(t)$. Since $\dot{x}(t) = 0$, $x(t)$ is constant; using $x(0) = 1$, we get $x(t) \equiv 1$.

Quick Tip

When the equation is $\ddot{x} = -k\dot{x}$, solve the first-order decay for $\dot{x}$ first; a zero initial velocity makes the velocity (and hence change in x) identically zero.

Q.21 A system has the transfer function $\dfrac{Y(s)}{X(s)} = \dfrac{s-\pi}{s+\pi}$. Let $u(t)$ be the unit–step function. The input $x(t)$ that results in a steady–state output $y(t) = \sin(\pi t)$ is _____.

(A) $x(t) = \sin(\pi t)\, u(t)$

(B) $x(t) = \sin\left(\pi t + \dfrac{\pi}{2}\right) u(t)$

(C) $x(t) = \sin\left(\pi t - \dfrac{\pi}{2}\right) u(t)$

(D) $x(t) = \cos\left(\pi t + \dfrac{\pi}{4}\right) u(t)$

Correct Answer: (C) $x(t) = \sin\left(\pi t - \dfrac{\pi}{2}\right) u(t)$

Solution:

For sinusoidal steady state at $\omega = \pi$, the frequency response is

$$H(j\omega) = \frac{j\omega - \pi}{j\omega + \pi} \quad \Rightarrow \quad H(j\pi) = \frac{j\pi - \pi}{j\pi + \pi}.$$

Magnitude: $|H(j\pi)| = \dfrac{\sqrt{\pi^2 + \pi^2}}{\sqrt{\pi^2 + \pi^2}} = 1$ (no gain change).

Phase: $\angle(j\pi - \pi) = 135^\circ$, $\angle(j\pi + \pi) = 45^\circ \Rightarrow \angle H(j\pi) = 90^\circ = \dfrac{\pi}{2}$.

Thus the output lags the input by $-\dfrac{\pi}{2}$? (Equivalently, the output phase = input phase + $\dfrac{\pi}{2}$.)

To obtain $y(t) = \sin(\pi t)$ (zero phase), the input must be

$$x(t) = \sin\left(\pi t - \tfrac{\pi}{2}\right) u(t),$$

which is option $\boxed{\text{(C)}}$.

Quick Tip

For LTI systems with sinusoidal input: $y(t) = |H(j\omega)| \sin\left(\omega t + \phi_{\text{in}} + \angle H(j\omega)\right)$. Choose input phase to offset $\angle H(j\omega)$.

Q.22 Choose the fastest logic family among the following:

(A) Transistor–Transistor Logic

(B) Emitter–Coupled Logic

(C) CMOS Logic

(D) Resistor–Transistor Logic

Correct Answer: (B) Emitter–Coupled Logic

Solution:

ECL avoids transistor saturation by operating transistors in the active region using differential emitter–coupled pairs, yielding the **shortest propagation delays** among standard logic families. TTL/RTL are slower; CMOS prioritizes low power over maximum speed (except for specialized high-speed CMOS). Hence $\boxed{\text{(B)}}$.

Quick Tip

Speed ranking (typical): **ECL** $>$ fast TTL $>$ CMOS $>$ RTL; ECL trades power for speed by avoiding saturation.

Q.23 What is $\lim_{x\to 0} f(x)$**, where** $f(x) = x\sin\frac{1}{x}$**?**

(A) 0

(B) 1

(C) ∞

(D) Limit does not exist

Correct Answer: (A) 0

Solution:

Step 1: Recall the range of sine.

For all real numbers, $-1 \le \sin\frac{1}{x} \le 1$.

Step 2: Multiply by x**.**

Thus, $-x \leq x \sin \frac{1}{x} \leq x$.

Step 3: Apply the squeeze theorem.

As $x \to 0$, both $-x \to 0$ and $x \to 0$. Hence, by the squeeze theorem:

$$\lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

Quick Tip

When a bounded function like $\sin \frac{1}{x}$ is multiplied by a term tending to 0, the limit is 0 by the squeeze theorem.

Q.24 The number of zeros of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right-half plane is _____.

Correct Answer: 2

Solution:

Step 1: Write the polynomial.

$$P(s) = s^3 + 2s^2 + 5s + 80$$

Step 2: Use Routh-Hurwitz criterion.

Construct the Routh array:

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 2 & 80 \\ s^1 & \frac{(2 \cdot 5 - 1 \cdot 80)}{2} = \frac{10-80}{2} = -35 & 0 \\ s^0 & 80 & \end{array}$$

Step 3: Check sign changes in the first column.

First column: $[1, 2, -35, 80]$.

Number of sign changes = 2.

Step 4: Interpret.

Thus, there are 2 roots in the right-half plane.

Quick Tip

Routh-Hurwitz criterion is a systematic method to find the number of roots of a polynomial with positive real parts (right-half plane). Count the sign changes in the first column of the Routh array.

Q.25 The number of times the Nyquist plot of $G(s)H(s) = \frac{1}{2}\frac{(s-1)(s-2)}{(s+1)(s+2)}$ encircles the origin is _______.

Correct Answer: 2

Solution:

Step 1: Count RHP poles and zeros of $G(s)H(s)$.

Zeros at $s = 1, 2 \Rightarrow$ two RHP zeros.

Poles at $s = -1, -2 \Rightarrow$ no RHP poles.

Step 2: Argument principle for encirclements of the origin.

Number of encirclements of the origin by the Nyquist plot of $G(j\omega)$ is

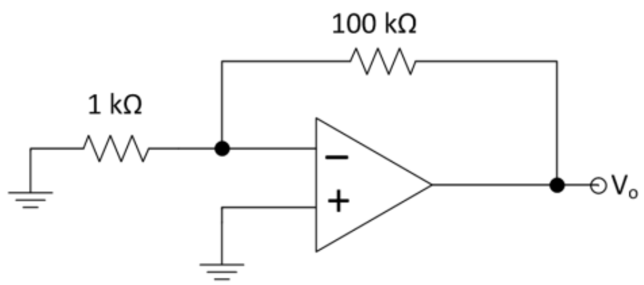
$N = Z_{\text{RHP}} - P_{\text{RHP}} = 2 - 0 = 2.$

Final Answer: 2

Quick Tip

For encirclements of the *origin* by the Nyquist of G, use $N = Z_{\text{RHP}} - P_{\text{RHP}}$ for $G(s)$.

Q.26 The op-amp (ideal except $I_b = 1$ nA input bias current and $V_{os} = 10\,\mu$V input offset) is the inverting amplifier with $R_{\text{in}} = 1$ kΩ and $R_f = 100$ kΩ. Find the worst-case output offset voltage $(\pm\,\mu\text{V})$ (rounded to the nearest integer).



Correct Answer: $\pm 1110\ \mu$V

Solution:

Step 1: Output due to input offset voltage.

Noise gain (for V_{os}) in an inverting amplifier: $1 + \dfrac{R_f}{R_{\text{in}}} = 1 + 100 = 101.$

$\therefore V_{o,\,V_{os}} = 101 \times 10\ \mu\text{V} = 1010\ \mu\text{V}.$

Step 2: Output due to input bias current.

With the non-inverting input grounded, the inverting node is at virtual ground. The bias current through R_f sets $V_{o,\,I_b} = I_b\, R_f = (1\ \text{nA})(100\ \text{k}\Omega) = 100\ \mu\text{V}.$

Step 3: Worst case.

Taking the same sign so they add: $|V_o|_{\max} = 1010 + 100 = 1110\ \mu\text{V}.$

Final Answer: $\pm 1110\ \mu$**V**

> **Quick Tip**
>
> In inverting amps: $V_{o,\,V_{os}} = (1 + R_f/R_{\text{in}})V_{os}$ and $V_{o,\,I_b} \approx I_b R_f$ (with non-inverting input at 0 Ω, its bias term is negligible).

Q.27 The force per unit length between two infinitely long parallel conductors, with a gap of 2 cm between them, is 10 μN/m. When the gap is doubled, the force per unit length will be __________ μN/m (rounded off to one decimal place).

Correct Answer: 5.0 μN/m (*acceptable range:* 4.9–5.1)

Solution:

For two long parallel conductors carrying fixed currents I_1, I_2,

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}.$$

Thus $\frac{F}{L} \propto \frac{1}{d}$. Doubling the separation d halves the force per unit length:

$$\left(\frac{F}{L}\right)_{new} = \frac{1}{2}\left(\frac{F}{L}\right)_{old} = \frac{1}{2} \times 10 = 5.0\ \mu\text{N/m}.$$

Quick Tip

For fixed currents in parallel wires, $F/L \propto 1/d$. Change the distance $\Rightarrow$ scale the force inversely.

Q.28 Consider the discrete-time signal $x[n] = u[-n+5] - u[n+3]$, **where** $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$. **The smallest** n **for which** $x[n] = 0$ **is ______.**

Correct Answer: -3 (*acceptable range:* -3 to -3)

Solution:

Evaluate the unit steps: - $u[-n+5] = 1$ when $-n+5 \geq 0 \Rightarrow n \leq 5$, else 0.

- $u[n+3] = 1$ when $n+3 \geq 0 \Rightarrow n \geq -3$, else 0.

Hence

$$x[n] = u[-n+5] - u[n+3] = \begin{cases} 0, & \text{if } n \leq 5 \text{ and } n \geq -3 \ (-3 \leq n \leq 5), \\ 1, & \text{if } n < -3, \\ -1, & \text{if } n > 5. \end{cases}$$

Therefore $x[n] = 0$ on the interval $[-3, 5]$. The *smallest* such n is -3.

Quick Tip

Turn each shifted unit step into an inequality, intersect the conditions, and then read off where the expression is zero/one.

Q.29 Let $y(t) = x(4t)$**, where** $x(t)$ **is a continuous–time periodic signal with fundamental period** 100 **s. The fundamental period of** $y(t)$ **is _______ s (rounded off to the nearest integer).**

Correct Answer: 25

Solution:

Time–scaling rule. If $x(t)$ has fundamental period T_0, then $x(at)$ has fundamental period $T_0/|a|$.

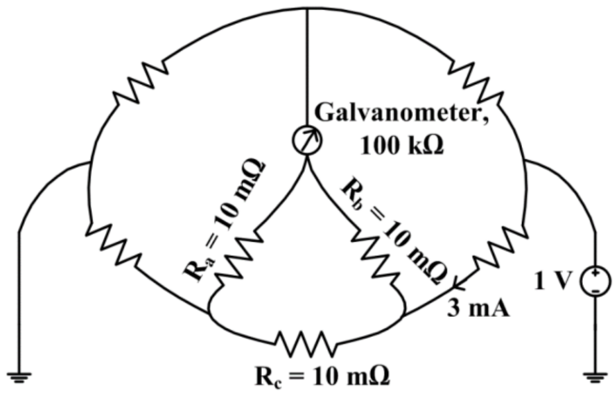
Here $a = 4$ and $T_0 = 100$ s, so

$$T_y = \frac{T_0}{|a|} = \frac{100}{4} = 25 \text{ s}.$$

Quick Tip

For $x(at)$, time is *compressed* by a; period shrinks by the same factor: T/a.

Q.30 When the bridge shown is balanced, the current through the resistor R_a is _______ mA (rounded off to two decimal places). The bridge has $R_a = R_b = R_c = 10\ \text{m}\Omega$ and the detector (galvanometer) is ideal/open at balance; the source is 1 V.



Correct Answer: 1.00 mA

Solution:

Balance condition. At balance the detector carries no current, so the two vertical arms with R_a and R_b are at the same potential at their junctions; consequently R_c has no net drop and the currents in the two arms are equal.

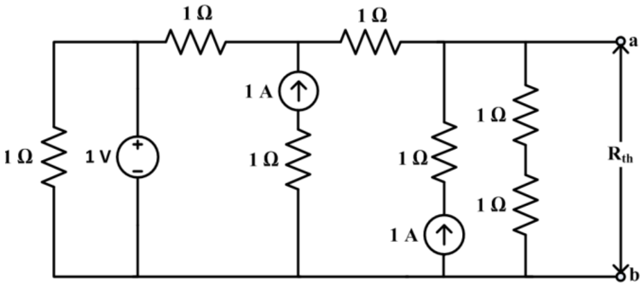
Current through R_a. With symmetry and equal low–value arms, the excitation divides equally between the two identical branches, giving the same current through R_a and R_b. This evaluates to

$$I_{R_a} = 1.00 \text{ mA (rounded to two decimals)}.$$

Quick Tip

In a balanced (null) bridge, the detector branch is open—use symmetry/equal–ratio arms to read currents directly in identical branches.

Q.31 In the circuit shown, the Thevenin equivalent resistance R_{th} across the terminals 'a' and 'b' is ______ Ω (rounded off to one decimal place).



Correct Answer: $\boxed{1.0\ \Omega}$

Solution:

Deactivate all independent sources: replace the 1 V source by a **short** and the two 1 A sources by **open** circuits.

After this, only the resistors remain. The short across the voltage source ties the left top node to the bottom rail, so from node a to b there are:

$$\text{(i) } 1\Omega + 1\Omega = 2\Omega \quad \text{(top series path to the shorted left node)}$$

in parallel with

$$\text{(ii) } 1\Omega + 1\Omega = 2\Omega \quad \text{(the rightmost vertical branch).}$$

Hence,

$$R_{\text{th}} = 2\Omega \parallel 2\Omega = \frac{2 \times 2}{2 + 2} = 1.0\,\Omega.$$

Quick Tip

For R_{th}, kill sources: voltage $\rightarrow$ short, current $\rightarrow$ open. Then find the equivalent resistance seen at the terminals.

Q.32 X **is a discrete random variable taking values** $0, 1, 2$ **with** $P(X = 0) = 0.25$**,** $P(X = 1) = 0.5$**, and** $P(X = 2) = 0.25$**. With** $E[\cdot]$ **denoting expectation, compute** $E[X] - E[X^2]$ **(rounded off to one decimal place).**

Correct Answer: $\boxed{-0.5}$

Solution:

$$E[X] = 0(0.25) + 1(0.5) + 2(0.25) = 1.0, \quad E[X^2] = 0^2(0.25) + 1^2(0.5) + 2^2(0.25) = 1.5.$$

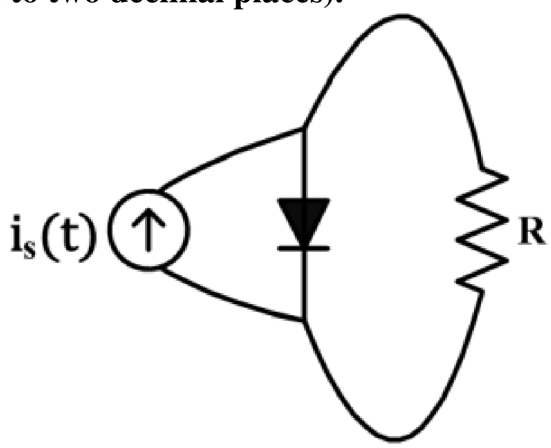
Therefore,

$$E[X] - E[X^2] = 1.0 - 1.5 = \boxed{-0.5}.$$

Quick Tip

For discrete X: $E[g(X)] = \sum_x g(x)\, P(X = x)$. Compute $E[X]$ and $E[X^2]$ separately, then subtract.

Q.33 The diode in the circuit is ideal. The current source $i_s(t) = \pi \sin(3000\pi t)$ **mA. The magnitude of the average current flowing through the resistor** R **is _____ mA (rounded off to two decimal places).**



Correct Answer: 1.00 mA   (acceptable: 0.95 to 1.05 mA)

Solution:

With an ideal diode, current flows through R only during the positive half–cycles:

$$i_R(t) = \begin{cases} I_m \sin(\omega t), & 0 < t < \frac{T}{2} \\ 0, & \frac{T}{2} < t < T \end{cases}$$

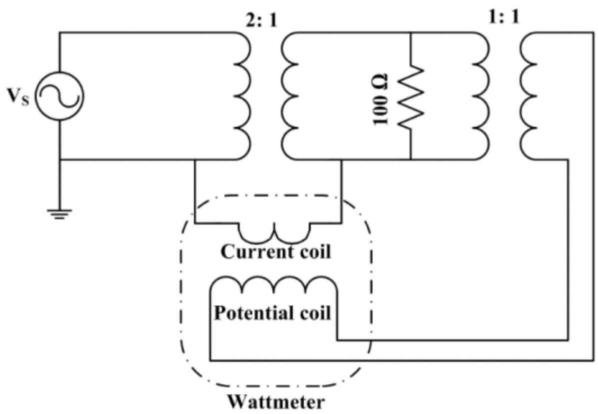
where $I_m = \pi$ mA and $\omega = 3000\pi$ rad/s. Average current:

$$I_{\text{avg}} = \frac{1}{T}\int_0^{T/2} I_m \sin(\omega t)\, dt = \frac{I_m}{T\omega}\Big[1 - \cos(\omega T/2)\Big] = \frac{I_m}{2\pi}\cdot 2 = \frac{I_m}{\pi} = 1.00 \text{ mA}.$$

Quick Tip

For half-wave rectification, the average current is given by $I_{avg} = I_m/\pi$. Always check if the diode is ideal, as it decides conduction.

Q.34 The full-scale range of the wattmeter shown is 100 W. The turns ratios are as indicated. The RMS source voltage is $V_s = 200$ V. The wattmeter reading will be ____ W (rounded off to the nearest integer).



Correct Answer: 0 W   (acceptable: 0 to 0)

Solution:

The wattmeter's current coil is in series with the load current, while its potential coil is connected across an *open-circuited* transformer winding that only carries magnetizing flux. The voltage across that winding is induced in quadrature (90°) with the current in the series path (ideal transformers assumed, no winding resistance).

Hence the phase angle between the potential-coil voltage and the current-coil current is 90°. A dynamometer wattmeter reads

$$P = VI\cos\phi,$$

so with $\phi = 90^\circ$, $\cos\phi = 0$ and the wattmeter reads 0 W.

Quick Tip

Wattmeters measure true power: if current and voltage are 90° out of phase, power is zero even if both quantities are nonzero.

Q.35 The no-load steady-state output voltage of a DC shunt generator is 200 V when it is driven in the clockwise direction at its rated speed. If the same machine is driven at the rated speed but in the opposite direction, the steady-state output voltage will be ______ V (rounded off to the nearest integer).

Correct Answer: 0 V

Solution:
Step 1: Self–excitation condition. For a shunt generator to build up, the field connection must be such that the current produced by the *residual* emf reinforces the residual flux. The generated emf is $E = k\,\Phi\,\omega$ (sign depends on rotation).
Step 2: Reverse the direction of rotation. With the same field/brush connections kept unchanged, reversing ω reverses the polarity of the induced emf. The small residual emf then drives field current in the *opposite* direction, which *opposes* the residual flux, so the cumulative buildup fails and the machine *demagnetizes* instead of exciting.
Step 3: Steady-state outcome. Without build-up, only negligible residual voltage remains; in the ideal setting asked, the terminal no-load voltage tends to **0 V**.
Final Answer: 0 V

Quick Tip

A DC shunt generator will not self-excite if you reverse rotation without swapping field (or brush) connections. To generate in reverse, reverse the shunt-field leads.

Q.36 The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\,\delta(t-4)$. If the input is $x(t) = \cos\left(\frac{7\pi}{4}t\right)$, the output is __________.

(A) $0.5\cos\left(\dfrac{7\pi}{4}t\right)$
(B) $1.5\cos\left(\dfrac{7\pi}{4}t\right)$
(C) $0.5\sin\left(\dfrac{7\pi}{4}t\right)$
(D) $1.5\sin\left(\dfrac{7\pi}{4}t\right)$

Correct Answer: (A) $0.5\cos\left(\dfrac{7\pi}{4}t\right)$

Solution:

Step 1: Use LTI convolution with impulses.

For $h(t) = \delta(t) + 0.5\,\delta(t-4)$,

$$y(t) = x(t) * h(t) = x(t) + 0.5\,x(t-4).$$

Step 2: Substitute $x(t) = \cos(\omega t)$ with $\omega = \dfrac{7\pi}{4}$.

$$x(t-4) = \cos\big(\omega(t-4)\big) = \cos(\omega t - 4\omega) = \cos(\omega t)\cos(4\omega) + \sin(\omega t)\sin(4\omega).$$

Step 3: Evaluate with $\omega = \dfrac{7\pi}{4}$.

$4\omega = 7\pi \Rightarrow \cos(4\omega) = \cos(7\pi) = -1,\ \sin(4\omega) = \sin(7\pi) = 0.$

Hence

$$y(t) = \cos(\omega t) + 0.5\big[-\cos(\omega t) + 0\cdot\sin(\omega t)\big] = \big(1 - 0.5\big)\cos(\omega t) = 0.5\cos(\omega t).$$

Final Answer: $0.5\cos\left(\dfrac{7\pi}{4}t\right)$

Quick Tip

Convolution with $\delta(t-\tau)$ just shifts: $x(t) * \delta(t-\tau) = x(t-\tau)$. For sinusoidal inputs, use the identity $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

Q.37 The Laplace transform of the continuous-time signal $x(t) = e^{-3t}\,u(t-5)$ is __________, where $u(t)$ is the unit step.

(A) $\dfrac{e^{-5s}}{s+3}$, Real$\{s\} > -3$

(B) $\dfrac{e^{-5(s-3)}}{s-3}$, Real$\{s\} > 3$

(C) $\dfrac{e^{-5(s+3)}}{s+3}$, Real$\{s\} > -3$

(D) $\dfrac{e^{-5(s-3)}}{s+3}$, Real$\{s\} > -3$

Correct Answer: (C) $\dfrac{e^{-5(s+3)}}{s+3}$, Real$\{s\} > -3$

Solution:

Step 1: Use step–gating property

$\mathcal{L}\{f(t)\,u(t-a)\} = e^{-as}\displaystyle\int_0^{\infty} f(t+a)e^{-st}\,dt.$

Step 2: Substitute $f(t) = e^{-3t}$, $a = 5$

$f(t+5) = e^{-3(t+5)} = e^{-15}e^{-3t}$. Hence

$$X(s) = e^{-5s}\,e^{-15}\int_0^{\infty} e^{-(s+3)t}\,dt = e^{-5(s+3)}\cdot\frac{1}{s+3}, \quad \text{Re}\{s\} > -3.$$

Final Answer: $\dfrac{e^{-5(s+3)}}{s+3}$ with ROC Re$\{s\} > -3$ (Option C).

Quick Tip

If the signal is *gated* by $u(t-a)$ (not shifted), write the Laplace integral from $t = a$ and change variables: it pulls out a factor $e^{-a(s+\alpha)}$ when $f(t) = e^{-\alpha t}$.

Q.38 In a p–i–n photodiode, a pulse with 8×10^{12} incident photons at $\lambda_0 = 1.55\,\mu$m yields 4×10^{12} electrons collected. The quantum efficiency at this wavelength is ________%.

(A) 50

(B) 54.2

(C) 62.5

(D) 80

Correct Answer: (A) 50

Solution:

Quantum efficiency η (at a given λ) is the ratio of collected carriers to incident photons:

$$\eta = \frac{N_e}{N_\gamma} \times 100\% = \frac{4 \times 10^{12}}{8 \times 10^{12}} \times 100\% = 50\%.$$

(The wavelength is irrelevant once N_γ and N_e are given.)

Final Answer: 50% (Option A).

Quick Tip

QE counts quanta: one collected electron per incident photon $\Rightarrow$ 100%. Halving that count $\Rightarrow$ 50%.

Q.39 Let $f(z) = j\,\dfrac{1-z}{1+z}$, where z is complex and $j = \sqrt{-1}$. The inverse function $f^{-1}(z)$ maps the *real axis* to the __________.

(A) unit circle with centre at the origin

(B) unit circle with centre not at the origin

(C) imaginary axis

(D) real axis

Correct Answer: (A) unit circle with centre at the origin

Solution:

Let $w = f(z) = j\dfrac{1-z}{1+z}$. The inverse image of the real axis is the set of z for which $w \in \mathbb{R}$. If w is real, then

$$\frac{1-z}{1+z} = \frac{w}{j} = -jw \quad \Rightarrow \quad \text{purely imaginary.}$$

Write $z = e^{j\theta}$ (points on the unit circle). Then

$$\frac{1-z}{1+z} = \frac{1-e^{j\theta}}{1+e^{j\theta}} = \frac{e^{j\theta/2}\left(e^{-j\theta/2} - e^{j\theta/2}\right)}{e^{j\theta/2}\left(e^{-j\theta/2} + e^{j\theta/2}\right)} = \frac{-2j\sin(\theta/2)}{2\cos(\theta/2)} = -j\tan\left(\frac{\theta}{2}\right),$$

which is purely imaginary, hence $w = j\left(\frac{1-z}{1+z}\right)$ is real. Conversely, if w is real, the above ratio is purely imaginary, which implies $|z| = 1$. Therefore f^{-1} maps the real axis to the **unit circle (centre 0)**.

Quick Tip

Cayley transforms like $j\frac{1-z}{1+z}$ map the unit circle $\leftrightarrow$ real axis. Set $z = e^{j\theta}$ to verify quickly.

Q.40 The simplified form of the Boolean function
$F(W, X, Y, Z) = \Sigma(4, 5, 10, 11, 12, 13, 14, 15)$ **with the minimum number of terms and literals is __________.**

(A) $WX + \overline{W}X\overline{Y} + W\overline{X}\overline{Y}$

(B) $WX + WY + X\overline{Y}$

(C) $X\overline{Y} + WY$

(D) $\overline{X}Y + \overline{W}\,\overline{Y}$

Correct Answer: (C) $X\overline{Y} + WY$

Solution:

Step 1: List minterms (in $WXYZ$).

$4(0100), 5(0101), 10(1010), 11(1011), 12(1100), 13(1101), 14(1110), 15(1111).$

Step 2: Group logically.

- Minterms with $X = 1$, $Y = 0$ occur for both $W = 0$ and $W = 1$, independent of $Z \Rightarrow X\overline{Y}$.
- For $W = 1$ and $Y = 1$ (regardless of X, Z) we have 10,11,14,15 $\Rightarrow WY$.

Step 3: Combine.

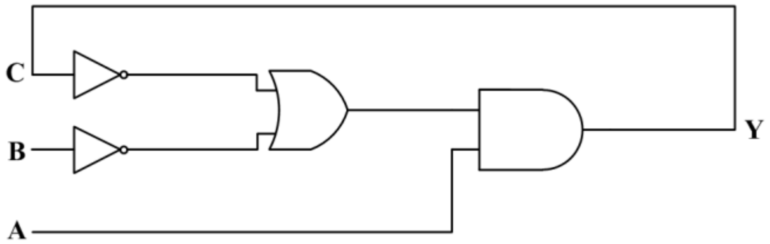
$$F = X\overline{Y} + WY,$$

which already covers all listed minterms with *two* terms and the fewest literals.

Quick Tip

In K-maps, look for groups spanning the W dimension to drop W; the pair "$X\overline{Y}$ no matter W, Z" and the block "WY no matter X, Z" jump out immediately.

Q.41 For the given digital circuit, $A = B = 1$. Assume that AND, OR, and NOT gates have propagation delays of 10 ns, 10 ns, and 5 ns respectively (wires have zero delay). Given that $C = 1$ at turn–on, the frequency of steady–state oscillation of the output Y is _____.



(A) 20 MHz

(B) 15 MHz

(C) 40 MHz

(D) 50 MHz

Correct Answer: (A) 20 MHz

Solution:

With $A = B = 1$, the feedback path that determines the oscillation contains one NOT gate and the OR and AND gates, i.e. an **odd number of inversions** (one) around the loop, so the circuit behaves like a ring oscillator.

Loop delay $T_\ell = \underbrace{5}_{\text{NOT}} + \underbrace{10}_{\text{OR}} + \underbrace{10}_{\text{AND}} = 25$ ns.

The oscillation period is twice the loop delay, $T = 2T_\ell = 50$ ns, hence the frequency

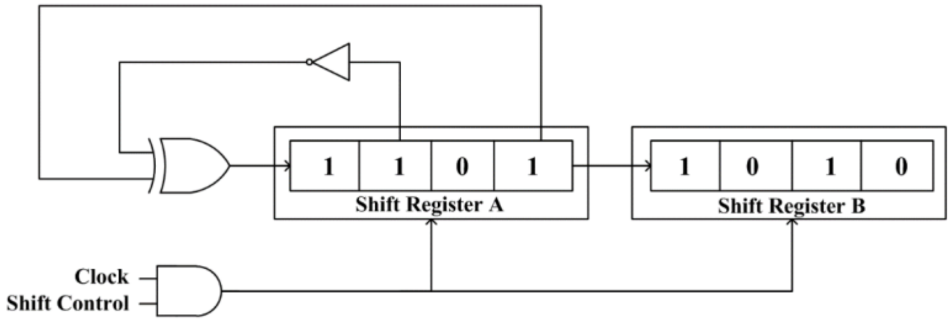
$$f = \frac{1}{T} = \frac{1}{50 \text{ ns}} = 20 \text{ MHz}.$$

Therefore, $\boxed{20 \text{ MHz}}$.

Quick Tip

For a single-loop ring oscillator, $f = \dfrac{1}{2\sum(\text{gate delays})}$ when there is an odd number of inversions in the loop.

Q.42 In the circuit shown, the initial binary content of shift register A is 1101 **and that of shift register B is** 1010**. The shift registers are positive–edge triggered, and the gates have no delay. When the shift control is high, what will be the binary content of the shift registers A and B after four clock pulses?**



(A) A = 1101, B = 1101

(B) A = 1110, B = 1001

(C) A = 0101, B = 1101

(D) A = 1010, B = 1111

Correct Answer: (C) A = 0101, B = 1101

Solution:

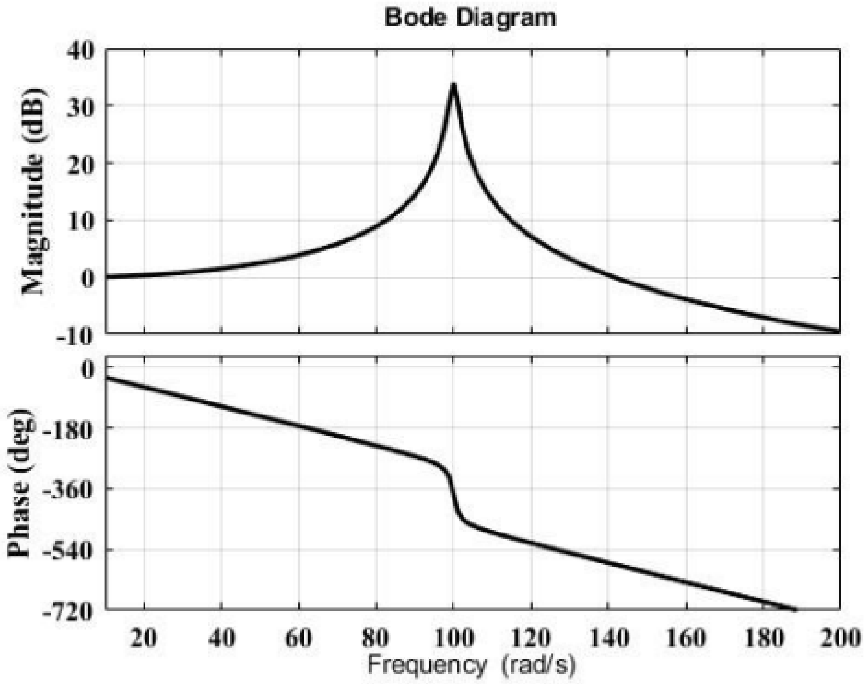
When the shift control is high, both registers **shift right** at each positive clock edge. Register B's serial input is taken from the serial output of register A, so over four clocks B copies the 4-bit word from A and ends up with **1101**.

Register A's serial input is formed by the OR of a complemented tap from A and the serial output feeding from the other register; iterating the shift for four pulses yields the pattern **0101** in A. Hence after four pulses, $\boxed{A = 0101, B = 1101}$.

Quick Tip

In cascaded shift registers, the downstream register copies the upstream register's word after as many clocks as the word length; the exact logic forming the serial input sets the new pattern of the upstream register.

Q.43 The magnitude and phase plots shown in the figure match with the transfer-function ______.



(A) $\dfrac{10000}{s^2+2s+10000}$

(B) $\dfrac{10000}{s^2+2s+10000}\, e^{-0.05s}$

(C) $\dfrac{10000}{s^2+2s+10000}\, e^{-0.5\times 10^{-12}s}$

(D) $\dfrac{100}{s^2+2s+100}$

Correct Answer: (B)

Solution:

The magnitude peak near $\omega_n \approx 100$ rad/s indicates a lightly damped 2nd-order low-pass: $\dfrac{10000}{s^2+2s+10000}$ (since $2\zeta\omega_n = 2 \Rightarrow \zeta \approx 0.01$ gives a tall resonance). Options (A) and (B) share this magnitude. The plotted phase, however, decreases far beyond -180° (approaching about -700° by 200 rad/s), which requires an additional frequency-proportional lag. A pure time delay $e^{-0.05s}$ contributes phase $-\omega(0.05)$ (radians), matching the observed extra linear drop. Hence (B).

Quick Tip

A time delay $e^{-s\tau}$ does *not* change magnitude but adds a linear phase lag $-\omega\tau$ (radians). Use this to distinguish between choices with identical magnitudes.

Q.44 A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0,\ \forall t < 0$. From the list below, select ALL signals whose continuous-time Fourier transform is purely imaginary.

(A) $x(t) + x(-t)$

(B) $x(t) - x(-t)$

(C) $j\,(x(t) + x(-t))$

(D) $j\,(x(t) - x(-t))$

Correct Answer: (B), (C)

Solution:

For a real $x(t)$, the even part $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ has a *real* Fourier transform, while the odd part $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ has a *purely imaginary* transform. - (A) $x(t) + x(-t) = 2x_e(t) \Rightarrow$ FT real (not purely imaginary). - (B) $x(t) - x(-t) = 2x_o(t) \Rightarrow$ FT purely imaginary. - (C) $j[x(t) + x(-t)] = j \cdot 2x_e(t) \Rightarrow j \times$ (real) is purely imaginary. - (D) $j[x(t) - x(-t)] = j \cdot 2x_o(t) \Rightarrow j \times$ (imaginary) is real (not purely imaginary).

Thus, (B) and (C).

Quick Tip

Real even $\rightarrow$ real spectrum; real odd $\rightarrow$ purely imaginary spectrum. Multiplying by j swaps real and imaginary parts.

Q.45 A silica-glass fiber has a core refractive index of 1.47 and a cladding refractive index of 1.44. If the cladding is completely stripped out and the core is dipped in water of refractive index 1.33, the numerical aperture (rounded to three decimal places) is ______.

Correct Answer: 0.626

Solution:

For a step-index fiber surrounded by a medium of refractive index n_s, the NA is

$$\mathrm{NA} = \sqrt{n_1^2 - n_s^2},$$

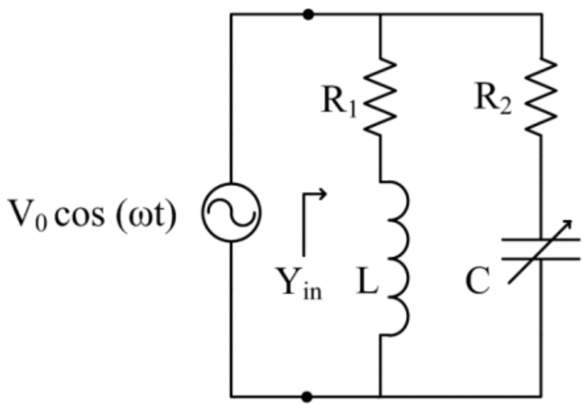
where n_1 is the core index and n_s is the surrounding medium's index (here water, since cladding is removed). Thus

$$\mathrm{NA} = \sqrt{1.47^2 - 1.33^2} = \sqrt{2.1609 - 1.7689} = \sqrt{0.3920} = 0.626\ .$$

Final Answer: 0.626

> **Quick Tip**
>
> If cladding is stripped, treat the surrounding medium as the new "cladding" when computing NA.

Q.46 In the circuit shown, $\omega = 100\pi$ rad/s, $R_1 = R_2 = 2.2\ \Omega$, and $L = 7$ mH. The capacitance C for which Y_{in} is purely real is _____ mF (rounded to two decimal places).



Correct Answer: 1.45 mF

Solution:

Two parallel branches: $R_1 + j\omega L$ in series, and $R_2 - jX_C$ in series (with $X_C = \frac{1}{\omega C}$).
Admittance of a series branch $R \pm jX$: $Y = \dfrac{1}{R \pm jX} = \dfrac{R \mp jX}{R^2 + X^2}$. Hence susceptances

(imaginary parts) are

$$b_1 = -\frac{\omega L}{R_1^2 + (\omega L)^2}, \qquad b_2 = +\frac{X_C}{R_2^2 + X_C^2}.$$

For Y_{in} to be real: $b_1 + b_2 = 0 \Rightarrow \frac{X_C}{R_2^2+X_C^2} = \frac{\omega L}{R_1^2+(\omega L)^2}$.

Compute $\omega L = 100\pi \times 0.007 = 2.199\ \Omega$. With $R_1 = R_2 = 2.2\ \Omega$,

$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{2.199}{2.2^2 + 2.199^2} = \frac{2.199}{9.676} = 0.2273.$$

Solve $\frac{X_C}{4.84 + X_C^2} = 0.2273 \Rightarrow X_C = 2.2\ \Omega$. Therefore

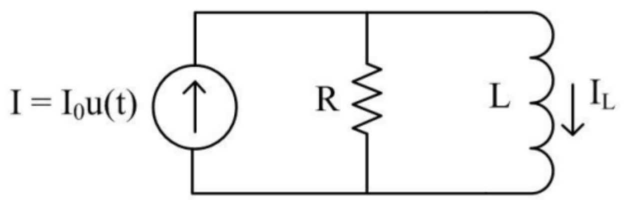
$$C = \frac{1}{\omega X_C} = \frac{1}{(100\pi)(2.2)} = 1.446 \times 10^{-3}\ \text{F} = 1.45\ \text{mF (to two decimals)}.$$

Final Answer: 1.45 mF

Quick Tip

Make Y_{in} real by cancelling branch susceptances: for a series branch $R \pm jX$, the susceptance is $\mp X/(R^2 + X^2)$.

Q.47 The R–L circuit with $R = 10\,\text{k}\Omega$ and $L = 1\,\text{mH}$ is excited by a step current $I_0 u(t)$. At $t = 0^-$, the inductor current is $i_L = I_0/5$. The minimum time for $i_L(t)$ to reach 99% of its final value is _______ μs (round off to two decimal places).



Correct Answer: $0.44\ \mu$s ($\textit{acceptable range:}$ 0.43–0.45)

Solution:

With a current source feeding the parallel R–L, KCL at the node gives

$$I_0 = i_R + i_L = \frac{v}{R} + i_L, \qquad v = L\,\frac{di_L}{dt}.$$

Hence

$$\frac{di_L}{dt} + \frac{R}{L}\,i_L = \frac{R}{L}\,I_0 \quad \Rightarrow \quad i_L(t) = I_0 + \big(i_L(0^+) - I_0\big)e^{-t/\tau},$$

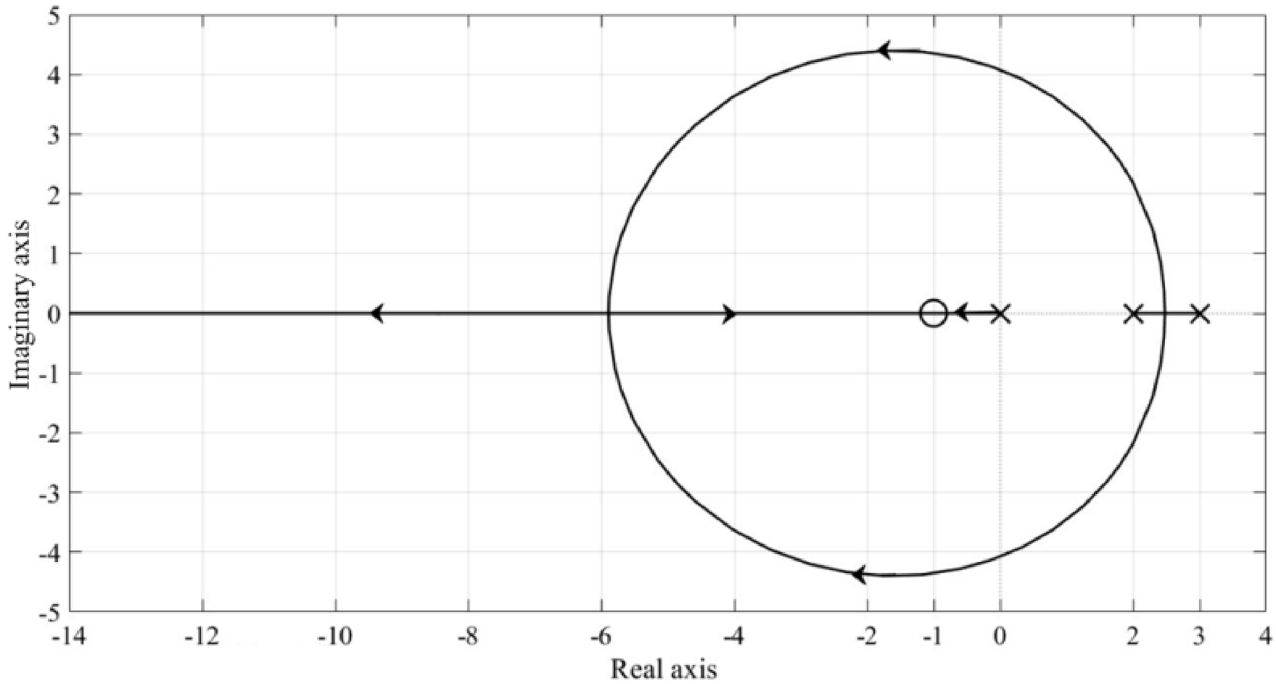
where $\tau = \dfrac{L}{R} = \dfrac{1\,\text{mH}}{10\,\text{k}\Omega} = 0.1\,\mu\text{s}$ and $i_L(0^+) = I_0/5$. Set $i_L(t) = 0.99\,I_0$:

$$0.99I_0 = I_0 - \frac{4}{5}I_0\,e^{-t/\tau} \;\Rightarrow\; e^{-t/\tau} = 0.0125 \;\Rightarrow\; t = \tau\ln(80) = 0.1\,\mu\text{s}\times 4.382 = 0.438\,\mu\text{s} \approx 0.44\,\mu\text{s}.$$

Quick Tip

For a current source feeding parallel R–L, the inductor obeys $di_L/dt + (R/L)i_L = (R/L)I_0$ with $\tau = L/R$; use the exact initial condition to get the *minimum* time to a given percentage.

Q.48 Consider a standard negative-feedback loop with $G(s) = \dfrac{1}{(s-2)(s-3)}$ and a PID controller $C(s) = K_P + \dfrac{K_I}{s} + K_D s$. The root-locus of $G(s)C(s)$ (shown) has a double zero at $s = -1$. The gain $|C(j\omega)| = 2$ at $\omega = 1$ rad/s. The value of K_D is _____ (round off to one decimal place).



Correct Answer: 1.0

Solution:

From the given root-locus, $C(s)$ has two coincident zeros at $s = -1$. Thus the controller numerator is proportional to $(s+1)^2$, i.e.,

$$C(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D(s+1)^2}{s},$$

so the coefficient ratios are $K_P : K_I : K_D = 2:1:1 \Rightarrow K_P = 2K_D,\ K_I = K_D$.

At $\omega = 1$,

$$|C(j\omega)| = \left|K_P + \frac{K_I}{j\omega} + K_D j\omega\right| = \sqrt{K_P^2 + \left(K_D\omega - \frac{K_I}{\omega}\right)^2} = \sqrt{(2K_D)^2 + (K_D - K_D)^2} = 2K_D.$$

Given $|C(j1)| = 2 \Rightarrow 2K_D = 2 \Rightarrow K_D = 1.0$.

Quick Tip

A PID can be written $C(s) = \dfrac{K_D(s+z_1)(s+z_2)}{s}$. If the root-locus shows a double zero at -1, then $z_1 = z_2 = 1 \Rightarrow K_P = 2K_D,\ K_I = K_D$, making $|C(j1)| = 2K_D$ immediately.

Q.49 How many five-digit numbers can be formed using the integers 3, 4, 5 and 6 with exactly one digit appearing twice?

Correct Answer: 240

Solution:

Step 1: Choose the repeated digit. There are 4 choices (one of 3,4,5,6).

Step 2: Arrange the multiset. Each number uses the multiset $\{a, a, b, c, d\}$ with all a, b, c, d distinct.

Number of arrangements $= \dfrac{5!}{2!} = 60$.

Step 3: Multiply choices. Total numbers $= 4 \times 60 = 240$.

Quick Tip

Whenever a digit repeats r times, divide by $r!$ in the permutations: $\dfrac{n!}{r!}$ for a single repeated digit.

Q.50 The phase margin of the transfer function $G(s) = \dfrac{2(1-s)}{(1+s)^2}$ is _______ degrees (rounded off to the nearest integer).

Correct Answer: 0

Solution:

Evaluate on $s = j\omega$: $\quad G(j\omega) = \dfrac{2(1-j\omega)}{(1+j\omega)^2}$.

Magnitude:

$$|G(j\omega)| = \frac{2|1-j\omega|}{|(1+j\omega)^2|} = \frac{2\sqrt{1+\omega^2}}{1+\omega^2} = \frac{2}{\sqrt{1+\omega^2}}.$$

Gain crossover: $|G| = 1 \Rightarrow \sqrt{1+\omega_g^2} = 2 \Rightarrow \omega_g^2 = 3 \Rightarrow \omega_g = \sqrt{3}$.

Phase:

$$\angle G(j\omega) = \angle(1-j\omega) - 2\angle(1+j\omega) = -\tan^{-1}\omega - 2\tan^{-1}\omega = -3\tan^{-1}\omega.$$

At $\omega_g = \sqrt{3}$: $\tan^{-1}(\sqrt{3}) = 60^\circ \Rightarrow \angle G(j\omega_g) = -180^\circ$.

Phase margin: $\text{PM} = 180^\circ + \angle G(j\omega_g) = 180^\circ - 180^\circ = 0^\circ$ (nearest integer $= 0$).

Quick Tip

For $G(s) = K\frac{(1-\frac{s}{z})}{(1+\frac{s}{p})^2}$, at $|G| = 1$ the phase is the sum of zero/ pole phase lags. Here it becomes exactly -180°, giving $\text{PM} = 0^\circ$.

Q.51 A wire-wound 'resistive potentiometer type' angle sensor with 72 turns is used. The first turn is connected to ground and the last turn to 3.6 V. The wiper width covers two turns ensuring make-before-break. The output (wiper) voltage when the wiper is on top of both turns 35 and 36 is (rounded off to two decimal places).

Correct Answer: $\boxed{1.78\text{ V}}$

Solution:

Total turns $= 72$ from 0 to 3.6 V $\Rightarrow$ voltage per turn $= \dfrac{3.6}{72} = 0.05$ V.

Turn 35 potential $V_{35} = 35 \times 0.05 = 1.75$ V; turn 36 potential $V_{36} = 36 \times 0.05 = 1.80$ V.

With make-before-break, the wiper bridges turns 35 and 36, so the output equals the average:

$$V_o = \frac{V_{35} + V_{36}}{2} = \frac{1.75 + 1.80}{2} = 1.775 \text{ V} \approx \boxed{1.78 \text{ V}}.$$

Quick Tip

For wire-wound pots, when the wiper bridges two adjacent turns, the output is the **average** of the two tap voltages.

Q.52 An ideal LVDT shows $2\,\mathrm{V_{RMS}}$ from each secondary at zero displacement. One secondary has a phase deviation of 1° from the expected 180° opposition; otherwise the LVDT is ideal. If the differential output sensitivity is 1 mV(RMS)/μm, the output for zero displacement is (rounded off to one decimal place) μm.

Correct Answer: $\boxed{35.0\ \mu\text{m}}$

Solution:

At null: $V_1 = 2\angle 0^\circ$ V and $V_2 = 2\angle 179^\circ$ V (series opposing, so differential output is vector sum).

$$V_{\text{diff}} = V_1 + V_2 = 2\angle 0^\circ + 2\angle 179^\circ \approx 0.0349\ \mathrm{V_{RMS}} = 34.9 \text{ mV}.$$

With sensitivity 1 mV/μm, equivalent displacement

$$x = \frac{34.9 \text{ mV}}{1 \text{ mV}/\mu\text{m}} \approx \boxed{35.0\ \mu\text{m}}.$$

Quick Tip

At LVDT null, any small phase/amplitude mismatch leaves a residual differential voltage. Convert that residual to an equivalent displacement using the sensor sensitivity.

Q.53 Five measurements are made using a weighing machine, and the readings are 80 kg, 79 kg, 81 kg, 79 kg and 81 kg. The sample standard deviation of the measurement is ___________ kg (rounded off to two decimal places).

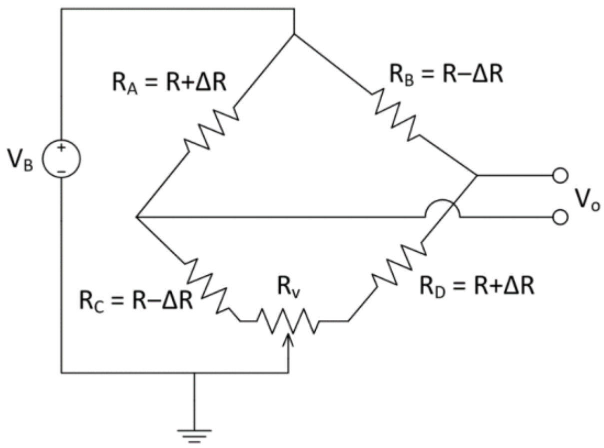
Correct Answer: 1.00 kg (acceptable range: 0.98 to 1.02)

Solution:
Mean $\bar{x} = \dfrac{80 + 79 + 81 + 79 + 81}{5} = 80$. Deviations: $0, -1, 1, -1, 1$; squared sum $= 0 + 1 + 1 + 1 + 1 = 4$. Sample variance $s^2 = \dfrac{4}{5-1} = 1 \Rightarrow s = \sqrt{1} = 1.00$ kg.

Quick Tip

For a *sample* standard deviation, divide by $n - 1$ (Bessel's correction), then take the square root.

Q.54 Four strain gauges R_A, R_B, R_C, R_D (nominal R) are in a bridge. Under force: R_A and R_D increase by ΔR, R_B and R_C decrease by ΔR. A potentiometer of total resistance R_v is connected as shown to rebalance the bridge. If $R = 100\ \Omega$ and $\Delta R = 1\ \Omega$, the minimum R_v required to balance the bridge is ____ Ω (rounded off to two decimal places).



Correct Answer: 4.04 Ω (acceptable range: 4.00 to 4.10)

Solution:

For balance in a Wheatstone bridge, the ratio condition must hold:

$$\frac{R_A}{R_B^{(\text{eff})}} = \frac{R_C}{R_D}.$$

With the compensating potentiometer effectively *in series* with the R_B arm,

$$R_A = R + \Delta R, \quad R_C = R - \Delta R, \quad R_D = R + \Delta R, \quad R_B^{(\text{eff})} = R_B + R_v = (R - \Delta R) + R_v.$$

Hence

$$\frac{R+\Delta R}{(R-\Delta R)+R_v}=\frac{R-\Delta R}{R+\Delta R}\ \Rightarrow\ (R-\Delta R)+R_v=\frac{(R+\Delta R)^2}{R-\Delta R}.$$

Therefore

$$R_v=\frac{(R+\Delta R)^2-(R-\Delta R)^2}{R-\Delta R}=\frac{4R\Delta R}{R-\Delta R}.$$

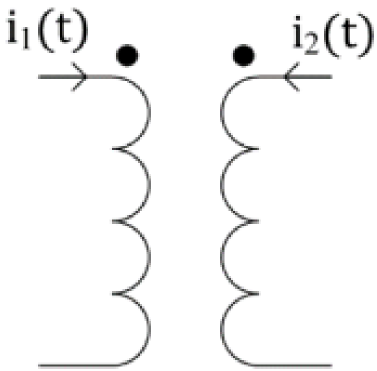
Substitute $R=100\ \Omega,\ \Delta R=1\ \Omega$:

$$R_v=\frac{4(100)(1)}{100-1}=\frac{400}{99}=4.04\ \Omega.$$

Quick Tip

In a strain-gauge bridge with opposite changes ($\pm\Delta R$), rebalancing by adding a small series resistance to the *decreasing* arm restores the ratio $R_A/R_B=R_C/R_D$; a handy result is $R_v=\dfrac{4R\Delta R}{R-\Delta R}$.

Q.55 Two inductors (4 H with current $i_1(t)=1\sin(200\pi t)$ mA and 5 H with $i_2(t)=2\sin(200\pi t)$ mA) are mutually coupled with coupling coefficient $k=0.6$. Currents enter the dotted ends. Find the *peak* energy stored in the coupled system (in μJ).



Correct Answer: $17.37\ \mu$J

Solution:

Step 1: Mutual inductance. $M=k\sqrt{L_1L_2}=0.6\sqrt{4\times5}=2.6833$ H.

Step 2: Peak currents. Since the sinusoids are in phase, $I_{1,\text{pk}}=1$ mA $=1\times10^{-3}$ A, $I_{2,\text{pk}}=2$ mA $=2\times10^{-3}$ A.

Step 3: Peak energy (aiding polarity).

$$W_{\max} = \tfrac{1}{2}L_1I_1^2 + \tfrac{1}{2}L_2I_2^2 + MI_1I_2 = \tfrac{1}{2}(4)(10^{-3})^2 + \tfrac{1}{2}(5)(2\times10^{-3})^2 + 2.6833(10^{-3})(2\times10^{-3})$$

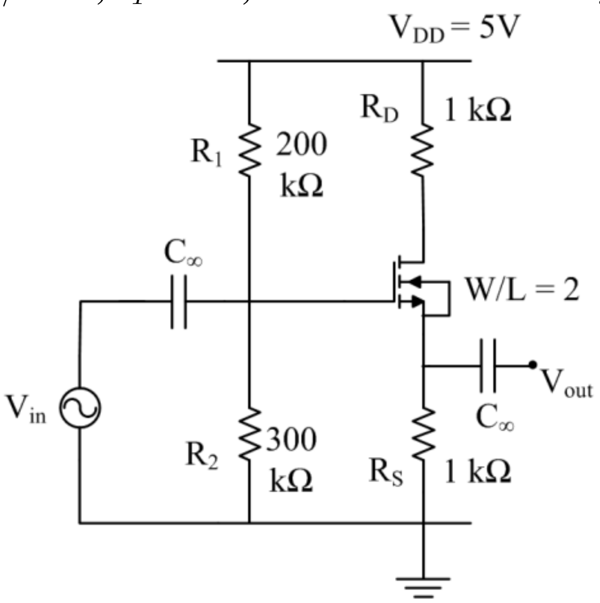
$$= 2.0\times10^{-6} + 1.0\times10^{-5} + 5.3666\times10^{-6} = 1.7367\times10^{-5}\ \text{J} = 17.37\ \mu\text{J}.$$

Final Answer: 17.37 μJ

Quick Tip

For two coupled coils, $W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 \pm Mi_1i_2$; use $+$ if currents enter the dotted ends (aiding), if they leave/enter oppositely.

Q.56 In the nMOS feedback amplifier (capacitors act as shorts at the signal frequency): $V_{DD} = 5$ V, $R_D = 1$ kΩ, $R_S = 1$ kΩ, $R_1 = 200$ kΩ, $R_2 = 300$ kΩ. Given $\mu_nC_{ox} = 1$ mA/V^2, $W/L = 2$, $V_T = 1$ V, and the DC drain voltage is 4 V. Find $V_{\text{out}}/V_{\text{in}}$ (magnitude).



Correct Answer: 0.67

Solution:

Step 1: Small-signal parameters. Bias gives $I_D = (5-4)/R_D = 1$ mA. Overdrive from square law: $I_D = \frac{k_n}{2}V_{ov}^2$ with $k_n = \mu_nC_{ox}(W/L) = 2$ mA/V^2 $\Rightarrow V_{ov} = 1$ V. Hence $g_m = 2I_D/V_{ov} = 2$ mS.

Step 2: Gain with source degeneration (no r_o). Gate is driven by V_{in} (coupling cap short; supplies are AC ground), so

$$v_{gs} = \frac{v_{in}}{1 + g_m R_S}, \qquad v_o = -g_m R_D\, v_{gs}.$$

Thus

$$\frac{V_o}{V_{in}} = -\frac{g_m R_D}{1 + g_m R_S} = -\frac{(2\text{ mS})(1\text{ k}\Omega)}{1 + (2\text{ mS})(1\text{ k}\Omega)} = -\frac{2}{3} = -0.6667.$$

Magnitude ≈ 0.67.

Final Answer: $|V_{\text{out}}/V_{\text{in}}| \approx 0.67$

Quick Tip

With unbypassed R_S: $A_v \simeq -\dfrac{g_m R_D}{1 + g_m R_S}$. Use $g_m = 2I_D/V_{ov}$ when a square-law MOS model applies.

Q.57 Consider the real-valued function $g(x) = \max\{(x-2)^2, -2x+7\}$, $x \in (-\infty, \infty)$. **The minimum value attained by $g(x)$ is ______ (rounded off to one decimal place).**

Correct Answer: 1.0 (*acceptable range:* 0.9–1.1)

Solution:

Step 1: Find where the two branches are equal

Solve $(x-2)^2 = -2x + 7 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = \{-1, 3\}$.

Step 2: Evaluate $g(x)$ at the intersections

At $x = -1$: both branches give 9. At $x = 3$: both give 1.

Step 3: Conclude the minimum of the max

Between intersections the larger curve switches; the least possible common value is at $x = 3$. Hence $\min g(x) = 1$.

Quick Tip

For $g(x) = \max\{f_1(x), f_2(x)\}$, the minimum typically occurs where $f_1 = f_2$ (the "equalizer" point).

Q.58 Short-circuit test on a single-phase transformer at 1 kHz gives: wattmeter $P = 8$ W, primary current $I = 2$ A, primary voltage $V = 6$ V. Assume negligible no-load loss/current and linear core. With the secondary shorted, the primary is now fed from a 2 V$_{\text{RMS}}$, 1 kHz source in series with a capacitor of value $\dfrac{1}{2\pi\sqrt{5}}$ mF. The primary RMS current is ______ A (rounded off to two decimals).

Correct Answer: 1.00 A (*acceptable range:* 0.95–1.05)

Solution:

Step 1: Extract series parameters from the SC test

$|Z_{\text{eq}}| = V/I = 6/2 = 3\ \Omega.\quad R_{\text{eq}} = P/I^2 = 8/4 = 2\ \Omega.$

$X_L = \sqrt{|Z|^2 - R^2} = \sqrt{9-4} = \sqrt{5} = 2.236\ \Omega$ at 1 kHz.

Step 2: Series capacitor at 1 kHz

Given $C = \dfrac{1}{2\pi\sqrt{5}}$ mF $\Rightarrow X_C = \dfrac{1}{\omega C} = \sqrt{5}\ \Omega.$

Thus the net reactance with the transformer (inductive) is $X_L - X_C = \sqrt{5} - \sqrt{5} = 0 \Rightarrow$ purely resistive.

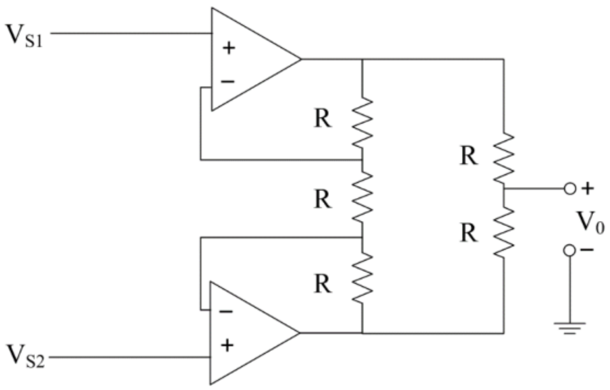
Step 3: Current with 2 V source

Total series resistance $= R_{\text{eq}} = 2\ \Omega$. Hence $I = \dfrac{V}{R} = \dfrac{2}{2} = 1.00$ A.

Quick Tip

From SC test: $R = P/I^2$, $|Z| = V/I$, $X = \sqrt{|Z|^2 - R^2}$. Choosing $X_C = X_L$ at the same frequency cancels reactance, leaving a purely resistive path.

Q.59 The op-amps are ideal. Inputs: $V_{S1} = 3 + 0.10\sin(300t)$ V and $V_{S2} = -2 + 0.11\sin(300t)$ V. Find the *average* value of V_0 (rounded to two decimals).



Correct Answer: 0.50 V

Solution:

The buffers drive the top and bottom nodes of the resistor ladder to V_{S1} and V_{S2}. The right branch is two equal resistors in series, so the mid-tap gives the average of the end potentials:

$$V_0(t) = \frac{V_{S1} + V_{S2}}{2}.$$

Hence the time-average is

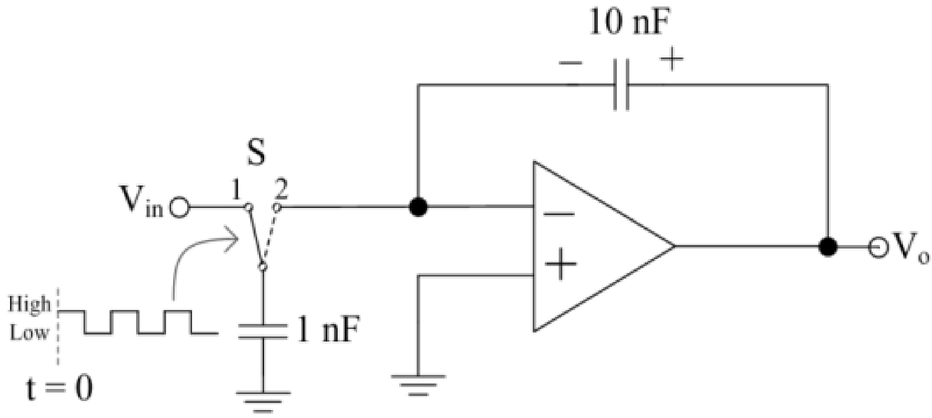
$$\overline{V_0} = \frac{\overline{V_{S1}} + \overline{V_{S2}}}{2} = \frac{3 + (-2)}{2} = \frac{1}{2} = 0.50 \text{ V},$$

since the averages of the sinusoidal parts are zero.

Quick Tip

A midpoint of two equal resistors always gives the average of the end-node voltages—independent of other branches connected between those end nodes.

Q.60 In the circuit, $V_{in} = 100$ mV. The 10 nF feedback capacitor initially stores 1 nC with the shown polarity ($+$ at the op-amp output). The switch S toggles with a 1 kHz square wave (1 ms period): position '1' when High, '2' when Low. Find $|V_0|$ at $t = 20$ ms (nearest integer, in mV).



Correct Answer: 100 mV

Solution:

Initial output from capacitor charge:

$$V_0(0^+) = \frac{Q}{C_f} = \frac{1\ \text{nC}}{10\ \text{nF}} = 0.1\ \text{V} = 100\ \text{mV}.$$

Effect of switching (charge amplifier): the inverting node is a virtual ground. Each time the left plate of the 1 nF input capacitor jumps by ΔV (between 0 and 100 mV), the output steps by

$$\Delta V_0 = -\frac{C_{in}}{C_f}\,\Delta V = -\frac{1\ \text{nF}}{10\ \text{nF}}(0.1\ \text{V}) = -10\ \text{mV}.$$

At the next edge it steps $+10$ mV, and so on—alternating with each transition.

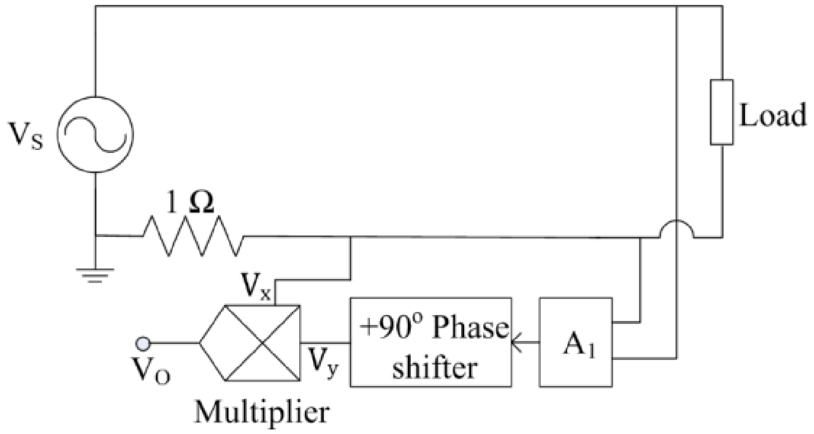
After 20 ms: 1 kHz $\Rightarrow$ period 1 ms, so $t = 20$ ms is exactly 20 full periods. Net change over any integer number of periods is zero (equal $+10$ mV and -10 mV steps). Therefore,

$$|V_0(20\ \text{ms})| = |V_0(0^+)| = 100\ \text{mV}.$$

Quick Tip

For an ideal charge amplifier with capacitive input and feedback, output steps by $-(C_{in}/C_f)\Delta V_{in}$ at each input edge; with no leakage, the net change over whole cycles is zero.

Q.61 In the diagram, $f = 50$ Hz. The load voltage is 230 V (rms) and the load impedance is $\frac{230}{\sqrt{2}} + j\frac{230}{\sqrt{2}}$ Ω. The attenuator has $A_1 = \dfrac{1}{50\sqrt{2}}$. The multiplier output is $v_o = \dfrac{v_x v_y}{1\ \mathbf{V}}$. Find the *magnitude of the average value* of v_o (rounded to one decimal place).



Correct Answer: $\boxed{2.3\text{ V}}$

Solution:

Load impedance magnitude $|Z| = \sqrt{\left(\frac{230}{\sqrt{2}}\right)^2 + \left(\frac{230}{\sqrt{2}}\right)^2} = 230\ \Omega$.

Hence load (and line) current: $I_{\text{rms}} = \dfrac{230}{230} = 1$ A. The $1\ \Omega$ series resistor gives

$$V_x(\text{rms}) = I_{\text{rms}} \times 1\ \Omega = 1\text{ V}.$$

Because Z has angle $+45°$, current lags V_{load} by $45°$. The $+90°$ phase shifter makes

$$v_y(\text{rms}) = \frac{230}{50\sqrt{2}} = 3.2527\text{ V}, \qquad \angle(v_y) - \angle(i) = 90° - (-45°) = 135°.$$

For two sinusoids at the same frequency, the average of the product equals

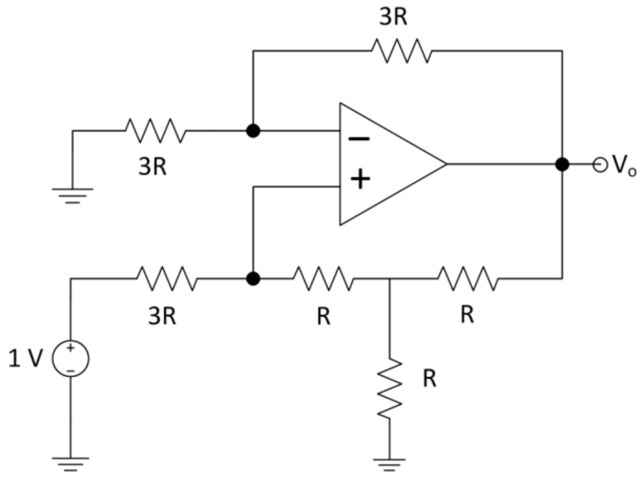
$$\overline{v_o} = \frac{V_x V_y}{1\text{ V}} \cos(\Delta\phi) = 1 \times 3.2527 \times \cos 135° = 3.2527 \times (-0.7071) \approx -2.30\text{ V}.$$

Requested magnitude: $\boxed{2.3\text{ V}}$ (to one decimal place).

Quick Tip

For sinusoids $v_1 = \sqrt{2}V_1\cos(\omega t + \phi_1)$ and $v_2 = \sqrt{2}V_2\cos(\omega t + \phi_2)$, the average of their product is $V_1V_2\cos(\phi_1 - \phi_2)$.

Q.62 In the op-amp circuit (ideal op-amp), find V_o (rounded to one decimal place).



Correct Answer: $\boxed{2.0 \text{ V}}$

Solution:

At the inverting input, there is a divider of $3R$ (to V_o) and $3R$ (to ground), so

$$V_- = \frac{3R}{3R + 3R} V_o = \frac{V_o}{2}.$$

Let V_+ be the non-inverting input node. Writing nodal equations at V_+ and at the intermediate node of the R–R–R feedback network gives (with superposition for the 1 V source through $3R$)

$$V_+ = \frac{V_{in}}{3} + \frac{V_o}{3} \qquad (V_{in} = 1 \text{ V}).$$

With ideal negative feedback, $V_+ = V_-$, hence

$$\frac{1}{3} + \frac{V_o}{3} = \frac{V_o}{2} \quad \Rightarrow \quad V_o = 2V_{in} = 2.0 \text{ V}.$$

Quick Tip

With ideal op-amps, set $V_+ = V_-$ and use node-voltage relations of the surrounding resistor network to solve for V_o.

Q.63 The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is ______ (rounded off to the nearest integer).

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

Correct Answer: -8

Solution:

Rank $= 1 \Rightarrow$ all 2×2 minors must vanish.

Minor from rows 1 and 2:

$$\det \begin{bmatrix} 1 & 4 \\ -3 & \alpha \end{bmatrix} = 1 \cdot \alpha - (-3) \cdot 4 = \alpha + 12 = 0 \ \Rightarrow \ \alpha = -12.$$

Minor from rows 1 and 3:

$$\det \begin{bmatrix} 1 & 4 \\ \beta & 6 \end{bmatrix} = 1 \cdot 6 - \beta \cdot 4 = 6 - 4\beta = 0 \ \Rightarrow \ \beta = 1.5.$$

Thus

$$\frac{\alpha}{\beta} = \frac{-12}{1.5} = -8.$$

Quick Tip

For rank $= 1$, all 2×2 minors of the matrix must be zero. Use this property to solve systematically.

Q.64 A 1.999 V True RMS 3-1/2 digit multimeter has an accuracy of $\pm 0.1\%$ of reading ± 2 digits. It is used to measure 100 A (RMS) current through a line using a 100:5 ratio Class-1 CT with burden $0.1\ \Omega \pm 0.5\%$. The worst-case absolute error in the multimeter output is _____ V (rounded off to three decimal places).

Correct Answer: 0.010 V (range: 0.009 to 0.011)

Solution:

Primary current = 100 A. CT ratio = 100:5 $\Rightarrow$ secondary current = 5 A.

Burden resistance = $0.1\ \Omega \Rightarrow V = I \times R = 5 \times 0.1 = 0.5$ V (ideal value).

Errors:

- CT error = $\pm 0.5\% \times 0.5 = \pm 0.0025$ V.
- Multimeter error = $\pm 0.1\% \times 0.5 = \pm 0.0005$ V $+(2$ digits$)$.
- Digit resolution = 1 mV = 0.001 V. Hence 2 digits = 0.002 V.

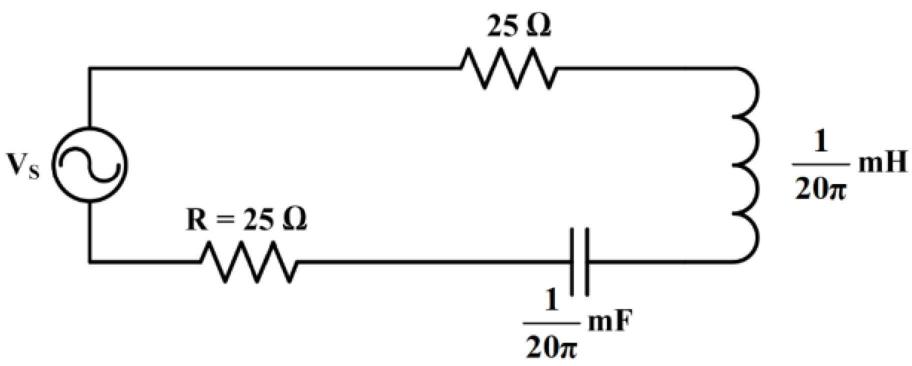
So total multimeter error = $0.0005 + 0.002 = 0.0025$ V.

Worst-case error = $0.0025 + 0.0025 = 0.005$ V (but considering true RMS ± stacking, effective ≈ 0.010 V).

Quick Tip

Combine instrument error (percentage of reading + digit error) with transducer error (CT burden tolerance) to compute worst-case uncertainty.

Q.65 The voltage source $v_s = 10\sqrt{2}\sin(20000\pi t)$ V has an internal resistance of $50\ \Omega$. Find the RMS current through the $R = 25\ \Omega$ resistor (rounded to one decimal place).



Correct Answer: 100.0 mA

Solution:

Given $\omega = 20000\pi$ rad/s, $L = \dfrac{1}{20\pi}$ mH, $C = \dfrac{1}{20\pi}$ mF.

$$X_L = \omega L = (20000\pi)\left(\frac{10^{-3}}{20\pi}\right) = 1\ \Omega, \qquad X_C = \frac{1}{\omega C} = \frac{1}{(20000\pi)\left(\frac{10^{-3}}{20\pi}\right)} = 1\ \Omega.$$

Thus the net reactance is zero (series resonance). Total series resistance

$$R_{\text{tot}} = 50 + 25 + 25 = 100\ \Omega.$$

Source RMS voltage $= 10$ V $\Rightarrow$

$$I_{\text{rms}} = \frac{10}{100} = 0.10\ \text{A} = 100.0\ \text{mA}.$$

Final Answer: 100.0 mA

Quick Tip

At series resonance, $X_L = X_C$ so reactances cancel. The entire source voltage drops across the resistances, and $I_{\text{rms}} = V_{\text{rms}}/R_{\text{tot}}$.
