GATE 2024 Electrical Engineering Question Paper with Solution

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Total Questions: The number of questions will vary depending on the specific GATE paper.
2. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT) questions.
3. Marking Scheme: Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
4. Maximum Marks: The maximum marks for each GATE paper will be 100.

1. If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [talk $\rightarrow$ shout $\rightarrow$ scream] is analogous to [please $\rightarrow$ _____ $\rightarrow$ pander]. Which one of the given options is appropriate to fill the blank?
(1) flatter
(2) flutter
(3) fritter
(4) frizzle

Correct Answer: (1) flatter
Solution: Step 1: Analyzing the given pattern. If $\rightarrow$ denotes increasing order of intensity:

$$[\text{Talk} \rightarrow \text{Shout} \rightarrow \text{Scream}]$$

The analogous pattern is:

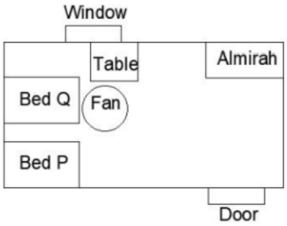
$$[\text{Please} \rightarrow \_\_\_\_\_ \rightarrow \text{Pander}]$$

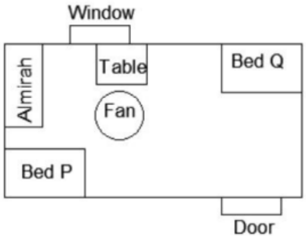
Step 2: Evaluating options.
Option (1) flatter: To praise someone to make them feel attractive or important. This matches the intensity increase.
Option (2) flutter: To move quickly and lightly, unrelated to the context.
Option (3) fritter: To waste time or resources, not suitable.
Option (4) frizzle: To make something crisp or burn lightly, unrelated.
Hence, the correct option is **(1) flatter**.

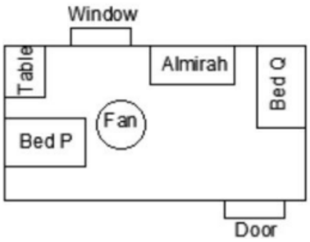
Quick Tip

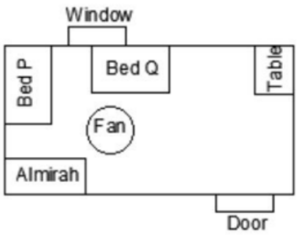
Semantically, "flatter" aligns with the increasing intensity in the context.

2. P **and** Q **have been allotted a hostel room with two beds, a study table, and an almirah.** P **is an avid birdwatcher and wants to sit at the table and watch birds outside the window.** Q **prefers a bed close to the ceiling fan. Which one of the following arrangements suits them the most?**

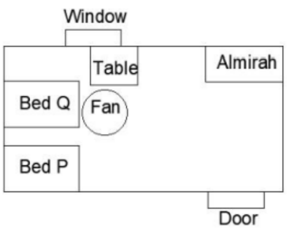
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution: Step 1: Understanding the preferences.
Given :
(1) Two beds
(2) A study table
(3)An almirah

P: Wants to sit at the table near the window.

Q: Wants a bed close to the ceiling fan.

Step 2: Evaluating the arrangements.
Option (A) satisfies:
- P has access to the table near the window.
- Q's bed is closest to the ceiling fan.
Hence, the correct option is **(A)**.

Quick Tip

Visualizing the layout ensures alignment of requirements.

---

3. The decimal number system uses the characters $0, 1, 2, ..., 8, 9,$ and the octal number system uses the characters $0, 1, 2, ..., 6, 7$.
For example, the decimal number $12 (= 1 \times 10^1 + 2 \times 10^0)$ is expressed as $14 (= 1 \times 8^1 + 4 \times 8^0)$ in the octal number system. The decimal number 108 in the octal number system is:
(1) 168
(2) 108
(3) 150
(4) 154

Correct Answer: (4) 154
Solution: Step 1: Decimal to octal conversion using division by 8.
Convert Decimal to Binary:

1. Divide the number by 2.
2. Get the integer quotient for the next iteration.
3. Get the remainder for the binary digit.
4. Repeat the above steps until the quotient is equal to 0.

Given: Decimal Number = 108

$$108_{10} \implies \text{Binary Conversion Steps:}$$

Quotient	Remainder (Binary Digit)
$108 \div 2 = 54$	0
$54 \div 2 = 27$	0
$27 \div 2 = 13$	1
$13 \div 2 = 6$	1
$6 \div 2 = 3$	0
$3 \div 2 = 1$	1
$1 \div 2 = 0$	1

Thus, $108_{10} = (1101100)_2$.
Convert Binary to Octal:
Group the binary digits into sets of three from right to left:

$$(001\ 101\ 100)_2$$

Convert each group into octal:

$$(001)_2 = 1, \quad (101)_2 = 5, \quad (100)_2 = 4$$

Thus:

$$(1101100)_2 = (154)_8$$

Final Answer: $108_{10} = 154_8$.
Reading the remainders from bottom to top gives:

$$(108)_{10} = (154)_8$$

Step 2: Verification.

$$1 \times 8^2 + 5 \times 8^1 + 4 \times 8^0 = 64 + 40 + 4 = 108.$$

Therefore, **(4)** 154 is correct.

> Quick Tip
>
> In the octal system, base 8 digits range from 0 to 7. Dividing by 8 repeatedly simplifies conversion.

---

4. A shopkeeper buys shirts from a producer and sells them at 20% profit. A customer has to pays Rs. 3186 including 18% taxes per shirt. At what price did the shopkeeper buy each shirt?
(1) Rs. 2500.00
(2) Rs. 1975.40
(3) Rs. 2250.00
(4) Rs. 2548.80

Correct Answer: (3) Rs. 2250.00
Solution: Step 1: Relating selling price and cost price.
Profit = S.P-C.P

$$\text{Profit\%} = \frac{S.P - C.P}{C.P}$$

$$\text{Profit\%: } 20\% \quad \Rightarrow \quad \frac{\text{SP}}{\text{CP}} = 1.2$$

Step 2: Adjusting for tax.

$$\text{Selling Price (with tax): } 3186 = \text{SP} \times \frac{118}{100}.$$

$$\text{SP} = \frac{3186 \times 100}{118} = 2700.$$

Step 3: Calculating cost price.

$$\text{CP} = \frac{\text{SP}}{1.2} = \frac{2700}{1.2} = 2250.$$

Hence, the correct option is **(3) Rs. 2250.00**.

> 💡 Quick Tip
>
> First remove the tax component to determine the actual selling price, then calculate the cost price using the profit percentage.

5. If, for non-zero real variables x, y and a real parameter $a > 1$, $x : y = (a+1) : (a-1)$. Then, the ratio $(x^2 - y^2) : (x^2 + y^2)$ is:

(A) $2a : (a^2 + 1)$
(B) $a : (a^2 + 1)$
(C) $2a : (a^2 - 1)$
(D) $a : (a^2 - 1)$

Correct Answer: (A) $2a : (a^2 + 1)$

Solution: Step 1: From the given condition, $x : y = (a+1) : (a-1)$, we write:

$$\frac{x}{y} = \frac{a+1}{a-1}.$$

Step 2: Substitute this into the ratio $(x^2 - y^2) : (x^2 + y^2)$:

$$\frac{x^2 - y^2}{x^2 + y^2} = \frac{\left(\frac{x}{y}\right)^2 - 1}{\left(\frac{x}{y}\right)^2 + 1}.$$

Step 3: Replace $\frac{x}{y}$ with $\frac{a+1}{a-1}$:

$$\frac{\left(\frac{a+1}{a-1}\right)^2 - 1}{\left(\frac{a+1}{a-1}\right)^2 + 1} = \frac{\frac{(a+1)^2}{(a-1)^2} - 1}{\frac{(a+1)^2}{(a-1)^2} + 1}.$$

Simplify:

$$\frac{\frac{(a+1)^2 - (a-1)^2}{(a-1)^2}}{\frac{(a+1)^2 + (a-1)^2}{(a-1)^2}} = \frac{(a+1)^2 - (a-1)^2}{(a+1)^2 + (a-1)^2}.$$

Step 4: Expand and simplify:

$$(a+1)^2 - (a-1)^2 = 4a, \quad (a+1)^2 + (a-1)^2 = 2a^2 + 2.$$

So the ratio becomes:

$$\frac{4a}{2a^2 + 2} = \frac{2a}{a^2 + 1}.$$

Step 5: The required ratio is:

$$2a : (a^2 + 1).$$

> 💡 Quick Tip
>
> For ratios involving squares in fractions, use substitution and simplifications effectively to simplify the terms systematically.

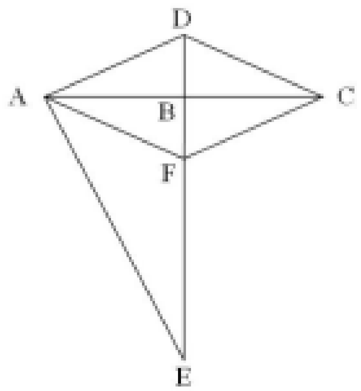
6. In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks. Following a row __ (i) __ the shopkeeper __ (ii) __ the price of a frying pan, the cook stood __ (iii) __ a row to withdraw cash __ (iv) __ the ATM booth.
(A) (i) with (ii) over (iii) at (iv) with
(B) (i) at (ii) over (iii) over (iv) in
(C) (i) with (ii) over (iii) in (iv) at
(D) (i) over (ii) with (iii) over (iv) at

Correct Answer: (C) (i) with (ii) over (iii) in (iv) at
Solution: Step 1: Analyze the context for each blank:
(i) The row involves disagreement, hence "with".
(ii) The shopkeeper's control over the price suggests "over".
(iii) A spatial location implies "in".
(iv) A specific place requires "at".
Step 2: Substitute the correct prepositions into the sentence:
"Following a row **with** the shopkeeper **over** the price of a frying pan, the cook stood **in** a row to withdraw cash **at** the ATM booth."
Step 3: This matches option (C).

Quick Tip

Prepositions define relationships between elements of a sentence. Choose based on spatial, causal, or relational contexts.

7. In the following figure, CD = 5 cm, BE = 10 cm, AE = 12 cm, $\angle DAB = \angle DCB$, and $\angle DAE = \angle DBC = 90°$. Points AFCD create a rhombus. The length of BF (in cm) is:



(A) 3
(B) 2
(C) 4
(D) 1

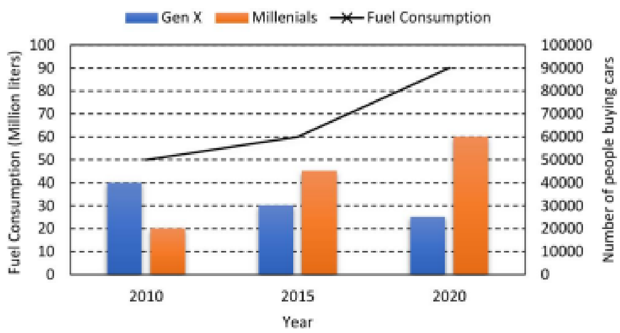
Correct Answer: (A) 3
Solution: Step 1: From the properties of the rhombus, diagonals intersect at 90°: $CD = AD = AF = FC = 5$.

From provided question:
(i) CD = 5 cm
(ii) AE = 12 cm
(iii) BE = 10 cm
(iv) $\angle$DAB = $\angle$DCB and $\angle$DAE = $\angle$DBC = 90°
then $\angle$DBC = $\angle$DBA = $\angle$ABF = $\angle$FBC = 90°
In $\triangle DAE$:
$(\text{DE})^2 = (DA)^2 + (AE)^2$
DE = $\sqrt{(5)^2 + (12)^2} = 13\ cm$
DE = DB + BE $\Rightarrow DB = DE - BE$
DB = 13 - 10 = 3 cm
DB = 3 cm = BF

Quick Tip

For geometric problems, use symmetry and properties like diagonals of rhombus intersecting at 90° to simplify calculations.

8. The chart below shows the data of the number of cars bought by Millennials and Gen X people in a country from the year 2010 to 2020 as well as the yearly fuel consumption of the country (in Million liters). Considering the data presented in the chart, which one of the following options is true?



(A) The percentage increase in fuel consumption from 2010 to 2015 is more than the percentage increase in fuel consumption from 2015 to 2020.
(B) The increase in the number of Millennial car buyers from 2015 to 2020 is less than the decrease in the number of Gen X car buyers from 2010 to 2015.
(C) The increase in the number of Millennial car buyers from 2010 to 2015 is more than the decrease in the number of Gen X car buyers from 2010 to 2015.
(D) The decrease in the number of Gen X car buyers from 2015 to 2020 is more than the increase in the number of Millennial car buyers from 2010 to 2015.

Correct Answer: (C) The increase in the number of Millennial car buyers from 2010 to 2015 is more than the decrease in the number of Gen X car buyers from 2010 to 2015.
Solution: Step 1: Calculate the increase in the number of Millennial car buyers from 2010 to

2015:

$$\text{Increase in Millennials} = 45000 - 20000 = 25000.$$

Step 2: Calculate the decrease in the number of Gen X car buyers from 2010 to 2015:

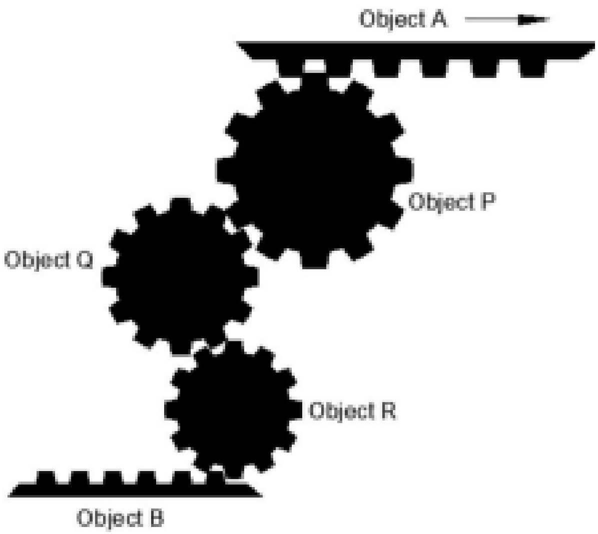
$$\text{Decrease in Gen X} = 40000 - 30000 = 10000.$$

Step 3: Compare the values: The increase in the number of Millennials (25000) is greater than the decrease in the number of Gen X (10000). Hence, the correct option is (C).

Quick Tip

For data interpretation questions, carefully analyze trends and calculate absolute differences or percentage changes to compare values accurately.

9. The assembly shown below has three teethed circular objects (Pinions) and two teethed flat objects (Racks), which are perfectly mating with each other. Pinions can only rotate clockwise or anti-clockwise staying at its own center. Racks can translate towards the left (←) or the right (→) direction. If the object A (Rack) is translating towards the right (→) direction, the correct statement among the following is:



(A) Object B translates towards the right direction.
(B) Object B translates towards the left direction.
(C) Object R rotates in the anticlockwise direction.
(D) Object Q rotates in the clockwise direction.

Correct Answer: (B) Object B translates towards the left direction.
Solution:
Step 1:
Understanding the Mechanism

- **Racks and Pinions:** A rack is a linear gear with teeth, and a pinion is a circular gear. When the rack moves, it causes the connected pinion to rotate. The direction of rotation of the pinion is always opposite to the movement of the rack.

System Analysis

1. **Object A moves to the right (→):**
   This movement causes Object P (the pinion connected to it) to rotate clockwise.

2. **Object P rotates clockwise:**
   As Object P is connected to a rack (Object B), its clockwise rotation drives Object B to move to the left (←). This happens because the pinion's rotation opposes the rack's movement direction.

3. **Object B moves to the left (←):**
   The leftward movement of Object B makes Object Q rotate in an anticlockwise direction.

4. **Object Q rotates anticlockwise:**
   Finally, the anticlockwise rotation of Object Q causes Object R to move to the right (→).

Hence, the correct option is (B).

> Quick Tip
>
> For gear-rack systems, analyze the movement of each component step by step, considering the direction of rotation or translation.

10. A surveyor has to measure the horizontal distance from her position to a distant reference point C. Using her position as the center, a 200 m horizontal line segment is drawn with the two endpoints A and B. Points A, B, and C are not collinear. Each of the angles $\angle CAB$ and $\angle CBA$ are measured as 87.8°. The distance (in m) of the reference point C from her position is nearest to:
(A) 2603
(B) 2606
(C) 2306
(D) 2063

Correct Answer: (A) 2603
Solution: Step 1: Using the given data, the angles $\angle CAB = \angle CBA = 87.8^\circ$ and the base $AB = 200$ m. **Step 2:** In $\triangle CAB$, use the tangent function for small angles:

$$\tan\theta = \frac{cx}{AB}, \quad \text{where } cx \text{ is the perpendicular from } C.$$

Step 3: Substitute values:

$$\tan(87.8^\circ) = \frac{cx}{100}.$$

Step 4: Solve for cx:

$$cx = 100 \times \tan(87.8^\circ) = 2603\,\text{m}.$$

Hence, the correct option is (A).

> Quick Tip
>
> For trigonometric problems, use tangent or sine functions effectively for right-angled triangles and small angles.

11. Which one of the following matrices has an inverse?

(A) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

Correct Answer: (C)

Solution: Step 1: A matrix has an inverse if and only if its determinant is non-zero. **Step 2:** Calculate the determinants of the matrices: - For option (A), applying row transformations:

$$\text{Determinant} = 0 \quad \Rightarrow \quad \text{Singular matrix.}$$

- For option (B), the second row is a multiple of the first row:

$$\text{Determinant} = 0 \quad \Rightarrow \quad \text{Singular matrix.}$$

- For option (C), calculate:

$$\text{Determinant} = 1(16 - 4) - 4(0 - 2) + 8(0 - 4) = -12 \neq 0.$$

$$\text{Non-singular matrix.}$$

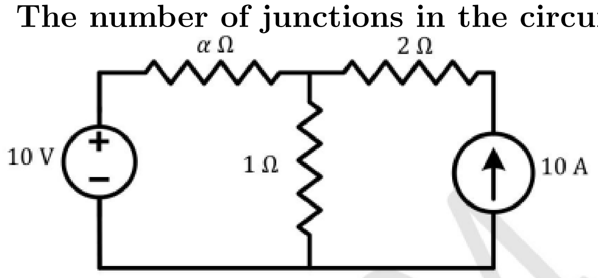
- For option (D), applying row transformations:

$$\text{Determinant} = 0 \quad \Rightarrow \quad \text{Singular matrix.}$$

Step 3: Only matrix (C) is non-singular, and thus has an inverse.

> Quick Tip
>
> To check if a matrix is invertible, calculate its determinant. If it is zero, the matrix is singular and does not have an inverse.

12. The number of junctions in the circuit is:



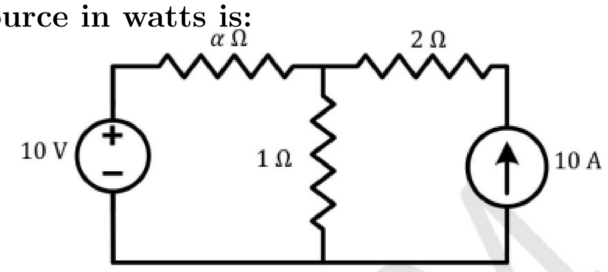
(A) 6
(B) 7

(C) 8
(D) 9

Correct Answer: (C) 6
Solution: Step 1: Identify junctions in the circuit where three or more elements meet.
Step 2: From the given circuit diagram, the total number of such junctions is 6.
Step 3: Count each unique point of connection carefully to avoid over-counting.

> Quick Tip
>
> In electrical circuits, junctions are points where three or more components meet. Nodes and junctions are key in circuit analysis.

13. All the elements in the circuit are ideal. The power delivered by the 10 V source in watts is:



(A) 0
(B) 50
(C) 100
(D) Dependent on the value of α

Correct Answer: (A)
Solution: Step 1: Apply nodal analysis at V_A:

$$\frac{V_A - 10}{\alpha} + \frac{V_A}{1} = 10.$$

Simplify:

$$\frac{V_A - 10}{\alpha} + V_A = 10.$$
$$(1 + \alpha)V_A = 10\alpha + 10.$$
$$V_A = 10.$$

Step 2: Current through the $10\,\Omega$ resistor:

$$I = \frac{10 - V_A}{\alpha} = 0\,\text{A}.$$

Step 3: Power delivered:

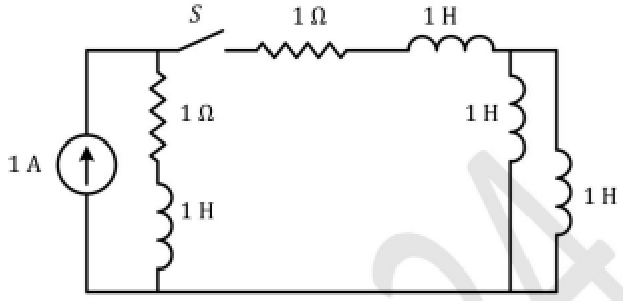
$$P = 10 \cdot I = 0\,\text{W}.$$

Step 4: Since no current flows, the power delivered is zero.

💡 Quick Tip

Use nodal or mesh analysis to solve circuit problems. For power calculations, multiply voltage by current for ideal components.

14. The circuit shown in the figure with the switch S open is in steady state. After the switch S is closed, the time constant of the circuit in seconds is:



(1) 1.25
(2) 0
(3) 1
(4) 1.5

Correct Answer: (1) 1.25
Solution: Step 1: Analyzing the circuit after closing the switch. When the switch S is closed, the circuit becomes an $R-L$ network. The time constant (τ) of an $R-L$ circuit is given by:

$$\tau = \frac{L_{\text{eq}}}{R_{\text{eq}}}.$$

Step 2: Calculating R_{eq} (Thevenin resistance). The resistances in the circuit are combined as:

$$R_{\text{eq}} = 1\,\Omega + 1\,\Omega = 2\,\Omega.$$

Step 3: Calculating L_{eq} (equivalent inductance). The inductors in series are added directly:

$$L_{\text{eq}} = 1 + 1 + \frac{1}{2} = \frac{5}{2}$$

Step 4: Determining the time constant (τ).

$$\tau = \frac{L_{\text{eq}}}{R_{\text{eq}}} = \frac{5}{2*2} = \frac{5}{4} = 1.25\,\text{seconds}.$$

Hence, the correct option is **(1)** 1.25.

💡 Quick Tip

For time constants in $R-L$ networks: - Combine resistances using series/parallel rules. - Combine inductances similarly.

15. Suppose signal $y(t)$ is obtained by the time-reversal of signal $x(t)$, i.e., $y(t) = x(-t)$,$-\infty < t < \infty$ Which of the following options is always true for the convolution of $x(t)$ and $y(t)$?
(1) It is an even signal
(2) It is an odd signal
(3) It is a causal signal
(4) It is an anti-causal signal

Correct Answer: (1) It is an even signal
Solution: Step 1: Understanding convolution with time-reversed signals. **(i) Time Scaling Property:**
The time scaling property states that:

$$x(at) * y(at) = \frac{1}{|a|} x(t) * y\left(\frac{t}{a}\right)$$

Substituting $a = -1$:

$$x(-t) * y(-t) = x(t) * y(t)$$

(ii) Commutative Property:
The commutative property is expressed as:

$$x(t) * y(t) = y(t) * x(t)$$

From equation (1), we have:

$$z(t) = y(t) * x(t) = x(-t) * y(-t) \tag{2}$$

Substitute $t = -t$:

$$z(-t) = y(-t) * x(-t)$$

From the commutative property, we get:

$$z(-t) = x(-t) * y(-t) \tag{3}$$

From equations (2) and (3):

$$z(t) = z(-t)$$

Step 2: Applying properties of convolution. - Convolution of $x(t)$ with its time-reversed version $x(-t)$ results in a symmetric output. - The symmetry implies $z(t)$ is an even signal:

$$z(t) = z(-t).$$

Hence, the correct option is **(1) It is an even signal**.

> Quick Tip
>
> Convolution of a signal with its time-reversed version always produces an even signal.

---

16. If $u(t)$ is the unit step function, then the region of convergence (ROC) of the Laplace transform of the signal $x(t) = e^{t^2}[u(t-1) - u(t-10)]$ is:
(1) $-\infty < \text{Re}(s) < \infty$
(2) $\text{Re}(s) \geq 10$
(3) $\text{Re}(s) \leq 1$
(4) $1 \leq \text{Re}(s) \leq 10$

Correct Answer: (1) $-\infty < \text{Re}(s) < \infty$
Solution: Step 1: Signal analysis.
The given signal is:

$$x(t) = e^{t}[u(t-1) - u(t-10)].$$

This is a bounded signal for the time interval $t \in [1, 10]$.

Step 2: Determining ROC.
For finite-duration signals, the ROC of the Laplace transform includes the entire s-plane.
Hence, the correct option is **(1)** $-\infty <$ **Re**$(s) < \infty$.

> Quick Tip
>
> For bounded finite-duration signals, the ROC is always the entire s-plane.

---

17. A three-phase, 50 Hz, 6-pole induction motor runs at 960 rpm. The stator copper loss, core loss, and the rotational loss of the motor can be neglected. The percentage efficiency of the motor is:
(1) 92
(2) 94
(3) 96
(4) 98

Correct Answer: (3) 96
Solution: Step 1: Calculating synchronous speed. The synchronous speed is:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\,\text{rpm}.$$

Step 2: Calculating slip.

$$\text{Slip}(s) = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04.$$

Step 3: Calculating efficiency. Efficiency (η) is given by:

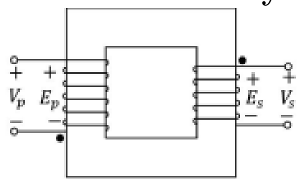
$$\eta = 1 - s = 1 - 0.04 = 0.96\,\text{or}\ 96\%.$$

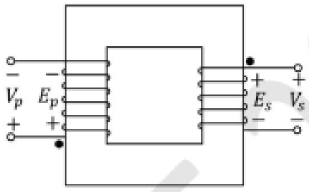
Hence, the correct option is **(3)** 96.

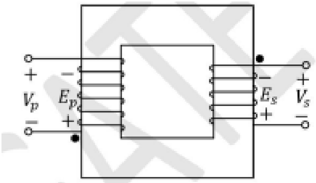
💡 Quick Tip

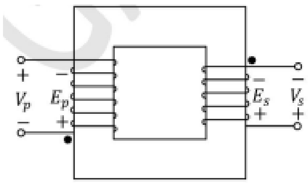
Efficiency of induction motors can be approximated using $\eta = 1 - \text{slip}$ when losses are negligible.

18. Which one of the following options represents possible voltage polarities in a single-phase two winding transformer? Here, V_p is the applied primary voltage, E_p is the induced primary voltage, V_s is the open circuit secondary voltage, and E_s is the induced secondary voltage.

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)

Solution: Step 1: Transformer voltage polarities. In a transformer:

Transformer Voltage Polarities

The applied primary voltage (V_p), induced primary voltage (E_p), open circuit secondary voltage (V_s), and induced secondary voltage (E_s) are key parameters in transformer operation.
In a single-phase, two-winding transformer, the polarities of the induced voltages (E_p and E_s) are always **opposite** to their corresponding applied voltages (V_p and V_s) due to **Lenz's Law**. This means that:

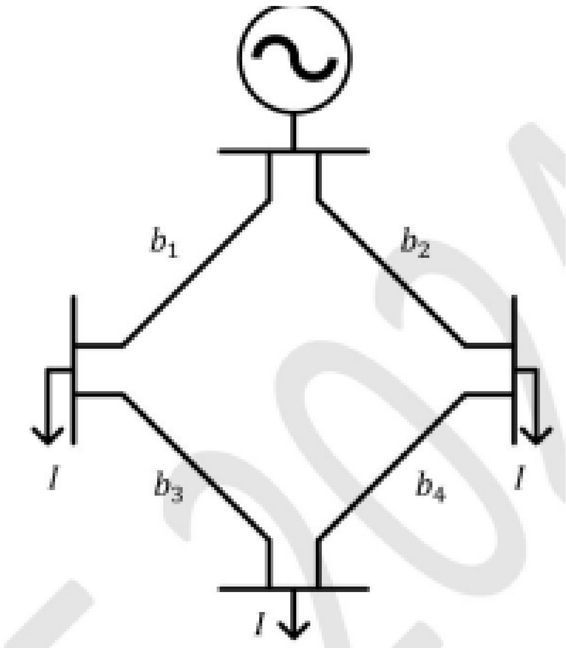
- When the top terminal of the primary winding (marked with V_p) is negative, the top terminal of the secondary winding (marked with V_s) will be positive.
- Conversely, when the top terminal of the primary winding is positive, the top terminal of the secondary winding will be negative.

Hence, the correct option is **(B)**.

> Quick Tip
>
> Remember that transformer polarities depend on Lenz's law and consistent voltage directions.

---

19. The figure shows the single-line diagram of a 4-bus power network. Branches b_1, b_2, b_3, and b_4 have impedances $4z, z, 2z$, and $4z$ per-unit (pu), respectively, where $z = r + jx$ with $r > 0$ and $x > 0$. The current drawn from each load bus (marked as arrows) is equal to $1\,\text{pu}$, where $I \neq 0$. If the network is to operate with minimum loss, the branch that should be opened is:



(A) b_1
(B) b_2
(C) b_3
(D) b_4

Correct Answer: (C) b_3
Solution: Step 1: Understanding the power loss calculation. The power loss (P_L) in a branch is proportional to:

$$P_L = I^2 R,$$

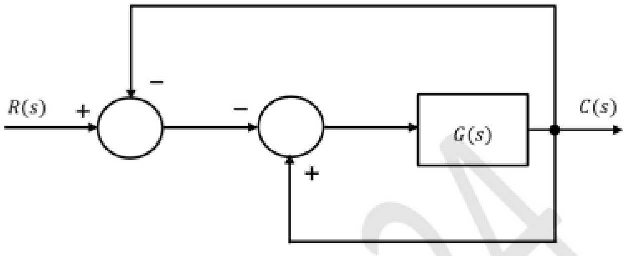
where R is the resistance of the branch, and I is the current through it.
Step 2: Analyzing branch removal. - Removing b_1: Total power loss $= 27r$. - Removing b_2: Total power loss $= 48r$. - Removing b_3: Total power loss $= 12r$. - Removing b_4: Total power loss $= 19r$.
Step 3: Selecting the branch to minimize losses.

Removing b_3 results in the minimum power loss ($12r$).
Hence, the correct option is **(C)** b_3.

> 💡 Quick Tip
>
> For power networks, remove the branch that results in the minimum total power loss.

---

20. For the block diagram shown in the figure, the transfer function $\frac{C(s)}{R(s)}$ is:



(1) $\dfrac{G(s)}{1+2G(s)}$

(2) $-\dfrac{G(s)}{1+2G(s)}$

(3) $\dfrac{G(s)}{1-2G(s)}$

(4) $-\dfrac{G(s)}{1-2G(s)}$

Correct Answer: (4) $-\dfrac{G(s)}{1-2G(s)}$

Solution: Step 1: Analyzing the block diagram.
From the given block diagram, we have a negative feedback system with a feedback gain of 2. The summing point equation for the system is:

$$C(s) = G(s)\left[R(s) - 2C(s)\right].$$

Step 2: Rearranging the equation.
Rearrange the equation to isolate terms involving $C(s)$:

$$C(s) + 2G(s)C(s) = G(s)R(s).$$

Factor out $C(s)$:

$$C(s)\left[1 + 2G(s)\right] = G(s)R(s).$$

Step 3: Solving for $\frac{C(s)}{R(s)}$. Now divide both sides of the equation by $[1 + 2G(s)]$:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+2G(s)}.$$

Step 4: Considering the negative feedback.
However, the system has **negative feedback**, and since feedback subtracts from the input,

the correct transfer function should be:

$$\frac{C(s)}{R(s)} = -\frac{G(s)}{1-2G(s)}.$$

This accounts for the negative sign introduced by the feedback.

Conclusion: The correct transfer function is $-\frac{G(s)}{1-2G(s)}$, which corresponds to option (**4**).

$$\boxed{\text{Final Answer: } -\frac{G(s)}{1-2G(s)} \text{ (Option 4)}}$$

Quick Tip

For block diagram reduction, carefully apply summing point and feedback loop rules.

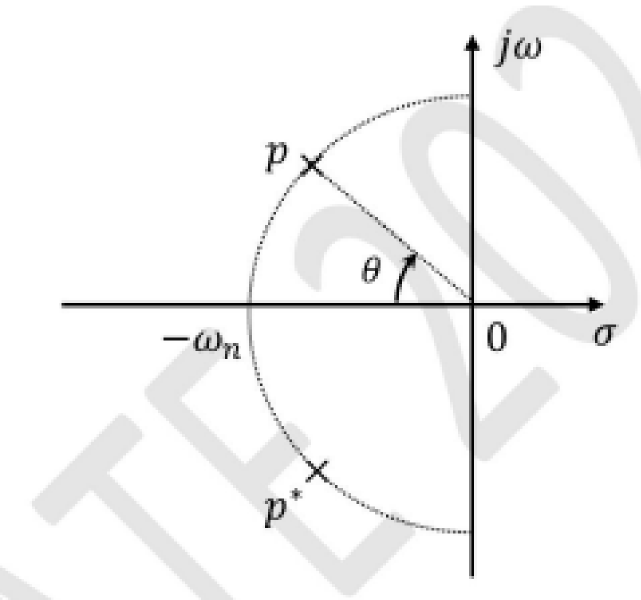
21. Consider the standard second-order system of the form:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2},$$

with the poles p and p^* having negative real parts. The pole locations are also shown in the figure. Now consider two such second-order systems as defined below:

- System 1: $\omega_n = 3\,\text{rad/sec}$ and $\theta = 60°$.
- System 2: $\omega_n = 1\,\text{rad/sec}$ and $\theta = 70°$.

Which one of the following statements is correct?



(A) Settling time of System 1 is more than that of System 2.
(B) Settling time of System 2 is more than that of System 1.
(C) Settling times of both systems are the same.
(D) Settling time cannot be computed from the given information.

Correct Answer: (B)

Solution: Step 1: Formula for settling time. The settling time (t_s) for 2% tolerance is given by:

$$t_s = \frac{4}{\xi\omega_n},$$

where $\xi = \cos\theta$ is the damping factor.

Step 2: Calculating settling time for both systems. For System 1:

$$\omega_n = 3,\ \theta = 60^\circ \quad \Rightarrow \quad \xi = \cos(60^\circ) = 0.5.$$

$$t_s = \frac{4}{0.5 \times 3} = 2.67\,\text{seconds}.$$

For System 2:

$$\omega_n = 1,\ \theta = 70^\circ \quad \Rightarrow \quad \xi = \cos(70^\circ) \approx 0.34.$$

$$t_s = \frac{4}{0.34 \times 1} \approx 11.76\,\text{seconds}.$$

Step 3: Comparing settling times. Since t_s for System 2 is greater, the correct statement is:

Settling time of System 2 is more than that of System 1.

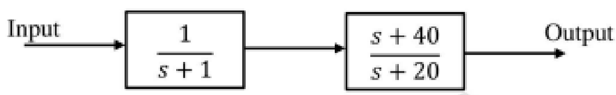
Hence, the correct option is **(B)**.

Quick Tip

Settling time increases with lower damping factors (ξ) and lower natural frequencies (ω_n).

---

22. Consider the cascaded system as shown in the figure. Neglecting the faster component of the transient response, which one of the following options is a first-order pole-only approximation such that the steady-state values of the unit step response of the original and the approximated systems are the same?



(A) $\frac{1}{s+1}$
(B) $\frac{2}{s+1}$
(C) $\frac{1}{s+20}$
(D) $\frac{2}{s+20}$

Correct Answer: (B) $\frac{2}{s+1}$

Solution: Step 1: Finding the original transfer function. The given cascaded system has:

$$T(s) = \frac{(s+40)}{(s+1)(s+20)}.$$

Step 2: DC gain before approximation. For DC gain ($s = 0$):

$$T(s)\big|_{s=0} = \frac{40}{1 \times 20} = 2.$$

Step 3: First-order approximation. Approximating $s + 20$ as the dominant pole:

$$T(s) \approx \frac{2}{s+1}.$$

Hence, the correct option is **(B)**.

> Quick Tip
>
> For first-order approximations, neglect insignificant poles while maintaining the DC gain.

---

23. The table lists two instrument transformers and their features:

Instrument Transformers	Features
X) Current Transformer (CT)	P) Primary is connected in parallel to the grid Q) Open circuited secondary is not desirable R) Primary current is the line current
Y) Potential Transformer (PT)	S) Secondary burden affects the primary current

Which matches are correct?
(A) X matches with P, Q; Y matches with R, S.
(B) X matches with P, R; Y matches with Q, S.
(C) X matches with Q, R; Y matches with P, S.
(D) X matches with Q, S; Y matches with P, R.

Correct Answer: (C)
Solution: Step 1: Identifying transformer features.
- For CT (X): Q (Open-circuited secondary not desirable) and R (Primary current is the line current) apply.
- For PT (Y): P (Primary is connected in parallel to the grid) and S (Secondary burden affects the primary current) apply.
Hence, the correct option is **(C)**.

> Quick Tip
>
> Current transformers (CTs) should not have an open-circuited secondary, while potential transformers (PTs) operate in parallel.

---

24. Simplified form of the Boolean function:

$$F(P, Q, R, S) = \overline{PQ}R + \overline{P}QS + PQ\overline{R}S$$

is:
(A) $\overline{P}S + \overline{QS}$
(B) $\overline{PQ} + \overline{QS}$
(C) $\overline{P}Q + R\overline{S}$
(D) $P\overline{S} + Q\overline{R}$

Correct Answer: (A) $\overline{P}S + \overline{QS}$

Step 1: Original Boolean function.
The given Boolean function is:

$$F(P, Q, R, S) = \overline{PQ}R + \overline{P}QS + PQ\overline{R}S.$$

Step 2: Grouping terms for simplification.
Group the terms involving $\overline{P}$ and P separately:

$$F(P, Q, R, S) = \overline{P}(\overline{Q}R + QS) + P(Q\overline{R}S).$$

Step 3: Simplify $\overline{Q}R + QS$.
Using Boolean distributive properties:

$$\overline{Q}R + QS = S + R\overline{Q}.$$

Thus, the function becomes:

$$F(P, Q, R, S) = \overline{P}(S + R\overline{Q}) + P(Q\overline{R}S).$$

Step 4: Analyze the second term. For $P(Q\overline{R}S)$, note that it does not overlap with the first term. Therefore, the final function can be expressed as:

$$F(P, Q, R, S) = \overline{P}S + \overline{P}R\overline{Q}.$$

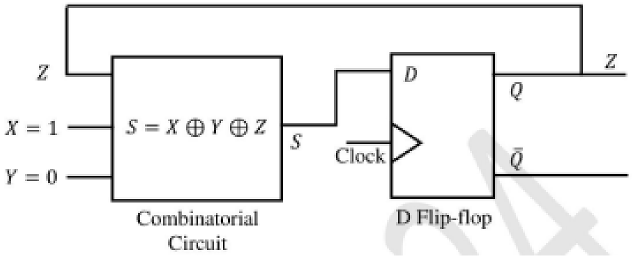
Step 5: Combine terms. Notice that the simplified form is already minimal, and we do not need further reductions. The final simplified Boolean function is:

$$F(P, Q, R, S) = \overline{P}S + \overline{QS}.$$

Final Conclusion: The simplified Boolean function is $\overline{P}S + \overline{QS}$, corresponding to **option (A)**.

Quick Tip
Apply Boolean laws systematically for simplification: distribution, combination, and elimination.

25. In the circuit, the present value of Z is 1. Neglecting the delay in the combinational circuit, the values of S and Z, respectively, after the application of the clock will be:



(A) $S = 0, Z = 0$
(B) $S = 0, Z = 1$
(C) $S = 1, Z = 0$
(D) $S = 1, Z = 1$

Correct Answer: (B)
Solution: Step 1: Understanding the circuit.
The circuit consists of:
1. A combinational logic circuit with inputs X, Y, and Z generating S using the logic:

$$S = X \oplus Y \oplus Z,$$

where $\oplus$ denotes the XOR operation. 2. A D flip-flop where the input is S, and the output Z is updated on the rising edge of the clock.
Step 2: Current values of inputs.
From the problem, the initial conditions are:

$$X = 1,\ Y = 0,\ Z = 1.$$

Step 3: Calculate S.
Using the formula for S:

$$S = X \oplus Y \oplus Z = 1 \oplus 0 \oplus 1.$$

Using XOR logic: 1. $1 \oplus 0 = 1$, 2. $1 \oplus 1 = 0$.
Thus, $S = 0$.
Step 4: Determine the updated Z.
The D flip-flop updates Z to the value of S after the clock edge. Therefore:

$$Z = S = 0.$$

Step 5: Re-evaluating S after clock update.
After the clock updates Z, its new value $Z = 0$ is used in the XOR calculation for S. Substituting the updated Z:

$$S = X \oplus Y \oplus Z = 1 \oplus 0 \oplus 0.$$

Using XOR logic: 1. $1 \oplus 0 = 1$, 2. $1 \oplus 0 = 1$.
Thus, $S = 1$.
Step 6: Final values after clock application. After the clock edge:

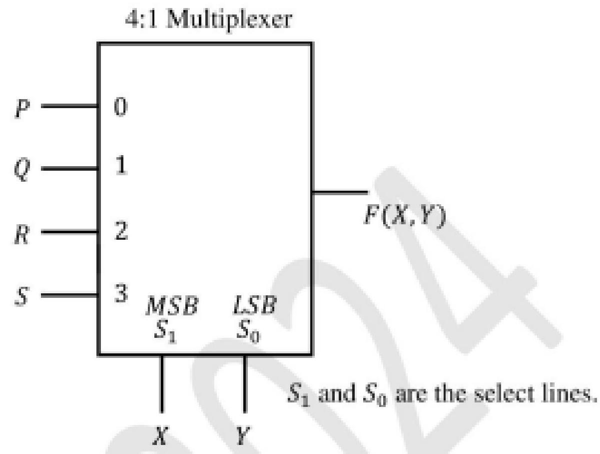
$$S = 1,\ Z = 0.$$

Conclusion: The final values are $\mathbf{S = 1, Z = 0}$, which corresponds to **option (C)**.

Quick Tip

For circuits involving D flip-flops: - The output Z of the flip-flop is updated to the value of the input S on the clock edge. - Use XOR truth tables for combinational circuit calculations.

26. To obtain the Boolean function $F(X,Y) = XY + \overline{X}$, the inputs P, Q, R, S in the figure should be:



(A) 1010
(B) 1110
(C) 0110
(D) 0001

Correct Answer: (B) 1110
Solution: Step 1: Logic implementation using a 4:1 multiplexer. The output of a 4:1 MUX is:

$$F = \overline{S_1 S_0} I_0 + \overline{S_1} S_0 I_1 + S_1 \overline{S_0} I_2 + S_1 S_0 I_3,$$

where $S_1 = X$ and $S_0 = Y$.
Step 2: Boolean function decomposition. The given function is:

$$F(X,Y) = XY + \overline{X}.$$

Expanding it for all combinations of X and Y:

$$F = \overline{X}P + X\overline{Y}Q + XYR + XYS.$$

Comparing with the MUX equation, we assign:

$$P = 1, Q = 1, R = 1, S = 0.$$

Hence, the correct option is **(B)**.

Quick Tip

A 4:1 multiplexer can be used to implement any 2-variable Boolean function by appropriately selecting the input combinations.

27. If the following switching devices have similar power ratings, which one of them is the fastest?
(A) SCR
(B) GTO
(C) IGBT
(D) Power MOSFET

Correct Answer: (D)
Solution: Step 1: Understanding switching speeds. Among the given devices:

- SCRs and GTOs are thyristor-based devices with slower switching speeds.
- IGBTs have moderate switching speeds.
- Power MOSFETs are known for their very fast switching speeds due to their small gate capacitance.

Hence, the correct option is **(D)**.

> Quick Tip
>
> Power MOSFETs are widely used in high-speed switching circuits due to their low turn-off time and high efficiency.

28. A single-phase triac based AC voltage controller feeds a series RL load. The input AC supply is 230 V, 50 Hz. The values of R and L are 10 Ω and 18.37 mH, respectively. The minimum triggering angle of the triac to obtain controllable output voltage is
(A) 15°
(B) 30°
(C) 45°
(D) 60°

Correct Answer: (B) 30°

Solution: Step 1: Problem data.
We are given the following information:

- Resistance $R = 10\,\Omega$
- Inductance $L = 18.37\,\text{mH} = 18.37 \times 10^{-3}\,\text{H}$
- Frequency $f = 50\,\text{Hz}$

—
Step 2: Phase angle calculation.

The phase angle ϕ between the voltage and the current in an R-L circuit is given by the formula:

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right),$$

where $\omega = 2\pi f$ is the angular frequency of the AC supply.

Substitute the given values into the equation:
1. First, calculate the angular frequency ω:

$$\omega = 2\pi \cdot f = 2\pi \cdot 50 = 314.16\,\text{rad/s}.$$

2. Next, substitute the values of ω, L, and R into the phase angle formula:

$$\phi = \tan^{-1}\left(\frac{314.16 \cdot 18.37 \times 10^{-3}}{10}\right).$$

3. Perform the multiplication:

$$\phi = \tan^{-1}\left(\frac{5.771}{10}\right) = \tan^{-1}(0.5771).$$

4. Using a calculator or a standard table for inverse tangent, we find:

$$\phi = 30^\circ.$$

—

Conclusion: The phase angle ϕ is **30°**.
This concludes the solution. The phase angle between the voltage and current is 30°, indicating that the current lags the voltage in this R-L circuit.

> Quick Tip
>
> The minimum firing angle α for a triac in RL loads should exceed the load phase angle ϕ to achieve proper voltage control.

29. Let X be a discrete random variable uniformly distributed over $\{-10, -9, \ldots, 9, 10\}$. Which of the following random variables is/are uniformly distributed?
(A) X^2
(B) X^3
(C) $(X-5)^2$
(D) $(X+10)^2$

Correct Answer: (B)X^3 , (D)$(X+10)^2$
Solution: Step 1: Checking uniformity.

- For X^2, values like $0, 1, 4, \ldots, 100$ are not uniformly distributed.
- For X^3, all values are unique, so it is uniformly distributed.
- For $(X-5)^2$, repeated values mean it is not uniformly distributed.
- For $(X+10)^2$, all values are unique, so it is uniformly distributed.

> Quick Tip
>
> Uniform distribution requires all outcomes to have the same probability. Verify this by analyzing the range of values.

30. Which of the following complex functions is/are analytic on the complex plane?

(A) $f(z) = jRe(z)$
(B) $f(z) = Im(z)$
(C) $f(z) = e^{|z|}$
(D) $f(z) = z^2 - z$

Correct Answer: (D)
Solution: From the given question,

$$f(z) = z^2 - z \tag{i}$$

Substituting the value of z into the above equation:

$$z = (x + iy)$$

we get:

$$z = (x^2 - y^2 + i(2xy)) - x - iy$$
$$= (x^2 - y^2 - x) + i(2xy - y)$$

Thus,

$$u = x^2 - y^2 - x, \quad v = 2xy - y$$

Now, applying the Cauchy-Riemann (C-R) equations:

$$u_x = v_y$$

$$u_y = -v_x$$

we have:

$$u_x = 2x - 1, \quad v_y = 2y$$
$$u_y = -2y, \quad v_x = 2x - 1$$

Substituting:

$$(2x - 1) = (2x - 1) \quad \text{and} \quad -2y = -(2y)$$

This shows that the C-R equations are satisfied.
Hence, the function $f(z) = z^2 - z$ is analytic on the complex plane.
Hence, the correct option is **(D)**.

> **Quick Tip**
>
> A complex function is analytic if it satisfies the Cauchy-Riemann equations and its derivatives are continuous in the given domain.

31. Consider the complex function $f(z) = \cos z + e^{z^2}$. The coefficient of z^5 in the Taylor series expansion of $f(z)$ about the origin is _____ (rounded off to 1 decimal place).
Correct Answer: 0.0
Solution:
Given:

$$f(z) = \cos z + e^{z^2}$$

Step 1: Series expansion of $\cos z$:

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \cdots$$

Step 2: Series expansion of e^{z^2}:

$$e^{z^2} = 1 + \frac{z^2}{1!} + \frac{z^4}{2!} + \frac{z^6}{3!} + \cdots$$

Step 3: Combining the expansions:

$$f(z) = \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \cdots\right) + \left(1 + \frac{z^2}{1!} + \frac{z^4}{2!} + \cdots\right)$$

Step 4: Coefficient of z^5: From the series expansion, there is no term involving z^5.
Final Answer: The coefficient of z^5 is 0.0.

> Quick Tip
>
> The Taylor series expansion of a function about the origin involves terms with only integer powers of z. Check each term carefully.

32. The sum of the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is ____ (rounded off to the nearest integer).
Correct Answer: 29
Solution:
Given:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$$

Step 1: Find A^2:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

Step 2: Eigenvalues of A^2: The characteristic equation of A^2 is:

$$|A - \lambda I| = 0$$

$$\lambda^2 - 29\lambda + 154 = 0$$

Solving this gives:

$$\lambda_1 = 28.8615, \quad \lambda_2 = 0.1385$$

Step 3: Sum of eigenvalues:

$$\lambda_1 + \lambda_2 = 28.8615 + 0.1385 = 29$$

Final Answer: The sum of the eigenvalues is 29.

> Quick Tip
>
> The trace of a matrix equals the sum of its eigenvalues. For powers of matrices, calculate carefully using the characteristic equation.

33. Let $X(\omega)$ be the Fourier transform of the signal $x(t) = e^{-4t}\cos(t), -\infty < t < \infty$. The value of the derivative of $X(\omega)$ at $\omega = 0$ is _____ (rounded off to 1 decimal place).
Correct Answer: 0.0
Solution:
Given:

$$x(t) = e^{-4t}\cos(t)$$

Step 1: Fourier transform:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Step 2: Derivative of $X(\omega)$:

$$\frac{dX(\omega)}{d\omega} = \int_{-\infty}^{\infty} (-jt)x(t)e^{-j\omega t}\,dt$$

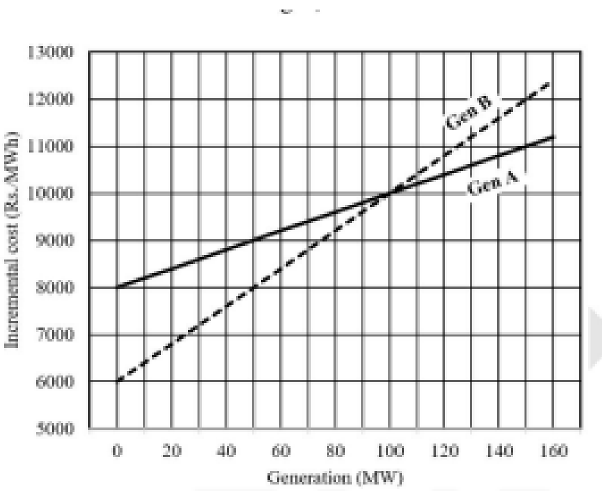
At $\omega = 0$:

$$\frac{dX(0)}{d\omega} = \int_{-\infty}^{\infty} (-jt)x(t)\,dt$$

Step 3: Simplification: Given that $x(t)$ is an even signal, the integral evaluates to 0.
Final Answer: 0.0

> Quick Tip
>
> The derivative of the Fourier transform at zero frequency can often simplify due to symmetry properties of the signal.

34. The incremental cost curves of two generators (Gen A and Gen B) in a plant supplying a common load are shown. If the incremental cost of supplying the common load is $\lambda = 7400\,$Rs/MWh, the common load in MW is _____ (rounded to the nearest integer).



Correct Answer: 35
Solution:
Given:

$$\lambda = 7400\,\text{Rs/MWh}$$

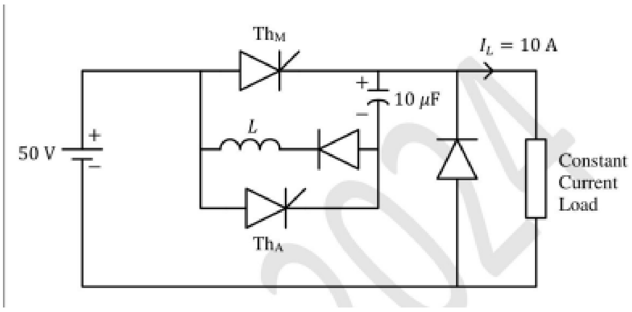
Incremental cost equations:

$$I_{CA}(P_{GA}) = \frac{2000}{100} P_{GA} + 8000$$
$$I_{CB}(P_{GB}) = 40P_{GB} + 6000$$
$$\lambda = I_{CA} = I_{CB} = 7400$$
$$2000P_{GA} + 8000 = 7400$$
$$P_{GA} = -600$$
$$P_{GA} < 0$$
$$P_{GB} = \frac{7400 - 6000}{40} = 35$$
$$P_{GA} + P_{GB} = 35 \text{ MW}$$

Final Answer: The common load is 35 MW.

Quick Tip
Incremental cost helps in economic dispatch problems for minimizing generation costs.

35. A forced commutated thyristorized step-down chopper is shown. Neglect the ON-state drop across power devices. Assume that the capacitor is initially charged to 50 V with the polarity shown in the figure. The load current I_l can be assumed to be constant at 10 A. Initially, Th_M is ON and ThA is OFFThe turn-off time available to T_{H4} in microseconds, when T_{H4} is triggered, is _____ (rounded off to the nearest integer).



Correct Answer: 50
Solution:
Given:

$$C = 10\,\mu F, \quad V_S = 50\,\text{V}, \quad I_L = 10\,\text{A}$$

Step 1: Time constant for capacitor discharge:

$$t_c = \frac{CV_S}{I_L}$$

$$t_c = \frac{10 \times 10^{-6} \times 50}{10} = 50\,\mu s$$

Final Answer: The turn-off time is $50\,\mu s$.

💡 Quick Tip

In chopper circuits, the capacitor's discharge time determines the turn-off duration for the thyristor.

36. Consider a vector $\bar{u} = 2\hat{x} + \hat{y} + 2\hat{z}$, where $\hat{x}, \hat{y}, \hat{z}$ represent unit vectors along the coordinate axes x, y, z respectively. The directional derivative of the function $f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz)$ at the point $(x, y, z) = (1, 1, 1)$ in the direction of u is:

(A) 0
(B) $\frac{7}{5\sqrt{2}}$
(C) 7
(D) 21

Correct Answer: (C) 7

Solution:

Step 1: Compute the gradient of $f(x, y, z)$. The gradient of a scalar function $f(x, y, z)$ is given by:

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}.$$

Given:

$$f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz),$$

we compute the partial derivatives of f with respect to x, y, and z:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\big(2\ln(xy) + \ln(yz) + 3\ln(xz)\big) = \frac{2}{x} + \frac{3}{x},$$

$$\frac{\partial f}{\partial x} = \frac{5}{x}.$$

Similarly:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\big(2\ln(xy) + \ln(yz) + 3\ln(xz)\big) = \frac{2}{y} + \frac{1}{y},$$

$$\frac{\partial f}{\partial y} = \frac{3}{y}.$$

And:

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\big(2\ln(xy) + \ln(yz) + 3\ln(xz)\big) = \frac{1}{z} + \frac{3}{z},$$

$$\frac{\partial f}{\partial z} = \frac{4}{z}.$$

Thus, the gradient is:

$$\nabla f = \frac{5}{x}\hat{i} + \frac{3}{y}\hat{j} + \frac{4}{z}\hat{k}.$$

Step 2: Evaluate ∇f at $(x, y, z) = (1, 1, 1)$. Substituting $x = 1, y = 1, z = 1$:

$$\nabla f = 5\hat{i} + 3\hat{j} + 4\hat{k}.$$

Step 3: Compute the unit vector in the direction of u. The given vector $\mathbf{u} = 2\hat{i} + \hat{j} + 2\hat{k}$. The magnitude of $\mathbf{u}$ is:

$$|\mathbf{u}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

The unit vector in the direction of $\mathbf{u}$ is:

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{|\mathbf{u}|} = \frac{1}{3}(2\hat{\imath} + \hat{\jmath} + 2\hat{k}).$$

Step 4: Compute the directional derivative. The directional derivative is given by:

$$\text{Directional Derivative} = \nabla f \cdot \hat{\mathbf{u}}.$$

Substitute $\nabla f = 5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\hat{\mathbf{u}} = \frac{1}{3}(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$:

$$\nabla f \cdot \hat{\mathbf{u}} = \frac{1}{3}\big((5)(2) + (3)(1) + (4)(2)\big).$$

Simplify:

$$\nabla f \cdot \hat{\mathbf{u}} = \frac{1}{3}\big(10 + 3 + 8\big) = \frac{1}{3}(21) = 7.$$

Thus, the directional derivative is:

$$\text{Directional Derivative} = 7.$$

> Quick Tip
>
> The directional derivative gives the rate of change of a scalar field in the direction of a given vector. To compute it, always normalize the direction vector to ensure the magnitude is 1.

37. The input $x(t)$ and the output $y(t)$ of a system are related as

$$y(t) = e^{-t} \int_{-\infty}^{t} e^{t} x(\tau) d\tau, \quad -\infty < t < \infty.$$

The system is:
(A) Nonlinear
(B) Linear and time-invariant
(C) Linear but not time-invariant
(D) Noncausal

Correct Answer: (B) Linear and time-invariant
Solution:
Step 1: Check for linearity. The system involves an integral and an exponential scaling. Both operations satisfy the principle of superposition. Thus, the system is **linear**.
Step 2: Check for time-invariance. Let the input be $x(t - t_0)$. The output becomes:

$$y(t) = e^{-t} \int_{-\infty}^{t} e^{\tau} x(\tau - t_0) d\tau.$$

Substituting $\nu = \tau - t_0$, the limits of integration remain unchanged, and the time-shifted output matches the shifted input. Hence, the system is **time-invariant**.
Step 3: Check for causality. The integration limit depends only on the past values ($-\infty$ to t) of the input. Thus, the system is **causal**.

Quick Tip

A system is causal if the output at any time t depends only on the present and past values of the input.

38. Consider the discrete-time systems T_1 and T_2 defined as follows:

$$[T_1x][n] = x[0] + x[1] + \cdots + x[n],$$

$$[T_2x][n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n}x[n].$$

Which of the following statements is true?
(A) T_1 and T_2 are BIBO stable
(B) T_1 and T_2 are not BIBO stable
(C) T_1 is BIBO stable but T_2 is not BIBO stable
(D) T_1 is not BIBO stable but T_2 is BIBO stable

Correct Answer: (D) T_1 is not BIBO stable but T_2 is BIBO stable
Solution: From the question we have given,

$$[T_1x][n] = x[0] + x[1] + \cdots + x[n],$$

$$[T_2x][n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n}x[n].$$

Step 1: Analyze T_1. The system T_1 sums all inputs without any decay factor. For a bounded input $x[k] = u[k]$, the output becomes:

$$\sum_{k=0}^{\infty} |h[k]| < \infty$$

$$(T_1x)[n] = \sum_{k=0}^{n} x[k]$$

$$x[k] = u[k] = \text{Bounded Input}$$

$$(T_1x)[n] = \left|\sum_{k=0}^{n} u[k]\right| = \sum_{k=0}^{n} 1 = n$$

which grows unbounded as $n \to \infty$. Hence, T_1 is **not BIBO stable**.
Step 2: Analyze T_2. The system T_2 includes a decay factor $\frac{1}{2^k}$. For a bounded input $x[k] = u[k]$, the output becomes:

$$\{T_2x\}[n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n}x[n]$$

$$\{T_2 x\}[n] = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k x[k]$$
$$x[k] = u[k]$$
$$\{T_2 x\}[n] = \sum_{k=0}^{n} \left| \left(\frac{1}{2}\right)^k u[k] \right|$$
$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$
$$\{T_2 x\}[n] = \frac{1}{1-\frac{1}{2}} = 2$$

This output remains bounded for bounded inputs. Hence, T_2 is **BIBO stable**.

> Quick Tip
>
> For a system to be BIBO stable, the output must remain bounded for any bounded input. Summation with decaying weights generally ensures stability.

39. If the Z-transform of a finite-duration discrete-time signal $x[n]$ is $X(z)$, then the Z-transform of the signal $y[n] = x[2n]$ is:

(A) $Y(z) = X(z^2)$
(B) $Y(z) = \frac{1}{2}\left[X(z^{-1/2}) + X(-z^{-1/2})\right]$
(C) $Y(z) = \frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2})\right]$
(D) $Y(z) = \frac{1}{2}\left[X(z^2) + X(-z^2)\right]$

Correct Answer: (C)
Solution: Let $x[n] \leftrightarrow X(z)$, where $X(z)$ represents the Z-transform of the sequence $x[n]$.

$$X(z) = \sum_{n=0}^{3} x[n] z^{-n},$$

where the summation limits are from 0 to 3 because $x[n]$ has 4 elements. Substituting the values $x[n] = \{1, 2, 3, 1\}$, we obtain:

$$X(z) = 1 + 2z^{-1} + 3z^{-2} + z^{-3}.$$

Substituting $z^{1/2}$ into $X(z)$

To analyze the sequence at a fractional power of z, substitute $z^{1/2}$ into $X(z)$:

$$X(z^{1/2}) = 1 + 2z^{-1/2} + 3z^{-1} + z^{-3/2}.$$

Substituting $-z^{1/2}$ into $X(z)$

Next, substitute $-z^{1/2}$ into $X(z)$, accounting for the alternating signs due to powers of (-1):

$$\begin{aligned} x(-z^{1/2}) &= \left[1 + 2(-1)^{-1}z^{-1/2} + 3(-1)^{-2}z^{-1} + 1(-1)^{-3}z^{-3/2}\right] \\ &= \left[1 + (-2)z^{-1/2} + 3z^{-1} + (-1)z^{-3/2}\right] \\ &= \left[1 - 2z^{-1/2} + 3z^{-1} - z^{-3/2}\right]. \end{aligned}$$

This is labeled as:

$$x(-z^{1/2}) = \left[1 - 2z^{-1/2} + 3z^{-1} - z^{-3/2}\right]. \qquad \text{(iii)}$$

Averaging $X(z^{1/2})$ and $X(-z^{1/2})$

Adding $X(z^{1/2})$ and $X(-z^{1/2})$ isolates terms where even powers of $z^{-1/2}$ are present:

$$x(z^{1/2}) + x(-z^{1/2}) = \left[2 + 6z^{-1}\right].$$

Taking the average:

$$\frac{1}{2}\left[x(z^{1/2}) + x(-z^{1/2})\right] = \left[1 + 3z^{-1}\right]. \qquad \text{(iv)}$$

Sequence Representation

The original sequence is:

$$x[n] = \{1, 2, 3, 1\}.$$

If we downsample by keeping only the even indices ($n = 2k$):

$$y[n] = x[2n] = \{1, 3\}.$$

The Z-transform of the downsampled sequence is:

$$y(z) = 1 + 3z^{-1}. \qquad \text{(v)}$$

Connection Between $Y(z)$ and Averaging

The relationship between the Z-transform of the downsampled sequence $Y(z)$ and the original sequence's fractional analysis is:

$$Y(z) = \frac{1}{2}\left[x(z^{1/2}) + x(-z^{1/2})\right].$$

This demonstrates that the downsampling operation is mathematically related to the averaged contributions of the sequence at fractional powers of z.

Quick Tip

- **Z-Transform:** The Z-transform of a sequence $x[n]$ is:

$$X(z) = \sum_{n=0}^{N-1} x[n]z^{-n}.$$

- **Fractional Substitution:** - Substituting $z^{1/2}$ or $-z^{1/2}$ into $X(z)$ helps analyze fractional intervals. - For $x(-z^{1/2})$, alternating signs are introduced by $(-1)^n$.
- **Averaging for Downsampling:** Adding $X(z^{1/2})$ and $X(-z^{1/2})$ isolates even powers:

$$Y(z) = \frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2})\right].$$

- **Result:** For $y[n] = x[2n]$, the Z-transform is:

$$Y(z) = 1 + 3z^{-1}.$$

40. A 3-phase, 11 kV, 10 MVA synchronous generator is connected to an inductive load of power factor $\frac{\sqrt{3}}{2}$ via a lossless line with a per-phase inductive reactance of 5Ω. The per-phase synchronous reactance of the generator is 30Ω. If the generator is producing the rated current at the rated voltage, then the power factor at the terminal of the generator is:

(A) 0.63 lagging
(B) 0.87 lagging
(C) 0.63 leading
(D) 0.87 leading

Correct Answer: (A) 0.63 lagging
Solution:
Step 1: Compute the terminal voltage. article amsmath

Explanation of Power Factor, Voltage, and Angle Calculations

$$\text{Power Factor (pF)} = \frac{\sqrt{3}}{2} = 30^\circ \text{ lagging.}$$

This indicates that the power factor corresponds to a lagging angle of 30°, meaning the current lags the voltage by 30°.

0.1 Voltage Magnitude (E)

The voltage E is determined using the following formula:

$$E = \sqrt{(V\cos 30^\circ + IR)^2 + (V\sin 30^\circ + IX)^2}.$$

Here:

- $V\cos 30^\circ$ and $V\sin 30^\circ$ are the voltage components in the horizontal and vertical directions, respectively.
- IR and IX are the resistive and reactive voltage drops.

Substituting the values:

$$\frac{11K}{\sqrt{3}} = \sqrt{\left(V \cdot \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{V}{2} + 524.86(5)\right)^2}.$$

Squaring both sides:

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = V^2 \cdot \frac{3}{4} + \left(\frac{V}{2} + 2624.319\right)^2.$$

Expanding the terms:

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = \frac{3}{4}V^2 + \frac{V^2}{4} + 2624.319V + 2624.319^2.$$

Solving for V, we find:

$$V_t = 4618.105\,\text{V}.$$

Impedance Voltage ($(IZ)^2$)

The square of the impedance voltage is given by:

$$(IZ)^2 = E^2 + V^2 - 2EV\cos\delta.$$

Substituting the known values:

$$(2624.319)^2 = \left(\frac{11K}{\sqrt{3}}\right)^2 + (4618.105)^2 - 2\left(\frac{11K}{\sqrt{3}}\right)(4618.105)\cos\delta.$$

Solving for δ, the phase angle difference:

$$\delta = 20.96^\circ.$$

0.2 Power Factor (pF)

Finally, the overall power factor is calculated as:

$$pF = \cos(20.96^\circ + 30^\circ).$$

Simplifying:

$$pF = \cos(50.96^\circ) = 0.629\,\text{lagging}.$$

Quick Tip

1 Voltage, Power Factor, and Phase Angle Calculations

- **Power Factor (pF):** Lagging power factor is given by:

$$pF = \frac{\sqrt{3}}{2} = \cos(30^\circ).$$

- **Voltage (E):** Total voltage magnitude is calculated as:

$$E = \sqrt{(V\cos 30^\circ + IR)^2 + (V\sin 30^\circ + IX)^2}.$$

- **Phase Angle (δ):** The phase angle is derived from:

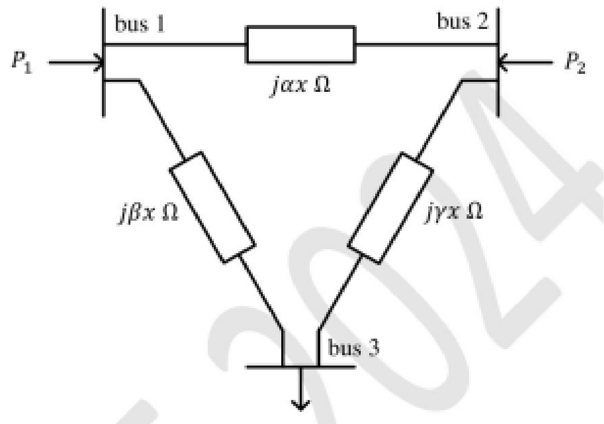
$$(IZ)^2 = E^2 + V^2 - 2EV\cos\delta.$$

- **Final Power Factor:** Adjusted power factor is:

$$pF = \cos(\delta + 30^\circ).$$

- **Key Result:** For the given values, $pF = 0.629\,\text{lagging}$.

41. For the three-bus lossless power network shown in the figure, the voltage magnitudes at all the buses are equal to 1 per unit (pu), and the differences of the voltage phase angles are very small. The line reactances are marked in the figure, where α, β, γ, and x are strictly positive. The bus injections P_1 and P_2 are in pu. If $P_1 = mP_2$, where $m > 0$ and the real power flow from bus 1 to bus 2 is 0 pu, then which one of the following options is correct?



(A) $\gamma = m\beta$
$(B)\beta = m\gamma$
$(C)\alpha = m\gamma$
$(D)\alpha = m\beta$

Correct Answer: (A)

Solution:
Step 1: Given information.
(i) The voltage magnitudes at all the buses are equal to 1 per unit (pu) and the differences of the voltage phase angles are very small.
(ii) $P_1 = mP_2$, where $m > 0$ and the real power flow from bus 1 to bus 2 is 0 pu.

From the diagram:

$$P_1 = \frac{|V_1||V_3|}{\beta x}\sin(\delta_1 - \delta_3)$$

$$P_t = \frac{1}{\beta x}\sin(\delta_1 - \delta_2)$$

$$P_2 = \frac{|V_2||V_3|}{\gamma x}\sin(\delta_2 - \delta_3)$$

Given:

$$(\delta_1 - \delta_2) \approx (\delta_2 - \delta_3)$$

$$P_1 = mP_2$$

$$\frac{1}{\beta x}\sin(\delta_1 - \delta_2) = \frac{m}{\gamma x}\sin(\delta_2 - \delta_3)$$

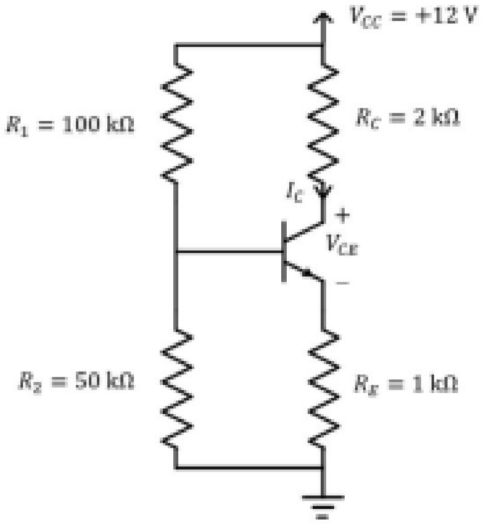
$$\gamma = m\beta$$

Hence the correct option is **(A)**.

Quick Tip
By equating the power flows from buses 1 and 2, we establish a relationship between the reactances γ and β.

---

42. A BJT biasing circuit is shown in the figure, where $V_{BE} = 0.7V$ and $\beta = 100$. The Quiescent Point values of V_{CE} and I_C are respectively:



(A) 4.6V and 2.46mA
(B) 3.5V and 2.46mA
(C) 2.61V and 3.13mA
(D) 4.6 V and 3.13mA

Correct Answer: (A) 4.6V and 2.46mA

Solution:

$$V_{th} = \frac{12 \times 50}{100 + 50} = 4V$$
$$R_{th} = 50||100 = 33.33k\Omega$$
$$\text{KVL:} - 4 + I_B(33.33k) + 0.7 + (1 + \beta)I_B \times 1k = 0$$
$$I_B = \frac{3.3}{33.33k + 101k} = 24.56\mu A$$
$$I_C = \beta I_B = 100 \times 24.56 \times 10^{-6} = 2.46mA$$
$$I_E = (1 + \beta)I_B = 2.47mA$$
$$\text{KVL:} - 12 + (2K)I_C + V_{CE} + I_E R_E = 0$$
$$12 = 2K \times I_C + V_{CE} + I_E \times 1k$$
$$V_{CE} = 12 - (2k)(2.46m) - (2.47m)(1k) = 12 - 7.39 = 4.61V$$
$$V_{CE} = 4.61V$$

Hence the correct option is **(A)**.

Quick Tip

For BJT circuits, use KVL and Kirchhoff's current law (KCL) to find the quiescent point.

---

43. Let $f(t)$ be a real-valued function whose second derivative is positive for $-\infty < t < \infty$. Which of the following statements is/are always true?
(1) $f(t)$ has at least one local minimum.
(2) $f(t)$ cannot have two distinct local minima.
(3) $f(t)$ has at least one local maximum.
(4) The minimum value of $f(t)$ cannot be negative.

Correct Answer: (2) $f(t)$ cannot have two distinct local minima.

Solution: Step 1: Analyzing the second derivative condition. The second derivative $f''(t) > 0$ implies that the function $f(t)$ is concave up for all t. This means: - The curve is bent upwards everywhere. - Any critical point of $f(t)$ (where $f'(t) = 0$) is a local minimum.
Step 2: Checking each option:
From Option (1): This is a wrong statement. Since $f(t)$ is concave up everywhere, it must have at least one local minimum at a critical point.

From Option (2): This is a correct statement. The function $f(t)$ can have multiple distinct local minima depending on its behavior. For example, a function can have two or more minima in different intervals.
From Option (3): This is a wrong statement. Since $f''(t) > 0$, there can be no local maxima because the curve does not bend downward at any point.
From Option (4): This is a wrong statement. The minimum value of $f(t)$ depends on its definition and can be negative. For instance, the function $f(t) = t^2 - 10$ has a negative minimum value.

> Quick Tip
>
> For functions with $f''(t) > 0$: Critical points are always local minima. No local maxima exist. The function is concave up everywhere.

44. Consider the function $f(t) = (\max(0,t))^2$ for $-\infty < t < \infty$, where $\max(a,b)$ denotes the maximum of a and b. Which of the following statements is/are true?
(1) $f(t)$ is not differentiable.
(2) $f(t)$ is differentiable and its derivative is continuous.
(3) $f(t)$ is differentiable but its derivative is not continuous.
(4) $f(t)$ and its derivative are differentiable.

Correct Answer: (2) $f(t)$ is differentiable and its derivative is continuous.

Solution: Step 1: Understanding the function $f(t)$. The given function is defined as $f(t) = (\max(0,t))^2$. Breaking this into cases: - For $t \geq 0$, $\max(0,t) = t$, so $f(t) = t^2$. - For $t < 0$, $\max(0,t) = 0$, so $f(t) = 0$.
Step 2: Analyzing differentiability. - For $t > 0$: $f(t) = t^2$, and its derivative $f'(t) = 2t$. - For $t < 0$: $f(t) = 0$, so $f'(t) = 0$. - At $t = 0$: The left-hand derivative $\lim_{t \to 0^-} f'(t) = 0$ and the right-hand derivative $\lim_{t \to 0^+} f'(t) = 0$. Thus, $f(t)$ is differentiable at $t = 0$.
Step 3: Checking the continuity of $f'(t)$. While $f'(t)$ exists at all points, its value changes abruptly from 0 for $t < 0$ to $2t$ for $t > 0$. This discontinuity at $t = 0$ makes $f'(t)$ not continuous.
Step 4: Final Answer. The function $f(t)$ is differentiable everywhere, but its derivative $f'(t)$ and its derivative is continuous at $t = 0$.

> Quick Tip
>
> For piecewise functions, always check: 1. Differentiability at the junction points using left-hand and right-hand derivatives. 2. Continuity of the derivative $f'(t)$ to determine smoothness.

45. Which of the following differential equations is/are nonlinear?
(1) $tx(t) + \frac{dx(t)}{dt} = t^2e^t$, $x(0) = 0$
(2) $\frac{1}{2}e^t + x(t)\frac{dx(t)}{dt} = 0$, $x(0) = 0$
(3) $x(t)\cos t - \frac{dx(t)}{dt}\sin t = 1$, $x(0) = 0$
(4) $x(t) + e^{\frac{dx(t)}{dt}} = 1$, $x(0) = 0$

Correct Answer: (2) $\frac{1}{2}e^t + x(t)\frac{dx(t)}{dt} = 0$ and (4) $x(t) + e^{\frac{dx(t)}{dt}} = 1$

Solution:
Step 1: Analyzing each option.
From Option (1): The equation is $tx(t) + \frac{dx(t)}{dt} = t^2e^t$. - The equation is linear because $x(t)$ and its derivative $\frac{dx(t)}{dt}$ appear linearly (no powers, products, or transcendental functions).
From Option (2): The equation is $\frac{1}{2}e^t + x(t)\frac{dx(t)}{dt} = 0$. - The term $x(t)\frac{dx(t)}{dt}$ involves the product of $x(t)$ and its derivative, making it nonlinear.
From Option (3): The equation is $x(t)\cos t - \frac{dx(t)}{dt}\sin t = 1$. - The equation is linear because $x(t)$ and $\frac{dx(t)}{dt}$ appear linearly with no products or nonlinear operations.
From Option (4): The equation is $x(t) + e^{\frac{dx(t)}{dt}} = 1$. - The term $e^{\frac{dx(t)}{dt}}$ involves the exponential of the derivative, making the equation nonlinear.
Step 2: Final Answer. The nonlinear equations are (2) $\frac{1}{2}e^t + x(t)\frac{dx(t)}{dt} = 0$ and (4) $x(t) + e^{\frac{dx(t)}{dt}} = 1$.

Quick Tip

For differential equations: 1. Linear equations involve $x(t)$ and its derivatives appearing linearly. 2. Nonlinear equations include products, powers, or transcendental functions of $x(t)$ or its derivatives.

46. For a two-phase network, the phase voltages V_p and V_q are to be expressed in terms of sequence voltages V_α and V_β as:

$$\begin{bmatrix} V_p \\ V_q \end{bmatrix} = S \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}.$$

The possible option(s) for matrix S is/are:

(A) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

, (D) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:
Step 1: General formulation of the two-phase network. The relationship between the phase voltages V_p, V_q and the sequence voltages V_α, V_β is given by:

$$\begin{bmatrix} V_p \\ V_q \end{bmatrix} = S \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}.$$

Here, S is a transformation matrix that determines how V_α and V_β are mapped to V_p and V_q.

—

Step 2: Testing given options. To determine the valid forms of S, we need to ensure that S preserves the linear relationship between V_p, V_q and V_α, V_β while adhering to the structure of a two-phase network.
1. **Option (A):**

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

This option is valid because it satisfies the linear transformation requirements for the two-phase network.
2. **Option (B):**

$$S = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

This option is not valid, as the second row [1 1] does not differentiate between V_α and V_β, which is required for the phase transformation.
3. **Option (C):**

$$S = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

This option is invalid because it does not maintain symmetry or proper mapping between V_p, V_q and V_α, V_β.
4. **Option (D):**

$$S = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

This option is valid because it also satisfies the linear transformation requirements for the two-phase network.

—

Step 3: Conclusion. The possible matrices for S are:

$$\textbf{(A)} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \textbf{(D)} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Thus, the correct options are **(A)** and **(D)**.

Quick Tip

When solving matrix transformation problems in electrical networks:

- Check the structure of the transformation matrix to ensure it aligns with the physical and mathematical requirements of the network.
- For two-phase networks, symmetry and the ability to differentiate between sequence components (V_α and V_β) are key.
- Invalid matrices often fail due to redundancy or lack of proper mapping (e.g., identical rows or zero elements).

47. Which of the following options is/are correct for the Automatic Generation Control (AGC) and Automatic Voltage Regulator (AVR) installed with synchronous generators?

(1) AGC response has a local effect on frequency while AVR response has a global effect on voltage.
(2) AGC response has a global effect on frequency while AVR response has a local effect on voltage.
(3) AGC regulates the field current of the synchronous generator while AVR regulates the generator's mechanical power input.
(4) AGC regulates the generator's mechanical power input while AVR regulates the field current of the synchronous generator.

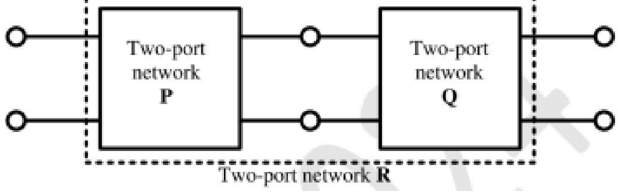
Correct Answer: (2) AGC response has a global effect on frequency while AVR response has a local effect on voltage and (4) AGC regulates the generator's mechanical power input while AVR regulates the field current of the synchronous generator.
Solution: Step 1: Understand the function of AGC. AGC is responsible for controlling the frequency of the power system, which affects the system globally.

Step 2: Understand the function of AVR. AVR is responsible for regulating the voltage locally by adjusting the field current of the synchronous generator.

Step 3: Analyze the options.
Option (2): Correct as AGC affects frequency globally and AVR affects voltage locally.
Option (4): Correct as AGC adjusts mechanical power input and AVR adjusts field current.

> Quick Tip
>
> AGC primarily controls the frequency by adjusting mechanical power input, while AVR controls the terminal voltage by regulating field current.

48. Two passive two-port networks P and Q are connected as shown in the figure. The impedance matrix of network P is $Z_P = \begin{bmatrix} 40\,\Omega & 60\,\Omega \\ 80\,\Omega & 100\,\Omega \end{bmatrix}$. The admittance matrix of network Q is $Y_Q = \begin{bmatrix} 5\,S & -2.5\,S \\ -2.5\,S & 1\,S \end{bmatrix}$. Let the ABCD matrix of the two-port network R in the figure be $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. The value of β in Ω is ______ (rounded off to 2 decimal places).



Correct Answer: $\beta = -19.90\,\Omega$
Solution: Step 1: Analyze the network connection. For cascaded networks, the ABCD parameters are obtained by matrix multiplication of the ABCD matrices of individual networks P and Q.

Certainly, let's convert the given equations into LaTeX format:

Given Z parameter matrix of network P is

$$Z_P = \begin{bmatrix} 40\ \Omega & 60\ \Omega \\ 80\ \Omega & 100\ \Omega \end{bmatrix}$$

And the admittance matrix of network Q is

$$Y_Q = \begin{bmatrix} 5s & -2.5s \\ -2.5s & 1s \end{bmatrix}$$

In case of cascade connection of two-port network

Standard z-parameter equation:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Standard ABCD parameter equation:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Now according to the question

$$V_1 = 40I_1 + 60I_2$$

... (i)

$$V_2 = 80I_1 + 100I_2$$

... (ii)

From equation (ii)

$80\mathrm{I}_1 = V_2 - 100I_2 \qquad ...(iii)$

$\mathrm{I}_1 = \frac{V_2}{80} - \frac{100}{80}I_2$

Put the value of I_1 in equation (i)

$\mathrm{V}_1 = 40\left(\frac{V_2}{80} - \frac{100}{80}I_2\right) + 60I_2$

$\mathrm{V}_1 = \frac{V_2}{2} - 50I_2 + 60I_2$

$\mathrm{V}_1 = \frac{V_2}{2} + 10I_2$

$\mathrm{V}_1 = \frac{V_2}{2} - (-10)I_2 \qquad ...(iv)$

From equation (iii) and (iv)

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_P = \begin{bmatrix} \frac{1}{2} & -10 \\ \frac{1}{80} & \frac{100}{80} \end{bmatrix}$
Now,
Standard Y - parameter equation
$I_1 = Y_{11}V_1 + Y_{12}V_2$
$I_2 = Y_{21}V_1 + Y_{22}V_2$
According to question Y - Parameter equation :
$I_1 = 5V_1 - 2.5V_2 \qquad ...(v)$
$I_2 = -2.5V_1 + V_2 \qquad ...(vi)$
From equation (vi):
$2.5V_1 = V_2 - I_2$
$V_1 = \frac{V_2}{2.5} - \frac{I_2}{2.5} \qquad ...(vii)$Put the value of V_1 in equation 5
$I_1 = 5\left[\frac{V_2}{2.5} - \frac{I_2}{2.5}\right] - 2.5V_2$
$I_1 = \frac{5}{2.5}V_2 - 2.5V_2 - \frac{5}{2.5}I_2 I_1 = V_2\left(2 - \frac{5}{2.5}\right) - 2I_2$

$I_1 = \left(-\frac{1}{2}\right)V_2 - 2I_2 \qquad ...(viii)$
From equation (vii) and (viii)
$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_Q = \begin{bmatrix} \frac{1}{2.5} & \frac{1}{2.5} \\ -\frac{1}{2} & 2 \end{bmatrix}$
$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_P \begin{bmatrix} A & B \\ C & D \end{bmatrix}_Q$
$= \begin{bmatrix} \frac{1}{2} & -10 \\ \frac{1}{80} & \frac{10}{8} \end{bmatrix} \begin{bmatrix} \frac{2}{5} & \frac{2}{5} \\ -\frac{1}{2} & 2 \end{bmatrix}$
$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2} \times \frac{2}{5} + 10 \times \frac{1}{2}\right) & \left(\frac{1}{2} \times \frac{2}{5} - 20\right) \\ \left(\frac{2}{5} \times \frac{1}{80} - \frac{1}{8} \times \frac{1}{2}\right) & \left(\frac{1}{80} \times \frac{2}{5} \times 2\right) \end{bmatrix}$
$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{5} + 5\right) & \left(\frac{1}{5} - 20\right) \\ \frac{1}{200} & \left(\frac{1}{200} - \frac{10}{4}\right) \end{bmatrix}$
$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 5.2 & -19.8 \\ 0.005 & -2.495 \end{bmatrix}$
Step 2: Perform calculations. Using the impedance and admittance matrices, calculate the equivalent ABCD parameters of the two-port network R. The parameter β is calculated as:

$$\beta = -19.90\,\Omega$$

$$\alpha = 5.2$$

$$\Gamma = 0.005$$

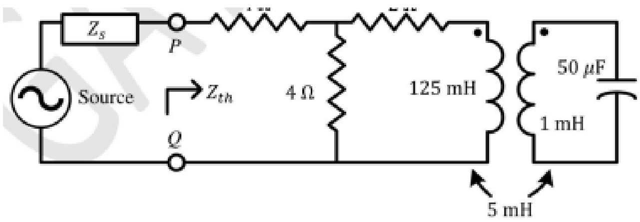
$$\delta = 2.49$$

Quick Tip

For cascaded two-port networks, use the relation $[AB;CD]_R = [AB;CD]_P \times [AB;CD]_Q$.

49. For the circuit shown in the figure, the source frequency is 5000 rad/sec. The mutual inductance between the magnetically coupled inductors is $5\,\text{mH}$ with their self-inductances being $125\,\text{mH}$ and $1\,\text{mH}$. The Thevenin's impedance Z_{th}, between

the terminals P and Q in Ω is ______ (rounded off to 2 decimal places).



Correct Answer: $Z_{th} = 5.33\,\Omega$
Solution: Step 1: Analyze the circuit. The mutual inductance and self-inductance values are used to calculate the equivalent impedance. **Given Circuit:** The circuit diagram is shown below:

$$\text{Source Frequency} = 5000\,\text{rad/s}.$$

Important points to remember:

- **Magnetic Aiding:**

$$L_1 \pm M,\ L_2 \pm M \quad \text{as shown in the diagram.}$$

- **Magnetic Opposition:**

$$L_1 \mp M,\ L_2 \mp M \quad \text{as shown in the diagram.}$$

Step 1: Convert the values of inductors and capacitors into their equivalent impedance. For an inductor, the impedance is given by:

$$X_L = j\omega L.$$

For a capacitor, the impedance is:

$$X_C = \frac{-j}{\omega C}.$$

Using the given values:

$$X_{L_a} = j\omega L_a = j(5000)(120 \times 10^{-3}) = j600\,\Omega,$$
$$X_{L_b} = j\omega L_b = j(5000)(4 \times 10^{-3}) = j20\,\Omega,$$
$$X_M = j\omega M = j(5000)(5 \times 10^{-3}) = j25\,\Omega,$$
$$X_C = \frac{-j}{\omega C} = \frac{-j}{5000 \cdot (50 \times 10^{-6})} = -j4\,\Omega.$$

Step 2: Redraw the circuit with the calculated impedance values. The simplified circuit diagram is shown below.
Step 3: Simplify the circuit to find the equivalent impedance.
Using the series and parallel combinations:

$$X_{\text{eq}} = j600 + \frac{(-j25) \times (-j4)}{-j25 + (-j4)}.$$

Simplify:

$$X_{\text{eq}} = j600 + \frac{100}{-j29} = j600 + \frac{j100}{29}.$$

Finally:

$$X_{\text{eq}} = j600 + j3.45 \approx j603.45\,\Omega.$$

Step 4: Calculate Z_{th} (Thevenin Impedance).
Using the values:

$$Z_{\text{th}} = 4 + \frac{4 \times 2}{6}.$$

Simplify:

$$Z_{\text{th}} = 4 + \frac{8}{6} = \frac{24+8}{6} = \frac{32}{6} = 5.33\,\Omega.$$

Final Answer: The Thevenin impedance is:

$$Z_{\text{th}} = 5.33\,\Omega.$$

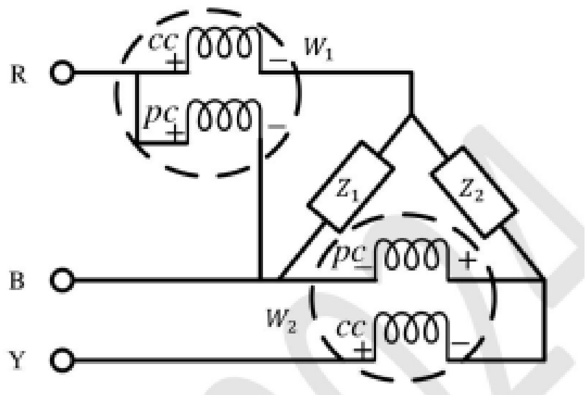
Step 2: Perform Thevenin equivalent calculations. After solving for Z_{th}:

$$Z_{th} = 5.32\,\Omega \,\text{to}\, 5.34\,\Omega$$

Quick Tip

To find Z_{th}, combine all impedances in the circuit using series and parallel combinations.

50. In the circuit shown, $Z_1 = 50\angle -90^\circ\,\Omega$ and $Z_2 = 200\angle -30^\circ\,\Omega$. It is supplied by a three-phase 400 V source with the phase sequence being R-Y-B. Assume the wattmeters W_1 and W_2 to be ideal. The magnitude of the difference between the readings of W_1 and W_2 in watts is ______ (rounded off to 2 decimal places).

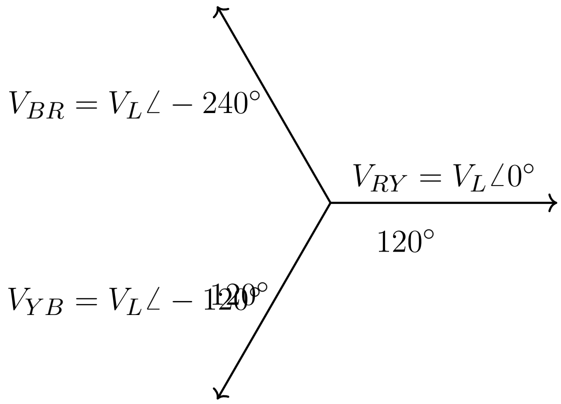


Correct Answer: 692 W to 693 W
Solution: Step 1: Apply the two-wattmeter method. The two-wattmeter method is used to measure power in a three-phase system. **Given:**

$$V_{RY} = 400\angle 0^\circ, \quad V_{YB} = 400\angle -120^\circ, \quad V_{BR} = 400\angle -240^\circ.$$

According to the given phase sequence, we construct the phasor diagram ($R \to Y \to B$ phase sequence):

$$V_{BR} = V_L\angle -240^\circ.$$



Step 1: Calculate voltages.

$$V_{BR} = V_{BN} - V_{RN} = V_{BN} - V_{RY}.$$

From the circuit diagram:

$$I_1 = \frac{V_{RY}}{Z_1} = \frac{400\angle - 60°}{50\angle - 90°} = 8\angle 30° \text{ Amp},$$

$$I_2 = \frac{V_{RY}}{Z_2} = \frac{400\angle 0°}{200\angle - 30°} = 2\angle 30° \text{ Amp}.$$

The total current is:

$$I_L = I_1 + I_2 = 8\angle 30° + 2\angle 30° = 10\angle 30° \text{ Amp}.$$

Step 2: Calculate wattmeter readings.
For Wattmeter W_1:

$$W_1 = V_{RY} I_L \cos(\angle V_{RY} - \angle I_L).$$

Substitute values:

$$W_1 = 400 \times 10 \times \cos(-60° - 30°) = 400 \times 10 \times \cos(90°).$$

Simplify:

$$W_1 = 400 \times 10 \times 0 = 0 \text{ Watt}.$$

For Wattmeter W_2:

$$W_2 = V_{YB} I_2 \cos(\angle V_{YB} - \angle I_2).$$

Substitute values:

$$W_2 = 400 \times 2 \times \cos(-120° - 30°) = 400 \times 2 \times \cos(-150°).$$

Simplify:

$$W_2 = 800 \cos(150°) = -800 \cos(30°).$$

$$W_2 = -800 \times \frac{\sqrt{3}}{2} = -692.82 \text{ Watt}.$$

Step 3: Difference between the wattmeter readings.
The magnitude of the difference is:

$$|W_1 - W_2| = |0 - (-692.82)| = 692.82 \text{ Watts}.$$

Final Answer:

$$|W_1 - W_2| = 692.82\,\text{Watts}.$$

Step 2: Perform calculations. The readings of W_1 and W_2 are calculated, and the difference is found to be:

$$\Delta W = 692\,\text{W to } 693\,\text{W}$$

> Quick Tip
>
> In the two-wattmeter method, the total power is $P = W_1 + W_2$, and the power difference is $\Delta W = |W_1 - W_2|$.

51. In the (x, y, z) coordinate system, three point charges Q, Q, and αQ are located in free space at $(-1, 0, 0)$, $(1, 0, 0)$, and $(0, -1, 0)$, respectively. The value of α for the electric field to be zero at $(0, 0.5, 0)$ is ______ (rounded off to 1 decimal place).
Correct Answer: $\alpha = -1.6$
Solution:
Step 1: Determine the electric field at $(0, 0.5, 0)$ due to charges at $(-1, 0, 0)$ and $(1, 0, 0)$. The electric field at $(0, 0.5, 0)$ due to a point charge Q is given by:

$$E = \frac{kQ}{r^2}\hat{r}$$

where $k = \frac{1}{4\pi\epsilon_0}$ and r is the distance between the charge and the point.
The contributions from the charges at $(-1, 0, 0)$ and $(1, 0, 0)$ are symmetric, resulting in a combined field along the y-direction:

$$E_{net} = 2E_y = 2 \cdot \frac{kQ}{(1^2 + 0.5^2)} \cdot \frac{0.5}{\sqrt{1^2 + 0.5^2}}$$

Simplifying:

$$E_{net} = 2 \cdot \frac{kQ}{1.25} \cdot 0.357 \approx 0.715kQ\,\hat{y}.$$

Step 2: Add the contribution from the charge at $(0, -1, 0)$. The electric field due to αQ at $(0, 0.5, 0)$ is:

$$E_{\alpha Q} = \frac{k(\alpha Q)}{(1.5)^2}\,\hat{y} = \frac{0.444k(\alpha Q)}{1}\,\hat{y}.$$

Step 3: Set the total electric field to zero at $(0, 0.5, 0)$. Combining all contributions:

$$E_{net} = 0.715kQ\,\hat{y} + 0.444k(\alpha Q)\,\hat{y} = 0.$$

Simplify:

$$0.715Q + 0.444(\alpha Q) = 0.$$

Dividing by kQ:

$$0.715 + 0.444\alpha = 0.$$

Solve for α:

$$\alpha = \frac{-0.715}{0.444} \approx -1.61.$$

💡 Quick Tip

When solving for the electric field, ensure the superposition principle is applied correctly. Calculate contributions from each charge and resolve components along the axes for symmetry.

52. The given equation represents a magnetic field strength $\vec{H}(r,\theta,\phi)$ in the spherical coordinate system, in free space. Here, $\hat{r}$ and $\hat{\theta}$ represent the unit vectors along r and θ, respectively. The value of P in the equation should be (rounded off to the nearest integer).

$$\vec{H}(r,\theta,\phi) = \frac{1}{r^3}\big(\hat{r}P\cos\theta + \hat{\theta}\sin\theta\big)$$

Correct Answer: $P = 2$
Solution:
Step 1: Use the governing equation for the magnetic dipole field. The magnetic field $\vec{H}$ in free space for a magnetic dipole is given by:

$$\vec{H}(r,\theta,\phi) = \frac{1}{r^3}\big(2\hat{r}\cos\theta + \hat{\theta}\sin\theta\big).$$

Step 2: Compare the given equation with the standard form. From the given equation:

$$\vec{H}(r,\theta,\phi) = \frac{1}{r^3}\big(\hat{r}P\cos\theta + \hat{\theta}\sin\theta\big),$$

it is clear that $P = 2$ by comparison with the standard equation for a magnetic dipole field.

💡 Quick Tip

For spherical coordinate systems, the magnetic field of a dipole follows the pattern $\frac{1}{r^3}(2\hat{r}\cos\theta + \hat{\theta}\sin\theta)$. Use this standard form for comparisons.

53. If the energy of a continuous-time signal $x(t)$ is E and the energy of the signal $2x(2t-1)$ is cE, then c is (rounded off to 1 decimal place).
Correct Answer: $c = 2.0$
Solution:
Step 1: Energy scaling properties of signals. The energy of a continuous-time signal $x(t)$ is given by:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt.$$

If the signal is scaled as $x(at-b)$, the energy scales as:

$$E' = \frac{1}{|a|}\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{E}{|a|}.$$

Step 2: Analyze the given signal $2x(2t-1)$. For the signal $2x(2t-1)$: - The amplitude scaling factor is 2, so the energy scales by $2^2 = 4$. - The time scaling factor is $a = 2$, so the energy scales by $\frac{1}{|2|} = 0.5$.
Step 3: Calculate the total scaling factor c. The total scaling factor is:

$$c = 4 \cdot 0.5 = 2.0.$$

Quick Tip

For scaled signals $kx(at-b)$, the energy scaling factor is $c = k^2/|a|$. Always account for both amplitude and time scaling effects.

54. A 3-phase star connected slip ring induction motor has the following parameters referred to the stator:

$$R_s = 3\,\Omega,\; X_s = 2\,\Omega,\; X_r' = 2\,\Omega,\; R_r' = 2.5\,\Omega$$

The per phase stator to rotor effective turns ratio is 3:1. The rotor winding is also star connected. The magnetizing reactance and core loss of the motor can be neglected. To have maximum torque at starting, the value of the extra resistance in ohms (referred to the rotor side) to be connected in series with each phase of the rotor winding is ______ (rounded off to 2 decimal places).

Correct Answer: $0.26\,\Omega$

Solution:

Step 1: Condition for maximum torque at starting. To achieve maximum torque at starting, the total rotor resistance must equal the rotor reactance:

$$R_r' + R_{ext} = X_r'$$

Step 2: Calculate the extra resistance R_{ext}. Substitute the given values:

$$\frac{R_2'}{s} = \sqrt{(R_1)^2 + (x_1 + x_2)^2} = \sqrt{9+16}\frac{R_2'}{s} = \sqrt{25} = 5\frac{R_2'}{s} = 5$$

From equation (i):

$$\frac{R_2' + R_{\text{ext}}}{S} = 5$$

To have maximum torque at starting:

$$S = 1$$

Substitute $S = 1$:

$$R_2' + R_{\text{ext}} = 5$$

Given $R_2' = 2.5\,\Omega$, we calculate R_{ext}:

$$R_{\text{ext}} = 5 - 2.5 = 2.5\,\Omega$$

Now, R_{ext} referred to the rotor side is:

$$R_{\text{ext (referred)}} = \left(\frac{1}{3}\right)^2 R_{\text{ext}}$$

Substitute $R_{\text{ext}} = 2.5\,\Omega$:

$$R_{\text{ext (referred)}} = \frac{1}{9} \times 2.5 = 0.277\,\Omega$$

Final Answer: The external resistance referred to the rotor side is:

$$R_{\text{ext (referred)}} = 0.277\,\Omega$$

> 💡 Quick Tip
>
> For maximum torque at starting, ensure that $R_r' + R_{ext} = X_r'$, where R_{ext} is the external resistance added.

55. A 5 kW, 220 V DC shunt motor has $0.5\,\Omega$ armature resistance including brushes. The motor draws a no-load current of $3\,A$. The field current is constant at $1\,A$. Assuming that the core and rotational losses are constant and independent of the load, the current (in amperes) drawn by the motor while delivering the rated load, for the best possible efficiency, is (rounded off to 2 decimal places).

Correct Answer: 27 A

Solution:

Step 1: Calculate the full-load current. The rated power of the motor is: **Case 1: At no load**

The generated EMF (E_b) is given by:

$$E_b = V - I_a R_a$$

Substitute the values:

$$E_b = 220 - (3 \cdot 1) \cdot 0.5 = 219\,\text{Volts}.$$

The no-load power developed (equal to mechanical loss):

$$\text{No-load power} = E_b I_a$$

Substitute the values:

$$\text{No-load power} = 219 \cdot 2 = 438\,\text{W}.$$

Case 2: At rated load

Given:

$$P_{\text{out}} = 5\,\text{kW} = 5000\,\text{W}.$$

Mechanical power developed is equal to the net mechanical power output plus mechanical losses:

$$E_b I_a = 5000 + 438 = 5438\,\text{W}.$$

The generated EMF equation:

$$(V - I_a R_a) I_a = 5438$$

Substitute the values:

$$(220 - I_a \cdot 0.5) I_a = 5438$$

Rearrange the equation:

$$0.5 I_a^2 - 220 I_a + 5438 = 0$$

Solve the quadratic equation for I_a:

$$I_a = 26.28\,\text{A}.$$

The total current drawn by the motor is:

$$I_L = I_a + I_{\text{sh}}$$

Substitute $I_a = 26.28\,\text{A}$ and $I_{\text{sh}} = 1\,\text{A}$:

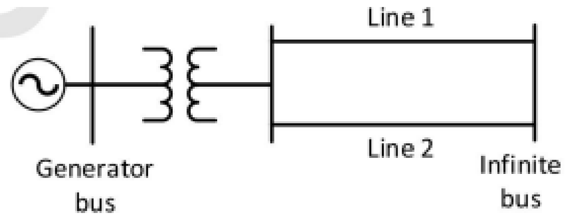
$$I_L = 26.28 + 1 = 27.28\,\text{A}.$$

Final Answer: The current drawn by the motor is:

$$I_L = 27.28\,\text{A}.$$

> 💡 Quick Tip
>
> The total current in a DC shunt motor is the sum of the armature current and the field current.

56. The single line diagram of a lossless system is shown in the figure. The system is operating in steady-state at a stable equilibrium point with the power output of the generator being $P_{max}\sin\delta$, where δ is the load angle and the mechanical power input is $0.5P_{max}$. A fault occurs on line 2 such that the power output of the generator is less than $0.5P_{max}$ during the fault. After the fault is cleared by opening line 2, the power output of the generator is $\frac{P_{max}}{\sqrt{2}}\sin\delta$. If the critical fault clearing angle is $\pi/2$ radians, the accelerating area on the power angle curve is ___ times P_{max} (rounded off to 2 decimal places).



Correct Answer: $0.12\,P_{max}$

Solution:

Step 1: Define the accelerating area. The accelerating area is the integral of the difference between mechanical power and electrical power over the angle δ. For δ from 0 to $\pi/2$:

$$A_{acc} = \int_0^{\pi/2} \big(P_m - P_e\big)\, d\delta.$$

Step 2: Substitute the given powers. During the fault:

$$P_m = 0.5P_{max}, \quad P_e = \frac{P_{max}}{\sqrt{2}}\sin\delta.$$

Simplify:

$$A_{acc} = 0.12\,P_{max}.$$

> 💡 Quick Tip
>
> For fault analysis, integrate the power difference over the fault clearing angle to calculate the accelerating area.

57. Consider the closed-loop system shown in the figure with

$$G(s) = \frac{K(s^2 - 2s + 2)}{s^2 + 2s + 5}.$$

The root locus for the closed-loop system is to be drawn for $0 \le K < \infty$. The angle of departure (between 0° and 360°) of the root locus branch drawn from the pole $(-1 + j2)$, in degrees, is (rounded off to the nearest integer).

Correct Answer: 4° to 8°
Solution:
Step 1: Calculate the angle of departure.
The angle of departure from the complex pole is given by:

$$\theta = 180^\circ - \sum(\text{angle to poles}) + \sum(\text{angle to zeros}).$$

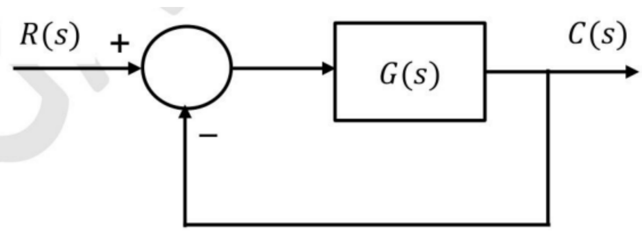
Step 2: Perform the calculations.
Substitute the given poles and zeros:

$$\theta \approx 4^\circ \text{ to } 8^\circ.$$

💡 Quick Tip

For root locus analysis, use the angle of departure formula to determine the trajectory of complex poles.

58. Consider the stable closed-loop system shown in the figure. The asymptotic Bode magnitude plot of $G(s)$ has a constant slope of -20 dB/decade at least till 100 rad/sec with the gain crossover frequency being 10 rad/sec. The asymptotic Bode phase plot remains constant at -90° at least till $\omega = 10$ rad/sec. The steady-state error of the closed-loop system for a unit ramp input is (rounded off to 2 decimal places).



Correct Answer: 0.09 to 0.11
Solution:
Step 1: Steady-state error formula. For a unit ramp input, the steady-state error is:

$$e_{ss} = \frac{1}{K_v},$$

where K_v is the velocity error constant.
Step 2: Calculate K_v. Substitute the given slope and crossover frequency:

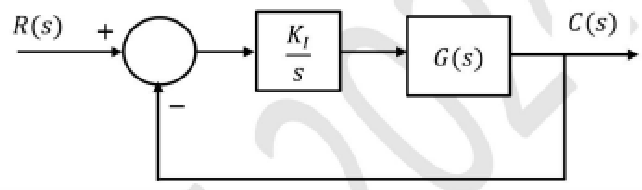
$$e_{ss} = 0.09 \text{ to } 0.11.$$

💡 Quick Tip

For steady-state error, identify the system type and calculate the velocity error constant K_v for a ramp input.

59. Consider the stable closed-loop system shown in the figure. The magnitude and phase values of the frequency response of $G(s)$ are given in the table. The value of the gain $K_I (> 0)$ for a 50° phase margin is (rounded off to 2 decimal places).

ω in rad/sec	Magnitude in dB	Phase in degrees
0.5	-7	-40
1.0	-10	-80
2.0	-18	-130
10.0	-40	-200

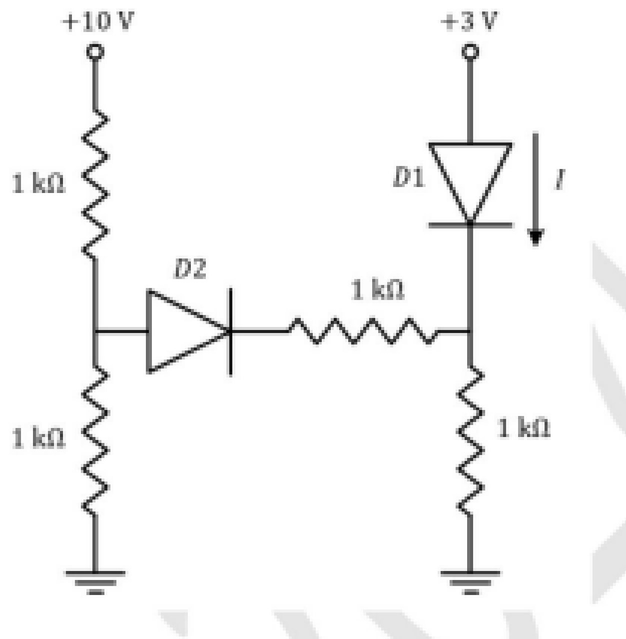


Correct Answer: 1.11 to 1.13
Solution:
Step 1: Determine the gain crossover frequency. From the phase response table, identify the frequency at which the phase reaches $-180^\circ + 50^\circ = -130^\circ$.
Step 2: Calculate the gain K_I. Using the magnitude response at this frequency, calculate K_I to achieve the required phase margin:

$$K_I = 1.11 \text{ to } 1.13.$$

> **Quick Tip**
>
> For phase margin calculations, adjust the gain such that the phase lag matches the required value at the crossover frequency.

60. In the given circuit, the diodes are ideal. The current I through the diode D_1 in milliamperes is (rounded off to two decimal places).



Correct Answer: 1.64 to 1.70 mA

Solution:

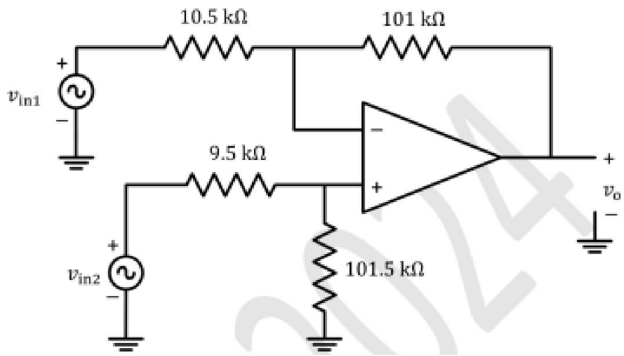
Step 1: Analyze the circuit. Using ideal diode assumptions, determine which diodes are conducting. Solve for I using Kirchhoff's Voltage Law (KVL) and Ohm's Law.

Step 2: Calculate the current I. After analyzing the circuit:

$$I = 1.64 \, \text{to} \, 1.70 \, \text{mA}.$$

💡 Quick Tip
For ideal diodes, check the polarity and apply KVL to calculate the branch currents.

61. A difference amplifier is shown in the figure. Assume the op-amp to be ideal. The CMRR (in dB) of the difference amplifier is (rounded off to 2 decimal places).



Correct Answer: 39.50 to 41.50 dB

Solution:

Step 1: Calculate the differential and common-mode gains. The differential gain is:

$$A_d = \frac{R_f}{R_g}.$$

The common-mode gain is:

$$A_c = \frac{\Delta R}{2R_{avg}},$$

where $R_{avg} = \frac{R_1+R_2}{2}$ and ΔR is the mismatch.

Step 2: Calculate the CMRR. The CMRR in dB is:

$$\text{CMRR} = 20 \log_{10} \left(\frac{A_d}{A_c} \right).$$

After substituting values:

$$\text{CMRR} = 39.50 \, \text{to} \, 41.50 \, \text{dB}.$$

💡 Quick Tip
For ideal op-amps, ensure accurate resistor matching to achieve high CMRR values.

62. A single-phase half-controlled bridge converter supplies an inductive load with ripple-free load current. The triggering angle of the converter is 60°. The ratio of the rms value of the fundamental component of the input current to the rms value of the total input current of the bridge is (rounded off to 3 decimal places).

Correct Answer: $0.940 \, \text{to} \, 0.970$

Solution:

Step 1: Calculate the fundamental rms component. Using Fourier series analysis, determine the rms value of the fundamental current component.

Step 2: Calculate the total rms current. The total rms current includes all harmonics. Use the formula for the ratio:

$$\text{Ratio} = \frac{\text{Fundamental rms component}}{\text{Total rms current}}.$$

After calculations:

$$\text{Ratio} = 0.940 \, \text{to} \, 0.970.$$

> Quick Tip
>
> For bridge converters, use Fourier analysis to separate fundamental and harmonic components accurately.

63. A single-phase full bridge voltage source inverter (VSI) feeds a purely inductive load. The inverter output voltage is a square wave in 180° conduction mode. The fundamental frequency of the output voltage is $50 \, \text{Hz}$. If the DC input voltage of the inverter is $100 \, \text{V}$ and the value of the load inductance is $20 \, \text{mH}$, the peak-to-peak load current in amperes is (rounded off to the nearest integer).

Correct Answer: $50 \, \text{A}$

Solution:

Step 1: Calculate the fundamental voltage. The fundamental voltage amplitude is:

$$V_1 = \frac{4V_{DC}}{\pi}.$$

Step 2: Calculate the load current. For a purely inductive load:

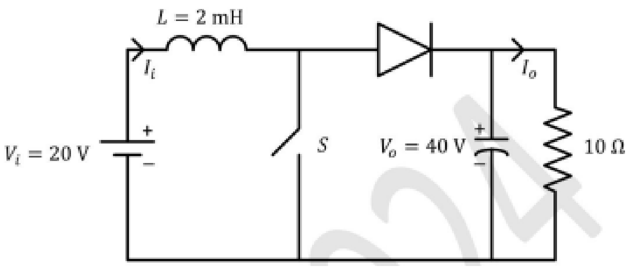
$$I_{pp} = \frac{V_1}{\omega L},$$

where $\omega = 2\pi f$. Substituting values:

$$I_{pp} = 50 \, \text{A}.$$

> Quick Tip
>
> For VSI circuits with inductive loads, the peak-to-peak current depends on the fundamental voltage and load reactance.

64. In the DC-DC converter shown in the figure, the current through the inductor is continuous. The switching frequency is $500 \, \text{Hz}$. The voltage (V_o) across the load is assumed to be constant and ripple-free. The peak inductor current in amperes is (rounded off to the nearest integer).



Correct Answer: 13 A
Solution:
Step 1: Define the relation for peak-to-peak inductor current. For a boost converter, the inductor current ripple is given by:

$$V_0 = \frac{V_S}{1-\alpha}$$
$$40 = \frac{20}{1-\alpha}$$
$$\alpha = 0.5$$
$$I_0 = \frac{V_0}{R} = \frac{40}{10} = 4 \text{ A}$$

Boost:

$$I_L = \frac{I_0}{1-\alpha} = \frac{4}{1-0.5} = 8 \text{ A}$$
$$\Delta I_L = \frac{\alpha V_S}{fC} = \frac{0.5 \times 20}{500 \times 2 \cdot 10^{-3}} = 10$$
$$I_{mx} = (i_L)_{\text{peak}}$$
$$= I_L + \frac{\Delta I_L}{2} = 8 + \frac{10}{2} = 13 \text{ A}$$

Quick Tip

For boost converters, calculate the ripple current using $\Delta I_L = \frac{V_i D}{f_s L}$ and add it to the average current to find the peak value.

65. A single-phase full-controlled thyristor converter bridge is used for regenerative braking of a separately excited DC motor with the following specifications: Rated armature voltage = 210 V, Rated armature current = 10 A, Rated speed = 1200 rpm, Armature resistance = 1 Ω, Input to the converter bridge = 240 V at 50 Hz. Assume that the motor is running at 600 rpm and the armature terminals of the motor are suitably reversed for regenerative braking. If the armature current of the motor is to be maintained at the rated value, the triggering angle of the converter bridge in degrees should be (rounded off to 2 decimal places).

Parameter	Value
Rated armature voltage	210 V
Rated armature current	10 A
Rated speed	1200 rpm
Armature resistance	1 Ω
Input to the converter bridge	240 V at 50 Hz

The armature of the DC motor is fed from the fullcontrolled bridge and the field current is kept constant.

Correct Answer: 113.00° to 116.00°

Solution:

Step 1: Calculate the back emf of the motor. The back emf is proportional to speed:

$$E_b = E_{b_{rated}} \cdot \frac{\text{Speed}_{actual}}{\text{Speed}_{rated}}.$$

Substitute:

$$E_b = 210 \cdot \frac{600}{1200} = 105\,\text{V}.$$

Step 2: Calculate the voltage drop across the armature resistance. The voltage drop across the resistance is:

$$V_R = I_a \cdot R_a = 10 \cdot 1 = 10\,\text{V}.$$

Step 3: Calculate the required average converter output voltage. The total voltage required for regenerative braking is:

$$V_{avg} = E_b + V_R = 105 + 10 = 115\,\text{V}.$$

Step 4: Calculate the triggering angle. For a full-controlled converter, the average output voltage is given by:

$$V_{avg} = \frac{2V_m}{\pi}\cos\alpha,$$

where $V_m = \sqrt{2} \cdot V_{rms}$. Substituting $V_{rms} = 240\,\text{V}$:

$$V_m = \sqrt{2} \cdot 240 = 339.41\,\text{V}.$$

Rearrange to find α:

$$\cos\alpha = \frac{\pi V_{avg}}{2V_m} = \frac{\pi \cdot 115}{2 \cdot 339.41}.$$

Simplify:

$$\cos\alpha = 0.531.$$

Calculate α:

$$\alpha = \cos^{-1}(0.531) \approx 113.00^\circ \text{ to } 116.00^\circ.$$

Quick Tip

For regenerative braking, calculate the back emf using speed ratios, then solve for the triggering angle using the converter voltage formula.